Computer Algebra
Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

4-Trig-functions/4.5-Secant/126-4.5.7-d-trig-^m-a+b-c-sec-^n-^p

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CHAPTER 1

Introduction

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This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [470]. This is test number [126].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	99.57 (468)	0.43 (2)
Mathematica	93.19 (438)	6.81 (32)
Maple	91.70 (431)	8.30 (39)
Fricas	88.51 (416)	11.49 (54)
Maxima	61.70 (290)	38.30 (180)
Giac	53.19 (250)	46.81 (220)
Mupad	51.70 (243)	48.30 (227)
Sympy	4.68 (22)	95.32 (448)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

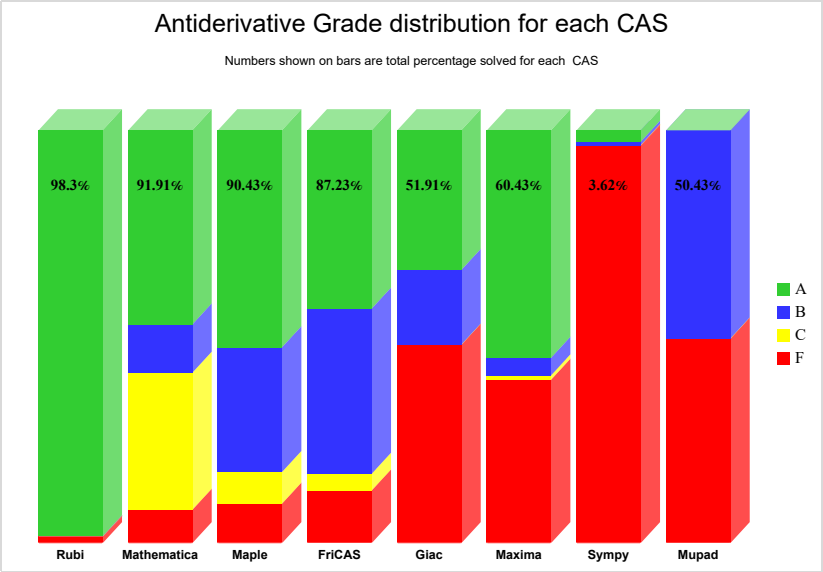
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

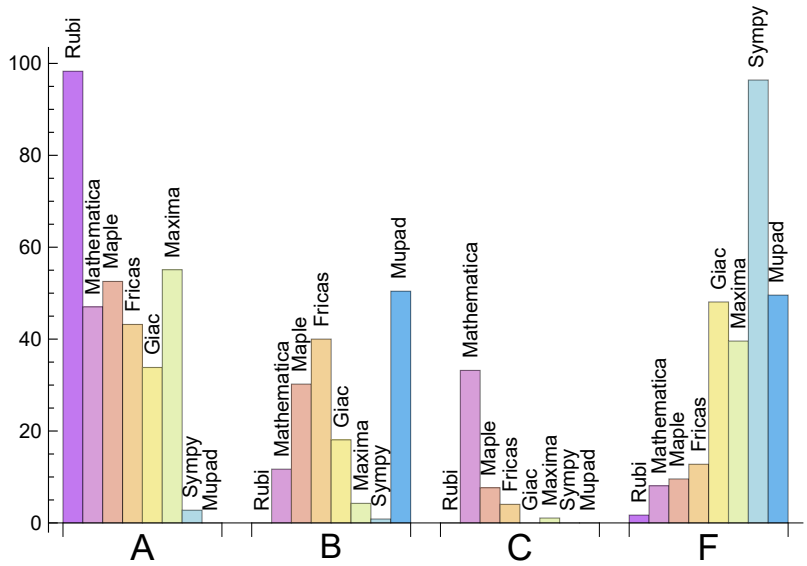
System	% A grade	% B grade	% C grade	% F grade
Rubi	98.298	0.000	0.000	1.702
Maxima	55.106	4.255	1.064	39.574
Maple	52.553	30.213	7.660	9.574
Mathematica	47.021	11.702	33.191	8.085
Fricas	43.191	40.000	4.043	12.766
Giac	33.830	18.085	0.000	48.085
Sympy	2.766	0.851	0.000	96.383
Mupad	0.000	50.426	0.000	49.574

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima

and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	2	100.00	0.00	0.00
Mathematica	32	100.00	0.00	0.00
Maple	39	100.00	0.00	0.00
Fricas	54	100.00	0.00	0.00
Maxima	180	88.33	11.67	0.00
Giac	220	97.73	0.00	2.27
Mupad	227	0.00	100.00	0.00
Sympy	448	72.99	27.01	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.34
Maxima	0.37
Giac	0.53
Fricas	0.93
Mathematica	4.12
Maple	4.99
Sympy	11.83
Mupad	18.49

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Sympy	73.41	1.68	63.50	1.31
Rubi	127.99	1.03	107.00	1.00
Maxima	223.76	2.81	101.50	1.17
Giac	256.05	2.64	149.50	1.36
Mathematica	428.96	3.68	168.00	1.55
Fricas	607.82	4.42	401.00	3.55
Mupad	1563.74	10.76	105.00	1.17
Maple	1754.82	9.02	147.00	1.29

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

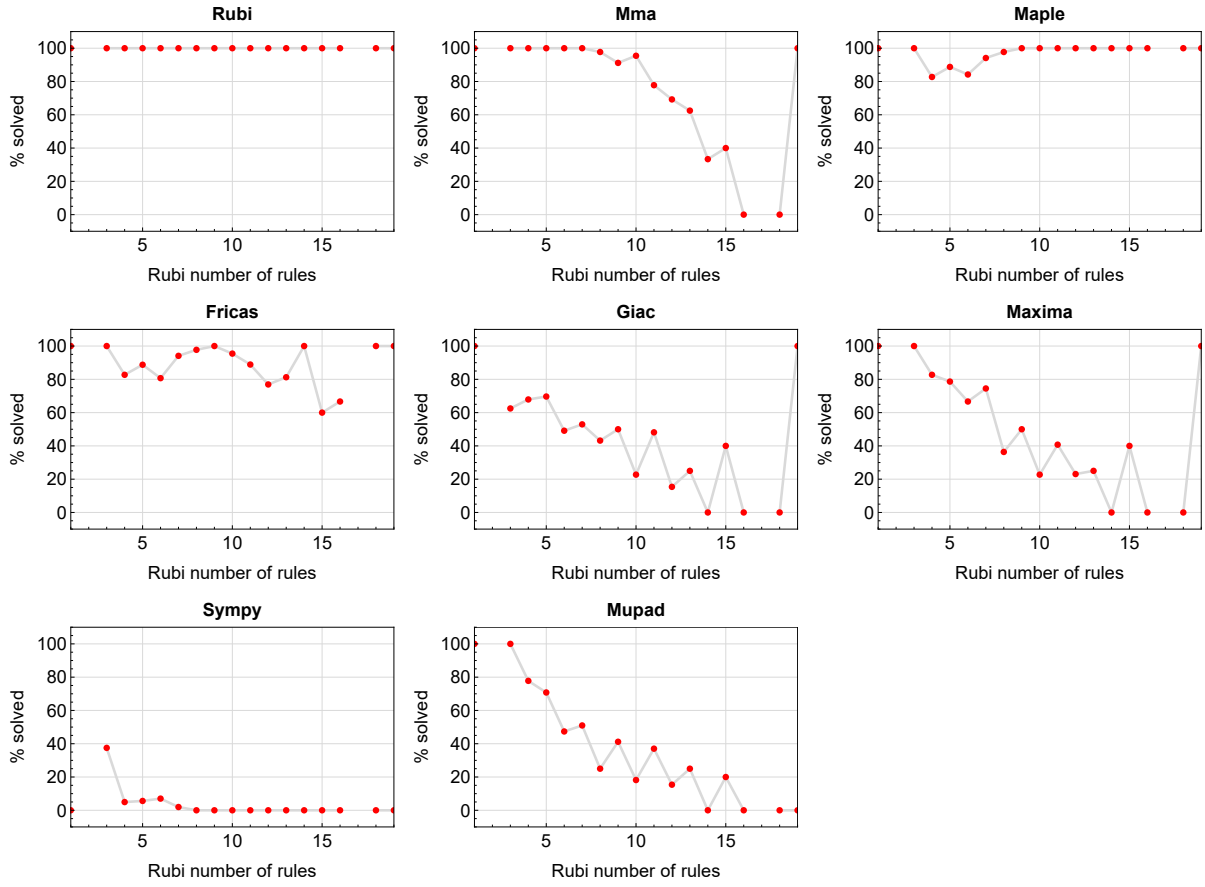


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

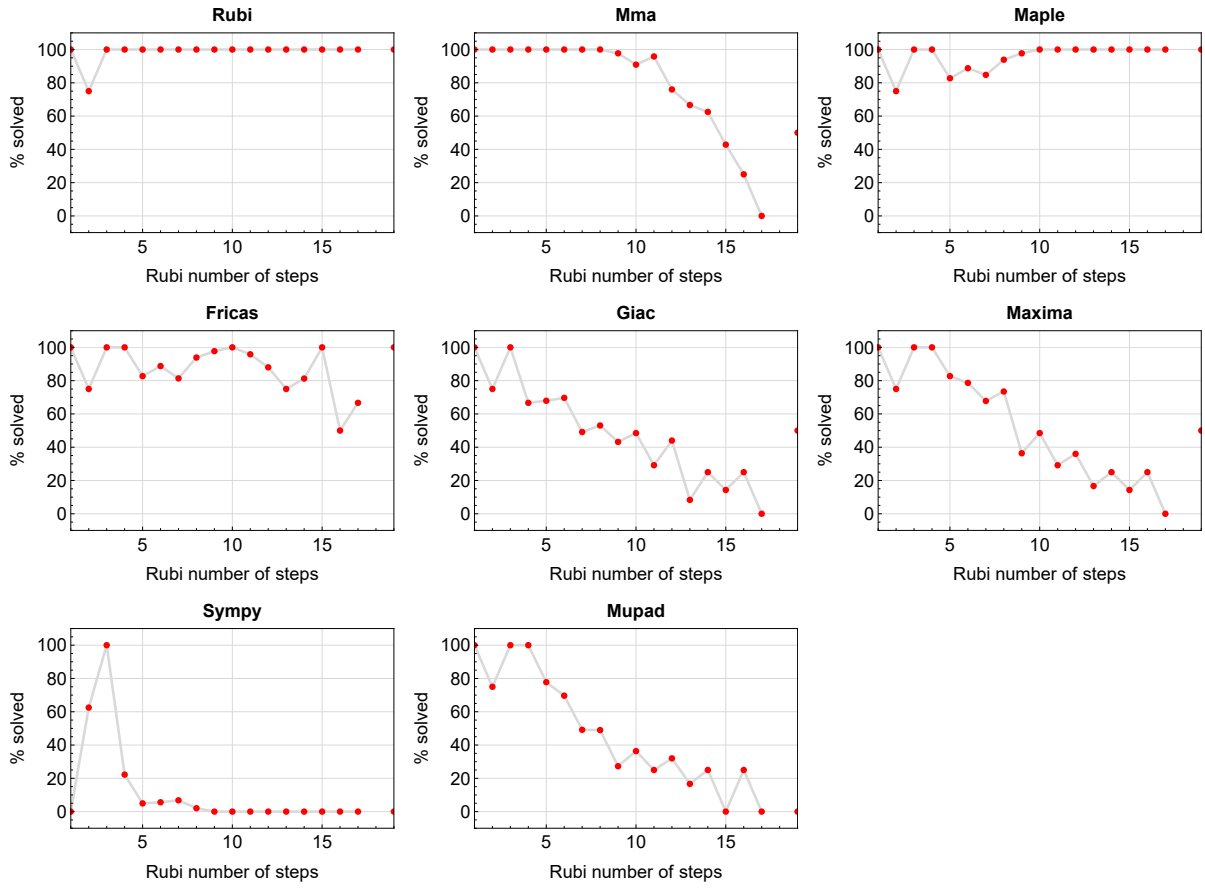


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

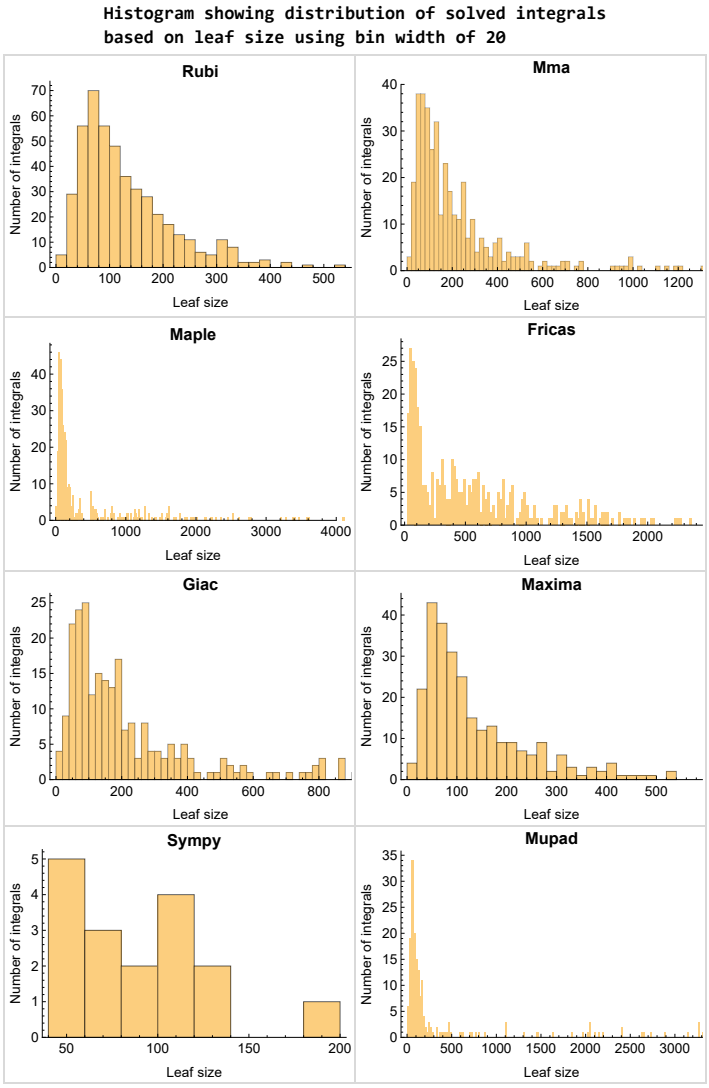


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

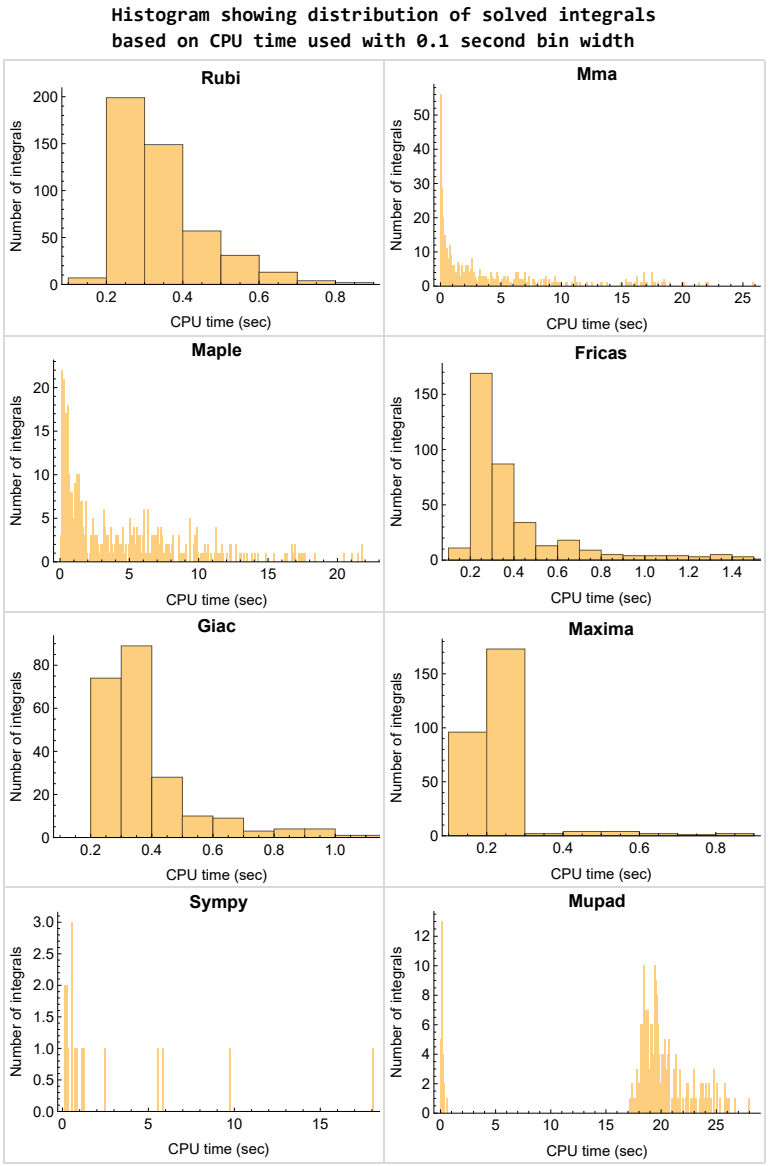


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

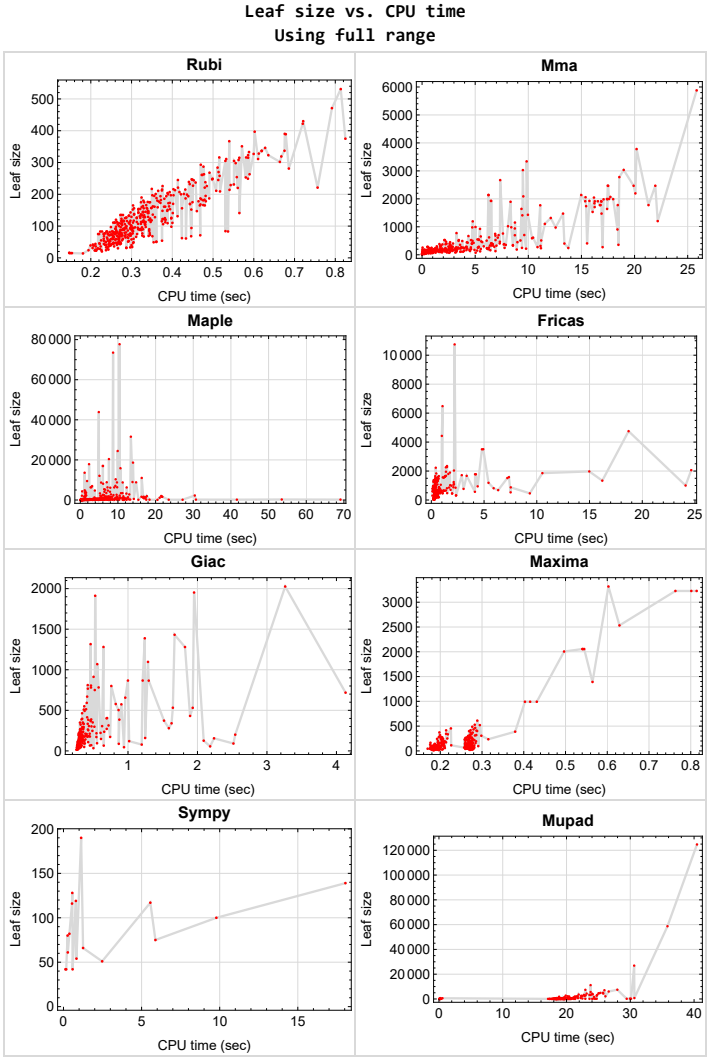


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{462, 466, 467, 468, 469, 470}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {311, 312, 324, 325, 326, 327, 337}

Mathematica {28, 29, 30, 34, 35, 36, 39, 40, 41, 42, 43, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 58, 59, 60, 61, 62, 63, 64, 65, 66, 78, 79, 88, 89, 125, 126, 128, 132, 136, 137, 138, 139, 140, 180, 193, 199, 202, 205, 208, 211, 213, 214, 215, 217, 218, 219, 235, 240, 247, 248, 250, 251, 254, 264, 268, 269, 276, 277, 280, 281, 282, 284, 289, 292, 293, 294, 295, 296, 298, 299, 300, 301, 302, 303, 307, 308, 309, 310, 348, 349, 356, 357, 358, 359, 360, 361, 362, 369, 370, 371, 372, 373, 374, 375, 380, 384, 392, 393, 394, 397, 398, 399, 428, 429, 430, 437, 441, 447, 448, 449, 450, 451, 463, 464}

Maple {72, 73, 74, 86, 87, 88, 89, 97, 111, 123, 124, 228, 229, 230, 231, 232, 233, 241, 244, 246, 247, 248, 250, 251, 254, 257, 258, 260, 261, 262, 265, 270, 271, 272, 273, 274, 275, 277, 283, 284, 285, 286, 287, 288, 380, 381, 392, 393, 394, 398, 399, 406, 407, 410, 418, 419, 420, 422, 425, 426, 427, 431, 432, 433, 434, 438, 439, 440}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

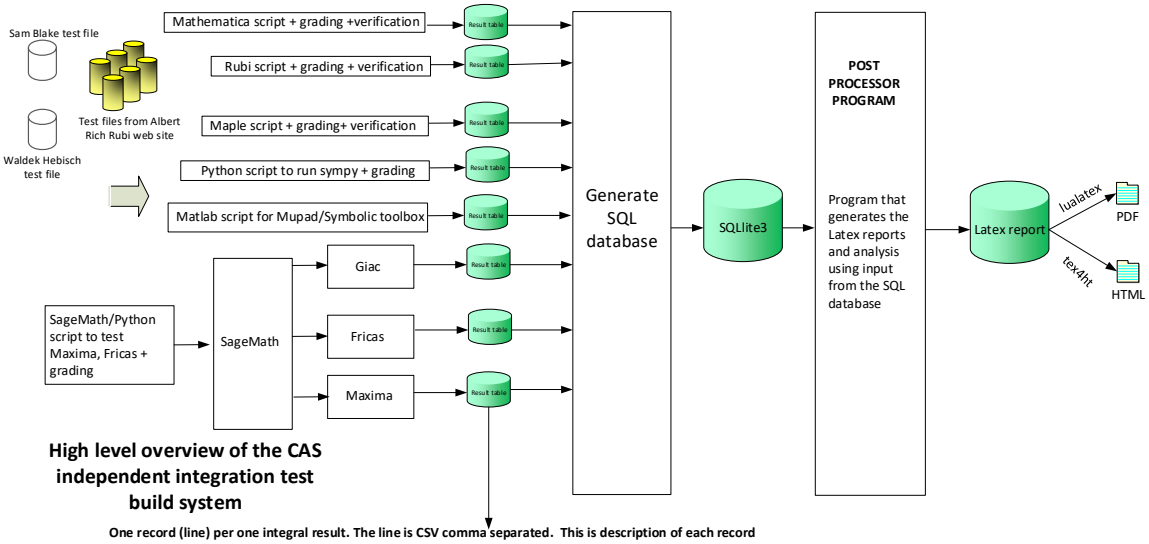
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



High level overview of the CAS independent integration test build system

One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,..}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

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June 27, 2023
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

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2.1 List of integrals sorted by grade for each CAS

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2.1.4 Fricas 23
2.1.5 Maxima 24
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2.1.7 Mupad 26
2.1.8 Sympy 27

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 463, 464, 465 }

B grade { }

C grade { }

F normal fail { 132, 298 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 20, 22, 23, 24, 26, 67, 68, 69, 70, 71, 72, 80, 81, 83, 84, 85, 87, 93, 94, 95, 96, 97, 99, 100, 101, 103, 104, 105, 106, 107, 108, 109, 112, 113, 114, 116, 117, 118, 119, 120, 121, 129, 130, 131, 133, 134, 135, 141, 142, 143, 144, 145, 146, 147, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 185, 188, 190, 191, 192, 194, 195, 196, 197, 198, 200, 203, 204, 206, 207, 209, 210, 212, 216, 220, 221, 222, 223, 224, 225, 226, 227, 231, 238, 239, 245, 252, 253, 259, 267, 271, 278, 283, 290, 304, 305, 306, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 324, 325, 326, 327, 328, 329, 333, 337, 338, 339, 340, 341, 342, 350, 351, 352, 353, 354, 355, 363, 364, 365, 366, 367, 368, 376, 377, 379, 382, 383, 386, 387, 388, 389, 390, 395, 396, 402, 403, 404, 408, 409, 412, 413, 414, 415, 416, 421, 422, 425, 426, 427, 438, 439, 440, 442, 443, 444, 445, 446, 452, 453, 454, 455, 456, 459, 463, 464, 465 }

B grade { 5, 6, 7, 18, 19, 21, 25, 27, 102, 115, 125, 126, 127, 132, 136, 137, 138, 139, 140, 236, 256, 265, 266, 279, 291, 297, 298, 299, 300, 301, 302, 303, 307, 308, 309, 310, 330, 331, 332, 334, 335, 336, 378, 411, 423, 424, 434, 435, 436, 441, 447, 448, 449, 450, 451 }

C grade { 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 75, 76, 77, 78, 79, 82, 88, 89, 90, 91, 92, 98, 110, 111, 122, 123, 124, 128, 148, 149, 150, 151, 180, 186, 187, 189, 193, 199, 201, 202, 205, 208, 211, 213, 214, 215, 217, 218, 219, 232, 234, 235, 237, 240, 246, 247, 248, 249, 250, 251, 254, 255, 260, 263, 264, 268, 269, 272, 276, 277, 280, 281, 282, 284, 289, 292, 293, 294, 295, 296, 321, 322, 323, 343, 344, 345, 346, 347, 348, 349, 356, 357, 358, 359, 360, 361, 362, 369, 370, 371, 372, 373, 374, 375, 380, 384, 385, 391, 392, 393, 394, 397, 398, 399, 400, 401, 405, 410, 417, 428, 429, 430, 437, 457, 458, 460, 461 }

F normal fail { 73, 74, 86, 228, 229, 230, 233, 241, 242, 243, 244, 257, 258, 261, 262, 270, 273, 274, 275, 285, 286, 287, 288, 381, 406, 407, 418, 419, 420, 431, 432, 433 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 69, 82, 93, 94, 95, 103, 104, 105, 106, 107, 108, 116, 117, 118, 119, 120, 121, 129, 130, 131, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 278, 290, 291, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 378, 391, 404, 417, 430, 452, 453, 454, 455, 456, 459, 461 }

B grade { 67, 68, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 96, 97, 98, 99, 100, 101, 102, 109, 110, 111, 112, 113, 114, 115, 122, 123, 124, 125, 126, 127, 128, 234, 235, 236, 237, 238, 239, 240, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 263, 264, 265, 266, 267, 268, 269, 276, 277, 279, 280, 281, 282, 289, 292, 293, 294, 295, 296, 297, 376, 377, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440 }

C grade { 26, 144, 145, 228, 229, 230, 231, 232, 233, 241, 242, 243, 244, 245, 246, 257, 258, 259, 260, 261, 262, 270, 271, 272, 273, 274, 275, 283, 284, 285, 286, 287, 288, 457, 458, 460 }

F normal fail { 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 463, 464, 465 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 3, 4, 8, 9, 10, 12, 13, 14, 15, 16, 17, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 35, 36, 37, 38, 41, 42, 43, 47, 48, 49, 54, 55, 56, 60, 61, 67, 68, 69, 70, 71, 73, 74, 80, 81, 82, 83, 84, 85, 86, 87, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 103, 104, 105, 106, 107, 108, 112, 113, 116, 117, 118, 119, 120, 121, 125, 126, 129, 144, 145, 146, 147, 148, 152, 153, 154, 155, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 189, 190, 191, 192, 193, 194, 195, 196, 198, 203, 204, 205, 218, 220, 221, 222, 223, 224, 225, 226, 227, 234, 235, 239, 240, 247, 248, 249, 253, 263, 268, 269, 276, 281, 282, 290, 295, 311, 312, 313, 314, 315, 316, 317, 318, 319, 321, 324, 325, 326, 327, 328, 329,

330, 331, 332, 333, 334, 337, 338, 339, 340, 341, 345, 346, 350, 351, 352, 353, 363, 364, 365, 382, 395, 396, 452, 453, 454, 455, 456, 459 }

B grade { 5, 6, 7, 11, 18, 19, 20, 33, 39, 40, 44, 45, 46, 50, 51, 52, 53, 57, 58, 59, 62, 63, 64, 65, 66, 72, 75, 76, 77, 78, 79, 88, 89, 101, 102, 109, 110, 111, 114, 115, 122, 123, 124, 127, 128, 130, 131, 149, 150, 151, 161, 186, 187, 188, 197, 199, 200, 201, 202, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 219, 236, 237, 238, 250, 251, 252, 254, 255, 256, 264, 265, 266, 267, 277, 278, 279, 280, 289, 291, 292, 293, 294, 296, 297, 320, 322, 323, 335, 336, 342, 343, 344, 347, 348, 349, 354, 355, 356, 357, 358, 359, 360, 361, 362, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440 }

C grade { 228, 229, 230, 241, 242, 243, 257, 258, 259, 270, 271, 272, 283, 284, 285, 457, 458, 460, 461 }

F normal fail { 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 231, 232, 233, 244, 245, 246, 260, 261, 262, 273, 274, 275, 286, 287, 288, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 463, 464, 465 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.5 Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 66, 67, 68, 69, 77, 78, 79, 80, 81, 82, 90, 91, 92, 93, 94, 95, 103, 104, 105, 106, 107, 108, 116, 117, 118, 119, 120, 121, 129, 130, 131, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 220, 221, 222, 223, 224, 225, 226, 227, 234, 235, 236, 247, 248, 249, 263, 264, 265, 276, 277, 278, 290, 291, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 369, 370, 371, 372, 373, 374, 375, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461 }

B grade { 33, 46, 58, 59, 64, 65, 102, 115, 219, 266, 279, 289, 297, 355, 367, 368, 411, 416, 423, 424 }

C grade { 76, 237, 256, 379, 385 }

F normal fail { 70, 71, 72, 73, 74, 75, 83, 84, 85, 86, 87, 88, 89, 96, 97, 98, 99, 100, 101, 109, 112, 113, 114, 122, 125, 126, 127, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 228, 229, 230, 231, 232, 233, 238, 239, 240, 241, 242, 243, 244, 245, 246, 250, 251, 252, 253, 254, 255, 257, 258, 259, 260, 261, 262, 267, 268, 269, 270, 271, 272, 273, 274, 275, 280, 281, 282, 283, 284, 285, 286, 287, 288, 293, 294, 295, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 376, 377, 378, 380, 381, 382, 383, 384, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 412, 413, 414, 415, 417, 418, 422, 425, 430, 431, 434, 438, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 463, 464, 465 }

F(-1) timedout fail { 110, 111, 123, 124, 128, 292, 296, 419, 420, 421, 426, 427, 428, 429, 432, 433, 435, 436, 437, 439, 440 }

F(-2) exception fail { }

2.1.6 Giac

A grade { 3, 4, 8, 9, 10, 11, 12, 13, 14, 16, 17, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 34, 35, 36, 37, 38, 39, 40, 42, 43, 47, 48, 49, 50, 51, 52, 53, 55, 56, 60, 61, 62, 63, 64, 65, 66, 69, 82, 95, 108, 121, 144, 145, 146, 147, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 225, 226, 227, 317, 318, 319, 320, 330, 331, 332, 333, 334, 343, 344, 345, 346, 347, 348, 349, 356, 357, 358, 359, 360, 361, 362, 369, 370, 371, 372, 373, 374, 375, 455 }

B grade { 1, 2, 5, 6, 7, 15, 18, 19, 20, 28, 32, 33, 41, 44, 45, 46, 54, 57, 58, 59, 67, 68, 70, 71, 72, 80, 81, 96, 109, 122, 148, 149, 150, 151, 222, 223, 311, 312, 313, 314, 315, 316, 321, 322, 323, 324, 325, 326, 327, 328, 329, 335, 336, 337, 338, 339, 340, 341, 342, 350, 351, 352, 353, 354, 355, 363, 364, 365, 366, 367, 368, 376, 377, 378, 379, 380, 381, 389, 390, 391, 392, 452, 453, 454, 456 }

C grade { }

F normal fail { 73, 74, 75, 76, 77, 78, 79, 86, 87, 88, 89, 90, 91, 92, 93, 94, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 224, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 382, 383, 384, 385, 386, 387, 388, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 457, 458, 459, 460, 461, 463, 464, 465 }

F(-1) timedout fail { }

F(-2) exception fail { 83, 84, 85, 393, 394 }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 69, 82, 95, 103, 104, 105, 108, 116, 117, 121, 129, 135, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 278, 290, 291, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 378, 391, 404, 417, 430, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461 }

C grade { }

F normal fail { }

F(-1) timedout fail { 67, 68, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 96, 97, 98, 99, 100, 101, 102, 106, 107, 109, 110, 111, 112, 113, 114, 115, 118, 119, 120, 122, 123, 124, 125, 126, 127, 128, 130, 131, 132, 133, 134, 136, 137, 138, 139, 140, 141, 142, 143, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 376, 377, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 463, 464, 465 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 162, 312, 313, 317, 318, 319, 325, 326, 417, 430, 452, 453, 454 }

B grade { 311, 324, 339, 459 }

C grade { }

F normal fail { 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 16, 17, 18, 19, 23, 24, 25, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 43, 44, 45, 48, 49, 50, 51, 52, 63, 69, 70, 71, 72, 74, 75, 76, 77, 78, 82, 83, 89, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 121, 122, 123, 124, 126, 127, 128, 129, 130, 135, 140, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 163, 165, 166, 167, 168, 169, 171, 172, 173, 174, 175, 176, 178, 179, 180, 181, 182, 183, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 199, 200, 201, 202, 203, 204, 206, 207, 208, 212, 213, 214, 215, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 234, 235, 236, 237, 238, 239, 241, 242, 243, 244, 247, 248, 249, 250, 254, 255, 256, 257, 258, 259, 260, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 289, 290, 291, 292, 293, 294, 296, 297, 298, 299, 300, 301, 306, 307, 314, 315, 316, 320, 321, 322, 323, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 353, 354, 355, 356, 357, 358, 359, 360, 361, 369, 370, 371, 372, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 395, 396, 397, 398, 399, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 443, 444, 445, 448, 449, 455, 456, 457, 458, 460, 461, 464, 465 }

F(-1) timedout fail { 1, 8, 14, 15, 20, 21, 22, 26, 27, 28, 29, 41, 42, 46, 47, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 64, 65, 66, 67, 68, 73, 79, 80, 81, 84, 85, 86, 87, 88, 90, 91, 92, 93, 94, 106, 107, 119, 120, 125, 131, 132, 133, 134, 136, 137, 138, 139, 141, 142, 143, 164, 170, 177, 184, 185, 197, 198, 205, 209, 210, 211, 216, 217, 218, 219, 220, 232, 233, 240, 245, 246, 251, 252, 253, 261, 262, 274, 275, 287, 288, 295, 302, 303, 304, 305, 308, 309, 310, 351, 352, 362, 363, 364, 365, 366, 367, 368, 373, 374, 375, 394, 400, 401, 441, 442, 446, 447, 450, 451, 463, 467 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	74	120	98	73	75	0	266	70
N.S.	1	0.89	1.45	1.18	0.88	0.90	0.00	3.20	0.84
time (sec)	N/A	0.243	0.404	4.138	0.195	0.251	0.000	0.325	0.090

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	59	88	81	58	60	0	197	55
N.S.	1	0.89	1.33	1.23	0.88	0.91	0.00	2.98	0.83
time (sec)	N/A	0.241	0.034	0.256	0.188	0.268	0.000	0.323	17.328

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	40	53	62	40	42	0	57	39
N.S.	1	0.91	1.20	1.41	0.91	0.95	0.00	1.30	0.89
time (sec)	N/A	0.224	0.028	0.175	0.195	0.257	0.000	0.291	0.071

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	23	35	25	25	27	0	26	25
N.S.	1	0.96	1.46	1.04	1.04	1.12	0.00	1.08	1.04
time (sec)	N/A	0.202	0.042	0.079	0.193	0.239	0.000	0.274	0.052

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	26	84	40	44	60	0	60	29
N.S.	1	0.96	3.11	1.48	1.63	2.22	0.00	2.22	1.07
time (sec)	N/A	0.202	0.073	0.119	0.194	0.248	0.000	0.276	0.101

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	60	236	90	76	124	0	193	62
N.S.	1	1.13	4.45	1.70	1.43	2.34	0.00	3.64	1.17
time (sec)	N/A	0.235	0.874	0.216	0.190	0.251	0.000	0.298	17.323

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	97	198	120	101	178	0	262	86
N.S.	1	1.20	2.44	1.48	1.25	2.20	0.00	3.23	1.06
time (sec)	N/A	0.264	6.264	0.300	0.178	0.257	0.000	0.310	17.538

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	124	78	89	111	86	0	104	105
N.S.	1	1.27	0.80	0.91	1.13	0.88	0.00	1.06	1.07
time (sec)	N/A	0.337	0.424	0.445	0.281	0.250	0.000	0.306	18.465

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	90	54	71	82	68	0	82	79
N.S.	1	1.29	0.77	1.01	1.17	0.97	0.00	1.17	1.13
time (sec)	N/A	0.254	0.371	0.273	0.269	0.253	0.000	0.294	17.106

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	55	54	46	47	50	0	47	35
N.S.	1	1.31	1.29	1.10	1.12	1.19	0.00	1.12	0.83
time (sec)	N/A	0.218	0.164	0.141	0.278	0.257	0.000	0.282	17.630

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	15	31	0	15	17
N.S.	1	1.00	1.00	1.07	1.00	2.07	0.00	1.00	1.13
time (sec)	N/A	0.142	0.010	0.147	0.185	0.242	0.000	0.260	17.710

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	24	36	43	26	39	0	26	28
N.S.	1	0.92	1.38	1.65	1.00	1.50	0.00	1.00	1.08
time (sec)	N/A	0.214	0.069	0.227	0.179	0.234	0.000	0.280	19.045

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	41	84	73	43	66	0	50	46
N.S.	1	0.89	1.83	1.59	0.93	1.43	0.00	1.09	1.00
time (sec)	N/A	0.226	0.054	0.487	0.179	0.254	0.000	0.288	19.230

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	60	128	89	64	91	0	76	60
N.S.	1	0.88	1.88	1.31	0.94	1.34	0.00	1.12	0.88
time (sec)	N/A	0.239	0.726	0.664	0.179	0.251	0.000	0.315	18.929

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	86	118	155	89	90	0	414	87
N.S.	1	0.89	1.22	1.60	0.92	0.93	0.00	4.27	0.90
time (sec)	N/A	0.273	2.333	0.460	0.177	0.258	0.000	0.412	18.291

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	65	83	69	67	67	0	91	66
N.S.	1	0.90	1.15	0.96	0.93	0.93	0.00	1.26	0.92
time (sec)	N/A	0.259	1.882	0.265	0.179	0.257	0.000	0.368	18.215

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	41	75	42	42	44	0	44	45
N.S.	1	0.89	1.63	0.91	0.91	0.96	0.00	0.96	0.98
time (sec)	N/A	0.221	0.103	0.175	0.175	0.253	0.000	0.311	0.093

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	48	108	94	82	101	0	172	53
N.S.	1	0.92	2.08	1.81	1.58	1.94	0.00	3.31	1.02
time (sec)	N/A	0.246	0.911	0.218	0.181	0.260	0.000	0.318	0.159

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	117	1021	163	126	193	0	341	96
N.S.	1	1.12	9.82	1.57	1.21	1.86	0.00	3.28	0.92
time (sec)	N/A	0.307	8.090	0.379	0.176	0.262	0.000	0.352	18.508

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	162	218	211	165	286	0	493	135
N.S.	1	1.15	1.55	1.50	1.17	2.03	0.00	3.50	0.96
time (sec)	N/A	0.341	9.790	0.558	0.177	0.257	0.000	0.376	18.296

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	168	499	178	164	131	0	183	163
N.S.	1	1.14	3.37	1.20	1.11	0.89	0.00	1.24	1.10
time (sec)	N/A	0.458	3.433	0.608	0.260	0.263	0.000	0.406	19.110

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	129	153	123	120	107	0	123	116
N.S.	1	1.13	1.34	1.08	1.05	0.94	0.00	1.08	1.02
time (sec)	N/A	0.331	2.230	0.407	0.270	0.250	0.000	0.372	18.780

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	76	126	71	67	81	0	67	94
N.S.	1	1.04	1.73	0.97	0.92	1.11	0.00	0.92	1.29
time (sec)	N/A	0.261	2.189	0.253	0.257	0.261	0.000	0.334	19.134

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	44	41	46	44	58	0	49	42
N.S.	1	1.10	1.02	1.15	1.10	1.45	0.00	1.22	1.05
time (sec)	N/A	0.211	0.373	0.126	0.170	0.240	0.000	0.281	18.820

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	45	109	96	54	71	0	60	56
N.S.	1	0.90	2.18	1.92	1.08	1.42	0.00	1.20	1.12
time (sec)	N/A	0.243	2.618	0.214	0.176	0.252	0.000	0.327	18.617

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	68	151	130	80	101	0	98	85
N.S.	1	0.89	1.99	1.71	1.05	1.33	0.00	1.29	1.12
time (sec)	N/A	0.262	5.183	0.293	0.183	0.238	0.000	0.345	18.494

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	92	353	154	107	139	0	141	108
N.S.	1	0.89	3.43	1.50	1.04	1.35	0.00	1.37	1.05
time (sec)	N/A	0.289	2.667	0.453	0.198	0.249	0.000	0.388	18.749

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F(-1)	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	98	91	425	115	102	229	0	349	123
N.S.	1	0.93	4.34	1.17	1.04	2.34	0.00	3.56	1.26
time (sec)	N/A	0.288	4.852	1.006	0.261	0.284	0.000	0.309	18.106

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	71	69	376	67	63	154	0	85	76
N.S.	1	0.97	5.30	0.94	0.89	2.17	0.00	1.20	1.07
time (sec)	N/A	0.244	2.207	0.486	0.276	0.268	0.000	0.292	0.117

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	47	46	329	44	40	118	0	42	39
N.S.	1	0.98	7.00	0.94	0.85	2.51	0.00	0.89	0.83
time (sec)	N/A	0.215	1.712	0.180	0.260	0.269	0.000	0.288	17.885

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	54	239	71	64	156	0	85	123
N.S.	1	0.98	4.35	1.29	1.16	2.84	0.00	1.55	2.24
time (sec)	N/A	0.236	1.239	0.144	0.264	0.281	0.000	0.292	0.247

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	99	371	115	128	327	0	209	392
N.S.	1	1.15	4.31	1.34	1.49	3.80	0.00	2.43	4.56
time (sec)	N/A	0.273	1.915	0.244	0.261	0.299	0.000	0.318	18.893

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	155	549	181	231	693	0	389	870
N.S.	1	1.20	4.26	1.40	1.79	5.37	0.00	3.02	6.74
time (sec)	N/A	0.325	5.212	0.720	0.261	0.325	0.000	0.332	22.212

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	166	199	357	170	209	428	0	237	1448
N.S.	1	1.20	2.15	1.02	1.26	2.58	0.00	1.43	8.72
time (sec)	N/A	0.422	3.575	2.537	0.261	0.298	0.000	0.306	19.652

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	117	141	303	119	137	332	0	152	494
N.S.	1	1.21	2.59	1.02	1.17	2.84	0.00	1.30	4.22
time (sec)	N/A	0.328	1.385	1.262	0.262	0.284	0.000	0.302	18.783

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	76	91	245	78	77	257	0	92	111
N.S.	1	1.20	3.22	1.03	1.01	3.38	0.00	1.21	1.46
time (sec)	N/A	0.272	0.607	0.574	0.261	0.285	0.000	0.298	18.391

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	182	46	44	231	0	65	460
N.S.	1	1.00	4.04	1.02	0.98	5.13	0.00	1.44	10.22
time (sec)	N/A	0.252	1.413	0.167	0.262	0.282	0.000	0.276	18.479

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	52	189	52	50	271	0	71	46
N.S.	1	0.96	3.50	0.96	0.93	5.02	0.00	1.31	0.85
time (sec)	N/A	0.232	0.886	0.293	0.263	0.295	0.000	0.295	18.116

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	76	77	226	69	82	397	0	103	80
N.S.	1	1.01	2.97	0.91	1.08	5.22	0.00	1.36	1.05
time (sec)	N/A	0.255	1.863	0.507	0.265	0.295	0.000	0.315	18.335

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	105	97	318	93	137	587	0	173	112
N.S.	1	0.92	3.03	0.89	1.30	5.59	0.00	1.65	1.07
time (sec)	N/A	0.321	1.777	0.785	0.263	0.294	0.000	0.339	19.026

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F(-1)	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	161	136	454	159	148	405	0	509	195
N.S.	1	0.84	2.82	0.99	0.92	2.52	0.00	3.16	1.21
time (sec)	N/A	0.326	5.661	4.609	0.261	0.303	0.000	0.364	0.164

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	114	107	403	102	104	297	0	136	130
N.S.	1	0.94	3.54	0.89	0.91	2.61	0.00	1.19	1.14
time (sec)	N/A	0.305	2.914	2.459	0.263	0.284	0.000	0.337	0.138

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	84	83	393	69	70	201	0	72	72
N.S.	1	0.99	4.68	0.82	0.83	2.39	0.00	0.86	0.86
time (sec)	N/A	0.231	2.584	1.030	0.259	0.285	0.000	0.309	18.816

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	106	384	103	138	390	0	272	2188
N.S.	1	1.07	3.88	1.04	1.39	3.94	0.00	2.75	22.10
time (sec)	N/A	0.274	1.633	0.578	0.262	0.321	0.000	0.331	20.176

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	157	468	148	231	698	0	550	1845
N.S.	1	1.07	3.18	1.01	1.57	4.75	0.00	3.74	12.55
time (sec)	N/A	0.334	2.259	0.805	0.258	0.327	0.000	0.371	18.841

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	221	450	213	369	1202	0	669	4338
N.S.	1	1.12	2.28	1.08	1.87	6.10	0.00	3.40	22.02
time (sec)	N/A	0.398	2.637	1.221	0.273	0.365	0.000	0.412	23.158

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	267	298	2468	204	302	674	0	294	1461
N.S.	1	1.12	9.24	0.76	1.13	2.52	0.00	1.10	5.47
time (sec)	N/A	0.530	19.912	5.733	0.267	0.326	0.000	0.362	20.400

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	191	213	1105	147	205	522	0	193	435
N.S.	1	1.12	5.79	0.77	1.07	2.73	0.00	1.01	2.28
time (sec)	N/A	0.400	11.557	3.168	0.260	0.306	0.000	0.343	19.422

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	130	145	699	109	126	441	0	149	816
N.S.	1	1.12	5.38	0.84	0.97	3.39	0.00	1.15	6.28
time (sec)	N/A	0.317	8.734	1.685	0.265	0.300	0.000	0.329	18.499

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	92	111	240	90	106	435	0	114	2056
N.S.	1	1.21	2.61	0.98	1.15	4.73	0.00	1.24	22.35
time (sec)	N/A	0.249	2.679	0.409	0.260	0.312	0.000	0.275	20.406

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	91	92	242	78	117	407	0	127	91
N.S.	1	1.01	2.66	0.86	1.29	4.47	0.00	1.40	1.00
time (sec)	N/A	0.256	2.543	0.576	0.262	0.309	0.000	0.352	18.054

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	123	125	303	106	193	663	0	185	141
N.S.	1	1.02	2.46	0.86	1.57	5.39	0.00	1.50	1.15
time (sec)	N/A	0.374	5.803	0.907	0.267	0.307	0.000	0.387	19.580

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	188	195	777	124	268	987	0	254	198
N.S.	1	1.04	4.13	0.66	1.43	5.25	0.00	1.35	1.05
time (sec)	N/A	0.461	3.241	1.326	0.262	0.322	0.000	0.429	20.341

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F(-1)	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	1641	190	204	579	0	781	255
N.S.	1	1.00	7.67	0.89	0.95	2.71	0.00	3.65	1.19
time (sec)	N/A	0.514	9.356	10.368	0.271	0.324	0.000	0.467	17.898

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	154	152	1153	125	149	439	0	174	172
N.S.	1	0.99	7.49	0.81	0.97	2.85	0.00	1.13	1.12
time (sec)	N/A	0.438	8.741	5.737	0.260	0.300	0.000	0.417	0.177

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	116	120	656	83	103	299	0	92	105
N.S.	1	1.03	5.66	0.72	0.89	2.58	0.00	0.79	0.91
time (sec)	N/A	0.251	5.522	2.949	0.260	0.289	0.000	0.380	0.154

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	173	447	156	261	779	0	592	3557
N.S.	1	1.12	2.90	1.01	1.69	5.06	0.00	3.84	23.10
time (sec)	N/A	0.362	4.217	1.471	0.260	0.396	0.000	0.407	21.902

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F(-1)	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	213	237	532	198	399	1332	0	750	2728
N.S.	1	1.11	2.50	0.93	1.87	6.25	0.00	3.52	12.81
time (sec)	N/A	0.454	5.356	1.760	0.269	0.430	0.000	0.522	19.873

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F(-1)	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	257	284	549	259	528	1833	0	1316	5613
N.S.	1	1.11	2.14	1.01	2.05	7.13	0.00	5.12	21.84
time (sec)	N/A	0.501	5.318	2.312	0.283	0.468	0.000	0.462	23.778

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	314	346	1639	247	418	930	0	351	2117
N.S.	1	1.10	5.22	0.79	1.33	2.96	0.00	1.12	6.74
time (sec)	N/A	0.601	17.416	11.695	0.272	0.359	0.000	0.492	22.332

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	238	261	2469	193	299	803	0	306	1317
N.S.	1	1.10	10.37	0.81	1.26	3.37	0.00	1.29	5.53
time (sec)	N/A	0.462	21.952	6.369	0.267	0.342	0.000	0.438	21.111

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	184	216	1915	155	272	815	0	208	2628
N.S.	1	1.17	10.41	0.84	1.48	4.43	0.00	1.13	14.28
time (sec)	N/A	0.394	15.357	4.212	0.273	0.334	0.000	0.406	22.316

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	144	175	332	147	231	819	0	196	3271
N.S.	1	1.22	2.31	1.02	1.60	5.69	0.00	1.36	22.72
time (sec)	N/A	0.320	6.393	1.535	0.266	0.327	0.000	0.288	22.736

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F(-1)	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	124	132	749	97	219	615	0	177	146
N.S.	1	1.06	6.04	0.78	1.77	4.96	0.00	1.43	1.18
time (sec)	N/A	0.281	6.880	1.373	0.266	0.308	0.000	0.452	19.490

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F(-1)	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	164	166	994	140	323	1009	0	264	207
N.S.	1	1.01	6.06	0.85	1.97	6.15	0.00	1.61	1.26
time (sec)	N/A	0.452	5.026	1.681	0.271	0.331	0.000	0.423	20.644

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	242	249	479	180	434	1423	0	367	267
N.S.	1	1.03	1.98	0.74	1.79	5.88	0.00	1.52	1.10
time (sec)	N/A	0.564	5.566	2.549	0.280	0.359	0.000	0.463	21.389

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	143	152	572	171	295	0	1281	0
N.S.	1	1.03	1.09	4.12	1.23	2.12	0.00	9.22	0.00
time (sec)	N/A	0.316	1.071	3.313	0.267	0.414	0.000	0.647	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	95	120	340	116	232	0	805	0
N.S.	1	0.95	1.20	3.40	1.16	2.32	0.00	8.05	0.00
time (sec)	N/A	0.272	0.364	0.381	0.264	0.386	0.000	0.467	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	64	99	93	88	182	0	55	87
N.S.	1	0.97	1.50	1.41	1.33	2.76	0.00	0.83	1.32
time (sec)	N/A	0.236	0.143	0.067	0.266	0.394	0.000	0.278	18.363

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	80	119	542	0	496	0	405	0
N.S.	1	0.98	1.45	6.61	0.00	6.05	0.00	4.94	0.00
time (sec)	N/A	0.282	0.298	0.560	0.000	0.337	0.000	0.699	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	131	163	2129	0	867	0	578	0
N.S.	1	1.06	1.31	17.17	0.00	6.99	0.00	4.66	0.00
time (sec)	N/A	0.321	0.662	0.444	0.000	0.396	0.000	0.824	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	183	207	209	4318	0	1476	0	867	0
N.S.	1	1.13	1.14	23.60	0.00	8.07	0.00	4.74	0.00
time (sec)	N/A	0.412	1.717	0.507	0.000	0.658	0.000	1.211	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	B	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	240	270	0	1180	0	1715	0	0	0
N.S.	1	1.12	0.00	4.92	0.00	7.15	0.00	0.00	0.00
time (sec)	N/A	0.505	0.000	12.638	0.000	2.887	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	B	F	A	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	181	197	0	791	0	1565	0	0	0
N.S.	1	1.09	0.00	4.37	0.00	8.65	0.00	0.00	0.00
time (sec)	N/A	0.393	0.000	9.838	0.000	1.085	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	127	432	516	0	1417	0	0	0
N.S.	1	1.03	3.51	4.20	0.00	11.52	0.00	0.00	0.00
time (sec)	N/A	0.309	4.291	7.378	0.000	0.606	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	C	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	77	284	353	3227	1227	0	0	0
N.S.	1	0.97	3.59	4.47	40.85	15.53	0.00	0.00	0.00
time (sec)	N/A	0.234	2.888	6.387	0.763	0.478	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	66	61	326	50	306	0	0	0
N.S.	1	0.97	0.90	4.79	0.74	4.50	0.00	0.00	0.00
time (sec)	N/A	0.246	0.198	5.342	0.180	0.310	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	105	100	285	1037	82	436	0	0	0
N.S.	1	0.95	2.71	9.88	0.78	4.15	0.00	0.00	0.00
time (sec)	N/A	0.273	2.761	7.325	0.184	0.473	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	B	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	149	157	941	1680	143	656	0	0	0
N.S.	1	1.05	6.32	11.28	0.96	4.40	0.00	0.00	0.00
time (sec)	N/A	0.325	8.732	7.177	0.187	1.387	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	175	188	802	277	351	0	1952	0
N.S.	1	0.89	0.96	4.09	1.41	1.79	0.00	9.96	0.00
time (sec)	N/A	0.321	1.155	7.029	0.265	0.527	0.000	1.951	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	144	164	552	250	278	0	1388	0
N.S.	1	0.89	1.01	3.41	1.54	1.72	0.00	8.57	0.00
time (sec)	N/A	0.308	0.557	6.934	0.270	0.497	0.000	1.242	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	98	73	121	142	231	0	87	61
N.S.	1	0.98	0.73	1.21	1.42	2.31	0.00	0.87	0.61
time (sec)	N/A	0.248	0.483	0.089	0.259	0.413	0.000	0.311	20.409

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	119	171	1437	0	715	0	0	0
N.S.	1	0.98	1.40	11.78	0.00	5.86	0.00	0.00	0.00
time (sec)	N/A	0.324	0.451	6.762	0.000	0.413	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	162	202	3470	0	984	0	0	0
N.S.	1	1.01	1.25	21.55	0.00	6.11	0.00	0.00	0.00
time (sec)	N/A	0.375	1.115	6.921	0.000	0.427	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	226	262	5131	0	1511	0	0	0
N.S.	1	1.04	1.20	23.54	0.00	6.93	0.00	0.00	0.00
time (sec)	N/A	0.473	2.604	6.610	0.000	0.468	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	B	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	298	311	0	1602	0	1855	0	0	0
N.S.	1	1.04	0.00	5.38	0.00	6.22	0.00	0.00	0.00
time (sec)	N/A	0.582	0.000	16.254	0.000	10.531	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	217	221	225	1186	0	1667	0	0	0
N.S.	1	1.02	1.04	5.47	0.00	7.68	0.00	0.00	0.00
time (sec)	N/A	0.471	3.567	16.911	0.000	3.352	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	161	159	493	900	0	1535	0	0	0
N.S.	1	0.99	3.06	5.59	0.00	9.53	0.00	0.00	0.00
time (sec)	N/A	0.360	3.784	11.543	0.000	1.180	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	118	116	527	709	0	1457	0	0	0
N.S.	1	0.98	4.47	6.01	0.00	12.35	0.00	0.00	0.00
time (sec)	N/A	0.275	3.456	9.876	0.000	0.615	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	103	64	741	98	370	0	0	0
N.S.	1	0.98	0.61	7.06	0.93	3.52	0.00	0.00	0.00
time (sec)	N/A	0.269	0.202	10.167	0.195	0.468	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	154	369	1141	243	472	0	0	0
N.S.	1	0.90	2.15	6.63	1.41	2.74	0.00	0.00	0.00
time (sec)	N/A	0.314	7.692	10.654	0.192	1.502	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	188	512	1851	273	682	0	0	0
N.S.	1	0.90	2.45	8.86	1.31	3.26	0.00	0.00	0.00
time (sec)	N/A	0.339	9.263	11.454	0.205	6.336	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	126	93	94	162	87	0	0	0
N.S.	1	1.02	0.76	0.76	1.32	0.71	0.00	0.00	0.00
time (sec)	N/A	0.302	0.972	1.170	0.199	0.269	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	72	64	58	83	57	0	0	0
N.S.	1	0.97	0.86	0.78	1.12	0.77	0.00	0.00	0.00
time (sec)	N/A	0.257	0.176	1.056	0.193	0.248	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	48	31	28	37	0	31	46
N.S.	1	1.00	1.60	1.03	0.93	1.23	0.00	1.03	1.53
time (sec)	N/A	0.211	0.101	0.271	0.188	0.248	0.000	0.299	17.825

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	86	247	0	140	0	305	0
N.S.	1	1.00	2.00	5.74	0.00	3.26	0.00	7.09	0.00
time (sec)	N/A	0.239	0.108	1.217	0.000	0.308	0.000	0.628	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	87	93	140	1215	0	305	0	0	0
N.S.	1	1.07	1.61	13.97	0.00	3.51	0.00	0.00	0.00
time (sec)	N/A	0.280	0.734	1.283	0.000	0.335	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	157	78	1760	0	491	0	0	0
N.S.	1	1.14	0.57	12.75	0.00	3.56	0.00	0.00	0.00
time (sec)	N/A	0.329	0.150	1.141	0.000	0.364	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	215	163	1136	0	639	0	0	0
N.S.	1	1.11	0.84	5.89	0.00	3.31	0.00	0.00	0.00
time (sec)	N/A	0.422	1.114	6.756	0.000	1.280	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	148	145	809	0	565	0	0	0
N.S.	1	1.10	1.07	5.99	0.00	4.19	0.00	0.00	0.00
time (sec)	N/A	0.317	0.335	5.557	0.000	0.565	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	89	125	509	0	497	0	0	0
N.S.	1	1.05	1.47	5.99	0.00	5.85	0.00	0.00	0.00
time (sec)	N/A	0.266	0.139	4.486	0.000	0.408	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	87	138	992	408	0	0	0
N.S.	1	1.00	2.23	3.54	25.44	10.46	0.00	0.00	0.00
time (sec)	N/A	0.188	0.060	2.726	0.431	0.387	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	55	48	33	47	0	0	74
N.S.	1	1.00	1.67	1.45	1.00	1.42	0.00	0.00	2.24
time (sec)	N/A	0.235	0.095	2.250	0.206	0.259	0.000	0.000	18.195

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	76	74	77	96	96	0	0	123
N.S.	1	0.97	0.95	0.99	1.23	1.23	0.00	0.00	1.58
time (sec)	N/A	0.261	0.156	3.635	0.210	0.303	0.000	0.000	22.952

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	137	100	120	191	171	0	0	723
N.S.	1	1.04	0.76	0.91	1.45	1.30	0.00	0.00	5.48
time (sec)	N/A	0.314	0.271	5.022	0.211	0.588	0.000	0.000	30.640

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	160	130	169	250	136	0	0	0
N.S.	1	0.94	0.76	0.99	1.46	0.80	0.00	0.00	0.00
time (sec)	N/A	0.338	6.908	5.043	0.217	0.326	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	106	93	115	141	98	0	0	0
N.S.	1	0.93	0.82	1.01	1.24	0.86	0.00	0.00	0.00
time (sec)	N/A	0.288	2.612	1.459	0.209	0.306	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	60	64	59	57	67	0	48	155
N.S.	1	0.97	1.03	0.95	0.92	1.08	0.00	0.77	2.50
time (sec)	N/A	0.234	0.941	0.101	0.211	0.277	0.000	0.369	24.732

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	78	113	1061	0	344	0	503	0
N.S.	1	0.98	1.41	13.26	0.00	4.30	0.00	6.29	0.00
time (sec)	N/A	0.268	0.565	1.177	0.000	0.330	0.000	0.864	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	134	97	1714	0	547	0	0	0
N.S.	1	1.06	0.77	13.60	0.00	4.34	0.00	0.00	0.00
time (sec)	N/A	0.332	0.263	1.086	0.000	0.376	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	177	200	100	3612	0	873	0	0	0
N.S.	1	1.13	0.56	20.41	0.00	4.93	0.00	0.00	0.00
time (sec)	N/A	0.384	0.383	1.234	0.000	0.419	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	268	256	1162	0	813	0	0	0
N.S.	1	1.11	1.06	4.80	0.00	3.36	0.00	0.00	0.00
time (sec)	N/A	0.486	6.295	9.328	0.000	5.907	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	194	229	833	0	703	0	0	0
N.S.	1	1.11	1.31	4.76	0.00	4.02	0.00	0.00	0.00
time (sec)	N/A	0.376	3.333	7.286	0.000	1.825	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	125	190	532	0	607	0	0	0
N.S.	1	1.03	1.57	4.40	0.00	5.02	0.00	0.00	0.00
time (sec)	N/A	0.312	1.102	5.306	0.000	0.674	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	75	168	515	2055	601	0	0	0
N.S.	1	0.97	2.18	6.69	26.69	7.81	0.00	0.00	0.00
time (sec)	N/A	0.213	1.052	3.687	0.545	0.430	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	66	76	84	64	102	0	0	2151
N.S.	1	0.97	1.12	1.24	0.94	1.50	0.00	0.00	31.63
time (sec)	N/A	0.251	1.274	3.986	0.210	0.343	0.000	0.000	26.102

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	117	102	142	156	189	0	0	124682
N.S.	1	0.95	0.83	1.15	1.27	1.54	0.00	0.00	1013.67
time (sec)	N/A	0.292	0.534	4.528	0.208	0.667	0.000	0.000	40.457

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	176	126	217	282	314	0	0	0
N.S.	1	0.96	0.69	1.19	1.54	1.72	0.00	0.00	0.00
time (sec)	N/A	0.346	0.739	7.085	0.199	2.338	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	195	182	217	334	189	0	0	0
N.S.	1	0.89	0.83	0.99	1.53	0.86	0.00	0.00	0.00
time (sec)	N/A	0.343	3.114	5.671	0.218	0.464	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	138	129	147	195	138	0	0	0
N.S.	1	0.95	0.88	1.01	1.34	0.95	0.00	0.00	0.00
time (sec)	N/A	0.299	2.189	1.531	0.209	0.385	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	88	90	86	101	0	70	26927
N.S.	1	1.00	0.91	0.93	0.89	1.04	0.00	0.72	277.60
time (sec)	N/A	0.242	0.859	0.108	0.193	0.314	0.000	0.471	30.609

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	134	108	3290	0	592	0	1097	0
N.S.	1	1.06	0.85	25.91	0.00	4.66	0.00	8.64	0.00
time (sec)	N/A	0.317	3.856	5.233	0.000	0.364	0.000	1.288	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	171	186	151	8689	0	941	0	0	0
N.S.	1	1.09	0.88	50.81	0.00	5.50	0.00	0.00	0.00
time (sec)	N/A	0.371	1.032	5.541	0.000	0.435	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	234	263	129	8115	0	1307	0	0	0
N.S.	1	1.12	0.55	34.68	0.00	5.59	0.00	0.00	0.00
time (sec)	N/A	0.468	1.311	6.063	0.000	0.557	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	288	315	1705	2211	0	1003	0	0	0
N.S.	1	1.09	5.92	7.68	0.00	3.48	0.00	0.00	0.00
time (sec)	N/A	0.560	16.243	10.455	0.000	24.107	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	227	246	1315	1611	0	873	0	0	0
N.S.	1	1.08	5.79	7.10	0.00	3.85	0.00	0.00	0.00
time (sec)	N/A	0.445	12.113	8.520	0.000	7.530	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	177	983	1592	0	879	0	0	0
N.S.	1	1.06	5.89	9.53	0.00	5.26	0.00	0.00	0.00
time (sec)	N/A	0.365	8.398	7.014	0.000	2.094	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	125	135	1927	1263	0	881	0	0	0
N.S.	1	1.08	15.42	10.10	0.00	7.05	0.00	0.00	0.00
time (sec)	N/A	0.278	15.750	5.537	0.000	0.711	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	108	108	136	94	192	0	0	336
N.S.	1	1.02	1.02	1.28	0.89	1.81	0.00	0.00	3.17
time (sec)	N/A	0.272	1.561	5.666	0.197	0.714	0.000	0.000	29.970

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	154	138	225	216	320	0	0	0
N.S.	1	0.97	0.87	1.42	1.37	2.03	0.00	0.00	0.00
time (sec)	N/A	0.315	4.513	7.221	0.203	2.353	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	211	173	332	373	460	0	0	0
N.S.	1	0.93	0.77	1.47	1.65	2.04	0.00	0.00	0.00
time (sec)	N/A	0.362	4.934	6.231	0.198	9.338	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD
size	123	0	286	0	0	0	0	0	0
N.S.	1	0.00	2.33	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	4.196	0.000	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	182	180	253	0	0	0	0	0	0
N.S.	1	0.99	1.39	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.345	3.665	0.000	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	117	115	178	0	0	0	0	0	0
N.S.	1	0.98	1.52	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.274	2.397	0.000	0.000	0.000	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	68	68	68	0	0	0	0	0	79
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.16
time (sec)	N/A	0.225	0.790	0.000	0.000	0.000	0.000	0.000	19.172

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	77	77	1532	0	0	0	0	0	0
N.S.	1	1.00	19.90	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.254	16.053	0.000	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	81	81	266	0	0	0	0	0	0
N.S.	1	1.00	3.28	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.261	3.224	0.000	0.000	0.000	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	5878	0	0	0	0	0	0
N.S.	1	1.00	66.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.278	25.843	0.000	0.000	0.000	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	3781	0	0	0	0	0	0
N.S.	1	1.00	42.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.264	20.190	0.000	0.000	0.000	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	83	83	2137	0	0	0	0	0	0
N.S.	1	1.00	25.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.224	14.980	0.000	0.000	0.000	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	72	0	0	0	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.252	0.998	0.000	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	128	126	132	0	0	0	0	0	0
N.S.	1	0.98	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.285	1.583	0.000	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	192	197	149	0	0	0	0	0	0
N.S.	1	1.03	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.345	1.443	0.000	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	62	72	97	129	82	0	66	61
N.S.	1	0.84	0.97	1.31	1.74	1.11	0.00	0.89	0.82
time (sec)	N/A	0.418	0.036	0.862	0.187	0.262	0.000	0.303	19.402

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	49	58	75	81	69	0	53	49
N.S.	1	0.88	1.04	1.34	1.45	1.23	0.00	0.95	0.88
time (sec)	N/A	0.342	0.027	0.550	0.197	0.240	0.000	0.286	19.124

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	32	42	47	45	56	0	39	33
N.S.	1	0.84	1.11	1.24	1.18	1.47	0.00	1.03	0.87
time (sec)	N/A	0.278	0.018	0.378	0.182	0.249	0.000	0.276	18.012

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	26	17	16	32	0	16	16
N.S.	1	1.00	1.62	1.06	1.00	2.00	0.00	1.00	1.00
time (sec)	N/A	0.144	0.002	0.171	0.197	0.246	0.000	0.261	18.292

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	20	31	24	26	31	0	45	19
N.S.	1	1.05	1.63	1.26	1.37	1.63	0.00	2.37	1.00
time (sec)	N/A	0.221	0.016	0.152	0.277	0.239	0.000	0.266	18.650

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	31	36	34	40	75	0	80	31
N.S.	1	0.84	0.97	0.92	1.08	2.03	0.00	2.16	0.84
time (sec)	N/A	0.280	0.020	0.190	0.273	0.243	0.000	0.274	18.372

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	50	36	44	50	109	0	111	41
N.S.	1	0.91	0.65	0.80	0.91	1.98	0.00	2.02	0.75
time (sec)	N/A	0.348	0.044	0.254	0.280	0.245	0.000	0.291	18.414

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	61	36	54	60	147	0	139	51
N.S.	1	0.84	0.49	0.74	0.82	2.01	0.00	1.90	0.70
time (sec)	N/A	0.417	0.016	0.324	0.281	0.240	0.000	0.314	19.074

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	94	137	108	126	114	0	121	102
N.S.	1	0.96	1.40	1.10	1.29	1.16	0.00	1.23	1.04
time (sec)	N/A	0.439	0.037	0.898	0.206	0.247	0.000	0.289	18.120

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	68	93	85	97	95	0	98	78
N.S.	1	0.97	1.33	1.21	1.39	1.36	0.00	1.40	1.11
time (sec)	N/A	0.345	0.023	0.585	0.192	0.250	0.000	0.285	0.146

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	48	55	58	72	0	60	41
N.S.	1	1.00	1.20	1.38	1.45	1.80	0.00	1.50	1.02
time (sec)	N/A	0.248	0.014	0.342	0.191	0.246	0.000	0.280	18.457

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	35	30	38	40	0	40	22
N.S.	1	1.00	1.46	1.25	1.58	1.67	0.00	1.67	0.92
time (sec)	N/A	0.237	0.041	0.199	0.198	0.248	0.000	0.266	18.342

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	50	31	27	28	0	34	28
N.S.	1	1.00	1.67	1.03	0.90	0.93	0.00	1.13	0.93
time (sec)	N/A	0.268	0.034	0.283	0.204	0.232	0.000	0.275	0.056

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	47	71	49	43	45	0	57	43
N.S.	1	0.94	1.42	0.98	0.86	0.90	0.00	1.14	0.86
time (sec)	N/A	0.299	0.021	0.431	0.198	0.257	0.000	0.280	18.146

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	70	81	78	60	74	0	79	56
N.S.	1	0.80	0.93	0.90	0.69	0.85	0.00	0.91	0.64
time (sec)	N/A	0.302	0.237	0.685	0.183	0.253	0.000	0.282	18.159

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	58	61	58	43	56	0	57	42
N.S.	1	0.89	0.94	0.89	0.66	0.86	0.00	0.88	0.65
time (sec)	N/A	0.293	0.151	0.475	0.191	0.236	0.000	0.274	18.416

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	36	35	34	37	0	34	28
N.S.	1	1.00	0.84	0.81	0.79	0.86	0.00	0.79	0.65
time (sec)	N/A	0.269	0.069	0.366	0.186	0.234	0.000	0.278	17.960

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	15	31	0	15	17
N.S.	1	1.00	1.00	1.07	1.00	2.07	0.00	1.00	1.13
time (sec)	N/A	0.143	0.001	0.161	0.183	0.246	0.000	0.267	18.417

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	33	24	37	28	51	37	25
N.S.	1	1.00	1.06	0.77	1.19	0.90	1.65	1.19	0.81
time (sec)	N/A	0.199	0.026	0.188	0.271	0.249	2.475	0.281	19.576

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	59	45	44	73	49	0	73	67
N.S.	1	0.97	0.74	0.72	1.20	0.80	0.00	1.20	1.10
time (sec)	N/A	0.268	0.104	0.333	0.274	0.257	0.000	0.274	18.806

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	85	68	61	103	68	0	96	91
N.S.	1	0.96	0.76	0.69	1.16	0.76	0.00	1.08	1.02
time (sec)	N/A	0.339	0.108	0.498	0.268	0.265	0.000	0.281	19.157

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	258	180	200	168	0	221	170
N.S.	1	1.00	1.56	1.09	1.21	1.02	0.00	1.34	1.03
time (sec)	N/A	0.303	0.048	1.380	0.194	0.261	0.000	0.302	18.832

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	136	187	147	166	143	0	183	134
N.S.	1	1.05	1.45	1.14	1.29	1.11	0.00	1.42	1.04
time (sec)	N/A	0.302	0.031	1.027	0.194	0.255	0.000	0.307	18.608

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	105	109	107	119	116	0	120	86
N.S.	1	1.15	1.20	1.18	1.31	1.27	0.00	1.32	0.95
time (sec)	N/A	0.261	0.026	0.645	0.203	0.258	0.000	0.290	18.690

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	59	80	69	87	94	0	79	55
N.S.	1	1.05	1.43	1.23	1.55	1.68	0.00	1.41	0.98
time (sec)	N/A	0.257	0.051	0.517	0.184	0.248	0.000	0.294	0.114

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	44	72	55	63	66	0	70	48
N.S.	1	0.90	1.47	1.12	1.29	1.35	0.00	1.43	0.98
time (sec)	N/A	0.245	0.052	0.418	0.198	0.251	0.000	0.287	17.975

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	48	106	67	55	59	0	76	44
N.S.	1	0.91	2.00	1.26	1.04	1.11	0.00	1.43	0.83
time (sec)	N/A	0.254	0.040	0.474	0.182	0.265	0.000	0.287	18.545

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	95	96	134	103	120	0	152	94
N.S.	1	0.90	0.91	1.26	0.97	1.13	0.00	1.43	0.89
time (sec)	N/A	0.293	0.350	1.094	0.200	0.250	0.000	0.328	18.233

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	72	75	104	81	94	0	114	70
N.S.	1	0.90	0.94	1.30	1.01	1.18	0.00	1.42	0.88
time (sec)	N/A	0.267	0.306	0.839	0.189	0.243	0.000	0.305	18.593

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	48	48	71	71	69	0	76	44
N.S.	1	0.91	0.91	1.34	1.34	1.30	0.00	1.43	0.83
time (sec)	N/A	0.250	0.208	0.619	0.190	0.246	0.000	0.302	18.245

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	44	41	46	44	58	0	49	42
N.S.	1	1.10	1.02	1.15	1.10	1.45	0.00	1.22	1.05
time (sec)	N/A	0.213	0.084	0.369	0.188	0.251	0.000	0.268	18.489

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	57	52	51	53	56	0	53	66
N.S.	1	1.21	1.11	1.09	1.13	1.19	0.00	1.13	1.40
time (sec)	N/A	0.252	0.163	0.421	0.278	0.271	0.000	0.289	18.780

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	100	58	54	87	62	0	87	76
N.S.	1	1.23	0.72	0.67	1.07	0.77	0.00	1.07	0.94
time (sec)	N/A	0.270	0.301	0.371	0.273	0.263	0.000	0.287	18.618

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	129	99	82	135	89	0	150	123
N.S.	1	1.08	0.83	0.69	1.13	0.75	0.00	1.26	1.03
time (sec)	N/A	0.298	0.269	0.493	0.273	0.253	0.000	0.296	19.077

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	74	70	84	83	90	0	91	73
N.S.	1	1.01	0.96	1.15	1.14	1.23	0.00	1.25	1.00
time (sec)	N/A	0.240	1.031	0.704	0.186	0.256	0.000	0.287	18.687

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	109	106	130	134	130	0	148	119
N.S.	1	0.98	0.95	1.17	1.21	1.17	0.00	1.33	1.07
time (sec)	N/A	0.269	3.596	1.082	0.187	0.246	0.000	0.307	18.709

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	86	95	1195	106	124	272	0	113	591
N.S.	1	1.10	13.90	1.23	1.44	3.16	0.00	1.31	6.87
time (sec)	N/A	0.290	4.768	0.788	0.270	0.289	0.000	0.287	0.552

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	53	49	63	83	157	0	74	456
N.S.	1	0.96	0.89	1.15	1.51	2.85	0.00	1.35	8.29
time (sec)	N/A	0.242	0.092	0.364	0.271	0.271	0.000	0.293	18.735

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	28	50	117	0	38	28
N.S.	1	1.00	1.00	0.78	1.39	3.25	0.00	1.06	0.78
time (sec)	N/A	0.211	0.053	0.215	0.267	0.266	0.000	0.274	0.124

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	50	52	45	67	164	0	53	44
N.S.	1	0.96	1.00	0.87	1.29	3.15	0.00	1.02	0.85
time (sec)	N/A	0.231	0.079	0.508	0.269	0.274	0.000	0.281	18.389

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	71	105	70	88	230	0	85	72
N.S.	1	0.93	1.38	0.92	1.16	3.03	0.00	1.12	0.95
time (sec)	N/A	0.272	0.226	0.987	0.269	0.272	0.000	0.294	18.602

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	100	136	110	117	305	0	129	111
N.S.	1	0.93	1.26	1.02	1.08	2.82	0.00	1.19	1.03
time (sec)	N/A	0.291	0.583	1.939	0.274	0.291	0.000	0.295	0.137

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	72	224	70	65	354	0	96	72
N.S.	1	0.94	2.91	0.91	0.84	4.60	0.00	1.25	0.94
time (sec)	N/A	0.272	2.791	0.971	0.273	0.279	0.000	0.293	18.549

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	50	192	45	45	286	0	66	44
N.S.	1	0.96	3.69	0.87	0.87	5.50	0.00	1.27	0.85
time (sec)	N/A	0.233	1.650	0.532	0.268	0.284	0.000	0.294	18.458

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	28	27	209	0	48	31
N.S.	1	1.00	1.00	0.78	0.75	5.81	0.00	1.33	0.86
time (sec)	N/A	0.223	0.033	0.218	0.270	0.261	0.000	0.290	18.856

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	182	46	44	231	0	65	460
N.S.	1	1.00	4.04	1.02	0.98	5.13	0.00	1.44	10.22
time (sec)	N/A	0.253	0.196	0.169	0.271	0.283	0.000	0.268	20.042

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	91	67	76	72	272	0	94	373
N.S.	1	1.21	0.89	1.01	0.96	3.63	0.00	1.25	4.97
time (sec)	N/A	0.274	0.537	0.683	0.271	0.302	0.000	0.295	20.228

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	141	95	116	126	343	0	141	1114
N.S.	1	1.21	0.81	0.99	1.08	2.93	0.00	1.21	9.52
time (sec)	N/A	0.334	3.202	1.371	0.269	0.289	0.000	0.289	20.259

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	197	133	165	189	424	0	216	1979
N.S.	1	1.21	0.82	1.01	1.16	2.60	0.00	1.33	12.14
time (sec)	N/A	0.398	1.855	2.619	0.269	0.301	0.000	0.290	21.549

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	102	112	980	110	146	392	0	126	2039
N.S.	1	1.10	9.61	1.08	1.43	3.84	0.00	1.24	19.99
time (sec)	N/A	0.291	4.729	1.060	0.276	0.312	0.000	0.305	21.326

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	72	71	68	98	262	0	76	62
N.S.	1	0.97	0.96	0.92	1.32	3.54	0.00	1.03	0.84
time (sec)	N/A	0.247	0.363	0.522	0.265	0.280	0.000	0.307	0.139

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	81	82	80	111	301	0	91	71
N.S.	1	0.98	0.99	0.96	1.34	3.63	0.00	1.10	0.86
time (sec)	N/A	0.237	0.300	0.549	0.271	0.268	0.000	0.306	19.736

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	96	89	92	133	391	0	113	94
N.S.	1	0.95	0.88	0.91	1.32	3.87	0.00	1.12	0.93
time (sec)	N/A	0.303	0.451	1.275	0.268	0.285	0.000	0.295	0.247

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	118	139	120	154	490	0	146	124
N.S.	1	0.94	1.10	0.95	1.22	3.89	0.00	1.16	0.98
time (sec)	N/A	0.328	0.834	3.095	0.277	0.308	0.000	0.300	19.791

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	146	171	158	183	583	0	188	173
N.S.	1	0.93	1.09	1.01	1.17	3.71	0.00	1.20	1.10
time (sec)	N/A	0.349	1.654	4.722	0.270	0.310	0.000	0.310	0.188

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	100	95	248	89	110	516	0	120	113
N.S.	1	0.95	2.48	0.89	1.10	5.16	0.00	1.20	1.13
time (sec)	N/A	0.304	3.164	1.211	0.268	0.305	0.000	0.305	19.570

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	80	84	76	88	406	0	89	70
N.S.	1	0.98	1.02	0.93	1.07	4.95	0.00	1.09	0.85
time (sec)	N/A	0.249	0.288	0.619	0.277	0.283	0.000	0.301	19.611

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	71	211	64	71	368	0	83	69
N.S.	1	0.97	2.89	0.88	0.97	5.04	0.00	1.14	0.95
time (sec)	N/A	0.236	2.519	0.474	0.266	0.277	0.000	0.303	19.350

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	92	111	240	90	106	435	0	114	2056
N.S.	1	1.21	2.61	0.98	1.15	4.73	0.00	1.24	22.35
time (sec)	N/A	0.253	1.410	0.351	0.273	0.295	0.000	0.286	21.719

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	161	103	120	175	544	0	192	2401
N.S.	1	1.13	0.73	0.85	1.23	3.83	0.00	1.35	16.91
time (sec)	N/A	0.352	1.233	1.876	0.269	0.319	0.000	0.312	22.474

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	232	138	160	268	656	0	195	2880
N.S.	1	1.14	0.68	0.79	1.32	3.23	0.00	0.96	14.19
time (sec)	N/A	0.435	1.565	3.492	0.276	0.328	0.000	0.316	22.995

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	278	316	499	210	369	789	0	270	3310
N.S.	1	1.14	1.79	0.76	1.33	2.84	0.00	0.97	11.91
time (sec)	N/A	0.512	4.748	6.040	0.278	0.313	0.000	0.313	23.683

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	113	90	108	179	472	0	117	113
N.S.	1	1.05	0.83	1.00	1.66	4.37	0.00	1.08	1.05
time (sec)	N/A	0.259	0.759	1.378	0.276	0.299	0.000	0.336	0.214

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	163	124	212	544	0	155	129
N.S.	1	1.00	1.30	0.99	1.70	4.35	0.00	1.24	1.03
time (sec)	N/A	0.276	0.701	1.711	0.269	0.297	0.000	0.343	19.600

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	144	156	927	142	233	613	0	178	149
N.S.	1	1.08	6.44	0.99	1.62	4.26	0.00	1.24	1.03
time (sec)	N/A	0.300	5.451	1.655	0.269	0.293	0.000	0.344	0.335

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	148	176	149	253	727	0	197	175
N.S.	1	0.95	1.13	0.96	1.62	4.66	0.00	1.26	1.12
time (sec)	N/A	0.369	1.364	3.144	0.275	0.331	0.000	0.340	19.719

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	170	194	177	272	856	0	229	256
N.S.	1	0.94	1.07	0.98	1.50	4.73	0.00	1.27	1.41
time (sec)	N/A	0.394	3.248	5.839	0.270	0.324	0.000	0.350	0.351

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	214	200	2670	214	303	995	0	271	257
N.S.	1	0.93	12.48	1.00	1.42	4.65	0.00	1.27	1.20
time (sec)	N/A	0.416	7.352	10.268	0.272	0.347	0.000	0.356	0.236

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	152	125	137	210	722	0	185	149
N.S.	1	1.07	0.88	0.96	1.48	5.08	0.00	1.30	1.05
time (sec)	N/A	0.312	0.827	1.474	0.276	0.317	0.000	0.336	20.045

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	283	118	187	654	0	163	125
N.S.	1	1.00	2.30	0.96	1.52	5.32	0.00	1.33	1.02
time (sec)	N/A	0.275	4.134	1.446	0.276	0.320	0.000	0.343	19.349

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	106	111	265	100	156	580	0	124	112
N.S.	1	1.05	2.50	0.94	1.47	5.47	0.00	1.17	1.06
time (sec)	N/A	0.258	4.593	1.580	0.284	0.297	0.000	0.334	19.073

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	144	175	332	147	231	819	0	196	3271
N.S.	1	1.22	2.31	1.02	1.60	5.69	0.00	1.36	22.72
time (sec)	N/A	0.326	6.650	1.372	0.279	0.324	0.000	0.298	23.592

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	239	156	177	338	970	0	228	3708
N.S.	1	1.19	0.78	0.88	1.68	4.83	0.00	1.13	18.45
time (sec)	N/A	0.441	4.487	4.398	0.275	0.330	0.000	0.338	24.104

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	269	305	1430	215	464	1129	0	464	4158
N.S.	1	1.13	5.32	0.80	1.72	4.20	0.00	1.72	15.46
time (sec)	N/A	0.521	9.964	6.905	0.284	0.347	0.000	0.340	24.036

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	352	397	1770	267	611	1296	0	350	4594
N.S.	1	1.13	5.03	0.76	1.74	3.68	0.00	0.99	13.05
time (sec)	N/A	0.597	11.122	12.974	0.289	0.395	0.000	0.350	25.065

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F(-1)	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	204	245	1411	229	401	1323	0	324	4506
N.S.	1	1.20	6.92	1.12	1.97	6.49	0.00	1.59	22.09
time (sec)	N/A	0.412	9.439	3.254	0.288	0.365	0.000	0.318	25.028

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	83	70	145	81	100	0	217	0
N.S.	1	0.62	0.52	1.08	0.60	0.75	0.00	1.62	0.00
time (sec)	N/A	0.525	2.544	5.155	0.277	0.268	0.000	0.546	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	68	60	135	62	87	0	182	0
N.S.	1	0.67	0.59	1.34	0.61	0.86	0.00	1.80	0.00
time (sec)	N/A	0.430	0.631	3.366	0.264	0.260	0.000	0.438	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	51	48	98	40	68	0	137	0
N.S.	1	0.80	0.75	1.53	0.62	1.06	0.00	2.14	0.00
time (sec)	N/A	0.350	0.151	0.764	0.272	0.271	0.000	0.357	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	38	21	53	0	141	0
N.S.	1	1.00	1.00	1.15	0.64	1.61	0.00	4.27	0.00
time (sec)	N/A	0.285	0.051	0.207	0.272	0.256	0.000	0.345	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	34	40	48	37	56	0	0	0
N.S.	1	1.06	1.25	1.50	1.16	1.75	0.00	0.00	0.00
time (sec)	N/A	0.293	0.083	0.200	0.278	0.256	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	54	57	87	60	94	0	108	0
N.S.	1	0.81	0.85	1.30	0.90	1.40	0.00	1.61	0.00
time (sec)	N/A	0.371	0.167	0.826	0.279	0.265	0.000	0.427	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	71	67	107	79	125	0	147	0
N.S.	1	0.71	0.67	1.07	0.79	1.25	0.00	1.47	0.00
time (sec)	N/A	0.447	0.306	0.866	0.281	0.282	0.000	0.612	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	84	79	117	94	162	0	171	0
N.S.	1	0.63	0.59	0.88	0.71	1.22	0.00	1.29	0.00
time (sec)	N/A	0.523	0.391	0.912	0.278	0.250	0.000	0.545	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	C	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	372	390	0	8814	0	887	0	0	0
N.S.	1	1.05	0.00	23.69	0.00	2.38	0.00	0.00	0.00
time (sec)	N/A	0.665	0.000	14.829	0.000	0.172	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	C	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	288	305	0	6342	0	782	0	0	0
N.S.	1	1.06	0.00	22.02	0.00	2.72	0.00	0.00	0.00
time (sec)	N/A	0.553	0.000	9.800	0.000	0.147	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	C	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	218	238	0	4676	0	603	0	0	0
N.S.	1	1.09	0.00	21.45	0.00	2.77	0.00	0.00	0.00
time (sec)	N/A	0.467	0.000	7.335	0.000	0.119	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	80	90	69	4727	0	0	0	0	0
N.S.	1	1.12	0.86	59.09	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.312	0.637	3.872	0.000	0.000	0.000	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	246	249	402	6300	0	0	0	0	0
N.S.	1	1.01	1.63	25.61	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.494	13.397	7.231	0.000	0.000	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	338	328	0	8763	0	0	0	0	0
N.S.	1	0.97	0.00	25.93	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.616	0.000	9.747	0.000	0.000	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	180	407	1375	317	468	0	0	0
N.S.	1	0.97	2.19	7.39	1.70	2.52	0.00	0.00	0.00
time (sec)	N/A	0.320	15.531	17.255	0.200	1.066	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	122	117	380	907	173	390	0	0	0
N.S.	1	0.96	3.11	7.43	1.42	3.20	0.00	0.00	0.00
time (sec)	N/A	0.268	7.729	11.174	0.214	0.425	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	74	210	537	69	320	0	0	0
N.S.	1	0.97	2.76	7.07	0.91	4.21	0.00	0.00	0.00
time (sec)	N/A	0.250	1.887	6.676	0.196	0.308	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	C	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	77	284	351	3227	1227	0	0	0
N.S.	1	0.97	3.59	4.44	40.85	15.53	0.00	0.00	0.00
time (sec)	N/A	0.238	0.489	2.109	0.801	0.497	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	86	136	313	0	499	0	0	0
N.S.	1	1.05	1.66	3.82	0.00	6.09	0.00	0.00	0.00
time (sec)	N/A	0.259	0.774	2.806	0.000	0.421	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	143	152	587	0	567	0	0	0
N.S.	1	1.02	1.09	4.19	0.00	4.05	0.00	0.00	0.00
time (sec)	N/A	0.289	1.381	5.099	0.000	0.546	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	196	215	1902	970	0	641	0	0	0
N.S.	1	1.10	9.70	4.95	0.00	3.27	0.00	0.00	0.00
time (sec)	N/A	0.367	16.882	7.745	0.000	1.169	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	C	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	450	471	0	11007	0	981	0	0	0
N.S.	1	1.05	0.00	24.46	0.00	2.18	0.00	0.00	0.00
time (sec)	N/A	0.770	0.000	16.319	0.000	0.201	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	C	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	371	389	0	8946	0	890	0	0	0
N.S.	1	1.05	0.00	24.11	0.00	2.40	0.00	0.00	0.00
time (sec)	N/A	0.674	0.000	13.873	0.000	0.163	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	C	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	290	308	0	7055	0	798	0	0	0
N.S.	1	1.06	0.00	24.33	0.00	2.75	0.00	0.00	0.00
time (sec)	N/A	0.559	0.000	9.322	0.000	0.141	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	224	244	0	5360	0	0	0	0	0
N.S.	1	1.09	0.00	23.93	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.455	0.000	6.353	0.000	0.000	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	245	179	6947	0	0	0	0	0
N.S.	1	1.02	0.74	28.83	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.499	2.739	3.142	0.000	0.000	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	319	323	356	8848	0	0	0	0	0
N.S.	1	1.01	1.12	27.74	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.633	18.481	7.028	0.000	0.000	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	243	217	512	1977	415	566	0	0	0
N.S.	1	0.89	2.11	8.14	1.71	2.33	0.00	0.00	0.00
time (sec)	N/A	0.351	11.216	21.487	0.214	4.133	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	165	152	450	1417	243	470	0	0	0
N.S.	1	0.92	2.73	8.59	1.47	2.85	0.00	0.00	0.00
time (sec)	N/A	0.294	8.974	16.720	0.207	1.074	0.000	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	109	84	951	104	390	0	0	0
N.S.	1	0.98	0.76	8.57	0.94	3.51	0.00	0.00	0.00
time (sec)	N/A	0.269	0.458	11.799	0.196	0.432	0.000	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	118	116	527	709	0	1457	0	0	0
N.S.	1	0.98	4.47	6.01	0.00	12.35	0.00	0.00	0.00
time (sec)	N/A	0.281	2.143	7.762	0.000	0.605	0.000	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	124	128	466	537	0	1403	0	0	0
N.S.	1	1.03	3.76	4.33	0.00	11.31	0.00	0.00	0.00
time (sec)	N/A	0.331	7.324	9.313	0.000	0.605	0.000	0.000	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	133	191	602	0	563	0	0	0
N.S.	1	1.06	1.53	4.82	0.00	4.50	0.00	0.00	0.00
time (sec)	N/A	0.288	1.241	3.575	0.000	0.574	0.000	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	190	165	986	0	647	0	0	0
N.S.	1	0.98	0.85	5.11	0.00	3.35	0.00	0.00	0.00
time (sec)	N/A	0.324	2.222	6.036	0.000	1.191	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	166	166	706	1137	0	1611	0	0	0
N.S.	1	1.00	4.25	6.85	0.00	9.70	0.00	0.00	0.00
time (sec)	N/A	0.334	9.620	21.085	0.000	1.291	0.000	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	109	189	0	160	0	0	0
N.S.	1	1.00	2.60	4.50	0.00	3.81	0.00	0.00	0.00
time (sec)	N/A	0.197	0.239	8.050	0.000	0.263	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	C	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	57	120	1391	131	0	0	0
N.S.	1	1.00	2.38	5.00	57.96	5.46	0.00	0.00	0.00
time (sec)	N/A	0.189	0.075	5.975	0.565	0.257	0.000	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	C	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	330	315	0	6274	0	784	0	0	0
N.S.	1	0.95	0.00	19.01	0.00	2.38	0.00	0.00	0.00
time (sec)	N/A	0.576	0.000	12.320	0.000	0.149	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	C	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	170	201	0	4103	0	594	0	0	0
N.S.	1	1.18	0.00	24.14	0.00	3.49	0.00	0.00	0.00
time (sec)	N/A	0.428	0.000	7.891	0.000	0.129	0.000	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	90	69	243	0	303	0	0	0
N.S.	1	1.12	0.86	3.04	0.00	3.79	0.00	0.00	0.00
time (sec)	N/A	0.350	0.882	3.490	0.000	0.102	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	105	115	274	4095	0	0	0	0	0
N.S.	1	1.10	2.61	39.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.319	10.867	6.407	0.000	0.000	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	255	258	0	6273	0	0	0	0	0
N.S.	1	1.01	0.00	24.60	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.506	0.000	9.019	0.000	0.000	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	345	337	0	8756	0	0	0	0	0
N.S.	1	0.98	0.00	25.38	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.606	0.000	11.261	0.000	0.000	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	142	326	1612	160	396	0	0	0
N.S.	1	1.04	2.38	11.77	1.17	2.89	0.00	0.00	0.00
time (sec)	N/A	0.311	8.552	16.828	0.202	0.435	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	81	79	326	1039	74	324	0	0	0
N.S.	1	0.98	4.02	12.83	0.91	4.00	0.00	0.00	0.00
time (sec)	N/A	0.266	6.998	11.293	0.202	0.320	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	39	39	87	241	23	215	0	0	0
N.S.	1	1.00	2.23	6.18	0.59	5.51	0.00	0.00	0.00
time (sec)	N/A	0.247	0.152	7.881	0.195	0.301	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	87	138	992	408	0	0	0
N.S.	1	1.00	2.23	3.54	25.44	10.46	0.00	0.00	0.00
time (sec)	N/A	0.205	0.068	3.019	0.403	0.367	0.000	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	91	126	507	0	502	0	0	0
N.S.	1	1.05	1.45	5.83	0.00	5.77	0.00	0.00	0.00
time (sec)	N/A	0.275	0.279	5.121	0.000	0.401	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	143	156	1840	809	0	567	0	0	0
N.S.	1	1.09	12.87	5.66	0.00	3.97	0.00	0.00	0.00
time (sec)	N/A	0.319	16.220	7.145	0.000	0.533	0.000	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	204	226	1739	1136	0	643	0	0	0
N.S.	1	1.11	8.52	5.57	0.00	3.15	0.00	0.00	0.00
time (sec)	N/A	0.375	15.442	9.682	0.000	1.140	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	C	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	289	316	0	10329	0	960	0	0	0
N.S.	1	1.09	0.00	35.74	0.00	3.32	0.00	0.00	0.00
time (sec)	N/A	0.575	0.000	9.398	0.000	0.163	0.000	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	150	187	113	2535	0	894	0	0	0
N.S.	1	1.25	0.75	16.90	0.00	5.96	0.00	0.00	0.00
time (sec)	N/A	0.432	3.659	6.144	0.000	0.171	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	229	247	905	2534	0	762	0	0	0
N.S.	1	1.08	3.95	11.07	0.00	3.33	0.00	0.00	0.00
time (sec)	N/A	0.488	18.390	5.982	0.000	0.146	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	240	261	0	6126	0	0	0	0	0
N.S.	1	1.09	0.00	25.52	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.471	0.000	7.970	0.000	0.000	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	335	336	0	15843	0	0	0	0	0
N.S.	1	1.00	0.00	47.29	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.605	0.000	10.670	0.000	0.000	0.000	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	436	422	0	18649	0	0	0	0	0
N.S.	1	0.97	0.00	42.77	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.704	0.000	14.006	0.000	0.000	0.000	0.000	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	138	143	259	3210	161	524	0	0	0
N.S.	1	1.04	1.88	23.26	1.17	3.80	0.00	0.00	0.00
time (sec)	N/A	0.319	3.393	12.250	0.203	0.475	0.000	0.000	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	77	75	405	1513	78	410	0	0	0
N.S.	1	0.97	5.26	19.65	1.01	5.32	0.00	0.00	0.00
time (sec)	N/A	0.268	7.307	8.133	0.189	0.333	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	57	49	30	65	0	0	199
N.S.	1	1.00	1.78	1.53	0.94	2.03	0.00	0.00	6.22
time (sec)	N/A	0.244	0.777	1.876	0.191	0.260	0.000	0.000	20.519

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	75	168	515	2055	601	0	0	0
N.S.	1	0.97	2.18	6.69	26.69	7.81	0.00	0.00	0.00
time (sec)	N/A	0.228	1.544	2.535	0.542	0.446	0.000	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	131	134	2059	809	0	699	0	0	0
N.S.	1	1.02	15.72	6.18	0.00	5.34	0.00	0.00	0.00
time (sec)	N/A	0.330	15.386	4.129	0.000	0.632	0.000	0.000	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	194	207	2046	1143	0	811	0	0	0
N.S.	1	1.07	10.55	5.89	0.00	4.18	0.00	0.00	0.00
time (sec)	N/A	0.382	16.178	6.399	0.000	1.479	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	271	293	2068	1499	0	941	0	0	0
N.S.	1	1.08	7.63	5.53	0.00	3.47	0.00	0.00	0.00
time (sec)	N/A	0.461	17.722	9.780	0.000	4.392	0.000	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	321	327	167	16966	0	1374	0	0	0
N.S.	1	1.02	0.52	52.85	0.00	4.28	0.00	0.00	0.00
time (sec)	N/A	0.595	4.177	6.008	0.000	0.221	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	319	323	1204	12063	0	1244	0	0	0
N.S.	1	1.01	3.77	37.82	0.00	3.90	0.00	0.00	0.00
time (sec)	N/A	0.573	22.179	5.086	0.000	0.207	0.000	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	C	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	327	333	0	16966	0	1293	0	0	0
N.S.	1	1.02	0.00	51.88	0.00	3.95	0.00	0.00	0.00
time (sec)	N/A	0.584	0.000	6.015	0.000	0.211	0.000	0.000	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	349	351	0	20403	0	0	0	0	0
N.S.	1	1.01	0.00	58.46	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.570	0.000	7.614	0.000	0.000	0.000	0.000	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	441	430	0	24449	0	0	0	0	0
N.S.	1	0.98	0.00	55.44	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.718	0.000	9.934	0.000	0.000	0.000	0.000	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	559	531	0	31577	0	0	0	0	0
N.S.	1	0.95	0.00	56.49	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.822	0.000	13.435	0.000	0.000	0.000	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	133	145	592	2355	275	688	0	0	0
N.S.	1	1.09	4.45	17.71	2.07	5.17	0.00	0.00	0.00
time (sec)	N/A	0.317	10.351	6.327	0.203	0.505	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	74	75	117	134	0	0	153
N.S.	1	1.00	0.94	0.95	1.48	1.70	0.00	0.00	1.94
time (sec)	N/A	0.251	5.042	3.141	0.184	0.343	0.000	0.000	29.436

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	69	215	84	61	134	0	0	172
N.S.	1	0.97	3.03	1.18	0.86	1.89	0.00	0.00	2.42
time (sec)	N/A	0.255	6.140	3.115	0.183	0.383	0.000	0.000	30.119

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	125	135	1927	1263	0	881	0	0	0
N.S.	1	1.08	15.42	10.10	0.00	7.05	0.00	0.00	0.00
time (sec)	N/A	0.274	6.499	3.763	0.000	0.732	0.000	0.000	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	187	204	1775	2143	0	1023	0	0	0
N.S.	1	1.09	9.49	11.46	0.00	5.47	0.00	0.00	0.00
time (sec)	N/A	0.392	16.736	5.352	0.000	1.784	0.000	0.000	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	261	287	1777	2770	0	1187	0	0	0
N.S.	1	1.10	6.81	10.61	0.00	4.55	0.00	0.00	0.00
time (sec)	N/A	0.469	18.420	8.145	0.000	5.422	0.000	0.000	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	332	367	1776	3399	0	1337	0	0	0
N.S.	1	1.11	5.35	10.24	0.00	4.03	0.00	0.00	0.00
time (sec)	N/A	0.527	21.312	10.807	0.000	16.210	0.000	0.000	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	179	201	1777	2018	0	1241	0	0	0
N.S.	1	1.12	9.93	11.27	0.00	6.93	0.00	0.00	0.00
time (sec)	N/A	0.334	16.808	5.457	0.000	2.122	0.000	0.000	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	37	59	388	53	0	0	0
N.S.	1	1.00	2.64	4.21	27.71	3.79	0.00	0.00	0.00
time (sec)	N/A	0.173	0.121	0.868	0.379	0.252	0.000	0.000	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	B	F	F	F	F	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD
size	111	0	2195	0	0	0	0	0	0
N.S.	1	0.00	19.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	20.076	0.000	0.000	0.000	0.000	0.000	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	103	113	1989	0	0	0	0	0	0
N.S.	1	1.10	19.31	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.346	17.989	0.000	0.000	0.000	0.000	0.000	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	103	113	1995	0	0	0	0	0	0
N.S.	1	1.10	19.37	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.311	17.416	0.000	0.000	0.000	0.000	0.000	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	101	111	1983	0	0	0	0	0	0
N.S.	1	1.10	19.63	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.293	17.082	0.000	0.000	0.000	0.000	0.000	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	103	113	1987	0	0	0	0	0	0
N.S.	1	1.10	19.29	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.327	17.433	0.000	0.000	0.000	0.000	0.000	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	103	113	1997	0	0	0	0	0	0
N.S.	1	1.10	19.39	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.338	17.602	0.000	0.000	0.000	0.000	0.000	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	216	212	149	0	0	0	0	0	0
N.S.	1	0.98	0.69	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.375	2.162	0.000	0.000	0.000	0.000	0.000	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	129	127	126	0	0	0	0	0	0
N.S.	1	0.98	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.281	2.142	0.000	0.000	0.000	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	72	72	71	0	0	0	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.244	1.035	0.000	0.000	0.000	0.000	0.000	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	83	83	2137	0	0	0	0	0	0
N.S.	1	1.00	25.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.234	6.248	0.000	0.000	0.000	0.000	0.000	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	83	83	1914	0	0	0	0	0	0
N.S.	1	1.00	23.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.256	16.549	0.000	0.000	0.000	0.000	0.000	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	83	83	1912	0	0	0	0	0	0
N.S.	1	1.00	23.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.257	16.255	0.000	0.000	0.000	0.000	0.000	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	83	83	1914	0	0	0	0	0	0
N.S.	1	1.00	23.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.255	16.860	0.000	0.000	0.000	0.000	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	63	55	55	95	69	116	280	52
N.S.	1	0.88	0.76	0.76	1.32	0.96	1.61	3.89	0.72
time (sec)	N/A	0.252	0.185	1.626	0.192	0.268	0.547	1.588	19.692

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	46	43	45	64	50	80	236	46
N.S.	1	0.94	0.88	0.92	1.31	1.02	1.63	4.82	0.94
time (sec)	N/A	0.244	0.084	0.736	0.190	0.259	0.263	0.658	19.536

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	28	30	26	33	37	42	190	32
N.S.	1	0.93	1.00	0.87	1.10	1.23	1.40	6.33	1.07
time (sec)	N/A	0.211	0.020	0.332	0.199	0.244	0.128	0.328	18.982

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	37	50	24	33	35	0	98	32
N.S.	1	1.32	1.79	0.86	1.18	1.25	0.00	3.50	1.14
time (sec)	N/A	0.240	0.029	0.327	0.194	0.258	0.000	0.276	19.680

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	41	52	39	29	50	0	146	51
N.S.	1	1.28	1.62	1.22	0.91	1.56	0.00	4.56	1.59
time (sec)	N/A	0.242	0.181	0.578	0.188	0.258	0.000	0.310	19.274

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	66	73	55	49	83	0	238	61
N.S.	1	1.29	1.43	1.08	0.96	1.63	0.00	4.67	1.20
time (sec)	N/A	0.268	0.029	1.253	0.183	0.259	0.000	0.354	19.878

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	60	73	57	56	89	66	56	51
N.S.	1	0.94	1.14	0.89	0.88	1.39	1.03	0.88	0.80
time (sec)	N/A	0.276	0.024	2.202	0.268	0.257	1.254	2.182	18.993

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	57	47	45	72	54	45	40
N.S.	1	1.00	1.19	0.98	0.94	1.50	1.12	0.94	0.83
time (sec)	N/A	0.268	0.017	1.098	0.267	0.254	0.831	0.943	19.463

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	36	41	37	33	53	42	33	29
N.S.	1	1.12	1.28	1.16	1.03	1.66	1.31	1.03	0.91
time (sec)	N/A	0.250	0.013	0.562	0.269	0.254	0.584	0.502	19.434

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	15	31	0	15	17
N.S.	1	1.00	1.00	1.07	1.00	2.07	0.00	1.00	1.13
time (sec)	N/A	0.151	0.002	0.271	0.184	0.251	0.000	0.275	19.591

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	26	43	33	25	34	0	53	19
N.S.	1	1.37	2.26	1.74	1.32	1.79	0.00	2.79	1.00
time (sec)	N/A	0.242	0.082	0.566	0.273	0.246	0.000	0.297	19.279

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	41	51	48	41	76	0	111	35
N.S.	1	1.24	1.55	1.45	1.24	2.30	0.00	3.36	1.06
time (sec)	N/A	0.259	0.017	0.886	0.269	0.252	0.000	0.310	19.443

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	49	51	63	52	110	0	170	46
N.S.	1	0.96	1.00	1.24	1.02	2.16	0.00	3.33	0.90
time (sec)	N/A	0.262	0.029	1.891	0.274	0.244	0.000	0.337	19.797

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	91	126	86	147	99	190	431	124
N.S.	1	0.91	1.26	0.86	1.47	0.99	1.90	4.31	1.24
time (sec)	N/A	0.299	0.531	9.059	0.195	0.274	1.130	1.894	19.519

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	71	107	76	114	79	128	385	92
N.S.	1	0.92	1.39	0.99	1.48	1.03	1.66	5.00	1.19
time (sec)	N/A	0.271	0.291	4.573	0.191	0.267	0.564	0.876	20.179

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	46	83	41	67	53	61	334	61
N.S.	1	0.96	1.73	0.85	1.40	1.10	1.27	6.96	1.27
time (sec)	N/A	0.242	0.120	2.223	0.190	0.261	0.277	0.420	19.686

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	55	84	50	64	79	0	247	58
N.S.	1	1.04	1.58	0.94	1.21	1.49	0.00	4.66	1.09
time (sec)	N/A	0.268	0.246	1.850	0.191	0.260	0.000	0.318	20.391

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	64	81	64	60	100	0	233	68
N.S.	1	1.12	1.42	1.12	1.05	1.75	0.00	4.09	1.19
time (sec)	N/A	0.277	0.195	2.618	0.188	0.260	0.000	0.329	20.281

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	69	77	71	61	97	0	332	83
N.S.	1	1.35	1.51	1.39	1.20	1.90	0.00	6.51	1.63
time (sec)	N/A	0.277	0.297	5.402	0.182	0.269	0.000	0.363	20.714

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	90	275	88	84	137	0	91	126
N.S.	1	0.95	2.89	0.93	0.88	1.44	0.00	0.96	1.33
time (sec)	N/A	0.324	3.708	12.310	0.269	0.269	0.000	2.517	19.692

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	75	395	78	71	113	0	78	97
N.S.	1	0.97	5.13	1.01	0.92	1.47	0.00	1.01	1.26
time (sec)	N/A	0.309	2.302	6.529	0.264	0.264	0.000	1.197	19.432

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	60	281	68	58	86	0	65	69
N.S.	1	1.02	4.76	1.15	0.98	1.46	0.00	1.10	1.17
time (sec)	N/A	0.299	1.710	3.204	0.264	0.262	0.000	0.651	19.412

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	44	41	46	44	58	0	49	42
N.S.	1	1.10	1.02	1.15	1.10	1.45	0.00	1.22	1.05
time (sec)	N/A	0.218	0.136	0.545	0.182	0.254	0.000	0.287	19.572

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	40	82	66	46	67	0	46	44
N.S.	1	1.11	2.28	1.83	1.28	1.86	0.00	1.28	1.22
time (sec)	N/A	0.292	3.125	1.889	0.266	0.251	0.000	0.323	19.059

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	49	160	73	59	98	0	176	53
N.S.	1	1.09	3.56	1.62	1.31	2.18	0.00	3.91	1.18
time (sec)	N/A	0.301	1.572	3.911	0.271	0.249	0.000	0.323	19.971

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	66	256	107	72	136	0	273	68
N.S.	1	1.02	3.94	1.65	1.11	2.09	0.00	4.20	1.05
time (sec)	N/A	0.308	3.887	7.660	0.280	0.271	0.000	0.354	19.656

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	63	99	67	81	84	0	373	103
N.S.	1	0.91	1.43	0.97	1.17	1.22	0.00	5.41	1.49
time (sec)	N/A	0.291	0.332	1.595	0.188	0.298	0.000	1.518	19.461

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	41	42	50	41	0	222	64
N.S.	1	1.00	0.91	0.93	1.11	0.91	0.00	4.93	1.42
time (sec)	N/A	0.272	0.118	0.798	0.190	0.291	0.000	0.605	20.163

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	26	35	26	21	117	129	63
N.S.	1	1.00	1.13	1.52	1.13	0.91	5.09	5.61	2.74
time (sec)	N/A	0.208	0.213	0.351	0.179	0.250	5.559	0.329	19.831

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	52	43	68	50	42	0	166	65
N.S.	1	1.13	0.93	1.48	1.09	0.91	0.00	3.61	1.41
time (sec)	N/A	0.272	0.118	0.648	0.186	0.299	0.000	0.296	20.645

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	77	100	119	87	126	0	294	98
N.S.	1	1.04	1.35	1.61	1.18	1.70	0.00	3.97	1.32
time (sec)	N/A	0.300	0.239	1.273	0.184	0.343	0.000	0.342	20.762

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	118	138	180	145	265	0	513	160
N.S.	1	1.09	1.28	1.67	1.34	2.45	0.00	4.75	1.48
time (sec)	N/A	0.344	0.717	2.611	0.195	0.458	0.000	0.397	20.663

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	98	229	101	95	373	0	126	1109
N.S.	1	1.18	2.76	1.22	1.14	4.49	0.00	1.52	13.36
time (sec)	N/A	0.389	3.953	2.321	0.273	0.306	0.000	2.090	19.875

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	70	206	72	66	297	0	87	410
N.S.	1	1.19	3.49	1.22	1.12	5.03	0.00	1.47	6.95
time (sec)	N/A	0.317	2.119	1.138	0.266	0.283	0.000	0.868	19.158

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	53	184	48	45	226	0	66	126
N.S.	1	1.15	4.00	1.04	0.98	4.91	0.00	1.43	2.74
time (sec)	N/A	0.288	0.755	0.632	0.266	0.279	0.000	0.452	19.656

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	182	46	44	231	0	65	460
N.S.	1	1.00	4.04	1.02	0.98	5.13	0.00	1.44	10.22
time (sec)	N/A	0.263	0.262	0.204	0.269	0.284	0.000	0.274	19.369

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	77	204	68	66	310	0	87	637
N.S.	1	1.24	3.29	1.10	1.06	5.00	0.00	1.40	10.27
time (sec)	N/A	0.325	2.248	0.947	0.266	0.283	0.000	0.312	21.799

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	86	109	390	90	106	533	0	134	2644
N.S.	1	1.27	4.53	1.05	1.23	6.20	0.00	1.56	30.74
time (sec)	N/A	0.385	4.688	1.806	0.268	0.308	0.000	0.352	24.728

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	120	147	671	118	161	833	0	212	4324
N.S.	1	1.22	5.59	0.98	1.34	6.94	0.00	1.77	36.03
time (sec)	N/A	0.456	4.363	3.663	0.271	0.318	0.000	0.386	25.759

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	70	109	72	98	118	0	532	170
N.S.	1	0.91	1.42	0.94	1.27	1.53	0.00	6.91	2.21
time (sec)	N/A	0.294	0.751	6.079	0.182	0.318	0.000	1.646	20.344

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	48	81	50	59	53	0	314	97
N.S.	1	0.94	1.59	0.98	1.16	1.04	0.00	6.16	1.90
time (sec)	N/A	0.271	2.341	3.282	0.195	0.256	0.000	0.720	20.325

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	44	79	59	57	52	0	384	90
N.S.	1	0.90	1.61	1.20	1.16	1.06	0.00	7.84	1.84
time (sec)	N/A	0.245	0.637	2.325	0.185	0.264	0.000	0.428	19.572

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	84	112	91	117	138	0	396	106
N.S.	1	1.01	1.35	1.10	1.41	1.66	0.00	4.77	1.28
time (sec)	N/A	0.302	0.569	3.656	0.188	0.377	0.000	0.341	19.758

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	108	130	141	192	312	0	811	160
N.S.	1	0.97	1.17	1.27	1.73	2.81	0.00	7.31	1.44
time (sec)	N/A	0.324	1.784	7.277	0.204	0.498	0.000	0.421	20.508

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	148	162	201	279	557	0	913	206
N.S.	1	1.06	1.16	1.44	1.99	3.98	0.00	6.52	1.47
time (sec)	N/A	0.377	2.956	13.267	0.196	0.797	0.000	0.505	21.174

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	119	131	286	125	127	514	0	155	765
N.S.	1	1.10	2.40	1.05	1.07	4.32	0.00	1.30	6.43
time (sec)	N/A	0.401	6.801	8.526	0.275	0.296	0.000	2.238	20.049

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	90	104	249	85	96	393	0	120	285
N.S.	1	1.16	2.77	0.94	1.07	4.37	0.00	1.33	3.17
time (sec)	N/A	0.334	3.735	4.548	0.291	0.294	0.000	1.017	19.786

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	85	95	346	77	75	458	0	94	711
N.S.	1	1.12	4.07	0.91	0.88	5.39	0.00	1.11	8.36
time (sec)	N/A	0.313	8.548	2.397	0.275	0.304	0.000	0.554	20.669

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	92	111	240	90	106	435	0	114	2056
N.S.	1	1.21	2.61	0.98	1.15	4.73	0.00	1.24	22.35
time (sec)	N/A	0.261	2.200	0.421	0.277	0.295	0.000	0.288	21.731

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	121	141	288	102	163	604	0	175	3146
N.S.	1	1.17	2.38	0.84	1.35	4.99	0.00	1.45	26.00
time (sec)	N/A	0.395	5.343	5.162	0.277	0.305	0.000	0.374	24.793

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	160	183	1896	124	235	979	0	212	4987
N.S.	1	1.14	11.85	0.78	1.47	6.12	0.00	1.32	31.17
time (sec)	N/A	0.472	8.329	9.732	0.315	0.320	0.000	0.449	25.728

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	207	231	3028	152	319	1505	0	298	6017
N.S.	1	1.12	14.63	0.73	1.54	7.27	0.00	1.44	29.07
time (sec)	N/A	0.551	9.491	18.385	0.286	0.360	0.000	0.516	26.616

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	73	136	80	112	116	0	531	166
N.S.	1	0.94	1.74	1.03	1.44	1.49	0.00	6.81	2.13
time (sec)	N/A	0.287	3.263	20.494	0.196	0.272	0.000	1.932	19.934

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	75	131	73	113	111	0	656	153
N.S.	1	0.93	1.62	0.90	1.40	1.37	0.00	8.10	1.89
time (sec)	N/A	0.292	1.118	11.325	0.226	0.273	0.000	0.961	19.468

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	69	129	81	102	102	0	782	142
N.S.	1	0.93	1.74	1.09	1.38	1.38	0.00	10.57	1.92
time (sec)	N/A	0.261	1.910	11.280	0.207	0.274	0.000	0.577	19.330

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	128	158	144	243	307	0	767	190
N.S.	1	0.98	1.22	1.11	1.87	2.36	0.00	5.90	1.46
time (sec)	N/A	0.348	1.422	17.523	0.201	0.611	0.000	0.422	20.768

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	151	176	193	344	584	0	1069	272
N.S.	1	0.98	1.14	1.25	2.23	3.79	0.00	6.94	1.77
time (sec)	N/A	0.380	1.981	30.707	0.214	0.929	0.000	0.558	21.188

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	194	208	254	454	859	0	1912	327
N.S.	1	1.01	1.08	1.32	2.36	4.47	0.00	9.96	1.70
time (sec)	N/A	0.434	5.525	53.496	0.225	1.684	0.000	0.528	22.487

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	147	170	760	134	193	664	0	200	615
N.S.	1	1.16	5.17	0.91	1.31	4.52	0.00	1.36	4.18
time (sec)	N/A	0.437	7.611	27.172	0.286	0.304	0.000	2.543	20.948

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	137	153	1473	124	163	763	0	159	1117
N.S.	1	1.12	10.75	0.91	1.19	5.57	0.00	1.16	8.15
time (sec)	N/A	0.384	16.892	15.427	0.278	0.308	0.000	1.247	21.390

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	138	161	1473	125	191	860	0	172	2405
N.S.	1	1.17	10.67	0.91	1.38	6.23	0.00	1.25	17.43
time (sec)	N/A	0.381	13.269	11.630	0.275	0.331	0.000	0.743	22.823

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	144	175	332	147	231	819	0	196	3271
N.S.	1	1.22	2.31	1.02	1.60	5.69	0.00	1.36	22.72
time (sec)	N/A	0.333	6.516	1.483	0.278	0.321	0.000	0.298	23.807

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	181	211	2089	149	311	1060	0	247	4890
N.S.	1	1.17	11.54	0.82	1.72	5.86	0.00	1.36	27.02
time (sec)	N/A	0.494	9.532	23.488	0.283	0.345	0.000	0.488	25.316

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	230	263	3340	171	409	1649	0	302	7057
N.S.	1	1.14	14.52	0.74	1.78	7.17	0.00	1.31	30.68
time (sec)	N/A	0.564	9.842	41.582	0.287	0.370	0.000	0.475	25.944

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	285	319	976	199	520	2229	0	388	7460
N.S.	1	1.12	3.42	0.70	1.82	7.82	0.00	1.36	26.18
time (sec)	N/A	0.650	12.570	69.090	0.295	0.420	0.000	0.471	27.969

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	107	139	568	0	456	0	1280	0
N.S.	1	0.96	1.25	5.12	0.00	4.11	0.00	11.53	0.00
time (sec)	N/A	0.317	2.828	12.017	0.000	1.972	0.000	1.820	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	79	74	338	0	386	0	800	0
N.S.	1	0.99	0.92	4.22	0.00	4.82	0.00	10.00	0.00
time (sec)	N/A	0.278	0.918	7.435	0.000	0.593	0.000	0.760	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	56	119	58	0	312	0	377	46
N.S.	1	1.04	2.20	1.07	0.00	5.78	0.00	6.98	0.85
time (sec)	N/A	0.235	0.561	0.155	0.000	0.371	0.000	0.463	20.345

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	C	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	72	70	540	3317	963	0	402	0
N.S.	1	1.03	1.00	7.71	47.39	13.76	0.00	5.74	0.00
time (sec)	N/A	0.290	0.328	1.350	0.603	0.418	0.000	0.692	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	B	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	109	113	527	2756	0	1342	0	574	0
N.S.	1	1.04	4.83	25.28	0.00	12.31	0.00	5.27	0.00
time (sec)	N/A	0.311	6.913	1.388	0.000	0.557	0.000	0.905	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	B	F	B	F	B	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	161	182	0	4323	0	1953	0	866	0
N.S.	1	1.13	0.00	26.85	0.00	12.13	0.00	5.38	0.00
time (sec)	N/A	0.369	0.000	1.359	0.000	1.388	0.000	1.299	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	229	263	1581	0	1775	0	0	0
N.S.	1	1.05	1.20	7.22	0.00	8.11	0.00	0.00	0.00
time (sec)	N/A	0.528	4.130	21.774	0.000	4.224	0.000	0.000	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	169	208	1065	0	1621	0	0	0
N.S.	1	1.02	1.26	6.45	0.00	9.82	0.00	0.00	0.00
time (sec)	N/A	0.429	3.055	16.737	0.000	1.312	0.000	0.000	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	118	115	526	661	0	1471	0	0	0
N.S.	1	0.97	4.46	5.60	0.00	12.47	0.00	0.00	0.00
time (sec)	N/A	0.361	4.606	12.297	0.000	0.619	0.000	0.000	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	C	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	77	284	351	3227	1227	0	0	0
N.S.	1	0.97	3.59	4.44	40.85	15.53	0.00	0.00	0.00
time (sec)	N/A	0.232	0.548	4.094	0.814	0.487	0.000	0.000	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	67	130	224	0	499	0	0	0
N.S.	1	0.97	1.88	3.25	0.00	7.23	0.00	0.00	0.00
time (sec)	N/A	0.312	0.731	3.215	0.000	0.429	0.000	0.000	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	112	176	596	0	629	0	0	0
N.S.	1	0.98	1.54	5.23	0.00	5.52	0.00	0.00	0.00
time (sec)	N/A	0.369	0.857	4.712	0.000	0.722	0.000	0.000	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	175	178	1016	0	849	0	0	0
N.S.	1	1.05	1.07	6.08	0.00	5.08	0.00	0.00	0.00
time (sec)	N/A	0.450	2.876	5.901	0.000	2.297	0.000	0.000	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	128	149	826	0	527	0	2026	0
N.S.	1	0.95	1.10	6.12	0.00	3.90	0.00	15.01	0.00
time (sec)	N/A	0.322	2.404	16.964	0.000	7.519	0.000	3.262	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	102	99	518	0	443	0	1431	0
N.S.	1	0.98	0.95	4.98	0.00	4.26	0.00	13.76	0.00
time (sec)	N/A	0.288	1.718	14.217	0.000	1.970	0.000	1.671	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	79	84	81	0	373	0	867	66
N.S.	1	1.01	1.08	1.04	0.00	4.78	0.00	11.12	0.85
time (sec)	N/A	0.255	0.333	0.141	0.000	0.599	0.000	1.000	23.605

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	B	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	91	91	506	1330	0	1075	0	718	0
N.S.	1	1.00	5.56	14.62	0.00	11.81	0.00	7.89	0.00
time (sec)	N/A	0.318	6.079	5.571	0.000	0.578	0.000	4.131	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	114	118	622	1569	0	1300	0	0	0
N.S.	1	1.04	5.46	13.76	0.00	11.40	0.00	0.00	0.00
time (sec)	N/A	0.345	6.308	1.263	0.000	0.605	0.000	0.000	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	159	172	684	4839	0	1801	0	0	0
N.S.	1	1.08	4.30	30.43	0.00	11.33	0.00	0.00	0.00
time (sec)	N/A	0.387	6.905	1.178	0.000	1.612	0.000	0.000	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	290	302	353	2229	0	1973	0	0	0
N.S.	1	1.04	1.22	7.69	0.00	6.80	0.00	0.00	0.00
time (sec)	N/A	0.658	7.880	30.399	0.000	14.978	0.000	0.000	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	220	258	1619	0	1777	0	0	0
N.S.	1	1.03	1.21	7.57	0.00	8.30	0.00	0.00	0.00
time (sec)	N/A	0.550	5.287	21.723	0.000	4.149	0.000	0.000	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	166	165	703	1119	0	1627	0	0	0
N.S.	1	0.99	4.23	6.74	0.00	9.80	0.00	0.00	0.00
time (sec)	N/A	0.451	7.206	17.357	0.000	1.269	0.000	0.000	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	118	116	527	709	0	1457	0	0	0
N.S.	1	0.98	4.47	6.01	0.00	12.35	0.00	0.00	0.00
time (sec)	N/A	0.271	2.095	8.033	0.000	0.623	0.000	0.000	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	111	106	410	539	0	1446	0	0	0
N.S.	1	0.95	3.69	4.86	0.00	13.03	0.00	0.00	0.00
time (sec)	N/A	0.377	6.497	9.300	0.000	0.650	0.000	0.000	0.000

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	110	100	369	0	597	0	0	0
N.S.	1	0.98	0.89	3.29	0.00	5.33	0.00	0.00	0.00
time (sec)	N/A	0.386	0.578	3.742	0.000	0.837	0.000	0.000	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	139	793	0	767	0	0	0
N.S.	1	1.00	0.84	4.81	0.00	4.65	0.00	0.00	0.00
time (sec)	N/A	0.464	1.497	5.016	0.000	3.056	0.000	0.000	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	87	107	303	0	410	0	0	0
N.S.	1	0.98	1.20	3.40	0.00	4.61	0.00	0.00	0.00
time (sec)	N/A	0.300	1.562	8.109	0.000	0.621	0.000	0.000	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	59	56	256	0	328	0	0	0
N.S.	1	1.05	1.00	4.57	0.00	5.86	0.00	0.00	0.00
time (sec)	N/A	0.276	0.604	4.970	0.000	0.371	0.000	0.000	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	42	0	261	0	0	27
N.S.	1	1.00	1.00	1.27	0.00	7.91	0.00	0.00	0.82
time (sec)	N/A	0.228	0.143	0.225	0.000	0.323	0.000	0.000	20.533

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	72	294	343	0	1015	0	0	0
N.S.	1	1.03	4.20	4.90	0.00	14.50	0.00	0.00	0.00
time (sec)	N/A	0.281	2.798	1.458	0.000	0.437	0.000	0.000	0.000

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	B	F	B	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	116	128	0	1944	0	1550	0	0	0
N.S.	1	1.10	0.00	16.76	0.00	13.36	0.00	0.00	0.00
time (sec)	N/A	0.327	0.000	1.378	0.000	0.618	0.000	0.000	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	B	F	B	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	166	194	0	3596	0	2257	0	0	0
N.S.	1	1.17	0.00	21.66	0.00	13.60	0.00	0.00	0.00
time (sec)	N/A	0.384	0.000	1.546	0.000	1.378	0.000	0.000	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	181	230	1930	0	1673	0	0	0
N.S.	1	1.05	1.33	11.16	0.00	9.67	0.00	0.00	0.00
time (sec)	N/A	0.462	3.645	17.678	0.000	1.390	0.000	0.000	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	122	196	1334	0	1507	0	0	0
N.S.	1	1.02	1.63	11.12	0.00	12.56	0.00	0.00	0.00
time (sec)	N/A	0.363	2.542	12.662	0.000	0.665	0.000	0.000	0.000

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	80	78	296	344	0	1259	0	0	0
N.S.	1	0.98	3.70	4.30	0.00	15.74	0.00	0.00	0.00
time (sec)	N/A	0.331	2.479	8.970	0.000	0.503	0.000	0.000	0.000

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	87	138	992	408	0	0	0
N.S.	1	1.00	2.23	3.54	25.44	10.46	0.00	0.00	0.00
time (sec)	N/A	0.195	0.117	3.317	0.415	0.372	0.000	0.000	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	72	127	507	0	525	0	0	0
N.S.	1	0.97	1.72	6.85	0.00	7.09	0.00	0.00	0.00
time (sec)	N/A	0.333	0.252	4.121	0.000	0.442	0.000	0.000	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	126	168	891	0	723	0	0	0
N.S.	1	1.06	1.41	7.49	0.00	6.08	0.00	0.00	0.00
time (sec)	N/A	0.393	1.494	5.279	0.000	0.677	0.000	0.000	0.000

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	187	199	1279	0	987	0	0	0
N.S.	1	1.09	1.16	7.44	0.00	5.74	0.00	0.00	0.00
time (sec)	N/A	0.455	3.337	6.782	0.000	1.910	0.000	0.000	0.000

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	121	562	0	458	0	0	0
N.S.	1	1.00	1.38	6.39	0.00	5.20	0.00	0.00	0.00
time (sec)	N/A	0.317	5.465	5.375	0.000	0.828	0.000	0.000	0.000

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	65	62	2293	2532	417	0	0	0
N.S.	1	1.03	0.98	36.40	40.19	6.62	0.00	0.00	0.00
time (sec)	N/A	0.286	0.829	1.587	0.629	0.385	0.000	0.000	0.000

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	59	382	62	0	392	75	0	49
N.S.	1	1.04	6.70	1.09	0.00	6.88	1.32	0.00	0.86
time (sec)	N/A	0.248	6.657	0.155	0.000	0.370	5.886	0.000	21.308

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	B	F	B	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	100	113	0	9462	0	1569	0	0	0
N.S.	1	1.13	0.00	94.62	0.00	15.69	0.00	0.00	0.00
time (sec)	N/A	0.326	0.000	1.806	0.000	0.610	0.000	0.000	0.000

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	B	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	153	173	0	13610	0	2347	0	0	0
N.S.	1	1.13	0.00	88.95	0.00	15.34	0.00	0.00	0.00
time (sec)	N/A	0.385	0.000	1.157	0.000	1.485	0.000	0.000	0.000

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	B	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	213	247	0	17868	0	3501	0	0	0
N.S.	1	1.16	0.00	83.89	0.00	16.44	0.00	0.00	0.00
time (sec)	N/A	0.449	0.000	2.335	0.000	4.800	0.000	0.000	0.000

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	178	247	2792	0	1895	0	0	0
N.S.	1	1.03	1.44	16.23	0.00	11.02	0.00	0.00	0.00
time (sec)	N/A	0.474	9.405	13.013	0.000	1.765	0.000	0.000	0.000

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	116	121	201	1186	0	1655	0	0	0
N.S.	1	1.04	1.73	10.22	0.00	14.27	0.00	0.00	0.00
time (sec)	N/A	0.371	4.615	10.586	0.000	0.715	0.000	0.000	0.000

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	69	169	282	2005	548	0	0	0
N.S.	1	0.97	2.38	3.97	28.24	7.72	0.00	0.00	0.00
time (sec)	N/A	0.338	2.612	4.092	0.497	0.434	0.000	0.000	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	75	168	515	2055	601	0	0	0
N.S.	1	0.97	2.18	6.69	26.69	7.81	0.00	0.00	0.00
time (sec)	N/A	0.221	1.452	4.431	0.540	0.432	0.000	0.000	0.000

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	119	124	182	1228	0	741	0	0	0
N.S.	1	1.04	1.53	10.32	0.00	6.23	0.00	0.00	0.00
time (sec)	N/A	0.395	4.462	5.738	0.000	0.731	0.000	0.000	0.000

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	174	184	224	1779	0	1061	0	0	0
N.S.	1	1.06	1.29	10.22	0.00	6.10	0.00	0.00	0.00
time (sec)	N/A	0.458	4.241	6.691	0.000	2.113	0.000	0.000	0.000

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	241	254	237	2449	0	1517	0	0	0
N.S.	1	1.05	0.98	10.16	0.00	6.29	0.00	0.00	0.00
time (sec)	N/A	0.543	6.093	8.818	0.000	7.207	0.000	0.000	0.000

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	97	96	187	6373	0	564	0	0	0
N.S.	1	0.99	1.93	65.70	0.00	5.81	0.00	0.00	0.00
time (sec)	N/A	0.323	8.911	2.938	0.000	0.973	0.000	0.000	0.000

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	89	93	613	6313	0	522	0	0	0
N.S.	1	1.04	6.89	70.93	0.00	5.87	0.00	0.00	0.00
time (sec)	N/A	0.298	10.448	2.750	0.000	0.838	0.000	0.000	0.000

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	A	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	83	87	613	86	0	494	100	0	68
N.S.	1	1.05	7.39	1.04	0.00	5.95	1.20	0.00	0.82
time (sec)	N/A	0.260	7.123	0.165	0.000	0.714	9.786	0.000	24.035

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	B	F	B	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	137	161	0	43855	0	2279	0	0	0
N.S.	1	1.18	0.00	320.11	0.00	16.64	0.00	0.00	0.00
time (sec)	N/A	0.369	0.000	4.924	0.000	1.483	0.000	0.000	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	B	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	200	227	0	73511	0	3507	0	0	0
N.S.	1	1.14	0.00	367.56	0.00	17.54	0.00	0.00	0.00
time (sec)	N/A	0.438	0.000	8.714	0.000	4.923	0.000	0.000	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	B	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	268	311	0	77709	0	4751	0	0	0
N.S.	1	1.16	0.00	289.96	0.00	17.73	0.00	0.00	0.00
time (sec)	N/A	0.524	0.000	10.432	0.000	18.704	0.000	0.000	0.000

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	157	178	316	1500	0	2035	0	0	0
N.S.	1	1.13	2.01	9.55	0.00	12.96	0.00	0.00	0.00
time (sec)	N/A	0.468	11.147	9.863	0.000	2.147	0.000	0.000	0.000

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	122	409	657	0	661	0	0	0
N.S.	1	1.02	3.41	5.48	0.00	5.51	0.00	0.00	0.00
time (sec)	N/A	0.377	6.554	4.586	0.000	0.952	0.000	0.000	0.000

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	120	410	1003	0	773	0	0	0
N.S.	1	1.01	3.45	8.43	0.00	6.50	0.00	0.00	0.00
time (sec)	N/A	0.378	4.692	5.600	0.000	0.825	0.000	0.000	0.000

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	125	135	1927	1263	0	881	0	0	0
N.S.	1	1.08	15.42	10.10	0.00	7.05	0.00	0.00	0.00
time (sec)	N/A	0.271	6.484	6.398	0.000	0.733	0.000	0.000	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	174	187	247	1862	0	1097	0	0	0
N.S.	1	1.07	1.42	10.70	0.00	6.30	0.00	0.00	0.00
time (sec)	N/A	0.481	7.765	7.589	0.000	2.145	0.000	0.000	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	236	253	234	2612	0	1579	0	0	0
N.S.	1	1.07	0.99	11.07	0.00	6.69	0.00	0.00	0.00
time (sec)	N/A	0.564	11.188	9.646	0.000	7.358	0.000	0.000	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	315	337	272	3418	0	2059	0	0	0
N.S.	1	1.07	0.86	10.85	0.00	6.54	0.00	0.00	0.00
time (sec)	N/A	0.662	16.993	11.213	0.000	24.643	0.000	0.000	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	105	105	259	0	0	0	0	0	0
N.S.	1	1.00	2.47	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.330	5.254	0.000	0.000	0.000	0.000	0.000	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	122	113	94	0	0	0	0	0	0
N.S.	1	0.93	0.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.309	0.903	0.000	0.000	0.000	0.000	0.000	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	86	83	61	0	0	0	0	0	0
N.S.	1	0.97	0.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.260	0.204	0.000	0.000	0.000	0.000	0.000	0.000

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.227	0.060	0.000	0.000	0.000	0.000	0.000	0.000

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	114	110	115	0	0	0	0	0	0
N.S.	1	0.96	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.286	1.903	0.000	0.000	0.000	0.000	0.000	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	157	163	139	0	0	0	0	0	0
N.S.	1	1.04	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.318	3.958	0.000	0.000	0.000	0.000	0.000	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	2777	0	0	0	0	0	0
N.S.	1	1.00	31.56	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.308	18.567	0.000	0.000	0.000	0.000	0.000	0.000

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	2465	0	0	0	0	0	0
N.S.	1	1.00	28.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.305	17.515	0.000	0.000	0.000	0.000	0.000	0.000

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	83	83	2137	0	0	0	0	0	0
N.S.	1	1.00	25.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.220	6.254	0.000	0.000	0.000	0.000	0.000	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	84	84	2469	0	0	0	0	0	0
N.S.	1	1.00	29.39	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.298	17.480	0.000	0.000	0.000	0.000	0.000	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	3033	0	0	0	0	0	0
N.S.	1	1.00	34.47	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.305	18.978	0.000	0.000	0.000	0.000	0.000	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	77	87	70	73	81	119	340	227
N.S.	1	0.84	0.95	0.76	0.79	0.88	1.29	3.70	2.47
time (sec)	N/A	0.292	0.584	3.185	0.181	0.266	0.794	1.627	24.339

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	55	59	49	51	59	82	271	167
N.S.	1	0.90	0.97	0.80	0.84	0.97	1.34	4.44	2.74
time (sec)	N/A	0.267	0.327	1.561	0.199	0.257	0.380	0.694	24.518

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	28	30	26	28	37	42	181	83
N.S.	1	0.93	1.00	0.87	0.93	1.23	1.40	6.03	2.77
time (sec)	N/A	0.213	0.015	0.722	0.190	0.261	0.174	0.352	20.762

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	51	69	42	45	61	0	87	72
N.S.	1	0.94	1.28	0.78	0.83	1.13	0.00	1.61	1.33
time (sec)	N/A	0.279	0.017	0.636	0.188	0.249	0.000	0.284	20.052

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	63	114	63	62	99	0	172	86
N.S.	1	0.88	1.58	0.88	0.86	1.38	0.00	2.39	1.19
time (sec)	N/A	0.263	1.176	1.473	0.183	0.253	0.000	0.298	20.234

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	209	251	233	218	4427	0	0	7402
N.S.	1	0.95	1.15	1.06	1.00	20.21	0.00	0.00	33.80
time (sec)	N/A	0.500	0.892	4.023	0.282	0.995	0.000	0.000	22.955

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	156	242	113	159	1052	0	0	1620
N.S.	1	0.94	1.46	0.68	0.96	6.34	0.00	0.00	9.76
time (sec)	N/A	0.413	0.644	4.378	0.275	0.893	0.000	0.000	24.316

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	35	21	21	139	0	114
N.S.	1	1.00	1.00	1.52	0.91	0.91	6.04	0.00	4.96
time (sec)	N/A	0.208	0.020	1.704	0.197	0.258	18.047	0.000	20.761

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	281	215	288	306	6482	0	0	11182
N.S.	1	0.95	0.73	0.98	1.04	21.97	0.00	0.00	37.91
time (sec)	N/A	0.674	0.690	3.434	0.298	1.063	0.000	0.000	23.775

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	393	375	336	374	488	10746	0	0	58699
N.S.	1	0.95	0.85	0.95	1.24	27.34	0.00	0.00	149.36
time (sec)	N/A	0.805	2.653	8.548	0.277	2.208	0.000	0.000	35.830

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	27	29	29	26	29	31
N.S.	1	1.00	1.07	1.00	1.07	1.07	0.96	1.07	1.15
time (sec)	N/A	0.234	5.725	0.832	5.460	0.257	104.479	0.827	21.065

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	226	221	245	0	0	0	0	0	0
N.S.	1	0.98	1.08	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.736	13.762	0.000	0.000	0.000	0.000	0.000	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	143	141	162	0	0	0	0	0	0
N.S.	1	0.99	1.13	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.554	6.427	0.000	0.000	0.000	0.000	0.000	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	59	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.254	0.114	0.000	0.000	0.000	0.000	0.000	0.000

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	25	22	25	27
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.96	1.09	1.17
time (sec)	N/A	0.218	4.534	0.602	3.783	0.260	12.664	0.795	20.712

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	27	27	0	27	29
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.00	1.08	1.16
time (sec)	N/A	0.225	31.508	0.622	9.322	0.257	0.000	1.040	21.969

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	27	27	24	27	29
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.96	1.08	1.16
time (sec)	N/A	0.228	3.319	0.459	5.858	0.258	19.172	1.015	21.034

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	15	18	20
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.94	1.12	1.25
time (sec)	N/A	0.185	1.738	0.479	3.951	0.258	1.725	0.520	20.775

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	27	27	24	27	29
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.96	1.08	1.16
time (sec)	N/A	0.229	2.788	0.602	6.345	0.263	73.546	0.975	21.323

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [227] had the largest ratio of [1.11765000000000003]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	4	0.89	21	0.190
2	A	5	4	0.89	21	0.190
3	A	5	4	0.91	21	0.190
4	A	5	4	0.96	19	0.211
5	A	5	4	0.96	19	0.211
6	A	7	6	1.13	21	0.286
7	A	9	8	1.20	21	0.381
8	A	10	9	1.27	21	0.429
9	A	7	6	1.29	21	0.286
10	A	7	6	1.31	21	0.286
11	A	1	1	1.00	12	0.083
12	A	5	4	0.92	21	0.190
13	A	5	4	0.89	21	0.190
14	A	5	4	0.88	21	0.190
15	A	5	4	0.89	23	0.174
16	A	5	4	0.90	23	0.174
17	A	5	4	0.89	21	0.190
18	A	5	4	0.92	21	0.190
19	A	8	7	1.12	23	0.304
20	A	10	9	1.15	23	0.391
21	A	9	8	1.14	23	0.348
22	A	7	6	1.13	23	0.261

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	7	6	1.04	23	0.261
24	A	5	4	1.10	14	0.286
25	A	5	4	0.90	23	0.174
26	A	5	4	0.89	23	0.174
27	A	5	4	0.89	23	0.174
28	A	5	4	0.93	23	0.174
29	A	6	5	0.97	23	0.217
30	A	5	4	0.98	21	0.190
31	A	6	5	0.98	21	0.238
32	A	7	6	1.15	23	0.261
33	A	8	7	1.20	23	0.304
34	A	10	9	1.20	23	0.391
35	A	8	7	1.21	23	0.304
36	A	7	6	1.20	23	0.261
37	A	6	5	1.00	14	0.357
38	A	5	4	0.96	23	0.174
39	A	6	5	1.01	23	0.217
40	A	5	4	0.92	23	0.174
41	A	8	7	0.84	23	0.304
42	A	6	5	0.94	23	0.217
43	A	6	5	0.99	21	0.238
44	A	7	6	1.07	21	0.286
45	A	9	8	1.07	23	0.348
46	A	11	10	1.12	23	0.435
47	A	12	11	1.12	23	0.478
48	A	11	10	1.12	23	0.435
49	A	9	8	1.12	23	0.348
50	A	7	6	1.21	14	0.429
51	A	6	5	1.01	23	0.217
52	A	7	6	1.02	23	0.261
53	A	8	7	1.04	23	0.304
54	A	8	7	1.00	23	0.304
55	A	8	7	0.99	23	0.304
56	A	7	6	1.03	21	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	8	7	1.12	21	0.333
58	A	12	11	1.11	23	0.478
59	A	12	11	1.11	23	0.478
60	A	14	13	1.10	23	0.565
61	A	12	11	1.10	23	0.478
62	A	10	9	1.17	23	0.391
63	A	8	7	1.22	14	0.500
64	A	7	6	1.06	23	0.261
65	A	8	7	1.01	23	0.304
66	A	9	8	1.03	23	0.348
67	A	9	8	1.03	25	0.320
68	A	8	7	0.95	25	0.280
69	A	6	5	0.97	23	0.217
70	A	9	8	0.98	23	0.348
71	A	9	8	1.06	25	0.320
72	A	12	11	1.13	25	0.440
73	A	12	11	1.12	25	0.440
74	A	10	9	1.09	25	0.360
75	A	9	8	1.03	25	0.320
76	A	8	7	0.97	16	0.438
77	A	6	5	0.97	25	0.200
78	A	7	6	0.95	25	0.240
79	A	8	7	1.05	25	0.280
80	A	10	9	0.89	25	0.360
81	A	9	8	0.89	25	0.320
82	A	7	6	0.98	23	0.261
83	A	11	10	0.98	23	0.435
84	A	11	10	1.01	25	0.400
85	A	15	14	1.04	25	0.560
86	A	14	13	1.04	25	0.520
87	A	13	12	1.02	25	0.480
88	A	11	10	0.99	25	0.400
89	A	9	8	0.98	16	0.500
90	A	7	6	0.98	25	0.240

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
91	A	8	7	0.90	25	0.280
92	A	10	9	0.90	25	0.360
93	A	7	6	1.02	25	0.240
94	A	6	5	0.97	25	0.200
95	A	4	3	1.00	23	0.130
96	A	6	5	1.00	23	0.217
97	A	7	6	1.07	25	0.240
98	A	9	8	1.14	25	0.320
99	A	9	8	1.11	25	0.320
100	A	8	7	1.10	25	0.280
101	A	7	6	1.05	25	0.240
102	A	5	4	1.00	16	0.250
103	A	4	3	1.00	25	0.120
104	A	5	4	0.97	25	0.160
105	A	6	5	1.04	25	0.200
106	A	8	7	0.94	25	0.280
107	A	7	6	0.93	25	0.240
108	A	5	4	0.97	23	0.174
109	A	7	6	0.98	23	0.261
110	A	9	8	1.06	25	0.320
111	A	12	11	1.13	25	0.440
112	A	11	10	1.11	25	0.400
113	A	10	9	1.11	25	0.360
114	A	8	7	1.03	25	0.280
115	A	6	5	0.97	16	0.312
116	A	5	4	0.97	25	0.160
117	A	6	5	0.95	25	0.200
118	A	7	6	0.96	25	0.240
119	A	9	8	0.89	25	0.320
120	A	8	7	0.95	25	0.280
121	A	6	5	1.00	23	0.217
122	A	9	8	1.06	23	0.348
123	A	11	10	1.09	25	0.400
124	A	14	13	1.12	25	0.520

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
125	A	14	13	1.09	25	0.520
126	A	11	10	1.08	25	0.400
127	A	10	9	1.06	25	0.360
128	A	8	7	1.08	16	0.438
129	A	6	5	1.02	25	0.200
130	A	7	6	0.97	25	0.240
131	A	8	7	0.93	25	0.280
132	F	0	0	N/A	0.000	N/A
133	A	8	7	0.99	23	0.304
134	A	7	6	0.98	23	0.261
135	A	5	4	1.00	21	0.190
136	A	6	5	1.00	21	0.238
137	A	5	4	1.00	23	0.174
138	A	5	4	1.00	23	0.174
139	A	5	4	1.00	23	0.174
140	A	5	4	1.00	14	0.286
141	A	5	4	1.00	23	0.174
142	A	6	5	0.98	23	0.217
143	A	7	6	1.03	23	0.261
144	A	11	11	0.84	15	0.733
145	A	9	9	0.88	15	0.600
146	A	7	7	0.84	15	0.467
147	A	1	1	1.00	13	0.077
148	A	5	5	1.05	15	0.333
149	A	7	7	0.84	15	0.467
150	A	9	9	0.91	15	0.600
151	A	11	11	0.84	15	0.733
152	A	8	8	0.96	21	0.381
153	A	6	6	0.97	21	0.286
154	A	4	4	1.00	19	0.211
155	A	4	4	1.00	19	0.211
156	A	6	5	1.00	21	0.238
157	A	7	6	0.94	21	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
158	A	6	5	0.80	21	0.238
159	A	6	5	0.89	21	0.238
160	A	6	5	1.00	21	0.238
161	A	1	1	1.00	12	0.083
162	A	3	3	1.00	21	0.143
163	A	5	5	0.97	21	0.238
164	A	7	7	0.96	21	0.333
165	A	9	8	1.00	23	0.348
166	A	8	7	1.05	23	0.304
167	A	7	6	1.15	21	0.286
168	A	5	4	1.05	21	0.190
169	A	5	4	0.90	23	0.174
170	A	5	4	0.91	23	0.174
171	A	5	4	0.90	23	0.174
172	A	5	4	0.90	23	0.174
173	A	5	4	0.91	23	0.174
174	A	5	4	1.10	14	0.286
175	A	5	4	1.21	23	0.174
176	A	6	5	1.23	23	0.217
177	A	7	6	1.08	23	0.261
178	A	5	4	1.01	14	0.286
179	A	5	4	0.98	14	0.286
180	A	8	7	1.10	23	0.304
181	A	6	5	0.96	23	0.217
182	A	4	3	1.00	21	0.143
183	A	5	4	0.96	21	0.190
184	A	5	4	0.93	23	0.174
185	A	5	4	0.93	23	0.174
186	A	5	4	0.94	23	0.174
187	A	5	4	0.96	23	0.174
188	A	4	3	1.00	23	0.130
189	A	6	5	1.00	14	0.357
190	A	8	7	1.21	23	0.304
191	A	10	9	1.21	23	0.391

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
192	A	12	11	1.21	23	0.478
193	A	8	7	1.10	23	0.304
194	A	5	4	0.97	23	0.174
195	A	5	4	0.98	21	0.190
196	A	5	4	0.95	21	0.190
197	A	5	4	0.94	23	0.174
198	A	5	4	0.93	23	0.174
199	A	5	4	0.95	23	0.174
200	A	5	4	0.98	23	0.174
201	A	5	4	0.97	23	0.174
202	A	7	6	1.21	14	0.429
203	A	10	9	1.13	23	0.391
204	A	12	11	1.14	23	0.478
205	A	14	13	1.14	23	0.565
206	A	6	5	1.05	23	0.217
207	A	6	5	1.00	23	0.217
208	A	7	6	1.08	21	0.286
209	A	5	4	0.95	21	0.190
210	A	5	4	0.94	23	0.174
211	A	5	4	0.93	23	0.174
212	A	6	5	1.07	23	0.217
213	A	6	5	1.00	23	0.217
214	A	6	5	1.05	23	0.217
215	A	8	7	1.22	14	0.500
216	A	11	10	1.19	23	0.435
217	A	14	13	1.13	23	0.565
218	A	16	15	1.13	23	0.652
219	A	10	9	1.20	14	0.643
220	A	12	12	0.62	17	0.706
221	A	10	10	0.67	17	0.588
222	A	8	8	0.80	17	0.471
223	A	6	6	1.00	17	0.353
224	A	7	7	1.06	17	0.412
225	A	11	11	0.81	17	0.647

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
226	A	15	15	0.71	17	0.882
227	A	19	19	0.63	17	1.118
228	A	17	16	1.05	25	0.640
229	A	15	14	1.06	25	0.560
230	A	13	12	1.09	23	0.522
231	A	7	6	1.12	23	0.261
232	A	12	11	1.01	25	0.440
233	A	14	13	0.97	25	0.520
234	A	9	8	0.97	25	0.320
235	A	7	6	0.96	25	0.240
236	A	6	5	0.97	25	0.200
237	A	8	7	0.97	16	0.438
238	A	6	5	1.05	25	0.200
239	A	7	6	1.02	25	0.240
240	A	11	10	1.10	25	0.400
241	A	19	18	1.05	25	0.720
242	A	17	16	1.05	25	0.640
243	A	15	14	1.06	23	0.609
244	A	13	12	1.09	23	0.522
245	A	11	10	1.02	25	0.400
246	A	13	12	1.01	25	0.480
247	A	10	9	0.89	25	0.360
248	A	8	7	0.92	25	0.280
249	A	7	6	0.98	25	0.240
250	A	9	8	0.98	16	0.500
251	A	9	8	1.03	25	0.320
252	A	7	6	1.06	25	0.240
253	A	8	7	0.98	25	0.280
254	A	10	9	1.00	16	0.562
255	A	9	8	1.00	10	0.800
256	A	7	6	1.00	10	0.600
257	A	15	14	0.95	25	0.560
258	A	10	9	1.18	25	0.360
259	A	7	6	1.12	23	0.261

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
260	A	7	6	1.10	23	0.261
261	A	12	11	1.01	25	0.440
262	A	14	13	0.98	25	0.520
263	A	8	7	1.04	25	0.280
264	A	6	5	0.98	25	0.200
265	A	5	4	1.00	25	0.160
266	A	5	4	1.00	16	0.250
267	A	6	5	1.05	25	0.200
268	A	10	9	1.09	25	0.360
269	A	11	10	1.11	25	0.400
270	A	15	14	1.09	25	0.560
271	A	9	8	1.25	25	0.320
272	A	12	11	1.08	23	0.478
273	A	12	11	1.09	23	0.478
274	A	14	13	1.00	25	0.520
275	A	16	15	0.97	25	0.600
276	A	7	6	1.04	25	0.240
277	A	6	5	0.97	25	0.200
278	A	4	3	1.00	25	0.120
279	A	6	5	0.97	16	0.312
280	A	9	8	1.02	25	0.320
281	A	11	10	1.07	25	0.400
282	A	13	12	1.08	25	0.480
283	A	14	13	1.02	25	0.520
284	A	14	13	1.01	25	0.520
285	A	14	13	1.02	23	0.565
286	A	13	12	1.01	23	0.522
287	A	16	15	0.98	25	0.600
288	A	17	16	0.95	25	0.640
289	A	7	6	1.09	25	0.240
290	A	5	4	1.00	25	0.160
291	A	5	4	0.97	25	0.160
292	A	8	7	1.08	16	0.438
293	A	10	9	1.09	25	0.360

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
294	A	12	11	1.10	25	0.440
295	A	14	13	1.11	25	0.520
296	A	9	8	1.12	16	0.500
297	A	5	4	1.00	10	0.400
298	F	0	0	N/A	0.000	N/A
299	A	7	6	1.10	23	0.261
300	A	7	6	1.10	21	0.286
301	A	7	6	1.10	21	0.286
302	A	7	6	1.10	23	0.261
303	A	7	6	1.10	23	0.261
304	A	8	7	0.98	23	0.304
305	A	6	5	0.98	23	0.217
306	A	5	4	1.00	23	0.174
307	A	5	4	1.00	14	0.286
308	A	5	4	1.00	23	0.174
309	A	5	4	1.00	23	0.174
310	A	5	4	1.00	23	0.174
311	A	6	5	0.88	21	0.238
312	A	6	5	0.94	21	0.238
313	A	5	4	0.93	19	0.211
314	A	6	5	1.32	19	0.263
315	A	6	5	1.28	21	0.238
316	A	6	5	1.29	21	0.238
317	A	7	6	0.94	21	0.286
318	A	7	6	1.00	21	0.286
319	A	7	6	1.12	21	0.286
320	A	1	1	1.00	12	0.083
321	A	6	5	1.37	21	0.238
322	A	7	6	1.24	21	0.286
323	A	8	7	0.96	21	0.333
324	A	6	5	0.91	23	0.217
325	A	6	5	0.92	23	0.217
326	A	6	5	0.96	21	0.238

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
327	A	6	5	1.04	21	0.238
328	A	6	5	1.12	23	0.217
329	A	6	5	1.35	23	0.217
330	A	6	5	0.95	23	0.217
331	A	6	5	0.97	23	0.217
332	A	6	5	1.02	23	0.217
333	A	5	4	1.10	14	0.286
334	A	6	5	1.11	23	0.217
335	A	6	5	1.09	23	0.217
336	A	6	5	1.02	23	0.217
337	A	6	5	0.91	23	0.217
338	A	6	5	1.00	23	0.217
339	A	4	3	1.00	21	0.143
340	A	6	5	1.13	21	0.238
341	A	6	5	1.04	23	0.217
342	A	6	5	1.09	23	0.217
343	A	10	9	1.18	23	0.391
344	A	8	7	1.19	23	0.304
345	A	7	6	1.15	23	0.261
346	A	6	5	1.00	14	0.357
347	A	9	8	1.24	23	0.348
348	A	10	9	1.27	23	0.391
349	A	12	11	1.22	23	0.478
350	A	6	5	0.91	23	0.217
351	A	6	5	0.94	23	0.217
352	A	6	5	0.90	21	0.238
353	A	6	5	1.01	21	0.238
354	A	6	5	0.97	23	0.217
355	A	6	5	1.06	23	0.217
356	A	9	8	1.10	23	0.348
357	A	8	7	1.16	23	0.304
358	A	8	7	1.12	23	0.304
359	A	7	6	1.21	14	0.429
360	A	9	8	1.17	23	0.348

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
361	A	11	10	1.14	23	0.435
362	A	13	12	1.12	23	0.522
363	A	6	5	0.94	23	0.217
364	A	6	5	0.93	23	0.217
365	A	6	5	0.93	21	0.238
366	A	6	5	0.98	21	0.238
367	A	6	5	0.98	23	0.217
368	A	6	5	1.01	23	0.217
369	A	10	9	1.16	23	0.391
370	A	10	9	1.12	23	0.391
371	A	9	8	1.17	23	0.348
372	A	8	7	1.22	14	0.500
373	A	10	9	1.17	23	0.391
374	A	12	11	1.14	23	0.478
375	A	14	13	1.12	23	0.565
376	A	6	5	0.96	25	0.200
377	A	9	8	0.99	25	0.320
378	A	7	6	1.04	23	0.261
379	A	8	7	1.03	23	0.304
380	A	9	8	1.04	25	0.320
381	A	12	11	1.13	25	0.440
382	A	13	12	1.05	25	0.480
383	A	12	11	1.02	25	0.440
384	A	10	9	0.97	25	0.360
385	A	8	7	0.97	16	0.438
386	A	9	8	0.97	25	0.320
387	A	9	8	0.98	25	0.320
388	A	11	10	1.05	25	0.400
389	A	6	5	0.95	25	0.200
390	A	10	9	0.98	25	0.360
391	A	8	7	1.01	23	0.304
392	A	10	9	1.00	23	0.391
393	A	9	8	1.04	25	0.320
394	A	12	11	1.08	25	0.440

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
395	A	15	14	1.04	25	0.560
396	A	14	13	1.03	25	0.520
397	A	12	11	0.99	25	0.440
398	A	9	8	0.98	16	0.500
399	A	11	10	0.95	25	0.400
400	A	10	9	0.98	25	0.360
401	A	12	11	1.00	25	0.440
402	A	6	5	0.98	25	0.200
403	A	8	7	1.05	25	0.280
404	A	6	5	1.00	23	0.217
405	A	8	7	1.03	23	0.304
406	A	9	8	1.10	25	0.320
407	A	12	11	1.17	25	0.440
408	A	11	10	1.05	25	0.400
409	A	10	9	1.02	25	0.360
410	A	9	8	0.98	25	0.320
411	A	5	4	1.00	16	0.250
412	A	9	8	0.97	25	0.320
413	A	10	9	1.06	25	0.360
414	A	11	10	1.09	25	0.400
415	A	6	5	1.00	25	0.200
416	A	8	7	1.03	25	0.280
417	A	7	6	1.04	23	0.261
418	A	10	9	1.13	23	0.391
419	A	11	10	1.13	25	0.400
420	A	14	13	1.16	25	0.520
421	A	11	10	1.03	25	0.400
422	A	10	9	1.04	25	0.360
423	A	7	6	0.97	25	0.240
424	A	6	5	0.97	16	0.312
425	A	9	8	1.04	25	0.320
426	A	10	9	1.06	25	0.360
427	A	11	10	1.05	25	0.400
428	A	6	5	0.99	25	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
429	A	9	8	1.04	25	0.320
430	A	8	7	1.05	23	0.304
431	A	12	11	1.18	23	0.478
432	A	13	12	1.14	25	0.480
433	A	16	15	1.16	25	0.600
434	A	13	12	1.13	25	0.480
435	A	9	8	1.02	25	0.320
436	A	9	8	1.01	25	0.320
437	A	8	7	1.08	16	0.438
438	A	10	9	1.07	25	0.360
439	A	12	11	1.07	25	0.440
440	A	13	12	1.07	25	0.480
441	A	6	5	1.00	25	0.200
442	A	6	5	0.93	23	0.217
443	A	7	6	0.97	23	0.261
444	A	5	4	1.00	21	0.190
445	A	8	7	0.96	21	0.333
446	A	9	8	1.04	23	0.348
447	A	6	5	1.00	23	0.217
448	A	6	5	1.00	23	0.217
449	A	5	4	1.00	14	0.286
450	A	6	5	1.00	23	0.217
451	A	6	5	1.00	23	0.217
452	A	5	4	0.84	21	0.190
453	A	5	4	0.90	21	0.190
454	A	5	4	0.93	19	0.211
455	A	5	4	0.94	19	0.211
456	A	8	7	0.88	21	0.333
457	A	5	4	0.95	23	0.174
458	A	13	12	0.94	23	0.522
459	A	4	3	1.00	21	0.143
460	A	5	4	0.95	21	0.190
461	A	5	4	0.95	23	0.174

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
462	N/A	2	0	1.00	27	0.000
463	A	5	4	0.98	25	0.160
464	A	6	5	0.99	25	0.200
465	A	7	6	1.00	23	0.261
466	N/A	2	0	1.00	23	0.000
467	N/A	2	0	1.00	25	0.000
468	N/A	2	0	1.00	25	0.000
469	N/A	2	0	1.00	16	0.000
470	N/A	2	0	1.00	25	0.000

CHAPTER 3

Listing of integrals

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3.170 $\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$ 1312
3.171 $\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$ 1317
3.172 $\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$ 1323
3.173 $\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$ 1329
3.174 $\int \left(a+b\sec^2(e+fx)\right)^2\,dx$ 1334
3.175 $\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$ 1339
3.176 $\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$ 1344
3.177 $\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$ 1350
3.178 $\int \left(a+b\sec^2(c+dx)\right)^3\,dx$ 1356
3.179 $\int \left(a+b\sec^2(c+dx)\right)^4\,dx$ 1361
3.180 $\int \frac{\sec^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$ 1367
3.181 $\int \frac{\sec^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$ 1375
3.182 $\int \frac{\sec(e+fx)}{a+b\sec^2(e+fx)}\,dx$ 1381
3.183 $\int \frac{\cos(e+fx)}{a+b\sec^2(e+fx)}\,dx$ 1386
3.184 $\int \frac{\cos^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$ 1391
3.185 $\int \frac{\cos^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$ 1396
3.186 $\int \frac{\sec^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$ 1402
3.187 $\int \frac{\sec^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$ 1408
3.188 $\int \frac{\sec^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$ 1413
3.189 $\int \frac{1}{a+b\sec^2(e+fx)}\,dx$ 1418
3.190 $\int \frac{\cos^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$ 1424
3.191 $\int \frac{\cos^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$ 1430
3.192 $\int \frac{\cos^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$ 1438
3.193 $\int \frac{\sec^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$ 1447
3.194 $\int \frac{\sec^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$ 1455
3.195 $\int \frac{\sec(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$ 1460
3.196 $\int \frac{\cos(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$ 1466
3.197 $\int \frac{\cos^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$ 1472
3.198 $\int \frac{\cos^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$ 1478
3.199 $\int \frac{\sec^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$ 1484

3.200 $\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$ . . . 1490
3.201 $\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$ . . . 1496
3.202 $\int \frac{1}{(a+b\sec^2(e+fx))^2}\,dx$ . . . 1502
3.203 $\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$ . . . 1509
3.204 $\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$ . . . 1517
3.205 $\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$ . . . 1526
3.206 $\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$ . . . 1536
3.207 $\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$ . . . 1542
3.208 $\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$ . . . 1549
3.209 $\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$ . . . 1557
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3.211 $\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$ . . . 1571
3.212 $\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$ . . . 1579
3.213 $\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$ . . . 1586
3.214 $\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$ . . . 1593
3.215 $\int \frac{1}{(a+b\sec^2(e+fx))^3}\,dx$ . . . 1600
3.216 $\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$ . . . 1608
3.217 $\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$ . . . 1617
3.218 $\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$ . . . 1628
3.219 $\int \frac{1}{(a+b\sec^2(c+dx))^4}\,dx$ . . . 1639
3.220 $\int (a-a\sec^2(c+dx))^{7/2}\,dx$ . . . 1648
3.221 $\int (a-a\sec^2(c+dx))^{5/2}\,dx$ . . . 1654
3.222 $\int (a-a\sec^2(c+dx))^{3/2}\,dx$ . . . 1660
3.223 $\int \sqrt{a-a\sec^2(c+dx)}\,dx$ . . . 1666
3.224 $\int \frac{1}{\sqrt{a-a\sec^2(c+dx)}}\,dx$ . . . 1671
3.225 $\int \frac{1}{(a-a\sec^2(c+dx))^{3/2}}\,dx$ . . . 1676
3.226 $\int \frac{1}{(a-a\sec^2(c+dx))^{5/2}}\,dx$ . . . 1682
3.227 $\int \frac{1}{(a-a\sec^2(c+dx))^{7/2}}\,dx$ . . . 1688
3.228 $\int \sec^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$ . . . 1695
3.229 $\int \sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$ . . . 1705
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3.232 $\int \cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$ . . . 1728
3.233 $\int \cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$ . . . 1736

3.234 $\int \sec^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$ . . . 1744
3.235 $\int \sec^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$ . . . 1752
3.236 $\int \sec^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$ . . . 1759
3.237 $\int \sqrt{a+b\sec^2(e+fx)}\,dx$ . . . 1765
3.238 $\int \cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$ . . . 1772
3.239 $\int \cos^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$ . . . 1778
3.240 $\int \cos^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$ . . . 1785
3.241 $\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$ . . . 1794
3.242 $\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$ . . . 1805
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3.244 $\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$ . . . 1824
3.245 $\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$ . . . 1832
3.246 $\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$ . . . 1839
3.247 $\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$ . . . 1847
3.248 $\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$ . . . 1855
3.249 $\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$ . . . 1863
3.250 $\int \left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$ . . . 1869
3.251 $\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$ . . . 1876
3.252 $\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$ . . . 1883
3.253 $\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$ . . . 1890
3.254 $\int \left(a+b\sec^2(c+dx)\right)^{5/2}\,dx$ . . . 1898
3.255 $\int \left(1+\sec^2(x)\right)^{3/2}\,dx$ . . . 1906
3.256 $\int \sqrt{1+\sec^2(x)}\,dx$ . . . 1912
3.257 $\int \frac{\sec^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$ . . . 1919
3.258 $\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$ . . . 1929
3.259 $\int \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$ . . . 1936
3.260 $\int \frac{\cos(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$ . . . 1942
3.261 $\int \frac{\cos^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$ . . . 1948
3.262 $\int \frac{\cos^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$ . . . 1956
3.263 $\int \frac{\sec^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$ . . . 1964
3.264 $\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$ . . . 1971
3.265 $\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$ . . . 1978
3.266 $\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx$ . . . 1983
3.267 $\int \frac{\cos^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$ . . . 1989
3.268 $\int \frac{\cos^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$ . . . 1995

3.269 $\int \frac{\cos^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$ . . . 2003
3.270 $\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$ . . . 2012
3.271 $\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$ . . . 2022
3.272 $\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$ . . . 2030
3.273 $\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$ . . . 2039
3.274 $\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$ . . . 2047
3.275 $\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$ . . . 2055
3.276 $\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$ . . . 2064
3.277 $\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$ . . . 2071
3.278 $\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$ . . . 2077
3.279 $\int \frac{1}{(a+b\sec^2(e+fx))^{3/2}}\,dx$ . . . 2082
3.280 $\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$ . . . 2089
3.281 $\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$ . . . 2097
3.282 $\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$ . . . 2105
3.283 $\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$ . . . 2114
3.284 $\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$ . . . 2123
3.285 $\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$ . . . 2133
3.286 $\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$ . . . 2142
3.287 $\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$ . . . 2150
3.288 $\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$ . . . 2161
3.289 $\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$ . . . 2172
3.290 $\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$ . . . 2179
3.291 $\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$ . . . 2184
3.292 $\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx$ . . . 2189
3.293 $\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$ . . . 2197
3.294 $\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$ . . . 2205
3.295 $\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$ . . . 2214
3.296 $\int \frac{1}{(a+b\sec^2(c+dx))^{7/2}}\,dx$ . . . 2224
3.297 $\int \frac{1}{\sqrt{1+\sec^2(x)}}\,dx$ . . . 2232
3.298 $\int (d\sec(e+fx))^m\,(a+b\sec^2(e+fx))^p\,dx$ . . . 2238
3.299 $\int \sec^3(e+fx)\,(a+b\sec^2(e+fx))^p\,dx$ . . . 2242

3.300 $\int \sec(e+fx)\,(a+b\sec^2(e+fx))^p\,dx$ . . . 2248
3.301 $\int \cos(e+fx)\,(a+b\sec^2(e+fx))^p\,dx$ . . . 2254
3.302 $\int \cos^3(e+fx)\,(a+b\sec^2(e+fx))^p\,dx$ . . . 2260
3.303 $\int \cos^5(e+fx)\,(a+b\sec^2(e+fx))^p\,dx$ . . . 2266
3.304 $\int \sec^6(e+fx)\,(a+b\sec^2(e+fx))^p\,dx$ . . . 2272
3.305 $\int \sec^4(e+fx)\,(a+b\sec^2(e+fx))^p\,dx$ . . . 2278
3.306 $\int \sec^2(e+fx)\,(a+b\sec^2(e+fx))^p\,dx$ . . . 2283
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3.309 $\int \cos^4(e+fx)\,(a+b\sec^2(e+fx))^p\,dx$ . . . 2298
3.310 $\int \cos^6(e+fx)\,(a+b\sec^2(e+fx))^p\,dx$ . . . 2303
3.311 $\int (a+b\sec^2(e+fx))\tan^5(e+fx)\,dx$ . . . 2308
3.312 $\int (a+b\sec^2(e+fx))\tan^3(e+fx)\,dx$ . . . 2314
3.313 $\int (a+b\sec^2(e+fx))\tan(e+fx)\,dx$ . . . 2320
3.314 $\int \cot(e+fx)\,(a+b\sec^2(e+fx))\,dx$ . . . 2325
3.315 $\int \cot^3(e+fx)\,(a+b\sec^2(e+fx))\,dx$ . . . 2330
3.316 $\int \cot^5(e+fx)\,(a+b\sec^2(e+fx))\,dx$ . . . 2335
3.317 $\int (a+b\sec^2(e+fx))\tan^6(e+fx)\,dx$ . . . 2341
3.318 $\int (a+b\sec^2(e+fx))\tan^4(e+fx)\,dx$ . . . 2347
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3.320 $\int (a+b\sec^2(e+fx))\,dx$ . . . 2359
3.321 $\int \cot^2(e+fx)\,(a+b\sec^2(e+fx))\,dx$ . . . 2363
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3.323 $\int \cot^6(e+fx)\,(a+b\sec^2(e+fx))\,dx$ . . . 2374
3.324 $\int (a+b\sec^2(e+fx))^2\tan^5(e+fx)\,dx$ . . . 2380
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3.328 $\int \cot^3(e+fx)\,(a+b\sec^2(e+fx))^2\,dx$ . . . 2405
3.329 $\int \cot^5(e+fx)\,(a+b\sec^2(e+fx))^2\,dx$ . . . 2411
3.330 $\int (a+b\sec^2(e+fx))^2\tan^6(e+fx)\,dx$ . . . 2417
3.331 $\int (a+b\sec^2(e+fx))^2\tan^4(e+fx)\,dx$ . . . 2423
3.332 $\int (a+b\sec^2(e+fx))^2\tan^2(e+fx)\,dx$ . . . 2429
3.333 $\int (a+b\sec^2(e+fx))^2\,dx$ . . . 2435
3.334 $\int \cot^2(e+fx)\,(a+b\sec^2(e+fx))^2\,dx$ . . . 2440
3.335 $\int \cot^4(e+fx)\,(a+b\sec^2(e+fx))^2\,dx$ . . . 2445
3.336 $\int \cot^6(e+fx)\,(a+b\sec^2(e+fx))^2\,dx$ . . . 2451
3.337 $\int \frac{\tan^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$ . . . 2457
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3.341 $\int \frac{\cot^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$ . . . 2478
3.342 $\int \frac{\cot^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$ . . . 2484
3.343 $\int \frac{\tan^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$ . . . 2490
3.344 $\int \frac{\tan^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$ . . . 2499
3.345 $\int \frac{\tan^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$ . . . 2506
3.346 $\int \frac{1}{a+b\sec^2(e+fx)}\,dx$ . . . 2513
3.347 $\int \frac{\cot^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$ . . . 2519
3.348 $\int \frac{\cot^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$ . . . 2526
3.349 $\int \frac{\cot^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$ . . . 2534
3.350 $\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$ . . . 2543
3.351 $\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$ . . . 2549
3.352 $\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$ . . . 2554
3.353 $\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$ . . . 2559
3.354 $\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$ . . . 2565
3.355 $\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$ . . . 2572
3.356 $\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$ . . . 2579
3.357 $\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$ . . . 2587
3.358 $\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$ . . . 2594
3.359 $\int \frac{1}{(a+b\sec^2(e+fx))^2}\,dx$ . . . 2601
3.360 $\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$ . . . 2608
3.361 $\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$ . . . 2616
3.362 $\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$ . . . 2625
3.363 $\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$ . . . 2635
3.364 $\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$ . . . 2641
3.365 $\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$ . . . 2647
3.366 $\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$ . . . 2653
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3.369 $\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$ . . . 2675
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3.372 $\int \frac{1}{(a+b\sec^2(e+fx))^3}\,dx$ . . . 2702
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3.375 $\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$ . . . 2729
3.376 $\int \sqrt{a+b\sec^2(e+fx)}\tan^5(e+fx)\,dx$ . . . 2740
3.377 $\int \sqrt{a+b\sec^2(e+fx)}\tan^3(e+fx)\,dx$ . . . 2747
3.378 $\int \sqrt{a+b\sec^2(e+fx)}\tan(e+fx)\,dx$ . . . 2754
3.379 $\int \cot(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$ . . . 2760
3.380 $\int \cot^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$ . . . 2767
3.381 $\int \cot^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$ . . . 2775
3.382 $\int \sqrt{a+b\sec^2(e+fx)}\tan^6(e+fx)\,dx$ . . . 2784
3.383 $\int \sqrt{a+b\sec^2(e+fx)}\tan^4(e+fx)\,dx$ . . . 2794
3.384 $\int \sqrt{a+b\sec^2(e+fx)}\tan^2(e+fx)\,dx$ . . . 2803
3.385 $\int \sqrt{a+b\sec^2(e+fx)}\,dx$ . . . 2811
3.386 $\int \cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$ . . . 2818
3.387 $\int \cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$ . . . 2825
3.388 $\int \cot^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$ . . . 2833
3.389 $\int (a+b\sec^2(e+fx))^{3/2}\tan^5(e+fx)\,dx$ . . . 2841
3.390 $\int (a+b\sec^2(e+fx))^{3/2}\tan^3(e+fx)\,dx$ . . . 2848
3.391 $\int (a+b\sec^2(e+fx))^{3/2}\tan(e+fx)\,dx$ . . . 2855
3.392 $\int \cot(e+fx)\,(a+b\sec^2(e+fx))^{3/2}\,dx$ . . . 2862
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3.395 $\int (a+b\sec^2(e+fx))^{3/2}\tan^6(e+fx)\,dx$ . . . 2885
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3.400 $\int \cot^4(e+fx)\,(a+b\sec^2(e+fx))^{3/2}\,dx$ . . . 2927
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3.403 $\int \frac{\tan^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$ . . . 2948
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3.405 $\int \frac{\cot(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$ . . . 2960
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3.408 $\int \frac{\tan^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$ . . . 2981
3.409 $\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$ . . . 2989
3.410 $\int \frac{\tan^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$ . . . 2997
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3.412 $\int \frac{\cot^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$ . . . 3010
3.413 $\int \frac{\cot^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$ . . . 3017
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3.1 $\int (a + b\sec^2(e+fx))\sin^7(e+fx)\,dx$

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3.1.1 Optimal result

Integrand size = 21, antiderivative size = 83

$$\int (a + b\sec^2(e+fx))\sin^7(e+fx)\,dx = -\frac{(a-3b)\cos(e+fx)}{f} + \frac{(a-b)\cos^3(e+fx)}{f} - \frac{(3a-b)\cos^5(e+fx)}{5f} + \frac{a\cos^7(e+fx)}{7f} + \frac{b\sec(e+fx)}{f}$$

output
```
-(a-3*b)*cos(f*x+e)/f+(a-b)*cos(f*x+e)^3/f-1/5*(3*a-b)*cos(f*x+e)^5/f+1/7*
a*cos(f*x+e)^7/f+b*sec(f*x+e)/f
```

3.1.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.45

$$\int (a + b\sec^2(e+fx))\sin^7(e+fx)\,dx = -\frac{35a\cos(e+fx)}{64f} + \frac{19b\cos(e+fx)}{8f} + \frac{7a\cos(3(e+fx))}{64f} - \frac{3b\cos(3(e+fx))}{16f} - \frac{7a\cos(5(e+fx))}{320f} + \frac{b\cos(5(e+fx))}{80f} + \frac{a\cos(7(e+fx))}{448f} + \frac{b\sec(e+fx)}{f}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^7,x]
```

output
```
(-35*a*Cos[e + f*x])/(64*f) + (19*b*Cos[e + f*x])/(8*f) + (7*a*Cos[3*(e +
f*x)])/(64*f) - (3*b*Cos[3*(e + f*x)])/(16*f) - (7*a*Cos[5*(e + f*x)])/(32
0*f) + (b*Cos[5*(e + f*x)])/(80*f) + (a*Cos[7*(e + f*x)])/(448*f) + (b*Sec
[e + f*x])/f
```

3.1.3 Rubi [A] (verified)

Time $= 0.24$ (sec) , antiderivative size $= 74$, normalized size of antiderivative $= 0.89$, number of steps used $= 5$, number of rules used $= 4$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used $= \{3042, 4621, 355, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^7(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \sin(e+fx)^7\left(a+b\sec(e+fx)^2\right)dx$$

$$\downarrow 4621$$

$$-\frac{\int \left(1-\cos^2(e+fx)\right)^3\left(a\cos^2(e+fx)+b\right)\sec^2(e+fx)d\cos(e+fx)}{f}$$

$$\downarrow 355$$

$$-\frac{\int \left(-a\cos^6(e+fx)+(3a-b)\cos^4(e+fx)-3(a-b)\cos^2(e+fx)+b\sec^2(e+fx)+a\left(1-\frac{3b}{a}\right)\right)d\cos(e+fx)}{f}$$

$$\downarrow 2009$$

$$-\frac{\frac{1}{5}(3a-b)\cos^5(e+fx)-(a-b)\cos^3(e+fx)+(a-3b)\cos(e+fx)-\frac{1}{7}a\cos^7(e+fx)-b\sec(e+fx)}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^7,x]
```

output
```
-(((a - 3*b)*Cos[e + f*x] - (a - b)*Cos[e + f*x]^3 + ((3*a - b)*Cos[e + f*
x]^5)/5 - (a*Cos[e + f*x]^7)/7 - b*Sec[e + f*x])/f)
```

3.1.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

3.1.4 Maple [A] (verified)

Time = 4.14 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.18

method	result
parallelrisch	$\frac{(-980a+4900b)\cos(2fx+2e)+(196a-392b)\cos(4fx+4e)+(-44a+28b)\cos(6fx+6e)+5a\cos(8fx+8e)+(-2048a+143}{4480f\cos(fx+e)}$
derivativedivides	$\frac{-\frac{a\left(\frac{16}{5}+\sin(fx+e)^6+\frac{6\sin(fx+e)^4}{5}+\frac{8\sin(fx+e)^2}{5}\right)\cos(fx+e)}{7}+b\left(\frac{\sin(fx+e)^8}{\cos(fx+e)}+\left(\frac{16}{5}+\sin(fx+e)^6+\frac{6\sin(fx+e)^4}{5}+\frac{8\sin(fx+e}{5}\right.\right.}{f}$
default	$\frac{-\frac{a\left(\frac{16}{5}+\sin(fx+e)^6+\frac{6\sin(fx+e)^4}{5}+\frac{8\sin(fx+e)^2}{5}\right)\cos(fx+e)}{7}+b\left(\frac{\sin(fx+e)^8}{\cos(fx+e)}+\left(\frac{16}{5}+\sin(fx+e)^6+\frac{6\sin(fx+e)^4}{5}+\frac{8\sin(fx+e}{5}\right.\right.}{f}$
parts	$-\frac{a\left(\frac{16}{5}+\sin(fx+e)^6+\frac{6\sin(fx+e)^4}{5}+\frac{8\sin(fx+e)^2}{5}\right)\cos(fx+e)}{7f}+\frac{b\left(\frac{\sin(fx+e)^8}{\cos(fx+e)}+\left(\frac{16}{5}+\sin(fx+e)^6+\frac{6\sin(fx+e)^4}{5}+\frac{8\sin}{f}\right.\right.}$
norman	$\frac{\frac{32a-224b}{35f}-\frac{32(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8}{f}+\frac{6(32a-224b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{35f}+\frac{2(32a-224b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{5f}+\frac{2(32a-224b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{5f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)^7}$
risch	$\frac{19\,e^{i(fx+e)}b}{16f}-\frac{35\,e^{i(fx+e)}a}{128f}+\frac{19\,e^{-i(fx+e)}b}{16f}-\frac{35\,e^{-i(fx+e)}a}{128f}+\frac{2be^{i(fx+e)}}{f\left(e^{2i(fx+e)}+1\right)}+\frac{a\cos(7fx+7e)}{448f}-\frac{7\cos(5f}{32}$

input
```
int((a+b*sec(f*x+e)^2)*sin(f*x+e)^7,x,method=_RETURNVERBOSE)
```

output
```
1/4480*((-980*a+4900*b)*cos(2*f*x+2*e)+(196*a-392*b)*cos(4*f*x+4*e)+(-44*a
+28*b)*cos(6*f*x+6*e)+5*a*cos(8*f*x+8*e)+(-2048*a+14336*b)*cos(f*x+e)-1225
*a+9800*b)/f/cos(f*x+e)
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

$$\int \left(a+b\sec^2(e+fx)\right)\sin^7(e+fx)\,dx$$
$$=\frac{5\,a\cos\left(fx+e\right)^8-7\left(3\,a-b\right)\cos\left(fx+e\right)^6+35\left(a-b\right)\cos\left(fx+e\right)^4-35\left(a-3\,b\right)\cos\left(fx+e\right)^2+35\,b}{35\,f\cos\left(fx+e\right)}$$

input
```
integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^7,x, algorithm="fricas")
```

output
```
1/35*(5*a*cos(f*x + e)^8 - 7*(3*a - b)*cos(f*x + e)^6 + 35*(a - b)*cos(f*x
 + e)^4 - 35*(a - 3*b)*cos(f*x + e)^2 + 35*b)/(f*cos(f*x + e))
```

3.1.6 Sympy [F(-1)]

Timed out.

$$\int (a + b\sec^2(e + fx))\sin^7(e + fx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e)**2)*sin(f*x+e)**7,x)
```

output
```
Timed out
```

3.1.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.88

$$\int (a + b\sec^2(e + fx))\sin^7(e + fx)\,dx$$
$$= \frac{5\,a\cos(fx+e)^7 - 7\,(3\,a-b)\cos(fx+e)^5 + 35\,(a-b)\cos(fx+e)^3 - 35\,(a-3\,b)\cos(fx+e) + \frac{35\,b}{\cos(fx+}}{35\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^7,x, algorithm="maxima")
```

output
```
1/35*(5*a*cos(f*x + e)^7 - 7*(3*a - b)*cos(f*x + e)^5 + 35*(a - b)*cos(f*x
 + e)^3 - 35*(a - 3*b)*cos(f*x + e) + 35*b/cos(f*x + e))/f
```

3.1.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. 2(79) = 158.

Time = 0.32 (sec) , antiderivative size = 266, normalized size of antiderivative = 3.20

$$\int (a + b\sec^2(e + fx))\sin^7(e + fx)\,dx$$
$$= \frac{2\left(\frac{35\,b}{\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1} + \frac{16\,a-77\,b-\frac{112\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{504\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{336\,a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{1337\,b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{560\,a(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}+\frac{1}{}}{\left(\frac{\cos(fx+e)-1}{\cos(fx+e)+1}-1\right)^7}\right.}{35\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^7,x, algorithm="giac")
```

output
```
2/35*(35*b/((cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1) + (16*a - 77*b - 11
2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 504*b*(cos(f*x + e) - 1)/(cos(
f*x + e) + 1) + 336*a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 1337*b*(
cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 560*a*(cos(f*x + e) - 1)^3/(cos
(f*x + e) + 1)^3 + 1680*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 1015
*b*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 280*b*(cos(f*x + e) - 1)^5/
(cos(f*x + e) + 1)^5 - 35*b*(cos(f*x + e) - 1)^6/(cos(f*x + e) + 1)^6)/((c
os(f*x + e) - 1)/(cos(f*x + e) + 1) - 1)^7)/f
```

3.1.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.84

$$\int \left(a+b\sec^2(e+fx)\right)\sin^7(e+fx)\,dx$$
$$=\frac{\frac{a\cos(e+fx)^7}{7}-\cos\left(e+f\,x\right)\,\left(a-3\,b\right)-\cos\left(e+f\,x\right)^5\,\left(\frac{3\,a}{5}-\frac{b}{5}\right)+\frac{b}{\cos(e+f\,x)}+\cos\left(e+f\,x\right)^3\,\left(a-b\right)}{f}$$

input
```
int(sin(e + f*x)^7*(a + b/cos(e + f*x)^2),x)
```

output
```
((a*cos(e + f*x)^7)/7 - cos(e + f*x)*(a - 3*b) - cos(e + f*x)^5*((3*a)/5 -
 b/5) + b/cos(e + f*x) + cos(e + f*x)^3*(a - b))/f
```

3.2 $\int (a + b\sec^2(e+fx))\sin^5(e+fx)\,dx$

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3.2.1 Optimal result

Integrand size = 21, antiderivative size = 66

$$\int (a + b\sec^2(e+fx))\sin^5(e+fx)\,dx = -\frac{(a-2b)\cos(e+fx)}{f} + \frac{(2a-b)\cos^3(e+fx)}{3f} - \frac{a\cos^5(e+fx)}{5f} + \frac{b\sec(e+fx)}{f}$$

output

```
-(a-2*b)*cos(f*x+e)/f+1/3*(2*a-b)*cos(f*x+e)^3/f-1/5*a*cos(f*x+e)^5/f+b*se
c(f*x+e)/f
```

3.2.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.33

$$\int (a + b\sec^2(e+fx))\sin^5(e+fx)\,dx = -\frac{5a\cos(e+fx)}{8f} + \frac{7b\cos(e+fx)}{4f} + \frac{5a\cos(3(e+fx))}{48f} - \frac{b\cos(3(e+fx))}{12f} - \frac{a\cos(5(e+fx))}{80f} + \frac{b\sec(e+fx)}{f}$$

input

```
Integrate[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^5,x]
```

output
```
(-5*a*Cos[e + f*x])/(8*f) + (7*b*Cos[e + f*x])/(4*f) + (5*a*Cos[3*(e + f*x
)])/(48*f) - (b*Cos[3*(e + f*x)])/(12*f) - (a*Cos[5*(e + f*x)])/(80*f) + (
b*Sec[e + f*x])/f
```

3.2.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4621, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \sin(e+fx)^5\left(a+b\sec(e+fx)^2\right)dx$$

$$\downarrow 4621$$

$$-\frac{\int \left(1-\cos^2(e+fx)\right)^2\left(a\cos^2(e+fx)+b\right)\sec^2(e+fx)d\cos(e+fx)}{f}$$

$$\downarrow 355$$

$$-\frac{\int \left(a\cos^4(e+fx)-(2a-b)\cos^2(e+fx)+b\sec^2(e+fx)+a\left(1-\frac{2b}{a}\right)\right)d\cos(e+fx)}{f}$$

$$\downarrow 2009$$

$$-\frac{-\frac{1}{3}(2a-b)\cos^3(e+fx)+(a-2b)\cos(e+fx)+\frac{1}{5}a\cos^5(e+fx)-b\sec(e+fx)}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^5,x]
```

output
```
-(((a - 2*b)*Cos[e + f*x] - ((2*a - b)*Cos[e + f*x]^3)/3 + (a*Cos[e + f*x]
^5)/5 - b*Sec[e + f*x])/f)
```

3.2.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

3.2.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.23

method	result
parallelrisch	$\frac{(-125a+400b)\cos(2fx+2e)+(22a-20b)\cos(4fx+4e)-3\cos(6fx+6e)a+(-256a+1280b)\cos(fx+e)-150a+900b}{480f\cos(fx+e)}$
derivativedivides	$\frac{-\frac{a\left(\frac{8}{3}+\sin(fx+e)^4+\frac{4\sin(fx+e)^2}{3}\right)\cos(fx+e)}{5}+b\left(\frac{\sin(fx+e)^6}{\cos(fx+e)}+\left(\frac{8}{3}+\sin(fx+e)^4+\frac{4\sin(fx+e)^2}{3}\right)\cos(fx+e)\right)}{f}$
default	$\frac{-\frac{a\left(\frac{8}{3}+\sin(fx+e)^4+\frac{4\sin(fx+e)^2}{3}\right)\cos(fx+e)}{5}+b\left(\frac{\sin(fx+e)^6}{\cos(fx+e)}+\left(\frac{8}{3}+\sin(fx+e)^4+\frac{4\sin(fx+e)^2}{3}\right)\cos(fx+e)\right)}{f}$
parts	$-\frac{a\left(\frac{8}{3}+\sin(fx+e)^4+\frac{4\sin(fx+e)^2}{3}\right)\cos(fx+e)}{5f}+\frac{b\left(\frac{\sin(fx+e)^6}{\cos(fx+e)}+\left(\frac{8}{3}+\sin(fx+e)^4+\frac{4\sin(fx+e)^2}{3}\right)\cos(fx+e)\right)}{f}$
norman	$\frac{\frac{16a-80b}{15f}-\frac{32(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{3f}+\frac{4(16a-80b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{15f}+\frac{(16a-80b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{3f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)^5}$
risch	$-\frac{5e^{i(fx+e)}a}{16f}+\frac{7e^{i(fx+e)}b}{8f}-\frac{5e^{-i(fx+e)}a}{16f}+\frac{7e^{-i(fx+e)}b}{8f}+\frac{2be^{i(fx+e)}}{f\left(e^{2i(fx+e)}+1\right)}-\frac{\cos(5fx+5e)a}{80f}+\frac{5\cos(3fx+$

input
```
int((a+b*sec(f*x+e)^2)*sin(f*x+e)^5,x,method=_RETURNVERBOSE)
```

output
```
1/480*((-125*a+400*b)*cos(2*f*x+2*e)+(22*a-20*b)*cos(4*f*x+4*e)-3*cos(6*f*
x+6*e)*a+(-256*a+1280*b)*cos(f*x+e)-150*a+900*b)/f/cos(f*x+e)
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.91

$$\int \left(a + b\sec^2(e+fx)\right)\sin^5(e+fx)\,dx$$
$$= -\frac{3\,a\cos\left(fx+e\right)^6 - 5\left(2\,a-b\right)\cos\left(fx+e\right)^4 + 15\left(a-2\,b\right)\cos\left(fx+e\right)^2 - 15\,b}{15\,f\cos\left(fx+e\right)}$$

input
```
integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^5,x, algorithm="fricas")
```

output
```
-1/15*(3*a*cos(f*x + e)^6 - 5*(2*a - b)*cos(f*x + e)^4 + 15*(a - 2*b)*cos(
f*x + e)^2 - 15*b)/(f*cos(f*x + e))
```

3.2.6 Sympy [F]

$$\int \left(a + b\sec^2(e+fx)\right)\sin^5(e+fx)\,dx = \int \left(a + b\sec^2\left(e+fx\right)\right)\sin^5\left(e+fx\right)\,dx$$

input
```
integrate((a+b*sec(f*x+e)**2)*sin(f*x+e)**5,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)*sin(e + f*x)**5, x)
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.88

$$\int (a + b\sec^2(e+fx))\sin^5(e+fx)\,dx$$
$$= -\frac{3\,a\cos(fx+e)^5 - 5\,(2\,a-b)\cos(fx+e)^3 + 15\,(a-2\,b)\cos(fx+e) - \frac{15\,b}{\cos(fx+e)}}{15\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^5,x, algorithm="maxima")
```

output
```
-1/15*(3*a*cos(f*x + e)^5 - 5*(2*a - b)*cos(f*x + e)^3 + 15*(a - 2*b)*cos(
f*x + e) - 15*b/cos(f*x + e))/f
```

3.2.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(62) = 124.

Time = 0.32 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.98

$$\int (a + b\sec^2(e+fx))\sin^5(e+fx)\,dx$$
$$= \frac{2\left(\frac{15\,b}{\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1} + \frac{8\,a-25\,b-\frac{40\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{110\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{80\,a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{160\,b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{90\,b(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}-\frac{15\,b(\cos}{\left(\frac{\cos(fx+e)-1}{\cos(fx+e)+1}-1\right)^5}\right.}{15\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^5,x, algorithm="giac")
```

output
```
2/15*(15*b/((cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1) + (8*a - 25*b - 40*
a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 110*b*(cos(f*x + e) - 1)/(cos(f*
x + e) + 1) + 80*a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 160*b*(cos(
f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 90*b*(cos(f*x + e) - 1)^3/(cos(f*x
+ e) + 1)^3 - 15*b*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4)/((cos(f*x +
e) - 1)/(cos(f*x + e) + 1) - 1)^5)/f
```

3.2.9 Mupad [B] (verification not implemented)

Time = 17.33 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.83

$$\int \left(a + b\sec^2(e+fx)\right)\sin^5(e+fx)\,dx$$
$$= \frac{\cos(e+fx)^3\left(\frac{2a}{3} - \frac{b}{3}\right) - \cos(e+fx)\,(a-2b) - \frac{a\cos(e+fx)^5}{5} + \frac{b}{\cos(e+fx)}}{f}$$

input
```
int(sin(e + f*x)^5*(a + b/cos(e + f*x)^2),x)
```

output
```
(cos(e + f*x)^3*((2*a)/3 - b/3) - cos(e + f*x)*(a - 2*b) - (a*cos(e + f*x)
^5)/5 + b/cos(e + f*x))/f
```

3.3 $\int (a + b\sec^2(e+fx))\sin^3(e+fx)\,dx$

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3.3.1 Optimal result

Integrand size = 21, antiderivative size = 44

$$\int (a + b\sec^2(e+fx))\sin^3(e+fx)\,dx = -\frac{(a-b)\cos(e+fx)}{f} + \frac{a\cos^3(e+fx)}{3f} + \frac{b\sec(e+fx)}{f}$$

output
```
-(a-b)*cos(f*x+e)/f+1/3*a*cos(f*x+e)^3/f+b*sec(f*x+e)/f
```

3.3.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.20

$$\int (a + b\sec^2(e+fx))\sin^3(e+fx)\,dx = -\frac{3a\cos(e+fx)}{4f} + \frac{b\cos(e+fx)}{f} + \frac{a\cos(3(e+fx))}{12f} + \frac{b\sec(e+fx)}{f}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^3,x]
```

output
```
(-3*a*Cos[e + f*x])/(4*f) + (b*Cos[e + f*x])/f + (a*Cos[3*(e + f*x)])/(12*
f) + (b*Sec[e + f*x])/f
```

3.3.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4621, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \sin(e+fx)^3\left(a+b\sec(e+fx)^2\right)dx$$

$$\downarrow 4621$$

$$-\frac{\int \left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)\sec^2(e+fx)d\cos(e+fx)}{f}$$

$$\downarrow 355$$

$$-\frac{\int \left(-a\cos^2(e+fx)+b\sec^2(e+fx)+a(1-\frac{b}{a})\right)d\cos(e+fx)}{f}$$

$$\downarrow 2009$$

$$-\frac{(a-b)\cos(e+fx)-\frac{1}{3}a\cos^3(e+fx)-b\sec(e+fx)}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^3,x]
```

output
```
-(((a - b)*Cos[e + f*x] - (a*Cos[e + f*x]^3)/3 - b*Sec[e + f*x])/f)
```

3.3.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

3.3.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.41

method	result	size
derivativedivides	$\frac{-\frac{a\left(2+\sin(fx+e)^2\right)\cos(fx+e)}{3}+b\left(\frac{\sin(fx+e)^4}{\cos(fx+e)}+\left(2+\sin(fx+e)^2\right)\cos(fx+e)\right)}{f}$	62
default	$\frac{-\frac{a\left(2+\sin(fx+e)^2\right)\cos(fx+e)}{3}+b\left(\frac{\sin(fx+e)^4}{\cos(fx+e)}+\left(2+\sin(fx+e)^2\right)\cos(fx+e)\right)}{f}$	62
parallelrisch	$\frac{(-8a+12b)\cos(2fx+2e)+\cos(4fx+4e)a+(-16a+48b)\cos(fx+e)-9a+36b}{24f\cos(fx+e)}$	63
parts	$-\frac{a\left(2+\sin(fx+e)^2\right)\cos(fx+e)}{3f}+\frac{b\left(\frac{\sin(fx+e)^4}{\cos(fx+e)}+\left(2+\sin(fx+e)^2\right)\cos(fx+e)\right)}{f}$	64
norman	$\frac{\frac{4a-12b}{3f}-\frac{4(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{f}+\frac{2(4a-12b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{3f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)^3}$	87
risch	$-\frac{3e^{i(fx+e)}a}{8f}+\frac{e^{i(fx+e)}b}{2f}-\frac{3e^{-i(fx+e)}a}{8f}+\frac{e^{-i(fx+e)}b}{2f}+\frac{2be^{i(fx+e)}}{f\left(e^{2i(fx+e)}+1\right)}+\frac{\cos(3fx+3e)a}{12f}$	105

input
```
int((a+b*sec(f*x+e)^2)*sin(f*x+e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/3*a*(2+sin(f*x+e)^2)*cos(f*x+e)+b*(sin(f*x+e)^4/cos(f*x+e)+(2+sin(
f*x+e)^2)*cos(f*x+e)))
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int (a + b\sec^2(e+fx))\sin^3(e+fx)\,dx = \frac{a\cos(fx+e)^4 - 3(a-b)\cos(fx+e)^2 + 3b}{3f\cos(fx+e)}$$

input
```
integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^3,x, algorithm="fricas")
```

output
```
1/3*(a*cos(f*x + e)^4 - 3*(a - b)*cos(f*x + e)^2 + 3*b)/(f*cos(f*x + e))
```

3.3.6 Sympy [F]

$$\int (a + b\sec^2(e+fx))\sin^3(e+fx)\,dx = \int (a + b\sec^2(e+fx))\sin^3(e+fx)\,dx$$

input
```
integrate((a+b*sec(f*x+e)**2)*sin(f*x+e)**3,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)*sin(e + f*x)**3, x)
```

3.3.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

$$\int (a + b\sec^2(e+fx))\sin^3(e+fx)\,dx = \frac{a\cos(fx+e)^3 - 3(a-b)\cos(fx+e) + \frac{3b}{\cos(fx+e)}}{3f}$$

input
```
integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^3,x, algorithm="maxima")
```

output
```
1/3*(a*cos(f*x + e)^3 - 3*(a - b)*cos(f*x + e) + 3*b/cos(f*x + e))/f
```

3.3.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.30

$$\int \left(a + b\sec^2(e+fx)\right)\sin^3(e+fx)\,dx$$
$$= \frac{b}{f\cos(fx+e)} + \frac{af^5\cos(fx+e)^3 - 3af^5\cos(fx+e) + 3bf^5\cos(fx+e)}{3f^6}$$

input
```
integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^3,x, algorithm="giac")
```

output
```
b/(f*cos(f*x + e)) + 1/3*(a*f^5*cos(f*x + e)^3 - 3*a*f^5*cos(f*x + e) + 3*
b*f^5*cos(f*x + e))/f^6
```

3.3.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int \left(a + b\sec^2(e+fx)\right)\sin^3(e+fx)\,dx = \frac{\frac{a\cos(e+fx)^3}{3} - \cos(e+fx)\,(a-b) + \frac{b}{\cos(e+fx)}}{f}$$

input
```
int(sin(e + f*x)^3*(a + b/cos(e + f*x)^2),x)
```

output
```
((a*cos(e + f*x)^3)/3 - cos(e + f*x)*(a - b) + b/cos(e + f*x))/f
```

3.4 $\int (a + b\sec^2(e+fx))\sin(e+fx)\,dx$

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3.4.1 Optimal result

Integrand size = 19, antiderivative size = 24

$$\int (a + b\sec^2(e+fx))\sin(e+fx)\,dx = -\frac{a\cos(e+fx)}{f} + \frac{b\sec(e+fx)}{f}$$

output
```
-a*cos(f*x+e)/f+b*sec(f*x+e)/f
```

3.4.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int (a + b\sec^2(e+fx))\sin(e+fx)\,dx = -\frac{a\cos(e)\cos(fx)}{f} + \frac{b\sec(e+fx)}{f} + \frac{a\sin(e)\sin(fx)}{f}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)*Sin[e + f*x],x]
```

output
```
-((a*Cos[e]*Cos[f*x])/f) + (b*Sec[e + f*x])/f + (a*Sin[e]*Sin[f*x])/f
```

3.4.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 4621, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \sin(e+fx)\left(a+b\sec(e+fx)^2\right)dx$$

$$\downarrow 4621$$

$$-\frac{\int \left(a\cos^2(e+fx)+b\right)\sec^2(e+fx)d\cos(e+fx)}{f}$$

$$\downarrow 244$$

$$-\frac{\int \left(b\sec^2(e+fx)+a\right)d\cos(e+fx)}{f}$$

$$\downarrow 2009$$

$$-\frac{a\cos(e+fx)-b\sec(e+fx)}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)*Sin[e + f*x],x]
```

output
```
-((a*Cos[e + f*x] - b*Sec[e + f*x])/f)
```

3.4.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

3.4.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{\sec(fx+e)b-\frac{a}{\sec(fx+e)}}{f}$	25
default	$\frac{\sec(fx+e)b-\frac{a}{\sec(fx+e)}}{f}$	25
parts	$-\frac{a\cos(fx+e)}{f}+\frac{b\sec(fx+e)}{f}$	25
parallelrisch	$\frac{-a\cos(2fx+2e)+(-2a+2b)\cos(fx+e)-a+2b}{2f\cos(fx+e)}$	47
risch	$-\frac{a\,\mathrm{e}^{3i(fx+e)}+(3a-4b)\cos(fx+e)+i(a-4b)\sin(fx+e)}{2f\left(\mathrm{e}^{2i(fx+e)}+1\right)}$	59
norman	$\frac{\frac{2a-2b}{f}-\frac{2(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)}$	63

input
```
int((a+b*sec(f*x+e)^2)*sin(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
1/f*(sec(f*x+e)*b-a/sec(f*x+e))
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.12

$$\int (a + b\sec^2(e + fx))\sin(e + fx)\,dx = -\frac{a\cos(fx + e)^2 - b}{f\cos(fx + e)}$$

input
```
integrate((a+b*sec(f*x+e)^2)*sin(f*x+e),x, algorithm="fricas")
```

output
```
-(a*cos(f*x + e)^2 - b)/(f*cos(f*x + e))
```

3.4.6 Sympy [F]

$$\int (a + b\sec^2(e + fx))\sin(e + fx)\,dx = \int (a + b\sec^2(e + fx))\sin(e + fx)\,dx$$

input
```
integrate((a+b*sec(f*x+e)**2)*sin(f*x+e),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)*sin(e + f*x), x)
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int (a + b\sec^2(e + fx))\sin(e + fx)\,dx = -\frac{a\cos(fx + e) - \frac{b}{\cos(fx+e)}}{f}$$

input
```
integrate((a+b*sec(f*x+e)^2)*sin(f*x+e),x, algorithm="maxima")
```

output
```
-(a*cos(f*x + e) - b/cos(f*x + e))/f
```

3.4.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (a + b\sec^2(e + fx))\sin(e + fx)\,dx = -\frac{a\cos(fx + e)}{f} + \frac{b}{f\cos(fx + e)}$$

input
```
integrate((a+b*sec(f*x+e)^2)*sin(f*x+e),x, algorithm="giac")
```

output
```
-a*cos(f*x + e)/f + b/(f*cos(f*x + e))
```

3.4.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int (a + b\sec^2(e + fx))\sin(e + fx)\,dx = -\frac{a\cos(e + f\,x) - \frac{b}{\cos(e+f\,x)}}{f}$$

input
```
int(sin(e + f*x)*(a + b/cos(e + f*x)^2),x)
```

output
```
-(a*cos(e + f*x) - b/cos(e + f*x))/f
```

3.5 $\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

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3.5.1 Optimal result

Integrand size = 19, antiderivative size = 27

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{(a+b)\operatorname{arctanh}(\cos(e+fx))}{f}+\frac{b\sec(e+fx)}{f}$$

output
```
-(a+b)*arctanh(cos(f*x+e))/f+b*sec(f*x+e)/f
```

3.5.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 84 vs. 2(27) = 54.

Time = 0.07 (sec) , antiderivative size = 84, normalized size of antiderivative = 3.11

$$\begin{aligned}\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = &-\frac{a\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f}-\frac{b\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{f}\\ &+\frac{a\log\left(\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f}\\ &+\frac{b\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}+\frac{b\sec(e+fx)}{f}\end{aligned}$$

input
```
Integrate[Csc[e + f*x]*(a + b*Sec[e + f*x]^2),x]
```

output
```
-((a*Log[Cos[e/2 + (f*x)/2]])/f) - (b*Log[Cos[(e + f*x)/2]])/f + (a*Log[Si
n[e/2 + (f*x)/2]])/f + (b*Log[Sin[(e + f*x)/2]])/f + (b*Sec[e + f*x])/f
```

3.5.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 4621, 359, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \frac{a+b\sec(e+fx)^2}{\sin(e+fx)}dx$$

$$\downarrow 4621$$

$$-\frac{\int \frac{(a\cos^2(e+fx)+b)\sec^2(e+fx)}{1-\cos^2(e+fx)}d\cos(e+fx)}{f}$$

$$\downarrow 359$$

$$-\frac{(a+b)\int \frac{1}{1-\cos^2(e+fx)}d\cos(e+fx)-b\sec(e+fx)}{f}$$

$$\downarrow 219$$

$$-\frac{(a+b)\text{arctanh}(\cos(e+fx))-b\sec(e+fx)}{f}$$

input
```
Int[Csc[e + f*x]*(a + b*Sec[e + f*x]^2),x]
```

output
```
-(((a + b)*ArcTanh[Cos[e + f*x]] - b*Sec[e + f*x])/f)
```

3.5.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

3.5.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.48

method	result	size
norman	$-\frac{2b}{f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)}+\frac{(a+b)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}$	40
parallelrisch	$\frac{\cos(fx+e)(a+b)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+b(1+\cos(fx+e))}{f\cos(fx+e)}$	44
derivativedivides	$\frac{a\ln(\csc(fx+e)-\cot(fx+e))+b\left(\frac{1}{\cos(fx+e)}+\ln(\csc(fx+e)-\cot(fx+e))\right)}{f}$	51
default	$\frac{a\ln(\csc(fx+e)-\cot(fx+e))+b\left(\frac{1}{\cos(fx+e)}+\ln(\csc(fx+e)-\cot(fx+e))\right)}{f}$	51
risch	$\frac{2be^{i(fx+e)}}{f\left(e^{2i(fx+e)}+1\right)}+\frac{\ln\left(e^{i(fx+e)}-1\right)a}{f}+\frac{\ln\left(e^{i(fx+e)}-1\right)b}{f}-\frac{\ln\left(e^{i(fx+e)}+1\right)a}{f}-\frac{\ln\left(e^{i(fx+e)}+1\right)b}{f}$	100

input
```
int(csc(f*x+e)*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
-2*b/f/(tan(1/2*f*x+1/2*e)^2-1)+(a+b)/f*ln(tan(1/2*f*x+1/2*e))
```

3.5.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(27) = 54$.

Time $= 0.25$ (sec) , antiderivative size $= 60$, normalized size of antiderivative $= 2.22$

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx =$$
$$-\frac{(a+b)\cos(fx+e)\log\left(\frac{1}{2}\cos(fx+e)+\frac{1}{2}\right)-(a+b)\cos(fx+e)\log\left(-\frac{1}{2}\cos(fx+e)+\frac{1}{2}\right)-2b}{2f\cos(fx+e)}$$

input
```
integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
-1/2*((a + b)*cos(f*x + e)*log(1/2*cos(f*x + e) + 1/2) - (a + b)*cos(f*x +
 e)*log(-1/2*cos(f*x + e) + 1/2) - 2*b)/(f*cos(f*x + e))
```

3.5.6 Sympy [F]

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \int \left(a+b\sec^2(e+fx)\right)\csc(e+fx)\,dx$$

input
```
integrate(csc(f*x+e)*(a+b*sec(f*x+e)**2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)*csc(e + f*x), x)
```

3.5.7 Maxima [A] (verification not implemented)

Time $= 0.19$ (sec) , antiderivative size $= 44$, normalized size of antiderivative $= 1.63$

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$= -\frac{(a+b)\log(\cos(fx+e)+1)-(a+b)\log(\cos(fx+e)-1)-\frac{2b}{\cos(fx+e)}}{2f}$$

input
```
integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
-1/2*((a + b)*log(cos(f*x + e) + 1) - (a + b)*log(cos(f*x + e) - 1) - 2*b/
cos(f*x + e))/f
```

3.5.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(27) = 54$.

Time $= 0.28$ (sec) , antiderivative size $= 60$, normalized size of antiderivative $= 2.22$

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{(a+b)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)+\frac{4b}{\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1}}{2f}$$

input
```
integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/2*((a + b)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1)) + 4*b/((cos
(f*x + e) - 1)/(cos(f*x + e) + 1) + 1))/f
```

3.5.9 Mupad [B] (verification not implemented)

Time $= 0.10$ (sec) , antiderivative size $= 29$, normalized size of antiderivative $= 1.07$

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{b}{f\,\cos\left(e+f\,x\right)} - \frac{\operatorname{atanh}(\cos\left(e+f\,x\right))\,\left(a+b\right)}{f}$$

input
```
int((a + b/cos(e + f*x)^2)/sin(e + f*x),x)
```

output
```
b/(f*cos(e + f*x)) - (atanh(cos(e + f*x))*(a + b))/f
```

3.6 $\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

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3.6.1 Optimal result

Integrand size = 21, antiderivative size = 53

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{(a+3b)\operatorname{arctanh}(\cos(e+fx))}{2f} - \frac{(a+b)\cot(e+fx)\csc(e+fx)}{2f} + \frac{b\sec(e+fx)}{f}$$

output

```
-1/2*(a+3*b)*arctanh(cos(f*x+e))/f-1/2*(a+b)*cot(f*x+e)*csc(f*x+e)/f+b*sec
(f*x+e)/f
```

3.6.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 236 vs. $2(53) = 106$.

Time $= 0.87$ (sec) , antiderivative size $= 236$, normalized size of antiderivative $= 4.45$

$$\begin{aligned}\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = &-\frac{a\csc^2\left(\frac{1}{2}(e+fx)\right)}{8f}-\frac{b\csc^2\left(\frac{1}{2}(e+fx)\right)}{8f}\\&-\frac{a\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{2f}\\&-\frac{3b\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{2f}\\&+\frac{a\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{2f}\\&+\frac{3b\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{2f}\\&+\frac{a\sec^2\left(\frac{1}{2}(e+fx)\right)}{8f}+\frac{b\sec^2\left(\frac{1}{2}(e+fx)\right)}{8f}\\&+\frac{b\sin\left(\frac{1}{2}(e+fx)\right)}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}\\&-\frac{b\sin\left(\frac{1}{2}(e+fx)\right)}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}\end{aligned}$$

input
```
Integrate[Csc[e + f*x]^3*(a + b*Sec[e + f*x]^2),x]
```

output
```
-1/8*(a*Csc[(e + f*x)/2]^2)/f - (b*Csc[(e + f*x)/2]^2)/(8*f) - (a*Log[Cos[
(e + f*x)/2]])/(2*f) - (3*b*Log[Cos[(e + f*x)/2]])/(2*f) + (a*Log[Sin[(e +
 f*x)/2]])/(2*f) + (3*b*Log[Sin[(e + f*x)/2]])/(2*f) + (a*Sec[(e + f*x)/2]
^2)/(8*f) + (b*Sec[(e + f*x)/2]^2)/(8*f) + (b*Sin[(e + f*x)/2])/(f*(Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2])) - (b*Sin[(e + f*x)/2])/(f*(Cos[(e + f*x)/2
] + Sin[(e + f*x)/2]))
```

3.6.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4621, 361, 25, 359, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \frac{a+b\sec(e+fx)^2}{\sin(e+fx)^3}dx$$

$$\downarrow 4621$$

$$-\frac{\int \frac{(a\cos^2(e+fx)+b)\sec^2(e+fx)}{(1-\cos^2(e+fx))^2}d\cos(e+fx)}{f}$$

$$\downarrow 361$$

$$-\frac{\frac{(a+b)\cos(e+fx)}{2(1-\cos^2(e+fx))}-\frac{1}{2}\int -\frac{((a+b)\cos^2(e+fx)+2b)\sec^2(e+fx)}{1-\cos^2(e+fx)}d\cos(e+fx)}{f}$$

$$\downarrow 25$$

$$-\frac{\frac{1}{2}\int \frac{((a+b)\cos^2(e+fx)+2b)\sec^2(e+fx)}{1-\cos^2(e+fx)}d\cos(e+fx)+\frac{(a+b)\cos(e+fx)}{2(1-\cos^2(e+fx))}}{f}$$

$$\downarrow 359$$

$$-\frac{\frac{1}{2}\left((a+3b)\int \frac{1}{1-\cos^2(e+fx)}d\cos(e+fx)-2b\sec(e+fx)\right)+\frac{(a+b)\cos(e+fx)}{2(1-\cos^2(e+fx))}}{f}$$

$$\downarrow 219$$

$$-\frac{\frac{1}{2}((a+3b)\operatorname{arctanh}(\cos(e+fx))-2b\sec(e+fx))+\frac{(a+b)\cos(e+fx)}{2(1-\cos^2(e+fx))}}{f}$$

input
```
Int[Csc[e + f*x]^3*(a + b*Sec[e + f*x]^2),x]
```

output
```
-((((a + b)*Cos[e + f*x])/(2*(1 - Cos[e + f*x]^2)) + ((a + 3*b)*ArcTanh[Co
s[e + f*x]] - 2*b*Sec[e + f*x])/2)/f)
```

3.6.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

3.6.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.70

method	result
derivativedivides	$\frac{a\left(-\frac{\csc(fx+e)\cot(fx+e)}{2}+\frac{\ln(\csc(fx+e)-\cot(fx+e))}{2}\right)+b\left(-\frac{1}{2\sin(fx+e)^2\cos(fx+e)}+\frac{3}{2\cos(fx+e)}+\frac{3\ln(\csc(fx+e)-\cot(fx+e))}{2}\right.}{f}$
default	$\frac{a\left(-\frac{\csc(fx+e)\cot(fx+e)}{2}+\frac{\ln(\csc(fx+e)-\cot(fx+e))}{2}\right)+b\left(-\frac{1}{2\sin(fx+e)^2\cos(fx+e)}+\frac{3}{2\cos(fx+e)}+\frac{3\ln(\csc(fx+e)-\cot(fx+e))}{2}\right.}{f}$
norman	$\frac{\frac{a+b}{8f}+\frac{(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{8f}-\frac{(a+9b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{4f}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)}+\frac{(a+3b)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2f}$
parallelrisch	$\frac{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)(a+3b)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+(a+b)\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^2-2a-18b}{8f\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-8f}$
risch	$\frac{e^{i(fx+e)}\left(a\,e^{4i(fx+e)}+3b\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}-2b\,e^{2i(fx+e)}+a+3b\right)}{f\left(e^{2i(fx+e)}+1\right)\left(e^{2i(fx+e)}-1\right)^2}-\frac{\ln\left(e^{i(fx+e)}+1\right)a}{2f}-\frac{3\ln\left(e^{i(fx+e)}+1\right)b}{2f}+$

input
```
int(csc(f*x+e)^3*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(a*(-1/2*csc(f*x+e)*cot(f*x+e)+1/2*ln(csc(f*x+e)-cot(f*x+e)))+b*(-1/2/
sin(f*x+e)^2/cos(f*x+e)+3/2/cos(f*x+e)+3/2*ln(csc(f*x+e)-cot(f*x+e))))
```

3.6.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(49) = 98.

Time = 0.25 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.34

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$= \frac{2\,(a+3\,b)\cos(fx+e)^2-\left((a+3\,b)\cos(fx+e)^3-(a+3\,b)\cos(fx+e)\right)\log\left(\frac{1}{2}\cos(fx+e)+\frac{1}{2}\right)+((a}{4\left(f\cos(fx+e)^3-f\cos(fx+}$$

input
```
integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/4*(2*(a + 3*b)*cos(f*x + e)^2 - ((a + 3*b)*cos(f*x + e)^3 - (a + 3*b)*co
s(f*x + e))*log(1/2*cos(f*x + e) + 1/2) + ((a + 3*b)*cos(f*x + e)^3 - (a +
 3*b)*cos(f*x + e))*log(-1/2*cos(f*x + e) + 1/2) - 4*b)/(f*cos(f*x + e)^3
- f*cos(f*x + e))
```

3.6.6 Sympy [F]

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \int \left(a+b\sec^2(e+fx)\right)\csc^3(e+fx)\,dx$$

input
```
integrate(csc(f*x+e)**3*(a+b*sec(f*x+e)**2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)*csc(e + f*x)**3, x)
```

3.6.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.43

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$= -\frac{(a+3b)\log(\cos(fx+e)+1) - (a+3b)\log(\cos(fx+e)-1) - \frac{2\left((a+3b)\cos(fx+e)^2-2b\right)}{\cos(fx+e)^3-\cos(fx+e)}}{4f}$$

input
```
integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
-1/4*((a + 3*b)*log(cos(f*x + e) + 1) - (a + 3*b)*log(cos(f*x + e) - 1) -
2*((a + 3*b)*cos(f*x + e)^2 - 2*b)/(cos(f*x + e)^3 - cos(f*x + e)))/f
```

3.6.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. 2(49) = 98.

Time = 0.30 (sec) , antiderivative size = 193, normalized size of antiderivative = 3.64

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$= \frac{2(a+3b)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right) - \frac{a(\cos(fx+e)-1)}{\cos(fx+e)+1} - \frac{b(\cos(fx+e)-1)}{\cos(fx+e)+1} + \frac{a+b+\frac{14\,b(\cos(fx+e)-1)}{\cos(fx+e)+1} - \frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2} - \frac{3\,b(\cos(fx+e)-}{(\cos(fx+e)+}}{\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+\frac{(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}}}{8f}$$

input
```
integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/8*(2*(a + 3*b)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1)) - a*(co
s(f*x + e) - 1)/(cos(f*x + e) + 1) - b*(cos(f*x + e) - 1)/(cos(f*x + e) +
1) + (a + b + 14*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - a*(cos(f*x + e)
 - 1)^2/(cos(f*x + e) + 1)^2 - 3*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)
^2)/((cos(f*x + e) - 1)/(cos(f*x + e) + 1) + (cos(f*x + e) - 1)^2/(cos(f*x
 + e) + 1)^2))/f
```

3.6.9 Mupad [B] (verification not implemented)

Time = 17.32 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.17

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{b-\cos\left(e+f\,x\right)^2\left(\frac{a}{2}+\frac{3\,b}{2}\right)}{f\,\left(\cos\left(e+f\,x\right)-\cos\left(e+f\,x\right)^3\right)} - \frac{\operatorname{atanh}(\cos\left(e+f\,x\right))\,\left(\frac{a}{2}+\frac{3\,b}{2}\right)}{f}$$

input
```
int((a + b/cos(e + f*x)^2)/sin(e + f*x)^3,x)
```

output
```
(b - cos(e + f*x)^2*(a/2 + (3*b)/2))/(f*(cos(e + f*x) - cos(e + f*x)^3)) -
 (atanh(cos(e + f*x))*(a/2 + (3*b)/2))/f
```

3.7 $\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

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3.7.1 Optimal result

Integrand size = 21, antiderivative size = 81

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{3(a+5b)\operatorname{arctanh}(\cos(e+fx))}{8f} - \frac{(3a+7b)\cot(e+fx)\csc(e+fx)}{8f} - \frac{(a+b)\cot(e+fx)\csc^3(e+fx)}{4f} + \frac{b\sec(e+fx)}{f}$$

output
```
-3/8*(a+5*b)*arctanh(cos(f*x+e))/f-1/8*(3*a+7*b)*cot(f*x+e)*csc(f*x+e)/f-1
/4*(a+b)*cot(f*x+e)*csc(f*x+e)^3/f+b*sec(f*x+e)/f
```

3.7.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 198 vs. 2(81) = 162.

Time = 6.26 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.44

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{-2(3a+7b)\csc^2\left(\frac{1}{2}(e+fx)\right) - (a+b)\csc^4\left(\frac{1}{2}(e+fx)\right) + \frac{2(-3(a+13b)+4\cos(e+fx)(8b+3(a+5b)\log(\cos(\frac{1}{2}(e+fx)))-}{64f}$$

input
```
Integrate[Csc[e + f*x]^5*(a + b*Sec[e + f*x]^2),x]
```

output
```
(-2*(3*a + 7*b)*Csc[(e + f*x)/2]^2 - (a + b)*Csc[(e + f*x)/2]^4 + (2*(-3*(
a + 13*b) + 4*Cos[e + f*x]*(8*b + 3*(a + 5*b)*Log[Cos[(e + f*x)/2]] - 3*(a
 + 5*b)*Log[Sin[(e + f*x)/2]]))*Sec[(e + f*x)/2]^2 - (a + b)*Sec[(e + f*x)
/2]^4 + (4*(a + 2*b) + (3*a + 7*b)*Cos[e + f*x])*Sec[(e + f*x)/2]^4*Tan[(e
 + f*x)/2]^2)/(-1 + Tan[(e + f*x)/2]^2))/(64*f)
```

3.7.3 Rubi [A] (verified)

Time $= 0.26$ (sec) , antiderivative size $= 97$, normalized size of antiderivative $= 1.20$, number of steps used $= 9$, number of rules used $= 8$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used $= \{3042, 4621, 361, 25, 361, 25, 359, 219\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \frac{a+b\sec(e+fx)^2}{\sin(e+fx)^5}dx$$

$$\downarrow 4621$$

$$-\frac{\int \frac{(a\cos^2(e+fx)+b)\sec^2(e+fx)}{(1-\cos^2(e+fx))^3}d\cos(e+fx)}{f}$$

$$\downarrow 361$$

$$-\frac{\frac{(a+b)\cos(e+fx)}{4(1-\cos^2(e+fx))^2}-\frac{1}{4}\int -\frac{(3(a+b)\cos^2(e+fx)+4b)\sec^2(e+fx)}{(1-\cos^2(e+fx))^2}d\cos(e+fx)}{f}$$

$$\downarrow 25$$

$$-\frac{\frac{1}{4}\int \frac{(3(a+b)\cos^2(e+fx)+4b)\sec^2(e+fx)}{(1-\cos^2(e+fx))^2}d\cos(e+fx)+\frac{(a+b)\cos(e+fx)}{4(1-\cos^2(e+fx))^2}}{f}$$

$$\downarrow 361$$

$$-\frac{\frac{1}{4}\left(\frac{(3a+7b)\cos(e+fx)}{2(1-\cos^2(e+fx))}-\frac{1}{2}\int -\frac{((3a+7b)\cos^2(e+fx)+8b)\sec^2(e+fx)}{1-\cos^2(e+fx)}d\cos(e+fx)\right)+\frac{(a+b)\cos(e+fx)}{4(1-\cos^2(e+fx))^2}}{f}$$

$$\downarrow 25$$

$$-\frac{\frac{1}{4}\left(\frac{1}{2}\int \frac{((3a+7b)\cos^2(e+fx)+8b)\sec^2(e+fx)}{1-\cos^2(e+fx)}d\cos(e+fx)+\frac{(3a+7b)\cos(e+fx)}{2(1-\cos^2(e+fx))}\right)+\frac{(a+b)\cos(e+fx)}{4(1-\cos^2(e+fx))^2}}{f}$$

$$\downarrow 359$$

$$-\frac{\frac{1}{4}\left(\frac{1}{2}\left(3(a+5b)\int \frac{1}{1-\cos^2(e+fx)}d\cos(e+fx)-8b\sec(e+fx)\right)+\frac{(3a+7b)\cos(e+fx)}{2(1-\cos^2(e+fx))}\right)+\frac{(a+b)\cos(e+fx)}{4(1-\cos^2(e+fx))^2}}{f}$$

$$\downarrow 219$$

$$-\frac{\frac{1}{4}\left(\frac{1}{2}(3(a+5b)\text{arctanh}(\cos(e+fx))-8b\sec(e+fx))+\frac{(3a+7b)\cos(e+fx)}{2(1-\cos^2(e+fx))}\right)+\frac{(a+b)\cos(e+fx)}{4(1-\cos^2(e+fx))^2}}{f}$$

input
```
Int[Csc[e + f*x]^5*(a + b*Sec[e + f*x]^2),x]
```

output
```
-((((a + b)*Cos[e + f*x])/(4*(1 - Cos[e + f*x]^2)^2) + (((3*a + 7*b)*Cos[e
 + f*x])/(2*(1 - Cos[e + f*x]^2)) + (3*(a + 5*b)*ArcTanh[Cos[e + f*x]] - 8
*b*Sec[e + f*x])/2)/4)/f)
```

3.7.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))  Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

3.7.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.48

method	result
derivativedivides	$\frac{a\left(\left(-\frac{\csc(fx+e)^3}{4}-\frac{3\csc(fx+e)}{8}\right)\cot(fx+e)+\frac{3\ln(\csc(fx+e)-\cot(fx+e))}{8}\right)+b\left(-\frac{1}{4\sin(fx+e)^4\cos(fx+e)}-\frac{5}{8\sin(fx+e)^2\cos(f$
default	$\frac{a\left(\left(-\frac{\csc(fx+e)^3}{4}-\frac{3\csc(fx+e)}{8}\right)\cot(fx+e)+\frac{3\ln(\csc(fx+e)-\cot(fx+e))}{8}\right)+b\left(-\frac{1}{4\sin(fx+e)^4\cos(fx+e)}-\frac{5}{8\sin(fx+e)^2\cos(f$
parallelrisch	$\frac{24\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)(a+5b)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6+(7a+15b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+(a+b$ $64f\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-64f$
norman	$\frac{\frac{a+b}{64f}+\frac{(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}}{64f}+\frac{(7a+15b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{64f}+\frac{(7a+15b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8}{64f}-\frac{(a+10b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{4f}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)}+\frac{3(a+5b)\ln\left(\tan\left(\frac{f}{2}\right.\right.}{8f}$
risch	$\frac{e^{i(fx+e)}\left(3a\,e^{8i(fx+e)}+15b\,e^{8i(fx+e)}-8a\,e^{6i(fx+e)}-40b\,e^{6i(fx+e)}-22a\,e^{4i(fx+e)}+18b\,e^{4i(fx+e)}-8a\,e^{2i(fx+e)}-40b\,e^{2i($ $4f\left(e^{2i(fx+e)}-1\right)^4\left(e^{2i(fx+e)}+1\right)$

input
```
int(csc(f*x+e)^5*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(a*((-1/4*csc(f*x+e)^3-3/8*csc(f*x+e))*cot(f*x+e)+3/8*ln(csc(f*x+e)-co
t(f*x+e)))+b*(-1/4/sin(f*x+e)^4/cos(f*x+e)-5/8/sin(f*x+e)^2/cos(f*x+e)+15/
8/cos(f*x+e)+15/8*ln(csc(f*x+e)-cot(f*x+e))))
```

3.7.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(75) = 150$.

Time $= 0.26$ (sec) , antiderivative size $= 178$, normalized size of antiderivative $= 2.20$

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$= \frac{6\,(a+5\,b)\cos(fx+e)^4 - 10\,(a+5\,b)\cos(fx+e)^2 - 3\left((a+5\,b)\cos(fx+e)^5 - 2\,(a+5\,b)\cos(fx+e)\right.}{}$$

input
```
integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/16*(6*(a + 5*b)*cos(f*x + e)^4 - 10*(a + 5*b)*cos(f*x + e)^2 - 3*((a + 5
*b)*cos(f*x + e)^5 - 2*(a + 5*b)*cos(f*x + e)^3 + (a + 5*b)*cos(f*x + e))*
log(1/2*cos(f*x + e) + 1/2) + 3*((a + 5*b)*cos(f*x + e)^5 - 2*(a + 5*b)*co
s(f*x + e)^3 + (a + 5*b)*cos(f*x + e))*log(-1/2*cos(f*x + e) + 1/2) + 16*b
)/(f*cos(f*x + e)^5 - 2*f*cos(f*x + e)^3 + f*cos(f*x + e))
```

3.7.6 Sympy [F]

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \int \left(a+b\sec^2(e+fx)\right)\csc^5(e+fx)\,dx$$

input
```
integrate(csc(f*x+e)**5*(a+b*sec(f*x+e)**2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)*csc(e + f*x)**5, x)
```

3.7.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.25

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx =$$

$$-\frac{3\,(a+5\,b)\log\left(\cos\left(fx+e\right)+1\right)-3\,(a+5\,b)\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\left(3\,(a+5\,b)\cos(fx+e)^4-5\,(a+5\,b)\cos(fx+e\right.}{\cos(fx+e)^5-2\,\cos(fx+e)^3+\cos(fx+}}{16\,f}$$

input
```
integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
-1/16*(3*(a + 5*b)*log(cos(f*x + e) + 1) - 3*(a + 5*b)*log(cos(f*x + e) -
1) - 2*(3*(a + 5*b)*cos(f*x + e)^4 - 5*(a + 5*b)*cos(f*x + e)^2 + 8*b)/(co
s(f*x + e)^5 - 2*cos(f*x + e)^3 + cos(f*x + e)))/f
```

3.7.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. 2(75) = 150.

Time = 0.31 (sec) , antiderivative size = 262, normalized size of antiderivative = 3.23

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$=\frac{12\,(a+5\,b)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)-\frac{\left(a+b-\frac{8\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{16\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{18\,a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{90\,b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)}{(\cos(fx+e)-1)^2}}{64\,f}$$

input
```
integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/64*(12*(a + 5*b)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1)) - (a
+ b - 8*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 16*b*(cos(f*x + e) - 1)/
(cos(f*x + e) + 1) + 18*a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 90*b
*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)^2/(cos(f*x
+ e) - 1)^2 - 8*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 16*b*(cos(f*x +
e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 +
 b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 128*b/((cos(f*x + e) - 1)/(
cos(f*x + e) + 1) + 1))/f
```

3.7.9 Mupad [B] (verification not implemented)

Time = 17.54 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.06

$$\begin{aligned}
&\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx \\
&= \frac{\left(\frac{3a}{8}+\frac{15b}{8}\right)\cos(e+fx)^4+\left(-\frac{5a}{8}-\frac{25b}{8}\right)\cos(e+fx)^2+b}{f\left(\cos(e+fx)^5-2\cos(e+fx)^3+\cos(e+fx)\right)} \\
&\quad -\frac{\operatorname{atanh}(\cos(e+fx))\left(\frac{3a}{8}+\frac{15b}{8}\right)}{f}
\end{aligned}$$

input
```
int((a + b/cos(e + f*x)^2)/sin(e + f*x)^5,x)
```

output
```
(b + cos(e + f*x)^4*((3*a)/8 + (15*b)/8) - cos(e + f*x)^2*((5*a)/8 + (25*b
)/8))/(f*(cos(e + f*x) - 2*cos(e + f*x)^3 + cos(e + f*x)^5)) - (atanh(cos(
e + f*x))*((3*a)/8 + (15*b)/8))/f
```

3.8 $\int (a + b\sec^2(e+fx))\sin^6(e+fx)\,dx$

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3.8.1 Optimal result

Integrand size = 21, antiderivative size = 98

$$\int (a + b\sec^2(e+fx))\sin^6(e+fx)\,dx = \frac{5}{16}(a-6b)x - \frac{(11a-18b)\cos(e+fx)\sin(e+fx)}{16f} + \frac{(13a-6b)\cos^3(e+fx)\sin(e+fx)}{24f} - \frac{a\cos^5(e+fx)\sin(e+fx)}{6f} + \frac{b\tan(e+fx)}{f}$$

output
```
5/16*(a-6*b)*x-1/16*(11*a-18*b)*cos(f*x+e)*sin(f*x+e)/f+1/24*(13*a-6*b)*co
s(f*x+e)^3*sin(f*x+e)/f-1/6*a*cos(f*x+e)^5*sin(f*x+e)/f+b*tan(f*x+e)/f
```

3.8.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.80

$$\int (a + b\sec^2(e+fx))\sin^6(e+fx)\,dx = \frac{60ae - 360be + 60afx - 360bfx + (-45a+96b)\sin(2(e+fx)) + (9a-6b)\sin(4(e+fx)) - a\sin(6(e+$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^6,x]
```

output
```
(60*a*e - 360*b*e + 60*a*f*x - 360*b*f*x + (-45*a + 96*b)*Sin[2*(e + f*x)]
 + (9*a - 6*b)*Sin[4*(e + f*x)] - a*Sin[6*(e + f*x)] + 192*b*Tan[e + f*x])
/(192*f)
```

3.8.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.27, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4620, 360, 25, 2345, 27, 1471, 299, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \sin(e+fx)^6\left(a+b\sec(e+fx)^2\right)dx$$

$$\downarrow 4620$$

$$\frac{\int \frac{\tan^6(e+fx)\left(b\tan^2(e+fx)+a+b\right)}{\left(\tan^2(e+fx)+1\right)^4}d\tan(e+fx)}{f}$$

$$\downarrow 360$$

$$\frac{-\frac{1}{6}\int -\frac{6b\tan^6(e+fx)+6a\tan^4(e+fx)-6a\tan^2(e+fx)+a}{\left(\tan^2(e+fx)+1\right)^3}d\tan(e+fx)-\frac{a\tan(e+fx)}{6\left(\tan^2(e+fx)+1\right)^3}}{f}$$

$$\downarrow 25$$

$$\frac{\frac{1}{6}\int \frac{6b\tan^6(e+fx)+6a\tan^4(e+fx)-6a\tan^2(e+fx)+a}{\left(\tan^2(e+fx)+1\right)^3}d\tan(e+fx)-\frac{a\tan(e+fx)}{6\left(\tan^2(e+fx)+1\right)^3}}{f}$$

$$\downarrow 2345$$

$$\frac{\frac{1}{6}\left(\frac{(13a-6b)\tan(e+fx)}{4\left(\tan^2(e+fx)+1\right)^2}-\frac{1}{4}\int \frac{3\left(-8b\tan^4(e+fx)-8(a-b)\tan^2(e+fx)+3a-2b\right)}{\left(\tan^2(e+fx)+1\right)^2}d\tan(e+fx)\right)-\frac{a\tan(e+fx)}{6\left(\tan^2(e+fx)+1\right)^3}}{f}$$

$$\downarrow 27$$

$$\frac{\frac{1}{6}\left(\frac{(13a-6b)\tan(e+fx)}{4\left(\tan^2(e+fx)+1\right)^2}-\frac{3}{4}\int \frac{-8b\tan^4(e+fx)-8(a-b)\tan^2(e+fx)+3a-2b}{\left(\tan^2(e+fx)+1\right)^2}d\tan(e+fx)\right)-\frac{a\tan(e+fx)}{6\left(\tan^2(e+fx)+1\right)^3}}{f}$$

$$\downarrow 1471$$

$$\frac{\frac{1}{6}\left(\frac{(13a-6b)\tan(e+fx)}{4(\tan^2(e+fx)+1)^2}-\frac{3}{4}\left(\frac{(11a-18b)\tan(e+fx)}{2(\tan^2(e+fx)+1)}-\frac{1}{2}\int\frac{16b\tan^2(e+fx)+5a-14b}{\tan^2(e+fx)+1}d\tan(e+fx)\right)\right)-\frac{a\tan(e+fx)}{6(\tan^2(e+fx)+1)^3}}{f}$$

$$\downarrow 299$$

$$\frac{\frac{1}{6}\left(\frac{(13a-6b)\tan(e+fx)}{4(\tan^2(e+fx)+1)^2}-\frac{3}{4}\left(\frac{1}{2}\left(-5(a-6b)\int\frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)-16b\tan(e+fx)\right)+\frac{(11a-18b)\tan(e+fx)}{2(\tan^2(e+fx)+1)}\right)\right)-}{f}$$

$$\downarrow 216$$

$$\frac{\frac{1}{6}\left(\frac{(13a-6b)\tan(e+fx)}{4(\tan^2(e+fx)+1)^2}-\frac{3}{4}\left(\frac{1}{2}(-5(a-6b)\arctan(\tan(e+fx))-16b\tan(e+fx))+\frac{(11a-18b)\tan(e+fx)}{2(\tan^2(e+fx)+1)}\right)\right)-\frac{a\tan(e}{6(\tan^2(e+}}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^6,x]
```

output
```
(-1/6*(a*Tan[e + f*x])/(1 + Tan[e + f*x]^2)^3 + (((13*a - 6*b)*Tan[e + f*x
])/(4*(1 + Tan[e + f*x]^2)^2) - (3*((-5*(a - 6*b)*ArcTan[Tan[e + f*x]] - 1
6*b*Tan[e + f*x])/2 + ((11*a - 18*b)*Tan[e + f*x])/(2*(1 + Tan[e + f*x]^2)
)))/4)/6)/f
```

3.8.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.8.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.91

method	result
parallelrisch	$\frac{(-36a+90b)\sin(3fx+3e)+(8a-6b)\sin(5fx+5e)-\sin(7fx+7e)a+120fx(a-6b)\cos(fx+e)-45\sin(fx+e)\left(a-\frac{32b}{3}\right)}{384f\cos(fx+e)}$
derivativedivides	$\frac{a\left(-\frac{\left(\sin(fx+e)^5+\frac{5\sin(fx+e)^3}{4}+\frac{15\sin(fx+e)}{8}\right)\cos(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)+b\left(\frac{\sin(fx+e)^7}{\cos(fx+e)}+\left(\sin(fx+e)^5+\frac{5\sin(fx+e)^3}{4}+\frac{15\sin}{}\right.\right.}{f}$
default	$\frac{a\left(-\frac{\left(\sin(fx+e)^5+\frac{5\sin(fx+e)^3}{4}+\frac{15\sin(fx+e)}{8}\right)\cos(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)+b\left(\frac{\sin(fx+e)^7}{\cos(fx+e)}+\left(\sin(fx+e)^5+\frac{5\sin(fx+e)^3}{4}+\frac{15\sin}{}\right.\right.}{f}$
parts	$\frac{a\left(-\frac{\left(\sin(fx+e)^5+\frac{5\sin(fx+e)^3}{4}+\frac{15\sin(fx+e)}{8}\right)\cos(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)}{f}+\frac{b\left(\frac{\sin(fx+e)^7}{\cos(fx+e)}+\left(\sin(fx+e)^5+\frac{5\sin(fx+e)^3}{4}+\frac{1}{}\right.\right.}{f}$
risch	$\frac{5ax}{16}-\frac{15xb}{8}+\frac{15ie^{2i(fx+e)}a}{128f}-\frac{ie^{2i(fx+e)}b}{4f}-\frac{15ie^{-2i(fx+e)}a}{128f}+\frac{ie^{-2i(fx+e)}b}{4f}+\frac{2ib}{f\left(e^{2i(fx+e)}+1\right)}-\frac{a\sin(6f}{192}$
norman	$\left(-\frac{5a}{16}+\frac{15b}{8}\right)x+\left(-\frac{45a}{16}+\frac{135b}{8}\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+\left(-\frac{25a}{16}+\frac{75b}{8}\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+\left(-\frac{25a}{16}+\frac{75b}{8}\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6+\left(\frac{5a}{16}-\frac{15}{8}\right.$

input
```
int((a+b*sec(f*x+e)^2)*sin(f*x+e)^6,x,method=_RETURNVERBOSE)
```

output
```
1/384*((-36*a+90*b)*sin(3*f*x+3*e)+(8*a-6*b)*sin(5*f*x+5*e)-sin(7*f*x+7*e)
*a+120*f*x*(a-6*b)*cos(f*x+e)-45*sin(f*x+e)*(a-32/3*b))/f/cos(f*x+e)
```

3.8.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.88

$$\int \left(a+b\sec^2(e+fx)\right)\sin^6(e+fx)\,dx$$
$$=\frac{15\,(a-6\,b)fx\cos\left(fx+e\right)-\left(8\,a\cos\left(fx+e\right)^6-2\,(13\,a-6\,b)\cos\left(fx+e\right)^4+3\,(11\,a-18\,b)\cos\left(fx+e\right.\right.}{48\,f\cos\left(fx+e\right)}$$

input
```
integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^6,x, algorithm="fricas")
```

output
```
1/48*(15*(a - 6*b)*f*x*cos(f*x + e) - (8*a*cos(f*x + e)^6 - 2*(13*a - 6*b)
*cos(f*x + e)^4 + 3*(11*a - 18*b)*cos(f*x + e)^2 - 48*b)*sin(f*x + e))/(f*
cos(f*x + e))
```

3.8.6 Sympy [F(-1)]

Timed out.

$$\int (a + b\sec^2(e + fx))\sin^6(e + fx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e)**2)*sin(f*x+e)**6,x)
```

output
```
Timed out
```

3.8.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.13

$$\int (a + b\sec^2(e + fx))\sin^6(e + fx)\,dx$$
$$= \frac{15\,(fx + e)(a - 6\,b) + 48\,b\tan(fx + e) - \frac{3\,(11\,a - 18\,b)\tan(fx+e)^5 + 8\,(5\,a - 12\,b)\tan(fx+e)^3 + 3\,(5\,a - 14\,b)\tan(fx+e)}{\tan(fx+e)^6 + 3\tan(fx+e)^4 + 3\tan(fx+e)^2 + 1}}{48\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^6,x, algorithm="maxima")
```

output
```
1/48*(15*(f*x + e)*(a - 6*b) + 48*b*tan(f*x + e) - (3*(11*a - 18*b)*tan(f*
x + e)^5 + 8*(5*a - 12*b)*tan(f*x + e)^3 + 3*(5*a - 14*b)*tan(f*x + e))/(t
an(f*x + e)^6 + 3*tan(f*x + e)^4 + 3*tan(f*x + e)^2 + 1))/f
```

3.8.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.06

$$\int \left(a + b\sec^2(e+fx)\right)\sin^6(e+fx)\,dx$$
$$= \frac{15\,(fx+e)(a-6\,b) + 48\,b\tan\left(fx+e\right) - \frac{33\,a\tan(fx+e)^5 - 54\,b\tan(fx+e)^5 + 40\,a\tan(fx+e)^3 - 96\,b\tan(fx+e)^3 + 15\,a\tan(fx+e) - 42\,b\tan(fx+e)}{\left(\tan(fx+e)^2+1\right)^3}}{48\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^6,x, algorithm="giac")
```

output
```
1/48*(15*(f*x + e)*(a - 6*b) + 48*b*tan(f*x + e) - (33*a*tan(f*x + e)^5 -
54*b*tan(f*x + e)^5 + 40*a*tan(f*x + e)^3 - 96*b*tan(f*x + e)^3 + 15*a*tan
(f*x + e) - 42*b*tan(f*x + e))/(tan(f*x + e)^2 + 1)^3)/f
```

3.8.9 Mupad [B] (verification not implemented)

Time = 18.47 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.07

$$\int \left(a + b\sec^2(e+fx)\right)\sin^6(e+fx)\,dx$$
$$= x\left(\frac{5\,a}{16} - \frac{15\,b}{8}\right) - \frac{\left(\frac{11\,a}{16} - \frac{9\,b}{8}\right)\tan(e+f\,x)^5 + \left(\frac{5\,a}{6} - 2\,b\right)\tan(e+f\,x)^3 + \left(\frac{5\,a}{16} - \frac{7\,b}{8}\right)\tan(e+f\,x)}{f\,\left(\tan\left(e+f\,x\right)^6 + 3\tan\left(e+f\,x\right)^4 + 3\tan\left(e+f\,x\right)^2 + 1\right)} + \frac{b\tan(e+f\,x)}{f}$$

input
```
int(sin(e + f*x)^6*(a + b/cos(e + f*x)^2),x)
```

output
```
x*((5*a)/16 - (15*b)/8) - (tan(e + f*x)^3*((5*a)/6 - 2*b) + tan(e + f*x)^5
*((11*a)/16 - (9*b)/8) + tan(e + f*x)*((5*a)/16 - (7*b)/8))/(f*(3*tan(e +
f*x)^2 + 3*tan(e + f*x)^4 + tan(e + f*x)^6 + 1)) + (b*tan(e + f*x))/f
```

3.9 $\int (a + b\sec^2(e+fx))\sin^4(e+fx)\,dx$

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3.9.1 Optimal result

Integrand size = 21, antiderivative size = 70

$$\int (a + b\sec^2(e+fx))\sin^4(e+fx)\,dx = \frac{3}{8}(a-4b)x - \frac{(5a-4b)\cos(e+fx)\sin(e+fx)}{8f} + \frac{a\cos^3(e+fx)\sin(e+fx)}{4f} + \frac{b\tan(e+fx)}{f}$$

output
```
3/8*(a-4*b)*x-1/8*(5*a-4*b)*cos(f*x+e)*sin(f*x+e)/f+1/4*a*cos(f*x+e)^3*sin
(f*x+e)/f+b*tan(f*x+e)/f
```

3.9.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.77

$$\int (a + b\sec^2(e+fx))\sin^4(e+fx)\,dx = \frac{12(a-4b)(e+fx) - 8(a-b)\sin(2(e+fx)) + a\sin(4(e+fx)) + 32b\tan(e+fx)}{32f}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^4,x]
```

output
```
(12*(a - 4*b)*(e + f*x) - 8*(a - b)*Sin[2*(e + f*x)] + a*Sin[4*(e + f*x)]
+ 32*b*Tan[e + f*x])/(32*f)
```

3.9.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4620, 360, 1471, 299, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \sin(e+fx)^4\left(a+b\sec(e+fx)^2\right)dx$$

$$\downarrow 4620$$

$$\frac{\int \frac{\tan^4(e+fx)(b\tan^2(e+fx)+a+b)}{(\tan^2(e+fx)+1)^3}d\tan(e+fx)}{f}$$

$$\downarrow 360$$

$$\frac{\frac{a\tan(e+fx)}{4(\tan^2(e+fx)+1)^2}-\frac{1}{4}\int \frac{-4b\tan^4(e+fx)-4a\tan^2(e+fx)+a}{(\tan^2(e+fx)+1)^2}d\tan(e+fx)}{f}$$

$$\downarrow 1471$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\int \frac{8b\tan^2(e+fx)+3a-4b}{\tan^2(e+fx)+1}d\tan(e+fx)-\frac{(5a-4b)\tan(e+fx)}{2(\tan^2(e+fx)+1)}\right)+\frac{a\tan(e+fx)}{4(\tan^2(e+fx)+1)^2}}{f}$$

$$\downarrow 299$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(3(a-4b)\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)+8b\tan(e+fx)\right)-\frac{(5a-4b)\tan(e+fx)}{2(\tan^2(e+fx)+1)}\right)+\frac{a\tan(e+fx)}{4(\tan^2(e+fx)+1)^2}}{f}$$

$$\downarrow 216$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}(3(a-4b)\arctan(\tan(e+fx))+8b\tan(e+fx))-\frac{(5a-4b)\tan(e+fx)}{2(\tan^2(e+fx)+1)}\right)+\frac{a\tan(e+fx)}{4(\tan^2(e+fx)+1)^2}}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^4,x]
```

output
```
((a*Tan[e + f*x])/(4*(1 + Tan[e + f*x]^2)^2) + ((3*(a - 4*b)*ArcTan[Tan[e
+ f*x]] + 8*b*Tan[e + f*x])/2 - ((5*a - 4*b)*Tan[e + f*x])/(2*(1 + Tan[e +
 f*x]^2)))/4)/f
```

3.9.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.9.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01

method	result
parallelrisch	$\frac{(-7a+8b)\sin(3fx+3e)+\sin(5fx+5e)a+24fx(a-4b)\cos(fx+e)-8\sin(fx+e)(a-9b)}{64f\cos(fx+e)}$
derivativedivides	$\frac{a\left(-\frac{\left(\sin(fx+e)^3+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+b\left(\frac{\sin(fx+e)^5}{\cos(fx+e)}+\left(\sin(fx+e)^3+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)-\frac{3fx}{2}-\frac{3e}{2}\right.}{f}$
default	$\frac{a\left(-\frac{\left(\sin(fx+e)^3+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+b\left(\frac{\sin(fx+e)^5}{\cos(fx+e)}+\left(\sin(fx+e)^3+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)-\frac{3fx}{2}-\frac{3e}{2}\right.}{f}$
parts	$\frac{a\left(-\frac{\left(\sin(fx+e)^3+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)}{f}+\frac{b\left(\frac{\sin(fx+e)^5}{\cos(fx+e)}+\left(\sin(fx+e)^3+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)-\frac{3fx}{2}-\right.}{f}$
risch	$\frac{3ax}{8}-\frac{3xb}{2}+\frac{ie^{2i(fx+e)}a}{8f}-\frac{ie^{2i(fx+e)}b}{8f}-\frac{ie^{-2i(fx+e)}a}{8f}+\frac{ie^{-2i(fx+e)}b}{8f}+\frac{2ib}{f\left(e^{2i(fx+e)}+1\right)}+\frac{\sin(4fx+4e)a}{32f}$
norman	$\frac{\left(-\frac{3a}{8}+\frac{3b}{2}\right)x+\left(-\frac{9a}{8}+\frac{9b}{2}\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+\left(-\frac{3a}{4}+3b\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+\left(\frac{3a}{4}-3b\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6+\left(\frac{3a}{8}-\frac{3b}{2}\right)x\tan\left(\frac{fx}{2}+\right.}{\left(\tan\right.}$

input

```
int((a+b*sec(f*x+e)^2)*sin(f*x+e)^4,x,method=_RETURNVERBOSE)
```

output

```
1/64*((-7*a+8*b)*sin(3*f*x+3*e)+sin(5*f*x+5*e)*a+24*f*x*(a-4*b)*cos(f*x+e)
-8*sin(f*x+e)*(a-9*b))/f/cos(f*x+e)
```

3.9.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97

$$\int (a + b\sec^2(e+fx))\sin^4(e+fx)\,dx$$
$$= \frac{3\,(a-4\,b)fx\cos(fx+e) + \left(2\,a\cos(fx+e)^4 - (5\,a-4\,b)\cos(fx+e)^2 + 8\,b\right)\sin(fx+e)}{8\,f\cos(fx+e)}$$

input
```
integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^4,x, algorithm="fricas")
```

output
```
1/8*(3*(a - 4*b)*f*x*cos(f*x + e) + (2*a*cos(f*x + e)^4 - (5*a - 4*b)*cos(
f*x + e)^2 + 8*b)*sin(f*x + e))/(f*cos(f*x + e))
```

3.9.6 Sympy [F]

$$\int (a + b\sec^2(e+fx))\sin^4(e+fx)\,dx = \int (a + b\sec^2(e+fx))\sin^4(e+fx)\,dx$$

input
```
integrate((a+b*sec(f*x+e)**2)*sin(f*x+e)**4,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)*sin(e + f*x)**4, x)
```

3.9.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.17

$$\int (a + b\sec^2(e+fx))\sin^4(e+fx)\,dx$$
$$= \frac{3\,(fx+e)(a-4\,b) + 8\,b\tan(fx+e) - \frac{(5\,a-4\,b)\tan(fx+e)^3 + (3\,a-4\,b)\tan(fx+e)}{\tan(fx+e)^4 + 2\tan(fx+e)^2 + 1}}{8\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^4,x, algorithm="maxima")
```

output
```
1/8*(3*(f*x + e)*(a - 4*b) + 8*b*tan(f*x + e) - ((5*a - 4*b)*tan(f*x + e)^
3 + (3*a - 4*b)*tan(f*x + e))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1))/f
```

3.9.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.17

$$\int \left(a + b\sec^2(e+fx)\right)\sin^4(e+fx)\,dx$$
$$= \frac{3\,(fx+e)(a-4\,b) + 8\,b\tan\left(fx+e\right) - \frac{5\,a\tan(fx+e)^3 - 4\,b\tan(fx+e)^3 + 3\,a\tan(fx+e) - 4\,b\tan(fx+e)}{\left(\tan(fx+e)^2+1\right)^2}}{8\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^4,x, algorithm="giac")
```

output
```
1/8*(3*(f*x + e)*(a - 4*b) + 8*b*tan(f*x + e) - (5*a*tan(f*x + e)^3 - 4*b*
tan(f*x + e)^3 + 3*a*tan(f*x + e) - 4*b*tan(f*x + e))/(tan(f*x + e)^2 + 1)
^2)/f
```

3.9.9 Mupad [B] (verification not implemented)

Time = 17.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.13

$$\int \left(a + b\sec^2(e+fx)\right)\sin^4(e+fx)\,dx = x\left(\frac{3\,a}{8} - \frac{3\,b}{2}\right) - \frac{\left(\frac{5\,a}{8} - \frac{b}{2}\right)\tan(e+f\,x)^3 + \left(\frac{3\,a}{8} - \frac{b}{2}\right)\tan(e+f\,x)}{f\,\left(\tan\left(e+f\,x\right)^4 + 2\tan\left(e+f\,x\right)^2 + 1\right)} + \frac{b\tan(e+f\,x)}{f}$$

input
```
int(sin(e + f*x)^4*(a + b/cos(e + f*x)^2),x)
```

output
```
x*((3*a)/8 - (3*b)/2) - (tan(e + f*x)^3*((5*a)/8 - b/2) + tan(e + f*x)*((3
*a)/8 - b/2))/(f*(2*tan(e + f*x)^2 + tan(e + f*x)^4 + 1)) + (b*tan(e + f*x
))/f
```

3.10 $\int (a + b\sec^2(e+fx))\sin^2(e+fx)\,dx$

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3.10.1 Optimal result

Integrand size = 21, antiderivative size = 42

$$\int (a + b\sec^2(e+fx))\sin^2(e+fx)\,dx = \frac{1}{2}(a-2b)x - \frac{a\cos(e+fx)\sin(e+fx)}{2f} + \frac{b\tan(e+fx)}{f}$$

output
```
1/2*(a-2*b)*x-1/2*a*cos(f*x+e)*sin(f*x+e)/f+b*tan(f*x+e)/f
```

3.10.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.29

$$\int (a + b\sec^2(e+fx))\sin^2(e+fx)\,dx = \frac{a(e+fx)}{2f} - \frac{b\arctan(\tan(e+fx))}{f} - \frac{a\sin(2(e+fx))}{4f} + \frac{b\tan(e+fx)}{f}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^2,x]
```

output
```
(a*(e + f*x))/(2*f) - (b*ArcTan[Tan[e + f*x]])/f - (a*Sin[2*(e + f*x)])/(4
*f) + (b*Tan[e + f*x])/f
```

3.10.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.31, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4620, 360, 25, 299, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \sin(e+fx)^2\left(a+b\sec(e+fx)^2\right)dx$$

$$\downarrow 4620$$

$$\frac{\int \frac{\tan^2(e+fx)(b\tan^2(e+fx)+a+b)}{(\tan^2(e+fx)+1)^2}d\tan(e+fx)}{f}$$

$$\downarrow 360$$

$$\frac{-\frac{1}{2}\int -\frac{2b\tan^2(e+fx)+a}{\tan^2(e+fx)+1}d\tan(e+fx) - \frac{a\tan(e+fx)}{2(\tan^2(e+fx)+1)}}{f}$$

$$\downarrow 25$$

$$\frac{\frac{1}{2}\int \frac{2b\tan^2(e+fx)+a}{\tan^2(e+fx)+1}d\tan(e+fx) - \frac{a\tan(e+fx)}{2(\tan^2(e+fx)+1)}}{f}$$

$$\downarrow 299$$

$$\frac{\frac{1}{2}\left((a-2b)\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx) + 2b\tan(e+fx)\right) - \frac{a\tan(e+fx)}{2(\tan^2(e+fx)+1)}}{f}$$

$$\downarrow 216$$

$$\frac{\frac{1}{2}((a-2b)\arctan(\tan(e+fx)) + 2b\tan(e+fx)) - \frac{a\tan(e+fx)}{2(\tan^2(e+fx)+1)}}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)*Sin[e + f*x]^2,x]
```

output
```
(((a - 2*b)*ArcTan[Tan[e + f*x]] + 2*b*Tan[e + f*x])/2 - (a*Tan[e + f*x])/
(2*(1 + Tan[e + f*x]^2)))/f
```

3.10.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.10.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{a\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+b(\tan(fx+e)-fx-e)}{f}$
default	$\frac{a\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+b(\tan(fx+e)-fx-e)}{f}$
parts	$\frac{a\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{b(\tan(fx+e)-fx-e)}{f}$
parallelrisch	$\frac{-\sin(3fx+3e)a+4fx(a-2b)\cos(fx+e)-\sin(fx+e)(a-8b)}{8f\cos(fx+e)}$
risch	$\frac{ax}{2}-xb+\frac{ie^{2i(fx+e)}a}{8f}-\frac{ie^{-2i(fx+e)}a}{8f}+\frac{2ib}{f\left(e^{2i(fx+e)}+1\right)}$
norman	$\frac{\left(-\frac{a}{2}+b\right)x+\frac{(a-2b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{(a-2b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{f}+\left(-\frac{a}{2}+b\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+\left(\frac{a}{2}-b\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+\left(\frac{a}{2}-b\right)x\tan\left(\frac{f}{}\right.}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)^2}$

input
```
int((a+b*sec(f*x+e)^2)*sin(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(a*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+b*(tan(f*x+e)-f*x-e))
```

3.10.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.19

$$\int \left(a + b\sec^2(e+fx)\right)\sin^2(e+fx)\,dx$$
$$= \frac{(a-2\,b)fx\cos\left(fx+e\right) - \left(a\cos\left(fx+e\right)^2 - 2\,b\right)\sin\left(fx+e\right)}{2\,f\cos\left(fx+e\right)}$$

input
```
integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^2,x, algorithm="fricas")
```

output
```
1/2*((a - 2*b)*f*x*cos(f*x + e) - (a*cos(f*x + e)^2 - 2*b)*sin(f*x + e))/(
f*cos(f*x + e))
```

3.10.6 Sympy [F]

$$\int (a + b \sec^2(e + fx)) \sin^2(e + fx)\, dx = \int (a + b \sec^2(e + fx)) \sin^2(e + fx)\, dx$$

input
```
integrate((a+b*sec(f*x+e)**2)*sin(f*x+e)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)*sin(e + f*x)**2, x)
```

3.10.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.12

$$\int (a + b \sec^2(e + fx)) \sin^2(e + fx)\, dx = \frac{(fx + e)(a - 2\,b) + 2\,b \tan(fx + e) - \frac{a \tan(fx+e)}{\tan(fx+e)^2+1}}{2\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^2,x, algorithm="maxima")
```

output
```
1/2*((f*x + e)*(a - 2*b) + 2*b*tan(f*x + e) - a*tan(f*x + e)/(tan(f*x + e)
^2 + 1))/f
```

3.10.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.12

$$\int (a + b \sec^2(e + fx)) \sin^2(e + fx)\, dx = \frac{(fx + e)(a - 2\,b) + 2\,b \tan(fx + e) - \frac{a \tan(fx+e)}{\tan(fx+e)^2+1}}{2\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)*sin(f*x+e)^2,x, algorithm="giac")
```

output
```
1/2*((f*x + e)*(a - 2*b) + 2*b*tan(f*x + e) - a*tan(f*x + e)/(tan(f*x + e)
^2 + 1))/f
```

3.10.9 Mupad [B] (verification not implemented)

Time = 17.63 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

$$\int \left(a + b\sec^2(e+fx)\right)\sin^2(e+fx)\,dx = \frac{b\tan(e+f\,x) - \frac{a\,\sin(2\,e+2\,f\,x)}{4} + f\,x\left(\frac{a}{2} - b\right)}{f}$$

input
```
int(sin(e + f*x)^2*(a + b/cos(e + f*x)^2),x)
```

output
```
(b*tan(e + f*x) - (a*sin(2*e + 2*f*x))/4 + f*x*(a/2 - b))/f
```

3.11 $\int (a + b\sec^2(e + fx))\, dx$

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3.11.1 Optimal result

Integrand size = 12, antiderivative size = 15

$$\int (a + b\sec^2(e + fx))\, dx = ax + \frac{b\tan(e + fx)}{f}$$

output
```
a*x+b*tan(f*x+e)/f
```

3.11.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int (a + b\sec^2(e + fx))\, dx = ax + \frac{b\tan(e + fx)}{f}$$

input
```
Integrate[a + b*Sec[e + f*x]^2,x]
```

output
```
a*x + (b*Tan[e + f*x])/f
```

3.11.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + b\sec^2(e+fx))\,dx$$

$$\downarrow \text{2009}$$

$$ax + \frac{b\tan(e+fx)}{f}$$

input
```
Int[a + b*Sec[e + f*x]^2,x]
```

output
```
a*x + (b*Tan[e + f*x])/f
```

3.11.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.11.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result	size
default	$ax + \frac{b\tan(fx+e)}{f}$	16
parts	$ax + \frac{b\tan(fx+e)}{f}$	16
derivativedivides	$\frac{(fx+e)a+b\tan(fx+e)}{f}$	21
risch	$ax + \frac{2ib}{f(e^{2i(fx+e)}+1)}$	25
parallelrisch	$-\frac{2b\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)} + ax$	35
norman	$\frac{ax\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2 - ax - \frac{2b\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1}$	51

input
```
int(a+b*sec(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
a*x+b*tan(f*x+e)/f
```

3.11.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(15) = 30$.

Time $= 0.24$ (sec) , antiderivative size $= 31$, normalized size of antiderivative $= 2.07$

$$\int \left(a + b\sec^2(e + fx)\right)\, dx = \frac{afx\cos\left(fx + e\right) + b\sin\left(fx + e\right)}{f\cos\left(fx + e\right)}$$

input
```
integrate(a+b*sec(f*x+e)^2,x, algorithm="fricas")
```

output
```
(a*f*x*cos(f*x + e) + b*sin(f*x + e))/(f*cos(f*x + e))
```

3.11.6 Sympy [F]

$$\int \left(a + b\sec^2(e + fx)\right)\, dx = \int \left(a + b\sec^2\left(e + fx\right)\right)\, dx$$

input
```
integrate(a+b*sec(f*x+e)**2,x)
```

output
```
Integral(a + b*sec(e + f*x)**2, x)
```

3.11.7 Maxima [A] (verification not implemented)

Time $= 0.19$ (sec) , antiderivative size $= 15$, normalized size of antiderivative $= 1.00$

$$\int \left(a + b\sec^2(e + fx)\right)\, dx = ax + \frac{b\tan\left(fx + e\right)}{f}$$

input
```
integrate(a+b*sec(f*x+e)^2,x, algorithm="maxima")
```

output
```
a*x + b*tan(f*x + e)/f
```

3.11.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \left(a + b\sec^2(e + fx)\right)\, dx = ax + \frac{b\tan\left(fx + e\right)}{f}$$

input
```
integrate(a+b*sec(f*x+e)^2,x, algorithm="giac")
```

output
```
a*x + b*tan(f*x + e)/f
```

3.11.9 Mupad [B] (verification not implemented)

Time = 17.71 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \left(a + b\sec^2(e + fx)\right)\, dx = \frac{b\tan(e + f\,x) + a\,f\,x}{f}$$

input
```
int(a + b/cos(e + f*x)^2,x)
```

output
```
(b*tan(e + f*x) + a*f*x)/f
```

3.12 $\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

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3.12.1 Optimal result

Integrand size = 21, antiderivative size = 26

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{(a+b)\cot(e+fx)}{f}+\frac{b\tan(e+fx)}{f}$$

output
```
-(a+b)*cot(f*x+e)/f+b*tan(f*x+e)/f
```

3.12.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.38

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{a\cot(e+fx)}{f}-\frac{b\cot(e+fx)}{f}+\frac{b\tan(e+fx)}{f}$$

input
```
Integrate[Csc[e + f*x]^2*(a + b*Sec[e + f*x]^2),x]
```

output
```
-((a*Cot[e + f*x])/f) - (b*Cot[e + f*x])/f + (b*Tan[e + f*x])/f
```

3.12.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4620, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \frac{a+b\sec(e+fx)^2}{\sin(e+fx)^2}dx$$

$$\downarrow 4620$$

$$\frac{\int \cot^2(e+fx)\left(b\tan^2(e+fx)+a+b\right)d\tan(e+fx)}{f}$$

$$\downarrow 244$$

$$\frac{\int \left((a+b)\cot^2(e+fx)+b\right)d\tan(e+fx)}{f}$$

$$\downarrow 2009$$

$$\frac{b\tan(e+fx)-(a+b)\cot(e+fx)}{f}$$

input
```
Int[Csc[e + f*x]^2*(a + b*Sec[e + f*x]^2),x]
```

output
```
(-((a + b)*Cot[e + f*x]) + b*Tan[e + f*x])/f
```

3.12.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.12.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.65

method	result	size
derivativedivides	$\frac{-\cot(fx+e)a+b\left(\frac{1}{\sin(fx+e)\cos(fx+e)}-2\cot(fx+e)\right)}{f}$	43
default	$\frac{-\cot(fx+e)a+b\left(\frac{1}{\sin(fx+e)\cos(fx+e)}-2\cot(fx+e)\right)}{f}$	43
risch	$-\frac{2i\left(a\,\mathrm{e}^{2i(fx+e)}+a+2b\right)}{f\left(\mathrm{e}^{2i(fx+e)}+1\right)\left(\mathrm{e}^{2i(fx+e)}-1\right)}$	49
parallelrisch	$\frac{\left((a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+(-2a-6b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+a+b\right)\cot\left(\frac{fx}{2}+\frac{e}{2}\right)}{2f\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-2f}$	68
norman	$\frac{\frac{a+b}{2f}+\frac{(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{2f}-\frac{(a+3b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{f}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)}$	77

input
```
int(csc(f*x+e)^2*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-cot(f*x+e)*a+b*(1/sin(f*x+e)/cos(f*x+e)-2*cot(f*x+e)))
```

3.12.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.50

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{(a+2\,b)\cos\left(fx+e\right)^2-b}{f\cos\left(fx+e\right)\sin\left(fx+e\right)}$$

input
```
integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
-((a + 2*b)*cos(f*x + e)^2 - b)/(f*cos(f*x + e)*sin(f*x + e))
```

3.12.6 Sympy [F]

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)\csc^2\left(e+fx\right)dx$$

input
```
integrate(csc(f*x+e)**2*(a+b*sec(f*x+e)**2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)*csc(e + f*x)**2, x)
```

3.12.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{b\tan\left(fx+e\right)-\frac{a+b}{\tan(fx+e)}}{f}$$

input
```
integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
(b*tan(f*x + e) - (a + b)/tan(f*x + e))/f
```

3.12.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{b\tan(fx+e)-\frac{a+b}{\tan(fx+e)}}{f}$$

input
```
integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
(b*tan(f*x + e) - (a + b)/tan(f*x + e))/f
```

3.12.9 Mupad [B] (verification not implemented)

Time = 19.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{b\tan(e+f\,x)}{f}-\frac{a+b}{f\tan(e+f\,x)}$$

input
```
int((a + b/cos(e + f*x)^2)/sin(e + f*x)^2,x)
```

output
```
(b*tan(e + f*x))/f - (a + b)/(f*tan(e + f*x))
```

3.13 $\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

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3.13.1 Optimal result

Integrand size = 21, antiderivative size = 46

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{(a+2b)\cot(e+fx)}{f} - \frac{(a+b)\cot^3(e+fx)}{3f} + \frac{b\tan(e+fx)}{f}$$

output
```
-(a+2*b)*cot(f*x+e)/f-1/3*(a+b)*cot(f*x+e)^3/f+b*tan(f*x+e)/f
```

3.13.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.83

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{2a\cot(e+fx)}{3f} - \frac{5b\cot(e+fx)}{3f} - \frac{a\cot(e+fx)\csc^2(e+fx)}{3f} - \frac{b\cot(e+fx)\csc^2(e+fx)}{3f} + \frac{b\tan(e+fx)}{f}$$

input
```
Integrate[Csc[e + f*x]^4*(a + b*Sec[e + f*x]^2),x]
```

output
```
(-2*a*Cot[e + f*x])/(3*f) - (5*b*Cot[e + f*x])/(3*f) - (a*Cot[e + f*x]*Csc
[e + f*x]^2)/(3*f) - (b*Cot[e + f*x]*Csc[e + f*x]^2)/(3*f) + (b*Tan[e + f*
x])/f
```

3.13.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4620, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \frac{a+b\sec(e+fx)^2}{\sin(e+fx)^4}dx$$

$$\downarrow 4620$$

$$\frac{\int \cot^4(e+fx)\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)d\tan(e+fx)}{f}$$

$$\downarrow 355$$

$$\frac{\int \left((a+b)\cot^4(e+fx)+(a+2b)\cot^2(e+fx)+b\right)d\tan(e+fx)}{f}$$

$$\downarrow 2009$$

$$\frac{-\frac{1}{3}(a+b)\cot^3(e+fx)-(a+2b)\cot(e+fx)+b\tan(e+fx)}{f}$$

input
```
Int[Csc[e + f*x]^4*(a + b*Sec[e + f*x]^2),x]
```

output
```
(-((a + 2*b)*Cot[e + f*x]) - ((a + b)*Cot[e + f*x]^3)/3 + b*Tan[e + f*x])/
f
```

3.13.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.13.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.59

method	result
derivativedivides	$\frac{a\left(-\frac{2}{3}-\frac{\csc(fx+e)^2}{3}\right)\cot(fx+e)+b\left(-\frac{1}{3\sin(fx+e)^3\cos(fx+e)}+\frac{4}{3\sin(fx+e)\cos(fx+e)}-\frac{8\cot(fx+e)}{3}\right)}{f}$
default	$\frac{a\left(-\frac{2}{3}-\frac{\csc(fx+e)^2}{3}\right)\cot(fx+e)+b\left(-\frac{1}{3\sin(fx+e)^3\cos(fx+e)}+\frac{4}{3\sin(fx+e)\cos(fx+e)}-\frac{8\cot(fx+e)}{3}\right)}{f}$
risch	$\frac{4i\left(3a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+8b\,e^{2i(fx+e)}-a-4b\right)}{3f\left(e^{2i(fx+e)}-1\right)^3\left(e^{2i(fx+e)}+1\right)}$
parallelrisch	$\frac{\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^3\left((a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8+(8a+20b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6+(-18a-90b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+(8a+20b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+a-\right.}{24f\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-24f}$
norman	$\frac{\frac{a+b}{24f}+\frac{(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8}{24f}-\frac{3(a+5b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{4f}+\frac{(2a+5b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{6f}+\frac{(2a+5b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{6f}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)}$

input
```
int(csc(f*x+e)^4*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(a*(-2/3-1/3*csc(f*x+e)^2)*cot(f*x+e)+b*(-1/3/sin(f*x+e)^3/cos(f*x+e)+
4/3/sin(f*x+e)/cos(f*x+e)-8/3*cot(f*x+e)))
```

3.13.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.43

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{2\,(a+4\,b)\cos(fx+e)^4-3\,(a+4\,b)\cos(fx+e)^2+3\,b}{3\left(f\cos(fx+e)^3-f\cos(fx+e)\right)\sin(fx+e)}$$

input
```
integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
-1/3*(2*(a + 4*b)*cos(f*x + e)^4 - 3*(a + 4*b)*cos(f*x + e)^2 + 3*b)/((f*c
os(f*x + e)^3 - f*cos(f*x + e))*sin(f*x + e))
```

3.13.6 Sympy [F]

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \int \left(a+b\sec^2(e+fx)\right)\csc^4(e+fx)\,dx$$

input
```
integrate(csc(f*x+e)**4*(a+b*sec(f*x+e)**2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)*csc(e + f*x)**4, x)
```

3.13.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{3\,b\tan(fx+e)-\frac{3\,(a+2\,b)\tan(fx+e)^2+a+b}{\tan(fx+e)^3}}{3\,f}$$

input
```
integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/3*(3*b*tan(f*x + e) - (3*(a + 2*b)*tan(f*x + e)^2 + a + b)/tan(f*x + e)^
3)/f
```

3.13.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{3\,b\tan\left(fx+e\right) - \frac{3\,a\tan(fx+e)^2+6\,b\tan(fx+e)^2+a+b}{\tan(fx+e)^3}}{3\,f}$$

input
```
integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/3*(3*b*tan(f*x + e) - (3*a*tan(f*x + e)^2 + 6*b*tan(f*x + e)^2 + a + b)/
tan(f*x + e)^3)/f
```

3.13.9 Mupad [B] (verification not implemented)

Time = 19.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{b\tan(e+f\,x)}{f} - \frac{(a+2\,b)\,\tan(e+f\,x)^2+\frac{a}{3}+\frac{b}{3}}{f\tan\left(e+f\,x\right)^3}$$

input
```
int((a + b/cos(e + f*x)^2)/sin(e + f*x)^4,x)
```

output
```
(b*tan(e + f*x))/f - (a/3 + b/3 + tan(e + f*x)^2*(a + 2*b))/(f*tan(e + f*x
)^3)
```

3.14 $\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

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3.14.1 Optimal result

Integrand size = 21, antiderivative size = 68

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{(a+3b)\cot(e+fx)}{f} - \frac{(2a+3b)\cot^3(e+fx)}{3f} - \frac{(a+b)\cot^5(e+fx)}{5f} + \frac{b\tan(e+fx)}{f}$$

output
```
-(a+3*b)*cot(f*x+e)/f-1/3*(2*a+3*b)*cot(f*x+e)^3/f-1/5*(a+b)*cot(f*x+e)^5/
f+b*tan(f*x+e)/f
```

3.14.2 Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.88

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{8a\cot(e+fx)}{15f} - \frac{11b\cot(e+fx)}{5f} - \frac{4a\cot(e+fx)\csc^2(e+fx)}{15f} - \frac{3b\cot(e+fx)\csc^2(e+fx)}{5f} - \frac{a\cot(e+fx)\csc^4(e+fx)}{5f} - \frac{b\cot(e+fx)\csc^4(e+fx)}{5f} + \frac{b\tan(e+fx)}{f}$$

input
```
Integrate[Csc[e + f*x]^6*(a + b*Sec[e + f*x]^2),x]
```

output
```
(-8*a*Cot[e + f*x])/(15*f) - (11*b*Cot[e + f*x])/(5*f) - (4*a*Cot[e + f*x]
*Csc[e + f*x]^2)/(15*f) - (3*b*Cot[e + f*x]*Csc[e + f*x]^2)/(5*f) - (a*Cot
[e + f*x]*Csc[e + f*x]^4)/(5*f) - (b*Cot[e + f*x]*Csc[e + f*x]^4)/(5*f) +
(b*Tan[e + f*x])/f
```

3.14.3 Rubi [A] (verified)

Time $= 0.24$ (sec) , antiderivative size $= 60$, normalized size of antiderivative $= 0.88$, number of steps used $= 5$, number of rules used $= 4$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used $= \{3042, 4620, 355, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \frac{a+b\sec(e+fx)^2}{\sin(e+fx)^6}dx$$

$$\downarrow 4620$$

$$\frac{\int \cot^6(e+fx)\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)d\tan(e+fx)}{f}$$

$$\downarrow 355$$

$$\frac{\int \left((a+b)\cot^6(e+fx)+(2a+3b)\cot^4(e+fx)+(a+3b)\cot^2(e+fx)+b\right)d\tan(e+fx)}{f}$$

$$\downarrow 2009$$

$$\frac{-\frac{1}{5}(a+b)\cot^5(e+fx)-\frac{1}{3}(2a+3b)\cot^3(e+fx)-(a+3b)\cot(e+fx)+b\tan(e+fx)}{f}$$

input
```
Int[Csc[e + f*x]^6*(a + b*Sec[e + f*x]^2),x]
```

output
```
(-((a + 3*b)*Cot[e + f*x]) - ((2*a + 3*b)*Cot[e + f*x]^3)/3 - ((a + b)*Cot
[e + f*x]^5)/5 + b*Tan[e + f*x])/f
```

3.14.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
 + 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.14.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.31

method	result
parallelrisch	$-\frac{\left((a+6b)\cos(2fx+2e)+\frac{4(-a-6b)\cos(4fx+4e)}{5}+\frac{(a+6b)\cos(6fx+6e)}{5}+2a\right)\sec\left(\frac{fx}{2}+\frac{e}{2}\right)^5\csc\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{384f\cos(fx+e)}$
risch	$-\frac{16i\left(10a\,e^{6i(fx+e)}+5a\,e^{4i(fx+e)}+30b\,e^{4i(fx+e)}-4a\,e^{2i(fx+e)}-24b\,e^{2i(fx+e)}+a+6b\right)}{15f\left(e^{2i(fx+e)}-1\right)^5\left(e^{2i(fx+e)}+1\right)}$
derivativedivides	$\frac{a\left(-\frac{8}{15}-\frac{\csc(fx+e)^4}{5}-\frac{4\csc(fx+e)^2}{15}\right)\cot(fx+e)+b\left(-\frac{1}{5\sin(fx+e)^5\cos(fx+e)}-\frac{2}{5\sin(fx+e)^3\cos(fx+e)}+\frac{8}{5\sin(fx+e)\cos(fx+}\right.}{f}$
default	$\frac{a\left(-\frac{8}{15}-\frac{\csc(fx+e)^4}{5}-\frac{4\csc(fx+e)^2}{15}\right)\cot(fx+e)+b\left(-\frac{1}{5\sin(fx+e)^5\cos(fx+e)}-\frac{2}{5\sin(fx+e)^3\cos(fx+e)}+\frac{8}{5\sin(fx+e)\cos(fx+}\right.}{f}$
norman	$\frac{\frac{a+b}{160f}+\frac{(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{12}}{160f}-\frac{5(a+7b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{8f}+\frac{5(5a+21b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{96f}+\frac{5(5a+21b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8}{96f}+\frac{(11a+21b)\tan\left(\frac{fx}{2}+}{240f}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)}$

input
```
int(csc(f*x+e)^6*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/384*((a+6*b)*cos(2*f*x+2*e)+4/5*(-a-6*b)*cos(4*f*x+4*e)+1/5*(a+6*b)*cos
(6*f*x+6*e)+2*a)*sec(1/2*f*x+1/2*e)^5*csc(1/2*f*x+1/2*e)^5/f/cos(f*x+e)
```

3.14.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.34

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$= -\frac{8\,(a+6\,b)\cos(fx+e)^6 - 20\,(a+6\,b)\cos(fx+e)^4 + 15\,(a+6\,b)\cos(fx+e)^2 - 15\,b}{15\left(f\cos(fx+e)^5 - 2\,f\cos(fx+e)^3 + f\cos(fx+e)\right)\sin(fx+e)}$$

input
```
integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
-1/15*(8*(a + 6*b)*cos(f*x + e)^6 - 20*(a + 6*b)*cos(f*x + e)^4 + 15*(a +
6*b)*cos(f*x + e)^2 - 15*b)/((f*cos(f*x + e)^5 - 2*f*cos(f*x + e)^3 + f*co
s(f*x + e))*sin(f*x + e))
```

3.14.6 Sympy [F(-1)]

Timed out.

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**6*(a+b*sec(f*x+e)**2),x)
```

output
```
Timed out
```

3.14.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.94

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{15\,b\tan(fx+e)-\frac{15\,(a+3\,b)\tan(fx+e)^4+5\,(2\,a+3\,b)\tan(fx+e)^2+3\,a+3\,b}{\tan(fx+e)^5}}{15\,f}$$

input
```
integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/15*(15*b*tan(f*x + e) - (15*(a + 3*b)*tan(f*x + e)^4 + 5*(2*a + 3*b)*tan
(f*x + e)^2 + 3*a + 3*b)/tan(f*x + e)^5)/f
```

3.14.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.12

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{15\,b\tan(fx+e)-\frac{15\,a\tan(fx+e)^4+45\,b\tan(fx+e)^4+10\,a\tan(fx+e)^2+15\,b\tan(fx+e)^2+3\,a+3\,b}{\tan(fx+e)^5}}{15\,f}$$

input
```
integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/15*(15*b*tan(f*x + e) - (15*a*tan(f*x + e)^4 + 45*b*tan(f*x + e)^4 + 10*
a*tan(f*x + e)^2 + 15*b*tan(f*x + e)^2 + 3*a + 3*b)/tan(f*x + e)^5)/f
```

3.14.9 Mupad [B] (verification not implemented)

Time = 18.93 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{b\tan(e+f\,x)}{f}-\frac{(a+3\,b)\,\tan(e+f\,x)^4+\left(\frac{2\,a}{3}+b\right)\,\tan(e+f\,x)^2+\frac{a}{5}+\frac{b}{5}}{f\,\tan(e+f\,x)^5}$$

input
```
int((a + b/cos(e + f*x)^2)/sin(e + f*x)^6,x)
```

output
```
(b*tan(e + f*x))/f - (a/5 + b/5 + tan(e + f*x)^2*((2*a)/3 + b) + tan(e + f
*x)^4*(a + 3*b))/(f*tan(e + f*x)^5)
```

3.15 $\int (a+b\sec^2(e+fx))^2 \sin^5(e+fx)\,dx$

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3.15.1 Optimal result

Integrand size = 23, antiderivative size = 97

$$\int (a+b\sec^2(e+fx))^2 \sin^5(e+fx)\,dx = -\frac{(a^2-4ab+b^2)\cos(e+fx)}{f} + \frac{2a(a-b)\cos^3(e+fx)}{3f} - \frac{a^2\cos^5(e+fx)}{5f} + \frac{2(a-b)b\sec(e+fx)}{f} + \frac{b^2\sec^3(e+fx)}{3f}$$

output
```
-(a^2-4*a*b+b^2)*cos(f*x+e)/f+2/3*a*(a-b)*cos(f*x+e)^3/f-1/5*a^2*cos(f*x+e
)^5/f+2*(a-b)*b*sec(f*x+e)/f+1/3*b^2*sec(f*x+e)^3/f
```

3.15.2 Mathematica [A] (verified)

Time = 2.33 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.22

$$\int (a+b\sec^2(e+fx))^2 \sin^5(e+fx)\,dx = -\frac{(425a^2-4400ab+2000b^2+24(22a^2-215ab+120b^2)\cos(2(e+fx))+12(7a^2-60ab+20b^2)\cos(4(e$$
$$1920f$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^5,x]
```

output
```
-1/1920*((425*a^2 - 4400*a*b + 2000*b^2 + 24*(22*a^2 - 215*a*b + 120*b^2)*
Cos[2*(e + f*x)] + 12*(7*a^2 - 60*a*b + 20*b^2)*Cos[4*(e + f*x)] - 16*a^2*
Cos[6*(e + f*x)] + 40*a*b*Cos[6*(e + f*x)] + 3*a^2*Cos[8*(e + f*x)])*Sec[e
 + f*x]^3)/f
```

3.15.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4621, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int \sin(e+fx)^5\left(a+b\sec(e+fx)^2\right)^2 dx$$

$$\downarrow 4621$$

$$-\frac{\int \left(1-\cos^2(e+fx)\right)^2\left(a\cos^2(e+fx)+b\right)^2\sec^4(e+fx)d\cos(e+fx)}{f}$$

$$\downarrow 355$$

$$-\frac{\int \left(a^2\cos^4(e+fx)-2a(a-b)\cos^2(e+fx)+b^2\sec^4(e+fx)+2(a-b)b\sec^2(e+fx)+a^2\left(\frac{b(b-4a)}{a^2}+1\right)\right)d\cos}{f}$$

$$\downarrow 2009$$

$$-\frac{\left(a^2-4ab+b^2\right)\cos(e+fx)+\frac{1}{5}a^2\cos^5(e+fx)-\frac{2}{3}a(a-b)\cos^3(e+fx)-2b(a-b)\sec(e+fx)-\frac{1}{3}b^2\sec^3(e+}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^5,x]
```

output
```
-(((a^2 - 4*a*b + b^2)*Cos[e + f*x] - (2*a*(a - b)*Cos[e + f*x]^3)/3 + (a^
2*Cos[e + f*x]^5)/5 - 2*(a - b)*b*Sec[e + f*x] - (b^2*Sec[e + f*x]^3)/3)/f
)
```

3.15.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

3.15.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.60

method	result
derivativedivides	$\frac{-\frac{a^2\left(\frac{8}{3}+\sin(fx+e)^4+\frac{4\sin(fx+e)^2}{3}\right)\cos(fx+e)}{5}+2ab\left(\frac{\sin(fx+e)^6}{\cos(fx+e)}+\left(\frac{8}{3}+\sin(fx+e)^4+\frac{4\sin(fx+e)^2}{3}\right)\cos(fx+e)\right)+b^2\left(\frac{\sin}{3\cos}\right.}{f}$
default	$\frac{-\frac{a^2\left(\frac{8}{3}+\sin(fx+e)^4+\frac{4\sin(fx+e)^2}{3}\right)\cos(fx+e)}{5}+2ab\left(\frac{\sin(fx+e)^6}{\cos(fx+e)}+\left(\frac{8}{3}+\sin(fx+e)^4+\frac{4\sin(fx+e)^2}{3}\right)\cos(fx+e)\right)+b^2\left(\frac{\sin}{3\cos}\right.}{f}$
parts	$-\frac{a^2\left(\frac{8}{3}+\sin(fx+e)^4+\frac{4\sin(fx+e)^2}{3}\right)\cos(fx+e)}{5f}+\frac{b^2\left(\frac{\sin(fx+e)^6}{3\cos(fx+e)^3}-\frac{\sin(fx+e)^6}{\cos(fx+e)}-\left(\frac{8}{3}+\sin(fx+e)^4+\frac{4\sin(fx+e)^2}{3}\right)\cos\right.}{f}$
parallelrisch	$\frac{\left(-256a^2+2560ab-1280b^2\right)\cos(3fx+3e)+\left(-528a^2+5160ab-2880b^2\right)\cos(2fx+2e)+\left(-84a^2+720ab-240b^2\right)\cos(4fx+}{480f(\cos(3fx+3e)}$
norman	$\frac{\frac{16a^2-160ab+80b^2}{15f}+\frac{16\left(5a^2-2ab-7b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8}{3f}+\frac{2\left(16a^2-160ab+80b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{15f}-\frac{2\left(16a^2-160ab+80b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{15f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)^5}$
risch	$-\frac{a^2e^{5i(fx+e)}}{160f}+\frac{5e^{3i(fx+e)}a^2}{96f}-\frac{e^{3i(fx+e)}ab}{12f}-\frac{5e^{i(fx+e)}a^2}{16f}+\frac{7e^{i(fx+e)}ab}{4f}-\frac{e^{i(fx+e)}b^2}{2f}-\frac{5e^{-i(fx+e)}a^2}{16f}-$

input
```
int((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^5,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/5*a^2*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+2*a*b*(sin(f*
x+e)^6/cos(f*x+e)+(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e))+b^2*(1/3
*sin(f*x+e)^6/cos(f*x+e)^3-sin(f*x+e)^6/cos(f*x+e)-(8/3+sin(f*x+e)^4+4/3*s
in(f*x+e)^2)*cos(f*x+e)))
```

3.15.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

$$\int \left(a + b\sec^2(e+fx)\right)^2 \sin^5(e+fx)\,dx =$$
$$-\frac{3\,a^2\cos(fx+e)^8 - 10\,(a^2-ab)\cos(fx+e)^6 + 15\,(a^2-4\,ab+b^2)\cos(fx+e)^4 - 30\,(ab-b^2)\cos(fx+e)^2 - 5\,b^2}{15\,f\cos(fx+e)^3}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^5,x, algorithm="fricas")
```

output
```
-1/15*(3*a^2*cos(f*x + e)^8 - 10*(a^2 - a*b)*cos(f*x + e)^6 + 15*(a^2 - 4*
a*b + b^2)*cos(f*x + e)^4 - 30*(a*b - b^2)*cos(f*x + e)^2 - 5*b^2)/(f*cos(
f*x + e)^3)
```

3.15.6 Sympy [F(-1)]

Timed out.

$$\int \left(a + b\sec^2(e+fx)\right)^2 \sin^5(e+fx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e)**2)**2*sin(f*x+e)**5,x)
```

output
```
Timed out
```

3.15.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.92

$$\int \left(a+b\sec^2(e+fx)\right)^2 \sin^5(e+fx)\,dx =$$

$$-\frac{3\,a^2\cos(fx+e)^5-10\,(a^2-ab)\cos(fx+e)^3+15\,(a^2-4\,ab+b^2)\cos(fx+e)-\frac{5\left(6\,(ab-b^2)\cos(fx+e)^2+\right.}{\cos(fx+e)^3}}{15\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^5,x, algorithm="maxima")
```

output
```
-1/15*(3*a^2*cos(f*x + e)^5 - 10*(a^2 - a*b)*cos(f*x + e)^3 + 15*(a^2 - 4*
a*b + b^2)*cos(f*x + e) - 5*(6*(a*b - b^2)*cos(f*x + e)^2 + b^2)/cos(f*x +
 e)^3)/f
```

3.15.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 414 vs. 2(91) = 182.

Time = 0.41 (sec) , antiderivative size = 414, normalized size of antiderivative = 4.27

$$\int \left(a+b\sec^2(e+fx)\right)^2 \sin^5(e+fx)\,dx$$

$$=\frac{2\left(\frac{5\left(6\,ab-5\,b^2+\frac{12\,ab(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{12\,b^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{6\,ab(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{3\,b^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{\left(\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right)^3}+\frac{8\,a^2-50\,ab+15\,b^2-\frac{40\,a^2(\cos(fx+e)-1)}{\cos(fx+e)+1}}{}\right.}{}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^5,x, algorithm="giac")
```

output
```
2/15*(5*(6*a*b - 5*b^2 + 12*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 12
*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 6*a*b*(cos(f*x + e) - 1)^2/(c
os(f*x + e) + 1)^2 - 3*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/((co
s(f*x + e) - 1)/(cos(f*x + e) + 1) + 1)^3 + (8*a^2 - 50*a*b + 15*b^2 - 40*
a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 220*a*b*(cos(f*x + e) - 1)/(co
s(f*x + e) + 1) - 60*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 80*a^2*(c
os(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 320*a*b*(cos(f*x + e) - 1)^2/(co
s(f*x + e) + 1)^2 + 90*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 180
*a*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 60*b^2*(cos(f*x + e) - 1)
^3/(cos(f*x + e) + 1)^3 - 30*a*b*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4
 + 15*b^2*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4)/((cos(f*x + e) - 1)/(
cos(f*x + e) + 1) - 1)^5)/f
```

3.15.9 Mupad [B] (verification not implemented)

Time = 18.29 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.90

$$\int \left(a + b\sec^2(e+fx)\right)^2 \sin^5(e+fx)\,dx$$
$$= \frac{\frac{b^2}{3} + \cos(e+fx)^2\,(2ab - 2b^2)}{\cos(e+fx)^3} - \cos(e+fx)\,(a^2 - 4ab + b^2) - \frac{a^2\cos(e+fx)^5}{5} + \frac{2a\cos(e+fx)^3\,(a-b)}{3}}{f}$$

input
```
int(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^2,x)
```

output
```
((b^2/3 + cos(e + f*x)^2*(2*a*b - 2*b^2))/cos(e + f*x)^3 - cos(e + f*x)*(a
^2 - 4*a*b + b^2) - (a^2*cos(e + f*x)^5)/5 + (2*a*cos(e + f*x)^3*(a - b))/
3)/f
```

3.16 $\int (a+b\sec^2(e+fx))^2 \sin^3(e+fx)\,dx$

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3.16.1 Optimal result

Integrand size = 23, antiderivative size = 72

$$\int (a+b\sec^2(e+fx))^2 \sin^3(e+fx)\,dx = -\frac{a(a-2b)\cos(e+fx)}{f} + \frac{a^2\cos^3(e+fx)}{3f} + \frac{(2a-b)b\sec(e+fx)}{f} + \frac{b^2\sec^3(e+fx)}{3f}$$

output
```
-a*(a-2*b)*cos(f*x+e)/f+1/3*a^2*cos(f*x+e)^3/f+(2*a-b)*b*sec(f*x+e)/f+1/3*
b^2*sec(f*x+e)^3/f
```

3.16.2 Mathematica [A] (verified)

Time = 1.88 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.15

$$\int (a+b\sec^2(e+fx))^2 \sin^3(e+fx)\,dx = \frac{(-26a^2+168ab-16b^2-3(11a^2-64ab+16b^2)\cos(2(e+fx))-6a(a-4b)\cos(4(e+fx))+a^2\cos(6(e$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^3,x]
```

output
```
((-26*a^2 + 168*a*b - 16*b^2 - 3*(11*a^2 - 64*a*b + 16*b^2)*Cos[2*(e + f*x
)] - 6*a*(a - 4*b)*Cos[4*(e + f*x)] + a^2*Cos[6*(e + f*x)])*Sec[e + f*x]^3
)/(96*f)
```

3.16.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4621, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int \sin(e+fx)^3\left(a+b\sec(e+fx)^2\right)^2 dx$$

$$\downarrow 4621$$

$$-\frac{\int \left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)^2\sec^4(e+fx)d\cos(e+fx)}{f}$$

$$\downarrow 355$$

$$-\frac{\int \left(b^2\sec^4(e+fx)+(2a-b)b\sec^2(e+fx)-a^2\cos^2(e+fx)+a(a-2b)\right)d\cos(e+fx)}{f}$$

$$\downarrow 2009$$

$$-\frac{-\frac{1}{3}a^2\cos^3(e+fx)+a(a-2b)\cos(e+fx)-b(2a-b)\sec(e+fx)-\frac{1}{3}b^2\sec^3(e+fx)}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^3,x]
```

output
```
-((a*(a - 2*b)*Cos[e + f*x] - (a^2*Cos[e + f*x]^3)/3 - (2*a - b)*b*Sec[e +
 f*x] - (b^2*Sec[e + f*x]^3)/3)/f)
```

3.16.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

3.16.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{\frac{b^2\sec(fx+e)^3}{3}+2ab\sec(fx+e)-b^2\sec(fx+e)+\frac{a^2}{3\sec(fx+e)^3}-\frac{a(a-2b)}{\sec(fx+e)}}{f}$
default	$\frac{\frac{b^2\sec(fx+e)^3}{3}+2ab\sec(fx+e)-b^2\sec(fx+e)+\frac{a^2}{3\sec(fx+e)^3}-\frac{a(a-2b)}{\sec(fx+e)}}{f}$
parts	$-\frac{a^2\left(2+\sin(fx+e)^2\right)\cos(fx+e)}{3f}+\frac{b^2\left(\frac{\sec(fx+e)^3}{3}-\sec(fx+e)\right)}{f}+\frac{2ab\left(\frac{\sin(fx+e)^4}{\cos(fx+e)}+\left(2+\sin(fx+e)^2\right)\cos(fx+e)\right)}{f}$
norman	$\frac{\frac{4a^2-24ab+4b^2}{3f}+\frac{32\left(a^2-b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{3f}-\frac{\left(4a^2+8ab+4b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8}{f}-\frac{\left(8a^2-16ab+8b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)^3}$
parallelrisch	$\frac{\left(-16a^2+96ab-16b^2\right)\cos(3fx+3e)+\left(-33a^2+192ab-48b^2\right)\cos(2fx+2e)-6a(a-4b)\cos(4fx+4e)+a^2\cos(6fx+6e)+(-}{24f(\cos(3fx+3e)+3\cos(fx+e))}$
risch	$\frac{e^{3i(fx+e)}a^2}{24f}-\frac{3e^{i(fx+e)}a^2}{8f}+\frac{e^{i(fx+e)}ab}{f}-\frac{3e^{-i(fx+e)}a^2}{8f}+\frac{e^{-i(fx+e)}ab}{f}+\frac{e^{-3i(fx+e)}a^2}{24f}-\frac{2e^{i(fx+e)}b(-6a}$

input
```
int((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/3*b^2*sec(f*x+e)^3+2*a*b*sec(f*x+e)-b^2*sec(f*x+e)+1/3*a^2/sec(f*x+
e)^3-a*(a-2*b)/sec(f*x+e))
```

3.16.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.93

$$\int (a+b\sec^2(e+fx))^2 \sin^3(e+fx)\,dx = \frac{a^2\cos(fx+e)^6 - 3(a^2-2ab)\cos(fx+e)^4 + 3(2ab-b^2)\cos(fx+e)^2 + b^2}{3f\cos(fx+e)^3}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^3,x, algorithm="fricas")
```

output
```
1/3*(a^2*cos(f*x + e)^6 - 3*(a^2 - 2*a*b)*cos(f*x + e)^4 + 3*(2*a*b - b^2)
*cos(f*x + e)^2 + b^2)/(f*cos(f*x + e)^3)
```

3.16.6 Sympy [F]

$$\int (a+b\sec^2(e+fx))^2 \sin^3(e+fx)\,dx = \int (a+b\sec^2(e+fx))^2 \sin^3(e+fx)\,dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**2*sin(f*x+e)**3,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**2*sin(e + f*x)**3, x)
```

3.16.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.93

$$\int (a+b\sec^2(e+fx))^2 \sin^3(e+fx)\,dx = \frac{a^2\cos(fx+e)^3 - 3(a^2-2ab)\cos(fx+e) + \frac{3(2ab-b^2)\cos(fx+e)^2+b^2}{\cos(fx+e)^3}}{3f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^3,x, algorithm="maxima")
```

output
```
1/3*(a^2*cos(f*x + e)^3 - 3*(a^2 - 2*a*b)*cos(f*x + e) + (3*(2*a*b - b^2)*
cos(f*x + e)^2 + b^2)/cos(f*x + e)^3)/f
```

3.16.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.26

$$\int \left(a+b\sec^2(e+fx)\right)^2 \sin^3(e+fx)\,dx$$
$$= \frac{6\,ab\cos\left(fx+e\right)^2 - 3\,b^2\cos\left(fx+e\right)^2 + b^2}{3\,f\cos\left(fx+e\right)^3}$$
$$+ \frac{a^2 f^{11}\cos\left(fx+e\right)^3 - 3\,a^2 f^{11}\cos\left(fx+e\right) + 6\,abf^{11}\cos\left(fx+e\right)}{3\,f^{12}}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^3,x, algorithm="giac")
```

output
```
1/3*(6*a*b*cos(f*x + e)^2 - 3*b^2*cos(f*x + e)^2 + b^2)/(f*cos(f*x + e)^3)
 + 1/3*(a^2*f^11*cos(f*x + e)^3 - 3*a^2*f^11*cos(f*x + e) + 6*a*b*f^11*cos
(f*x + e))/f^12
```

3.16.9 Mupad [B] (verification not implemented)

Time = 18.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.92

$$\int \left(a+b\sec^2(e+fx)\right)^2 \sin^3(e+fx)\,dx$$
$$= \frac{\frac{b^2}{3}+\cos(e+fx)^2\,(2\,a\,b-b^2)}{\cos(e+fx)^3} + \frac{a^2\cos(e+fx)^3}{3} - a\,\cos\left(e+f\,x\right)\,\left(a-2\,b\right)}{f}$$

input
```
int(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^2,x)
```

output
```
((b^2/3 + cos(e + f*x)^2*(2*a*b - b^2))/cos(e + f*x)^3 + (a^2*cos(e + f*x)
^3)/3 - a*cos(e + f*x)*(a - 2*b))/f
```

3.17 $\int (a+b\sec^2(e+fx))^2 \sin(e+fx)\,dx$

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3.17.1 Optimal result

Integrand size = 21, antiderivative size = 46

$$\int (a+b\sec^2(e+fx))^2 \sin(e+fx)\,dx = -\frac{a^2\cos(e+fx)}{f} + \frac{2ab\sec(e+fx)}{f} + \frac{b^2\sec^3(e+fx)}{3f}$$

output
```
-a^2*cos(f*x+e)/f+2*a*b*sec(f*x+e)/f+1/3*b^2*sec(f*x+e)^3/f
```

3.17.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.63

$$\int (a+b\sec^2(e+fx))^2 \sin(e+fx)\,dx = \frac{4(b+a\cos^2(e+fx))^2\,(b^2+6ab\cos^2(e+fx)-3a^2\cos^4(e+fx))\sec^3(e+fx)}{3f(a+2b+a\cos(2(e+fx)))^2}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x],x]
```

output
```
(4*(b + a*Cos[e + f*x]^2)^2*(b^2 + 6*a*b*Cos[e + f*x]^2 - 3*a^2*Cos[e + f*
x]^4)*Sec[e + f*x]^3)/(3*f*(a + 2*b + a*Cos[2*(e + f*x)])^2)
```

3.17.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4621, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int \sin(e+fx)\left(a+b\sec(e+fx)^2\right)^2 dx$$

$$\downarrow 4621$$

$$-\frac{\int \left(a\cos^2(e+fx)+b\right)^2 \sec^4(e+fx)\,d\cos(e+fx)}{f}$$

$$\downarrow 244$$

$$-\frac{\int \left(b^2\sec^4(e+fx)+2ab\sec^2(e+fx)+a^2\right) d\cos(e+fx)}{f}$$

$$\downarrow 2009$$

$$-\frac{a^2\cos(e+fx)-2ab\sec(e+fx)-\frac{1}{3}b^2\sec^3(e+fx)}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x],x]
```

output
```
-((a^2*Cos[e + f*x] - 2*a*b*Sec[e + f*x] - (b^2*Sec[e + f*x]^3)/3)/f)
```

3.17.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

3.17.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{\frac{b^2\sec(fx+e)^3}{3}+2ab\sec(fx+e)-\frac{a^2}{\sec(fx+e)}}{f}$
default	$\frac{\frac{b^2\sec(fx+e)^3}{3}+2ab\sec(fx+e)-\frac{a^2}{\sec(fx+e)}}{f}$
parts	$-\frac{a^2\cos(fx+e)}{f}+\frac{2ab\sec(fx+e)}{f}+\frac{b^2\sec(fx+e)^3}{3f}$
risch	$-\frac{e^{i(fx+e)}a^2}{2f}-\frac{e^{-i(fx+e)}a^2}{2f}+\frac{4b\left(3a\,e^{5i(fx+e)}+6a\,e^{3i(fx+e)}+2b\,e^{3i(fx+e)}+3a\,e^{i(fx+e)}\right)}{3f\left(e^{2i(fx+e)}+1\right)^3}$
parallelrisch	$\frac{2(-3a^2+6ab+b^2)\cos(3fx+3e)-12a(a-2b)\cos(2fx+2e)-3a^2\cos(4fx+4e)+6(-3a^2+6ab+b^2)\cos(fx+e)-9a^2+24ab}{6f(\cos(3fx+3e)+3\cos(fx+e))}$
norman	$\frac{\frac{6a^2-12ab-2b^2}{3f}+\frac{2\left(3a^2+2ab-b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{f}-\frac{2\left(a^2+2ab+b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{f}-\frac{2\left(9a^2-6ab+b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{3f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)}$

input
```
int((a+b*sec(f*x+e)^2)^2*sin(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/3*b^2*sec(f*x+e)^3+2*a*b*sec(f*x+e)-a^2/sec(f*x+e))
```

3.17.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \left(a + b\sec^2(e+fx)\right)^2 \sin(e+fx)\,dx = -\frac{3\,a^2\cos\left(fx+e\right)^4 - 6\,ab\cos\left(fx+e\right)^2 - b^2}{3\,f\cos\left(fx+e\right)^3}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e),x, algorithm="fricas")
```

output
```
-1/3*(3*a^2*cos(f*x + e)^4 - 6*a*b*cos(f*x + e)^2 - b^2)/(f*cos(f*x + e)^3
)
```

3.17.6 Sympy [F]

$$\int \left(a + b\sec^2(e+fx)\right)^2 \sin(e+fx)\,dx = \int \left(a + b\sec^2\left(e+fx\right)\right)^2 \sin\left(e+fx\right)\,dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**2*sin(f*x+e),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**2*sin(e + f*x), x)
```

3.17.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \left(a + b\sec^2(e+fx)\right)^2 \sin(e+fx)\,dx = -\frac{3\,a^2\cos\left(fx+e\right) - \frac{6\,ab}{\cos(fx+e)} - \frac{b^2}{\cos(fx+e)^3}}{3\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e),x, algorithm="maxima")
```

output
```
-1/3*(3*a^2*cos(f*x + e) - 6*a*b/cos(f*x + e) - b^2/cos(f*x + e)^3)/f
```

3.17.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int (a + b\sec^2(e + fx))^2 \sin(e + fx)\, dx = -\frac{a^2 \cos(fx + e)}{f} + \frac{6\, ab \cos(fx + e)^2 + b^2}{3\, f \cos(fx + e)^3}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e),x, algorithm="giac")
```

output
```
-a^2*cos(f*x + e)/f + 1/3*(6*a*b*cos(f*x + e)^2 + b^2)/(f*cos(f*x + e)^3)
```

3.17.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int (a + b\sec^2(e + fx))^2 \sin(e + fx)\, dx = \frac{\frac{b^2}{3} + 2\, a\, b \cos(e + f\, x)^2}{f \cos(e + f\, x)^3} - \frac{a^2 \cos(e + f\, x)}{f}$$

input
```
int(sin(e + f*x)*(a + b/cos(e + f*x)^2)^2,x)
```

output
```
(b^2/3 + 2*a*b*cos(e + f*x)^2)/(f*cos(e + f*x)^3) - (a^2*cos(e + f*x))/f
```

3.18 $\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

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3.18.1 Optimal result

Integrand size = 21, antiderivative size = 52

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{(a+b)^2\operatorname{arctanh}(\cos(e+fx))}{f} + \frac{b(2a+b)\sec(e+fx)}{f} + \frac{b^2\sec^3(e+fx)}{3f}$$

output
```
-(a+b)^2*arctanh(cos(f*x+e))/f+b*(2*a+b)*sec(f*x+e)/f+1/3*b^2*sec(f*x+e)^3
/f
```

3.18.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 108 vs. 2(52) = 104.

Time = 0.91 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.08

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{4(b+a\cos^2(e+fx))^2\left(-b^2-3b(2a+b)\cos^2(e+fx)+3(a+b)^2\cos^3(e+fx)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)\right.\right.}{3f(a+2b+a\cos(2(e+fx)))^2}$$

input
```
Integrate[Csc[e + f*x]*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(-4*(b + a*Cos[e + f*x]^2)^2*(-b^2 - 3*b*(2*a + b)*Cos[e + f*x]^2 + 3*(a +
 b)^2*Cos[e + f*x]^3*(Log[Cos[(e + f*x)/2]] - Log[Sin[(e + f*x)/2]]))*Sec[
e + f*x]^3)/(3*f*(a + 2*b + a*Cos[2*(e + f*x)])^2)
```

3.18.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4621, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^2}{\sin(e+fx)}dx$$

$$\downarrow 4621$$

$$-\frac{\int \frac{\left(a\cos^2(e+fx)+b\right)^2\sec^4(e+fx)}{1-\cos^2(e+fx)}d\cos(e+fx)}{f}$$

$$\downarrow 364$$

$$-\frac{\int \left(b^2\sec^4(e+fx)+b(2a+b)\sec^2(e+fx)-\frac{(a+b)^2}{\cos^2(e+fx)-1}\right)d\cos(e+fx)}{f}$$

$$\downarrow 2009$$

$$-\frac{(a+b)^2\text{arctanh}(\cos(e+fx))-b(2a+b)\sec(e+fx)-\frac{1}{3}b^2\sec^3(e+fx)}{f}$$

input
```
Int[Csc[e + f*x]*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
-(((a + b)^2*ArcTanh[Cos[e + f*x]] - b*(2*a + b)*Sec[e + f*x] - (b^2*Sec[e
 + f*x]^3)/3)/f)
```

3.18.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

3.18.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.81

method	result
derivativedivides	$\frac{a^2\ln(\csc(fx+e)-\cot(fx+e))+2ab\left(\frac{1}{\cos(fx+e)}+\ln(\csc(fx+e)-\cot(fx+e))\right)+b^2\left(\frac{1}{3\cos(fx+e)^3}+\frac{1}{\cos(fx+e)}+\ln(\csc(fx+$
default	$\frac{a^2\ln(\csc(fx+e)-\cot(fx+e))+2ab\left(\frac{1}{\cos(fx+e)}+\ln(\csc(fx+e)-\cot(fx+e))\right)+b^2\left(\frac{1}{3\cos(fx+e)^3}+\frac{1}{\cos(fx+e)}+\ln(\csc(fx+$
norman	$\frac{\left(8ab+4b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{f}-\frac{12ab+8b^2}{3f}-\frac{\left(4ab+4b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{f}+\frac{\left(a^2+2ab+b^2\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}$ $\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3$
parallelrisch	$\frac{3\left(\frac{\cos(3fx+3e)}{3}+\cos(fx+e)\right)(a+b)^2\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+6b\left(\frac{\left(a+\frac{2b}{3}\right)\cos(3fx+3e)}{3}+\frac{(2a+b)\cos(2fx+2e)}{3}+\left(a+\frac{2b}{3}\right)\cos(fx+$ $f(\cos(3fx+3e)+3\cos(fx+e))$
risch	$\frac{2b\,e^{i(fx+e)}\left(6a\,e^{4i(fx+e)}+3b\,e^{4i(fx+e)}+12a\,e^{2i(fx+e)}+10b\,e^{2i(fx+e)}+6a+3b\right)}{3f\left(e^{2i(fx+e)}+1\right)^3}+\frac{\ln\left(e^{i(fx+e)}-1\right)a^2}{f}+\frac{2\ln\left(e^{i(fx+e)}-\right.}{f}$

input
```
int(csc(f*x+e)*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(a^2*ln(csc(f*x+e)-cot(f*x+e))+2*a*b*(1/cos(f*x+e)+ln(csc(f*x+e)-cot(f
*x+e)))+b^2*(1/3/cos(f*x+e)^3+1/cos(f*x+e)+ln(csc(f*x+e)-cot(f*x+e))))
```

3.18.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(50) = 100$.

Time $= 0.26$ (sec) , antiderivative size $= 101$, normalized size of antiderivative $= 1.94$

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx =$$

$$-\frac{3\left(a^2+2\,ab+b^2\right)\cos\left(fx+e\right)^3\log\left(\frac{1}{2}\cos\left(fx+e\right)+\frac{1}{2}\right)-3\left(a^2+2\,ab+b^2\right)\cos\left(fx+e\right)^3\log\left(-\frac{1}{2}\cos\left(\right.\right.}{6\,f\cos\left(fx+e\right)^3}$$

input
```
integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
-1/6*(3*(a^2 + 2*a*b + b^2)*cos(f*x + e)^3*log(1/2*cos(f*x + e) + 1/2) - 3
*(a^2 + 2*a*b + b^2)*cos(f*x + e)^3*log(-1/2*cos(f*x + e) + 1/2) - 6*(2*a*
b + b^2)*cos(f*x + e)^2 - 2*b^2)/(f*cos(f*x + e)^3)
```

3.18.6 Sympy [F]

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)^2\csc\left(e+fx\right)dx$$

input
```
integrate(csc(f*x+e)*(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**2*csc(e + f*x), x)
```

3.18.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.58

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx =$$

$$-\frac{3\left(a^2+2ab+b^2\right)\log\left(\cos\left(fx+e\right)+1\right)-3\left(a^2+2ab+b^2\right)\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\left(3\left(2ab+b^2\right)\cos(fx+e)\right.}{\cos(fx+e)^3}}{6f}$$

input
```
integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/6*(3*(a^2 + 2*a*b + b^2)*log(cos(f*x + e) + 1) - 3*(a^2 + 2*a*b + b^2)*
log(cos(f*x + e) - 1) - 2*(3*(2*a*b + b^2)*cos(f*x + e)^2 + b^2)/cos(f*x +
 e)^3)/f
```

3.18.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(50) = 100.

Time = 0.32 (sec) , antiderivative size = 172, normalized size of antiderivative = 3.31

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$

$$=\frac{3\left(a^2+2ab+b^2\right)\log\left(\frac{\left|-\cos(fx+e)+1\right|}{\left|\cos(fx+e)+1\right|}\right)+\frac{8\left(3ab+2b^2+\frac{6ab(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{3b^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{3ab(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{3b^2(\cos(fx+e)-1}{(\cos(fx+e)+1)^2}\right.}{\left(\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right)^3}}{6f}$$

input
```
integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/6*(3*(a^2 + 2*a*b + b^2)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1
)) + 8*(3*a*b + 2*b^2 + 6*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 3*b^
2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 3*a*b*(cos(f*x + e) - 1)^2/(cos(
f*x + e) + 1)^2 + 3*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/((cos(f
*x + e) - 1)/(cos(f*x + e) + 1) + 1)^3)/f
```

3.18.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{\cos(e+fx)^2\left(b^2+2ab\right)+\frac{b^2}{3}}{f\cos(e+fx)^3} - \frac{\operatorname{atanh}(\cos(e+fx))\,(a+b)^2}{f}$$

input
```
int((a + b/cos(e + f*x)^2)^2/sin(e + f*x),x)
```

output
```
(cos(e + f*x)^2*(2*a*b + b^2) + b^2/3)/(f*cos(e + f*x)^3) - (atanh(cos(e +
 f*x))*(a + b)^2)/f
```

3.19 $\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

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3.19.1 Optimal result

Integrand size = 23, antiderivative size = 104

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{(a+b)(a+5b)\operatorname{arctanh}(\cos(e+fx))}{2f} - \frac{(3a^2+6ab+5b^2)\cot(e+fx)\csc(e+fx)}{6f} + \frac{b(6a+5b)\sec(e+fx)}{3f} + \frac{b^2\csc^2(e+fx)\sec^3(e+fx)}{3f}$$

output
```
-1/2*(a+b)*(a+5*b)*arctanh(cos(f*x+e))/f-1/6*(3*a^2+6*a*b+5*b^2)*cot(f*x+e
)*csc(f*x+e)/f+1/3*b*(6*a+5*b)*sec(f*x+e)/f+1/3*b^2*csc(f*x+e)^2*sec(f*x+e
)^3/f
```

3.19.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1021 vs. 2(104) = 208.

Time = 8.09 (sec) , antiderivative size = 1021, normalized size of antiderivative = 9.82

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$

$$\begin{aligned}
&= \frac{(-a^2-2ab-b^2)\cos^4(e+fx)\csc^2\left(\frac{e}{2}+\frac{fx}{2}\right)\left(a+b\sec^2(e+fx)\right)^2}{2f(a+2b+a\cos(2e+2fx))^2} \\
&\quad - \frac{2(a^2+6ab+5b^2)\cos^4(e+fx)\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)\left(a+b\sec^2(e+fx)\right)^2}{f(a+2b+a\cos(2e+2fx))^2} \\
&\quad + \frac{2(a^2+6ab+5b^2)\cos^4(e+fx)\log\left(\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)\left(a+b\sec^2(e+fx)\right)^2}{f(a+2b+a\cos(2e+2fx))^2} \\
&\quad + \frac{2b(12a+13b)\cos^4(e+fx)\sec(e)\left(a+b\sec^2(e+fx)\right)^2}{3f(a+2b+a\cos(2e+2fx))^2} \\
&\quad + \frac{(a^2+2ab+b^2)\cos^4(e+fx)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)\left(a+b\sec^2(e+fx)\right)^2}{2f(a+2b+a\cos(2e+2fx))^2} \\
&\quad + \frac{2b^2\cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\sin\left(\frac{fx}{2}\right)}{3f(a+2b+a\cos(2e+2fx))^2\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)-\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)^3} \\
&\quad + \frac{\cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\left(b^2\cos\left(\frac{e}{2}\right)+b^2\sin\left(\frac{e}{2}\right)\right)}{3f(a+2b+a\cos(2e+2fx))^2\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)-\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)^2} \\
&\quad + \frac{2\cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\left(12ab\sin\left(\frac{fx}{2}\right)+13b^2\sin\left(\frac{fx}{2}\right)\right)}{3f(a+2b+a\cos(2e+2fx))^2\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)-\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)} \\
&\quad - \frac{2b^2\cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\sin\left(\frac{fx}{2}\right)}{3f(a+2b+a\cos(2e+2fx))^2\left(\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)+\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)^3} \\
&\quad + \frac{\cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\left(b^2\cos\left(\frac{e}{2}\right)-b^2\sin\left(\frac{e}{2}\right)\right)}{3f(a+2b+a\cos(2e+2fx))^2\left(\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)+\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)^2} \\
&\quad - \frac{2\cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\left(12ab\sin\left(\frac{fx}{2}\right)+13b^2\sin\left(\frac{fx}{2}\right)\right)}{3f(a+2b+a\cos(2e+2fx))^2\left(\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)+\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}
\end{aligned}$$

input
```
Integrate[Csc[e + f*x]^3*(a + b*Sec[e + f*x]^2)^2,x]
```

output

```
((-a^2 - 2*a*b - b^2)*Cos[e + f*x]^4*Csc[e/2 + (f*x)/2]^2*(a + b*Sec[e + f
*x]^2)^2)/(2*f*(a + 2*b + a*Cos[2*e + 2*f*x])^2) - (2*(a^2 + 6*a*b + 5*b^2
)*Cos[e + f*x]^4*Log[Cos[e/2 + (f*x)/2]]*(a + b*Sec[e + f*x]^2)^2)/(f*(a +
 2*b + a*Cos[2*e + 2*f*x])^2) + (2*(a^2 + 6*a*b + 5*b^2)*Cos[e + f*x]^4*Lo
g[Sin[e/2 + (f*x)/2]]*(a + b*Sec[e + f*x]^2)^2)/(f*(a + 2*b + a*Cos[2*e +
2*f*x])^2) + (2*b*(12*a + 13*b)*Cos[e + f*x]^4*Sec[e]*(a + b*Sec[e + f*x]^
2)^2)/(3*f*(a + 2*b + a*Cos[2*e + 2*f*x])^2) + ((a^2 + 2*a*b + b^2)*Cos[e
+ f*x]^4*Sec[e/2 + (f*x)/2]^2*(a + b*Sec[e + f*x]^2)^2)/(2*f*(a + 2*b + a*
Cos[2*e + 2*f*x])^2) + (2*b^2*Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2*Sin[
(f*x)/2])/(3*f*(a + 2*b + a*Cos[2*e + 2*f*x])^2*(Cos[e/2] - Sin[e/2])*(Cos
[e/2 + (f*x)/2] - Sin[e/2 + (f*x)/2])^3) + (Cos[e + f*x]^4*(a + b*Sec[e +
f*x]^2)^2*(b^2*Cos[e/2] + b^2*Sin[e/2]))/(3*f*(a + 2*b + a*Cos[2*e + 2*f*x
])^2*(Cos[e/2] - Sin[e/2])*(Cos[e/2 + (f*x)/2] - Sin[e/2 + (f*x)/2])^2) +
(2*Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2*(12*a*b*Sin[(f*x)/2] + 13*b^2*S
in[(f*x)/2]))/(3*f*(a + 2*b + a*Cos[2*e + 2*f*x])^2*(Cos[e/2] - Sin[e/2])*
(Cos[e/2 + (f*x)/2] - Sin[e/2 + (f*x)/2])) - (2*b^2*Cos[e + f*x]^4*(a + b*
Sec[e + f*x]^2)^2*Sin[(f*x)/2])/(3*f*(a + 2*b + a*Cos[2*e + 2*f*x])^2*(Cos
[e/2] + Sin[e/2])*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])^3) + (Cos[e +
f*x]^4*(a + b*Sec[e + f*x]^2)^2*(b^2*Cos[e/2] - b^2*Sin[e/2]))/(3*f*(a + 2
*b + a*Cos[2*e + 2*f*x])^2*(Cos[e/2] + Sin[e/2])*(Cos[e/2 + (f*x)/2] + ...
```

3.19.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4621, 365, 361, 25, 359, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^2}{\sin(e+fx)^3} dx$$

$$\downarrow 4621$$

$$-\frac{\int \frac{\left(a\cos^2(e+fx)+b\right)^2 \sec^4(e+fx)}{\left(1-\cos^2(e+fx)\right)^2} d\cos(e+fx)}{f}$$

$$\downarrow 365$$

$$-\frac{\frac{1}{3}\int \frac{\left(3a^2\cos^2(e+fx)+b(6a+5b)\right)\sec^2(e+fx)}{\left(1-\cos^2(e+fx)\right)^2}d\cos(e+fx)-\frac{b^2\sec^3(e+fx)}{3(1-\cos^2(e+fx))}}{f}$$

$$\downarrow 361$$

$$-\frac{\frac{1}{3}\left(\frac{\left(3a^2+6ab+5b^2\right)\cos(e+fx)}{2(1-\cos^2(e+fx))}-\frac{1}{2}\int -\frac{\left(\left(3a^2+6ba+5b^2\right)\cos^2(e+fx)+2b(6a+5b)\right)\sec^2(e+fx)}{1-\cos^2(e+fx)}d\cos(e+fx)\right)-\frac{b^2\sec^3(e+fx)}{3(1-\cos^2(e+fx))}}{f}$$

$$\downarrow 25$$

$$-\frac{\frac{1}{3}\left(\frac{1}{2}\int \frac{\left(\left(3a^2+6ba+5b^2\right)\cos^2(e+fx)+2b(6a+5b)\right)\sec^2(e+fx)}{1-\cos^2(e+fx)}d\cos(e+fx)+\frac{\left(3a^2+6ab+5b^2\right)\cos(e+fx)}{2(1-\cos^2(e+fx))}\right)-\frac{b^2\sec^3(e+fx)}{3(1-\cos^2(e+fx))}}{f}$$

$$\downarrow 359$$

$$-\frac{\frac{1}{3}\left(\frac{1}{2}\left(3(a+b)(a+5b)\int \frac{1}{1-\cos^2(e+fx)}d\cos(e+fx)-2b(6a+5b)\sec(e+fx)\right)+\frac{\left(3a^2+6ab+5b^2\right)\cos(e+fx)}{2(1-\cos^2(e+fx))}\right)-\frac{b^2\sec}{3(1-c}}{f}$$

$$\downarrow 219$$

$$-\frac{\frac{1}{3}\left(\frac{\left(3a^2+6ab+5b^2\right)\cos(e+fx)}{2(1-\cos^2(e+fx))}+\frac{1}{2}(3(a+b)(a+5b)\operatorname{arctanh}(\cos(e+fx))-2b(6a+5b)\sec(e+fx))\right)-\frac{b^2\sec^3(e+fx)}{3(1-\cos^2(e+fx}}{f}$$

input
```
Int[Csc[e + f*x]^3*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
-((-1/3*(b^2*Sec[e + f*x]^3)/(1 - Cos[e + f*x]^2) + (((3*a^2 + 6*a*b + 5*b
^2)*Cos[e + f*x])/(2*(1 - Cos[e + f*x]^2)) + (3*(a + b)*(a + 5*b)*ArcTanh[
Cos[e + f*x]] - 2*b*(6*a + 5*b)*Sec[e + f*x])/2)/3)/f)
```

3.19.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

3.19.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.57

method	result
derivativedivides	$\frac{a^2\left(-\frac{\csc(fx+e)\cot(fx+e)}{2}+\frac{\ln(\csc(fx+e)-\cot(fx+e))}{2}\right)+2ab\left(-\frac{1}{2\sin(fx+e)^2\cos(fx+e)}+\frac{3}{2\cos(fx+e)}+\frac{3\ln(\csc(fx+e)-\cot(fx+$
default	$\frac{a^2\left(-\frac{\csc(fx+e)\cot(fx+e)}{2}+\frac{\ln(\csc(fx+e)-\cot(fx+e))}{2}\right)+2ab\left(-\frac{1}{2\sin(fx+e)^2\cos(fx+e)}+\frac{3}{2\cos(fx+e)}+\frac{3\ln(\csc(fx+e)-\cot(fx+$
norman	$\frac{\frac{a^2+2ab+b^2}{8f}+\frac{\left(a^2+2ab+b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}}{8f}-\frac{\left(7a^2+46ab+55b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{8f}-\frac{\left(9a^2+66ab+65b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{12f}+\frac{\left(11a^2+86ab-$
parallelrisch	$\frac{3\left(\frac{\cos(3fx+3e)}{3}+\cos(fx+e)\right)(a+5b)(a+b)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}-\frac{3\csc\left(\frac{fx}{2}+\frac{e}{2}\right)^2\left(\left(a^2+\frac{22}{3}ab+\frac{65}{9}b^2\right)\cos(3fx+3e)+\frac{32\left(a^2+2ab+\frac{5}{3}b^2\right)}{3}\right.}{f(\cos(3f$
risch	$\frac{e^{i(fx+e)}\left(3a^2e^{8i(fx+e)}+18ab\,e^{8i(fx+e)}+15b^2e^{8i(fx+e)}+12a^2e^{6i(fx+e)}+24ab\,e^{6i(fx+e)}+20b^2e^{6i(fx+e)}+18a^2e^{4i(fx+e)}\right.}{3f\left(e^{2i(fx+e)}+1\right)^3\left(e^{2i(fx+}$

input
```
int(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(a^2*(-1/2*csc(f*x+e)*cot(f*x+e)+1/2*ln(csc(f*x+e)-cot(f*x+e)))+2*a*b*
(-1/2/sin(f*x+e)^2/cos(f*x+e)+3/2/cos(f*x+e)+3/2*ln(csc(f*x+e)-cot(f*x+e))
)+b^2*(1/3/sin(f*x+e)^2/cos(f*x+e)^3-5/6/sin(f*x+e)^2/cos(f*x+e)+5/2/cos(f
*x+e)+5/2*ln(csc(f*x+e)-cot(f*x+e))))
```

3.19.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. 2(96) = 192.

Time = 0.26 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.86

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$= \frac{6\left(a^2+6ab+5b^2\right)\cos(fx+e)^4-4\left(6ab+5b^2\right)\cos(fx+e)^2-4b^2-3\left(\left(a^2+6ab+5b^2\right)\cos(fx+e)^5\right.}{}$$

input
```
integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/12*(6*(a^2 + 6*a*b + 5*b^2)*cos(f*x + e)^4 - 4*(6*a*b + 5*b^2)*cos(f*x +
 e)^2 - 4*b^2 - 3*((a^2 + 6*a*b + 5*b^2)*cos(f*x + e)^5 - (a^2 + 6*a*b + 5
*b^2)*cos(f*x + e)^3)*log(1/2*cos(f*x + e) + 1/2) + 3*((a^2 + 6*a*b + 5*b^
2)*cos(f*x + e)^5 - (a^2 + 6*a*b + 5*b^2)*cos(f*x + e)^3)*log(-1/2*cos(f*x
 + e) + 1/2))/(f*cos(f*x + e)^5 - f*cos(f*x + e)^3)
```

3.19.6 Sympy [F]

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \int \left(a+b\sec^2(e+fx)\right)^2 \csc^3(e+fx)\,dx$$

input
```
integrate(csc(f*x+e)**3*(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**2*csc(e + f*x)**3, x)
```

3.19.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.21

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx =$$

$$-\frac{3\left(a^2+6ab+5b^2\right)\log\left(\cos(fx+e)+1\right)-3\left(a^2+6ab+5b^2\right)\log\left(\cos(fx+e)-1\right)-\frac{2\left(3\left(a^2+6ab+5b^2\right)\right.}{12f}$$

input
```
integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/12*(3*(a^2 + 6*a*b + 5*b^2)*log(cos(f*x + e) + 1) - 3*(a^2 + 6*a*b + 5*
b^2)*log(cos(f*x + e) - 1) - 2*(3*(a^2 + 6*a*b + 5*b^2)*cos(f*x + e)^4 - 2
*(6*a*b + 5*b^2)*cos(f*x + e)^2 - 2*b^2)/(cos(f*x + e)^5 - cos(f*x + e)^3)
)/f
```

3.19.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(96) = 192$.

Time $= 0.35$ (sec) , antiderivative size $= 341$, normalized size of antiderivative $= 3.28$

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx =$$

$$-\frac{\frac{3a^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{6ab(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{3b^2(\cos(fx+e)-1)}{\cos(fx+e)+1}-6\left(a^2+6ab+5b^2\right)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)-\frac{3\left(a^2+2ab-\right.}{}$$

input
```
integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-1/24*(3*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 6*a*b*(cos(f*x + e) -
 1)/(cos(f*x + e) + 1) + 3*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 6*(
a^2 + 6*a*b + 5*b^2)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1)) - 3
*(a^2 + 2*a*b + b^2 - 2*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 12*a*b
*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 10*b^2*(cos(f*x + e) - 1)/(cos(f*
x + e) + 1))*(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - 16*(6*a*b + 7*b^2 + 1
2*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 12*b^2*(cos(f*x + e) - 1)/(c
os(f*x + e) + 1) + 6*a*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 9*b^2
*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/((cos(f*x + e) - 1)/(cos(f*x +
 e) + 1) + 1)^3)/f
```

3.19.9 Mupad [B] (verification not implemented)

Time $= 18.51$ (sec) , antiderivative size $= 96$, normalized size of antiderivative $= 0.92$

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx$$
$$= \frac{\frac{b^2}{3}+\cos(e+fx)^2\left(\frac{5b^2}{3}+2ab\right)-\cos(e+fx)^4\left(\frac{a^2}{2}+3ab+\frac{5b^2}{2}\right)}{f\left(\cos(e+fx)^3-\cos(e+fx)^5\right)}$$
$$-\frac{\operatorname{atanh}(\cos(e+fx))\,(a+b)\,(a+5b)}{2f}$$

input
```
int((a + b/cos(e + f*x)^2)^2/sin(e + f*x)^3,x)
```

output
```
(b^2/3 + cos(e + f*x)^2*(2*a*b + (5*b^2)/3) - cos(e + f*x)^4*(3*a*b + a^2/
2 + (5*b^2)/2))/(f*(cos(e + f*x)^3 - cos(e + f*x)^5)) - (atanh(cos(e + f*x
))*(a + b)*(a + 5*b))/(2*f)
```

3.20 $\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

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3.20.1 Optimal result

Integrand size = 23, antiderivative size = 141

$$\begin{aligned}\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = &-\frac{(3a^2+30ab+35b^2)\operatorname{arctanh}(\cos(e+fx))}{8f}\\ &-\frac{(3a+7b)^2\cot(e+fx)\csc(e+fx)}{24f}\\ &-\frac{(3a^2+6ab+7b^2)\cot(e+fx)\csc^3(e+fx)}{12f}\\ &+\frac{b(6a+7b)\sec(e+fx)}{3f}\\ &+\frac{b^2\csc^4(e+fx)\sec^3(e+fx)}{3f}\end{aligned}$$

output

```
-1/8*(3*a^2+30*a*b+35*b^2)*arctanh(cos(f*x+e))/f-1/24*(3*a+7*b)^2*cot(f*x+
e)*csc(f*x+e)/f-1/12*(3*a^2+6*a*b+7*b^2)*cot(f*x+e)*csc(f*x+e)^3/f+1/3*b*(
6*a+7*b)*sec(f*x+e)/f+1/3*b^2*csc(f*x+e)^4*sec(f*x+e)^3/f
```

3.20.2 Mathematica [A] (verified)

Time = 9.79 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.55

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx =$$

$$-\frac{(b+a\cos^2(e+fx))^2\left((90a^2+132ab-102b^2+(6a^2+60ab+70b^2)\cos(4(e+fx))-3(3a^2+30ab+3\right.}{}$$

input
```
Integrate[Csc[e + f*x]^5*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
-1/192*((b + a*Cos[e + f*x]^2)^2*((90*a^2 + 132*a*b - 102*b^2 + (6*a^2 + 6
0*a*b + 70*b^2)*Cos[4*(e + f*x)] - 3*(3*a^2 + 30*a*b + 35*b^2)*Cos[6*(e +
f*x)])*Cot[e + f*x]*Csc[e + f*x]^3 + ((105*a^2 + 282*a*b + 329*b^2)*(Cos[e
 + f*x] + Cos[3*(e + f*x)])*Csc[e + f*x]^4)/2 + 96*(3*a^2 + 30*a*b + 35*b^
2)*Cos[e + f*x]^4*(Log[Cos[(e + f*x)/2]] - Log[Sin[(e + f*x)/2]]))*Sec[e +
 f*x]^4)/(f*(a + 2*b + a*Cos[2*(e + f*x)])^2)
```

3.20.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.15, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4621, 365, 361, 25, 361, 25, 359, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^2}{\sin(e+fx)^5} dx$$

$$\downarrow 4621$$

$$-\frac{\int \frac{\left(a\cos^2(e+fx)+b\right)^2\sec^4(e+fx)}{\left(1-\cos^2(e+fx)\right)^3} d\cos(e+fx)}{f}$$

$$\downarrow 365$$

$$-\frac{\frac{1}{3}\int \frac{\left(3a^2\cos^2(e+fx)+b(6a+7b)\right)\sec^2(e+fx)}{\left(1-\cos^2(e+fx)\right)^3}d\cos(e+fx)-\frac{b^2\sec^3(e+fx)}{3\left(1-\cos^2(e+fx)\right)^2}}{f}$$

$$\downarrow 361$$

$$-\frac{\frac{1}{3}\left(\frac{\left(3a^2+6ab+7b^2\right)\cos(e+fx)}{4\left(1-\cos^2(e+fx)\right)^2}-\frac{1}{4}\int -\frac{\left(3\left(3a^2+6ba+7b^2\right)\cos^2(e+fx)+4b(6a+7b)\right)\sec^2(e+fx)}{\left(1-\cos^2(e+fx)\right)^2}d\cos(e+fx)\right)-\frac{b^2\sec^3(e+fx)}{3\left(1-\cos^2(e+fx)\right)^2}}{f}$$

$$\downarrow 25$$

$$-\frac{\frac{1}{3}\left(\frac{1}{4}\int \frac{\left(3\left(3a^2+6ba+7b^2\right)\cos^2(e+fx)+4b(6a+7b)\right)\sec^2(e+fx)}{\left(1-\cos^2(e+fx)\right)^2}d\cos(e+fx)+\frac{\left(3a^2+6ab+7b^2\right)\cos(e+fx)}{4\left(1-\cos^2(e+fx)\right)^2}\right)-\frac{b^2\sec^3(e+fx)}{3\left(1-\cos^2(e+fx)\right)^2}}{f}$$

$$\downarrow 361$$

$$-\frac{\frac{1}{3}\left(\frac{1}{4}\left(\frac{(3a+7b)^2\cos(e+fx)}{2\left(1-\cos^2(e+fx)\right)}-\frac{1}{2}\int -\frac{\left((3a+7b)^2\cos^2(e+fx)+8b(6a+7b)\right)\sec^2(e+fx)}{1-\cos^2(e+fx)}d\cos(e+fx)\right)+\frac{\left(3a^2+6ab+7b^2\right)\cos(e+fx)}{4\left(1-\cos^2(e+fx)\right)^2}\right)-}{f}$$

$$\downarrow 25$$

$$-\frac{\frac{1}{3}\left(\frac{1}{4}\left(\frac{1}{2}\int \frac{\left((3a+7b)^2\cos^2(e+fx)+8b(6a+7b)\right)\sec^2(e+fx)}{1-\cos^2(e+fx)}d\cos(e+fx)+\frac{(3a+7b)^2\cos(e+fx)}{2\left(1-\cos^2(e+fx)\right)}\right)+\frac{\left(3a^2+6ab+7b^2\right)\cos(e+fx)}{4\left(1-\cos^2(e+fx)\right)^2}\right)-}{f}$$

$$\downarrow 359$$

$$-\frac{\frac{1}{3}\left(\frac{1}{4}\left(\frac{1}{2}\left(3\left(3a^2+30ab+35b^2\right)\int \frac{1}{1-\cos^2(e+fx)}d\cos(e+fx)-8b(6a+7b)\sec(e+fx)\right)+\frac{(3a+7b)^2\cos(e+fx)}{2\left(1-\cos^2(e+fx)\right)}\right)+}{f}$$

$$\downarrow 219$$

$$-\frac{\frac{1}{3}\left(\frac{1}{4}\left(\frac{1}{2}\left(3\left(3a^2+30ab+35b^2\right)\operatorname{arctanh}(\cos(e+fx))-8b(6a+7b)\sec(e+fx)\right)+\frac{(3a+7b)^2\cos(e+fx)}{2\left(1-\cos^2(e+fx)\right)}\right)+\frac{\left(3a^2+6ab+\right.}{4(1-\mathrm{co}}}{f}$$

input
```
Int[Csc[e + f*x]^5*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
-((-1/3*(b^2*Sec[e + f*x]^3)/(1 - Cos[e + f*x]^2)^2 + (((3*a^2 + 6*a*b + 7
*b^2)*Cos[e + f*x])/(4*(1 - Cos[e + f*x]^2)^2) + (((3*a + 7*b)^2*Cos[e + f
*x])/(2*(1 - Cos[e + f*x]^2)) + (3*(3*a^2 + 30*a*b + 35*b^2)*ArcTanh[Cos[e
 + f*x]] - 8*b*(6*a + 7*b)*Sec[e + f*x])/2)/4)/3)/f)
```

3.20. $\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx$

3.20.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

3.20.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.50

method	result
derivativedivides	$\frac{a^2\left(\left(-\frac{\csc(fx+e)^3}{4}-\frac{3\csc(fx+e)}{8}\right)\cot(fx+e)+\frac{3\ln(\csc(fx+e)-\cot(fx+e))}{8}\right)+2ab\left(-\frac{1}{4\sin(fx+e)^4\cos(fx+e)}-\frac{5}{8\sin(fx+e)^2\cos(fx+e)}+\frac{15}{8\cos(fx+e)}+\frac{15\ln(\csc(fx+e)-\cot(fx+e))}{8}\right)+b^2\left(-\frac{1}{4\sin(fx+e)^4\cos(fx+e)^3}+\frac{7}{12\sin(fx+e)^2\cos(fx+e)^3}-\frac{35}{24\sin(fx+e)^2\cos(fx+e)}+\frac{35}{8\cos(fx+e)}+\frac{35\ln(\csc(fx+e)-\cot(fx+e))}{8}\right)}{f}$
default	$\frac{a^2\left(\left(-\frac{\csc(fx+e)^3}{4}-\frac{3\csc(fx+e)}{8}\right)\cot(fx+e)+\frac{3\ln(\csc(fx+e)-\cot(fx+e))}{8}\right)+2ab\left(-\frac{1}{4\sin(fx+e)^4\cos(fx+e)}-\frac{5}{8\sin(fx+e)^2\cos(fx+e)}+\frac{15}{8\cos(fx+e)}+\frac{15\ln(\csc(fx+e)-\cot(fx+e))}{8}\right)+b^2\left(-\frac{1}{4\sin(fx+e)^4\cos(fx+e)^3}+\frac{7}{12\sin(fx+e)^2\cos(fx+e)^3}-\frac{35}{24\sin(fx+e)^2\cos(fx+e)}+\frac{35}{8\cos(fx+e)}+\frac{35\ln(\csc(fx+e)-\cot(fx+e))}{8}\right)}{f}$
norman	$\frac{\frac{a^2+2ab+b^2}{64f}+\frac{\left(a^2+2ab+b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{14}}{64f}+\frac{\left(5a^2+26ab+21b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{64f}+\frac{\left(5a^2+26ab+21b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{12}}{64f}-\frac{\left(3a^2+29ab+\right.}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right.}$
parallelrisch	$\frac{55296\left(\frac{\cos(3fx+3e)}{3}+\cos(fx+e)\right)\left(a^2+10ab+\frac{35}{3}b^2\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-63\left(\left(3a^2+\frac{218}{7}ab+\frac{121}{3}b^2\right)\cos(3fx+3e)+\left(\frac{80}{3}a^2+\right.}{f}$
risch	$\frac{e^{i(fx+e)}\left(9a^2e^{12i(fx+e)}+90ab\,e^{12i(fx+e)}+105b^2e^{12i(fx+e)}-6a^2e^{10i(fx+e)}-60ab\,e^{10i(fx+e)}-70b^2e^{10i(fx+e)}-105a^2e^{8i}\right.}{f}$

input
```
int(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(a^2*((-1/4*csc(f*x+e)^3-3/8*csc(f*x+e))*cot(f*x+e)+3/8*ln(csc(f*x+e)-
cot(f*x+e)))+2*a*b*(-1/4/sin(f*x+e)^4/cos(f*x+e)-5/8/sin(f*x+e)^2/cos(f*x+
e)+15/8/cos(f*x+e)+15/8*ln(csc(f*x+e)-cot(f*x+e)))+b^2*(-1/4/sin(f*x+e)^4/
cos(f*x+e)^3+7/12/sin(f*x+e)^2/cos(f*x+e)^3-35/24/sin(f*x+e)^2/cos(f*x+e)+
35/8/cos(f*x+e)+35/8*ln(csc(f*x+e)-cot(f*x+e))))
```

3.20.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. 2(131) = 262.

Time = 0.26 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.03

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$= \frac{6\left(3\,a^2+30\,ab+35\,b^2\right)\cos\left(fx+e\right)^6-10\left(3\,a^2+30\,ab+35\,b^2\right)\cos\left(fx+e\right)^4+16\left(6\,ab+7\,b^2\right)\cos\left(fx+\right.}{}$$

input
```
integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/48*(6*(3*a^2 + 30*a*b + 35*b^2)*cos(f*x + e)^6 - 10*(3*a^2 + 30*a*b + 35
*b^2)*cos(f*x + e)^4 + 16*(6*a*b + 7*b^2)*cos(f*x + e)^2 + 16*b^2 - 3*((3*
a^2 + 30*a*b + 35*b^2)*cos(f*x + e)^7 - 2*(3*a^2 + 30*a*b + 35*b^2)*cos(f*
x + e)^5 + (3*a^2 + 30*a*b + 35*b^2)*cos(f*x + e)^3)*log(1/2*cos(f*x + e)
+ 1/2) + 3*((3*a^2 + 30*a*b + 35*b^2)*cos(f*x + e)^7 - 2*(3*a^2 + 30*a*b +
 35*b^2)*cos(f*x + e)^5 + (3*a^2 + 30*a*b + 35*b^2)*cos(f*x + e)^3)*log(-1
/2*cos(f*x + e) + 1/2))/(f*cos(f*x + e)^7 - 2*f*cos(f*x + e)^5 + f*cos(f*x
 + e)^3)
```

3.20.6 Sympy [F(-1)]

Timed out.

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**5*(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Timed out
```

3.20.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.17

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx =$$

$$-\frac{3\left(3\,a^2+30\,ab+35\,b^2\right)\log\left(\cos\left(fx+e\right)+1\right)-3\left(3\,a^2+30\,ab+35\,b^2\right)\log\left(\cos\left(fx+e\right)-1\right)-\frac{2\left(3\left(3\,a^2\right.\right.}{48\,f}$$

input
```
integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/48*(3*(3*a^2 + 30*a*b + 35*b^2)*log(cos(f*x + e) + 1) - 3*(3*a^2 + 30*a
*b + 35*b^2)*log(cos(f*x + e) - 1) - 2*(3*(3*a^2 + 30*a*b + 35*b^2)*cos(f*
x + e)^6 - 5*(3*a^2 + 30*a*b + 35*b^2)*cos(f*x + e)^4 + 8*(6*a*b + 7*b^2)*
cos(f*x + e)^2 + 8*b^2)/(cos(f*x + e)^7 - 2*cos(f*x + e)^5 + cos(f*x + e)^
3))/f
```

3.20.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 493 vs. $2(131) = 262$.

Time = 0.38 (sec) , antiderivative size = 493, normalized size of antiderivative = 3.50

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx =$$

$$-\frac{\frac{24a^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{96ab(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{72b^2(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{3a^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{6ab(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}-\frac{3b^2(\cos(fx+e)}{(\cos(fx+e)+}$$

input
```
integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-1/192*(24*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 96*a*b*(cos(f*x + e
) - 1)/(cos(f*x + e) + 1) + 72*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) -
 3*a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 6*a*b*(cos(f*x + e) - 1
)^2/(cos(f*x + e) + 1)^2 - 3*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2
 - 12*(3*a^2 + 30*a*b + 35*b^2)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e
) + 1)) + 3*(a^2 + 2*a*b + b^2 - 8*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) +
1) - 32*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 24*b^2*(cos(f*x + e) -
 1)/(cos(f*x + e) + 1) + 18*a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2
+ 180*a*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 210*b^2*(cos(f*x + e
) - 1)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)^2/(cos(f*x + e) - 1)^2 -
 256*(3*a*b + 5*b^2 + 6*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 9*b^2*
(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 3*a*b*(cos(f*x + e) - 1)^2/(cos(f*
x + e) + 1)^2 + 6*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/((cos(f*x
 + e) - 1)/(cos(f*x + e) + 1) + 1)^3)/f
```

3.20.9 Mupad [B] (verification not implemented)

Time = 18.30 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.96

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx$$

$$=\frac{\frac{b^2}{3}+\cos(e+fx)^2\left(\frac{7b^2}{3}+2ab\right)+\cos(e+fx)^6\left(\frac{3a^2}{8}+\frac{15ab}{4}+\frac{35b^2}{8}\right)-\cos(e+fx)^4\left(\frac{5a^2}{8}+\frac{25ab}{4}+\frac{175b}{24}\right.}{f\left(\cos(e+fx)^7-2\cos(e+fx)^5+\cos(e+fx)^3\right)}$$

$$-\frac{\operatorname{atanh}(\cos(e+fx))\left(\frac{3a^2}{8}+\frac{15ab}{4}+\frac{35b^2}{8}\right)}{f}$$

input
```
int((a + b/cos(e + f*x)^2)^2/sin(e + f*x)^5,x)
```

output
```
(b^2/3 + cos(e + f*x)^2*(2*a*b + (7*b^2)/3) + cos(e + f*x)^6*((15*a*b)/4 +
 (3*a^2)/8 + (35*b^2)/8) - cos(e + f*x)^4*((25*a*b)/4 + (5*a^2)/8 + (175*b
^2)/24))/(f*(cos(e + f*x)^3 - 2*cos(e + f*x)^5 + cos(e + f*x)^7)) - (atanh
(cos(e + f*x))*((15*a*b)/4 + (3*a^2)/8 + (35*b^2)/8))/f
```

3.21 $\int (a+b\sec^2(e+fx))^2 \sin^6(e+fx)\,dx$

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3.21.1 Optimal result

Integrand size = 23, antiderivative size = 148

$$\begin{aligned}\int (a+b\sec^2(e+fx))^2 \sin^6(e+fx)\,dx = {}& \frac{5}{16}\left(a^2-12ab+8b^2\right)x \\ & -\frac{\left(3a^2-36ab+8b^2\right)\cos(e+fx)\sin(e+fx)}{16f} \\ & +\frac{a(a-12b)\cos^3(e+fx)\sin(e+fx)}{24f} \\ & -\frac{\left(a^2-12ab+12b^2\right)\tan(e+fx)}{6f} \\ & +\frac{a^2\sin^6(e+fx)\tan(e+fx)}{6f}+\frac{b^2\tan^3(e+fx)}{3f}\end{aligned}$$

output

```
5/16*(a^2-12*a*b+8*b^2)*x-1/16*(3*a^2-36*a*b+8*b^2)*cos(f*x+e)*sin(f*x+e)/
f+1/24*a*(a-12*b)*cos(f*x+e)^3*sin(f*x+e)/f-1/6*(a^2-12*a*b+12*b^2)*tan(f*
x+e)/f+1/6*a^2*sin(f*x+e)^6*tan(f*x+e)/f+1/3*b^2*tan(f*x+e)^3/f
```

3.21.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 499 vs. $2(148) = 296$.

Time $= 3.43$ (sec) , antiderivative size $= 499$, normalized size of antiderivative $= 3.37$

$$\int \left(a + b\sec^2(e+fx)\right)^2 \sin^6(e+fx)\,dx$$
$$= \frac{\left(b + a\cos^2(e+fx)\right)^2 \sec(e)\sec^3(e+fx)\left(360(a^2 - 12ab + 8b^2)\,fx\cos(fx) + 360(a^2 - 12ab + 8b^2)\,fx\cos\right.}{}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^6,x]
```

output
```
((b + a*Cos[e + f*x]^2)^2*Sec[e]*Sec[e + f*x]^3*(360*(a^2 - 12*a*b + 8*b^2
)*f*x*Cos[f*x] + 360*(a^2 - 12*a*b + 8*b^2)*f*x*Cos[2*e + f*x] + 120*a^2*f
*x*Cos[2*e + 3*f*x] - 1440*a*b*f*x*Cos[2*e + 3*f*x] + 960*b^2*f*x*Cos[2*e
+ 3*f*x] + 120*a^2*f*x*Cos[4*e + 3*f*x] - 1440*a*b*f*x*Cos[4*e + 3*f*x] +
960*b^2*f*x*Cos[4*e + 3*f*x] - 81*a^2*Sin[f*x] + 3444*a*b*Sin[f*x] - 3168*
b^2*Sin[f*x] - 81*a^2*Sin[2*e + f*x] - 1164*a*b*Sin[2*e + f*x] + 2208*b^2*
Sin[2*e + f*x] - 109*a^2*Sin[2*e + 3*f*x] + 2076*a*b*Sin[2*e + 3*f*x] - 19
36*b^2*Sin[2*e + 3*f*x] - 109*a^2*Sin[4*e + 3*f*x] + 540*a*b*Sin[4*e + 3*f
*x] - 144*b^2*Sin[4*e + 3*f*x] - 21*a^2*Sin[4*e + 5*f*x] + 156*a*b*Sin[4*e
 + 5*f*x] - 48*b^2*Sin[4*e + 5*f*x] - 21*a^2*Sin[6*e + 5*f*x] + 156*a*b*Si
n[6*e + 5*f*x] - 48*b^2*Sin[6*e + 5*f*x] + 6*a^2*Sin[6*e + 7*f*x] - 12*a*b
*Sin[6*e + 7*f*x] + 6*a^2*Sin[8*e + 7*f*x] - 12*a*b*Sin[8*e + 7*f*x] - a^2
*Sin[8*e + 9*f*x] - a^2*Sin[10*e + 9*f*x]))/(768*f*(a + 2*b + a*Cos[2*(e +
 f*x)])^2)
```

3.21.3 Rubi [A] (verified)

Time $= 0.46$ (sec) , antiderivative size $= 168$, normalized size of antiderivative $= 1.14$, number of steps used $= 9$, number of rules used $= 8$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used $= \{3042, 4620, 366, 360, 25, 2345, 1467, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^6(e+fx)\left(a + b\sec^2(e+fx)\right)^2 dx$$
$$\downarrow 3042$$

$$\int \sin(e+fx)^6 \left(a+b\sec(e+fx)^2\right)^2 dx$$

$$\Big\downarrow \text{4620}$$

$$\frac{\int \frac{\tan^6(e+fx)\left(b\tan^2(e+fx)+a+b\right)^2}{\left(\tan^2(e+fx)+1\right)^4}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{366}$$

$$\frac{\frac{a^2\tan^7(e+fx)}{6\left(\tan^2(e+fx)+1\right)^3}-\frac{1}{6}\int \frac{\tan^6(e+fx)\left(7a^2-6(a+b)^2-6b^2\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{360}$$

$$\frac{\frac{1}{6}\left(\frac{1}{4}\int -\frac{-24b^2\tan^6(e+fx)+4a(a-12b)\tan^4(e+fx)-4a(a-12b)\tan^2(e+fx)+a(a-12b)}{\left(\tan^2(e+fx)+1\right)^2}d\tan(e+fx)+\frac{a(a-12b)\tan(e+fx)}{4\left(\tan^2(e+fx)+1\right)^2}\right)+\frac{a^2\,\mathrm{t}}{6(\tan^2}}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{6}\left(\frac{a(a-12b)\tan(e+fx)}{4\left(\tan^2(e+fx)+1\right)^2}-\frac{1}{4}\int \frac{-24b^2\tan^6(e+fx)+4a(a-12b)\tan^4(e+fx)-4a(a-12b)\tan^2(e+fx)+a(a-12b)}{\left(\tan^2(e+fx)+1\right)^2}d\tan(e+fx)\right)+\frac{a^2\tan}{6(\tan^2(e}}{f}$$

$$\Big\downarrow \text{2345}$$

$$\frac{\frac{1}{6}\left(\frac{1}{4}\left(\frac{1}{2}\int \frac{48b^2\tan^4(e+fx)-8\left(a^2-12ba+6b^2\right)\tan^2(e+fx)+7a^2+24b^2-84ab}{\tan^2(e+fx)+1}d\tan(e+fx)-\frac{3\left(3a^2-36ab+8b^2\right)\tan(e+fx)}{2\left(\tan^2(e+fx)+1\right)}\right)+\frac{a(a-12b)}{4(\tan^2(e}\right.}{f}$$

$$\Big\downarrow \text{1467}$$

$$\frac{\frac{1}{6}\left(\frac{1}{4}\left(\frac{1}{2}\int \left(48b^2\tan^2(e+fx)-8\left(a^2-12ba+12b^2\right)+\frac{15\left(a^2-12ba+8b^2\right)}{\tan^2(e+fx)+1}\right)d\tan(e+fx)-\frac{3\left(3a^2-36ab+8b^2\right)\tan(e+fx)}{2\left(\tan^2(e+fx)+1\right)}\right)\right.}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{6}\left(\frac{1}{4}\left(\frac{1}{2}\left(15\left(a^2-12ab+8b^2\right)\arctan(\tan(e+fx))-8\left(a^2-12ab+12b^2\right)\tan(e+fx)+16b^2\tan^3(e+fx)\right)-\frac{3(3a}{}\right.\right.}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^6,x]
```

3.21. $\int \left(a+b\sec^2(e+fx)\right)^2\sin^6(e+fx)\,dx$

output
```
((a^2*Tan[e + f*x]^7)/(6*(1 + Tan[e + f*x]^2)^3) + ((a*(a - 12*b)*Tan[e +
f*x])/(4*(1 + Tan[e + f*x]^2)^2) + ((-3*(3*a^2 - 36*a*b + 8*b^2)*Tan[e + f
*x])/(2*(1 + Tan[e + f*x]^2)) + (15*(a^2 - 12*a*b + 8*b^2)*ArcTan[Tan[e +
f*x]] - 8*(a^2 - 12*a*b + 12*b^2)*Tan[e + f*x] + 16*b^2*Tan[e + f*x]^3)/2)
/4)/6)/f
```

3.21.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
 + 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.21.4 Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.20

method	result
parallelrisch	$\frac{120fx(a^2-12ab+8b^2)\cos(3fx+3e)+(-109a^2+1308ab-1040b^2)\sin(3fx+3e)+(-21a^2+156ab-48b^2)\sin(5fx+5e)+6}{384f(\cos(3fx+3e)+3}$
derivativedivides	$a^2\left(-\frac{\left(\sin(fx+e)^5+\frac{5\sin(fx+e)^3}{4}+\frac{15\sin(fx+e)}{8}\right)\cos(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)+2ab\left(\frac{\sin(fx+e)^7}{\cos(fx+e)}+\left(\sin(fx+e)^5+\frac{5\sin(fx+e)^3}{4}+\right.\right.$
default	$a^2\left(-\frac{\left(\sin(fx+e)^5+\frac{5\sin(fx+e)^3}{4}+\frac{15\sin(fx+e)}{8}\right)\cos(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)+2ab\left(\frac{\sin(fx+e)^7}{\cos(fx+e)}+\left(\sin(fx+e)^5+\frac{5\sin(fx+e)^3}{4}+\right.\right.$
parts	$\frac{a^2\left(-\frac{\left(\sin(fx+e)^5+\frac{5\sin(fx+e)^3}{4}+\frac{15\sin(fx+e)}{8}\right)\cos(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)}{f}+\frac{b^2\left(\frac{\sin(fx+e)^7}{3\cos(fx+e)^3}-\frac{4\sin(fx+e)^7}{3\cos(fx+e)}-\frac{4\left(\sin(fx+e)^5\right.\right.}{}$
risch	$\frac{5a^2x}{16}-\frac{15xab}{4}+\frac{5xb^2}{2}-\frac{3ie^{4i(fx+e)}a^2}{128f}+\frac{ie^{4i(fx+e)}ab}{32f}+\frac{15ie^{2i(fx+e)}a^2}{128f}-\frac{ie^{2i(fx+e)}ab}{2f}+\frac{ie^{2i(fx+e)}b^2}{8f}-$

input
```
int((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^6,x,method=_RETURNVERBOSE)
```

output
```
1/384*(120*f*x*(a^2-12*a*b+8*b^2)*cos(3*f*x+3*e)+(-109*a^2+1308*a*b-1040*b
^2)*sin(3*f*x+3*e)+(-21*a^2+156*a*b-48*b^2)*sin(5*f*x+5*e)+6*a*(a-2*b)*sin
(7*f*x+7*e)-a^2*sin(9*f*x+9*e)+360*f*x*(a^2-12*a*b+8*b^2)*cos(f*x+e)-81*(a
^2-380/27*a*b+160/27*b^2)*sin(f*x+e))/f/(cos(3*f*x+3*e)+3*cos(f*x+e))
```

3.21.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.89

$$\int \left(a + b\sec^2(e+fx)\right)^2 \sin^6(e+fx)\,dx$$
$$= \frac{15\,(a^2 - 12\,ab + 8\,b^2)fx\cos\left(fx+e\right)^3 - \left(8\,a^2\cos\left(fx+e\right)^8 - 2\,(13\,a^2 - 12\,ab)\cos\left(fx+e\right)^6 + 3\,(11\,a^2 -}{48\,f\cos\left(fx+e\right)^3}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^6,x, algorithm="fricas")
```

output
```
1/48*(15*(a^2 - 12*a*b + 8*b^2)*f*x*cos(f*x + e)^3 - (8*a^2*cos(f*x + e)^8
 - 2*(13*a^2 - 12*a*b)*cos(f*x + e)^6 + 3*(11*a^2 - 36*a*b + 8*b^2)*cos(f*
x + e)^4 - 16*(6*a*b - 7*b^2)*cos(f*x + e)^2 - 16*b^2)*sin(f*x + e))/(f*co
s(f*x + e)^3)
```

3.21.6 Sympy [F(-1)]

Timed out.

$$\int \left(a + b\sec^2(e+fx)\right)^2 \sin^6(e+fx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e)**2)**2*sin(f*x+e)**6,x)
```

output
```
Timed out
```

3.21.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.11

$$\int \left(a+b\sec^2(e+fx)\right)^2 \sin^6(e+fx)\,dx$$

$$= \frac{16\,b^2\tan(fx+e)^3 + 15\,(a^2-12\,ab+8\,b^2)(fx+e) + 96\,(ab-b^2)\tan(fx+e) - \frac{3\,(11\,a^2-36\,ab+8\,b^2)\tan(fx+e)}{\tan(}}{48\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^6,x, algorithm="maxima")
```

output
```
1/48*(16*b^2*tan(f*x + e)^3 + 15*(a^2 - 12*a*b + 8*b^2)*(f*x + e) + 96*(a*
b - b^2)*tan(f*x + e) - (3*(11*a^2 - 36*a*b + 8*b^2)*tan(f*x + e)^5 + 8*(5
*a^2 - 24*a*b + 6*b^2)*tan(f*x + e)^3 + 3*(5*a^2 - 28*a*b + 8*b^2)*tan(f*x
 + e))/(tan(f*x + e)^6 + 3*tan(f*x + e)^4 + 3*tan(f*x + e)^2 + 1))/f
```

3.21.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.24

$$\int \left(a+b\sec^2(e+fx)\right)^2 \sin^6(e+fx)\,dx$$

$$= \frac{16\,b^2\tan(fx+e)^3 + 96\,ab\tan(fx+e) - 96\,b^2\tan(fx+e) + 15\,(a^2-12\,ab+8\,b^2)(fx+e) - \frac{33\,a^2\tan(fx-}{}}{}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^6,x, algorithm="giac")
```

output
```
1/48*(16*b^2*tan(f*x + e)^3 + 96*a*b*tan(f*x + e) - 96*b^2*tan(f*x + e) +
15*(a^2 - 12*a*b + 8*b^2)*(f*x + e) - (33*a^2*tan(f*x + e)^5 - 108*a*b*tan
(f*x + e)^5 + 24*b^2*tan(f*x + e)^5 + 40*a^2*tan(f*x + e)^3 - 192*a*b*tan(
f*x + e)^3 + 48*b^2*tan(f*x + e)^3 + 15*a^2*tan(f*x + e) - 84*a*b*tan(f*x
+ e) + 24*b^2*tan(f*x + e))/(tan(f*x + e)^2 + 1)^3)/f
```

3.21.9 Mupad [B] (verification not implemented)

Time = 19.11 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.10

$$\int \left(a+b\sec^2(e+fx)\right)^2 \sin^6(e+fx)\,dx = x\left(\frac{5\,a^2}{16}-\frac{15\,a\,b}{4}+\frac{5\,b^2}{2}\right) - \frac{\left(\frac{11\,a^2}{16}-\frac{9\,a\,b}{4}+\frac{b^2}{2}\right)\tan(e+f\,x)^5+\left(\frac{5\,a^2}{6}-4\,a\,b+b^2\right)\tan(e+f\,x)^3+\left(\frac{5\,a^2}{16}-\frac{7\,a\,b}{4}+\frac{b^2}{2}\right)\tan(e+f\,x)}{f\left(\tan(e+f\,x)^6+3\tan(e+f\,x)^4+3\tan(e+f\,x)^2+1\right)} + \frac{b^2\tan(e+f\,x)^3}{3\,f}-\frac{\tan(e+f\,x)\left(4\,b^2-2\,b\,(a+b)\right)}{f}$$

input
```
int(sin(e + f*x)^6*(a + b/cos(e + f*x)^2)^2,x)
```

output
```
x*((5*a^2)/16 - (15*a*b)/4 + (5*b^2)/2) - (tan(e + f*x)*((5*a^2)/16 - (7*a
*b)/4 + b^2/2) + tan(e + f*x)^3*((5*a^2)/6 - 4*a*b + b^2) + tan(e + f*x)^5
*((11*a^2)/16 - (9*a*b)/4 + b^2/2))/(f*(3*tan(e + f*x)^2 + 3*tan(e + f*x)^
4 + tan(e + f*x)^6 + 1)) + (b^2*tan(e + f*x)^3)/(3*f) - (tan(e + f*x)*(4*b
^2 - 2*b*(a + b)))/f
```

3.22 $\int (a+b\sec^2(e+fx))^2 \sin^4(e+fx)\,dx$

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3.22.1 Optimal result

Integrand size = 23, antiderivative size = 114

$$\int (a+b\sec^2(e+fx))^2 \sin^4(e+fx)\,dx = \frac{1}{8}\left(3a^2-24ab+8b^2\right)x - \frac{a(a-8b)\cos(e+fx)\sin(e+fx)}{8f} - \frac{\left(a^2-8ab+4b^2\right)\tan(e+fx)}{4f} + \frac{a^2\sin^4(e+fx)\tan(e+fx)}{4f} + \frac{b^2\tan^3(e+fx)}{3f}$$

output

```
1/8*(3*a^2-24*a*b+8*b^2)*x-1/8*a*(a-8*b)*cos(f*x+e)*sin(f*x+e)/f-1/4*(a^2-
8*a*b+4*b^2)*tan(f*x+e)/f+1/4*a^2*sin(f*x+e)^4*tan(f*x+e)/f+1/3*b^2*tan(f*
x+e)^3/f
```

3.22.2 Mathematica [A] (verified)

Time = 2.23 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.34

$$\int (a+b\sec^2(e+fx))^2 \sin^4(e+fx)\,dx = \frac{(b+a\cos^2(e+fx))^2 \sec^3(e+fx)\left(32b^2\sec(e)\sin(fx)+64(3a-2b)b\cos^2(e+fx)\sec(e)\sin(fx)+3\cos^{3}\right.}{24f(a+2b+}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^4,x]
```

output
```
((b + a*Cos[e + f*x]^2)^2*Sec[e + f*x]^3*(32*b^2*Sec[e]*Sin[f*x] + 64*(3*a
 - 2*b)*b*Cos[e + f*x]^2*Sec[e]*Sin[f*x] + 3*Cos[e + f*x]^3*(4*(3*a^2 - 24
*a*b + 8*b^2)*f*x - 8*a*(a - 2*b)*Sin[2*(e + f*x)] + a^2*Sin[4*(e + f*x)])
 + 32*b^2*Cos[e + f*x]*Tan[e]))/(24*f*(a + 2*b + a*Cos[2*(e + f*x)])^2)
```

3.22.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4620, 366, 360, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int \sin(e+fx)^4\left(a+b\sec(e+fx)^2\right)^2 dx$$

$$\downarrow 4620$$

$$\frac{\int \frac{\tan^4(e+fx)\left(b\tan^2(e+fx)+a+b\right)^2}{\left(\tan^2(e+fx)+1\right)^3} d\tan(e+fx)}{f}$$

$$\downarrow 366$$

$$\frac{\frac{a^2\tan^5(e+fx)}{4\left(\tan^2(e+fx)+1\right)^2} - \frac{1}{4}\int \frac{\tan^4(e+fx)\left(5a^2-4(a+b)^2-4b^2\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)^2} d\tan(e+fx)}{f}$$

$$\downarrow 360$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\int \frac{8b^2\tan^4(e+fx)-2a(a-8b)\tan^2(e+fx)+a(a-8b)}{\tan^2(e+fx)+1} d\tan(e+fx) - \frac{a(a-8b)\tan(e+fx)}{2\left(\tan^2(e+fx)+1\right)}\right) + \frac{a^2\tan^5(e+fx)}{4\left(\tan^2(e+fx)+1\right)^2}}{f}$$

$$\downarrow 1467$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\int \left(8b^2\tan^2(e+fx) - 2\left(a^2-8ba+4b^2\right) + \frac{3a^2-24ba+8b^2}{\tan^2(e+fx)+1}\right) d\tan(e+fx) - \frac{a(a-8b)\tan(e+fx)}{2\left(\tan^2(e+fx)+1\right)}\right) + \frac{a^2\tan^5(e+fx)}{4\left(\tan^2(e+fx)+1\right)^2}}{f}$$

$$\downarrow 2009$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(\left(3a^2-24ab+8b^2\right)\arctan(\tan(e+fx))-2\left(a^2-8ab+4b^2\right)\tan(e+fx)+\frac{8}{3}b^2\tan^3(e+fx)\right)-\frac{a(a-8b)\tan(e}{2(\tan^2(e+fx}\right.}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^4,x]
```

output
```
((a^2*Tan[e + f*x]^5)/(4*(1 + Tan[e + f*x]^2)^2) + (-1/2*(a*(a - 8*b)*Tan[
e + f*x])/(1 + Tan[e + f*x]^2) + ((3*a^2 - 24*a*b + 8*b^2)*ArcTan[Tan[e +
f*x]] - 2*(a^2 - 8*a*b + 4*b^2)*Tan[e + f*x] + (8*b^2*Tan[e + f*x]^3)/3)/2
)/4)/f
```

3.22.3.1 Defintions of rubi rules used

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.22.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{a^2\left(-\frac{\left(\sin(fx+e)^3+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+2ab\left(\frac{\sin(fx+e)^5}{\cos(fx+e)}+\left(\sin(fx+e)^3+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)-\frac{3fx}{2}-$
default	$\frac{a^2\left(-\frac{\left(\sin(fx+e)^3+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+2ab\left(\frac{\sin(fx+e)^5}{\cos(fx+e)}+\left(\sin(fx+e)^3+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)-\frac{3fx}{2}-$
parts	$\frac{a^2\left(-\frac{\left(\sin(fx+e)^3+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)}{f}+\frac{b^2\left(\frac{\tan(fx+e)^3}{3}-\tan(fx+e)+fx+e\right)}{f}+\frac{2ab\left(\frac{\sin(fx+e)^5}{\cos(fx+e)}+$
parallelrisch	$\frac{72fx\left(a^2-8ab+\frac{8}{3}b^2\right)\cos(3fx+3e)+\left(-63a^2+528ab-256b^2\right)\sin(3fx+3e)+\left(-15a^2+48ab\right)\sin(5fx+5e)+3a^2\sin(7fx+7}{192f(\cos(3fx+3e)+3\cos(fx+e))}$
risch	$\frac{3a^2x}{8}-3xab+x\,b^2-\frac{ie^{4i(fx+e)}a^2}{64f}+\frac{ie^{2i(fx+e)}a^2}{8f}-\frac{ie^{2i(fx+e)}ab}{4f}-\frac{ie^{-2i(fx+e)}a^2}{8f}+\frac{ie^{-2i(fx+e)}ab}{4f}+$
norman	$\frac{\left(-\frac{3}{8}a^2+3ab-b^2\right)x+\left(-\frac{9}{8}a^2+9ab-3b^2\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8+\left(-\frac{9}{8}a^2+9ab-3b^2\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}+\left(-\frac{3}{8}a^2+3ab-b^2\right)x\tan\left(\frac{f}{2}\right.$

input
```
int((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^4,x,method=_RETURNVERBOSE)
```

output
```
1/f*(a^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+2*a
*b*(sin(f*x+e)^5/cos(f*x+e)+(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)-3/2*f
*x-3/2*e)+b^2*(1/3*tan(f*x+e)^3-tan(f*x+e)+f*x+e))
```

3.22.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.94

$$\int \left(a+b\sec^2(e+fx)\right)^2 \sin^4(e+fx)\,dx$$
$$= \frac{3\left(3a^2-24ab+8b^2\right)fx\cos\left(fx+e\right)^3+\left(6a^2\cos\left(fx+e\right)^6-3\left(5a^2-8ab\right)\cos\left(fx+e\right)^4+16\left(3ab-2b\right.\right.}{24f\cos\left(fx+e\right)^3}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^4,x, algorithm="fricas")
```

output
```
1/24*(3*(3*a^2 - 24*a*b + 8*b^2)*f*x*cos(f*x + e)^3 + (6*a^2*cos(f*x + e)^
6 - 3*(5*a^2 - 8*a*b)*cos(f*x + e)^4 + 16*(3*a*b - 2*b^2)*cos(f*x + e)^2 +
 8*b^2)*sin(f*x + e))/(f*cos(f*x + e)^3)
```

3.22.6 Sympy [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^2 \sin^4(e+fx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e)**2)**2*sin(f*x+e)**4,x)
```

output
```
Timed out
```

3.22.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.05

$$\int \left(a+b\sec^2(e+fx)\right)^2 \sin^4(e+fx)\,dx$$
$$= \frac{8b^2\tan\left(fx+e\right)^3+3\left(3a^2-24ab+8b^2\right)(fx+e)+24\left(2ab-b^2\right)\tan\left(fx+e\right)-\frac{3\left(\left(5a^2-8ab\right)\tan(fx+e)^3+(3\right.}{\tan(fx+e)^4+2\tan}}{24f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^4,x, algorithm="maxima")
```

output
```
1/24*(8*b^2*tan(f*x + e)^3 + 3*(3*a^2 - 24*a*b + 8*b^2)*(f*x + e) + 24*(2*
a*b - b^2)*tan(f*x + e) - 3*((5*a^2 - 8*a*b)*tan(f*x + e)^3 + (3*a^2 - 8*a
*b)*tan(f*x + e))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1))/f
```

3.22.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.08

$$\int \left(a+b\sec^2(e+fx)\right)^2 \sin^4(e+fx)\,dx$$

$$= \frac{8\,b^2\tan(fx+e)^3 + 48\,ab\tan(fx+e) - 24\,b^2\tan(fx+e) + 3\,(3\,a^2 - 24\,ab + 8\,b^2)(fx+e) - \frac{3\left(5\,a^2\tan(fx}{}}{24\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^4,x, algorithm="giac")
```

output
```
1/24*(8*b^2*tan(f*x + e)^3 + 48*a*b*tan(f*x + e) - 24*b^2*tan(f*x + e) + 3
*(3*a^2 - 24*a*b + 8*b^2)*(f*x + e) - 3*(5*a^2*tan(f*x + e)^3 - 8*a*b*tan(
f*x + e)^3 + 3*a^2*tan(f*x + e) - 8*a*b*tan(f*x + e))/(tan(f*x + e)^2 + 1)
^2)/f
```

3.22.9 Mupad [B] (verification not implemented)

Time = 18.78 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.02

$$\int \left(a+b\sec^2(e+fx)\right)^2 \sin^4(e+fx)\,dx$$

$$= x\left(\frac{3\,a^2}{8} - 3\,a\,b + b^2\right) + \frac{\left(a\,b - \frac{5\,a^2}{8}\right)\tan(e+f\,x)^3 + \left(a\,b - \frac{3\,a^2}{8}\right)\tan(e+f\,x)}{f\,\left(\tan(e+f\,x)^4 + 2\tan(e+f\,x)^2 + 1\right)} + \frac{b^2\tan(e+f\,x)^3}{3\,f} - \frac{\tan(e+f\,x)\,\left(3\,b^2 - 2\,b\,(a+b)\right)}{f}$$

input
```
int(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^2,x)
```

output
```
x*((3*a^2)/8 - 3*a*b + b^2) + (tan(e + f*x)*(a*b - (3*a^2)/8) + tan(e + f*
x)^3*(a*b - (5*a^2)/8))/(f*(2*tan(e + f*x)^2 + tan(e + f*x)^4 + 1)) + (b^2
*tan(e + f*x)^3)/(3*f) - (tan(e + f*x)*(3*b^2 - 2*b*(a + b)))/f
```

3.23 $\int (a + b\sec^2(e+fx))^2 \sin^2(e+fx)\,dx$

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3.23.1 Optimal result

Integrand size = 23, antiderivative size = 73

$$\int (a + b\sec^2(e+fx))^2 \sin^2(e+fx)\,dx = \frac{1}{2}a(a-4b)x - \frac{a(a-4b)\tan(e+fx)}{2f} + \frac{a^2\sin^2(e+fx)\tan(e+fx)}{2f} + \frac{b^2\tan^3(e+fx)}{3f}$$

output

```
1/2*a*(a-4*b)*x-1/2*a*(a-4*b)*tan(f*x+e)/f+1/2*a^2*sin(f*x+e)^2*tan(f*x+e)
/f+1/3*b^2*tan(f*x+e)^3/f
```

3.23.2 Mathematica [A] (verified)

Time = 2.19 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.73

$$\int (a + b\sec^2(e+fx))^2 \sin^2(e+fx)\,dx = -\frac{(b + a\cos^2(e+fx))^2 \sec^3(e+fx)\,(-4b^2\sec(e)\sin(fx) - 4(6a-b)b\cos^2(e+fx)\sec(e)\sin(fx) + 3a\,\text{co}}{3f(a+2b+a\cos(2(e+fx)))^2}$$

input

```
Integrate[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^2,x]
```

output

```
-1/3*((b + a*Cos[e + f*x]^2)^2*Sec[e + f*x]^3*(-4*b^2*Sec[e]*Sin[f*x] - 4*
(6*a - b)*b*Cos[e + f*x]^2*Sec[e]*Sin[f*x] + 3*a*Cos[e + f*x]^3*(-2*(a - 4
*b)*f*x + a*Sin[2*(e + f*x)]) - 4*b^2*Cos[e + f*x]*Tan[e]))/(f*(a + 2*b +
a*Cos[2*(e + f*x)])^2)
```

3.23.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4620, 366, 363, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int \sin(e+fx)^2\left(a+b\sec(e+fx)^2\right)^2 dx$$

$$\downarrow 4620$$

$$\frac{\int \frac{\tan^2(e+fx)\left(b\tan^2(e+fx)+a+b\right)^2}{\left(\tan^2(e+fx)+1\right)^2} d\tan(e+fx)}{f}$$

$$\downarrow 366$$

$$\frac{\frac{a^2\tan^3(e+fx)}{2(\tan^2(e+fx)+1)} - \frac{1}{2}\int \frac{\tan^2(e+fx)(3a^2-2(a+b)^2-2b^2\tan^2(e+fx))}{\tan^2(e+fx)+1} d\tan(e+fx)}{f}$$

$$\downarrow 363$$

$$\frac{\frac{1}{2}\left(\frac{2}{3}b^2\tan^3(e+fx) - a(a-4b)\int \frac{\tan^2(e+fx)}{\tan^2(e+fx)+1} d\tan(e+fx)\right) + \frac{a^2\tan^3(e+fx)}{2(\tan^2(e+fx)+1)}}{f}$$

$$\downarrow 262$$

$$\frac{\frac{1}{2}\left(\frac{2}{3}b^2\tan^3(e+fx) - a(a-4b)\left(\tan(e+fx) - \int \frac{1}{\tan^2(e+fx)+1} d\tan(e+fx)\right)\right) + \frac{a^2\tan^3(e+fx)}{2(\tan^2(e+fx)+1)}}{f}$$

$$\downarrow 216$$

$$\frac{\frac{a^2\tan^3(e+fx)}{2(\tan^2(e+fx)+1)} + \frac{1}{2}\left(\frac{2}{3}b^2\tan^3(e+fx) - a(a-4b)(\tan(e+fx) - \arctan(\tan(e+fx)))\right)}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^2*Sin[e + f*x]^2,x]
```

output
```
((a^2*Tan[e + f*x]^3)/(2*(1 + Tan[e + f*x]^2)) + ((2*b^2*Tan[e + f*x]^3)/3
 - a*(a - 4*b)*(-ArcTan[Tan[e + f*x]] + Tan[e + f*x]))/2)/f
```

3.23.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.23.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{a^2\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+2ab(\tan(fx+e)-fx-e)+\frac{b^2\sin(fx+e)^3}{3\cos(fx+e)^3}}{f}$
default	$\frac{a^2\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+2ab(\tan(fx+e)-fx-e)+\frac{b^2\sin(fx+e)^3}{3\cos(fx+e)^3}}{f}$
parts	$\frac{a^2\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{b^2\sin(fx+e)^3}{3f\cos(fx+e)^3}+\frac{2ab(\tan(fx+e)-fx-e)}{f}$
risch	$\frac{a^2x}{2}-2xab+\frac{ie^{2i(fx+e)}a^2}{8f}-\frac{ie^{-2i(fx+e)}a^2}{8f}-\frac{2ib\left(-6a\,e^{4i(fx+e)}+3b\,e^{4i(fx+e)}-12a\,e^{2i(fx+e)}-6a+b\right)}{3f\left(e^{2i(fx+e)}+1\right)^3}$
parallelrisch	$\frac{12afx(a-4b)\cos(3fx+3e)+\left(-9a^2+48ab-8b^2\right)\sin(3fx+3e)-3\sin(5fx+5e)a^2+36afx(a-4b)\cos(fx+e)-6\sin(fx+e)}{24f(\cos(3fx+3e)+3\cos(fx+e))}$
norman	$\frac{(-\frac{1}{2}a^2+2ab)x+(-\frac{1}{2}a^2+2ab)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8+(\frac{1}{2}a^2-2ab)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+(\frac{1}{2}a^2-2ab)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}+(-a^2+4ab)x}$

input
```
int((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(a^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+2*a*b*(tan(f*x+e)-f*x-
e)+1/3*b^2*sin(f*x+e)^3/cos(f*x+e)^3)
```

3.23.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.11

$$\int \left(a+b\sec^2(e+fx)\right)^2\sin^2(e+fx)\,dx$$
$$=\frac{3\left(a^2-4ab\right)fx\cos\left(fx+e\right)^3-\left(3a^2\cos\left(fx+e\right)^4-2\left(6ab-b^2\right)\cos\left(fx+e\right)^2-2b^2\right)\sin\left(fx+e\right)}{6f\cos\left(fx+e\right)^3}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^2,x, algorithm="fricas")
```

output
```
1/6*(3*(a^2 - 4*a*b)*f*x*cos(f*x + e)^3 - (3*a^2*cos(f*x + e)^4 - 2*(6*a*b
 - b^2)*cos(f*x + e)^2 - 2*b^2)*sin(f*x + e))/(f*cos(f*x + e)^3)
```

3.23.6 Sympy [F]

$$\int \left(a + b\sec^2(e+fx)\right)^2 \sin^2(e+fx)\,dx = \int \left(a + b\sec^2(e+fx)\right)^2 \sin^2(e+fx)\,dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**2*sin(f*x+e)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**2*sin(e + f*x)**2, x)
```

3.23.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.92

$$\int \left(a + b\sec^2(e+fx)\right)^2 \sin^2(e+fx)\,dx$$
$$= \frac{2\,b^2\tan(fx+e)^3 + 12\,ab\tan(fx+e) + 3\,(a^2 - 4\,ab)(fx+e) - \frac{3\,a^2\tan(fx+e)}{\tan(fx+e)^2+1}}{6\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^2,x, algorithm="maxima")
```

output
```
1/6*(2*b^2*tan(f*x + e)^3 + 12*a*b*tan(f*x + e) + 3*(a^2 - 4*a*b)*(f*x + e
) - 3*a^2*tan(f*x + e)/(tan(f*x + e)^2 + 1))/f
```

3.23.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.92

$$\int \left(a + b\sec^2(e+fx)\right)^2 \sin^2(e+fx)\,dx$$
$$= \frac{2\,b^2\tan(fx+e)^3 + 12\,ab\tan(fx+e) + 3\,(a^2 - 4\,ab)(fx+e) - \frac{3\,a^2\tan(fx+e)}{\tan(fx+e)^2+1}}{6\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*sin(f*x+e)^2,x, algorithm="giac")
```

output
```
1/6*(2*b^2*tan(f*x + e)^3 + 12*a*b*tan(f*x + e) + 3*(a^2 - 4*a*b)*(f*x + e
) - 3*a^2*tan(f*x + e)/(tan(f*x + e)^2 + 1))/f
```

3.23.9 Mupad [B] (verification not implemented)

Time = 19.13 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.29

$$\int \left(a+b\sec^2(e+fx)\right)^2 \sin^2(e+fx)\,dx = \frac{b^2\tan(e+f\,x)^3}{3\,f} - \frac{a^2\,\sin(2\,e+2\,f\,x)}{4\,f} - \frac{\tan(e+f\,x)\,\left(2\,b^2-2\,b\,(a+b)\right)}{f} - \frac{a\,\mathrm{atan}\left(\frac{a\,\tan(e+f\,x)\,(a-4\,b)}{2\left(2\,a\,b-\frac{a^2}{2}\right)}\right)\,(a-4\,b)}{2\,f}$$

input
```
int(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^2,x)
```

output
```
(b^2*tan(e + f*x)^3)/(3*f) - (a^2*sin(2*e + 2*f*x))/(4*f) - (tan(e + f*x)*
(2*b^2 - 2*b*(a + b)))/f - (a*atan((a*tan(e + f*x)*(a - 4*b))/(2*(2*a*b -
a^2/2)))*(a - 4*b))/(2*f)
```

3.24 $\int (a + b\sec^2(e + fx))^2\, dx$

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3.24.1 Optimal result

Integrand size = 14, antiderivative size = 40

$$\int (a + b\sec^2(e + fx))^2\, dx = a^2x + \frac{b(2a + b)\tan(e + fx)}{f} + \frac{b^2\tan^3(e + fx)}{3f}$$

output
```
a^2*x+b*(2*a+b)*tan(f*x+e)/f+1/3*b^2*tan(f*x+e)^3/f
```

3.24.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int (a + b\sec^2(e + fx))^2\, dx = \frac{3a^2fx + 3b(2a + b)\tan(e + fx) + b^2\tan^3(e + fx)}{3f}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(3*a^2*f*x + 3*b*(2*a + b)*Tan[e + f*x] + b^2*Tan[e + f*x]^3)/(3*f)
```

3.24.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4616, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + b\sec^2(e+fx))^2\,dx$$

$$\downarrow 3042$$

$$\int (a + b\sec(e+fx)^2)^2\,dx$$

$$\downarrow 4616$$

$$\frac{\int \frac{(b\tan^2(e+fx)+a+b)^2}{\tan^2(e+fx)+1} d\tan(e+fx)}{f}$$

$$\downarrow 300$$

$$\frac{\int \left(\frac{a^2}{\tan^2(e+fx)+1} + b^2\tan^2(e+fx) + b(2a+b)\right) d\tan(e+fx)}{f}$$

$$\downarrow 2009$$

$$\frac{a^2\arctan(\tan(e+fx)) + b(2a+b)\tan(e+fx) + \frac{1}{3}b^2\tan^3(e+fx)}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(a^2*ArcTan[Tan[e + f*x]] + b*(2*a + b)*Tan[e + f*x] + (b^2*Tan[e + f*x]^3
)/3)/f
```

3.24.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f  Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

3.24.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

method	result
parts	$a^2x-\frac{b^2\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)}{f}+\frac{2ab\tan(fx+e)}{f}$
derivativedivides	$\frac{a^2(fx+e)+2ab\tan(fx+e)-b^2\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)}{f}$
default	$\frac{a^2(fx+e)+2ab\tan(fx+e)-b^2\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)}{f}$
risch	$a^2x+\frac{4ib\left(3a\,\mathrm{e}^{4i(fx+e)}+6a\,\mathrm{e}^{2i(fx+e)}+3b\,\mathrm{e}^{2i(fx+e)}+3a+b\right)}{3f\left(\mathrm{e}^{2i(fx+e)}+1\right)^3}$
norman	$\frac{a^2x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6-a^2x+3a^2x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-3a^2x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4-\frac{2b(2a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{2b(2a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{f}+\frac{4b(6a}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3}$
parallelrisch	$\frac{3x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6a^2f+(-12ab-6b^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5-9x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4a^2f+(24ab+4b^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3+9x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{3f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}$

input
```
int((a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x-b^2/f*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+2*a*b/f*tan(f*x+e)
```

3.24.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.45

$$\int \left(a + b\sec^2(e+fx)\right)^2 dx$$
$$= \frac{3\,a^2 f x \cos\left(fx+e\right)^3 + \left(2\left(3\,ab+b^2\right)\cos\left(fx+e\right)^2 + b^2\right)\sin\left(fx+e\right)}{3\,f\cos\left(fx+e\right)^3}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/3*(3*a^2*f*x*cos(f*x + e)^3 + (2*(3*a*b + b^2)*cos(f*x + e)^2 + b^2)*sin
(f*x + e))/(f*cos(f*x + e)^3)
```

3.24.6 Sympy [F]

$$\int \left(a + b\sec^2(e+fx)\right)^2 dx = \int \left(a + b\sec^2\left(e+fx\right)\right)^2 dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**2, x)
```

3.24.7 Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10

$$\int \left(a + b\sec^2(e+fx)\right)^2 dx = a^2 x + \frac{\left(\tan\left(fx+e\right)^3 + 3\,\tan\left(fx+e\right)\right)b^2}{3\,f} + \frac{2\,ab\tan\left(fx+e\right)}{f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
a^2*x + 1/3*(tan(f*x + e)^3 + 3*tan(f*x + e))*b^2/f + 2*a*b*tan(f*x + e)/f
```

3.24.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.22

$$\int \left(a + b\sec^2(e+fx)\right)^2 dx$$
$$= \frac{b^2\tan(fx+e)^3 + 3(fx+e)a^2 + 6ab\tan(fx+e) + 3b^2\tan(fx+e)}{3f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/3*(b^2*tan(f*x + e)^3 + 3*(f*x + e)*a^2 + 6*a*b*tan(f*x + e) + 3*b^2*tan
(f*x + e))/f
```

3.24.9 Mupad [B] (verification not implemented)

Time = 18.82 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \left(a + b\sec^2(e+fx)\right)^2 dx = \frac{\frac{b^2\tan(e+fx)^3}{3} - \tan(e+fx)\left(b^2 - 2b(a+b)\right) + a^2 fx}{f}$$

input
```
int((a + b/cos(e + f*x)^2)^2,x)
```

output
```
((b^2*tan(e + f*x)^3)/3 - tan(e + f*x)*(b^2 - 2*b*(a + b)) + a^2*f*x)/f
```

3.25 $\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

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3.25.1 Optimal result

Integrand size = 23, antiderivative size = 50

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{(a+b)^2\cot(e+fx)}{f} + \frac{2b(a+b)\tan(e+fx)}{f} + \frac{b^2\tan^3(e+fx)}{3f}$$

output
```
-(a+b)^2*cot(f*x+e)/f+2*b*(a+b)*tan(f*x+e)/f+1/3*b^2*tan(f*x+e)^3/f
```

3.25.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 109 vs. 2(50) = 100.

Time = 2.62 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.18

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{4(b+a\cos^2(e+fx))^2\sec^3(e+fx)\left(b^2\sec(e)\sin(fx)+\cos^2(e+fx)\left(3(a+b)^2\cot(e+fx)\csc(e)+b(6a\right.\right.}{3f(a+2b+a\cos(2(e+fx)))^2}$$

input
```
Integrate[Csc[e + f*x]^2*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(4*(b + a*Cos[e + f*x]^2)^2*Sec[e + f*x]^3*(b^2*Sec[e]*Sin[f*x] + Cos[e +
f*x]^2*(3*(a + b)^2*Cot[e + f*x]*Csc[e] + b*(6*a + 5*b)*Sec[e])*Sin[f*x] +
 b^2*Cos[e + f*x]*Tan[e]))/(3*f*(a + 2*b + a*Cos[2*(e + f*x)])^2)
```

3.25.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4620, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^2}{\sin(e+fx)^2} dx$$

$$\downarrow 4620$$

$$\frac{\int \cot^2(e+fx)\left(b\tan^2(e+fx)+a+b\right)^2 d\tan(e+fx)}{f}$$

$$\downarrow 244$$

$$\frac{\int \left((a+b)^2\cot^2(e+fx)+b^2\tan^2(e+fx)+2b(a+b)\right) d\tan(e+fx)}{f}$$

$$\downarrow 2009$$

$$\frac{2b(a+b)\tan(e+fx)-(a+b)^2\cot(e+fx)+\frac{1}{3}b^2\tan^3(e+fx)}{f}$$

input
```
Int[Csc[e + f*x]^2*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(-((a + b)^2*Cot[e + f*x]) + 2*b*(a + b)*Tan[e + f*x] + (b^2*Tan[e + f*x]^
3)/3)/f
```

3.25.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.25.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.92

method	result
derivativedivides	$\frac{-a^2\cot(fx+e)+2ab\left(\frac{1}{\sin(fx+e)\cos(fx+e)}-2\cot(fx+e)\right)+b^2\left(\frac{1}{3\sin(fx+e)\cos(fx+e)^3}+\frac{4}{3\sin(fx+e)\cos(fx+e)}-\frac{8\cot(fx+e)}{3}\right.}{f}$
default	$\frac{-a^2\cot(fx+e)+2ab\left(\frac{1}{\sin(fx+e)\cos(fx+e)}-2\cot(fx+e)\right)+b^2\left(\frac{1}{3\sin(fx+e)\cos(fx+e)^3}+\frac{4}{3\sin(fx+e)\cos(fx+e)}-\frac{8\cot(fx+e)}{3}\right.}{f}$
risch	$-\frac{2i\left(3a^2e^{6i(fx+e)}+9a^2e^{4i(fx+e)}+12ab\,e^{4i(fx+e)}+9a^2e^{2i(fx+e)}+24ab\,e^{2i(fx+e)}+16b^2e^{2i(fx+e)}+3a^2+12ab+8b^2\right)}{3f\left(e^{2i(fx+e)}+1\right)^3\left(e^{2i(fx+e)}-1\right)}$
parallelrisch	$\frac{\cot\left(\frac{fx}{2}+\frac{e}{2}\right)\left((a+b)^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8-4(a+3b)(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6+\left(6a^2+28ab+\frac{50}{3}b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4-4(a+3b)(a+b)\ta\right.}{2f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}$
norman	$\frac{\frac{a^2+2ab+b^2}{2f}+\frac{\left(a^2+2ab+b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8}{2f}-\frac{2\left(a^2+4ab+3b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{f}-\frac{2\left(a^2+4ab+3b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{f}+\frac{\left(9a^2+42ab+25b^2\right)}{3f}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3}$

input
```
int(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-a^2*cot(f*x+e)+2*a*b*(1/sin(f*x+e)/cos(f*x+e)-2*cot(f*x+e))+b^2*(1/3
/sin(f*x+e)/cos(f*x+e)^3+4/3/sin(f*x+e)/cos(f*x+e)-8/3*cot(f*x+e)))
```

3.25.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.42

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{\left(3a^2+12ab+8b^2\right)\cos(fx+e)^4-2\left(3ab+2b^2\right)\cos(fx+e)^2-b^2}{3f\cos(fx+e)^3\sin(fx+e)}$$

input
```
integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
-1/3*((3*a^2 + 12*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 2*b^2)*cos(f*x
+ e)^2 - b^2)/(f*cos(f*x + e)^3*sin(f*x + e))
```

3.25.6 Sympy [F]

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \int \left(a+b\sec^2(e+fx)\right)^2\csc^2(e+fx)\,dx$$

input
```
integrate(csc(f*x+e)**2*(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**2*csc(e + f*x)**2, x)
```

3.25.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{b^2\tan(fx+e)^3+6\left(ab+b^2\right)\tan(fx+e)-\frac{3\left(a^2+2ab+b^2\right)}{\tan(fx+e)}}{3f}$$

input
```
integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/3*(b^2*tan(f*x + e)^3 + 6*(a*b + b^2)*tan(f*x + e) - 3*(a^2 + 2*a*b + b^
2)/tan(f*x + e))/f
```

3.25.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.20

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{b^2\tan(fx+e)^3+6\,ab\tan(fx+e)+6\,b^2\tan(fx+e)-\frac{3\,(a^2+2\,ab+b^2)}{\tan(fx+e)}}{3\,f}$$

input
```
integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/3*(b^2*tan(f*x + e)^3 + 6*a*b*tan(f*x + e) + 6*b^2*tan(f*x + e) - 3*(a^2
 + 2*a*b + b^2)/tan(f*x + e))/f
```

3.25.9 Mupad [B] (verification not implemented)

Time = 18.62 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.12

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{b^2\tan(e+f\,x)^3}{3\,f}-\frac{a^2+2\,a\,b+b^2}{f\tan(e+f\,x)}+\frac{2\,b\tan(e+f\,x)\,(a+b)}{f}$$

input
```
int((a + b/cos(e + f*x)^2)^2/sin(e + f*x)^2,x)
```

output
```
(b^2*tan(e + f*x)^3)/(3*f) - (2*a*b + a^2 + b^2)/(f*tan(e + f*x)) + (2*b*t
an(e + f*x)*(a + b))/f
```

3.26 $\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

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3.26.1 Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{(a+b)(a+3b)\cot(e+fx)}{f} - \frac{(a+b)^2\cot^3(e+fx)}{3f} + \frac{b(2a+3b)\tan(e+fx)}{f} + \frac{b^2\tan^3(e+fx)}{3f}$$

output
```
-(a+b)*(a+3*b)*cot(f*x+e)/f-1/3*(a+b)^2*cot(f*x+e)^3/f+b*(2*a+3*b)*tan(f*x
+e)/f+1/3*b^2*tan(f*x+e)^3/f
```

3.26.2 Mathematica [A] (verified)

Time = 5.18 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.99

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{\csc(2e)\csc^3(2(e+fx))\left(8a(a+2b)\sin(2e) - 6(a+2b)^2\sin(2fx) - 3a^2\sin(2(e+fx)) - 6ab\sin(2(e+\right.$$

input
```
Integrate[Csc[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
-1/6*(Csc[2*e]*Csc[2*(e + f*x)]^3*(8*a*(a + 2*b)*Sin[2*e] - 6*(a + 2*b)^2*
Sin[2*f*x] - 3*a^2*Sin[2*(e + f*x)] - 6*a*b*Sin[2*(e + f*x)] + a^2*Sin[6*(
e + f*x)] + 2*a*b*Sin[6*(e + f*x)] + 3*a^2*Sin[4*e + 2*f*x] + a^2*Sin[4*e
+ 6*f*x] + 8*a*b*Sin[4*e + 6*f*x] + 8*b^2*Sin[4*e + 6*f*x]))/f
```

3.26.3 Rubi [A] (verified)

Time $= 0.26$ (sec) , antiderivative size $= 68$, normalized size of antiderivative $= 0.89$, number of steps used $= 5$, number of rules used $= 4$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used $= \{3042, 4620, 355, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^2}{\sin(e+fx)^4}dx$$

$$\downarrow 4620$$

$$\frac{\int \cot^4(e+fx)\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2 d\tan(e+fx)}{f}$$

$$\downarrow 355$$

$$\frac{\int \left((a+b)^2\cot^4(e+fx)+(a+b)(a+3b)\cot^2(e+fx)+b^2\tan^2(e+fx)+b(2a+3b)\right)d\tan(e+fx)}{f}$$

$$\downarrow 2009$$

$$\frac{b(2a+3b)\tan(e+fx)-\frac{1}{3}(a+b)^2\cot^3(e+fx)-(a+b)(a+3b)\cot(e+fx)+\frac{1}{3}b^2\tan^3(e+fx)}{f}$$

input
```
Int[Csc[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(-((a + b)*(a + 3*b)*Cot[e + f*x]) - ((a + b)^2*Cot[e + f*x]^3)/3 + b*(2*a
 + 3*b)*Tan[e + f*x] + (b^2*Tan[e + f*x]^3)/3)/f
```

3.26.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
 + 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.26.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.71

method	result
risch	$\frac{4i\left(3a^2e^{8i(fx+e)}+8a^2e^{6i(fx+e)}+16abe^{6i(fx+e)}+6a^2e^{4i(fx+e)}+24abe^{4i(fx+e)}+24b^2e^{4i(fx+e)}-a^2-8ab-8b^2\right)}{3f\left(e^{2i(fx+e)}-1\right)^3\left(e^{2i(fx+e)}+1\right)^3}$
parallelrisch	$-\frac{(9\cos(2fx+2e)a^2+24\cos(2fx+2e)ab+24\cos(2fx+2e)b^2-a^2\cos(6fx+6e)-8ab\cos(6fx+6e)-8b^2\cos(6fx+6e)+8a}{96f(\cos(3fx+3e)+3\cos(fx+e))}$
derivativedivides	$\frac{a^2\left(-\frac{2}{3}-\frac{\csc(fx+e)^2}{3}\right)\cot(fx+e)+2ab\left(-\frac{1}{3\sin(fx+e)^3\cos(fx+e)}+\frac{4}{3\sin(fx+e)\cos(fx+e)}-\frac{8\cot(fx+e)}{3}\right)+b^2\left(\frac{1}{3\sin(fx+e)^3c}\right.}{f}$
default	$\frac{a^2\left(-\frac{2}{3}-\frac{\csc(fx+e)^2}{3}\right)\cot(fx+e)+2ab\left(-\frac{1}{3\sin(fx+e)^3\cos(fx+e)}+\frac{4}{3\sin(fx+e)\cos(fx+e)}-\frac{8\cot(fx+e)}{3}\right)+b^2\left(\frac{1}{3\sin(fx+e)^3c}\right.}{f}$
norman	$\frac{a^2+2ab+b^2}{24f}+\frac{\left(a^2+2ab+b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{12}}{24f}+\frac{\left(a^2+6ab+5b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{4f}+\frac{\left(a^2+6ab+5b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}}{4f}-\frac{\left(11a^2+86ab+91b^2\right.}{8f}$ $\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3$

input
```
int(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
4/3*I*(3*a^2*exp(8*I*(f*x+e))+8*a^2*exp(6*I*(f*x+e))+16*a*b*exp(6*I*(f*x+e
))+6*a^2*exp(4*I*(f*x+e))+24*a*b*exp(4*I*(f*x+e))+24*b^2*exp(4*I*(f*x+e))-
a^2-8*a*b-8*b^2)/f/(exp(2*I*(f*x+e))-1)^3/(exp(2*I*(f*x+e))+1)^3
```

3.26.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.33

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{2\left(a^2+8\,ab+8\,b^2\right)\cos\left(fx+e\right)^6-3\left(a^2+8\,ab+8\,b^2\right)\cos\left(fx+e\right)^4+6\left(ab+b^2\right)\cos\left(fx+e\right)^2+b^2}{3\left(f\cos\left(fx+e\right)^5-f\cos\left(fx+e\right)^3\right)\sin\left(fx+e\right)}$$

input
```
integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
-1/3*(2*(a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^6 - 3*(a^2 + 8*a*b + 8*b^2)*cos
(f*x + e)^4 + 6*(a*b + b^2)*cos(f*x + e)^2 + b^2)/((f*cos(f*x + e)^5 - f*c
os(f*x + e)^3)*sin(f*x + e))
```

3.26.6 Sympy [F(-1)]

Timed out.

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**4*(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Timed out
```

3.26.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.05

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{b^2\tan(fx+e)^3+3\left(2ab+3b^2\right)\tan(fx+e)-\frac{3\left(a^2+4ab+3b^2\right)\tan(fx+e)^2+a^2+2ab+b^2}{\tan(fx+e)^3}}{3f}$$

input
```
integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/3*(b^2*tan(f*x + e)^3 + 3*(2*a*b + 3*b^2)*tan(f*x + e) - (3*(a^2 + 4*a*b
 + 3*b^2)*tan(f*x + e)^2 + a^2 + 2*a*b + b^2)/tan(f*x + e)^3)/f
```

3.26.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.29

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{b^2\tan(fx+e)^3+6ab\tan(fx+e)+9b^2\tan(fx+e)-\frac{3a^2\tan(fx+e)^2+12ab\tan(fx+e)^2+9b^2\tan(fx+e)^2+a^2+2ab+}{\tan(fx+e)^3}}{3f}$$

input
```
integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/3*(b^2*tan(f*x + e)^3 + 6*a*b*tan(f*x + e) + 9*b^2*tan(f*x + e) - (3*a^2
*tan(f*x + e)^2 + 12*a*b*tan(f*x + e)^2 + 9*b^2*tan(f*x + e)^2 + a^2 + 2*a
*b + b^2)/tan(f*x + e)^3)/f
```

3.26.9 Mupad [B] (verification not implemented)

Time = 18.49 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.12

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{b^2\tan(e+fx)^3}{3f}-\frac{\frac{2ab}{3}+\tan(e+fx)^2\left(a^2+4ab+3b^2\right)+\frac{a^2}{3}+\frac{b^2}{3}}{f\tan(e+fx)^3}$$
$$+\frac{b\tan(e+fx)\,(2a+3b)}{f}$$

input
```
int((a + b/cos(e + f*x)^2)^2/sin(e + f*x)^4,x)
```

output
```
(b^2*tan(e + f*x)^3)/(3*f) - ((2*a*b)/3 + tan(e + f*x)^2*(4*a*b + a^2 + 3*
b^2) + a^2/3 + b^2/3)/(f*tan(e + f*x)^3) + (b*tan(e + f*x)*(2*a + 3*b))/f
```

3.27 $\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

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3.27.1 Optimal result

Integrand size = 23, antiderivative size = 103

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{(a^2+6ab+6b^2)\cot(e+fx)}{f} - \frac{2(a+b)(a+2b)\cot^3(e+fx)}{3f} - \frac{(a+b)^2\cot^5(e+fx)}{5f} + \frac{2b(a+2b)\tan(e+fx)}{f} + \frac{b^2\tan^3(e+fx)}{3f}$$

output
```
-(a^2+6*a*b+6*b^2)*cot(f*x+e)/f-2/3*(a+b)*(a+2*b)*cot(f*x+e)^3/f-1/5*(a+b)
^2*cot(f*x+e)^5/f+2*b*(a+2*b)*tan(f*x+e)/f+1/3*b^2*tan(f*x+e)^3/f
```

3.27.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 353 vs. 2(103) = 206.

Time = 2.67 (sec) , antiderivative size = 353, normalized size of antiderivative = 3.43

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{\csc(e)\csc^5(e+fx)\sec(e)\sec^3(e+fx)\left(20a(5a+12b)\sin(2e)-32(2a^2+9ab+12b^2)\sin(2fx)-24a^2\right.}{}$$

input
```
Integrate[Csc[e + f*x]^6*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
-1/1920*(Csc[e]*Csc[e + f*x]^5*Sec[e]*Sec[e + f*x]^3*(20*a*(5*a + 12*b)*Si
n[2*e] - 32*(2*a^2 + 9*a*b + 12*b^2)*Sin[2*f*x] - 24*a^2*Sin[2*(e + f*x)]
- 108*a*b*Sin[2*(e + f*x)] - 54*b^2*Sin[2*(e + f*x)] + 8*a^2*Sin[4*(e + f*
x)] + 36*a*b*Sin[4*(e + f*x)] + 18*b^2*Sin[4*(e + f*x)] + 8*a^2*Sin[6*(e +
 f*x)] + 36*a*b*Sin[6*(e + f*x)] + 18*b^2*Sin[6*(e + f*x)] - 4*a^2*Sin[8*(
e + f*x)] - 18*a*b*Sin[8*(e + f*x)] - 9*b^2*Sin[8*(e + f*x)] + 8*a^2*Sin[2
*(e + 2*f*x)] + 96*a*b*Sin[2*(e + 2*f*x)] + 128*b^2*Sin[2*(e + 2*f*x)] + 4
0*a^2*Sin[4*e + 2*f*x] + 8*a^2*Sin[4*e + 6*f*x] + 96*a*b*Sin[4*e + 6*f*x]
+ 128*b^2*Sin[4*e + 6*f*x] - 4*a^2*Sin[6*e + 8*f*x] - 48*a*b*Sin[6*e + 8*f
*x] - 64*b^2*Sin[6*e + 8*f*x]))/f
```

3.27.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4620, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^2}{\sin(e+fx)^6}dx$$

$$\downarrow 4620$$

$$\frac{\int \cot^6(e+fx)\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^2 d\tan(e+fx)}{f}$$

$$\downarrow 355$$

$$\frac{\int \left((a+b)^2\cot^6(e+fx)+2(a+b)(a+2b)\cot^4(e+fx)+\left(a^2+6ba+6b^2\right)\cot^2(e+fx)+b^2\tan^2(e+fx)+2b(\right.}{f}$$

$$\downarrow 2009$$

$$\frac{-\left(a^2+6ab+6b^2\right)\cot(e+fx)+2b(a+2b)\tan(e+fx)-\frac{1}{5}(a+b)^2\cot^5(e+fx)-\frac{2}{3}(a+b)(a+2b)\cot^3(e+fx}{f}$$

input
```
Int[Csc[e + f*x]^6*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(-((a^2 + 6*a*b + 6*b^2)*Cot[e + f*x]) - (2*(a + b)*(a + 2*b)*Cot[e + f*x]
^3)/3 - ((a + b)^2*Cot[e + f*x]^5)/5 + 2*b*(a + 2*b)*Tan[e + f*x] + (b^2*T
an[e + f*x]^3)/3)/f
```

3.27.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.27.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.50

method	result
parallelrisch	$\frac{\left(\left(-13a^2-36ab-48b^2\right)\cos(2fx+2e)+\left(a^2+12ab+16b^2\right)\cos(4fx+4e)+\left(a^2+12ab+16b^2\right)\cos(6fx+6e)+\left(-\frac{1}{2}a^2-6ab-8b\right.\right.}{960f(\cos(3fx+3e)+3\cos(fx+e))}$
derivativedivides	$\frac{a^2\left(-\frac{8}{15}-\frac{\csc(fx+e)^4}{5}-\frac{4\csc(fx+e)^2}{15}\right)\cot(fx+e)+2ab\left(-\frac{1}{5\sin(fx+e)^5\cos(fx+e)}-\frac{2}{5\sin(fx+e)^3\cos(fx+e)}+\frac{8}{5\sin(fx+e)\cos(}\right.}{}$
default	$\frac{a^2\left(-\frac{8}{15}-\frac{\csc(fx+e)^4}{5}-\frac{4\csc(fx+e)^2}{15}\right)\cot(fx+e)+2ab\left(-\frac{1}{5\sin(fx+e)^5\cos(fx+e)}-\frac{2}{5\sin(fx+e)^3\cos(fx+e)}+\frac{8}{5\sin(fx+e)\cos(}\right.}{}$
risch	$-\frac{16i\left(10a^2e^{10i(fx+e)}+25a^2e^{8i(fx+e)}+60ab\,e^{8i(fx+e)}+16a^2e^{6i(fx+e)}+72ab\,e^{6i(fx+e)}+96b^2e^{6i(fx+e)}-2a^2e^{4i(fx+e)}-\right.}{15f\left(e^{2i(fx+e)}-1\right)^5\left(e^{2i(fx+}\right.$

input
```
int(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/960*((-13*a^2-36*a*b-48*b^2)*cos(2*f*x+2*e)+(a^2+12*a*b+16*b^2)*cos(4*f*
x+4*e)+(a^2+12*a*b+16*b^2)*cos(6*f*x+6*e)+(-1/2*a^2-6*a*b-8*b^2)*cos(8*f*x
+8*e)-25/2*a^2-30*a*b)*csc(1/2*f*x+1/2*e)^5*sec(1/2*f*x+1/2*e)^5/f/(cos(3*
f*x+3*e)+3*cos(f*x+e))
```

3.27.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.35

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx =$$
$$-\frac{8\left(a^2+12\,ab+16\,b^2\right)\cos\left(fx+e\right)^8-20\left(a^2+12\,ab+16\,b^2\right)\cos\left(fx+e\right)^6+15\left(a^2+12\,ab+16\,b^2\right)\cos}{15\left(f\cos\left(fx+e\right)^7-2\,f\cos\left(fx+e\right)^5+f\cos\left(fx+e\right)^3\right)\sin(}$$

input
```
integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
-1/15*(8*(a^2 + 12*a*b + 16*b^2)*cos(f*x + e)^8 - 20*(a^2 + 12*a*b + 16*b^
2)*cos(f*x + e)^6 + 15*(a^2 + 12*a*b + 16*b^2)*cos(f*x + e)^4 - 10*(3*a*b
+ 4*b^2)*cos(f*x + e)^2 - 5*b^2)/((f*cos(f*x + e)^7 - 2*f*cos(f*x + e)^5 +
 f*cos(f*x + e)^3)*sin(f*x + e))
```

3.27.6 Sympy [F(-1)]

Timed out.

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**6*(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Timed out
```

3.27.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.04

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$= \frac{5\,b^2\tan(fx+e)^3+30\,(ab+2\,b^2)\tan(fx+e)-\frac{15\,(a^2+6\,ab+6\,b^2)\tan(fx+e)^4+10\,(a^2+3\,ab+2\,b^2)\tan(fx+e)^2+3\,a^2+6\,ab+}{\tan(fx+e)^5}}{15\,f}$$

input
```
integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/15*(5*b^2*tan(f*x + e)^3 + 30*(a*b + 2*b^2)*tan(f*x + e) - (15*(a^2 + 6*
a*b + 6*b^2)*tan(f*x + e)^4 + 10*(a^2 + 3*a*b + 2*b^2)*tan(f*x + e)^2 + 3*
a^2 + 6*a*b + 3*b^2)/tan(f*x + e)^5)/f
```

3.27.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.37

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$= \frac{5\,b^2\tan(fx+e)^3+30\,ab\tan(fx+e)+60\,b^2\tan(fx+e)-\frac{15\,a^2\tan(fx+e)^4+90\,ab\tan(fx+e)^4+90\,b^2\tan(fx+e)^4+10}{t}}{15\,f}$$

input
```
integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/15*(5*b^2*tan(f*x + e)^3 + 30*a*b*tan(f*x + e) + 60*b^2*tan(f*x + e) - (
15*a^2*tan(f*x + e)^4 + 90*a*b*tan(f*x + e)^4 + 90*b^2*tan(f*x + e)^4 + 10
*a^2*tan(f*x + e)^2 + 30*a*b*tan(f*x + e)^2 + 20*b^2*tan(f*x + e)^2 + 3*a^
2 + 6*a*b + 3*b^2)/tan(f*x + e)^5)/f
```

3.27.9 Mupad [B] (verification not implemented)

Time = 18.75 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.05

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$= \frac{b^2\tan(e+fx)^3}{3f}$$
$$- \frac{\frac{2ab}{5}+\tan(e+fx)^4\left(a^2+6ab+6b^2\right)+\frac{a^2}{5}+\frac{b^2}{5}+\tan(e+fx)^2\left(\frac{2a^2}{3}+2ab+\frac{4b^2}{3}\right)}{f\tan(e+fx)^5}$$
$$+ \frac{2b\tan(e+fx)\,(a+2b)}{f}$$

input
```
int((a + b/cos(e + f*x)^2)^2/sin(e + f*x)^6,x)
```

output
```
(b^2*tan(e + f*x)^3)/(3*f) - ((2*a*b)/5 + tan(e + f*x)^4*(6*a*b + a^2 + 6*
b^2) + a^2/5 + b^2/5 + tan(e + f*x)^2*(2*a*b + (2*a^2)/3 + (4*b^2)/3))/(f*
tan(e + f*x)^5) + (2*b*tan(e + f*x)*(a + 2*b))/f
```

3.28 $\int \frac{\sin^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$

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3.28.1 Optimal result

Integrand size = 23, antiderivative size = 98

$$\int \frac{\sin^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{b}(a+b)^2\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a^{7/2}f} - \frac{(a+b)^2\cos(e+fx)}{a^3 f} + \frac{(2a+b)\cos^3(e+fx)}{3a^2 f} - \frac{\cos^5(e+fx)}{5af}$$

output
```
-(a+b)^2*cos(f*x+e)/a^3/f+1/3*(2*a+b)*cos(f*x+e)^3/a^2/f-1/5*cos(f*x+e)^5/
a/f+(a+b)^2*arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*b^(1/2)/a^(7/2)/f
```

3.28.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.85 (sec) , antiderivative size = 425, normalized size of antiderivative = 4.34

$$\int \frac{\sin^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{(a+2b+a\cos(2(e+fx)))\left(15(5a^3+64a^2b+128ab^2+64b^3)\arctan\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\,\text{ta}}{\cdots}\right.\right.}{\cdots}$$

input
```
Integrate[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*(15*(5*a^3 + 64*a^2*b + 128*a*b^2 + 64*b^3
)*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Ta
n[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan
[(f*x)/2]))/Sqrt[b]] + 15*(5*a^3 + 64*a^2*b + 128*a*b^2 + 64*b^3)*ArcTan[(
(-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2]
 + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2])
)/Sqrt[b]] - 75*a^3*ArcTan[(Sqrt[a] - Sqrt[a + b]*Tan[(e + f*x)/2])/Sqrt[b
]] - 75*a^3*ArcTan[(Sqrt[a] + Sqrt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]] - 8*S
qrt[a]*Sqrt[b]*Cos[e + f*x]*(89*a^2 + 220*a*b + 120*b^2 - 4*a*(7*a + 5*b)*
Cos[2*(e + f*x)] + 3*a^2*Cos[4*(e + f*x)]))*Sec[e + f*x]^2)/(1920*a^(7/2)*
Sqrt[b]*f*(a + b*Sec[e + f*x]^2))
```

3.28.3 Rubi [A] (verified)

Time $= 0.29$ (sec) , antiderivative size $= 91$, normalized size of antiderivative $= 0.93$, number of steps used $= 5$, number of rules used $= 4$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used $= \{3042, 4621, 364, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sin(e+fx)^5}{a+b\sec(e+fx)^2}\,dx$$

$$\downarrow 4621$$

$$-\frac{\int \frac{\cos^2(e+fx)\left(1-\cos^2(e+fx)\right)^2}{a\cos^2(e+fx)+b}\,d\cos(e+fx)}{f}$$

$$\downarrow 364$$

$$-\frac{\int \left(\frac{\cos^4(e+fx)}{a}-\frac{(2a+b)\cos^2(e+fx)}{a^2}+\frac{(a+b)^2}{a^3}+\frac{-b^3-2ab^2-a^2b}{a^3(a\cos^2(e+fx)+b)}\right)d\cos(e+fx)}{f}$$

$$\downarrow 2009$$

$$-\frac{-\frac{\sqrt{b}(a+b)^2\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a^{7/2}}+\frac{(a+b)^2\cos(e+fx)}{a^3}-\frac{(2a+b)\cos^3(e+fx)}{3a^2}+\frac{\cos^5(e+fx)}{5a}}{f}$$

input
```
Int[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

output
```
-((-((Sqrt[b]*(a + b)^2*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/a^(7/2)) +
 ((a + b)^2*Cos[e + f*x])/a^3 - ((2*a + b)*Cos[e + f*x]^3)/(3*a^2) + Cos[e
 + f*x]^5/(5*a))/f)
```

3.28.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

3.28.4 Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.17

method	result
derivativedivides	$\frac{-\frac{\cos(fx+e)^5a^2}{5}-\frac{2a^2\cos(fx+e)^3}{3}-\frac{a\cos(fx+e)^3b}{3}+a^2\cos(fx+e)+2ab\cos(fx+e)+b^2\cos(fx+e)}{a^3}+\frac{b\left(a^2+2ab+b^2\right)\arctan\left(\frac{a\cos(fx}{\sqrt{ab}}\right.}{a^3\sqrt{ab}}$ / f
default	$\frac{-\frac{\cos(fx+e)^5a^2}{5}-\frac{2a^2\cos(fx+e)^3}{3}-\frac{a\cos(fx+e)^3b}{3}+a^2\cos(fx+e)+2ab\cos(fx+e)+b^2\cos(fx+e)}{a^3}+\frac{b\left(a^2+2ab+b^2\right)\arctan\left(\frac{a\cos(fx}{\sqrt{ab}}\right.}{a^3\sqrt{ab}}$ / f
risch	$-\frac{5e^{i(fx+e)}}{16af}-\frac{7e^{i(fx+e)}b}{8a^2f}-\frac{e^{i(fx+e)}b^2}{2a^3f}-\frac{5e^{-i(fx+e)}}{16af}-\frac{7e^{-i(fx+e)}b}{8a^2f}-\frac{e^{-i(fx+e)}b^2}{2a^3f}-\frac{i\sqrt{ab}\ln\left(e^{2i(fx+e)}-\right.}{2a}$

input
```
int(sin(f*x+e)^5/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/a^3*(1/5*cos(f*x+e)^5*a^2-2/3*a^2*cos(f*x+e)^3-1/3*a*cos(f*x+e)^3*
b+a^2*cos(f*x+e)+2*a*b*cos(f*x+e)+b^2*cos(f*x+e))+b*(a^2+2*a*b+b^2)/a^3/(a
*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2)))
```

3.28.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.34

$$\int \frac{\sin^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$= \left[-\frac{6\,a^2\cos(fx+e)^5 - 10\,(2\,a^2+ab)\cos(fx+e)^3 - 15\,(a^2+2\,ab+b^2)\sqrt{-\frac{b}{a}}\log\left(-\frac{a\cos(fx+e)^2+2\,a\sqrt{-\frac{b}{a}}\cos}{a\cos(fx+e)^2+b}\right.}{30\,a^3 f}\right.$$

$$\left. -\frac{3\,a^2\cos(fx+e)^5 - 5\,(2\,a^2+ab)\cos(fx+e)^3 - 15\,(a^2+2\,ab+b^2)\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}\cos(fx+e)}{b}\right) + 15\,(a}{15\,a^3 f}\right.$$

input
```
integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[-1/30*(6*a^2*cos(f*x + e)^5 - 10*(2*a^2 + a*b)*cos(f*x + e)^3 - 15*(a^2 +
 2*a*b + b^2)*sqrt(-b/a)*log(-(a*cos(f*x + e)^2 + 2*a*sqrt(-b/a)*cos(f*x +
 e) - b)/(a*cos(f*x + e)^2 + b)) + 30*(a^2 + 2*a*b + b^2)*cos(f*x + e))/(a
^3*f), -1/15*(3*a^2*cos(f*x + e)^5 - 5*(2*a^2 + a*b)*cos(f*x + e)^3 - 15*(
a^2 + 2*a*b + b^2)*sqrt(b/a)*arctan(a*sqrt(b/a)*cos(f*x + e)/b) + 15*(a^2
+ 2*a*b + b^2)*cos(f*x + e))/(a^3*f)]
```

3.28.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**5/(a+b*sec(f*x+e)**2),x)
```

output
```
Timed out
```

3.28.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.04

$$\int \frac{\sin^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{15\,(a^2b+2\,ab^2+b^3)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}a^3} - \frac{3\,a^2\cos(fx+e)^5 - 5\,(2\,a^2+ab)\cos(fx+e)^3 + 15\,(a^2+2\,ab+b^2)\cos(fx+e)}{a^3}}{15\,f}$$

input
```
integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/15*(15*(a^2*b + 2*a*b^2 + b^3)*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*
b)*a^3) - (3*a^2*cos(f*x + e)^5 - 5*(2*a^2 + a*b)*cos(f*x + e)^3 + 15*(a^2
 + 2*a*b + b^2)*cos(f*x + e))/a^3)/f
```

3.28.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. 2(86) = 172.

Time = 0.31 (sec) , antiderivative size = 349, normalized size of antiderivative = 3.56

$$\int \frac{\sin^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{15\,(a^2b+2\,ab^2+b^3)\arctan\left(-\frac{a\cos(fx+e)-b}{\sqrt{ab}\cos(fx+e)+\sqrt{ab}}\right)}{\sqrt{ab}a^3} - \frac{2\left(8\,a^2+25\,ab+15\,b^2-\frac{40\,a^2(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{110\,ab(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{60\,b^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{8}{}\right.}{}$$

input
```
integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
-1/15*(15*(a^2*b + 2*a*b^2 + b^3)*arctan(-(a*cos(f*x + e) - b)/(sqrt(a*b)*
cos(f*x + e) + sqrt(a*b)))/(sqrt(a*b)*a^3) - 2*(8*a^2 + 25*a*b + 15*b^2 -
40*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 110*a*b*(cos(f*x + e) - 1)/
(cos(f*x + e) + 1) - 60*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 80*a^2
*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 160*a*b*(cos(f*x + e) - 1)^2/
(cos(f*x + e) + 1)^2 + 90*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 -
90*a*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 60*b^2*(cos(f*x + e) -
1)^3/(cos(f*x + e) + 1)^3 + 15*a*b*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)
^4 + 15*b^2*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4)/(a^3*((cos(f*x + e)
 - 1)/(cos(f*x + e) + 1) - 1)^5))/f
```

3.28.9 Mupad [B] (verification not implemented)

Time = 18.11 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.26

$$\int \frac{\sin^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\cos(e+fx)^3\left(\frac{b}{3a^2}+\frac{2}{3a}\right)}{f} - \frac{\cos(e+fx)^5}{5af} - \frac{\cos(e+fx)\left(\frac{1}{a}+\frac{b\left(\frac{b}{a^2}+\frac{2}{a}\right)}{a}\right)}{f} + \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{a}\sqrt{b}\cos(e+fx)(a+b)^2}{a^2b+2ab^2+b^3}\right)(a+b)^2}{a^{7/2}f}$$

input
```
int(sin(e + f*x)^5/(a + b/cos(e + f*x)^2),x)
```

output
```
(cos(e + f*x)^3*(b/(3*a^2) + 2/(3*a)))/f - cos(e + f*x)^5/(5*a*f) - (cos(e
 + f*x)*(1/a + (b*(b/a^2 + 2/a))/a))/f + (b^(1/2)*atan((a^(1/2)*b^(1/2)*co
s(e + f*x)*(a + b)^2)/(2*a*b^2 + a^2*b + b^3))*(a + b)^2)/(a^(7/2)*f)
```

3.29 $\int \frac{\sin^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$

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3.29.1 Optimal result

Integrand size = 23, antiderivative size = 71

$$\int \frac{\sin^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{b}(a+b)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a^{5/2}f} - \frac{(a+b)\cos(e+fx)}{a^2 f} + \frac{\cos^3(e+fx)}{3af}$$

output
```
-(a+b)*cos(f*x+e)/a^2/f+1/3*cos(f*x+e)^3/a/f+(a+b)*arctan(cos(f*x+e)*a^(1/
2)/b^(1/2))*b^(1/2)/a^(5/2)/f
```

3.29.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.21 (sec) , antiderivative size = 376, normalized size of antiderivative = 5.30

$$\int \frac{\sin^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{(a+2b+a\cos(2(e+fx)))\left(3(a^2+8ab+8b^2)\arctan\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cos(e)\left(\sqrt{a}-\right.}{\sqrt{b}}\right.\right.$$

input
```
Integrate[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

output

```
((a + 2*b + a*Cos[2*(e + f*x)])*(3*(a^2 + 8*a*b + 8*b^2)*ArcTan[((-Sqrt[a]
 - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]
*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]
] + 3*(a^2 + 8*a*b + 8*b^2)*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e]
 - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[
(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]] - 3*a^2*ArcTan[(Sqrt[a] - S
qrt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]] - 3*a^2*ArcTan[(Sqrt[a] + Sqrt[a + b
]*Tan[(e + f*x)/2])/Sqrt[b]] + 4*Sqrt[a]*Sqrt[b]*Cos[e + f*x]*(-5*a - 6*b
+ a*Cos[2*(e + f*x)]))*Sec[e + f*x]^2)/(48*a^(5/2)*Sqrt[b]*f*(a + b*Sec[e
+ f*x]^2))
```

3.29.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4621, 363, 262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sin(e+fx)^3}{a+b\sec(e+fx)^2}\,dx$$

$$\downarrow 4621$$

$$-\frac{\int \frac{\cos^2(e+fx)\left(1-\cos^2(e+fx)\right)}{a\cos^2(e+fx)+b}\,d\cos(e+fx)}{f}$$

$$\downarrow 363$$

$$-\frac{\frac{(a+b)\int \frac{\cos^2(e+fx)}{a\cos^2(e+fx)+b}\,d\cos(e+fx)}{a}-\frac{\cos^3(e+fx)}{3a}}{f}$$

$$\downarrow 262$$

$$-\frac{\frac{(a+b)\left(\frac{\cos(e+fx)}{a}-\frac{b\int \frac{1}{a\cos^2(e+fx)+b}\,d\cos(e+fx)}{a}\right)}{a}-\frac{\cos^3(e+fx)}{3a}}{f}$$

$$\downarrow 218$$

$$-\frac{\frac{(a+b)\left(\frac{\cos(e+fx)}{a}-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a^{3/2}}\right)}{a}-\frac{\cos^3(e+fx)}{3a}}{f}$$

input
```
Int[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

output
```
-((-1/3*Cos[e + f*x]^3/a + ((a + b)*(-((Sqrt[b]*ArcTan[(Sqrt[a]*Cos[e + f*
x])/Sqrt[b]])/a^(3/2)) + Cos[e + f*x]/a))/a)/f)
```

3.29.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

3.29.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{\frac{a\cos(fx+e)^3}{3}-\cos(fx+e)a-b\cos(fx+e)}{a^2}+\frac{b(a+b)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{a^2\sqrt{ab}}}{f}$
default	$\frac{\frac{a\cos(fx+e)^3}{3}-\cos(fx+e)a-b\cos(fx+e)}{a^2}+\frac{b(a+b)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{a^2\sqrt{ab}}}{f}$
risch	$-\frac{3e^{i(fx+e)}}{8af}-\frac{e^{i(fx+e)}b}{2a^2f}-\frac{3e^{-i(fx+e)}}{8af}-\frac{e^{-i(fx+e)}b}{2a^2f}+\frac{i\sqrt{ab}\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{ab}e^{i(fx+e)}}{a}+1\right)}{2a^2f}+\frac{i\sqrt{ab}\ln\left(e$

input
```
int(sin(f*x+e)^3/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/a^2*(1/3*a*cos(f*x+e)^3-cos(f*x+e)*a-b*cos(f*x+e))+b*(a+b)/a^2/(a*b
)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2)))
```

3.29.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.17

$$\int \frac{\sin^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$=\left[\frac{2\,a\cos(fx+e)^3+3\,(a+b)\sqrt{-\frac{b}{a}}\log\left(-\frac{a\cos(fx+e)^2+2\,a\sqrt{-\frac{b}{a}}\cos(fx+e)-b}{a\cos(fx+e)^2+b}\right)-6\,(a+b)\cos(fx+e)}{6\,a^2f},\ \frac{a\cos(f\right.$$

input
```
integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/6*(2*a*cos(f*x + e)^3 + 3*(a + b)*sqrt(-b/a)*log(-(a*cos(f*x + e)^2 + 2
*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) - 6*(a + b)*cos(f*
x + e))/(a^2*f), 1/3*(a*cos(f*x + e)^3 + 3*(a + b)*sqrt(b/a)*arctan(a*sqrt
(b/a)*cos(f*x + e)/b) - 3*(a + b)*cos(f*x + e))/(a^2*f)]
```

3.29.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**3/(a+b*sec(f*x+e)**2),x)
```

output
```
Timed out
```

3.29.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

$$\int \frac{\sin^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{3\,(ab+b^2)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}a^2} + \frac{a\cos(fx+e)^3 - 3\,(a+b)\cos(fx+e)}{a^2}}{3\,f}$$

input
```
integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/3*(3*(a*b + b^2)*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^2) + (a*c
os(f*x + e)^3 - 3*(a + b)*cos(f*x + e))/a^2)/f
```

3.29.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.20

$$\int \frac{\sin^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{(ab+b^2)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}a^2f} + \frac{a^2f^5\cos(fx+e)^3 - 3\,a^2f^5\cos(fx+e) - 3\,abf^5\cos(fx+e)}{3\,a^3f^6}$$

input
```
integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
(a*b + b^2)*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^2*f) + 1/3*(a^2*
f^5*cos(f*x + e)^3 - 3*a^2*f^5*cos(f*x + e) - 3*a*b*f^5*cos(f*x + e))/(a^3
*f^6)
```

3.29.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.07

$$\int \frac{\sin^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\cos(e+f\,x)^3}{3\,a\,f} - \frac{\cos(e+f\,x)\,\left(\frac{b}{a^2}+\frac{1}{a}\right)}{f} + \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,\cos(e+f\,x)\,(a+b)}{b^2+a\,b}\right)\,(a+b)}{a^{5/2}\,f}$$

input
```
int(sin(e + f*x)^3/(a + b/cos(e + f*x)^2),x)
```

output
```
cos(e + f*x)^3/(3*a*f) - (cos(e + f*x)*(b/a^2 + 1/a))/f + (b^(1/2)*atan((a
^(1/2)*b^(1/2)*cos(e + f*x)*(a + b))/(a*b + b^2))*(a + b))/(a^(5/2)*f)
```

3.30 $\int \frac{\sin(e+fx)}{a+b\sec^2(e+fx)}\,dx$

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3.30.1 Optimal result

Integrand size = 21, antiderivative size = 47

$$\int \frac{\sin(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a^{3/2}f} - \frac{\cos(e+fx)}{af}$$

output
```
-cos(f*x+e)/a/f+arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*b^(1/2)/a^(3/2)/f
```

3.30.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.71 (sec) , antiderivative size = 329, normalized size of antiderivative = 7.00

$$\int \frac{\sin(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{\left((a+4b)\arctan\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cos(e)\left(\sqrt{a}-\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{b}}\right)+(a+}{}$$

input
```
Integrate[Sin[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

output
```
(((a + 4*b)*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])
*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[
e])^2]*Tan[(f*x)/2]))/Sqrt[b]] + (a + 4*b)*ArcTan[((-Sqrt[a] + I*Sqrt[a +
b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sq
rt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]] - a*ArcTan[(
Sqrt[a] - Sqrt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]] - a*ArcTan[(Sqrt[a] + Sqr
t[a + b]*Tan[(e + f*x)/2])/Sqrt[b]] - 4*Sqrt[a]*Sqrt[b]*Cos[e + f*x])*(a +
 2*b + a*Cos[2*(e + f*x)]))/(8*a^(3/2)*Sqrt[b]*f*(b + a*Cos[e + f*x]^2))
```

3.30.3 Rubi [A] (verified)

Time $= 0.22$ (sec) , antiderivative size $= 46$, normalized size of antiderivative $= 0.98$, number of steps used $= 5$, number of rules used $= 4$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used $= \{3042, 4621, 262, 218\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sin(e+fx)}{a+b\sec(e+fx)^2}dx$$

$$\downarrow 4621$$

$$-\frac{\int \frac{\cos^2(e+fx)}{a\cos^2(e+fx)+b}d\cos(e+fx)}{f}$$

$$\downarrow 262$$

$$-\frac{\frac{\cos(e+fx)}{a}-\frac{b\int \frac{1}{a\cos^2(e+fx)+b}d\cos(e+fx)}{a}}{f}$$

$$\downarrow 218$$

$$-\frac{\frac{\cos(e+fx)}{a}-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a^{3/2}}}{f}$$

input
```
Int[Sin[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

output
```
-((-((Sqrt[b]*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/a^(3/2)) + Cos[e + f
*x]/a)/f)
```

3.30.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

3.30.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{-\frac{1}{a\sec(fx+e)}-\frac{b\arctan\left(\frac{\sec(fx+e)b}{\sqrt{ab}}\right)}{a\sqrt{ab}}}{f}$
default	$\frac{-\frac{1}{a\sec(fx+e)}-\frac{b\arctan\left(\frac{\sec(fx+e)b}{\sqrt{ab}}\right)}{a\sqrt{ab}}}{f}$
risch	$-\frac{e^{i(fx+e)}}{2af}-\frac{e^{-i(fx+e)}}{2af}-\frac{i\sqrt{ab}\,\ln\left(e^{2i(fx+e)}-\frac{2i\sqrt{ab}\,e^{i(fx+e)}}{a}+1\right)}{2a^2f}+\frac{i\sqrt{ab}\,\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{ab}\,e^{i(fx+e)}}{a}+1\right)}{2a^2f}$

input
```
int(sin(f*x+e)/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/a/sec(f*x+e)-b/a/(a*b)^(1/2)*arctan(sec(f*x+e)*b/(a*b)^(1/2)))
```

3.30.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.51

$$\int \frac{\sin(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \left[\frac{\sqrt{-\frac{b}{a}}\log\left(-\frac{a\cos(fx+e)^2+2\,a\sqrt{-\frac{b}{a}}\cos(fx+e)-b}{a\cos(fx+e)^2+b}\right)-2\,\cos(fx+e)}{2\,af},\ \frac{\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}\cos(fx+e)}{b}\right)-\cos(fx+e)}{af}\right.$$

input
```
integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(-b/a)*log(-(a*cos(f*x + e)^2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)
/(a*cos(f*x + e)^2 + b)) - 2*cos(f*x + e))/(a*f), (sqrt(b/a)*arctan(a*sqrt
(b/a)*cos(f*x + e)/b) - cos(f*x + e))/(a*f)]
```

3.30.6 Sympy [F]

$$\int \frac{\sin(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\sin(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

input
```
integrate(sin(f*x+e)/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(sin(e + f*x)/(a + b*sec(e + f*x)**2), x)
```

3.30.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int \frac{\sin(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{b\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}a} - \frac{\cos(fx+e)}{a}}{f}$$

input
```
integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
(b*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a) - cos(f*x + e)/a)/f
```

3.30.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \frac{\sin(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{b\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}af} - \frac{\cos(fx+e)}{af}$$

input
```
integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
b*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a*f) - cos(f*x + e)/(a*f)
```

3.30.9 Mupad [B] (verification not implemented)

Time = 17.89 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \frac{\sin(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{b}\operatorname{atan}\left(\frac{\sqrt{a}\cos(e+f\,x)}{\sqrt{b}}\right)}{a^{3/2}\,f} - \frac{\cos(e+f\,x)}{a\,f}$$

input
```
int(sin(e + f*x)/(a + b/cos(e + f*x)^2),x)
```

output
```
(b^(1/2)*atan((a^(1/2)*cos(e + f*x))/b^(1/2)))/(a^(3/2)*f) - cos(e + f*x)/
(a*f)
```

3.31 $\int \frac{\csc(e+fx)}{a+b\sec^2(e+fx)}\,dx$

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3.31.1 Optimal result

Integrand size = 21, antiderivative size = 55

$$\int \frac{\csc(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{\sqrt{a}(a+b)f} - \frac{\operatorname{arctanh}(\cos(e+fx))}{(a+b)f}$$

output

```
-arctanh(cos(f*x+e))/(a+b)/f+arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*b^(1/2)/(a
+b)/f/a^(1/2)
```

3.31.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.24 (sec) , antiderivative size = 239, normalized size of antiderivative = 4.35

$$\int \frac{\csc(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{\sqrt{b}\arctan\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cos(e)\left(\sqrt{a}-\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{b}}\right)}{\sqrt{a}} + \frac{\sqrt{b}\arctan\left(\frac{\left(-\sqrt{a}+i\sqrt{a+b}\sqrt{(\cos(e}\right.\right.}{(a+b$$

input

```
Integrate[Csc[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

output
```
((Sqrt[b]*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*S
in[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e]
)^2]*Tan[(f*x)/2]))/Sqrt[b]])/Sqrt[a] + (Sqrt[b]*ArcTan[((-Sqrt[a] + I*Sqr
t[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a
] + Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]])/Sqrt[
a] - Log[Cos[(e + f*x)/2]] + Log[Sin[(e + f*x)/2]])/((a + b)*f)
```

3.31.3 Rubi [A] (verified)

Time $= 0.24$ (sec) , antiderivative size $= 54$, normalized size of antiderivative $= 0.98$, number of steps used $= 6$, number of rules used $= 5$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used $= \{3042, 4621, 383, 218, 219\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sin(e+fx)\left(a+b\sec(e+fx)^2\right)}dx$$

$$\downarrow 4621$$

$$-\frac{\int \frac{\cos^2(e+fx)}{(1-\cos^2(e+fx))(a\cos^2(e+fx)+b)}d\cos(e+fx)}{f}$$

$$\downarrow 383$$

$$-\frac{\frac{\int \frac{1}{1-\cos^2(e+fx)}d\cos(e+fx)}{a+b} - \frac{b\int \frac{1}{a\cos^2(e+fx)+b}d\cos(e+fx)}{a+b}}{f}$$

$$\downarrow 218$$

$$-\frac{\frac{\int \frac{1}{1-\cos^2(e+fx)}d\cos(e+fx)}{a+b} - \frac{\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{\sqrt{a}(a+b)}}{f}$$

$$\downarrow 219$$

$$-\frac{\frac{\operatorname{arctanh}(\cos(e+fx))}{a+b} - \frac{\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{\sqrt{a}(a+b)}}{f}$$

input
```
Int[Csc[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

output
```
-((-((Sqrt[b]*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(Sqrt[a]*(a + b))) +
 ArcTanh[Cos[e + f*x]]/(a + b))/f)
```

3.31.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 383
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Sym
bol] :> Simp[(-a)*(e^2/(b*c - a*d))   Int[(e*x)^(m - 2)/(a + b*x^2), x], x]
 + Simp[c*(e^2/(b*c - a*d))   Int[(e*x)^(m - 2)/(c + d*x^2), x], x] /; Free
Q[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LeQ[2, m, 3]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

3.31.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.29

method	result
derivativedivides	$\frac{\frac{\ln(-1+\cos(fx+e))}{2a+2b}+\frac{b\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{(a+b)\sqrt{ab}}-\frac{\ln(1+\cos(fx+e))}{2a+2b}}{f}$
default	$\frac{\frac{\ln(-1+\cos(fx+e))}{2a+2b}+\frac{b\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{(a+b)\sqrt{ab}}-\frac{\ln(1+\cos(fx+e))}{2a+2b}}{f}$
risch	$-\frac{\ln(e^{i(fx+e)}+1)}{f(a+b)}+\frac{\ln(e^{i(fx+e)}-1)}{f(a+b)}+\frac{i\sqrt{ab}\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{ab}\,e^{i(fx+e)}}{a}+1\right)}{2a(a+b)f}-\frac{i\sqrt{ab}\ln\left(e^{2i(fx+e)}-\frac{2i\sqrt{ab}\,e^{i(f}{a}}{2a(a+b)f}$

input
```
int(csc(f*x+e)/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/(2*a+2*b)*ln(-1+cos(f*x+e))+b/(a+b)/(a*b)^(1/2)*arctan(a*cos(f*x+e)
/(a*b)^(1/2))-1/(2*a+2*b)*ln(1+cos(f*x+e)))
```

3.31.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.84

$$\int \frac{\csc(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \left[\frac{\sqrt{-\frac{b}{a}}\log\left(-\frac{a\cos(fx+e)^2+2a\sqrt{-\frac{b}{a}}\cos(fx+e)-b}{a\cos(fx+e)^2+b}\right)-\log\left(\frac{1}{2}\cos(fx+e)+\frac{1}{2}\right)+\log\left(-\frac{1}{2}\cos(fx+e)+\frac{1}{2}\right)}{2(a+b)f}, \frac{2\sqrt{}}{}\right]$$

input
```
integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(-b/a)*log(-(a*cos(f*x + e)^2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)
/(a*cos(f*x + e)^2 + b)) - log(1/2*cos(f*x + e) + 1/2) + log(-1/2*cos(f*x
+ e) + 1/2))/((a + b)*f), 1/2*(2*sqrt(b/a)*arctan(a*sqrt(b/a)*cos(f*x + e)
/b) - log(1/2*cos(f*x + e) + 1/2) + log(-1/2*cos(f*x + e) + 1/2))/((a + b)
*f)]
```

3.31.6 Sympy [F]

$$\int \frac{\csc(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\csc(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

input
```
integrate(csc(f*x+e)/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(csc(e + f*x)/(a + b*sec(e + f*x)**2), x)
```

3.31.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.16

$$\int \frac{\csc(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{2\,b\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}(a+b)} - \frac{\log(\cos(fx+e)+1)}{a+b} + \frac{\log(\cos(fx+e)-1)}{a+b}}{2\,f}$$

input
```
integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/2*(2*b*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*(a + b)) - log(cos(f*
x + e) + 1)/(a + b) + log(cos(f*x + e) - 1)/(a + b))/f
```

3.31.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.55

$$\int \frac{\csc(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{2\,b\arctan\left(-\frac{a\cos(fx+e)-b}{\sqrt{ab}\cos(fx+e)+\sqrt{ab}}\right)}{\sqrt{ab}(a+b)} - \frac{\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a+b}}{2\,f}$$

input
```
integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
-1/2*(2*b*arctan(-(a*cos(f*x + e) - b)/(sqrt(a*b)*cos(f*x + e) + sqrt(a*b)
))/(sqrt(a*b)*(a + b)) - log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1))
/(a + b))/f
```

3.31.9 Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.24

$$\int \frac{\csc(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\operatorname{atanh}\left(\frac{\cos(e+fx)\left(2a^3+2ab^2\right)-\frac{\cos(e+fx)\left(8a^5+8a^4b-8a^3b^2-8a^2b^3\right)}{4(a+b)^2}}{2ab(a+b)}\right)}{f\,(a+b)} - \frac{\operatorname{atanh}\left(\frac{\cos(e+fx)\sqrt{-ab}}{b}\right)\sqrt{-ab}}{f\,(a^2+ba)}$$

input
```
int(1/(sin(e + f*x)*(a + b/cos(e + f*x)^2)),x)
```

output
```
- atanh((cos(e + f*x)*(2*a*b^2 + 2*a^3) - (cos(e + f*x)*(8*a^4*b + 8*a^5 -
 8*a^2*b^3 - 8*a^3*b^2))/(4*(a + b)^2))/(2*a*b*(a + b)))/(f*(a + b)) - (at
anh((cos(e + f*x)*(-a*b)^(1/2))/b)*(-a*b)^(1/2))/(f*(a*b + a^2))
```

3.32 $\int \frac{\csc^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$

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3.32.1 Optimal result

Integrand size = 23, antiderivative size = 86

$$\int \frac{\csc^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{a}\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{(a+b)^2 f} - \frac{(a-b)\operatorname{arctanh}(\cos(e+fx))}{2(a+b)^2 f} - \frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f}$$

output
```
-1/2*(a-b)*arctanh(cos(f*x+e))/(a+b)^2/f-1/2*cot(f*x+e)*csc(f*x+e)/(a+b)/f
+arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*a^(1/2)*b^(1/2)/(a+b)^2/f
```

3.32.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.92 (sec) , antiderivative size = 371, normalized size of antiderivative = 4.31

$$\int \frac{\csc^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))\left(-8\sqrt{a}\sqrt{b}\arctan\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cos(e)\left(\sqrt{a}-\sqrt{a+b}\sqrt{}\right.\right.\right.}{\sqrt{b}}$$

input
```
Integrate[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

output

```
-1/16*((a + 2*b + a*Cos[2*(e + f*x)])*(-8*Sqrt[a]*Sqrt[b]*ArcTan[((-Sqrt[a
] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e
]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b
]] - 8*Sqrt[a]*Sqrt[b]*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*
Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[
e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]] + a*Csc[(e + f*x)/2]^2 + b*Csc[(
e + f*x)/2]^2 + 4*a*Log[Cos[(e + f*x)/2]] - 4*b*Log[Cos[(e + f*x)/2]] - 4*
a*Log[Sin[(e + f*x)/2]] + 4*b*Log[Sin[(e + f*x)/2]] - a*Sec[(e + f*x)/2]^2
 - b*Sec[(e + f*x)/2]^2)*Sec[e + f*x]^2)/((a + b)^2*f*(a + b*Sec[e + f*x]^
2))
```

3.32.3 Rubi [A] (verified)

Time $= 0.27$ (sec) , antiderivative size $= 99$, normalized size of antiderivative $= 1.15$, number of steps used $= 7$, number of rules used $= 6$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used $= \{3042, 4621, 373, 397, 218, 219\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sin(e+fx)^3\left(a+b\sec(e+fx)^2\right)}dx$$

$$\downarrow 4621$$

$$-\frac{\int \frac{\cos^2(e+fx)}{\left(1-\cos^2(e+fx)\right)^2\left(a\cos^2(e+fx)+b\right)}d\cos(e+fx)}{f}$$

$$\downarrow 373$$

$$-\frac{\frac{\cos(e+fx)}{2(a+b)(1-\cos^2(e+fx))}-\frac{\int \frac{b-a\cos^2(e+fx)}{\left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)}d\cos(e+fx)}{2(a+b)}}{f}$$

$$\downarrow 397$$

$$-\frac{\frac{\cos(e+fx)}{2(a+b)(1-\cos^2(e+fx))}-\frac{\frac{2ab\int \frac{1}{a\cos^2(e+fx)+b}d\cos(e+fx)}{a+b}-\frac{(a-b)\int \frac{1}{1-\cos^2(e+fx)}d\cos(e+fx)}{a+b}}{2(a+b)}}{f}$$

$$\downarrow 218$$

$$\frac{-\frac{\cos(e+fx)}{2(a+b)(1-\cos^2(e+fx))}-\frac{\frac{2\sqrt{a}\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a+b}-\frac{(a-b)\int\frac{1}{1-\cos^2(e+fx)}d\cos(e+fx)}{a+b}}{2(a+b)}}{f}$$

$$\downarrow 219$$

$$\frac{-\frac{\cos(e+fx)}{2(a+b)(1-\cos^2(e+fx))}-\frac{\frac{2\sqrt{a}\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a+b}-\frac{(a-b)\operatorname{arctanh}(\cos(e+fx))}{a+b}}{2(a+b)}}{f}$$

input
```
Int[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

output
```
-((-1/2*((2*Sqrt[a]*Sqrt[b]*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(a + b
) - ((a - b)*ArcTanh[Cos[e + f*x]])/(a + b))/(a + b) + Cos[e + f*x]/(2*(a
+ b)*(1 - Cos[e + f*x]^2)))/f)
```

3.32.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

3.32.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.34

method	result
derivativedivides	$\frac{\frac{1}{(4a+4b)(-1+\cos(fx+e))}+\frac{(a-b)\ln(-1+\cos(fx+e))}{4(a+b)^2}+\frac{ab\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{(a+b)^2\sqrt{ab}}+\frac{1}{(4a+4b)(1+\cos(fx+e))}+\frac{(-a+b)\ln(1+\cos(fx+e)}{4(a+b)^2}}{f}$
default	$\frac{\frac{1}{(4a+4b)(-1+\cos(fx+e))}+\frac{(a-b)\ln(-1+\cos(fx+e))}{4(a+b)^2}+\frac{ab\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{(a+b)^2\sqrt{ab}}+\frac{1}{(4a+4b)(1+\cos(fx+e))}+\frac{(-a+b)\ln(1+\cos(fx+e)}{4(a+b)^2}}{f}$
risch	$\frac{e^{3i(fx+e)}+e^{i(fx+e)}}{f(a+b)\left(e^{2i(fx+e)}-1\right)^2}-\frac{\ln\left(e^{i(fx+e)}+1\right)a}{2f(a^2+2ab+b^2)}+\frac{\ln\left(e^{i(fx+e)}+1\right)b}{2f(a^2+2ab+b^2)}+\frac{\ln\left(e^{i(fx+e)}-1\right)a}{2f(a^2+2ab+b^2)}-\frac{\ln\left(e^{i(fx+e)}-1\right)b}{2f(a^2+2ab+b^2)}-\frac{i\sqrt{ab}}{}$

input
```
int(csc(f*x+e)^3/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/(4*a+4*b)/(-1+cos(f*x+e))+1/4*(a-b)/(a+b)^2*ln(-1+cos(f*x+e))+a*b/(
a+b)^2/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2))+1/(4*a+4*b)/(1+cos(f*x
+e))+1/4/(a+b)^2*(-a+b)*ln(1+cos(f*x+e)))
```

3.32.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 327, normalized size of antiderivative = 3.80

$$\int \frac{\csc^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \left[\frac{2\sqrt{-ab}\left(\cos(fx+e)^2-1\right)\log\left(-\frac{a\cos(fx+e)^2+2\sqrt{-ab}\cos(fx+e)-b}{a\cos(fx+e)^2+b}\right)+2(a+b)\cos(fx+e)-\left((a-b)\cos(\right.\right.}{4\left((a^2+2ab+b^2)f\cos(fx\right.}$$

input
```
integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(-a*b)*(cos(f*x + e)^2 - 1)*log(-(a*cos(f*x + e)^2 + 2*sqrt(-a
*b)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) + 2*(a + b)*cos(f*x + e) - (
(a - b)*cos(f*x + e)^2 - a + b)*log(1/2*cos(f*x + e) + 1/2) + ((a - b)*cos
(f*x + e)^2 - a + b)*log(-1/2*cos(f*x + e) + 1/2))/((a^2 + 2*a*b + b^2)*f*
cos(f*x + e)^2 - (a^2 + 2*a*b + b^2)*f), 1/4*(4*sqrt(a*b)*(cos(f*x + e)^2
- 1)*arctan(sqrt(a*b)*cos(f*x + e)/b) + 2*(a + b)*cos(f*x + e) - ((a - b)*
cos(f*x + e)^2 - a + b)*log(1/2*cos(f*x + e) + 1/2) + ((a - b)*cos(f*x + e
)^2 - a + b)*log(-1/2*cos(f*x + e) + 1/2))/((a^2 + 2*a*b + b^2)*f*cos(f*x
+ e)^2 - (a^2 + 2*a*b + b^2)*f)]
```

3.32.6 Sympy [F]

$$\int \frac{\csc^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\csc^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

input
```
integrate(csc(f*x+e)**3/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(csc(e + f*x)**3/(a + b*sec(e + f*x)**2), x)
```

3.32.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.49

$$\int \frac{\csc^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{4\,ab\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{(a^2+2\,ab+b^2)\sqrt{ab}} - \frac{(a-b)\log(\cos(fx+e)+1)}{a^2+2\,ab+b^2} + \frac{(a-b)\log(\cos(fx+e)-1)}{a^2+2\,ab+b^2} + \frac{2\cos(fx+e)}{(a+b)\cos(fx+e)^2-a-b}}{4\,f}$$

input
```
integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/4*(4*a*b*arctan(a*cos(f*x + e)/sqrt(a*b))/((a^2 + 2*a*b + b^2)*sqrt(a*b)
) - (a - b)*log(cos(f*x + e) + 1)/(a^2 + 2*a*b + b^2) + (a - b)*log(cos(f*
x + e) - 1)/(a^2 + 2*a*b + b^2) + 2*cos(f*x + e)/((a + b)*cos(f*x + e)^2 -
 a - b))/f
```

3.32.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(74) = 148$.

Time $= 0.32$ (sec) , antiderivative size $= 209$, normalized size of antiderivative $= 2.43$

$$\int \frac{\csc^3(e+fx)}{a+b\sec^2(e+fx)}\,dx =$$

$$-\frac{\frac{8ab\arctan\left(-\frac{a\cos(fx+e)-b}{\sqrt{ab}\cos(fx+e)+\sqrt{ab}}\right)}{(a^2+2ab+b^2)\sqrt{ab}} - \frac{2(a-b)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a^2+2ab+b^2} - \frac{\left(a+b-\frac{2a(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{2b(\cos(fx+e)-1)}{\cos(fx+e)+1}\right)(\cos(fx+e)+1)}{(a^2+2ab+b^2)(\cos(fx+e)-1)} + \frac{(a+}{}}{8f}$$

input
```
integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
-1/8*(8*a*b*arctan(-(a*cos(f*x + e) - b)/(sqrt(a*b)*cos(f*x + e) + sqrt(a*
b)))/((a^2 + 2*a*b + b^2)*sqrt(a*b)) - 2*(a - b)*log(abs(-cos(f*x + e) + 1
)/abs(cos(f*x + e) + 1))/(a^2 + 2*a*b + b^2) - (a + b - 2*a*(cos(f*x + e)
- 1)/(cos(f*x + e) + 1) + 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1))*(cos(
f*x + e) + 1)/((a^2 + 2*a*b + b^2)*(cos(f*x + e) - 1)) + (cos(f*x + e) - 1
)/((a + b)*(cos(f*x + e) + 1)))/f
```

3.32.9 Mupad [B] (verification not implemented)

Time $= 18.89$ (sec) , antiderivative size $= 392$, normalized size of antiderivative $= 4.56$

$$\int \frac{\csc^3(e+fx)}{a+b\sec^2(e+fx)}\,dx =$$

$$-\frac{2a\cos(e+fx)+2b\cos(e+fx)-a\ln(\cos(e+fx)-1)+a\ln(\cos(e+fx)+1)+b\ln(\cos(e+}{}$$

input
```
int(1/(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)),x)
```

output

```
-(2*a*cos(e + f*x) + 2*b*cos(e + f*x) - atan((a^3*cos(e + f*x)*(-a*b)^(1/2
)*1i + a*b^2*cos(e + f*x)*(-a*b)^(1/2)*1i + a^2*b*cos(e + f*x)*(-a*b)^(1/2
)*2i)/(a*b^3 + a^3*b + 2*a^2*b^2))*(-a*b)^(1/2)*4i - a*log(cos(e + f*x) -
1) + a*log(cos(e + f*x) + 1) + b*log(cos(e + f*x) - 1) - b*log(cos(e + f*x
) + 1) + cos(e + f*x)^2*atan((a^3*cos(e + f*x)*(-a*b)^(1/2)*1i + a*b^2*cos
(e + f*x)*(-a*b)^(1/2)*1i + a^2*b*cos(e + f*x)*(-a*b)^(1/2)*2i)/(a*b^3 + a
^3*b + 2*a^2*b^2))*(-a*b)^(1/2)*4i + a*log(cos(e + f*x) - 1)*cos(e + f*x)^
2 - a*log(cos(e + f*x) + 1)*cos(e + f*x)^2 - b*log(cos(e + f*x) - 1)*cos(e
 + f*x)^2 + b*log(cos(e + f*x) + 1)*cos(e + f*x)^2)/(4*a^2*f + 4*b^2*f - 4
*a^2*f*cos(e + f*x)^2 - 4*b^2*f*cos(e + f*x)^2 + 8*a*b*f - 8*a*b*f*cos(e +
 f*x)^2)
```

3.33 $\int \frac{\csc^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$

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3.33.1 Optimal result

Integrand size = 23, antiderivative size = 129

$$\int \frac{\csc^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{a^{3/2}\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{(a+b)^3 f} - \frac{(3a^2-6ab-b^2)\operatorname{arctanh}(\cos(e+fx))}{8(a+b)^3 f} - \frac{(3a-b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2 f} - \frac{\cot(e+fx)\csc^3(e+fx)}{4(a+b)f}$$

output
```
-1/8*(3*a^2-6*a*b-b^2)*arctanh(cos(f*x+e))/(a+b)^3/f-1/8*(3*a-b)*cot(f*x+e
)*csc(f*x+e)/(a+b)^2/f-1/4*cot(f*x+e)*csc(f*x+e)^3/(a+b)/f+a^(3/2)*arctan(
cos(f*x+e)*a^(1/2)/b^(1/2))*b^(1/2)/(a+b)^3/f
```

3.33.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.21 (sec) , antiderivative size = 549, normalized size of antiderivative = 4.26

$$\int \frac{\csc^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))\left(-64a^{3/2}\sqrt{b}\arctan\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cos(e)\left(\sqrt{a}-\sqrt{a+}\right.\right.\right.}{\sqrt{b}}$$

input
```
Integrate[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

output
```
-1/128*((a + 2*b + a*Cos[2*(e + f*x)])*(-64*a^(3/2)*Sqrt[b]*ArcTan[((-Sqrt
[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos
[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt
[b]] - 64*a^(3/2)*Sqrt[b]*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] -
 I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(C
os[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]] + 6*a^2*Csc[(e + f*x)/2]^2 +
4*a*b*Csc[(e + f*x)/2]^2 - 2*b^2*Csc[(e + f*x)/2]^2 + a^2*Csc[(e + f*x)/2]
^4 + 2*a*b*Csc[(e + f*x)/2]^4 + b^2*Csc[(e + f*x)/2]^4 + 24*a^2*Log[Cos[(e
 + f*x)/2]] - 48*a*b*Log[Cos[(e + f*x)/2]] - 8*b^2*Log[Cos[(e + f*x)/2]] -
 24*a^2*Log[Sin[(e + f*x)/2]] + 48*a*b*Log[Sin[(e + f*x)/2]] + 8*b^2*Log[S
in[(e + f*x)/2]] - 6*a^2*Sec[(e + f*x)/2]^2 - 4*a*b*Sec[(e + f*x)/2]^2 + 2
*b^2*Sec[(e + f*x)/2]^2 - a^2*Sec[(e + f*x)/2]^4 - 2*a*b*Sec[(e + f*x)/2]^
4 - b^2*Sec[(e + f*x)/2]^4)*Sec[e + f*x]^2)/((a + b)^3*f*(a + b*Sec[e + f*
x]^2))
```

3.33.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.20, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4621, 373, 402, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sin(e+fx)^5\left(a+b\sec(e+fx)^2\right)}dx$$

$$\downarrow 4621$$

$$-\frac{\int \frac{\cos^2(e+fx)}{(1-\cos^2(e+fx))^3(a\cos^2(e+fx)+b)}d\cos(e+fx)}{f}$$

$$\downarrow 373$$

$$-\frac{\frac{\cos(e+fx)}{4(a+b)(1-\cos^2(e+fx))^2}-\frac{\int \frac{b-3a\cos^2(e+fx)}{\left(1-\cos^2(e+fx)\right)^2\left(a\cos^2(e+fx)+b\right)}d\cos(e+fx)}{4(a+b)}}{f}$$

$$\downarrow 402$$

$$-\frac{\frac{\cos(e+fx)}{4(a+b)(1-\cos^2(e+fx))^2}-\frac{\frac{\int \frac{b(5a+b)-a(3a-b)\cos^2(e+fx)}{\left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)}d\cos(e+fx)}{2(a+b)}-\frac{(3a-b)\cos(e+fx)}{2(a+b)\left(1-\cos^2(e+fx)\right)}}{4(a+b)}}{f}$$

$$\downarrow 397$$

$$-\frac{\frac{\cos(e+fx)}{4(a+b)(1-\cos^2(e+fx))^2}-\frac{\frac{8a^2b\int \frac{1}{a\cos^2(e+fx)+b}d\cos(e+fx)}{a+b}-\frac{\left(3a^2-6ab-b^2\right)\int \frac{1}{1-\cos^2(e+fx)}d\cos(e+fx)}{a+b}}{2(a+b)}-\frac{(3a-b)\cos(e+fx)}{2(a+b)\left(1-\cos^2(e+fx)\right)}}{4(a+b)}}{f}$$

$$\downarrow 218$$

$$-\frac{\frac{\cos(e+fx)}{4(a+b)(1-\cos^2(e+fx))^2}-\frac{\frac{8a^{3/2}\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a+b}-\frac{\left(3a^2-6ab-b^2\right)\int \frac{1}{1-\cos^2(e+fx)}d\cos(e+fx)}{a+b}}{2(a+b)}-\frac{(3a-b)\cos(e+fx)}{2(a+b)\left(1-\cos^2(e+fx)\right)}}{4(a+b)}}{f}$$

$$\downarrow 219$$

$$-\frac{\frac{\cos(e+fx)}{4(a+b)(1-\cos^2(e+fx))^2}-\frac{\frac{8a^{3/2}\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a+b}-\frac{\left(3a^2-6ab-b^2\right)\operatorname{arctanh}(\cos(e+fx))}{a+b}}{2(a+b)}-\frac{(3a-b)\cos(e+fx)}{2(a+b)\left(1-\cos^2(e+fx)\right)}}{4(a+b)}}{f}$$

input
```
Int[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

output
```
-((Cos[e + f*x]/(4*(a + b)*(1 - Cos[e + f*x]^2)^2) - (((8*a^(3/2)*Sqrt[b]*
ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(a + b) - ((3*a^2 - 6*a*b - b^2)*A
rcTanh[Cos[e + f*x]])/(a + b))/(2*(a + b)) - ((3*a - b)*Cos[e + f*x])/(2*(
a + b)*(1 - Cos[e + f*x]^2)))/(4*(a + b)))/f)
```

3.33.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

3.33.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.40

method	result
derivativedivides	$\frac{-\frac{1}{2(8a+8b)(-1+\cos(fx+e))^2}-\frac{-3a+b}{16(a+b)^2(-1+\cos(fx+e))}+\frac{\left(3a^2-6ab-b^2\right)\ln(-1+\cos(fx+e))}{16(a+b)^3}+\frac{a^2b\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{(a+b)^3\sqrt{ab}}+\frac{}{2(8a+}}{f}$
default	$\frac{-\frac{1}{2(8a+8b)(-1+\cos(fx+e))^2}-\frac{-3a+b}{16(a+b)^2(-1+\cos(fx+e))}+\frac{\left(3a^2-6ab-b^2\right)\ln(-1+\cos(fx+e))}{16(a+b)^3}+\frac{a^2b\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{(a+b)^3\sqrt{ab}}+\frac{}{2(8a+}}{f}$
risch	$\frac{3a\,e^{7i(fx+e)}-b\,e^{7i(fx+e)}-11a\,e^{5i(fx+e)}-7b\,e^{5i(fx+e)}-11a\,e^{3i(fx+e)}-7b\,e^{3i(fx+e)}+3a\,e^{i(fx+e)}-b\,e^{i(fx+e)}}{4f(a+b)^2\left(e^{2i(fx+e)}-1\right)^4}-\frac{3\ln(}{8f(a^3-}$

input
```
int(csc(f*x+e)^5/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/2/(8*a+8*b)/(-1+cos(f*x+e))^2-1/16*(-3*a+b)/(a+b)^2/(-1+cos(f*x+e)
)+1/16*(3*a^2-6*a*b-b^2)/(a+b)^3*ln(-1+cos(f*x+e))+a^2*b/(a+b)^3/(a*b)^(1/
2)*arctan(a*cos(f*x+e)/(a*b)^(1/2))+1/2/(8*a+8*b)/(1+cos(f*x+e))^2-1/16*(-
3*a+b)/(a+b)^2/(1+cos(f*x+e))+1/16/(a+b)^3*(-3*a^2+6*a*b+b^2)*ln(1+cos(f*x
+e)))
```

3.33.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 331 vs. 2(115) = 230.

Time = 0.32 (sec) , antiderivative size = 693, normalized size of antiderivative = 5.37

$$\int \frac{\csc^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \left[\frac{2\left(3\,a^2+2\,ab-b^2\right)\cos\left(fx+e\right)^3+8\left(a\cos\left(fx+e\right)^4-2\,a\cos\left(fx+e\right)^2+a\right)\sqrt{-ab}\log\left(-\frac{a\cos(fx+e)^2+2\sqrt{}}{a\cos(f}\right.\right.$$

input
```
integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/16*(2*(3*a^2 + 2*a*b - b^2)*cos(f*x + e)^3 + 8*(a*cos(f*x + e)^4 - 2*a*
cos(f*x + e)^2 + a)*sqrt(-a*b)*log(-(a*cos(f*x + e)^2 + 2*sqrt(-a*b)*cos(f
*x + e) - b)/(a*cos(f*x + e)^2 + b)) - 2*(5*a^2 + 6*a*b + b^2)*cos(f*x + e
) - ((3*a^2 - 6*a*b - b^2)*cos(f*x + e)^4 - 2*(3*a^2 - 6*a*b - b^2)*cos(f*
x + e)^2 + 3*a^2 - 6*a*b - b^2)*log(1/2*cos(f*x + e) + 1/2) + ((3*a^2 - 6*
a*b - b^2)*cos(f*x + e)^4 - 2*(3*a^2 - 6*a*b - b^2)*cos(f*x + e)^2 + 3*a^2
 - 6*a*b - b^2)*log(-1/2*cos(f*x + e) + 1/2))/((a^3 + 3*a^2*b + 3*a*b^2 +
b^3)*f*cos(f*x + e)^4 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*cos(f*x + e)^2
 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f), 1/16*(2*(3*a^2 + 2*a*b - b^2)*cos(f
*x + e)^3 + 16*(a*cos(f*x + e)^4 - 2*a*cos(f*x + e)^2 + a)*sqrt(a*b)*arcta
n(sqrt(a*b)*cos(f*x + e)/b) - 2*(5*a^2 + 6*a*b + b^2)*cos(f*x + e) - ((3*a
^2 - 6*a*b - b^2)*cos(f*x + e)^4 - 2*(3*a^2 - 6*a*b - b^2)*cos(f*x + e)^2
+ 3*a^2 - 6*a*b - b^2)*log(1/2*cos(f*x + e) + 1/2) + ((3*a^2 - 6*a*b - b^2
)*cos(f*x + e)^4 - 2*(3*a^2 - 6*a*b - b^2)*cos(f*x + e)^2 + 3*a^2 - 6*a*b
- b^2)*log(-1/2*cos(f*x + e) + 1/2))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*co
s(f*x + e)^4 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*cos(f*x + e)^2 + (a^3 +
 3*a^2*b + 3*a*b^2 + b^3)*f)]
```

3.33.6 Sympy [F]

$$\int \frac{\csc^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\csc^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

input
```
integrate(csc(f*x+e)**5/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(csc(e + f*x)**5/(a + b*sec(e + f*x)**2), x)
```

3.33.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(115) = 230$.

Time $= 0.26$ (sec) , antiderivative size $= 231$, normalized size of antiderivative $= 1.79$

$$\int \frac{\csc^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{\frac{16\,a^2 b \arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{ab}} - \frac{(3\,a^2-6\,ab-b^2)\log(\cos(fx+e)+1)}{a^3+3\,a^2b+3\,ab^2+b^3} + \frac{(3\,a^2-6\,ab-b^2)\log(\cos(fx+e)-1)}{a^3+3\,a^2b+3\,ab^2+b^3} + \frac{2\left((3\,a-b)\cos(fx+e)\right.}{(a^2+2\,ab+b^2)\cos(fx+e)^4-2\,(a^2+}}{16\,f}$$

input
```
integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/16*(16*a^2*b*arctan(a*cos(f*x + e)/sqrt(a*b))/((a^3 + 3*a^2*b + 3*a*b^2
+ b^3)*sqrt(a*b)) - (3*a^2 - 6*a*b - b^2)*log(cos(f*x + e) + 1)/(a^3 + 3*a
^2*b + 3*a*b^2 + b^3) + (3*a^2 - 6*a*b - b^2)*log(cos(f*x + e) - 1)/(a^3 +
 3*a^2*b + 3*a*b^2 + b^3) + 2*((3*a - b)*cos(f*x + e)^3 - (5*a + b)*cos(f*
x + e))/((a^2 + 2*a*b + b^2)*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f*
x + e)^2 + a^2 + 2*a*b + b^2))/f
```

3.33.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 389 vs. $2(115) = 230$.

Time $= 0.33$ (sec) , antiderivative size $= 389$, normalized size of antiderivative $= 3.02$

$$\int \frac{\csc^5(e+fx)}{a+b\sec^2(e+fx)}\,dx =$$

$$-\frac{\frac{64\,a^2b\arctan\left(-\frac{a\cos(fx+e)-b}{\sqrt{ab}\cos(fx+e)+\sqrt{ab}}\right)}{(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{ab}} - \frac{4\left(3\,a^2-6\,ab-b^2\right)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a^3+3\,a^2b+3\,ab^2+b^3} + \frac{\frac{8\,a(\cos(fx+e)-1)}{\cos(fx+e)+1} - \frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2} - \frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}}{a^2+2\,ab+b^2} +$$

$$64\,f$$

input
```
integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
-1/64*(64*a^2*b*arctan(-(a*cos(f*x + e) - b)/(sqrt(a*b)*cos(f*x + e) + sqr
t(a*b)))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*b)) - 4*(3*a^2 - 6*a*b -
b^2)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1))/(a^3 + 3*a^2*b + 3*
a*b^2 + b^3) + (8*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - a*(cos(f*x + e
) - 1)^2/(cos(f*x + e) + 1)^2 - b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^
2)/(a^2 + 2*a*b + b^2) + (a^2 + 2*a*b + b^2 - 8*a^2*(cos(f*x + e) - 1)/(co
s(f*x + e) + 1) - 8*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 18*a^2*(co
s(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 36*a*b*(cos(f*x + e) - 1)^2/(cos(
f*x + e) + 1)^2 - 6*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)*(cos(f*
x + e) + 1)^2/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(cos(f*x + e) - 1)^2))/f
```

3.33.9 Mupad [B] (verification not implemented)

Time = 22.21 (sec) , antiderivative size = 870, normalized size of antiderivative = 6.74

$$\int \frac{\csc^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$= \frac{3\,a^2\cos(e+f\,x)^3 - b^2\,\cos(e+f\,x) - 5\,a^2\,\cos(e+f\,x) - b^2\cos(e+f\,x)^3 - 3\,a^2\,\mathrm{atanh}(\cos(e+f\,x)) +$$

input
```
int(1/(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)),x)
```

output
```
(atan((a^5*cos(e + f*x)*(-a^3*b)^(1/2)*9i + a^2*b^3*cos(e + f*x)*(-a^3*b)^
(1/2)*12i + a^3*b^2*cos(e + f*x)*(-a^3*b)^(1/2)*30i + a*b^4*cos(e + f*x)*(
-a^3*b)^(1/2)*1i + a^4*b*cos(e + f*x)*(-a^3*b)^(1/2)*28i)/(9*a^6*b + a^2*b
^5 + 12*a^3*b^4 + 30*a^4*b^3 + 28*a^5*b^2))*(-a^3*b)^(1/2)*8i - 5*a^2*cos(
e + f*x) - b^2*cos(e + f*x) + 3*a^2*cos(e + f*x)^3 - b^2*cos(e + f*x)^3 -
3*a^2*atanh(cos(e + f*x)) + b^2*atanh(cos(e + f*x)) - atan((a^5*cos(e + f*
x)*(-a^3*b)^(1/2)*9i + a^2*b^3*cos(e + f*x)*(-a^3*b)^(1/2)*12i + a^3*b^2*c
os(e + f*x)*(-a^3*b)^(1/2)*30i + a*b^4*cos(e + f*x)*(-a^3*b)^(1/2)*1i + a^
4*b*cos(e + f*x)*(-a^3*b)^(1/2)*28i)/(9*a^6*b + a^2*b^5 + 12*a^3*b^4 + 30*
a^4*b^3 + 28*a^5*b^2))*cos(e + f*x)^2*(-a^3*b)^(1/2)*16i + atan((a^5*cos(e
 + f*x)*(-a^3*b)^(1/2)*9i + a^2*b^3*cos(e + f*x)*(-a^3*b)^(1/2)*12i + a^3*
b^2*cos(e + f*x)*(-a^3*b)^(1/2)*30i + a*b^4*cos(e + f*x)*(-a^3*b)^(1/2)*1i
 + a^4*b*cos(e + f*x)*(-a^3*b)^(1/2)*28i)/(9*a^6*b + a^2*b^5 + 12*a^3*b^4
+ 30*a^4*b^3 + 28*a^5*b^2))*cos(e + f*x)^4*(-a^3*b)^(1/2)*8i - 6*a*b*cos(e
 + f*x) + 6*a^2*cos(e + f*x)^2*atanh(cos(e + f*x)) - 3*a^2*cos(e + f*x)^4*
atanh(cos(e + f*x)) - 2*b^2*cos(e + f*x)^2*atanh(cos(e + f*x)) + b^2*cos(e
 + f*x)^4*atanh(cos(e + f*x)) + 2*a*b*cos(e + f*x)^3 + 6*a*b*atanh(cos(e +
 f*x)) - 12*a*b*cos(e + f*x)^2*atanh(cos(e + f*x)) + 6*a*b*cos(e + f*x)^4*
atanh(cos(e + f*x)))/(8*a^3*f + 8*b^3*f - 16*a^3*f*cos(e + f*x)^2 + 8*a^3*
f*cos(e + f*x)^4 - 16*b^3*f*cos(e + f*x)^2 + 8*b^3*f*cos(e + f*x)^4 + 2...
```

3.34 $\int \frac{\sin^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$

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3.34.1 Optimal result

Integrand size = 23, antiderivative size = 166

$$\int \frac{\sin^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{(5a^3+30a^2b+40ab^2+16b^3)\,x}{16a^4} - \frac{\sqrt{b}(a+b)^{5/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^4 f} - \frac{(11a^2+18ab+8b^2)\cos(e+fx)\sin(e+fx)}{16a^3 f} + \frac{(3a+2b)\cos^3(e+fx)\sin(e+fx)}{8a^2 f} + \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af}$$

output

```
1/16*(5*a^3+30*a^2*b+40*a*b^2+16*b^3)*x/a^4-1/16*(11*a^2+18*a*b+8*b^2)*cos
(f*x+e)*sin(f*x+e)/a^3/f+1/8*(3*a+2*b)*cos(f*x+e)^3*sin(f*x+e)/a^2/f+1/6*c
os(f*x+e)^3*sin(f*x+e)^3/a/f-(a+b)^(5/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(
1/2))*b^(1/2)/a^4/f
```

3.34.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.57 (sec) , antiderivative size = 357, normalized size of antiderivative = 2.15

$$\int \frac{\sin^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(3\sqrt{b}(9a^4+136a^3b+384a^2b^2+384ab^3+128b^4)\arctan\left(\frac{\sec(fx)(}{}\right.\right.$$

input
```
Integrate[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(3*Sqrt[b]*(9*a^4 + 136*a^3
*b + 384*a^2*b^2 + 384*a*b^3 + 128*b^4)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin
[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(
Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]) + Sqrt[b*(Cos[e] - I*Sin[e
])^4]*(3*a^3*(9*a + 8*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]] + 2*Sq
rt[b]*Sqrt[a + b]*(-12*a^3*e + 60*a^3*f*x + 360*a^2*b*f*x + 480*a*b^2*f*x
+ 192*b^3*f*x - 3*a*(15*a^2 + 32*a*b + 16*b^2)*Sin[2*(e + f*x)] + 3*a^2*(3
*a + 2*b)*Sin[4*(e + f*x)] - a^3*Sin[6*(e + f*x)])))/(768*a^4*Sqrt[b]*Sqr
t[a + b]*f*(a + b*Sec[e + f*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

3.34.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.20, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4620, 372, 27, 440, 402, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\sin(e+fx)^6}{a+b\sec(e+fx)^2}dx$$
$$\downarrow 4620$$

$$\frac{\int \frac{\tan^6(e+fx)}{(\tan^2(e+fx)+1)^4(b\tan^2(e+fx)+a+b)}d\tan(e+fx)}{f}$$

$$\downarrow 372$$

$$\frac{\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3}-\frac{\int \frac{3\tan^2(e+fx)\left(-\left((2a+b)\tan^2(e+fx)\right)+a+b\right)}{\left(\tan^2(e+fx)+1\right)^3\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{6a}}{f}$$

$$\downarrow 27$$

$$\frac{\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3}-\frac{\int \frac{\tan^2(e+fx)\left(-\left((2a+b)\tan^2(e+fx)\right)+a+b\right)}{\left(\tan^2(e+fx)+1\right)^3\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2a}}{f}$$

$$\downarrow 440$$

$$\frac{\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3}-\frac{\frac{\int \frac{(a+b)(3a+2b)-\left(8a^2+13ba+6b^2\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{4a}-\frac{(3a+2b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2}}{2a}}{f}$$

$$\downarrow 402$$

$$\frac{\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3}-\frac{\frac{\left(11a^2+18ab+8b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)}-\frac{\int \frac{(a+b)(a+2b)(5a+4b)-b\left(11a^2+18ba+8b^2\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2a}}{4a}-\frac{(3a+2b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2}}{2a}}{f}$$

$$\downarrow 397$$

$$\frac{\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3}-\frac{\frac{\left(11a^2+18ab+8b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)}-\frac{\left(5a^3+30a^2b+40ab^2+16b^3\right)\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}-16b(a+b)^3\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+f}{2a}}{4a}}{2a}}{f}$$

$$\downarrow 216$$

$$\frac{\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3}-\frac{\frac{\left(11a^2+18ab+8b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)}-\frac{\left(5a^3+30a^2b+40ab^2+16b^3\right)\arctan(\tan(e+fx))}{a}-16b(a+b)^3\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{2a}}{4a}}{2a}}{f}$$

3.34. $\int \frac{\sin^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$

$$\downarrow 218$$

$$\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3} - \frac{\frac{\left(11a^2+18ab+8b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)} - \frac{\frac{\left(5a^3+30a^2b+40ab^2+16b^3\right)\arctan(\tan(e+fx))}{a} - \frac{16\sqrt{b}(a+b)^{5/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a}}{2a}}{4a} - \frac{(3a+2b)\,t}{4a\left(\tan^2(e}}{2a}}{f}$$

input
```
Int[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

output
```
(Tan[e + f*x]^3/(6*a*(1 + Tan[e + f*x]^2)^3) - (-1/4*((3*a + 2*b)*Tan[e +
f*x])/(a*(1 + Tan[e + f*x]^2)^2) + (-1/2*(((5*a^3 + 30*a^2*b + 40*a*b^2 +
16*b^3)*ArcTan[Tan[e + f*x]])/a - (16*Sqrt[b]*(a + b)^(5/2)*ArcTan[(Sqrt[b
]*Tan[e + f*x])/Sqrt[a + b]])/a)/a + ((11*a^2 + 18*a*b + 8*b^2)*Tan[e + f*
x])/(2*a*(1 + Tan[e + f*x]^2)))/(4*a))/(2*a))/f
```

3.34.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.34.4 Maple [A] (verified)

Time = 2.54 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.02

method	result
derivativedivides	$-\frac{b(a+b)^3 \arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a^4\sqrt{(a+b)b}}+\frac{\left(-\frac{9}{8}a^2b-\frac{1}{2}ab^2-\frac{11}{16}a^3\right)\tan(fx+e)^5+\left(-2a^2b-ab^2-\frac{5}{6}a^3\right)\tan(fx+e)^3+\left(-\frac{5}{16}a^3-\frac{7}{8}a^2b-\frac{1}{2}ab^2\right.}{\left(1+\tan(fx+e)^2\right)^3}}{f}$
default	$-\frac{b(a+b)^3 \arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a^4\sqrt{(a+b)b}}+\frac{\left(-\frac{9}{8}a^2b-\frac{1}{2}ab^2-\frac{11}{16}a^3\right)\tan(fx+e)^5+\left(-2a^2b-ab^2-\frac{5}{6}a^3\right)\tan(fx+e)^3+\left(-\frac{5}{16}a^3-\frac{7}{8}a^2b-\frac{1}{2}ab^2\right.}{\left(1+\tan(fx+e)^2\right)^3}}{f}$
risch	$\frac{5x}{16a}+\frac{15xb}{8a^2}+\frac{5xb^2}{2a^3}+\frac{xb^3}{a^4}+\frac{15ie^{2i(fx+e)}}{128af}+\frac{ie^{2i(fx+e)}b}{4a^2f}-\frac{ie^{-2i(fx+e)}b^2}{8a^3f}-\frac{ie^{-2i(fx+e)}b}{4a^2f}-\frac{15ie^{-2i(fx+e)}}{128af}$

input
```
int(sin(f*x+e)^6/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-b/a^4*(a+b)^3/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))+1
/a^4*(((-9/8*a^2*b-1/2*a*b^2-11/16*a^3)*tan(f*x+e)^5+(-2*a^2*b-a*b^2-5/6*a
^3)*tan(f*x+e)^3+(-5/16*a^3-7/8*a^2*b-1/2*a*b^2)*tan(f*x+e))/(1+tan(f*x+e)
^2)^3+1/16*(5*a^3+30*a^2*b+40*a*b^2+16*b^3)*arctan(tan(f*x+e))))
```

3.34.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 428, normalized size of antiderivative = 2.58

$$\int \frac{\sin^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \left[\frac{3\left(5a^3+30a^2b+40ab^2+16b^3\right)fx+12\left(a^2+2ab+b^2\right)\sqrt{-ab-b^2}\log\left(\frac{\left(a^2+8ab+8b^2\right)\cos(fx+e)^4-2\left(3ab+4b^2\right.}{}\right.}{}\right.$$

input
```
integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/48*(3*(5*a^3 + 30*a^2*b + 40*a*b^2 + 16*b^3)*f*x + 12*(a^2 + 2*a*b + b^
2)*sqrt(-a*b - b^2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b +
 4*b^2)*cos(f*x + e)^2 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqr
t(-a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)
^2 + b^2)) - (8*a^3*cos(f*x + e)^5 - 2*(13*a^3 + 6*a^2*b)*cos(f*x + e)^3 +
 3*(11*a^3 + 18*a^2*b + 8*a*b^2)*cos(f*x + e))*sin(f*x + e))/(a^4*f), 1/48
*(3*(5*a^3 + 30*a^2*b + 40*a*b^2 + 16*b^3)*f*x + 24*(a^2 + 2*a*b + b^2)*sq
rt(a*b + b^2)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a*b + b^2)*c
os(f*x + e)*sin(f*x + e))) - (8*a^3*cos(f*x + e)^5 - 2*(13*a^3 + 6*a^2*b)*
cos(f*x + e)^3 + 3*(11*a^3 + 18*a^2*b + 8*a*b^2)*cos(f*x + e))*sin(f*x + e
))/(a^4*f)]
```

3.34.6 Sympy [F]

$$\int \frac{\sin^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\sin^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

input
```
integrate(sin(f*x+e)**6/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(sin(e + f*x)**6/(a + b*sec(e + f*x)**2), x)
```

3.34.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.26

$$\int \frac{\sin^6(e+fx)}{a+b\sec^2(e+fx)}\,dx =$$

$$-\frac{\frac{3\left(11a^2+18ab+8b^2\right)\tan(fx+e)^5+8\left(5a^2+12ab+6b^2\right)\tan(fx+e)^3+3\left(5a^2+14ab+8b^2\right)\tan(fx+e)}{a^3\tan(fx+e)^6+3a^3\tan(fx+e)^4+3a^3\tan(fx+e)^2+a^3} - \frac{3\left(5a^3+30a^2b+40ab^2+16b^3\right)(fx+}{a^4}}{48f}$$

input
```
integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
-1/48*((3*(11*a^2 + 18*a*b + 8*b^2)*tan(f*x + e)^5 + 8*(5*a^2 + 12*a*b + 6
*b^2)*tan(f*x + e)^3 + 3*(5*a^2 + 14*a*b + 8*b^2)*tan(f*x + e))/(a^3*tan(f
*x + e)^6 + 3*a^3*tan(f*x + e)^4 + 3*a^3*tan(f*x + e)^2 + a^3) - 3*(5*a^3
+ 30*a^2*b + 40*a*b^2 + 16*b^3)*(f*x + e)/a^4 + 48*(a^3*b + 3*a^2*b^2 + 3*
a*b^3 + b^4)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^4))
/f
```

3.34.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.43

$$\int \frac{\sin^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$= \frac{\frac{3\left(5\,a^3+30\,a^2b+40\,ab^2+16\,b^3\right)(fx+e)}{a^4} - \frac{48\left(a^3b+3\,a^2b^2+3\,ab^3+b^4\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{\sqrt{ab+b^2}a^4} - \frac{33\,a^2\tan(fx+e)^5+54\,ab}{}}{48}$$

input
```
integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/48*(3*(5*a^3 + 30*a^2*b + 40*a*b^2 + 16*b^3)*(f*x + e)/a^4 - 48*(a^3*b +
 3*a^2*b^2 + 3*a*b^3 + b^4)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(
b*tan(f*x + e)/sqrt(a*b + b^2)))/(sqrt(a*b + b^2)*a^4) - (33*a^2*tan(f*x +
 e)^5 + 54*a*b*tan(f*x + e)^5 + 24*b^2*tan(f*x + e)^5 + 40*a^2*tan(f*x + e
)^3 + 96*a*b*tan(f*x + e)^3 + 48*b^2*tan(f*x + e)^3 + 15*a^2*tan(f*x + e)
+ 42*a*b*tan(f*x + e) + 24*b^2*tan(f*x + e))/((tan(f*x + e)^2 + 1)^3*a^3))
/f
```

3.34.9 Mupad [B] (verification not implemented)

Time = 19.65 (sec) , antiderivative size = 1448, normalized size of antiderivative = 8.72

$$\int \frac{\sin^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
int(sin(e + f*x)^6/(a + b/cos(e + f*x)^2),x)
```

output

```
(atanh((25*b^3*tan(e + f*x)*(- 5*a*b^5 - a^5*b - b^6 - 10*a^2*b^4 - 10*a^3
*b^3 - 5*a^4*b^2)^(1/2))/(128*((227*a*b^5)/128 + (217*b^6)/128 + (119*a^2*
b^4)/128 + (25*a^3*b^3)/128 + (13*b^7)/(16*a) + (5*b^8)/(32*a^2))) + (11*b
^4*tan(e + f*x)*(- 5*a*b^5 - a^5*b - b^6 - 10*a^2*b^4 - 10*a^3*b^3 - 5*a^4
*b^2)^(1/2))/(32*((217*a*b^6)/128 + (13*b^7)/16 + (227*a^2*b^5)/128 + (119
*a^3*b^4)/128 + (25*a^4*b^3)/128 + (5*b^8)/(32*a))) + (5*b^5*tan(e + f*x)*
(- 5*a*b^5 - a^5*b - b^6 - 10*a^2*b^4 - 10*a^3*b^3 - 5*a^4*b^2)^(1/2))/(32
*((13*a*b^7)/16 + (5*b^8)/32 + (217*a^2*b^6)/128 + (227*a^3*b^5)/128 + (11
9*a^4*b^4)/128 + (25*a^5*b^3)/128)))*(-b*(a + b)^5)^(1/2))/(a^4*f) - (atan
(((((((512*a^8*b^5 + 1408*a^9*b^4 + 1216*a^10*b^3 + 320*a^11*b^2)/(256*a^9
) - (tan(e + f*x)*(2048*a^8*b^3 + 1024*a^9*b^2)*(a*b^2*40i + a^2*b*30i + a
^3*5i + b^3*16i))/(4096*a^10))*(a*b^2*40i + a^2*b*30i + a^3*5i + b^3*16i))
/(32*a^4) - (tan(e + f*x)*(2816*a*b^8 + 512*b^9 + 6400*a^2*b^7 + 7680*a^3*
b^6 + 5140*a^4*b^5 + 1836*a^5*b^4 + 281*a^6*b^3))/(128*a^6))*(a*b^2*40i +
a^2*b*30i + a^3*5i + b^3*16i)*1i)/(32*a^4) - (((((512*a^8*b^5 + 1408*a^9*b
^4 + 1216*a^10*b^3 + 320*a^11*b^2)/(256*a^9) + (tan(e + f*x)*(2048*a^8*b^3
 + 1024*a^9*b^2)*(a*b^2*40i + a^2*b*30i + a^3*5i + b^3*16i))/(4096*a^10))*
(a*b^2*40i + a^2*b*30i + a^3*5i + b^3*16i))/(32*a^4) + (tan(e + f*x)*(2816
*a*b^8 + 512*b^9 + 6400*a^2*b^7 + 7680*a^3*b^6 + 5140*a^4*b^5 + 1836*a^5*b
^4 + 281*a^6*b^3))/(128*a^6))*(a*b^2*40i + a^2*b*30i + a^3*5i + b^3*16i...
```

3.35 $\int \frac{\sin^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$

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3.35.1 Optimal result

Integrand size = 23, antiderivative size = 117

$$\int \frac{\sin^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{(3a^2+12ab+8b^2)\,x}{8a^3} - \frac{\sqrt{b}(a+b)^{3/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^3 f} - \frac{(5a+4b)\cos(e+fx)\sin(e+fx)}{8a^2 f} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af}$$

output

```
1/8*(3*a^2+12*a*b+8*b^2)*x/a^3-1/8*(5*a+4*b)*cos(f*x+e)*sin(f*x+e)/a^2/f+1
/4*cos(f*x+e)^3*sin(f*x+e)/a/f-(a+b)^(3/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)
^(1/2))*b^(1/2)/a^3/f
```

3.35.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.38 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.59

$$\int \frac{\sin^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(\sqrt{b}(3a^3+34a^2b+64ab^2+32b^3)\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-}{2\sqrt{a+b}\sqrt{b}(c}\right.\right.$$

input
```
Integrate[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(Sqrt[b]*(3*a^3 + 34*a^2*b
+ 64*a*b^2 + 32*b^3)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)
*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^
4])]*(Cos[2*e] - I*Sin[2*e]) + Sqrt[b*(Cos[e] - I*Sin[e])^4]*(a^2*(3*a + 2
*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]] + Sqrt[b]*Sqrt[a + b]*(-2*a
^2*e + 12*a^2*f*x + 48*a*b*f*x + 32*b^2*f*x - 8*a*(a + b)*Sin[2*(e + f*x)]
 + a^2*Sin[4*(e + f*x)])))/(64*a^3*Sqrt[b]*Sqrt[a + b]*f*(a + b*Sec[e + f
*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

3.35.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.21, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4620, 372, 402, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sin(e+fx)^4}{a+b\sec(e+fx)^2}\,dx$$

$$\downarrow 4620$$

$$\frac{\int \frac{\tan^4(e+fx)}{(\tan^2(e+fx)+1)^3(b\tan^2(e+fx)+a+b)}\,d\tan(e+fx)}{f}$$

$$\downarrow 372$$

$$\frac{\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2} - \frac{\int \frac{(b-4(a+b))\tan^2(e+fx)+a+b}{(\tan^2(e+fx)+1)^2(b\tan^2(e+fx)+a+b)}\,d\tan(e+fx)}{4a}}{f}$$

$$\downarrow 402$$

$$\frac{\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2} - \frac{\frac{(5a+4b)\tan(e+fx)}{2a(\tan^2(e+fx)+1)} - \frac{\int \frac{(a+b)(3a+4b)-b(5a+4b)\tan^2(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)}\,d\tan(e+fx)}{2a}}{4a}}{f}$$

$$\downarrow 397$$

$$\frac{\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2}-\frac{\frac{(5a+4b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)}-\frac{\frac{\left(3a^2+12ab+8b^2\right)\int\frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}-\frac{8b(a+b)^2\int\frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{2a}}{4a}}{f}$$

$$\downarrow 216$$

$$\frac{\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2}-\frac{\frac{(5a+4b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)}-\frac{\frac{\left(3a^2+12ab+8b^2\right)\arctan(\tan(e+fx))}{a}-\frac{8b(a+b)^2\int\frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{2a}}{4a}}{f}$$

$$\downarrow 218$$

$$\frac{\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2}-\frac{\frac{(5a+4b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)}-\frac{\frac{\left(3a^2+12ab+8b^2\right)\arctan(\tan(e+fx))}{a}-\frac{8\sqrt{b}(a+b)^{3/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a}}{2a}}{4a}}{f}$$

input
```
Int[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

output
```
(Tan[e + f*x]/(4*a*(1 + Tan[e + f*x]^2)^2) - (-1/2*(((3*a^2 + 12*a*b + 8*b
^2)*ArcTan[Tan[e + f*x]])/a - (8*Sqrt[b]*(a + b)^(3/2)*ArcTan[(Sqrt[b]*Tan
[e + f*x])/Sqrt[a + b]])/a)/a + ((5*a + 4*b)*Tan[e + f*x])/(2*a*(1 + Tan[e
 + f*x]^2)))/(4*a))/f
```

3.35.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.35.4 Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{-\frac{(a+b)^2 b \arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a^3\sqrt{(a+b)b}}+\frac{\frac{\left(-\frac{1}{2}ab-\frac{5}{8}a^2\right)\tan(fx+e)^3+\left(-\frac{3}{8}a^2-\frac{1}{2}ab\right)\tan(fx+e)}{\left(1+\tan(fx+e)^2\right)^2}+\frac{\left(3a^2+12ab+8b^2\right)\arctan(\tan(fx+e))}{8}}{a^3}}{f}$
default	$\frac{-\frac{(a+b)^2 b \arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a^3\sqrt{(a+b)b}}+\frac{\frac{\left(-\frac{1}{2}ab-\frac{5}{8}a^2\right)\tan(fx+e)^3+\left(-\frac{3}{8}a^2-\frac{1}{2}ab\right)\tan(fx+e)}{\left(1+\tan(fx+e)^2\right)^2}+\frac{\left(3a^2+12ab+8b^2\right)\arctan(\tan(fx+e))}{8}}{a^3}}{f}$
risch	$\frac{3x}{8a}+\frac{3xb}{2a^2}+\frac{x\,b^2}{a^3}+\frac{i\mathrm{e}^{2i(fx+e)}}{8af}+\frac{i\mathrm{e}^{2i(fx+e)}b}{8a^2f}-\frac{i\mathrm{e}^{-2i(fx+e)}}{8af}-\frac{i\mathrm{e}^{-2i(fx+e)}b}{8a^2f}+\frac{\sqrt{-ab-b^2}\,\ln\left(\mathrm{e}^{2i(fx+e)}+\frac{2i\sqrt{\cdots}}{\cdots}\right.}{2f\,a^2}$

input
```
int(sin(f*x+e)^4/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-(a+b)^2*b/a^3/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))+1
/a^3*(((-1/2*a*b-5/8*a^2)*tan(f*x+e)^3+(-3/8*a^2-1/2*a*b)*tan(f*x+e))/(1+t
an(f*x+e)^2)^2+1/8*(3*a^2+12*a*b+8*b^2)*arctan(tan(f*x+e))))
```

3.35.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.84

$$\int \frac{\sin^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \left[\frac{(3\,a^2+12\,ab+8\,b^2)fx+2\,\sqrt{-ab-b^2}(a+b)\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos(fx+e)^2+4\,\left((a+2\,b)\cos(\cdots\right.}{a^2\cos(fx+e)^4+2\,ab\cos(fx+e\cdots}\right.}{8\,a^3 f\cdots}\right.$$

input
```
integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/8*((3*a^2 + 12*a*b + 8*b^2)*f*x + 2*sqrt(-a*b - b^2)*(a + b)*log(((a^2
+ 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a
 + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b - b^2)*sin(f*x + e) + b
^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) + (2*a^2*cos(f*x +
e)^3 - (5*a^2 + 4*a*b)*cos(f*x + e))*sin(f*x + e))/(a^3*f), 1/8*((3*a^2 +
12*a*b + 8*b^2)*f*x + 4*sqrt(a*b + b^2)*(a + b)*arctan(1/2*((a + 2*b)*cos(
f*x + e)^2 - b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e))) + (2*a^2*cos(
f*x + e)^3 - (5*a^2 + 4*a*b)*cos(f*x + e))*sin(f*x + e))/(a^3*f)]
```

3.35.6 Sympy [F]

$$\int \frac{\sin^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\sin^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

input
```
integrate(sin(f*x+e)**4/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(sin(e + f*x)**4/(a + b*sec(e + f*x)**2), x)
```

3.35.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.17

$$\int \frac{\sin^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= -\frac{\frac{(5a+4b)\tan(fx+e)^3+(3a+4b)\tan(fx+e)}{a^2\tan(fx+e)^4+2a^2\tan(fx+e)^2+a^2} - \frac{(3a^2+12ab+8b^2)(fx+e)}{a^3} + \frac{8(a^2b+2ab^2+b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}a^3}}{8f}$$

input
```
integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
-1/8*(((5*a + 4*b)*tan(f*x + e)^3 + (3*a + 4*b)*tan(f*x + e))/(a^2*tan(f*x
 + e)^4 + 2*a^2*tan(f*x + e)^2 + a^2) - (3*a^2 + 12*a*b + 8*b^2)*(f*x + e)
/a^3 + 8*(a^2*b + 2*a*b^2 + b^3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(s
qrt((a + b)*b)*a^3))/f
```

3.35.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.30

$$\int \frac{\sin^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$= \frac{\frac{(3a^2+12ab+8b^2)(fx+e)}{a^3} - \frac{8\,(a^2b+2ab^2+b^3)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor \mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{\sqrt{ab+b^2}a^3} - \frac{5a\tan(fx+e)^3+4b\tan(fx+e)^3+3a\tan(fx+}{\left(\tan(fx+e)^2+1\right)^2a^2}}{8f}$$

input
```
integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/8*((3*a^2 + 12*a*b + 8*b^2)*(f*x + e)/a^3 - 8*(a^2*b + 2*a*b^2 + b^3)*(p
i*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)
))/(sqrt(a*b + b^2)*a^3) - (5*a*tan(f*x + e)^3 + 4*b*tan(f*x + e)^3 + 3*a*
tan(f*x + e) + 4*b*tan(f*x + e))/((tan(f*x + e)^2 + 1)^2*a^2))/f
```

3.35.9 Mupad [B] (verification not implemented)

Time = 18.78 (sec) , antiderivative size = 494, normalized size of antiderivative = 4.22

$$\int \frac{\sin^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$= \frac{\mathrm{atanh}\left(\frac{9b^3\tan(e+fx)\sqrt{-a^3b-3a^2b^2-3ab^3-b^4}}{32\left(\frac{13ab^4}{16}+\frac{25b^5}{32}+\frac{9a^2b^3}{32}+\frac{b^6}{4a}\right)}+\frac{b^4\tan(e+fx)\sqrt{-a^3b-3a^2b^2-3ab^3-b^4}}{4\left(\frac{9a^3b^3}{32}+\frac{13a^2b^4}{16}+\frac{25ab^5}{32}+\frac{b^6}{4}\right)}\right)\sqrt{-b\,(a+b)^3}}{a^3f}$$

$$- \frac{\frac{\tan(e+fx)\,(3a+4b)}{8a^2}+\frac{\tan(e+fx)^3\,(5a+4b)}{8a^2}}{f\,\left(\tan(e+fx)^4+2\tan(e+fx)^2+1\right)}$$

$$- \frac{\mathrm{atan}\left(\frac{159b^3\tan(e+fx)}{256\left(\frac{27ab^2}{256}+\frac{159b^3}{256}+\frac{75b^4}{64a}+\frac{29b^5}{32a^2}+\frac{b^6}{4a^3}\right)}+\frac{75b^4\tan(e+fx)}{64\left(\frac{159ab^3}{256}+\frac{75b^4}{64}+\frac{27a^2b^2}{256}+\frac{29b^5}{32a}+\frac{b^6}{4a^2}\right)}+\frac{29b^5\tan(e+fx)}{32\left(\frac{75ab^4}{64}+\frac{29b^5}{32}+\frac{159a^2b^3}{256}+\frac{27a^3b^2}{256}+\right.}\right.}{8a^3}$$

input
```
int(sin(e + f*x)^4/(a + b/cos(e + f*x)^2),x)
```

output

```
(atanh((9*b^3*tan(e + f*x)*(- 3*a*b^3 - a^3*b - b^4 - 3*a^2*b^2)^(1/2))/(3
2*((13*a*b^4)/16 + (25*b^5)/32 + (9*a^2*b^3)/32 + b^6/(4*a))) + (b^4*tan(e
 + f*x)*(- 3*a*b^3 - a^3*b - b^4 - 3*a^2*b^2)^(1/2))/(4*((25*a*b^5)/32 + b
^6/4 + (13*a^2*b^4)/16 + (9*a^3*b^3)/32)))*(-b*(a + b)^3)^(1/2))/(a^3*f) -
 ((tan(e + f*x)*(3*a + 4*b))/(8*a^2) + (tan(e + f*x)^3*(5*a + 4*b))/(8*a^2
))/(f*(2*tan(e + f*x)^2 + tan(e + f*x)^4 + 1)) - (atan((159*b^3*tan(e + f*
x))/(256*((27*a*b^2)/256 + (159*b^3)/256 + (75*b^4)/(64*a) + (29*b^5)/(32*
a^2) + b^6/(4*a^3))) + (75*b^4*tan(e + f*x))/(64*((159*a*b^3)/256 + (75*b^
4)/64 + (27*a^2*b^2)/256 + (29*b^5)/(32*a) + b^6/(4*a^2))) + (29*b^5*tan(e
 + f*x))/(32*((75*a*b^4)/64 + (29*b^5)/32 + (159*a^2*b^3)/256 + (27*a^3*b^
2)/256 + b^6/(4*a))) + (b^6*tan(e + f*x))/(4*((29*a*b^5)/32 + b^6/4 + (75*
a^2*b^4)/64 + (159*a^3*b^3)/256 + (27*a^4*b^2)/256)) + (27*b^2*tan(e + f*x
))/(256*((27*b^2)/256 + (159*b^3)/(256*a) + (75*b^4)/(64*a^2) + (29*b^5)/(
32*a^3) + b^6/(4*a^4))))*(a*b*12i + a^2*3i + b^2*8i)*1i)/(8*a^3*f)
```

3.36 $\int \frac{\sin^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$

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3.36.1 Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \frac{\sin^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{(a+2b)x}{2a^2} - \frac{\sqrt{b}\sqrt{a+b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^2 f} - \frac{\cos(e+fx)\sin(e+fx)}{2af}$$

output
```
1/2*(a+2*b)*x/a^2-1/2*cos(f*x+e)*sin(f*x+e)/a/f-arctan(b^(1/2)*tan(f*x+e)/
(a+b)^(1/2))*b^(1/2)*(a+b)^(1/2)/a^2/f
```

3.36.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 245, normalized size of antiderivative = 3.22

$$\int \frac{\sin^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(\frac{\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{b}\sqrt{a+b}f} - \frac{-4(a+2b)x - \frac{\left(a^2+8ab+8b^2\right)\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e)}{2\sqrt{a+b}\sqrt{}}\right.}{\sqrt{a+b}f\sqrt{b(}}}{}\right.}{16\left(a+b\sec^2(e+fx)\right)}$$

input
```
Integrate[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(ArcTan[(Sqrt[b]*Tan[e + f*
x])/Sqrt[a + b]]/(Sqrt[b]*Sqrt[a + b]*f) - (-4*(a + 2*b)*x - ((a^2 + 8*a*b
 + 8*b^2)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x])
+ a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2
*e] - I*Sin[2*e]))/(Sqrt[a + b]*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (2*a*Co
s[2*f*x]*Sin[2*e])/f + (2*a*Cos[2*e]*Sin[2*f*x])/f)/a^2))/(16*(a + b*Sec[e
 + f*x]^2))
```

3.36.3 Rubi [A] (verified)

Time $= 0.27$ (sec) , antiderivative size $= 91$, normalized size of antiderivative $= 1.20$, number of steps used $= 7$, number of rules used $= 6$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used $= \{3042, 4620, 373, 397, 216, 218\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sin(e+fx)^2}{a+b\sec(e+fx)^2}\,dx$$

$$\downarrow 4620$$

$$\frac{\int \frac{\tan^2(e+fx)}{(\tan^2(e+fx)+1)^2(b\tan^2(e+fx)+a+b)}\,d\tan(e+fx)}{f}$$

$$\downarrow 373$$

$$\frac{\int \frac{-b\tan^2(e+fx)+a+b}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)}\,d\tan(e+fx)}{2a} - \frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)}}{f}$$

$$\downarrow 397$$

$$\frac{\frac{(a+2b)\int \frac{1}{\tan^2(e+fx)+1}\,d\tan(e+fx)}{a} - \frac{2b(a+b)\int \frac{1}{b\tan^2(e+fx)+a+b}\,d\tan(e+fx)}{a}}{2a} - \frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)}}{f}$$

$$\downarrow 216$$

$$\frac{\frac{(a+2b)\arctan(\tan(e+fx))}{a}-\frac{2b(a+b)\int\frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{2a}-\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)}}{f}$$

$$\downarrow 218$$

$$\frac{\frac{(a+2b)\arctan(\tan(e+fx))}{a}-\frac{2\sqrt{b}\sqrt{a+b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a}}{2a}-\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)}}{f}$$

input
```
Int[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

output
```
((((a + 2*b)*ArcTan[Tan[e + f*x]])/a - (2*Sqrt[b]*Sqrt[a + b]*ArcTan[(Sqrt
[b]*Tan[e + f*x])/Sqrt[a + b]])/a)/(2*a) - Tan[e + f*x]/(2*a*(1 + Tan[e +
f*x]^2)))/f
```

3.36.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.36.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.03

method	result
derivativedivides	$\frac{-\frac{(a+b)b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a^2\sqrt{(a+b)b}}+\frac{-\frac{a\tan(fx+e)}{2\left(1+\tan(fx+e)^2\right)}+\frac{(a+2b)\arctan(\tan(fx+e))}{2}}{a^2}}{f}$
default	$\frac{-\frac{(a+b)b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a^2\sqrt{(a+b)b}}+\frac{-\frac{a\tan(fx+e)}{2\left(1+\tan(fx+e)^2\right)}+\frac{(a+2b)\arctan(\tan(fx+e))}{2}}{a^2}}{f}$
risch	$\frac{x}{2a}+\frac{xb}{a^2}+\frac{ie^{2i(fx+e)}}{8af}-\frac{ie^{-2i(fx+e)}}{8af}-\frac{\sqrt{-ab-b^2}\ln\left(e^{2i(fx+e)}-\frac{2i\sqrt{-ab-b^2}-a-2b}{a}\right)}{2fa^2}+\frac{\sqrt{-ab-b^2}\ln\left(e^{2i(fx+e)}\right.}{2f}$

input
```
int(sin(f*x+e)^2/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-(a+b)*b/a^2/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))+1/a
^2*(-1/2*a*tan(f*x+e)/(1+tan(f*x+e)^2)+1/2*(a+2*b)*arctan(tan(f*x+e))))
```

3.36.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 257, normalized size of antiderivative = 3.38

$$\int \frac{\sin^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \left[\frac{2\,(a+2\,b)fx-2\,a\cos\left(fx+e\right)\sin\left(fx+e\right)+\sqrt{-ab-b^2}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos(fx+e)^2}{a^2\cos(fx+e)}\right.}{4\,a^2f}\right.$$

input
```
integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/4*(2*(a + 2*b)*f*x - 2*a*cos(f*x + e)*sin(f*x + e) + sqrt(-a*b - b^2)*l
og(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^
2 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b - b^2)*sin(f*x
 + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)))/(a^2*f),
1/2*((a + 2*b)*f*x - a*cos(f*x + e)*sin(f*x + e) + sqrt(a*b + b^2)*arctan(
1/2*((a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x +
 e))))/(a^2*f)]
```

3.36.6 Sympy [F]

$$\int \frac{\sin^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\sin^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

input
```
integrate(sin(f*x+e)**2/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(sin(e + f*x)**2/(a + b*sec(e + f*x)**2), x)
```

3.36.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.01

$$\int \frac{\sin^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{(fx+e)(a+2\,b)}{a^2} - \frac{\tan(fx+e)}{a\tan(fx+e)^2+a} - \frac{2\,(ab+b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}a^2}}{2\,f}$$

input
```
integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/2*((f*x + e)*(a + 2*b)/a^2 - tan(f*x + e)/(a*tan(f*x + e)^2 + a) - 2*(a*
b + b^2)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^2))/f
```

3.36.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.21

$$\int \frac{\sin^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{\frac{(fx+e)(a+2b)}{a^2} - \frac{2\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)\sqrt{ab+b^2}}{a^2} - \frac{\tan(fx+e)}{\left(\tan(fx+e)^2+1\right)a}}{2f}$$

input
```
integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/2*((f*x + e)*(a + 2*b)/a^2 - 2*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + ar
ctan(b*tan(f*x + e)/sqrt(a*b + b^2)))*sqrt(a*b + b^2)/a^2 - tan(f*x + e)/(
(tan(f*x + e)^2 + 1)*a))/f
```

3.36.9 Mupad [B] (verification not implemented)

Time = 18.39 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.46

$$\int \frac{\sin^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{\operatorname{atanh}\left(\frac{\sin(e+fx)\sqrt{-b^2-ab}}{a\cos(e+fx)+b\cos(e+fx)}\right)\sqrt{-b^2-ab} - a\left(\frac{\sin(2e+2fx)}{4} - \frac{\operatorname{atan}\left(\frac{\sin(e+fx)}{\cos(e+fx)}\right)}{2}\right) + b\operatorname{atan}\left(\frac{\sin(e+fx)}{\cos(e+fx)}\right)}{a^2 f}$$

input
```
int(sin(e + f*x)^2/(a + b/cos(e + f*x)^2),x)
```

output
```
(atanh((sin(e + f*x)*(- a*b - b^2)^(1/2))/(a*cos(e + f*x) + b*cos(e + f*x)
))*(- a*b - b^2)^(1/2) - a*(sin(2*e + 2*f*x)/4 - atan(sin(e + f*x)/cos(e +
 f*x))/2) + b*atan(sin(e + f*x)/cos(e + f*x)))/(a^2*f)
```

3.37 $\int \frac{1}{a+b\sec^2(e+fx)}\,dx$

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3.37.1 Optimal result

Integrand size = 14, antiderivative size = 45

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx = \frac{x}{a} + \frac{\sqrt{b}\arctan\left(\frac{\sqrt{a+b}\cot(e+fx)}{\sqrt{b}}\right)}{a\sqrt{a+b}f}$$

output
```
x/a+arctan(cot(f*x+e)*(a+b)^(1/2)/b^(1/2))*b^(1/2)/a/f/(a+b)^(1/2)
```

3.37.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.41 (sec) , antiderivative size = 182, normalized size of antiderivative = 4.04

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(\sqrt{a+b}fx\sqrt{b(\cos(e)-i\sin(e))^4}+b\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e}{2\sqrt{a+b}}\right.\right.}{2a\sqrt{a+b}f\,(a+b\sec^2(e+fx))\sqrt{b(\cos(e)-i\sin(e))^4}}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(-1),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(Sqrt[a + b]*f*x*Sqrt[b*(Co
s[e] - I*Sin[e])^4] + b*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2
*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e
])^4])]*(Cos[2*e] - I*Sin[2*e])))/(2*a*Sqrt[a + b]*f*(a + b*Sec[e + f*x]^2
)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

3.37.3 Rubi [A] (verified)

Time $= 0.25$ (sec) , antiderivative size $= 45$, normalized size of antiderivative $= 1.00$, number of steps used $= 6$, number of rules used $= 5$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used $= \{3042, 4615, 3042, 3660, 218\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a + b\sec^2(e+fx)}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{a + b\sec(e+fx)^2}\,dx$$
$$\downarrow 4615$$
$$\frac{x}{a} - \frac{b\int \frac{1}{a\cos^2(e+fx)+b}\,dx}{a}$$
$$\downarrow 3042$$
$$\frac{x}{a} - \frac{b\int \frac{1}{a\sin\left(e+fx+\frac{\pi}{2}\right)^2+b}\,dx}{a}$$
$$\downarrow 3660$$
$$\frac{b\int \frac{1}{(a+b)\cot^2(e+fx)+b}\,d\cot(e+fx)}{af} + \frac{x}{a}$$
$$\downarrow 218$$
$$\frac{\sqrt{b}\arctan\left(\frac{\sqrt{a+b}\cot(e+fx)}{\sqrt{b}}\right)}{af\sqrt{a+b}} + \frac{x}{a}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^(-1),x]
```

output
```
x/a + (Sqrt[b]*ArcTan[(Sqrt[a + b]*Cot[e + f*x])/Sqrt[b]])/(a*Sqrt[a + b]*
f)
```

3.37.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f  Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 4615
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> Simp[x/a, x
] - Simp[b/a  Int[1/(b + a*Cos[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f},
x] && NeQ[a + b, 0]
```

3.37.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

method	result	size
derivativedivides	$\frac{-\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a\sqrt{(a+b)b}}+\frac{\arctan(\tan(fx+e))}{a}}{f}$	46
default	$\frac{-\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a\sqrt{(a+b)b}}+\frac{\arctan(\tan(fx+e))}{a}}{f}$	46
risch	$\frac{x}{a}+\frac{\sqrt{-(a+b)b}\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-(a+b)b}+a+2b}{a}\right)}{2(a+b)fa}-\frac{\sqrt{-(a+b)b}\ln\left(e^{2i(fx+e)}-\frac{2i\sqrt{-(a+b)b}-a-2b}{a}\right)}{2(a+b)fa}$	114

input
```
int(1/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-b/a/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))+1/a*arctan(
tan(f*x+e)))
```

3.37.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 231, normalized size of antiderivative = 5.13

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx$$
$$= \left[\frac{4\,fx+\sqrt{-\frac{b}{a+b}}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos(fx+e)^2+4\left((a^2+3\,ab+2\,b^2)\cos(fx+e)^3-(ab+b^2)\cos(fx+e)\right)\sqrt{-}}{a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2}\right.}{4\,af}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/4*(4*f*x + sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 -
 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^
3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*co
s(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)))/(a*f), 1/2*(2*f*x + sqrt(b/(a
 + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*
x + e)*sin(f*x + e))))/(a*f)]
```

3.37.6 Sympy [F]

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx = \int \frac{1}{a+b\sec^2(e+fx)}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(1/(a + b*sec(e + f*x)**2), x)
```

3.37.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}a} - \frac{fx+e}{a}}{f}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
-(b*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a) - (f*x + e)
/a)/f
```

3.37.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.44

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b}{\sqrt{ab+b^2}a} - \frac{fx+e}{a}}{f}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
-((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b +
b^2)))*b/(sqrt(a*b + b^2)*a) - (f*x + e)/a)/f
```

3.37.9 Mupad [B] (verification not implemented)

Time = 18.48 (sec) , antiderivative size = 460, normalized size of antiderivative = 10.22

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx = \frac{x}{a}$$

$$-\frac{\operatorname{atan}\left(\frac{\frac{\left(2b^3\tan(e+fx)-\frac{\left(2a^2b^2-\frac{\tan(e+fx)\left(8a^3b^2+16a^2b^3\right)\sqrt{-b(a+b)}}{4\left(a^2+ba\right)}\right)\sqrt{-b(a+b)}}{2\left(a^2+ba\right)}\right)\sqrt{-b(a+b)}\,1\mathrm{i}}{a^2+ba}+\frac{\left(2b^3\tan(e+fx)+\frac{\left(2a^2b^2+\frac{\tan(e+fx)\left(8\right.}{}\right.}{}\right.}{}}{\frac{\left(2b^3\tan(e+fx)-\frac{\left(2a^2b^2-\frac{\tan(e+fx)\left(8a^3b^2+16a^2b^3\right)\sqrt{-b(a+b)}}{4\left(a^2+ba\right)}\right)\sqrt{-b(a+b)}}{2\left(a^2+ba\right)}\right)\sqrt{-b(a+b)}}{a^2+ba}-\frac{\left(2b^3\tan(e+fx)+\frac{\left(2a^2b^2+\frac{\tan(e+fx)\left(8\right.}{}\right.}{}\right.}{}}{f\left(a^2+ba\right)}$$

input
```
int(1/(a + b/cos(e + f*x)^2),x)
```

output
```
x/a - (atan((((2*b^3*tan(e + f*x) - ((2*a^2*b^2 - (tan(e + f*x)*(16*a^2*b^
3 + 8*a^3*b^2)*(-b*(a + b))^(1/2))/(4*(a*b + a^2)))*(-b*(a + b))^(1/2))/(2
*(a*b + a^2)))*(-b*(a + b))^(1/2)*1i)/(a*b + a^2) + ((2*b^3*tan(e + f*x) +
 ((2*a^2*b^2 + (tan(e + f*x)*(16*a^2*b^3 + 8*a^3*b^2)*(-b*(a + b))^(1/2))/
(4*(a*b + a^2)))*(-b*(a + b))^(1/2))/(2*(a*b + a^2)))*(-b*(a + b))^(1/2)*1
i)/(a*b + a^2))/(((2*b^3*tan(e + f*x) - ((2*a^2*b^2 - (tan(e + f*x)*(16*a^
2*b^3 + 8*a^3*b^2)*(-b*(a + b))^(1/2))/(4*(a*b + a^2)))*(-b*(a + b))^(1/2)
)/(2*(a*b + a^2)))*(-b*(a + b))^(1/2))/(a*b + a^2) - ((2*b^3*tan(e + f*x)
+ ((2*a^2*b^2 + (tan(e + f*x)*(16*a^2*b^3 + 8*a^3*b^2)*(-b*(a + b))^(1/2))
/(4*(a*b + a^2)))*(-b*(a + b))^(1/2))/(2*(a*b + a^2)))*(-b*(a + b))^(1/2))
/(a*b + a^2)))*(-b*(a + b))^(1/2)*1i)/(f*(a*b + a^2))
```

3.38 $\int \frac{\csc^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$

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3.38.1 Optimal result

Integrand size = 23, antiderivative size = 54

$$\int \frac{\csc^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}f} - \frac{\cot(e+fx)}{(a+b)f}$$

output
```
-cot(f*x+e)/(a+b)/f-arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(1/2)/(a+b)^(
3/2)/f
```

3.38.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.89 (sec) , antiderivative size = 189, normalized size of antiderivative = 3.50

$$\int \frac{\csc^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(b\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(\cos(2$$
$$2(a+b)^{3/2}f\,(a+b\sec^2(e+fx))\,\sqrt{b(\cos(e$$

input
```
Integrate[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(b*ArcTan[(Sec[f*x]*(Cos[2*
e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b
]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]) + Sqrt[a + b]*Cs
c[e]*Csc[e + f*x]*Sqrt[b*(Cos[e] - I*Sin[e])^4]*Sin[f*x]))/(2*(a + b)^(3/2
)*f*(a + b*Sec[e + f*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

3.38.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4620, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sin(e+fx)^2\left(a+b\sec(e+fx)^2\right)}dx$$

$$\downarrow 4620$$

$$\frac{\int \frac{\cot^2(e+fx)}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{f}$$

$$\downarrow 264$$

$$\frac{-\frac{b\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a+b} - \frac{\cot(e+fx)}{a+b}}{f}$$

$$\downarrow 218$$

$$\frac{-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} - \frac{\cot(e+fx)}{a+b}}{f}$$

input
```
Int[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

output
```
(-((Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(3/2)) - C
ot[e + f*x]/(a + b))/f
```

3.38.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.38.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{-\frac{1}{(a+b)\tan(fx+e)}-\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a+b)\sqrt{(a+b)b}}}{f}$
default	$\frac{-\frac{1}{(a+b)\tan(fx+e)}-\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a+b)\sqrt{(a+b)b}}}{f}$
risch	$-\frac{2i}{f(a+b)\left(e^{2i(fx+e)}-1\right)}-\frac{\sqrt{-(a+b)b}\ln\left(e^{2i(fx+e)}-\frac{2i\sqrt{-(a+b)b}-a-2b}{a}\right)}{2(a+b)^2f}+\frac{\sqrt{-(a+b)b}\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-(a+b)b}}{a}\right.}{2(a+b)^2f}$

input
```
int(csc(f*x+e)^2/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/(a+b)/tan(f*x+e)-b/(a+b)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b
)*b)^(1/2)))
```

3.38.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 271, normalized size of antiderivative = 5.02

$$\int \frac{\csc^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$= \left[\frac{\sqrt{-\frac{b}{a+b}}\log\left(\frac{(a^2+8ab+8b^2)\cos(fx+e)^4-2(3ab+4b^2)\cos(fx+e)^2+4\left((a^2+3ab+2b^2)\cos(fx+e)^3-(ab+b^2)\cos(fx+e)\right)\sqrt{-\frac{b}{a+b}}\sin(f}{a^2\cos(fx+e)^4+2ab\cos(fx+e)^2+b^2}\right.}{4(a+b)f\sin(fx+e)}\right.$$

input
```
integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*
b + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b
 + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x +
e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) - 4*cos(f*x + e))/((a + b
)*f*sin(f*x + e)), 1/2*(sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)
^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) - 2*co
s(f*x + e))/((a + b)*f*sin(f*x + e))]
```

3.38.6 Sympy [F]

$$\int \frac{\csc^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\csc^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

input
```
integrate(csc(f*x+e)**2/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(csc(e + f*x)**2/(a + b*sec(e + f*x)**2), x)
```

3.38.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \frac{\csc^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b(a+b)}} + \frac{1}{(a+b)\tan(fx+e)}}{f}$$

input
```
integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
-(b*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*(a + b)) + 1/(
(a + b)*tan(f*x + e)))/f
```

3.38.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.31

$$\int \frac{\csc^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b}{\sqrt{ab+b^2}(a+b)} + \frac{1}{(a+b)\tan(fx+e)}}{f}$$

input
```
integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
-((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b +
b^2)))*b/(sqrt(a*b + b^2)*(a + b)) + 1/((a + b)*tan(f*x + e)))/f
```

3.38.9 Mupad [B] (verification not implemented)

Time = 18.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{\csc^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\cot(e+f\,x)}{f\,(a+b)} - \frac{\sqrt{b}\,\operatorname{atan}\left(\frac{\sqrt{b}\tan(e+f\,x)}{\sqrt{a+b}}\right)}{f\,(a+b)^{3/2}}$$

input
```
int(1/(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)),x)
```

output
```
- cot(e + f*x)/(f*(a + b)) - (b^(1/2)*atan((b^(1/2)*tan(e + f*x))/(a + b)^
(1/2)))/(f*(a + b)^(3/2))
```

3.39 $\int \frac{\csc^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$

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3.39.1 Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \frac{\csc^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{a\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}f} - \frac{a\cot(e+fx)}{(a+b)^2 f} - \frac{\cot^3(e+fx)}{3(a+b)f}$$

output
```
-a*cot(f*x+e)/(a+b)^2/f-1/3*cot(f*x+e)^3/(a+b)/f-a*arctan(b^(1/2)*tan(f*x+
e)/(a+b)^(1/2))*b^(1/2)/(a+b)^(5/2)/f
```

3.39.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.86 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.97

$$\int \frac{\csc^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(3ab\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(\cos}{6(a+b)^{5/2}f\,(a}$$

input
```
Integrate[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(3*a*b*ArcTan[(Sec[f*x]*(Co
s[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a
 + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]) + (Sqrt[a +
b]*Csc[e]*Csc[e + f*x]^3*Sqrt[b*(Cos[e] - I*Sin[e])^4]*(6*a*Sin[f*x] - 3*b
*Sin[2*e + f*x] + (-2*a + b)*Sin[2*e + 3*f*x]))/4))/(6*(a + b)^(5/2)*f*(a
+ b*Sec[e + f*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

3.39.3 Rubi [A] (verified)

Time $= 0.26$ (sec) , antiderivative size $= 77$, normalized size of antiderivative $= 1.01$, number of steps used $= 6$, number of rules used $= 5$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used $= \{3042, 4620, 359, 264, 218\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sin(e+fx)^4\left(a+b\sec(e+fx)^2\right)}dx$$

$$\downarrow 4620$$

$$\frac{\int \frac{\cot^4(e+fx)(\tan^2(e+fx)+1)}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{f}$$

$$\downarrow 359$$

$$\frac{\frac{a\int \frac{\cot^2(e+fx)}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a+b} - \frac{\cot^3(e+fx)}{3(a+b)}}{f}$$

$$\downarrow 264$$

$$\frac{\frac{a\left(-\frac{b\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a+b} - \frac{\cot(e+fx)}{a+b}\right)}{a+b} - \frac{\cot^3(e+fx)}{3(a+b)}}{f}$$

$$\downarrow 218$$

$$\frac{\frac{a\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} - \frac{\cot(e+fx)}{a+b}\right)}{a+b} - \frac{\cot^3(e+fx)}{3(a+b)}}{f}$$

input
```
Int[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

output
```
(-1/3*Cot[e + f*x]^3/(a + b) + (a*(-((Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x]
)/Sqrt[a + b]])/(a + b)^(3/2)) - Cot[e + f*x]/(a + b)))/(a + b))/f
```

3.39.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.39.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{-\frac{ab\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a+b)^2\sqrt{(a+b)b}}-\frac{1}{3(a+b)\tan(fx+e)^3}-\frac{a}{(a+b)^2\tan(fx+e)}}{f}$
default	$\frac{-\frac{ab\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a+b)^2\sqrt{(a+b)b}}-\frac{1}{3(a+b)\tan(fx+e)^3}-\frac{a}{(a+b)^2\tan(fx+e)}}{f}$
risch	$\frac{2i\left(3b\,e^{4i(fx+e)}+6a\,e^{2i(fx+e)}-2a+b\right)}{3f(a+b)^2\left(e^{2i(fx+e)}-1\right)^3}+\frac{\sqrt{-(a+b)b}\,a\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-(a+b)b}+a+2b}{a}\right)}{2(a+b)^3f}-\frac{\sqrt{-(a+b)b}\,a\ln\left(e^{2i(fx+$ $2(a+$

input
```
int(csc(f*x+e)^4/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-a*b/(a+b)^2/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))-1/3
/(a+b)/tan(f*x+e)^3-a/(a+b)^2/tan(f*x+e))
```

3.39.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. 2(66) = 132.

Time = 0.30 (sec) , antiderivative size = 397, normalized size of antiderivative = 5.22

$$\int \frac{\csc^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$= \left[-\frac{4\,(2\,a-b)\cos(fx+e)^3-3\left(a\cos(fx+e)^2-a\right)\sqrt{-\frac{b}{a+b}}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos(fx+e)}{a^2\,\mathrm{co}}\right.}{12\left((a^2+2\,ab+b^2)f\cos(fx+e)^2-(\right.} \right.$$

$$\left. -\frac{2\,(2\,a-b)\cos(fx+e)^3-3\left(a\cos(fx+e)^2-a\right)\sqrt{\frac{b}{a+b}}\arctan\left(\frac{\left((a+2\,b)\cos(fx+e)^2-b\right)\sqrt{\frac{b}{a+b}}}{2\,b\cos(fx+e)\sin(fx+e)}\right)\sin(fx+e)}{6\left((a^2+2\,ab+b^2)f\cos(fx+e)^2-(a^2+2\,ab+b^2)f\right)\sin(fx+e)} \right]$$

input
```
integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[-1/12*(4*(2*a - b)*cos(f*x + e)^3 - 3*(a*cos(f*x + e)^2 - a)*sqrt(-b/(a +
 b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x
 + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x +
e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f
*x + e)^2 + b^2))*sin(f*x + e) - 12*a*cos(f*x + e))/(((a^2 + 2*a*b + b^2)*
f*cos(f*x + e)^2 - (a^2 + 2*a*b + b^2)*f)*sin(f*x + e)), -1/6*(2*(2*a - b)
*cos(f*x + e)^3 - 3*(a*cos(f*x + e)^2 - a)*sqrt(b/(a + b))*arctan(1/2*((a
+ 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e)))*
sin(f*x + e) - 6*a*cos(f*x + e))/(((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2 -
(a^2 + 2*a*b + b^2)*f)*sin(f*x + e))]
```

3.39.6 Sympy [F]

$$\int \frac{\csc^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\csc^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

input
```
integrate(csc(f*x+e)**4/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(csc(e + f*x)**4/(a + b*sec(e + f*x)**2), x)
```

3.39.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.08

$$\int \frac{\csc^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{3\,ab\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^2+2\,ab+b^2)\sqrt{(a+b)b}} + \frac{3\,a\tan(fx+e)^2+a+b}{(a^2+2\,ab+b^2)\tan(fx+e)^3}}{3\,f}$$

input
```
integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
-1/3*(3*a*b*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^2 + 2*a*b + b^2)*sq
rt((a + b)*b)) + (3*a*tan(f*x + e)^2 + a + b)/((a^2 + 2*a*b + b^2)*tan(f*x
 + e)^3))/f
```

3.39.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.36

$$\int \frac{\csc^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{3\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)ab}{(a^2+2ab+b^2)\sqrt{ab+b^2}}+\frac{3a\tan(fx+e)^2+a+b}{(a^2+2ab+b^2)\tan(fx+e)^3}}{3f}$$

input
```
integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
-1/3*(3*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(
a*b + b^2)))*a*b/((a^2 + 2*a*b + b^2)*sqrt(a*b + b^2)) + (3*a*tan(f*x + e)
^2 + a + b)/((a^2 + 2*a*b + b^2)*tan(f*x + e)^3))/f
```

3.39.9 Mupad [B] (verification not implemented)

Time = 18.33 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.05

$$\int \frac{\csc^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{1}{3\,(a+b)}+\frac{a\tan(e+f\,x)^2}{(a+b)^2}}{f\tan\left(e+f\,x\right)^3}-\frac{a\sqrt{b}\,\operatorname{atan}\left(\frac{\sqrt{b}\tan(e+f\,x)\,(a^2+2\,a\,b+b^2)}{(a+b)^{5/2}}\right)}{f\,(a+b)^{5/2}}$$

input
```
int(1/(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)),x)
```

output
```
- (1/(3*(a + b)) + (a*tan(e + f*x)^2)/(a + b)^2)/(f*tan(e + f*x)^3) - (a*b
^(1/2)*atan((b^(1/2)*tan(e + f*x)*(2*a*b + a^2 + b^2))/(a + b)^(5/2)))/(f*
(a + b)^(5/2))
```

3.40 $\int \frac{\csc^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$

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3.40.1 Optimal result

Integrand size = 23, antiderivative size = 105

$$\int \frac{\csc^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{a^2\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{7/2}f} - \frac{a^2\cot(e+fx)}{(a+b)^3 f} - \frac{(2a+b)\cot^3(e+fx)}{3(a+b)^2 f} - \frac{\cot^5(e+fx)}{5(a+b)f}$$

output

```
-a^2*cot(f*x+e)/(a+b)^3/f-1/3*(2*a+b)*cot(f*x+e)^3/(a+b)^2/f-1/5*cot(f*x+e
)^5/(a+b)/f-a^2*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(1/2)/(a+b)^(7/2)
/f
```

3.40.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.78 (sec) , antiderivative size = 318, normalized size of antiderivative = 3.03

$$\int \frac{\csc^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(240a^2b\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)\right.}{}$$

input
```
Integrate[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(240*a^2*b*ArcTan[(Sec[f*x]
*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sq
rt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]) + Sqrt[a
 + b]*Csc[e]*Csc[e + f*x]^5*Sqrt[b*(Cos[e] - I*Sin[e])^4]*(10*(8*a^2 + b^2
)*Sin[f*x] - 30*b*(3*a + b)*Sin[2*e + f*x] - 40*a^2*Sin[2*e + 3*f*x] + 30*
a*b*Sin[2*e + 3*f*x] + 10*b^2*Sin[2*e + 3*f*x] + 15*a*b*Sin[4*e + 3*f*x] +
 8*a^2*Sin[4*e + 5*f*x] - 9*a*b*Sin[4*e + 5*f*x] - 2*b^2*Sin[4*e + 5*f*x])
))/(480*(a + b)^(7/2)*f*(a + b*Sec[e + f*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^
4])
```

3.40.3 Rubi [A] (verified)

Time $= 0.32$ (sec) , antiderivative size $= 97$, normalized size of antiderivative $= 0.92$, number of steps used $= 5$, number of rules used $= 4$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used $= \{3042, 4620, 364, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sin(e+fx)^6\left(a+b\sec(e+fx)^2\right)}dx$$

$$\downarrow 4620$$

$$\frac{\int \frac{\cot^6(e+fx)\left(\tan^2(e+fx)+1\right)^2}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{f}$$

$$\downarrow 364$$

$$\frac{\int \left(\frac{\cot^6(e+fx)}{a+b}+\frac{(2a+b)\cot^4(e+fx)}{(a+b)^2}+\frac{a^2\cot^2(e+fx)}{(a+b)^3}-\frac{a^2b}{(a+b)^3(b\tan^2(e+fx)+a+b)}\right)d\tan(e+fx)}{f}$$

$$\downarrow 2009$$

$$\frac{-\frac{a^2\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{7/2}}-\frac{a^2\cot(e+fx)}{(a+b)^3}-\frac{\cot^5(e+fx)}{5(a+b)}-\frac{(2a+b)\cot^3(e+fx)}{3(a+b)^2}}{f}$$

input
```
Int[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

output
```
(-((a^2*Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(7/2))
 - (a^2*Cot[e + f*x])/(a + b)^3 - ((2*a + b)*Cot[e + f*x]^3)/(3*(a + b)^2)
 - Cot[e + f*x]^5/(5*(a + b)))/f
```

3.40.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.40.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{-\frac{a^2 b \arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a+b)^3\sqrt{(a+b)b}} - \frac{1}{5(a+b)\tan(fx+e)^5} - \frac{a^2}{(a+b)^3\tan(fx+e)} - \frac{2a+b}{3(a+b)^2\tan(fx+e)^3}}{f}$
default	$\frac{-\frac{a^2 b \arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a+b)^3\sqrt{(a+b)b}} - \frac{1}{5(a+b)\tan(fx+e)^5} - \frac{a^2}{(a+b)^3\tan(fx+e)} - \frac{2a+b}{3(a+b)^2\tan(fx+e)^3}}{f}$
risch	$\frac{2i\left(15ab\,e^{8i(fx+e)} - 90ab\,e^{6i(fx+e)} - 30b^2e^{6i(fx+e)} - 80a^2e^{4i(fx+e)} - 10b^2e^{4i(fx+e)} + 40a^2e^{2i(fx+e)} - 30ab\,e^{2i(fx+e)} - 10b\right.}{15f(a+b)^3\left(e^{2i(fx+e)}-1\right)^5}$

input
```
int(csc(f*x+e)^6/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-a^2*b/(a+b)^3/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))-1
/5/(a+b)/tan(f*x+e)^5-a^2/(a+b)^3/tan(f*x+e)-1/3*(2*a+b)/(a+b)^2/tan(f*x+e
)^3)
```

3.40.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. 2(93) = 186.

Time = 0.29 (sec) , antiderivative size = 587, normalized size of antiderivative = 5.59

$$\int \frac{\csc^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$= \left[-\frac{4\left(8a^2-9ab-2b^2\right)\cos(fx+e)^5 - 20\left(4a^2-3ab-b^2\right)\cos(fx+e)^3 - 15\left(a^2\cos(fx+e)^4 - 2a^2\,\mathrm{co}\right.}{60\left(\left(a^3+3a^2b+3ab^2+b^3\right)f\cos\right.}\right.$$

$$\left. -\frac{2\left(8a^2-9ab-2b^2\right)\cos(fx+e)^5 - 10\left(4a^2-3ab-b^2\right)\cos(fx+e)^3 - 15\left(a^2\cos(fx+e)^4 - 2a^2\,\mathrm{co}\right.}{30\left(\left(a^3+3a^2b+3ab^2+b^3\right)f\cos(fx+e)^4 - 2\left(a^3+3a^2b+3ab^2+b^3\right.\right.}\right]$$

input
```
integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[-1/60*(4*(8*a^2 - 9*a*b - 2*b^2)*cos(f*x + e)^5 - 20*(4*a^2 - 3*a*b - b^2
)*cos(f*x + e)^3 - 15*(a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x + e)^2 + a^2)*sq
rt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^
2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*
cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2
*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) + 60*a^2*cos(f*x + e))/(((a^3 + 3
*a^2*b + 3*a*b^2 + b^3)*f*cos(f*x + e)^4 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^
3)*f*cos(f*x + e)^2 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f)*sin(f*x + e)), -1
/30*(2*(8*a^2 - 9*a*b - 2*b^2)*cos(f*x + e)^5 - 10*(4*a^2 - 3*a*b - b^2)*c
os(f*x + e)^3 - 15*(a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x + e)^2 + a^2)*sqrt(
b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*co
s(f*x + e)*sin(f*x + e)))*sin(f*x + e) + 30*a^2*cos(f*x + e))/(((a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*f*cos(f*x + e)^4 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
*f*cos(f*x + e)^2 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f)*sin(f*x + e))]
```

3.40.6 Sympy [F]

$$\int \frac{\csc^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\csc^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

input
```
integrate(csc(f*x+e)**6/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(csc(e + f*x)**6/(a + b*sec(e + f*x)**2), x)
```

3.40.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.30

$$\int \frac{\csc^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{15\,a^2 b \arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{(a+b)b}} + \frac{15\,a^2\tan(fx+e)^4+5\,(2\,a^2+3\,ab+b^2)\tan(fx+e)^2+3\,a^2+6\,ab+3\,b^2}{(a^3+3\,a^2b+3\,ab^2+b^3)\tan(fx+e)^5}}{15\,f}$$

input
```
integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
-1/15*(15*a^2*b*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^3 + 3*a^2*b + 3
*a*b^2 + b^3)*sqrt((a + b)*b)) + (15*a^2*tan(f*x + e)^4 + 5*(2*a^2 + 3*a*b
 + b^2)*tan(f*x + e)^2 + 3*a^2 + 6*a*b + 3*b^2)/((a^3 + 3*a^2*b + 3*a*b^2
+ b^3)*tan(f*x + e)^5))/f
```

3.40.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.65

$$\int \frac{\csc^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{15\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)a^2b}{(a^3+3a^2b+3ab^2+b^3)\sqrt{ab+b^2}}+\frac{15a^2\tan(fx+e)^4+10a^2\tan(fx+e)^2+15ab\tan(fx+e)^2+5b^2\tan(fx+e)^2+3a^2+6a}{(a^3+3a^2b+3ab^2+b^3)\tan(fx+e)^5}}{15f}$$

input
```
integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
-1/15*(15*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqr
t(a*b + b^2)))*a^2*b/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*b + b^2)) + (
15*a^2*tan(f*x + e)^4 + 10*a^2*tan(f*x + e)^2 + 15*a*b*tan(f*x + e)^2 + 5*
b^2*tan(f*x + e)^2 + 3*a^2 + 6*a*b + 3*b^2)/((a^3 + 3*a^2*b + 3*a*b^2 + b^
3)*tan(f*x + e)^5))/f
```

3.40.9 Mupad [B] (verification not implemented)

Time = 19.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.07

$$\int \frac{\csc^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{1}{5\,(a+b)}+\frac{\tan(e+fx)^2\,(2\,a+b)}{3\,(a+b)^2}+\frac{a^2\tan(e+fx)^4}{(a+b)^3}}{f\tan(e+fx)^5} - \frac{a^2\sqrt{b}\operatorname{atan}\left(\frac{\sqrt{b}\tan(e+fx)\,(a^3+3a^2b+3ab^2+b^3)}{(a+b)^{7/2}}\right)}{f\,(a+b)^{7/2}}$$

input
```
int(1/(sin(e + f*x)^6*(a + b/cos(e + f*x)^2)),x)
```

output
```
- (1/(5*(a + b)) + (tan(e + f*x)^2*(2*a + b))/(3*(a + b)^2) + (a^2*tan(e +
 f*x)^4)/(a + b)^3)/(f*tan(e + f*x)^5) - (a^2*b^(1/2)*atan((b^(1/2)*tan(e
+ f*x)*(3*a*b^2 + 3*a^2*b + a^3 + b^3))/(a + b)^(7/2)))/(f*(a + b)^(7/2))
```

3.41 $\int \frac{\sin^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

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3.41.1 Optimal result

Integrand size = 23, antiderivative size = 161

$$\int \frac{\sin^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\sqrt{b}(a+b)(3a+7b)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2a^{9/2}f} - \frac{(a+b)(3a+7b)\cos(e+fx)}{2a^4 f} + \frac{(a+b)(3a+7b)\cos^3(e+fx)}{6a^3 bf} - \frac{\cos^5(e+fx)}{5a^2 f} - \frac{(a+b)^2\cos^5(e+fx)}{2a^2 bf\left(b+a\cos^2(e+fx)\right)}$$

output

```
-1/2*(a+b)*(3*a+7*b)*cos(f*x+e)/a^4/f+1/6*(a+b)*(3*a+7*b)*cos(f*x+e)^3/a^3
/b/f-1/5*cos(f*x+e)^5/a^2/f-1/2*(a+b)^2*cos(f*x+e)^5/a^2/b/f/(b+a*cos(f*x+
e)^2)+1/2*(a+b)*(3*a+7*b)*arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*b^(1/2)/a^(9/
2)/f
```

3.41.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.66 (sec) , antiderivative size = 454, normalized size of antiderivative = 2.82

$$\int \frac{\sin^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$= \frac{15\left(3a^4+384a^2b^2+1280ab^3+896b^4\right)\arctan\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cos(e)\left(\sqrt{a}-\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{b}}\right)}{b^{3/2}} +$$

input
```
Integrate[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((15*(3*a^4 + 384*a^2*b^2 + 1280*a*b^3 + 896*b^4)*ArcTan[((-Sqrt[a] - I*Sq
rt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[
a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]])/b^(3
/2) + (15*(3*a^4 + 384*a^2*b^2 + 1280*a*b^3 + 896*b^4)*ArcTan[((-Sqrt[a] +
 I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(
Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]])
/b^(3/2) - (45*a^4*ArcTan[(Sqrt[a] - Sqrt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]
])/b^(3/2) - (45*a^4*ArcTan[(Sqrt[a] + Sqrt[a + b]*Tan[(e + f*x)/2])/Sqrt[
b]])/b^(3/2) - (16*Sqrt[a]*Cos[e + f*x]*(150*a^3 + 1436*a^2*b + 2960*a*b^2
 + 1680*b^3 + a*(125*a^2 + 688*a*b + 560*b^2)*Cos[2*(e + f*x)] - 2*a^2*(11
*a + 14*b)*Cos[4*(e + f*x)] + 3*a^3*Cos[6*(e + f*x)]))/(a + 2*b + a*Cos[2*
(e + f*x)]))/(3840*a^(9/2)*f)
```

3.41.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.84, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4621, 366, 25, 363, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$\downarrow \text{3042}$$

$$\int \frac{\sin(e+fx)^5}{(a+b\sec(e+fx)^2)^2}dx$$

$$\downarrow 4621$$

$$-\frac{\int \frac{\cos^4(e+fx)(1-\cos^2(e+fx))^2}{(a\cos^2(e+fx)+b)^2}d\cos(e+fx)}{f}$$

$$\downarrow 366$$

$$-\frac{\frac{(a+b)^2\cos^5(e+fx)}{2a^2b(a\cos^2(e+fx)+b)} - \frac{\int -\frac{\cos^4(e+fx)\left(2a^2+2b\cos^2(e+fx)a-5(a+b)^2\right)}{a\cos^2(e+fx)+b}d\cos(e+fx)}{2a^2b}}{f}$$

$$\downarrow 25$$

$$-\frac{\frac{\int \frac{\cos^4(e+fx)\left(2a^2+2b\cos^2(e+fx)a-5(a+b)^2\right)}{a\cos^2(e+fx)+b}d\cos(e+fx)}{2a^2b} + \frac{(a+b)^2\cos^5(e+fx)}{2a^2b(a\cos^2(e+fx)+b)}}{f}$$

$$\downarrow 363$$

$$-\frac{\frac{\frac{2}{5}b\cos^5(e+fx)-(a+b)(3a+7b)\int \frac{\cos^4(e+fx)}{a\cos^2(e+fx)+b}d\cos(e+fx)}{2a^2b} + \frac{(a+b)^2\cos^5(e+fx)}{2a^2b(a\cos^2(e+fx)+b)}}{f}$$

$$\downarrow 254$$

$$-\frac{\frac{\frac{2}{5}b\cos^5(e+fx)-(a+b)(3a+7b)\int \left(\frac{b^2}{a^2\left(a\cos^2(e+fx)+b\right)}-\frac{b}{a^2}+\frac{\cos^2(e+fx)}{a}\right)d\cos(e+fx)}{2a^2b} + \frac{(a+b)^2\cos^5(e+fx)}{2a^2b(a\cos^2(e+fx)+b)}}{f}$$

$$\downarrow 2009$$

$$-\frac{\frac{(a+b)^2\cos^5(e+fx)}{2a^2b(a\cos^2(e+fx)+b)} + \frac{\frac{2}{5}b\cos^5(e+fx)-(a+b)(3a+7b)\left(\frac{b^{3/2}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a^{5/2}}-\frac{b\cos(e+fx)}{a^2}+\frac{\cos^3(e+fx)}{3a}\right)}{2a^2b}}{f}$$

input
```
Int[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
-((((a + b)^2*Cos[e + f*x]^5)/(2*a^2*b*(b + a*Cos[e + f*x]^2)) + ((2*b*Cos
[e + f*x]^5)/5 - (a + b)*(3*a + 7*b)*((b^(3/2)*ArcTan[(Sqrt[a]*Cos[e + f*x
])/Sqrt[b]])/a^(5/2) - (b*Cos[e + f*x])/a^2 + Cos[e + f*x]^3/(3*a)))/(2*a^
2*b))/f)
```

3.41.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

3.41.4 Maple [A] (verified)

Time = 4.61 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.99

method	result
derivativedivides	$\frac{-\frac{\cos(fx+e)^5 a^2}{5}-\frac{2a^2\cos(fx+e)^3}{3}-\frac{2a\cos(fx+e)^3 b}{3}+a^2\cos(fx+e)+4ab\cos(fx+e)+3b^2\cos(fx+e)}{a^4}+\frac{b\left(\frac{\left(-\frac{1}{2}a^2-ab-\frac{1}{2}b^2\right)\cos(fx+}{b+a\cos(fx+e)^2}\right.}{f}$
default	$\frac{-\frac{\cos(fx+e)^5 a^2}{5}-\frac{2a^2\cos(fx+e)^3}{3}-\frac{2a\cos(fx+e)^3 b}{3}+a^2\cos(fx+e)+4ab\cos(fx+e)+3b^2\cos(fx+e)}{a^4}+\frac{b\left(\frac{\left(-\frac{1}{2}a^2-ab-\frac{1}{2}b^2\right)\cos(fx+}{b+a\cos(fx+e)^2}\right.}{f}$
risch	$\frac{5\,e^{3i(fx+e)}}{96a^2f}+\frac{e^{3i(fx+e)}b}{12a^3f}-\frac{5\,e^{i(fx+e)}}{16fa^2}-\frac{7\,e^{i(fx+e)}b}{4fa^3}-\frac{3\,e^{i(fx+e)}b^2}{2fa^4}-\frac{5\,e^{-i(fx+e)}}{16fa^2}-\frac{7\,e^{-i(fx+e)}b}{4fa^3}-\frac{3\,e^{-i(f}}{2f}$

input
```
int(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/a^4*(1/5*cos(f*x+e)^5*a^2-2/3*a^2*cos(f*x+e)^3-2/3*a*cos(f*x+e)^3*
b+a^2*cos(f*x+e)+4*a*b*cos(f*x+e)+3*b^2*cos(f*x+e))+b/a^4*((-1/2*a^2-a*b-1
/2*b^2)*cos(f*x+e)/(b+a*cos(f*x+e)^2)+1/2*(3*a^2+10*a*b+7*b^2)/(a*b)^(1/2)
*arctan(a*cos(f*x+e)/(a*b)^(1/2))))
```

3.41.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.52

$$\int \frac{\sin^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$= \left[-\frac{12\,a^3\cos(fx+e)^7 - 4\,(10\,a^3+7\,a^2b)\cos(fx+e)^5 + 20\,(3\,a^3+10\,a^2b+7\,ab^2)\cos(fx+e)^3 - 15\,(3\,a\right.$$

$$\left. -\frac{6\,a^3\cos(fx+e)^7 - 2\,(10\,a^3+7\,a^2b)\cos(fx+e)^5 + 10\,(3\,a^3+10\,a^2b+7\,ab^2)\cos(fx+e)^3 - 15\,(3\,a^2b}{30\,(a}\right.$$

input
```
integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[-1/60*(12*a^3*cos(f*x + e)^7 - 4*(10*a^3 + 7*a^2*b)*cos(f*x + e)^5 + 20*(
3*a^3 + 10*a^2*b + 7*a*b^2)*cos(f*x + e)^3 - 15*(3*a^2*b + 10*a*b^2 + 7*b^
3 + (3*a^3 + 10*a^2*b + 7*a*b^2)*cos(f*x + e)^2)*sqrt(-b/a)*log(-(a*cos(f*
x + e)^2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) + 30*(
3*a^2*b + 10*a*b^2 + 7*b^3)*cos(f*x + e))/(a^5*f*cos(f*x + e)^2 + a^4*b*f)
, -1/30*(6*a^3*cos(f*x + e)^7 - 2*(10*a^3 + 7*a^2*b)*cos(f*x + e)^5 + 10*(
3*a^3 + 10*a^2*b + 7*a*b^2)*cos(f*x + e)^3 - 15*(3*a^2*b + 10*a*b^2 + 7*b^
3 + (3*a^3 + 10*a^2*b + 7*a*b^2)*cos(f*x + e)^2)*sqrt(b/a)*arctan(a*sqrt(b
/a)*cos(f*x + e)/b) + 15*(3*a^2*b + 10*a*b^2 + 7*b^3)*cos(f*x + e))/(a^5*f
*cos(f*x + e)^2 + a^4*b*f)]
```

3.41.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**5/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Timed out
```

3.41.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.92

$$\int \frac{\sin^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx =$$

$$-\frac{\frac{15\left(a^2b+2ab^2+b^3\right)\cos(fx+e)}{a^5\cos(fx+e)^2+a^4b} - \frac{15\left(3a^2b+10ab^2+7b^3\right)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}a^4} + \frac{2\left(3a^2\cos(fx+e)^5-10\left(a^2+ab\right)\cos(fx+e)^3+15\left(a^2+4a\right.\right.}{a^4}}{30\,f}$$

input
```
integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/30*(15*(a^2*b + 2*a*b^2 + b^3)*cos(f*x + e)/(a^5*cos(f*x + e)^2 + a^4*b
) - 15*(3*a^2*b + 10*a*b^2 + 7*b^3)*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt
(a*b)*a^4) + 2*(3*a^2*cos(f*x + e)^5 - 10*(a^2 + a*b)*cos(f*x + e)^3 + 15*
(a^2 + 4*a*b + 3*b^2)*cos(f*x + e))/a^4)/f
```

3.41.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 509 vs. $2(143) = 286$.

Time $= 0.36$ (sec) , antiderivative size $= 509$, normalized size of antiderivative $= 3.16$

$$\int \frac{\sin^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx =$$

$$-\frac{15\left(3a^2b+10ab^2+7b^3\right)\arctan\left(-\frac{a\cos(fx+e)-b}{\sqrt{ab}\cos(fx+e)+\sqrt{ab}}\right)}{\sqrt{ab}a^4}+\frac{30\left(a^2b+2ab^2+b^3+\frac{a^2b(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{b^3(\cos(fx+e)-1)}{\cos(fx+e)+1}\right)}{\left(a+b+\frac{2a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)a^4}$$

input
```
integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-1/30*(15*(3*a^2*b + 10*a*b^2 + 7*b^3)*arctan(-(a*cos(f*x + e) - b)/(sqrt(
a*b)*cos(f*x + e) + sqrt(a*b)))/(sqrt(a*b)*a^4) + 30*(a^2*b + 2*a*b^2 + b^
3 + a^2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - b^3*(cos(f*x + e) - 1)/(
cos(f*x + e) + 1))/((a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2
*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x
 + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)*a^4) - 4*(8*a^
2 + 50*a*b + 45*b^2 - 40*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 220*a
*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 180*b^2*(cos(f*x + e) - 1)/(cos
(f*x + e) + 1) + 80*a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 320*a*
b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 270*b^2*(cos(f*x + e) - 1)^2
/(cos(f*x + e) + 1)^2 - 180*a*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3
 - 180*b^2*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 30*a*b*(cos(f*x + e)
 - 1)^4/(cos(f*x + e) + 1)^4 + 45*b^2*(cos(f*x + e) - 1)^4/(cos(f*x + e) +
 1)^4)/(a^4*((cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 1)^5))/f
```

3.41.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.21

$$\int \frac{\sin^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\cos(e+fx)^3\left(\frac{2b}{3a^3}+\frac{2}{3a^2}\right)}{f} - \frac{\cos(e+fx)^5}{5a^2f} - \frac{\cos(e+fx)\left(\frac{1}{a^2}-\frac{b^2}{a^4}+\frac{2b\left(\frac{2b}{a^3}+\frac{2}{a^2}\right)}{a}\right)}{f} - \frac{\cos(e+fx)\left(\frac{a^2b}{2}+ab^2+\frac{b^3}{2}\right)}{f\left(a^5\cos(e+fx)^2+ba^4\right)} + \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{a}\sqrt{b}\cos(e+fx)(a+b)(3a+7b)}{3a^2b+10ab^2+7b^3}\right)(a+b)(3a+7b)}{2a^{9/2}f}$$

input
```
int(sin(e + f*x)^5/(a + b/cos(e + f*x)^2)^2,x)
```

output
```
(cos(e + f*x)^3*((2*b)/(3*a^3) + 2/(3*a^2)))/f - cos(e + f*x)^5/(5*a^2*f)
- (cos(e + f*x)*(1/a^2 - b^2/a^4 + (2*b*((2*b)/a^3 + 2/a^2))/a))/f - (cos(
e + f*x)*(a*b^2 + (a^2*b)/2 + b^3/2))/(f*(a^4*b + a^5*cos(e + f*x)^2)) + (
b^(1/2)*atan((a^(1/2)*b^(1/2)*cos(e + f*x)*(a + b)*(3*a + 7*b))/(10*a*b^2
+ 3*a^2*b + 7*b^3))*(a + b)*(3*a + 7*b))/(2*a^(9/2)*f)
```

3.42 $\int \frac{\sin^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

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3.42.1 Optimal result

Integrand size = 23, antiderivative size = 114

$$\int \frac{\sin^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\sqrt{b}(3a+5b)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2a^{7/2}f} - \frac{(a+2b)\cos(e+fx)}{a^3 f} + \frac{\cos^3(e+fx)}{3a^2 f} - \frac{b(a+b)\cos(e+fx)}{2a^3 f\left(b+a\cos^2(e+fx)\right)}$$

output
```
-(a+2*b)*cos(f*x+e)/a^3/f+1/3*cos(f*x+e)^3/a^2/f-1/2*b*(a+b)*cos(f*x+e)/a^
3/f/(b+a*cos(f*x+e)^2)+1/2*(3*a+5*b)*arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*b^
(1/2)/a^(7/2)/f
```

3.42.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.91 (sec) , antiderivative size = 403, normalized size of antiderivative = 3.54

$$\int \frac{\sin^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{3\left(3a^3+192ab^2+320b^3\right)\arctan\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cos(e)\left(\sqrt{a}-\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{b}}\right)}{b^{3/2}} + \frac{3\left(3a^3+192a\right.}{}$$

input
```
Integrate[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((3*(3*a^3 + 192*a*b^2 + 320*b^3)*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(
Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]
*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]])/b^(3/2) + (3*(3*a^3
+ 192*a*b^2 + 320*b^3)*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*
Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[
e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]])/b^(3/2) - (9*a^3*ArcTan[(Sqrt[a
] - Sqrt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]])/b^(3/2) - (9*a^3*ArcTan[(Sqrt[
a] + Sqrt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]])/b^(3/2) - (32*Sqrt[a]*Cos[e +
 f*x]*(9*a^2 + 56*a*b + 60*b^2 + 4*a*(2*a + 5*b)*Cos[2*(e + f*x)] - a^2*Co
s[4*(e + f*x)]))/(a + 2*b + a*Cos[2*(e + f*x)]))/(384*a^(7/2)*f)
```

3.42.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4621, 360, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sin(e+fx)^3}{\left(a+b\sec(e+fx)^2\right)^2}\,dx$$

$$\downarrow 4621$$

$$-\frac{\int \frac{\cos^4(e+fx)\left(1-\cos^2(e+fx)\right)}{\left(a\cos^2(e+fx)+b\right)^2}\,d\cos(e+fx)}{f}$$

$$\downarrow 360$$

$$-\frac{\frac{b(a+b)\cos(e+fx)}{2a^3\left(a\cos^2(e+fx)+b\right)}-\frac{\int \frac{2a^2\cos^4(e+fx)-2a(a+b)\cos^2(e+fx)+b(a+b)}{a\cos^2(e+fx)+b}\,d\cos(e+fx)}{2a^3}}{f}$$

$$\downarrow 1467$$

$$\frac{-\frac{b(a+b)\cos(e+fx)}{2a^3(a\cos^2(e+fx)+b)}-\frac{\int\left(2a\cos^2(e+fx)-2(a+2b)+\frac{5b^2+3ab}{a\cos^2(e+fx)+b}\right)d\cos(e+fx)}{2a^3}}{f}$$

$$\downarrow 2009$$

$$\frac{-\frac{b(a+b)\cos(e+fx)}{2a^3(a\cos^2(e+fx)+b)}-\frac{\frac{\sqrt{b}(3a+5b)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{\sqrt{a}}-2(a+2b)\cos(e+fx)+\frac{2}{3}a\cos^3(e+fx)}{2a^3}}{f}$$

input
```
Int[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
-(((b*(a + b)*Cos[e + f*x])/(2*a^3*(b + a*Cos[e + f*x]^2)) - ((Sqrt[b]*(3*
a + 5*b)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/Sqrt[a] - 2*(a + 2*b)*Cos
[e + f*x] + (2*a*Cos[e + f*x]^3)/3)/(2*a^3))/f)
```

3.42.3.1 Defintions of rubi rules used

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

3.42.4 Maple [A] (verified)

Time = 2.46 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{\frac{a\cos(fx+e)^3}{3}-\cos(fx+e)a-2b\cos(fx+e)}{a^3}+\frac{b\left(\frac{\left(-\frac{a}{2}-\frac{b}{2}\right)\cos(fx+e)}{b+a\cos(fx+e)^2}+\frac{(3a+5b)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^3}}{f}$
default	$\frac{\frac{a\cos(fx+e)^3}{3}-\cos(fx+e)a-2b\cos(fx+e)}{a^3}+\frac{b\left(\frac{\left(-\frac{a}{2}-\frac{b}{2}\right)\cos(fx+e)}{b+a\cos(fx+e)^2}+\frac{(3a+5b)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^3}}{f}$
risch	$\frac{e^{3i(fx+e)}}{24a^2f}-\frac{3e^{i(fx+e)}}{8fa^2}-\frac{e^{i(fx+e)}b}{fa^3}-\frac{3e^{-i(fx+e)}}{8fa^2}-\frac{e^{-i(fx+e)}b}{fa^3}+\frac{e^{-3i(fx+e)}}{24a^2f}-\frac{(a+b)b(e^{3i(fx+e)}+$ $a^3f(ae^{4i(fx+e)}+2ae^{2i(fx+$

input

```
int(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output

```
1/f*(1/a^3*(1/3*a*cos(f*x+e)^3-cos(f*x+e)*a-2*b*cos(f*x+e))+b/a^3*((-1/2*a
-1/2*b)*cos(f*x+e)/(b+a*cos(f*x+e)^2)+1/2*(3*a+5*b)/(a*b)^(1/2)*arctan(a*c
os(f*x+e)/(a*b)^(1/2))))
```

3.42.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 297, normalized size of antiderivative = 2.61

$$\int \frac{\sin^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$= \left[\frac{4\,a^2\cos\left(fx+e\right)^5-4\left(3\,a^2+5\,ab\right)\cos\left(fx+e\right)^3+3\left(\left(3\,a^2+5\,ab\right)\cos\left(fx+e\right)^2+3\,ab+5\,b^2\right)\sqrt{-\frac{b}{a}}\log\right.$$
$$\left.\frac{}{12\left(a^4f\cos\left(fx+e\right)^2+a^3bf\right)}\right.$$

input
```
integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[1/12*(4*a^2*cos(f*x + e)^5 - 4*(3*a^2 + 5*a*b)*cos(f*x + e)^3 + 3*((3*a^2
 + 5*a*b)*cos(f*x + e)^2 + 3*a*b + 5*b^2)*sqrt(-b/a)*log(-(a*cos(f*x + e)^
2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) - 6*(3*a*b +
5*b^2)*cos(f*x + e))/(a^4*f*cos(f*x + e)^2 + a^3*b*f), 1/6*(2*a^2*cos(f*x
+ e)^5 - 2*(3*a^2 + 5*a*b)*cos(f*x + e)^3 + 3*((3*a^2 + 5*a*b)*cos(f*x + e
)^2 + 3*a*b + 5*b^2)*sqrt(b/a)*arctan(a*sqrt(b/a)*cos(f*x + e)/b) - 3*(3*a
*b + 5*b^2)*cos(f*x + e))/(a^4*f*cos(f*x + e)^2 + a^3*b*f)]
```

3.42.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**3/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Timed out
```

3.42.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.91

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\frac{3\,(ab+b^2)\cos(fx+e)}{a^4\cos(fx+e)^2+a^3b} - \frac{3\,(3\,ab+5\,b^2)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}a^3} - \frac{2\left(a\cos(fx+e)^3-3\,(a+2\,b)\cos(fx+e)\right)}{a^3}}{6\,f}$$

input
```
integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/6*(3*(a*b + b^2)*cos(f*x + e)/(a^4*cos(f*x + e)^2 + a^3*b) - 3*(3*a*b +
 5*b^2)*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^3) - 2*(a*cos(f*x +
e)^3 - 3*(a + 2*b)*cos(f*x + e))/a^3)/f
```

3.42.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.19

$$\int \frac{\sin^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$
$$= \frac{\left(3\,ab+5\,b^2\right)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}a^3 f} - \frac{\frac{ab\cos(fx+e)}{f}+\frac{b^2\cos(fx+e)}{f}}{2\left(a\cos\left(fx+e\right)^2+b\right)a^3}$$
$$+ \frac{a^4 f^{11}\cos\left(fx+e\right)^3 - 3\,a^4 f^{11}\cos\left(fx+e\right) - 6\,a^3 b f^{11}\cos\left(fx+e\right)}{3\,a^6 f^{12}}$$

input
```
integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/2*(3*a*b + 5*b^2)*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^3*f) - 1
/2*(a*b*cos(f*x + e)/f + b^2*cos(f*x + e)/f)/((a*cos(f*x + e)^2 + b)*a^3)
+ 1/3*(a^4*f^11*cos(f*x + e)^3 - 3*a^4*f^11*cos(f*x + e) - 6*a^3*b*f^11*co
s(f*x + e))/(a^6*f^12)
```

3.42.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.14

$$\int \frac{\sin^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\cos\left(e+f\,x\right)^3}{3\,a^2 f} - \frac{\cos\left(e+f\,x\right)\left(\frac{2\,b}{a^3}+\frac{1}{a^2}\right)}{f}$$
$$- \frac{\cos\left(e+f\,x\right)\left(\frac{b^2}{2}+\frac{a\,b}{2}\right)}{f\left(a^4\cos\left(e+f\,x\right)^2+b\,a^3\right)}$$
$$+ \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,\cos(e+f\,x)\,(3\,a+5\,b)}{5\,b^2+3\,a\,b}\right)\,(3\,a+5\,b)}{2\,a^{7/2}\,f}$$

input
```
int(sin(e + f*x)^3/(a + b/cos(e + f*x)^2)^2,x)
```

output
```
cos(e + f*x)^3/(3*a^2*f) - (cos(e + f*x)*((2*b)/a^3 + 1/a^2))/f - (cos(e +
 f*x)*((a*b)/2 + b^2/2))/(f*(a^3*b + a^4*cos(e + f*x)^2)) + (b^(1/2)*atan(
(a^(1/2)*b^(1/2)*cos(e + f*x)*(3*a + 5*b))/(3*a*b + 5*b^2))*(3*a + 5*b))/(
2*a^(7/2)*f)
```

3.43 $\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

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3.43.1 Optimal result

Integrand size = 21, antiderivative size = 84

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{3\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2a^{5/2}f} - \frac{3\cos(e+fx)}{2a^2 f} + \frac{\cos^3(e+fx)}{2af\left(b+a\cos^2(e+fx)\right)}$$

output
```
-3/2*cos(f*x+e)/a^2/f+1/2*cos(f*x+e)^3/a/f/(b+a*cos(f*x+e)^2)+3/2*arctan(c
os(f*x+e)*a^(1/2)/b^(1/2))*b^(1/2)/a^(5/2)/f
```

3.43.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.58 (sec) , antiderivative size = 393, normalized size of antiderivative = 4.68

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\left(a+2b+a\cos(2(e+fx))\right)^2 \left(\frac{\left(a^2+24b^2\right)\arctan\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cos(e)\left(\sqrt{a}-\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)}{\sqrt{b}} \right)}{b^{3/2}} \right.$$

input
```
Integrate[Sin[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])^2*(((a^2 + 24*b^2)*ArcTan[((-Sqrt[a] - I*S
qrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt
[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]])/b^(
3/2) + ((a^2 + 24*b^2)*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*
Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[
e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]])/b^(3/2) - (a^2*ArcTan[(Sqrt[a]
- Sqrt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]])/b^(3/2) - (a^2*ArcTan[(Sqrt[a] +
 Sqrt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]])/b^(3/2) - (16*Sqrt[a]*Cos[e + f*x
]*(a + 3*b + a*Cos[2*(e + f*x)]))/(a + 2*b + a*Cos[2*(e + f*x)]))*Sec[e +
f*x]^4)/(64*a^(5/2)*f*(a + b*Sec[e + f*x]^2)^2)
```

3.43.3 Rubi [A] (verified)

Time $= 0.23$ (sec) , antiderivative size $= 83$, normalized size of antiderivative $= 0.99$, number of steps used $= 6$, number of rules used $= 5$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used $= \{3042, 4621, 252, 262, 218\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sin(e+fx)}{(a+b\sec(e+fx)^2)^2}\,dx$$

$$\downarrow 4621$$

$$-\frac{\int \frac{\cos^4(e+fx)}{(a\cos^2(e+fx)+b)^2}\,d\cos(e+fx)}{f}$$

$$\downarrow 252$$

$$-\frac{\frac{3\int \frac{\cos^2(e+fx)}{a\cos^2(e+fx)+b}\,d\cos(e+fx)}{2a} - \frac{\cos^3(e+fx)}{2a(a\cos^2(e+fx)+b)}}{f}$$

$$\downarrow 262$$

$$-\frac{\frac{3\left(\frac{\cos(e+fx)}{a}-\frac{b\int\frac{1}{a\cos^2(e+fx)+b}d\cos(e+fx)}{a}\right)}{2a}-\frac{\cos^3(e+fx)}{2a(a\cos^2(e+fx)+b)}}{f}$$

$$\downarrow 218$$

$$-\frac{\frac{3\left(\frac{\cos(e+fx)}{a}-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a^{3/2}}\right)}{2a}-\frac{\cos^3(e+fx)}{2a(a\cos^2(e+fx)+b)}}{f}$$

input
```
Int[Sin[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
-(((3*(-((Sqrt[b]*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/a^(3/2)) + Cos[e
 + f*x]/a))/(2*a) - Cos[e + f*x]^3/(2*a*(b + a*Cos[e + f*x]^2)))/f)
```

3.43.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

3.43.4 Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{\sec(fx+e)}{2a+2b\sec(fx+e)^2}+\frac{3\arctan\left(\frac{\sec(fx+e)b}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^2}-\frac{1}{a^2\sec(fx+e)}}{f}$
default	$\frac{-\frac{b\left(\frac{\sec(fx+e)}{2a+2b\sec(fx+e)^2}+\frac{3\arctan\left(\frac{\sec(fx+e)b}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^2}-\frac{1}{a^2\sec(fx+e)}}{f}$
risch	$-\frac{e^{i(fx+e)}}{2fa^2}-\frac{e^{-i(fx+e)}}{2fa^2}-\frac{b\left(e^{3i(fx+e)}+e^{i(fx+e)}\right)}{a^2f\left(ae^{4i(fx+e)}+2ae^{2i(fx+e)}+4be^{2i(fx+e)}+a\right)}-\frac{3i\sqrt{ab}\ln\left(e^{2i(fx+e)}-\frac{2i\sqrt{ab}e^{i(fx+e)}}{a}\right.}{4a^3f}$

input

```
int(sin(f*x+e)/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output

```
1/f*(-b/a^2*(1/2*sec(f*x+e)/(a+b*sec(f*x+e)^2)+3/2/(a*b)^(1/2)*arctan(sec(
f*x+e)*b/(a*b)^(1/2)))-1/a^2/sec(f*x+e))
```

3.43.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.39

$$\int \frac{\sin(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$= \left[-\frac{4\,a\cos\left(fx+e\right)^3 - 3\left(a\cos\left(fx+e\right)^2+b\right)\sqrt{-\frac{b}{a}}\log\left(-\frac{a\cos(fx+e)^2+2\,a\sqrt{-\frac{b}{a}}\cos(fx+e)-b}{a\cos(fx+e)^2+b}\right) + 6\,b\cos\left(fx+e\right)}{4\left(a^3 f\cos\left(fx+e\right)^2 + a^2 b f\right)}\right.$$

$$\left. -\frac{2\,a\cos\left(fx+e\right)^3 - 3\left(a\cos\left(fx+e\right)^2+b\right)\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}\cos(fx+e)}{b}\right) + 3\,b\cos\left(fx+e\right)}{2\left(a^3 f\cos\left(fx+e\right)^2 + a^2 b f\right)}\right]$$

input
```
integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[-1/4*(4*a*cos(f*x + e)^3 - 3*(a*cos(f*x + e)^2 + b)*sqrt(-b/a)*log(-(a*co
s(f*x + e)^2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) +
6*b*cos(f*x + e))/(a^3*f*cos(f*x + e)^2 + a^2*b*f), -1/2*(2*a*cos(f*x + e)
^3 - 3*(a*cos(f*x + e)^2 + b)*sqrt(b/a)*arctan(a*sqrt(b/a)*cos(f*x + e)/b)
 + 3*b*cos(f*x + e))/(a^3*f*cos(f*x + e)^2 + a^2*b*f)]
```

3.43.6 Sympy [F]

$$\int \frac{\sin(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \int \frac{\sin\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^2}\,dx$$

input
```
integrate(sin(f*x+e)/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral(sin(e + f*x)/(a + b*sec(e + f*x)**2)**2, x)
```

3.43.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.83

$$\int \frac{\sin(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{\frac{b\cos(fx+e)}{a^3\cos(fx+e)^2+a^2b} - \frac{3\,b\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}a^2} + \frac{2\cos(fx+e)}{a^2}}{2f}$$

input
```
integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/2*(b*cos(f*x + e)/(a^3*cos(f*x + e)^2 + a^2*b) - 3*b*arctan(a*cos(f*x +
 e)/sqrt(a*b))/(sqrt(a*b)*a^2) + 2*cos(f*x + e)/a^2)/f
```

3.43.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.86

$$\int \frac{\sin(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{3\,b\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}a^2f} - \frac{\cos(fx+e)}{a^2f} - \frac{b\cos(fx+e)}{2\left(a\cos(fx+e)^2+b\right)a^2f}$$

input
```
integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
3/2*b*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^2*f) - cos(f*x + e)/(a
^2*f) - 1/2*b*cos(f*x + e)/((a*cos(f*x + e)^2 + b)*a^2*f)
```

3.43.9 Mupad [B] (verification not implemented)

Time = 18.82 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.86

$$\int \frac{\sin(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{3\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\cos(e+f\,x)}{\sqrt{b}}\right)}{2\,a^{5/2}\,f} - \frac{b\,\cos(e+f\,x)}{2\,f\,\left(a^3\cos(e+f\,x)^2+b\,a^2\right)} - \frac{\cos(e+f\,x)}{a^2\,f}$$

input
```
int(sin(e + f*x)/(a + b/cos(e + f*x)^2)^2,x)
```

output
```
(3*b^(1/2)*atan((a^(1/2)*cos(e + f*x))/b^(1/2)))/(2*a^(5/2)*f) - (b*cos(e
+ f*x))/(2*f*(a^2*b + a^3*cos(e + f*x)^2)) - cos(e + f*x)/(a^2*f)
```

3.44 $\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

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3.44.1 Optimal result

Integrand size = 21, antiderivative size = 99

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\sqrt{b}(3a+b)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2a^{3/2}(a+b)^2 f} - \frac{\operatorname{arctanh}(\cos(e+fx))}{(a+b)^2 f} - \frac{b\cos(e+fx)}{2a(a+b)f\left(b+a\cos^2(e+fx)\right)}$$

output

```
-arctanh(cos(f*x+e))/(a+b)^2/f-1/2*b*cos(f*x+e)/a/(a+b)/f/(b+a*cos(f*x+e)^
2)+1/2*(3*a+b)*arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*b^(1/2)/a^(3/2)/(a+b)^2/
f
```

3.44.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.63 (sec) , antiderivative size = 384, normalized size of antiderivative = 3.88

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{(a+2b+a\cos(2(e+fx)))\sec^3(e+fx)\left(-\frac{2b(a+b)}{a} + \frac{\sqrt{b}(3a+b)\arctan\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cos}{\sqrt{b}}\right.}{}\right.}{}$$

input
```
Integrate[Csc[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*((-2*b*(a + b))/a + (Sqrt[b
]*(3*a + b)*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])
*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[
e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]
)/a^(3/2) + (Sqrt[b]*(3*a + b)*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos
[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sq
rt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e +
f*x)])*Sec[e + f*x])/a^(3/2) - 2*(a + 2*b + a*Cos[2*(e + f*x)])*Log[Cos[(e
 + f*x)/2]]*Sec[e + f*x] + 2*(a + 2*b + a*Cos[2*(e + f*x)])*Log[Sin[(e + f
*x)/2]]*Sec[e + f*x]))/(8*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^2)
```

3.44.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4621, 372, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sin(e+fx)\left(a+b\sec(e+fx)^2\right)^2}dx$$

$$\downarrow 4621$$

$$-\frac{\int \frac{\cos^4(e+fx)}{(1-\cos^2(e+fx))(a\cos^2(e+fx)+b)^2}d\cos(e+fx)}{f}$$

$$\downarrow 372$$

$$-\frac{\frac{b\cos(e+fx)}{2a(a+b)(a\cos^2(e+fx)+b)}-\frac{\int \frac{b-(2a+b)\cos^2(e+fx)}{\left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)}d\cos(e+fx)}{2a(a+b)}}{f}$$

$$\downarrow 397$$

$$-\frac{\frac{b\cos(e+fx)}{2a(a+b)(a\cos^2(e+fx)+b)}-\frac{\frac{b(3a+b)\int\frac{1}{a\cos^2(e+fx)+b}d\cos(e+fx)}{a+b}-\frac{2a\int\frac{1}{1-\cos^2(e+fx)}d\cos(e+fx)}{a+b}}{2a(a+b)}}{f}$$

$$\downarrow 218$$

$$-\frac{\frac{b\cos(e+fx)}{2a(a+b)(a\cos^2(e+fx)+b)}-\frac{\frac{\sqrt{b}(3a+b)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{\sqrt{a}(a+b)}-\frac{2a\int\frac{1}{1-\cos^2(e+fx)}d\cos(e+fx)}{a+b}}{2a(a+b)}}{f}$$

$$\downarrow 219$$

$$-\frac{\frac{b\cos(e+fx)}{2a(a+b)(a\cos^2(e+fx)+b)}-\frac{\frac{\sqrt{b}(3a+b)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{\sqrt{a}(a+b)}-\frac{2a\text{arctanh}(\cos(e+fx))}{a+b}}{2a(a+b)}}{f}$$

input
```
Int[Csc[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
-((-1/2*((Sqrt[b]*(3*a + b)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(Sqrt[
a]*(a + b)) - (2*a*ArcTanh[Cos[e + f*x]])/(a + b))/(a*(a + b)) + (b*Cos[e
+ f*x])/(2*a*(a + b)*(b + a*Cos[e + f*x]^2)))/f)
```

3.44.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)  Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)  Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
  Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

3.44.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.04

method	result
derivativedivides	$\frac{\frac{\ln(-1+\cos(fx+e))}{2(a+b)^2}+\frac{b\left(-\frac{(a+b)\cos(fx+e)}{2a\left(b+a\cos(fx+e)^2\right)}+\frac{(3a+b)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{(a+b)^2}-\frac{\ln(1+\cos(fx+e))}{2(a+b)^2}}{f}$
default	$\frac{\frac{\ln(-1+\cos(fx+e))}{2(a+b)^2}+\frac{b\left(-\frac{(a+b)\cos(fx+e)}{2a\left(b+a\cos(fx+e)^2\right)}+\frac{(3a+b)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{(a+b)^2}-\frac{\ln(1+\cos(fx+e))}{2(a+b)^2}}{f}$
risch	$-\frac{b\left(e^{3i(fx+e)}+e^{i(fx+e)}\right)}{af(a+b)\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)}+\frac{\ln\left(e^{i(fx+e)}-1\right)}{f(a^2+2ab+b^2)}-\frac{\ln\left(e^{i(fx+e)}+1\right)}{f(a^2+2ab+b^2)}+\frac{3i\sqrt{ab}\,\ln\left(e^{2i(fx+$

input
```
int(csc(f*x+e)/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/2/(a+b)^2*ln(-1+cos(f*x+e))+b/(a+b)^2*(-1/2*(a+b)/a*cos(f*x+e)/(b+a
*cos(f*x+e)^2)+1/2*(3*a+b)/a/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2)))
-1/2/(a+b)^2*ln(1+cos(f*x+e)))
```

3.44.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(87) = 174$.

Time = 0.32 (sec) , antiderivative size = 390, normalized size of antiderivative = 3.94

$$\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$= \left[\frac{\left(\left(3\,a^2+ab\right)\cos\left(fx+e\right)^2+3\,ab+b^2\right)\sqrt{-\frac{b}{a}}\log\left(-\frac{a\cos(fx+e)^2+2\,a\sqrt{-\frac{b}{a}}\cos(fx+e)-b}{a\cos(fx+e)^2+b}\right)-2\left(ab+b^2\right)\cos\left(fx-\right.}{4\left(\left(a^4+2\,a^3b+a^2b^2\right)f\cos\left(\right.\right.}\right.$$

input
```
integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[1/4*(((3*a^2 + a*b)*cos(f*x + e)^2 + 3*a*b + b^2)*sqrt(-b/a)*log(-(a*cos(
f*x + e)^2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) - 2*
(a*b + b^2)*cos(f*x + e) - 2*(a^2*cos(f*x + e)^2 + a*b)*log(1/2*cos(f*x +
e) + 1/2) + 2*(a^2*cos(f*x + e)^2 + a*b)*log(-1/2*cos(f*x + e) + 1/2))/((a
^4 + 2*a^3*b + a^2*b^2)*f*cos(f*x + e)^2 + (a^3*b + 2*a^2*b^2 + a*b^3)*f),
 1/2*(((3*a^2 + a*b)*cos(f*x + e)^2 + 3*a*b + b^2)*sqrt(b/a)*arctan(a*sqrt
(b/a)*cos(f*x + e)/b) - (a*b + b^2)*cos(f*x + e) - (a^2*cos(f*x + e)^2 + a
*b)*log(1/2*cos(f*x + e) + 1/2) + (a^2*cos(f*x + e)^2 + a*b)*log(-1/2*cos(
f*x + e) + 1/2))/((a^4 + 2*a^3*b + a^2*b^2)*f*cos(f*x + e)^2 + (a^3*b + 2*
a^2*b^2 + a*b^3)*f)]
```

3.44.6 Sympy [F]

$$\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \int \frac{\csc\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^2}\,dx$$

input
```
integrate(csc(f*x+e)/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral(csc(e + f*x)/(a + b*sec(e + f*x)**2)**2, x)
```

3.44.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.39

$$\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$
$$= -\frac{\frac{b\cos(fx+e)}{a^2b+ab^2+(a^3+a^2b)\cos(fx+e)^2} - \frac{(3ab+b^2)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{(a^3+2a^2b+ab^2)\sqrt{ab}} + \frac{\log(\cos(fx+e)+1)}{a^2+2ab+b^2} - \frac{\log(\cos(fx+e)-1)}{a^2+2ab+b^2}}{2f}$$

input
```
integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/2*(b*cos(f*x + e)/(a^2*b + a*b^2 + (a^3 + a^2*b)*cos(f*x + e)^2) - (3*a
*b + b^2)*arctan(a*cos(f*x + e)/sqrt(a*b))/((a^3 + 2*a^2*b + a*b^2)*sqrt(a
*b)) + log(cos(f*x + e) + 1)/(a^2 + 2*a*b + b^2) - log(cos(f*x + e) - 1)/(
a^2 + 2*a*b + b^2))/f
```

3.44.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. 2(87) = 174.

Time = 0.33 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.75

$$\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx =$$
$$-\frac{\frac{(3ab+b^2)\arctan\left(-\frac{a\cos(fx+e)-b}{\sqrt{ab}\cos(fx+e)+\sqrt{ab}}\right)}{(a^3+2a^2b+ab^2)\sqrt{ab}} - \frac{\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a^2+2ab+b^2} + \frac{2\left(ab+b^2+\frac{ab(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{b^2(\cos(fx+e)-1)}{\cos(fx+e)+1}\right.}{(a^3+2a^2b+ab^2)\left(a+b+\frac{2a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+}{(\cos(fx+}}$$
$$2f$$

input
```
integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-1/2*((3*a*b + b^2)*arctan(-(a*cos(f*x + e) - b)/(sqrt(a*b)*cos(f*x + e) +
 sqrt(a*b)))/((a^3 + 2*a^2*b + a*b^2)*sqrt(a*b)) - log(abs(-cos(f*x + e) +
 1)/abs(cos(f*x + e) + 1))/(a^2 + 2*a*b + b^2) + 2*(a*b + b^2 + a*b*(cos(f
*x + e) - 1)/(cos(f*x + e) + 1) - b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1
))/((a^3 + 2*a^2*b + a*b^2)*(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e)
+ 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/
(cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)))/f
```

3.44.9 Mupad [B] (verification not implemented)

Time = 20.18 (sec) , antiderivative size = 2188, normalized size of antiderivative = 22.10

$$\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(e + f*x)*(a + b/cos(e + f*x)^2)^2),x)
```

output
```
(atan((((3*a + b)*(-a^3*b)^(1/2)*((cos(e + f*x)*(6*a*b^3 + 4*a^4 + b^4 + 9
*a^2*b^2))/(2*(a*b^2 + 2*a^2*b + a^3)) + ((3*a + b)*(-a^3*b)^(1/2)*((2*a^6
*b + 2*a^2*b^5 + 8*a^3*b^4 + 12*a^4*b^3 + 8*a^5*b^2)/(a*b^3 + 3*a^3*b + a^
4 + 3*a^2*b^2) - (cos(e + f*x)*(3*a + b)*(-a^3*b)^(1/2)*(48*a^7*b + 16*a^8
 - 16*a^3*b^5 - 48*a^4*b^4 - 32*a^5*b^3 + 32*a^6*b^2))/(8*(a*b^2 + 2*a^2*b
 + a^3)*(2*a^4*b + a^5 + a^3*b^2))))/(4*(2*a^4*b + a^5 + a^3*b^2)))*1i)/(4
*(2*a^4*b + a^5 + a^3*b^2)) + ((3*a + b)*(-a^3*b)^(1/2)*((cos(e + f*x)*(6*
a*b^3 + 4*a^4 + b^4 + 9*a^2*b^2))/(2*(a*b^2 + 2*a^2*b + a^3)) - ((3*a + b)
*(-a^3*b)^(1/2)*((2*a^6*b + 2*a^2*b^5 + 8*a^3*b^4 + 12*a^4*b^3 + 8*a^5*b^2
)/(a*b^3 + 3*a^3*b + a^4 + 3*a^2*b^2) + (cos(e + f*x)*(3*a + b)*(-a^3*b)^(
1/2)*(48*a^7*b + 16*a^8 - 16*a^3*b^5 - 48*a^4*b^4 - 32*a^5*b^3 + 32*a^6*b^
2))/(8*(a*b^2 + 2*a^2*b + a^3)*(2*a^4*b + a^5 + a^3*b^2))))/(4*(2*a^4*b +
a^5 + a^3*b^2)))*1i)/(4*(2*a^4*b + a^5 + a^3*b^2)))/(((5*a*b^2)/2 + 3*a^2*
b + b^3/2)/(a*b^3 + 3*a^3*b + a^4 + 3*a^2*b^2) - ((3*a + b)*(-a^3*b)^(1/2)
*((cos(e + f*x)*(6*a*b^3 + 4*a^4 + b^4 + 9*a^2*b^2))/(2*(a*b^2 + 2*a^2*b +
 a^3)) + ((3*a + b)*(-a^3*b)^(1/2)*((2*a^6*b + 2*a^2*b^5 + 8*a^3*b^4 + 12*
a^4*b^3 + 8*a^5*b^2)/(a*b^3 + 3*a^3*b + a^4 + 3*a^2*b^2) - (cos(e + f*x)*(
3*a + b)*(-a^3*b)^(1/2)*(48*a^7*b + 16*a^8 - 16*a^3*b^5 - 48*a^4*b^4 - 32*
a^5*b^3 + 32*a^6*b^2))/(8*(a*b^2 + 2*a^2*b + a^3)*(2*a^4*b + a^5 + a^3*b^2
))))/(4*(2*a^4*b + a^5 + a^3*b^2))))/(4*(2*a^4*b + a^5 + a^3*b^2)) + ((...
```

3.45 $\int \frac{\csc^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

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3.45.1 Optimal result

Integrand size = 23, antiderivative size = 147

$$\begin{aligned}\int \frac{\csc^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx &= \frac{(3a-b)\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2\sqrt{a}(a+b)^3 f}\\ &- \frac{(a-3b)\operatorname{arctanh}(\cos(e+fx))}{2(a+b)^3 f}\\ &+ \frac{(a-b)\cos(e+fx)}{2(a+b)^2 f\left(b+a\cos^2(e+fx)\right)}\\ &- \frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\left(b+a\cos^2(e+fx)\right)}\end{aligned}$$

output
```
-1/2*(a-3*b)*arctanh(cos(f*x+e))/(a+b)^3/f+1/2*(a-b)*cos(f*x+e)/(a+b)^2/f/
(b+a*cos(f*x+e)^2)-1/2*cot(f*x+e)*csc(f*x+e)/(a+b)/f/(b+a*cos(f*x+e)^2)+1/
2*(3*a-b)*arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*b^(1/2)/(a+b)^3/f/a^(1/2)
```

3.45.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.26 (sec) , antiderivative size = 468, normalized size of antiderivative = 3.18

$$\int \frac{\csc^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^3(e+fx)\left(-8b(a+b)-\frac{4\sqrt{b}(-3a+b)\arctan\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{1}{2}\right.\right.}{}\right.}{}$$

input
```
Integrate[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(-8*b*(a + b) - (4*Sqrt[b]*
(-3*a + b)*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*
Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e
])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x])
/Sqrt[a] - (4*Sqrt[b]*(-3*a + b)*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(C
os[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*
Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e
+ f*x)])*Sec[e + f*x])/Sqrt[a] - (a + b)*(a + 2*b + a*Cos[2*(e + f*x)])*Cs
c[(e + f*x)/2]^2*Sec[e + f*x] - 4*(a - 3*b)*(a + 2*b + a*Cos[2*(e + f*x)])
*Log[Cos[(e + f*x)/2]]*Sec[e + f*x] + 4*(a - 3*b)*(a + 2*b + a*Cos[2*(e +
f*x)])*Log[Sin[(e + f*x)/2]]*Sec[e + f*x] + (a + b)*(a + 2*b + a*Cos[2*(e
+ f*x)])*Sec[(e + f*x)/2]^2*Sec[e + f*x]))/(32*(a + b)^3*f*(a + b*Sec[e +
f*x]^2)^2)
```

3.45.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4621, 372, 402, 27, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sin(e+fx)^3\left(a+b\sec(e+fx)^2\right)^2}dx$$

$$\downarrow 4621$$

$$-\frac{\int \frac{\cos^4(e+fx)}{(1-\cos^2(e+fx))^2(a\cos^2(e+fx)+b)^2}d\cos(e+fx)}{f}$$

$$\downarrow 372$$

$$-\frac{\frac{\cos(e+fx)}{2(a+b)(1-\cos^2(e+fx))(a\cos^2(e+fx)+b)}-\frac{\int \frac{b-(a-2b)\cos^2(e+fx)}{\left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)^2}d\cos(e+fx)}{2(a+b)}}{f}$$

$$\downarrow 402$$

$$-\frac{\frac{\cos(e+fx)}{2(a+b)(1-\cos^2(e+fx))(a\cos^2(e+fx)+b)}-\frac{\frac{(a-b)\cos(e+fx)}{(a+b)\left(a\cos^2(e+fx)+b\right)}-\frac{\int -\frac{2b\left(2b-(a-b)\cos^2(e+fx)\right)}{\left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)}d\cos(e+fx)}{2b(a+b)}}{2(a+b)}}{f}$$

$$\downarrow 27$$

$$-\frac{\frac{\cos(e+fx)}{2(a+b)(1-\cos^2(e+fx))(a\cos^2(e+fx)+b)}-\frac{\frac{\int \frac{2b-(a-b)\cos^2(e+fx)}{\left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)}d\cos(e+fx)}{a+b}+\frac{(a-b)\cos(e+fx)}{(a+b)\left(a\cos^2(e+fx)+b\right)}}{2(a+b)}}{f}$$

$$\downarrow 397$$

$$-\frac{\frac{\cos(e+fx)}{2(a+b)(1-\cos^2(e+fx))(a\cos^2(e+fx)+b)}-\frac{\frac{b(3a-b)\int \frac{1}{a\cos^2(e+fx)+b}d\cos(e+fx)}{a+b}-\frac{(a-3b)\int \frac{1}{1-\cos^2(e+fx)}d\cos(e+fx)}{a+b}}{a+b}+\frac{(a-b)\cos(e+fx)}{(a+b)\left(a\cos^2(e+fx)+b\right)}}{2(a+b)}}{f}$$

$$\downarrow 218$$

$$-\frac{\frac{\cos(e+fx)}{2(a+b)(1-\cos^2(e+fx))(a\cos^2(e+fx)+b)}-\frac{\frac{\sqrt{b}(3a-b)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{\sqrt{a}(a+b)}-\frac{(a-3b)\int \frac{1}{1-\cos^2(e+fx)}d\cos(e+fx)}{a+b}}{a+b}+\frac{(a-b)\cos(e+fx)}{(a+b)\left(a\cos^2(e+fx)+b\right)}}{2(a+b)}}{f}$$

$$\downarrow 219$$

$$-\frac{\cos(e+fx)}{2(a+b)(1-\cos^2(e+fx))(a\cos^2(e+fx)+b)}-\frac{\frac{\frac{\sqrt{b}(3a-b)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{\sqrt{a}(a+b)}-\frac{(a-3b)\operatorname{arctanh}(\cos(e+fx))}{a+b}}{a+b}+\frac{(a-b)\cos(e+fx)}{(a+b)\left(a\cos^2(e+fx)+b\right)}}{2(a+b)}}{f}$$

input
```
Int[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
-((Cos[e + f*x]/(2*(a + b)*(1 - Cos[e + f*x]^2)*(b + a*Cos[e + f*x]^2)) -
((((3*a - b)*Sqrt[b]*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(Sqrt[a]*(a +
 b)) - ((a - 3*b)*ArcTanh[Cos[e + f*x]])/(a + b))/(a + b) + ((a - b)*Cos[e
 + f*x])/((a + b)*(b + a*Cos[e + f*x]^2)))/(2*(a + b)))/f)
```

3.45.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

3.45.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{\frac{1}{4(a+b)^2(-1+\cos(fx+e))}+\frac{(a-3b)\ln(-1+\cos(fx+e))}{4(a+b)^3}+\frac{b\left(\frac{\left(-\frac{a}{2}-\frac{b}{2}\right)\cos(fx+e)}{b+a\cos(fx+e)^2}+\frac{(3a-b)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{(a+b)^3}+\frac{1}{4(a+b)^2(1+\cos}}{f}$
default	$\frac{\frac{1}{4(a+b)^2(-1+\cos(fx+e))}+\frac{(a-3b)\ln(-1+\cos(fx+e))}{4(a+b)^3}+\frac{b\left(\frac{\left(-\frac{a}{2}-\frac{b}{2}\right)\cos(fx+e)}{b+a\cos(fx+e)^2}+\frac{(3a-b)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{(a+b)^3}+\frac{1}{4(a+b)^2(1+\cos}}{f}$
risch	$\frac{a\,e^{7i(fx+e)}-b\,e^{7i(fx+e)}+3a\,e^{5i(fx+e)}+5b\,e^{5i(fx+e)}+3a\,e^{3i(fx+e)}+5b\,e^{3i(fx+e)}+a\,e^{i(fx+e)}-b\,e^{i(fx+e)}}{f(a+b)^2\left(e^{2i(fx+e)}-1\right)^2\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)}+\frac{\ln\left(e^{i(fx}{2f(a^3+3a^2}$

input
```
int(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/4/(a+b)^2/(-1+cos(f*x+e))+1/4*(a-3*b)/(a+b)^3*ln(-1+cos(f*x+e))+b/(
a+b)^3*((-1/2*a-1/2*b)*cos(f*x+e)/(b+a*cos(f*x+e)^2)+1/2*(3*a-b)/(a*b)^(1/
2)*arctan(a*cos(f*x+e)/(a*b)^(1/2)))+1/4/(a+b)^2/(1+cos(f*x+e))+1/4/(a+b)^
3*(-a+3*b)*ln(1+cos(f*x+e)))
```

3.45.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(131) = 262$.

Time $= 0.33$ (sec) , antiderivative size $= 698$, normalized size of antiderivative $= 4.75$

$$\int \frac{\csc^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$= \left[\frac{2\left(a^2-b^2\right)\cos\left(fx+e\right)^3 - \left(\left(3\,a^2-ab\right)\cos\left(fx+e\right)^4 - \left(3\,a^2-4\,ab+b^2\right)\cos\left(fx+e\right)^2 - 3\,ab+b^2\right)\sqrt{-}}{}\right.$$

input
```
integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[1/4*(2*(a^2 - b^2)*cos(f*x + e)^3 - ((3*a^2 - a*b)*cos(f*x + e)^4 - (3*a^
2 - 4*a*b + b^2)*cos(f*x + e)^2 - 3*a*b + b^2)*sqrt(-b/a)*log((a*cos(f*x +
 e)^2 - 2*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) + 4*(a*b
+ b^2)*cos(f*x + e) - ((a^2 - 3*a*b)*cos(f*x + e)^4 - (a^2 - 4*a*b + 3*b^2
)*cos(f*x + e)^2 - a*b + 3*b^2)*log(1/2*cos(f*x + e) + 1/2) + ((a^2 - 3*a*
b)*cos(f*x + e)^4 - (a^2 - 4*a*b + 3*b^2)*cos(f*x + e)^2 - a*b + 3*b^2)*lo
g(-1/2*cos(f*x + e) + 1/2))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x
 + e)^4 - (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*f*cos(f*x + e)^2 - (a^3*b + 3*a^
2*b^2 + 3*a*b^3 + b^4)*f), 1/4*(2*(a^2 - b^2)*cos(f*x + e)^3 + 2*((3*a^2 -
 a*b)*cos(f*x + e)^4 - (3*a^2 - 4*a*b + b^2)*cos(f*x + e)^2 - 3*a*b + b^2)
*sqrt(b/a)*arctan(a*sqrt(b/a)*cos(f*x + e)/b) + 4*(a*b + b^2)*cos(f*x + e)
 - ((a^2 - 3*a*b)*cos(f*x + e)^4 - (a^2 - 4*a*b + 3*b^2)*cos(f*x + e)^2 -
a*b + 3*b^2)*log(1/2*cos(f*x + e) + 1/2) + ((a^2 - 3*a*b)*cos(f*x + e)^4 -
 (a^2 - 4*a*b + 3*b^2)*cos(f*x + e)^2 - a*b + 3*b^2)*log(-1/2*cos(f*x + e)
 + 1/2))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^4 - (a^4 + 2*
a^3*b - 2*a*b^3 - b^4)*f*cos(f*x + e)^2 - (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b
^4)*f)]
```

3.45.6 Sympy [F]

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

input
```
integrate(csc(f*x+e)**3/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral(csc(e + f*x)**3/(a + b*sec(e + f*x)**2)**2, x)
```

3.45.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.57

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx =$$
$$-\frac{\frac{(a-3\,b)\log(\cos(fx+e)+1)}{a^3+3\,a^2b+3\,ab^2+b^3} - \frac{(a-3\,b)\log(\cos(fx+e)-1)}{a^3+3\,a^2b+3\,ab^2+b^3} - \frac{2\,(3\,ab-b^2)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{ab}} - \frac{2\left((a-b)\cos(fx+e)^3-\right.}{(a^3+2\,a^2b+ab^2)\cos(fx+e)^4-a^2b-2\,ab^2-}}{4\,f}$$

input
```
integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/4*((a - 3*b)*log(cos(f*x + e) + 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (a
 - 3*b)*log(cos(f*x + e) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 2*(3*a*b -
 b^2)*arctan(a*cos(f*x + e)/sqrt(a*b))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sq
rt(a*b)) - 2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))/((a^3 + 2*a^2*b +
 a*b^2)*cos(f*x + e)^4 - a^2*b - 2*a*b^2 - b^3 - (a^3 + a^2*b - a*b^2 - b^
3)*cos(f*x + e)^2))/f
```

3.45.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 550 vs. 2(131) = 262.

Time = 0.37 (sec) , antiderivative size = 550, normalized size of antiderivative = 3.74

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$
$$= \frac{\frac{6\,(a-3\,b)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a^3+3\,a^2b+3\,ab^2+b^3} - \frac{12\,(3\,ab-b^2)\arctan\left(-\frac{a\cos(fx+e)-b}{\sqrt{ab}\cos(fx+e)+\sqrt{ab}}\right)}{(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{ab}} + \frac{3\,a^2+6\,ab+3\,b^2+\frac{4\,a^2(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{20\,ab(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{24\,b}{}}{(a^3+3\,a^2b+3\,ab^2+b^3)\left(\frac{a(co}{co}}$$

input
```
integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/24*(6*(a - 3*b)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1))/(a^3 +
 3*a^2*b + 3*a*b^2 + b^3) - 12*(3*a*b - b^2)*arctan(-(a*cos(f*x + e) - b)/
(sqrt(a*b)*cos(f*x + e) + sqrt(a*b)))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqr
t(a*b)) + (3*a^2 + 6*a*b + 3*b^2 + 4*a^2*(cos(f*x + e) - 1)/(cos(f*x + e)
+ 1) - 20*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 24*b^2*(cos(f*x + e)
 - 1)/(cos(f*x + e) + 1) - a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 -
 2*a*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 15*b^2*(cos(f*x + e) -
1)^2/(cos(f*x + e) + 1)^2 - 2*a^2*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^
3 + 4*a*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 6*b^2*(cos(f*x + e)
- 1)^3/(cos(f*x + e) + 1)^3)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(a*(cos(f*x
+ e) - 1)/(cos(f*x + e) + 1) + b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 2
*a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 2*b*(cos(f*x + e) - 1)^2/(c
os(f*x + e) + 1)^2 + a*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + b*(cos(
f*x + e) - 1)^3/(cos(f*x + e) + 1)^3)) - 3*(cos(f*x + e) - 1)/((a^2 + 2*a*
b + b^2)*(cos(f*x + e) + 1)))/f
```

3.45.9 Mupad [B] (verification not implemented)

Time = 18.84 (sec) , antiderivative size = 1845, normalized size of antiderivative = 12.55

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^2),x)
```

output

```
- ((cos(e + f*x)^3*(a - b))/(2*(2*a*b + a^2 + b^2)) + (b*cos(e + f*x))/(2*
a*b + a^2 + b^2))/(f*(b - a*cos(e + f*x)^4 + cos(e + f*x)^2*(a - b))) - (l
og(cos(e + f*x) - 1)*(b/(a + b)^3 - 1/(4*(a + b)^2)))/f - (log(cos(e + f*x
) + 1)*(a - 3*b))/(4*f*(a + b)^3) - (atan((((-a*b)^(1/2)*((cos(e + f*x)*(a
*b^4 - 6*a^4*b + a^5 - 6*a^2*b^3 + 18*a^3*b^2))/(2*(4*a*b^3 + 4*a^3*b + a^
4 + b^4 + 6*a^2*b^2)) + ((-a*b)^(1/2)*((4*a^8*b + 4*a^2*b^7 + 24*a^3*b^6 +
 60*a^4*b^5 + 80*a^5*b^4 + 60*a^6*b^3 + 24*a^7*b^2)/(6*a*b^5 + 6*a^5*b + a
^6 + b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2) - (cos(e + f*x)*(-a*b)^(1
/2)*(3*a - b)*(80*a^8*b + 16*a^9 - 16*a^2*b^7 - 80*a^3*b^6 - 144*a^4*b^5 -
 80*a^5*b^4 + 80*a^6*b^3 + 144*a^7*b^2))/(8*(a*b^3 + 3*a^3*b + a^4 + 3*a^2
*b^2)*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)))*(3*a - b))/(4*(a*b^3 +
 3*a^3*b + a^4 + 3*a^2*b^2)))*(3*a - b)*1i)/(4*(a*b^3 + 3*a^3*b + a^4 + 3*
a^2*b^2)) + ((-a*b)^(1/2)*((cos(e + f*x)*(a*b^4 - 6*a^4*b + a^5 - 6*a^2*b^
3 + 18*a^3*b^2))/(2*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)) - ((-a*b)
^(1/2)*((4*a^8*b + 4*a^2*b^7 + 24*a^3*b^6 + 60*a^4*b^5 + 80*a^5*b^4 + 60*a
^6*b^3 + 24*a^7*b^2)/(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 20*a^3*
b^3 + 15*a^4*b^2) + (cos(e + f*x)*(-a*b)^(1/2)*(3*a - b)*(80*a^8*b + 16*a^
9 - 16*a^2*b^7 - 80*a^3*b^6 - 144*a^4*b^5 - 80*a^5*b^4 + 80*a^6*b^3 + 144*
a^7*b^2))/(8*(a*b^3 + 3*a^3*b + a^4 + 3*a^2*b^2)*(4*a*b^3 + 4*a^3*b + a^4
+ b^4 + 6*a^2*b^2)))*(3*a - b))/(4*(a*b^3 + 3*a^3*b + a^4 + 3*a^2*b^2))...
```

3.46 $\int \frac{\csc^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

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3.46.1 Optimal result

Integrand size = 23, antiderivative size = 197

$$\begin{aligned}\int \frac{\csc^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = {}& \frac{3\sqrt{a}(a-b)\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{2(a+b)^4 f} \\ & - \frac{3(a^2-6ab+b^2)\operatorname{arctanh}(\cos(e+fx))}{8(a+b)^4 f} \\ & + \frac{3a(a-3b)\cos(e+fx)}{8(a+b)^3 f\left(b+a\cos^2(e+fx)\right)} \\ & - \frac{(a-5b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2 f\left(b+a\cos^2(e+fx)\right)} \\ & - \frac{\cot(e+fx)\csc^3(e+fx)}{4(a+b)f\left(b+a\cos^2(e+fx)\right)}\end{aligned}$$

output

```
-3/8*(a^2-6*a*b+b^2)*arctanh(cos(f*x+e))/(a+b)^4/f+3/8*a*(a-3*b)*cos(f*x+e
)/(a+b)^3/f/(b+a*cos(f*x+e)^2)-1/8*(a-5*b)*cot(f*x+e)*csc(f*x+e)/(a+b)^2/f
/(b+a*cos(f*x+e)^2)-1/4*cot(f*x+e)*csc(f*x+e)^3/(a+b)/f/(b+a*cos(f*x+e)^2)
+3/2*(a-b)*arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*a^(1/2)*b^(1/2)/(a+b)^4/f
```

3.46.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.64 (sec) , antiderivative size = 450, normalized size of antiderivative = 2.28

$$\int \frac{\csc^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\left(96\sqrt{a}(a-b)\sqrt{b}\arctan\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cos(e)\left(\sqrt{a}-\sqrt{}\right.\right.}{\sqrt{b}}\right.$$

input
```
Integrate[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*(96*Sqrt[a]*(a - b)*Sqrt[b]*ArcTan[((-Sqrt
[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos
[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt
[b]]*(a + 2*b + a*Cos[2*(e + f*x)]) + 96*Sqrt[a]*(a - b)*Sqrt[b]*ArcTan[((
-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2]
+ Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))
/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)]) - 2*(a + b)*(11*a^2 + 43*a*b - 4*
b^2 + 4*(2*a^2 - 5*a*b + 5*b^2)*Cos[2*(e + f*x)] - 3*a*(a - 3*b)*Cos[4*(e
+ f*x)])*Cot[e + f*x]*Csc[e + f*x]^3 - 24*(a^2 - 6*a*b + b^2)*(a + 2*b + a
*Cos[2*(e + f*x)])*Log[Cos[(e + f*x)/2]] + 24*(a^2 - 6*a*b + b^2)*(a + 2*b
 + a*Cos[2*(e + f*x)])*Log[Sin[(e + f*x)/2]])*Sec[e + f*x]^4)/(256*(a + b)
^4*f*(a + b*Sec[e + f*x]^2)^2)
```

3.46.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.12, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 4621, 372, 402, 27, 402, 27, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sin(e+fx)^5\left(a+b\sec(e+fx)^2\right)^2}dx$$

$$\downarrow 4621$$

$$-\frac{\int \frac{\cos^4(e+fx)}{(1-\cos^2(e+fx))^3(a\cos^2(e+fx)+b)^2}d\cos(e+fx)}{f}$$

$$\downarrow 372$$

$$-\frac{\cos(e+fx)}{4(a+b)(1-\cos^2(e+fx))^2(a\cos^2(e+fx)+b)}-\frac{\int \frac{b-(a-4b)\cos^2(e+fx)}{\left(1-\cos^2(e+fx)\right)^2\left(a\cos^2(e+fx)+b\right)^2}d\cos(e+fx)}{4(a+b)}}{f}$$

$$\downarrow 402$$

$$-\frac{\cos(e+fx)}{4(a+b)(1-\cos^2(e+fx))^2(a\cos^2(e+fx)+b)}-\frac{\int \frac{3\left((a-b)b-a(a-5b)\cos^2(e+fx)\right)}{\left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)^2}d\cos(e+fx)}{2(a+b)}-\frac{(a-5b)\cos(e+fx)}{2(a+b)\left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)}}{4(a+b)}}{f}$$

$$\downarrow 27$$

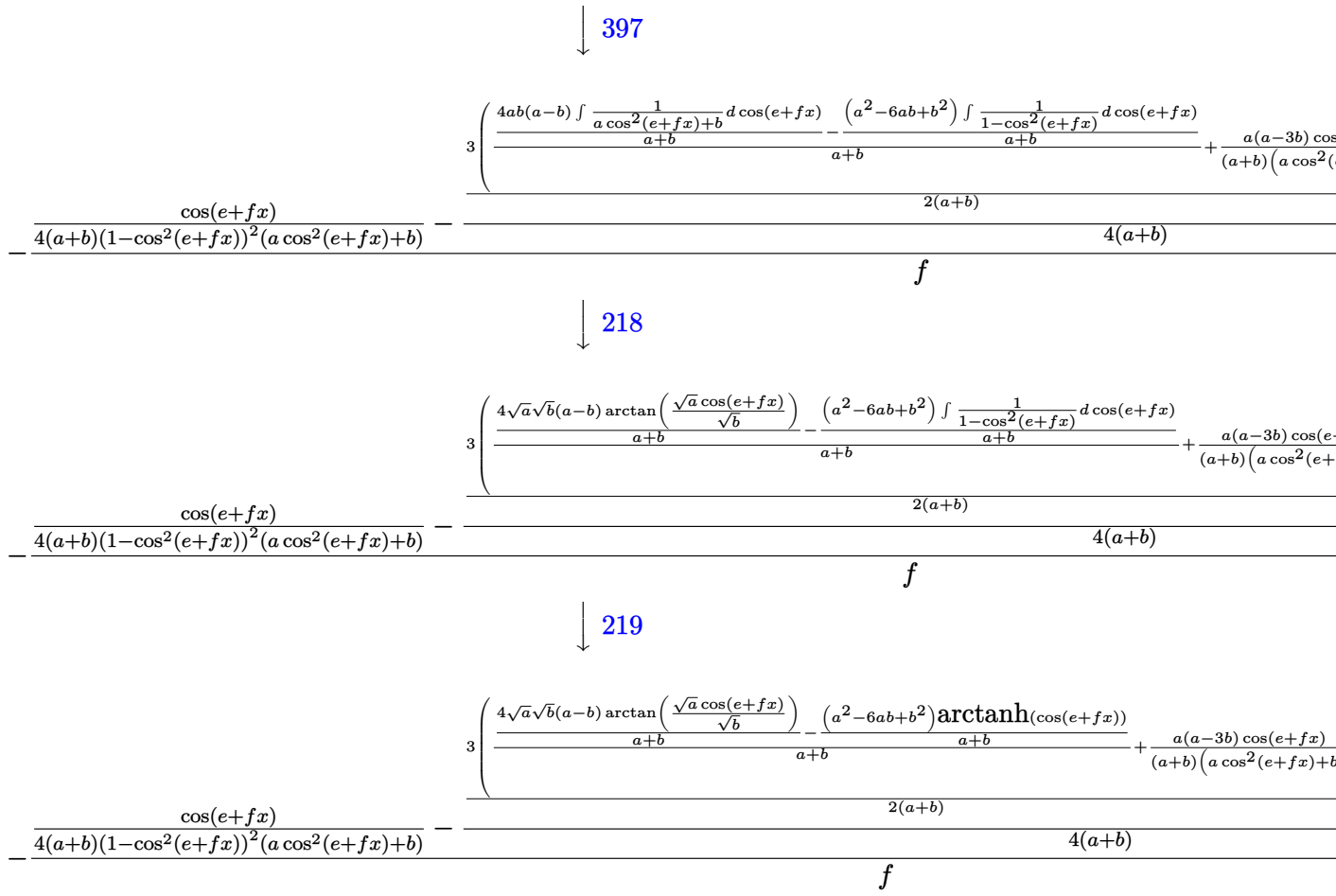
$$-\frac{\cos(e+fx)}{4(a+b)(1-\cos^2(e+fx))^2(a\cos^2(e+fx)+b)}-\frac{\frac{3\int \frac{(a-b)b-a(a-5b)\cos^2(e+fx)}{\left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)^2}d\cos(e+fx)}{2(a+b)}-\frac{(a-5b)\cos(e+fx)}{2(a+b)\left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)}}{4(a+b)}}{f}$$

$$\downarrow 402$$

$$-\frac{\cos(e+fx)}{4(a+b)(1-\cos^2(e+fx))^2(a\cos^2(e+fx)+b)}-\frac{3\left(\frac{a(a-3b)\cos(e+fx)}{(a+b)\left(a\cos^2(e+fx)+b\right)}-\frac{\int -\frac{2b\left((3a-b)b-a(a-3b)\cos^2(e+fx)\right)}{\left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)}d\cos(e+fx)}{2b(a+b)}\right)}{2(a+b)}-\frac{(a-5b)}{2(a+b)\left(1-\cos^2(e+f}}{4(a+b)}}{f}$$

$$\downarrow 27$$

$$-\frac{\cos(e+fx)}{4(a+b)(1-\cos^2(e+fx))^2(a\cos^2(e+fx)+b)}-\frac{3\left(\frac{\int \frac{(3a-b)b-a(a-3b)\cos^2(e+fx)}{\left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)}d\cos(e+fx)}{a+b}+\frac{a(a-3b)\cos(e+fx)}{(a+b)\left(a\cos^2(e+fx)+b\right)}\right)}{2(a+b)}-\frac{(a-5b)\cos}{2(a+b)\left(1-\cos^2(e+fx}}{4(a+b)}}{f}$$



input
```
Int[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
-((Cos[e + f*x]/(4*(a + b)*(1 - Cos[e + f*x]^2)^2*(b + a*Cos[e + f*x]^2))
- (-1/2*((a - 5*b)*Cos[e + f*x])/((a + b)*(1 - Cos[e + f*x]^2)*(b + a*Cos[
e + f*x]^2)) + (3*(((4*Sqrt[a]*(a - b)*Sqrt[b]*ArcTan[(Sqrt[a]*Cos[e + f*x
])/Sqrt[b]])/(a + b) - ((a^2 - 6*a*b + b^2)*ArcTanh[Cos[e + f*x]])/(a + b)
)/(a + b) + (a*(a - 3*b)*Cos[e + f*x])/((a + b)*(b + a*Cos[e + f*x]^2))))/
(2*(a + b)))/(4*(a + b)))/f)
```

3.46.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

3.46.4 Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.08

method	result
derivativedivides	$-\frac{1}{16(a+b)^2(-1+\cos(fx+e))^2}-\frac{-3a+5b}{16(a+b)^3(-1+\cos(fx+e))}+\frac{\left(3a^2-18ab+3b^2\right)\ln(-1+\cos(fx+e))}{16(a+b)^4}+\frac{ab\left(\frac{\left(-\frac{a}{2}-\frac{b}{2}\right)\cos(fx+e)}{b+a\cos(fx+e)^2}+\frac{3(a-}{(a+b)}\right.}{f}$
default	$-\frac{1}{16(a+b)^2(-1+\cos(fx+e))^2}-\frac{-3a+5b}{16(a+b)^3(-1+\cos(fx+e))}+\frac{\left(3a^2-18ab+3b^2\right)\ln(-1+\cos(fx+e))}{16(a+b)^4}+\frac{ab\left(\frac{\left(-\frac{a}{2}-\frac{b}{2}\right)\cos(fx+e)}{b+a\cos(fx+e)^2}+\frac{3(a-}{(a+b)}\right.}{f}$
risch	$-\frac{-3a^2e^{11i(fx+e)}+9abe^{11i(fx+e)}+5a^2e^{9i(fx+e)}-11abe^{9i(fx+e)}+20b^2e^{9i(fx+e)}+30a^2e^{7i(fx+e)}+66abe^{7i(fx+e)}+12b}{4f(a+b)^3\left(e^{2i(fx+e)}-1\right)^4(}$

input

```
int(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output

```
1/f*(-1/16/(a+b)^2/(-1+cos(f*x+e))^2-1/16*(-3*a+5*b)/(a+b)^3/(-1+cos(f*x+e
))+1/16/(a+b)^4*(3*a^2-18*a*b+3*b^2)*ln(-1+cos(f*x+e))+a*b/(a+b)^4*((-1/2*
a-1/2*b)*cos(f*x+e)/(b+a*cos(f*x+e)^2)+3/2*(a-b)/(a*b)^(1/2)*arctan(a*cos(
f*x+e)/(a*b)^(1/2)))+1/16/(a+b)^2/(1+cos(f*x+e))^2-1/16*(-3*a+5*b)/(a+b)^3
/(1+cos(f*x+e))+1/16/(a+b)^4*(-3*a^2+18*a*b-3*b^2)*ln(1+cos(f*x+e)))
```

3.46.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 586 vs. $2(179) = 358$.

Time = 0.37 (sec) , antiderivative size = 1202, normalized size of antiderivative = 6.10

$$\int \frac{\csc^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \text{Too large to display}$$

input

```
integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output

```
[1/16*(6*(a^3 - 2*a^2*b - 3*a*b^2)*cos(f*x + e)^5 - 2*(5*a^3 - 9*a^2*b - 9
*a*b^2 + 5*b^3)*cos(f*x + e)^3 - 12*((a^2 - a*b)*cos(f*x + e)^6 - (2*a^2 -
 3*a*b + b^2)*cos(f*x + e)^4 + (a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^2 + a*b
- b^2)*sqrt(-a*b)*log((a*cos(f*x + e)^2 - 2*sqrt(-a*b)*cos(f*x + e) - b)/(
a*cos(f*x + e)^2 + b)) - 6*(3*a^2*b + 2*a*b^2 - b^3)*cos(f*x + e) - 3*((a^
3 - 6*a^2*b + a*b^2)*cos(f*x + e)^6 - (2*a^3 - 13*a^2*b + 8*a*b^2 - b^3)*c
os(f*x + e)^4 + a^2*b - 6*a*b^2 + b^3 + (a^3 - 8*a^2*b + 13*a*b^2 - 2*b^3)
*cos(f*x + e)^2)*log(1/2*cos(f*x + e) + 1/2) + 3*((a^3 - 6*a^2*b + a*b^2)*
cos(f*x + e)^6 - (2*a^3 - 13*a^2*b + 8*a*b^2 - b^3)*cos(f*x + e)^4 + a^2*b
 - 6*a*b^2 + b^3 + (a^3 - 8*a^2*b + 13*a*b^2 - 2*b^3)*cos(f*x + e)^2)*log(
-1/2*cos(f*x + e) + 1/2))/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)
*f*cos(f*x + e)^6 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 - b
^5)*f*cos(f*x + e)^4 + (a^5 + 2*a^4*b - 2*a^3*b^2 - 8*a^2*b^3 - 7*a*b^4 -
2*b^5)*f*cos(f*x + e)^2 + (a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*
f), 1/16*(6*(a^3 - 2*a^2*b - 3*a*b^2)*cos(f*x + e)^5 - 2*(5*a^3 - 9*a^2*b
- 9*a*b^2 + 5*b^3)*cos(f*x + e)^3 + 24*((a^2 - a*b)*cos(f*x + e)^6 - (2*a^
2 - 3*a*b + b^2)*cos(f*x + e)^4 + (a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^2 + a
*b - b^2)*sqrt(a*b)*arctan(sqrt(a*b)*cos(f*x + e)/b) - 6*(3*a^2*b + 2*a*b^
2 - b^3)*cos(f*x + e) - 3*((a^3 - 6*a^2*b + a*b^2)*cos(f*x + e)^6 - (2*a^3
 - 13*a^2*b + 8*a*b^2 - b^3)*cos(f*x + e)^4 + a^2*b - 6*a*b^2 + b^3 + (...
```

3.46.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Timed out}$$

input

```
integrate(csc(f*x+e)**5/(a+b*sec(f*x+e)**2)**2,x)
```

output

```
Timed out
```

3.46.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 369 vs. 2(179) = 358.

Time = 0.27 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.87

$$\int \frac{\csc^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx =$$

$$-\frac{\frac{3\left(a^2-6ab+b^2\right)\log(\cos(fx+e)+1)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4}-\frac{3\left(a^2-6ab+b^2\right)\log(\cos(fx+e)-1)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4}-\frac{24\left(a^2b-ab^2\right)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{\left(a^4+4a^3b+6a^2b^2+4ab^3+b^4\right)\sqrt{ab}}-\frac{}{(a^4+3a^3b+3a^2b^2+ab^3}}{16\,f}$$

input
```
integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/16*(3*(a^2 - 6*a*b + b^2)*log(cos(f*x + e) + 1)/(a^4 + 4*a^3*b + 6*a^2*
b^2 + 4*a*b^3 + b^4) - 3*(a^2 - 6*a*b + b^2)*log(cos(f*x + e) - 1)/(a^4 +
4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - 24*(a^2*b - a*b^2)*arctan(a*cos(f*x
 + e)/sqrt(a*b))/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*sqrt(a*b)) -
 2*(3*(a^2 - 3*a*b)*cos(f*x + e)^5 - (5*a^2 - 14*a*b + 5*b^2)*cos(f*x + e)
^3 - 3*(3*a*b - b^2)*cos(f*x + e))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*co
s(f*x + e)^6 - (2*a^4 + 5*a^3*b + 3*a^2*b^2 - a*b^3 - b^4)*cos(f*x + e)^4
+ a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4 + (a^4 + a^3*b - 3*a^2*b^2 - 5*a*b^3 -
 2*b^4)*cos(f*x + e)^2))/f
```

3.46.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 669 vs. 2(179) = 358.

Time = 0.41 (sec) , antiderivative size = 669, normalized size of antiderivative = 3.40

$$\int \frac{\csc^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$=\frac{\frac{12\left(a^2-6ab+b^2\right)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4}-\frac{96\left(a^2b-ab^2\right)\arctan\left(-\frac{a\cos(fx+e)-b}{\sqrt{ab}\cos(fx+e)+\sqrt{ab}}\right)}{\left(a^4+4a^3b+6a^2b^2+4ab^3+b^4\right)\sqrt{ab}}-\frac{\frac{8a^2(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{8b^2(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{a^2(\cos(fx+e)}{(\cos(fx+e)+}}{a^4+4a^3b+6a^2b^2-}$$

input
```
integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/64*(12*(a^2 - 6*a*b + b^2)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) +
 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - 96*(a^2*b - a*b^2)*arct
an(-(a*cos(f*x + e) - b)/(sqrt(a*b)*cos(f*x + e) + sqrt(a*b)))/((a^4 + 4*a
^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*sqrt(a*b)) - (8*a^2*(cos(f*x + e) - 1)/(
cos(f*x + e) + 1) - 8*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - a^2*(cos
(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 2*a*b*(cos(f*x + e) - 1)^2/(cos(f*
x + e) + 1)^2 - b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/(a^4 + 4*a^
3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - (a^2 + 2*a*b + b^2 - 8*a^2*(cos(f*x + e
) - 1)/(cos(f*x + e) + 1) + 8*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) +
18*a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 108*a*b*(cos(f*x + e) -
 1)^2/(cos(f*x + e) + 1)^2 + 18*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1
)^2)*(cos(f*x + e) + 1)^2/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(co
s(f*x + e) - 1)^2) - 64*(a^2*b + a*b^2 + a^2*b*(cos(f*x + e) - 1)/(cos(f*x
 + e) + 1) - a*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1))/((a^4 + 4*a^3*b
+ 6*a^2*b^2 + 4*a*b^3 + b^4)*(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e)
 + 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2
/(cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)))/f
```

3.46.9 Mupad [B] (verification not implemented)

Time = 23.16 (sec) , antiderivative size = 4338, normalized size of antiderivative = 22.02

$$\int \frac{\csc^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^2),x)
```

output

```
(atan((((-a*b)^(1/2)*((cos(e + f*x)*(9*a^7 - 108*a^6*b + 153*a^3*b^4 - 396
*a^4*b^3 + 486*a^5*b^2))/(32*(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 +
 20*a^3*b^3 + 15*a^4*b^2)) + (3*(-a*b)^(1/2)*(a - b)*(((9*a^11*b)/2 - (3*a
^2*b^10)/2 - (15*a^3*b^9)/2 - 6*a^4*b^8 + 42*a^5*b^7 + 147*a^6*b^6 + 231*a
^7*b^5 + 210*a^8*b^4 + 114*a^9*b^3 + (69*a^10*b^2)/2)/(9*a*b^8 + 9*a^8*b +
 a^9 + b^9 + 36*a^2*b^7 + 84*a^3*b^6 + 126*a^4*b^5 + 126*a^5*b^4 + 84*a^6*
b^3 + 36*a^7*b^2) - (3*cos(e + f*x)*(-a*b)^(1/2)*(a - b)*(1792*a^10*b + 25
6*a^11 - 256*a^2*b^9 - 1792*a^3*b^8 - 5120*a^4*b^7 - 7168*a^5*b^6 - 3584*a
^6*b^5 + 3584*a^7*b^4 + 7168*a^8*b^3 + 5120*a^9*b^2))/(128*(4*a*b^3 + 4*a^
3*b + a^4 + b^4 + 6*a^2*b^2)*(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 +
 20*a^3*b^3 + 15*a^4*b^2))))/(4*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2
)))*(a - b)*3i)/(4*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)) + ((-a*b)^
(1/2)*((cos(e + f*x)*(9*a^7 - 108*a^6*b + 153*a^3*b^4 - 396*a^4*b^3 + 486*
a^5*b^2))/(32*(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 1
5*a^4*b^2)) - (3*(-a*b)^(1/2)*(a - b)*(((9*a^11*b)/2 - (3*a^2*b^10)/2 - (1
5*a^3*b^9)/2 - 6*a^4*b^8 + 42*a^5*b^7 + 147*a^6*b^6 + 231*a^7*b^5 + 210*a^
8*b^4 + 114*a^9*b^3 + (69*a^10*b^2)/2)/(9*a*b^8 + 9*a^8*b + a^9 + b^9 + 36
*a^2*b^7 + 84*a^3*b^6 + 126*a^4*b^5 + 126*a^5*b^4 + 84*a^6*b^3 + 36*a^7*b^
2) + (3*cos(e + f*x)*(-a*b)^(1/2)*(a - b)*(1792*a^10*b + 256*a^11 - 256*a^
2*b^9 - 1792*a^3*b^8 - 5120*a^4*b^7 - 7168*a^5*b^6 - 3584*a^6*b^5 + 358...
```

3.47 $\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

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3.47.1 Optimal result

Integrand size = 23, antiderivative size = 267

$$\begin{aligned}\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx &= \frac{\left(5a^3+60a^2b+120ab^2+64b^3\right)x}{16a^5} \\ &- \frac{\sqrt{b}(a+b)^{3/2}(3a+8b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^5f} \\ &- \frac{\left(33a^2+82ab+48b^2\right)\cos(e+fx)\sin(e+fx)}{48a^3f\left(a+b+b\tan^2(e+fx)\right)} \\ &+ \frac{(9a+8b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)} \\ &+ \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)} \\ &- \frac{b\left(19a^2+52ab+32b^2\right)\tan(e+fx)}{16a^4f\left(a+b+b\tan^2(e+fx)\right)}\end{aligned}$$

output

```
1/16*(5*a^3+60*a^2*b+120*a*b^2+64*b^3)*x/a^5-1/2*(a+b)^(3/2)*(3*a+8*b)*arc
tan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(1/2)/a^5/f-1/48*(33*a^2+82*a*b+48*b
^2)*cos(f*x+e)*sin(f*x+e)/a^3/f/(a+b+b*tan(f*x+e)^2)+1/24*(9*a+8*b)*cos(f*
x+e)^3*sin(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)+1/6*cos(f*x+e)^3*sin(f*x+e)^3
/a/f/(a+b+b*tan(f*x+e)^2)-1/16*b*(19*a^2+52*a*b+32*b^2)*tan(f*x+e)/a^4/f/(
a+b+b*tan(f*x+e)^2)
```

3.47.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 19.91 (sec) , antiderivative size = 2468, normalized size of antiderivative = 9.24

$$\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
-1/512*((a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[e + f*x]^4*(16*x + ((-a^3 + 6
*a^2*b + 24*a*b^2 + 16*b^3)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a
 + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*S
in[e])^4])]*(Cos[2*e] - I*Sin[2*e]))/(b*(a + b)^(3/2)*f*Sqrt[b*(Cos[e] - I
*Sin[e])^4]) + ((a^2 + 8*a*b + 8*b^2)*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))
/(b*(a + b)*f*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[e] - Sin[e])*(Cos[e] + S
in[e])))/(a^2*(a + b*Sec[e + f*x]^2)^2) + (3*(a + 2*b + a*Cos[2*e + 2*f*x
])^2*Sec[e + f*x]^4*(-64*(a + 2*b)*x + ((a^4 - 16*a^3*b - 144*a^2*b^2 - 25
6*a*b^3 - 128*b^4)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*S
in[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]
)]*(Cos[2*e] - I*Sin[2*e]))/(b*(a + b)^(3/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^
4]) + (16*a*Cos[2*f*x]*Sin[2*e])/f + (16*a*Cos[2*e]*Sin[2*f*x])/f - ((a^3
+ 18*a^2*b + 48*a*b^2 + 32*b^3)*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/(b*(a
 + b)*f*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[e] - Sin[e])*(Cos[e] + Sin[e])
)))/(4096*a^3*(a + b*Sec[e + f*x]^2)^2) + (3*(a + 2*b + a*Cos[2*e + 2*f*x]
)^2*Sec[e + f*x]^4*(((a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])
/(a + b)^(3/2) - (a*Sqrt[b]*Sin[2*(e + f*x)])/((a + b)*(a + 2*b + a*Cos[2*
(e + f*x)])))/(2048*b^(3/2)*f*(a + b*Sec[e + f*x]^2)^2) - ((a + 2*b + a*C
os[2*e + 2*f*x])^2*Sec[e + f*x]^4*(-((a*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt
[a + b]])/(a + b)^(3/2)) + (Sqrt[b]*(a + 2*b)*Sin[2*(e + f*x)])/((a + b...
```

3.47.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.12, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 4620, 372, 440, 402, 27, 402, 27, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sin(e+fx)^6}{\left(a+b\sec(e+fx)^2\right)^2}\,dx$$

$$\downarrow 4620$$

$$\frac{\int \frac{\tan^6(e+fx)}{\left(\tan^2(e+fx)+1\right)^4\left(b\tan^2(e+fx)+a+b\right)^2}\,d\tan(e+fx)}{f}$$

$$\downarrow 372$$

$$\frac{\frac{\tan^3(e+fx)}{6a\left(\tan^2(e+fx)+1\right)^3\left(a+b\tan^2(e+fx)+b\right)} - \frac{\int \frac{\tan^2(e+fx)\left((b-6(a+b))\tan^2(e+fx)+3(a+b)\right)}{\left(\tan^2(e+fx)+1\right)^3\left(b\tan^2(e+fx)+a+b\right)^2}\,d\tan(e+fx)}{6a}}{f}$$

$$\downarrow 440$$

$$\frac{\frac{\tan^3(e+fx)}{6a\left(\tan^2(e+fx)+1\right)^3\left(a+b\tan^2(e+fx)+b\right)} - \frac{\frac{\int \frac{(a+b)(9a+8b)-\left(24a^2+65ba+40b^2\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^2}\,d\tan(e+fx)}{4a} - \frac{(9a+8b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\left(a+b\tan^2(e+fx)+b\right)}}{6a}}{f}$$

$$\downarrow 402$$

$$\frac{\frac{\tan^3(e+fx)}{6a\left(\tan^2(e+fx)+1\right)^3\left(a+b\tan^2(e+fx)+b\right)} - \frac{\frac{\left(33a^2+82ab+48b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)} - \frac{\int \frac{3\left((a+b)\left(5a^2+22ba+16b^2\right)-b\left(33a^2+82ba+48b^2\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}\,d}{2a}}{4a}}{6a}}{f}$$

$$\downarrow 27$$

$$\frac{\frac{\tan^3(e+fx)}{6a\left(\tan^2(e+fx)+1\right)^3\left(a+b\tan^2(e+fx)+b\right)} - \frac{\frac{\left(33a^2+82ab+48b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)} - \frac{3\int \frac{(a+b)\left(5a^2+22ba+16b^2\right)-b\left(33a^2+82ba+48b^2\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}\,d\ta}{2a}}{4a}}{6a}}{f}$$

$$\downarrow 402$$

3.47. $\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

$$\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)} - \frac{\frac{\left(33a^2+82ab+48b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)} - \frac{3\left(\int \frac{2(a+b)\left((a+b)\left(5a^2+36ba+32b^2\right)-b\left(19a^2+52ba+32b^2\right)\tan^2(}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}}{2a(a+b)}\right)}{2a}}{4a}}{6a}$$

$$f$$

$$\downarrow 27$$

$$\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)} - \frac{\frac{\left(33a^2+82ab+48b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)} - \frac{3\left(\int \frac{(a+b)\left(5a^2+36ba+32b^2\right)-b\left(19a^2+52ba+32b^2\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\right.}{a}}{2a}}{4a}}{6a}$$

$$f$$

$$\downarrow 397$$

$$\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)} - \frac{\frac{\left(33a^2+82ab+48b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)} - \frac{3\left(\frac{\left(5a^3+60a^2b+120ab^2+64b^3\right)\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a} - \frac{8b}{a}\right.}{4a}}{6a}$$

$$f$$

$$\downarrow 216$$

$$\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)} - \frac{\frac{\left(33a^2+82ab+48b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)} - \frac{3\left(\frac{\left(5a^3+60a^2b+120ab^2+64b^3\right)\arctan(\tan(e+fx))}{a} - \frac{8b(a+b)^2(3a-}{a}\right.}{4a}}{6a}$$

$$f$$

$$\downarrow 218$$

$$\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)} - \frac{\frac{\left(33a^2+82ab+48b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)} - \frac{3\left(\frac{\left(5a^3+60a^2b+120ab^2+64b^3\right)\arctan(\tan(e+fx))}{a} - \frac{8\sqrt{b}(a+b)^{3/2}}{a}\right.}{4a}}{6a}$$

$$f$$

3.47. $\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

input
```
Int[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(Tan[e + f*x]^3/(6*a*(1 + Tan[e + f*x]^2)^3*(a + b + b*Tan[e + f*x]^2)) -
(-1/4*((9*a + 8*b)*Tan[e + f*x])/(a*(1 + Tan[e + f*x]^2)^2*(a + b + b*Tan[
e + f*x]^2)) + (((33*a^2 + 82*a*b + 48*b^2)*Tan[e + f*x])/(2*a*(1 + Tan[e
+ f*x]^2)*(a + b + b*Tan[e + f*x]^2)) - (3*((((5*a^3 + 60*a^2*b + 120*a*b^
2 + 64*b^3)*ArcTan[Tan[e + f*x]])/a - (8*Sqrt[b]*(a + b)^(3/2)*(3*a + 8*b)
*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/a)/a - (b*(19*a^2 + 52*a*b +
32*b^2)*Tan[e + f*x])/(a*(a + b + b*Tan[e + f*x]^2))))/(2*a))/(4*a))/(6*a)
)/f
```

3.47.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.47.4 Maple [A] (verified)

Time = 5.73 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.76

method	result
derivativedivides	$-\frac{(a+b)^2 b\left(\frac{a\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{(3a+8b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{a^5}+\frac{\left(-\frac{9}{4}a^2b-\frac{3}{2}ab^2-\frac{11}{16}a^3\right)\tan(fx+e)^5+\left(-4a^2b-3ab^2-\frac{5}{6}a^3\right.}{\left(1+\tan(fx+e\right.}$
default	$-\frac{(a+b)^2 b\left(\frac{a\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{(3a+8b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{a^5}+\frac{\left(-\frac{9}{4}a^2b-\frac{3}{2}ab^2-\frac{11}{16}a^3\right)\tan(fx+e)^5+\left(-4a^2b-3ab^2-\frac{5}{6}a^3\right.}{\left(1+\tan(fx+e\right.}$
risch	$\frac{5x}{16a^2}+\frac{15xb}{4a^3}+\frac{15xb^2}{2a^4}+\frac{4xb^3}{a^5}-\frac{3ie^{4i(fx+e)}}{128a^2f}-\frac{ie^{-2i(fx+e)}b}{2a^3f}-\frac{15ie^{-2i(fx+e)}}{128a^2f}+\frac{3ie^{-4i(fx+e)}}{128a^2f}+\frac{ie^{-4i(fx}}{32a^3f}$

input
```
int(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-(a+b)^2*b/a^5*(1/2*a*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(3*a+8*b)/(
(a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))+1/a^5*(((-9/4*a^2*b-3
/2*a*b^2-11/16*a^3)*tan(f*x+e)^5+(-4*a^2*b-3*a*b^2-5/6*a^3)*tan(f*x+e)^3+(
-5/16*a^3-7/4*a^2*b-3/2*a*b^2)*tan(f*x+e))/(1+tan(f*x+e)^2)^3+1/16*(5*a^3+
60*a^2*b+120*a*b^2+64*b^3)*arctan(tan(f*x+e))))
```

3.47.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 674, normalized size of antiderivative = 2.52

$$\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$= \left[\frac{3\left(5\,a^4+60\,a^3b+120\,a^2b^2+64\,ab^3\right)fx\cos\left(fx+e\right)^2+3\left(5\,a^3b+60\,a^2b^2+120\,ab^3+64\,b^4\right)fx+6\left(3\,a^2b\right.}{}\right.$$

input
```
integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[1/48*(3*(5*a^4 + 60*a^3*b + 120*a^2*b^2 + 64*a*b^3)*f*x*cos(f*x + e)^2 +
3*(5*a^3*b + 60*a^2*b^2 + 120*a*b^3 + 64*b^4)*f*x + 6*(3*a^2*b + 11*a*b^2
+ 8*b^3 + (3*a^3 + 11*a^2*b + 8*a*b^2)*cos(f*x + e)^2)*sqrt(-a*b - b^2)*lo
g(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2
 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b - b^2)*sin(f*x
+ e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) - (8*a^4*co
s(f*x + e)^7 - 2*(13*a^4 + 8*a^3*b)*cos(f*x + e)^5 + (33*a^4 + 82*a^3*b +
48*a^2*b^2)*cos(f*x + e)^3 + 3*(19*a^3*b + 52*a^2*b^2 + 32*a*b^3)*cos(f*x
+ e))*sin(f*x + e))/(a^6*f*cos(f*x + e)^2 + a^5*b*f), 1/48*(3*(5*a^4 + 60*
a^3*b + 120*a^2*b^2 + 64*a*b^3)*f*x*cos(f*x + e)^2 + 3*(5*a^3*b + 60*a^2*b
^2 + 120*a*b^3 + 64*b^4)*f*x + 12*(3*a^2*b + 11*a*b^2 + 8*b^3 + (3*a^3 + 1
1*a^2*b + 8*a*b^2)*cos(f*x + e)^2)*sqrt(a*b + b^2)*arctan(1/2*((a + 2*b)*c
os(f*x + e)^2 - b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e))) - (8*a^4*c
os(f*x + e)^7 - 2*(13*a^4 + 8*a^3*b)*cos(f*x + e)^5 + (33*a^4 + 82*a^3*b +
 48*a^2*b^2)*cos(f*x + e)^3 + 3*(19*a^3*b + 52*a^2*b^2 + 32*a*b^3)*cos(f*x
 + e))*sin(f*x + e))/(a^6*f*cos(f*x + e)^2 + a^5*b*f)]
```

3.47.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**6/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Timed out
```

3.47.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.13

$$\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx =$$

$$-\frac{3\left(19a^2b+52ab^2+32b^3\right)\tan(fx+e)^7+\left(33a^3+253a^2b+516ab^2+288b^3\right)\tan(fx+e)^5+\left(40a^3+319a^2b+564ab^2+288b^3\right)\tan(fx+e)^3+3\left(5a^3\right.}{a^4b\tan(fx+e)^8+\left(a^5+4a^4b\right)\tan(fx+e)^6+a^5+a^4b+3\left(a^5+2a^4b\right)\tan(fx+e)^4+\left(3a^5+4a^4b\right)\tan(fx+e)^2}$$

$$48$$

input
```
integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/48*((3*(19*a^2*b + 52*a*b^2 + 32*b^3)*tan(f*x + e)^7 + (33*a^3 + 253*a^
2*b + 516*a*b^2 + 288*b^3)*tan(f*x + e)^5 + (40*a^3 + 319*a^2*b + 564*a*b^
2 + 288*b^3)*tan(f*x + e)^3 + 3*(5*a^3 + 41*a^2*b + 68*a*b^2 + 32*b^3)*tan
(f*x + e))/(a^4*b*tan(f*x + e)^8 + (a^5 + 4*a^4*b)*tan(f*x + e)^6 + a^5 +
a^4*b + 3*(a^5 + 2*a^4*b)*tan(f*x + e)^4 + (3*a^5 + 4*a^4*b)*tan(f*x + e)^
2) - 3*(5*a^3 + 60*a^2*b + 120*a*b^2 + 64*b^3)*(f*x + e)/a^5 + 24*(3*a^3*b
 + 14*a^2*b^2 + 19*a*b^3 + 8*b^4)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(
sqrt((a + b)*b)*a^5))/f
```

3.47.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.10

$$\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$= \frac{\frac{3\left(5a^3+60a^2b+120ab^2+64b^3\right)(fx+e)}{a^5} - \frac{24\left(3a^3b+14a^2b^2+19ab^3+8b^4\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{\sqrt{ab+b^2}a^5} - \frac{24\left(a^2b\tan(fx+e\right.}{\left(bt\right.}}{}$$

input
```
integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/48*(3*(5*a^3 + 60*a^2*b + 120*a*b^2 + 64*b^3)*(f*x + e)/a^5 - 24*(3*a^3*
b + 14*a^2*b^2 + 19*a*b^3 + 8*b^4)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) +
arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/(sqrt(a*b + b^2)*a^5) - 24*(a^2*b*
tan(f*x + e) + 2*a*b^2*tan(f*x + e) + b^3*tan(f*x + e))/((b*tan(f*x + e)^2
 + a + b)*a^4) - (33*a^2*tan(f*x + e)^5 + 108*a*b*tan(f*x + e)^5 + 72*b^2*
tan(f*x + e)^5 + 40*a^2*tan(f*x + e)^3 + 192*a*b*tan(f*x + e)^3 + 144*b^2*
tan(f*x + e)^3 + 15*a^2*tan(f*x + e) + 84*a*b*tan(f*x + e) + 72*b^2*tan(f*
x + e))/((tan(f*x + e)^2 + 1)^3*a^4))/f
```

3.47.9 Mupad [B] (verification not implemented)

Time = 20.40 (sec) , antiderivative size = 1461, normalized size of antiderivative = 5.47

$$\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(sin(e + f*x)^6/(a + b/cos(e + f*x)^2)^2,x)
```

output

```
(atanh((75*b^3*tan(e + f*x)*(- 3*a*b^3 - a^3*b - b^4 - 3*a^2*b^2)^(1/2))/(
256*((211*a*b^4)/128 + (811*b^5)/256 + (75*a^2*b^3)/256 + (41*b^6)/(16*a)
+ (3*b^7)/(4*a^2))) + (17*b^4*tan(e + f*x)*(- 3*a*b^3 - a^3*b - b^4 - 3*a^
2*b^2)^(1/2))/(16*((811*a*b^5)/256 + (41*b^6)/16 + (211*a^2*b^4)/128 + (75
*a^3*b^3)/256 + (3*b^7)/(4*a))) + (3*b^5*tan(e + f*x)*(- 3*a*b^3 - a^3*b -
 b^4 - 3*a^2*b^2)^(1/2))/(4*((41*a*b^6)/16 + (3*b^7)/4 + (811*a^2*b^5)/256
 + (211*a^3*b^4)/128 + (75*a^4*b^3)/256)))*(-b*(a + b)^3)^(1/2)*(3*a + 8*b
))/(2*a^5*f) - (atan((((((8*a^10*b^5 + 17*a^11*b^4 + (41*a^12*b^3)/4 + (5
*a^13*b^2)/4)/a^12 - (tan(e + f*x)*(2048*a^10*b^3 + 1024*a^11*b^2)*(a*b^2*
120i + a^2*b*60i + a^3*5i + b^3*64i))/(4096*a^13))*(a*b^2*120i + a^2*b*60i
 + a^3*5i + b^3*64i))/(32*a^5) - (tan(e + f*x)*(34816*a*b^8 + 8192*b^9 + 5
9520*a^2*b^7 + 52160*a^3*b^6 + 24640*a^4*b^5 + 5976*a^5*b^4 + 601*a^6*b^3)
)/(128*a^8))*(a*b^2*120i + a^2*b*60i + a^3*5i + b^3*64i)*1i)/(32*a^5) - ((
(((8*a^10*b^5 + 17*a^11*b^4 + (41*a^12*b^3)/4 + (5*a^13*b^2)/4)/a^12 + (ta
n(e + f*x)*(2048*a^10*b^3 + 1024*a^11*b^2)*(a*b^2*120i + a^2*b*60i + a^3*5
i + b^3*64i))/(4096*a^13))*(a*b^2*120i + a^2*b*60i + a^3*5i + b^3*64i))/(3
2*a^5) + (tan(e + f*x)*(34816*a*b^8 + 8192*b^9 + 59520*a^2*b^7 + 52160*a^3
*b^6 + 24640*a^4*b^5 + 5976*a^5*b^4 + 601*a^6*b^3))/(128*a^8))*(a*b^2*120i
 + a^2*b*60i + a^3*5i + b^3*64i)*1i)/(32*a^5))/((376*a*b^10 + 64*b^11 + 93
7*a^2*b^9 + (10285*a^3*b^8)/8 + (33701*a^4*b^7)/32 + (8333*a^5*b^6)/16 ...
```

3.48 $\int \frac{\sin^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

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3.48.1 Optimal result

Integrand size = 23, antiderivative size = 191

$$\int \frac{\sin^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{3(a^2+8ab+8b^2)\,x}{8a^4} - \frac{3\sqrt{b}\sqrt{a+b}(a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^4 f} - \frac{(5a+6b)\cos(e+fx)\sin(e+fx)}{8a^2 f\left(a+b+b\tan^2(e+fx)\right)} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)} - \frac{3b(3a+4b)\tan(e+fx)}{8a^3 f\left(a+b+b\tan^2(e+fx)\right)}$$

output

```
3/8*(a^2+8*a*b+8*b^2)*x/a^4-3/2*(a+2*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1
/2))*b^(1/2)*(a+b)^(1/2)/a^4/f-1/8*(5*a+6*b)*cos(f*x+e)*sin(f*x+e)/a^2/f/(
a+b+b*tan(f*x+e)^2)+1/4*cos(f*x+e)^3*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)-3
/8*b*(3*a+4*b)*tan(f*x+e)/a^3/f/(a+b+b*tan(f*x+e)^2)
```

3.48.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.56 (sec) , antiderivative size = 1105, normalized size of antiderivative = 5.79

$$\int \frac{\sin^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx =$$

$$-\frac{(a+2b+a\cos(2e+2fx))^2\sec^4(e+fx)\left(16x+\frac{\left(-a^3+6a^2b+24ab^2+16b^3\right)\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e)}}\right.}{b(a+b)^{3/2}f\sqrt{b(\cos(e)-i\sin(e))^4}}\right.}{256a^2\left(a+b\sec^2(e+fx)\right.}$$

$$+\frac{3(a+2b+a\cos(2e+2fx))^2\sec^4(e+fx)\left(\frac{(a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{a\sqrt{b}\sin(2(e+fx))}{(a+b)(a+2b+a\cos(2(e+fx)))}\right)}{1024b^{3/2}f\left(a+b\sec^2(e+fx)\right)^2}$$

$$+\frac{(a+2b+a\cos(2e+2fx))^2\sec^4(e+fx)\left(-\frac{\left(a^5-30a^4b-480a^3b^2-1600a^2b^3-1920ab^4-768b^5\right)\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(}{2\sqrt{a+}}\right.}{\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e)}}\right.}{}$$

input
```
Integrate[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

output

```
-1/256*((a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[e + f*x]^4*(16*x + ((-a^3 + 6
*a^2*b + 24*a*b^2 + 16*b^3)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a
 + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*S
in[e])^4])]*(Cos[2*e] - I*Sin[2*e]))/(b*(a + b)^(3/2)*f*Sqrt[b*(Cos[e] - I
*Sin[e])^4]) + ((a^2 + 8*a*b + 8*b^2)*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))
/(b*(a + b)*f*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[e] - Sin[e])*(Cos[e] + S
in[e])))/(a^2*(a + b*Sec[e + f*x]^2)^2) + (3*(a + 2*b + a*Cos[2*e + 2*f*x
])^2*Sec[e + f*x]^4*(((a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]]
)/(a + b)^(3/2) - (a*Sqrt[b]*Sin[2*(e + f*x)])/((a + b)*(a + 2*b + a*Cos[2
*(e + f*x)])))/(1024*b^(3/2)*f*(a + b*Sec[e + f*x]^2)^2) + ((a + 2*b + a*
Cos[2*e + 2*f*x])^2*Sec[e + f*x]^4*(-(((a^5 - 30*a^4*b - 480*a^3*b^2 - 160
0*a^2*b^3 - 1920*a*b^4 - 768*b^5)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])
*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e]
 - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I
*Sin[e])^4])) + (Sec[2*e]*(32*b*(5*a^4 + 39*a^3*b + 106*a^2*b^2 + 120*a*b^
3 + 48*b^4)*f*x*Cos[2*e] + 16*a*b*(5*a^3 + 29*a^2*b + 48*a*b^2 + 24*b^3)*f
*x*Cos[2*f*x] + 80*a^4*b*f*x*Cos[4*e + 2*f*x] + 464*a^3*b^2*f*x*Cos[4*e +
2*f*x] + 768*a^2*b^3*f*x*Cos[4*e + 2*f*x] + 384*a*b^4*f*x*Cos[4*e + 2*f*x]
 + a^5*Sin[2*e] + 34*a^4*b*Sin[2*e] + 224*a^3*b^2*Sin[2*e] + 576*a^2*b^3*S
in[2*e] + 640*a*b^4*Sin[2*e] + 256*b^5*Sin[2*e] - a^5*Sin[2*f*x] - 62*a...
```

3.48.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.12, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 4620, 372, 402, 27, 402, 27, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sin(e+fx)^4}{\left(a+b\sec(e+fx)^2\right)^2}\,dx$$

$$\downarrow 4620$$

$$\frac{\int \frac{\tan^4(e+fx)}{(\tan^2(e+fx)+1)^3\left(b\tan^2(e+fx)+a+b\right)^2}\,d\tan(e+fx)}{f}$$

$$\downarrow 372$$

$$\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)} - \frac{\int \frac{-\left((4a+5b)\tan^2(e+fx)\right)+a+b}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^2}\,d\tan(e+fx)}{4a}}{f}$$

$$\downarrow 402$$

$$\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)} - \frac{\frac{(5a+6b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)} - \frac{\int \frac{3\left((a+b)(a+2b)-b(5a+6b)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}\,d\tan(e+fx)}{2a}}{4a}}{f}$$

$$\downarrow 27$$

$$\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)} - \frac{\frac{(5a+6b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)} - \frac{3\int \frac{(a+b)(a+2b)-b(5a+6b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}\,d\tan(e+fx)}{2a}}{4a}}{f}$$

$$\downarrow 402$$

$$\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)} - \frac{\frac{(5a+6b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)} - \frac{3\left(\frac{\int \frac{2(a+b)\left((a+b)(a+4b)-b(3a+4b)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}\,d\tan(e+fx)}{2a(a+b)} - \frac{}{a}\right.}{2a}}{4a}}{f}$$

$$\downarrow 27$$

$$\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)} - \frac{\frac{(5a+6b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)} - \frac{3\left(\frac{\int \frac{(a+b)(a+4b)-b(3a+4b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}\,d\tan(e+fx)}{a} - \frac{b(3a+}{a\left(a+b\right.}\right.}{2a}}{4a}}{f}$$

$$\downarrow 397$$

$$\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)} - \frac{\frac{(5a+6b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)} - \frac{3\left(\frac{\left(a^2+8ab+8b^2\right)\int \frac{1}{\tan^2(e+fx)+1}\,d\tan(e+fx)}{a} - \frac{4b(a+b)(a+2b)}{a}\right.}{2a}}{4a}}{f}$$

3.48. $\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

$$\downarrow 216$$

$$\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)} - \frac{\frac{(5a+6b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)} - \frac{3\left(\frac{\left(a^2+8ab+8b^2\right)\arctan(\tan(e+fx))}{a} - \frac{4b(a+b)(a+2b)\int \frac{}{b\tan^2(e+}}{a}\right.}{2a}}{4a}}{f}$$

$$\downarrow 218$$

$$\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)} - \frac{\frac{(5a+6b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)} - \frac{3\left(\frac{\left(a^2+8ab+8b^2\right)\arctan(\tan(e+fx))}{a} - \frac{4\sqrt{b}\sqrt{a+b}(a+2b)\arctan\left(}{a}\right.}{2a}}{4a}}{f}$$

input
```
Int[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(Tan[e + f*x]/(4*a*(1 + Tan[e + f*x]^2)^2*(a + b + b*Tan[e + f*x]^2)) - ((
(5*a + 6*b)*Tan[e + f*x])/(2*a*(1 + Tan[e + f*x]^2)*(a + b + b*Tan[e + f*x
]^2)) - (3*((((a^2 + 8*a*b + 8*b^2)*ArcTan[Tan[e + f*x]])/a - (4*Sqrt[b]*S
qrt[a + b]*(a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/a)/a - (b
*(3*a + 4*b)*Tan[e + f*x])/(a*(a + b + b*Tan[e + f*x]^2))))/(2*a))/(4*a))/
f
```

3.48.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.48.4 Maple [A] (verified)

Time = 3.17 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.77

method	result
derivativedivides	$-\frac{(a+b)b\left(\frac{a\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{3(a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{a^4}+\frac{\left(-ab-\frac{5}{8}a^2\right)\tan(fx+e)^3+\left(-\frac{3}{8}a^2-ab\right)\tan(fx+e)}{\left(1+\tan(fx+e)^2\right)^2}+\frac{3\left(a^2+8$ f
default	$-\frac{(a+b)b\left(\frac{a\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{3(a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{a^4}+\frac{\left(-ab-\frac{5}{8}a^2\right)\tan(fx+e)^3+\left(-\frac{3}{8}a^2-ab\right)\tan(fx+e)}{\left(1+\tan(fx+e)^2\right)^2}+\frac{3\left(a^2+8$ f
risch	$\frac{3x}{8a^2}+\frac{3xb}{a^3}+\frac{3xb^2}{a^4}-\frac{ie^{4i(fx+e)}}{64a^2f}+\frac{ie^{2i(fx+e)}}{8a^2f}+\frac{ie^{2i(fx+e)}b}{4a^3f}-\frac{ie^{-2i(fx+e)}}{8a^2f}-\frac{ie^{-2i(fx+e)}b}{4a^3f}+\frac{ie^{-4i(fx+e)}}{64a^2f}$

input
```
int(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-(a+b)*b/a^4*(1/2*a*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+3/2*(a+2*b)/((a+b
)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))+1/a^4*(((-a*b-5/8*a^2)*ta
n(f*x+e)^3+(-3/8*a^2-a*b)*tan(f*x+e))/(1+tan(f*x+e)^2)^2+3/8*(a^2+8*a*b+8*
b^2)*arctan(tan(f*x+e))))
```

3.48.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 522, normalized size of antiderivative = 2.73

$$\int \frac{\sin^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$= \left[\frac{3\left(a^3+8\,a^2b+8\,ab^2\right)fx\cos\left(fx+e\right)^2+3\left(a^2b+8\,ab^2+8\,b^3\right)fx+3\left(\left(a^2+2\,ab\right)\cos\left(fx+e\right)^2+ab+2\right.}{}\right.$$

input
```
integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[1/8*(3*(a^3 + 8*a^2*b + 8*a*b^2)*f*x*cos(f*x + e)^2 + 3*(a^2*b + 8*a*b^2
+ 8*b^3)*f*x + 3*((a^2 + 2*a*b)*cos(f*x + e)^2 + a*b + 2*b^2)*sqrt(-a*b -
b^2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x
 + e)^2 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b - b^2)*s
in(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) + (2
*a^3*cos(f*x + e)^5 - (5*a^3 + 6*a^2*b)*cos(f*x + e)^3 - 3*(3*a^2*b + 4*a*
b^2)*cos(f*x + e))*sin(f*x + e))/(a^5*f*cos(f*x + e)^2 + a^4*b*f), 1/8*(3*
(a^3 + 8*a^2*b + 8*a*b^2)*f*x*cos(f*x + e)^2 + 3*(a^2*b + 8*a*b^2 + 8*b^3)
*f*x + 6*((a^2 + 2*a*b)*cos(f*x + e)^2 + a*b + 2*b^2)*sqrt(a*b + b^2)*arct
an(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*
x + e))) + (2*a^3*cos(f*x + e)^5 - (5*a^3 + 6*a^2*b)*cos(f*x + e)^3 - 3*(3
*a^2*b + 4*a*b^2)*cos(f*x + e))*sin(f*x + e))/(a^5*f*cos(f*x + e)^2 + a^4*
b*f)]
```

3.48.6 Sympy [F]

$$\int \frac{\sin^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \int \frac{\sin^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

input
```
integrate(sin(f*x+e)**4/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral(sin(e + f*x)**4/(a + b*sec(e + f*x)**2)**2, x)
```

3.48.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.07

$$\int \frac{\sin^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx =$$

$$-\frac{\frac{3\,(3\,ab+4\,b^2)\tan(fx+e)^5+(5\,a^2+24\,ab+24\,b^2)\tan(fx+e)^3+3\,(a^2+5\,ab+4\,b^2)\tan(fx+e)}{a^3b\tan(fx+e)^6+(a^4+3\,a^3b)\tan(fx+e)^4+a^4+a^3b+(2\,a^4+3\,a^3b)\tan(fx+e)^2}-\frac{3\,(a^2+8\,ab+8\,b^2)(fx+e)}{a^4}+\frac{12\,(a^2b+3\,ab^2+2}{v}}{8\,f}$$

input
```
integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/8*((3*(3*a*b + 4*b^2)*tan(f*x + e)^5 + (5*a^2 + 24*a*b + 24*b^2)*tan(f*
x + e)^3 + 3*(a^2 + 5*a*b + 4*b^2)*tan(f*x + e))/(a^3*b*tan(f*x + e)^6 + (
a^4 + 3*a^3*b)*tan(f*x + e)^4 + a^4 + a^3*b + (2*a^4 + 3*a^3*b)*tan(f*x +
e)^2) - 3*(a^2 + 8*a*b + 8*b^2)*(f*x + e)/a^4 + 12*(a^2*b + 3*a*b^2 + 2*b^
3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^4))/f
```

3.48.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.01

$$\int \frac{\sin^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$= \frac{\frac{3\,(a^2+8\,ab+8\,b^2)(fx+e)}{a^4} - \frac{12\,(a^2b+3\,ab^2+2\,b^3)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{\sqrt{ab+b^2}\,a^4} - \frac{4\,(ab\tan(fx+e)+b^2\tan(fx+e))}{\left(b\tan(fx+e)^2+a+b\right)a^3} - \frac{5\,a\tan}{}}{8\,f}$$

input
```
integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/8*(3*(a^2 + 8*a*b + 8*b^2)*(f*x + e)/a^4 - 12*(a^2*b + 3*a*b^2 + 2*b^3)*
(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^
2)))/(sqrt(a*b + b^2)*a^4) - 4*(a*b*tan(f*x + e) + b^2*tan(f*x + e))/((b*t
an(f*x + e)^2 + a + b)*a^3) - (5*a*tan(f*x + e)^3 + 8*b*tan(f*x + e)^3 + 3
*a*tan(f*x + e) + 8*b*tan(f*x + e))/((tan(f*x + e)^2 + 1)^2*a^3))/f
```

3.48.9 Mupad [B] (verification not implemented)

Time = 19.42 (sec) , antiderivative size = 435, normalized size of antiderivative = 2.28

$$\int \frac{\sin^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$= \frac{3\,\operatorname{atanh}\left(\frac{27\,b^3\tan(e+f\,x)\,\sqrt{-b^2-a\,b}}{64\left(\frac{27\,a\,b^3}{64}+\frac{81\,b^4}{64}+\frac{27\,b^5}{32\,a}\right)}+\frac{27\,b^4\tan(e+f\,x)\,\sqrt{-b^2-a\,b}}{32\left(\frac{27\,a^2\,b^3}{64}+\frac{81\,a\,b^4}{64}+\frac{27\,b^5}{32}\right)}\right)\,(a+2\,b)\,\sqrt{-b\,(a+b)}}{2\,a^4\,f}$$
$$- \frac{\frac{3\tan(e+f\,x)^5\,(4\,b^2+3\,a\,b)}{8\,a^3}+\frac{\tan(e+f\,x)^3\,(5\,a^2+24\,a\,b+24\,b^2)}{8\,a^3}+\frac{3\tan(e+f\,x)\,(a^2+5\,a\,b+4\,b^2)}{8\,a^3}}{f\,\left(b\tan(e+f\,x)^6+(a+3\,b)\,\tan(e+f\,x)^4+(2\,a+3\,b)\,\tan(e+f\,x)^2+a+b\right)}$$
$$- \frac{\operatorname{atan}\left(\frac{27\,b^2\tan(e+f\,x)}{256\left(\frac{27\,b^2}{256}+\frac{243\,b^3}{256\,a}+\frac{27\,b^4}{16\,a^2}+\frac{27\,b^5}{32\,a^3}\right)}+\frac{243\,b^3\tan(e+f\,x)}{256\left(\frac{27\,a\,b^2}{256}+\frac{243\,b^3}{256}+\frac{27\,b^4}{16\,a}+\frac{27\,b^5}{32\,a^2}\right)}+\frac{27\,b^4\tan(e+f\,x)}{16\left(\frac{243\,a\,b^3}{256}+\frac{27\,b^4}{16}+\frac{27\,a^2\,b^2}{256}+\frac{27\,b^5}{32\,a}\right)}+\frac{2}{32\left(\frac{27\,a^3\,b^2}{256}\right.}\right.}{8\,a^4\,f}$$

input
```
int(sin(e + f*x)^4/(a + b/cos(e + f*x)^2)^2,x)
```

output
```
(3*atanh((27*b^3*tan(e + f*x)*(- a*b - b^2)^(1/2))/(64*((27*a*b^3)/64 + (8
1*b^4)/64 + (27*b^5)/(32*a))) + (27*b^4*tan(e + f*x)*(- a*b - b^2)^(1/2))/
(32*((81*a*b^4)/64 + (27*b^5)/32 + (27*a^2*b^3)/64)))*(a + 2*b)*(-b*(a + b
))^(1/2))/(2*a^4*f) - (atan((27*b^2*tan(e + f*x))/(256*((27*b^2)/256 + (24
3*b^3)/(256*a) + (27*b^4)/(16*a^2) + (27*b^5)/(32*a^3))) + (243*b^3*tan(e
+ f*x))/(256*((27*a*b^2)/256 + (243*b^3)/256 + (27*b^4)/(16*a) + (27*b^5)/
(32*a^2))) + (27*b^4*tan(e + f*x))/(16*((243*a*b^3)/256 + (27*b^4)/16 + (2
7*a^2*b^2)/256 + (27*b^5)/(32*a))) + (27*b^5*tan(e + f*x))/(32*((27*a*b^4)
/16 + (27*b^5)/32 + (243*a^2*b^3)/256 + (27*a^3*b^2)/256)))*(a*b*8i + a^2*
1i + b^2*8i)*3i)/(8*a^4*f) - ((3*tan(e + f*x)^5*(3*a*b + 4*b^2))/(8*a^3) +
 (tan(e + f*x)^3*(24*a*b + 5*a^2 + 24*b^2))/(8*a^3) + (3*tan(e + f*x)*(5*a
*b + a^2 + 4*b^2))/(8*a^3))/(f*(a + b + tan(e + f*x)^2*(2*a + 3*b) + b*tan
(e + f*x)^6 + tan(e + f*x)^4*(a + 3*b)))
```

3.49 $\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

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3.49.1 Optimal result

Integrand size = 23, antiderivative size = 130

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{(a+4b)x}{2a^3} - \frac{\sqrt{b}(3a+4b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^3\sqrt{a+b}f} - \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)} - \frac{b\tan(e+fx)}{a^2f\left(a+b+b\tan^2(e+fx)\right)}$$

output
```
1/2*(a+4*b)*x/a^3-1/2*(3*a+4*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(
1/2)/a^3/f/(a+b)^(1/2)-1/2*cos(f*x+e)*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)-
b*tan(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)
```

3.49.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 8.73 (sec) , antiderivative size = 699, normalized size of antiderivative = 5.38

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$= (a+2b+a\cos(2(e+fx)))^2\sec^4(e+fx)\left(-\frac{2\left(16x+\frac{\left(-a^3+6a^2b+24ab^2+16b^3\right)\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx)}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right.\right.\right.}{b(a+b)^{3/2}f\sqrt{b(\cos(e)-i\sin(e))^4}}$$

input
```
Integrate[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])^2*Sec[e + f*x]^4*((-2*(16*x + ((-a^3 + 6*a
^2*b + 24*a*b^2 + 16*b^3)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a +
 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin
[e])^4])]*(Cos[2*e] - I*Sin[2*e]))/(b*(a + b)^(3/2)*f*Sqrt[b*(Cos[e] - I*S
in[e])^4]) + ((a^2 + 8*a*b + 8*b^2)*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/(
b*(a + b)*f*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[e] - Sin[e])*(Cos[e] + Sin
[e])))/a^2 - (-64*(a + 2*b)*x + ((a^4 - 16*a^3*b - 144*a^2*b^2 - 256*a*b^
3 - 128*b^4)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x
]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Co
s[2*e] - I*Sin[2*e]))/(b*(a + b)^(3/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) +
(16*a*Cos[2*f*x]*Sin[2*e])/f + (16*a*Cos[2*e]*Sin[2*f*x])/f - ((a^3 + 18*a
^2*b + 48*a*b^2 + 32*b^3)*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/(b*(a + b)*
f*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[e] - Sin[e])*(Cos[e] + Sin[e])))/a^3
 + (2*(((a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(3/2
) - (a*Sqrt[b]*Sin[2*(e + f*x)])/((a + b)*(a + 2*b + a*Cos[2*(e + f*x)])))
)/(b^(3/2)*f) + (-((a*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^
(3/2)) + (Sqrt[b]*(a + 2*b)*Sin[2*(e + f*x)])/((a + b)*(a + 2*b + a*Cos[2*
(e + f*x)])))/(b^(3/2)*f)))/(256*(a + b*Sec[e + f*x]^2)^2)
```

3.49.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4620, 373, 402, 27, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sin(e+fx)^2}{\left(a+b\sec(e+fx)^2\right)^2}\,dx$$

$$\downarrow 4620$$

$$\frac{\int \frac{\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^2}\,d\tan(e+fx)}{f}$$

$$\downarrow 373$$

$$\frac{\int \frac{-3b\tan^2(e+fx)+a+b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}\,d\tan(e+fx)}{2a} - \frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)}$$

$$f$$

$$\downarrow 402$$

$$\frac{\int \frac{2(a+b)\left(-2b\tan^2(e+fx)+a+2b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}\,d\tan(e+fx)}{2a(a+b)} - \frac{2b\tan(e+fx)}{a\left(a+b\tan^2(e+fx)+b\right)} - \frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)}$$

$$f$$

$$\downarrow 27$$

$$\frac{\int \frac{-2b\tan^2(e+fx)+a+2b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}\,d\tan(e+fx)}{a} - \frac{2b\tan(e+fx)}{a\left(a+b\tan^2(e+fx)+b\right)} - \frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)}$$

$$f$$

$$\downarrow 397$$

$$\frac{\frac{(a+4b)\int \frac{1}{\tan^2(e+fx)+1}\,d\tan(e+fx)}{a} - \frac{b(3a+4b)\int \frac{1}{b\tan^2(e+fx)+a+b}\,d\tan(e+fx)}{a}}{a} - \frac{2b\tan(e+fx)}{a\left(a+b\tan^2(e+fx)+b\right)} - \frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)}$$

$$f$$

$$\downarrow 216$$

$$\frac{\frac{(a+4b)\arctan(\tan(e+fx))}{a} - \frac{b(3a+4b)\int \frac{1}{b\tan^2(e+fx)+a+b}\,d\tan(e+fx)}{a}}{a} - \frac{2b\tan(e+fx)}{a\left(a+b\tan^2(e+fx)+b\right)} - \frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)}$$

$$f$$

$$\downarrow 218$$

$$\frac{\frac{(a+4b)\arctan(\tan(e+fx))}{a} - \frac{\sqrt{b}(3a+4b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{a} - \frac{2b\tan(e+fx)}{a\left(a+b\tan^2(e+fx)+b\right)} - \frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)}$$

$$f$$

input
```
Int[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

3.49. $\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

output
```
(-1/2*Tan[e + f*x]/(a*(1 + Tan[e + f*x]^2)*(a + b + b*Tan[e + f*x]^2)) + (
(((a + 4*b)*ArcTan[Tan[e + f*x]])/a - (Sqrt[b]*(3*a + 4*b)*ArcTan[(Sqrt[b]
*Tan[e + f*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/a - (2*b*Tan[e + f*x])/(a*(a
 + b + b*Tan[e + f*x]^2)))/(2*a))/f
```

3.49.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.49.4 Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{a\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{(3a+4b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{a^3}+\frac{-\frac{a\tan(fx+e)}{2\left(1+\tan(fx+e)^2\right)}+\frac{(a+4b)\arctan(\tan(fx+e))}{2}}{a^3}}{f}$
default	$\frac{-\frac{b\left(\frac{a\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{(3a+4b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{a^3}+\frac{-\frac{a\tan(fx+e)}{2\left(1+\tan(fx+e)^2\right)}+\frac{(a+4b)\arctan(\tan(fx+e))}{2}}{a^3}}{f}$
risch	$\frac{x}{2a^2}+\frac{2xb}{a^3}+\frac{ie^{2i(fx+e)}}{8a^2f}-\frac{ie^{-2i(fx+e)}}{8a^2f}-\frac{ib\left(ae^{2i(fx+e)}+2be^{2i(fx+e)}+a\right)}{a^3f\left(ae^{4i(fx+e)}+2ae^{2i(fx+e)}+4be^{2i(fx+e)}+a\right)}-\frac{3\sqrt{-(a+b)b}\ln\left(e^{}\right.}{}$

input
```
int(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/a^3*b*(1/2*a*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(3*a+4*b)/((a+b)*
b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))+1/a^3*(-1/2*a*tan(f*x+e)/(1
+tan(f*x+e)^2)+1/2*(a+4*b)*arctan(tan(f*x+e))))
```

3.49.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 441, normalized size of antiderivative = 3.39

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$= \left[\frac{4\left(a^2+4\,ab\right)fx\cos\left(fx+e\right)^2+4\left(ab+4\,b^2\right)fx+\left(\left(3\,a^2+4\,ab\right)\cos\left(fx+e\right)^2+3\,ab+4\,b^2\right)\sqrt{-\frac{b}{a+b}}\log\left(\right.\right.$$

input
```
integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[1/8*(4*(a^2 + 4*a*b)*f*x*cos(f*x + e)^2 + 4*(a*b + 4*b^2)*f*x + ((3*a^2 +
 4*a*b)*cos(f*x + e)^2 + 3*a*b + 4*b^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b
 + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*
a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*s
in(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) - 4*
(a^2*cos(f*x + e)^3 + 2*a*b*cos(f*x + e))*sin(f*x + e))/(a^4*f*cos(f*x + e
)^2 + a^3*b*f), 1/4*(2*(a^2 + 4*a*b)*f*x*cos(f*x + e)^2 + 2*(a*b + 4*b^2)*
f*x + ((3*a^2 + 4*a*b)*cos(f*x + e)^2 + 3*a*b + 4*b^2)*sqrt(b/(a + b))*arc
tan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin
(f*x + e))) - 2*(a^2*cos(f*x + e)^3 + 2*a*b*cos(f*x + e))*sin(f*x + e))/(a
^4*f*cos(f*x + e)^2 + a^3*b*f)]
```

3.49.6 Sympy [F]

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \int \frac{\sin^2\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^2}\,dx$$

input
```
integrate(sin(f*x+e)**2/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral(sin(e + f*x)**2/(a + b*sec(e + f*x)**2)**2, x)
```

3.49.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.97

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$
$$= -\frac{\frac{2\,b\tan(fx+e)^3+(a+2\,b)\tan(fx+e)}{a^2b\tan(fx+e)^4+a^3+a^2b+(a^3+2\,a^2b)\tan(fx+e)^2} - \frac{(fx+e)(a+4\,b)}{a^3} + \frac{(3\,ab+4\,b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}a^3}}{2\,f}$$

input
```
integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/2*((2*b*tan(f*x + e)^3 + (a + 2*b)*tan(f*x + e))/(a^2*b*tan(f*x + e)^4
+ a^3 + a^2*b + (a^3 + 2*a^2*b)*tan(f*x + e)^2) - (f*x + e)*(a + 4*b)/a^3
+ (3*a*b + 4*b^2)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*
a^3))/f
```

3.49.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.15

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$
$$= \frac{\frac{(fx+e)(a+4\,b)}{a^3} - \frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(3\,ab+4\,b^2)}{\sqrt{ab+b^2}a^3} - \frac{2\,b\tan(fx+e)^3+a\tan(fx+e)+2\,b\tan(fx+e)}{\left(b\tan(fx+e)^4+a\tan(fx+e)^2+2\,b\tan(fx+e)^2+a+b\right)a^2}}{2\,f}$$

input
```
integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/2*((f*x + e)*(a + 4*b)/a^3 - (pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arct
an(b*tan(f*x + e)/sqrt(a*b + b^2)))*(3*a*b + 4*b^2)/(sqrt(a*b + b^2)*a^3)
- (2*b*tan(f*x + e)^3 + a*tan(f*x + e) + 2*b*tan(f*x + e))/((b*tan(f*x + e
)^4 + a*tan(f*x + e)^2 + 2*b*tan(f*x + e)^2 + a + b)*a^2))/f
```

3.49.9 Mupad [B] (verification not implemented)

Time = 18.50 (sec) , antiderivative size = 816, normalized size of antiderivative = 6.28

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{\frac{\tan(e+fx)\,(a+2b)}{2a^2}+\frac{b\tan(e+fx)^3}{a^2}}{f\left(b\tan(e+fx)^4+(a+2b)\tan(e+fx)^2+a+b\right)} - \frac{\operatorname{atan}\left(\frac{b^2\tan(e+fx)}{4\left(\frac{b^2}{4}+\frac{b^3}{a}\right)}+\frac{b^3\tan(e+fx)}{b^3+\frac{a\,b^2}{4}}\right)(a\,1\mathrm{i}+b\,4\mathrm{i})\,1\mathrm{i}}{2a^3f}$$

$$+\operatorname{atan}\left(\frac{\left(\frac{\tan(e+fx)\left(5a^2b^3+16ab^4+16b^5\right)}{a^4}+\frac{\left(\frac{2a^7b^2+4a^6b^3}{a^6}+\frac{\tan(e+fx)\left(8a^7b^2+16a^6b^3\right)\left(\frac{3a}{4}+b\right)\sqrt{-b(a+b)}}{a^4\left(a^4+ba^3\right)}\right)\left(\frac{3a}{4}+b\right)\sqrt{-b(a+b)}}{a^4+ba^3}\right)(\cdots}{\frac{3a^2b^3}{2}+8ab^4+8b^5}{a^6}+\frac{\left(\frac{\tan(e+fx)\left(5a^2b^3+16ab^4+16b^5\right)}{a^4}+\frac{\left(\frac{2a^7b^2+4a^6b^3}{a^6}+\frac{\tan(e+fx)\left(8a^7b^2+16a^6b^3\right)\left(\frac{3a}{4}+b\right)\sqrt{-b(a+b)}}{a^4\left(a^4+ba^3\right)}\right)\left(\frac{3a}{4}+b\right)\sqrt{\cdots}}{a^4+ba^3}\right)$$

input
```
int(sin(e + f*x)^2/(a + b/cos(e + f*x)^2)^2,x)
```

output
```
(atan(((((tan(e + f*x)*(16*a*b^4 + 16*b^5 + 5*a^2*b^3))/a^4 + (((4*a^6*b^3
 + 2*a^7*b^2)/a^6 + (tan(e + f*x)*(16*a^6*b^3 + 8*a^7*b^2)*((3*a)/4 + b)*(
-b*(a + b))^(1/2))/(a^4*(a^3*b + a^4)))*((3*a)/4 + b)*(-b*(a + b))^(1/2))/
(a^3*b + a^4))*((3*a)/4 + b)*(-b*(a + b))^(1/2)*1i)/(a^3*b + a^4) + (((tan
(e + f*x)*(16*a*b^4 + 16*b^5 + 5*a^2*b^3))/a^4 - (((4*a^6*b^3 + 2*a^7*b^2)
/a^6 - (tan(e + f*x)*(16*a^6*b^3 + 8*a^7*b^2)*((3*a)/4 + b)*(-b*(a + b))^(
1/2))/(a^4*(a^3*b + a^4)))*((3*a)/4 + b)*(-b*(a + b))^(1/2))/(a^3*b + a^4)
)*((3*a)/4 + b)*(-b*(a + b))^(1/2)*1i)/(a^3*b + a^4))/((8*a*b^4 + 8*b^5 +
(3*a^2*b^3)/2)/a^6 + (((tan(e + f*x)*(16*a*b^4 + 16*b^5 + 5*a^2*b^3))/a^4
+ (((4*a^6*b^3 + 2*a^7*b^2)/a^6 + (tan(e + f*x)*(16*a^6*b^3 + 8*a^7*b^2)*(
(3*a)/4 + b)*(-b*(a + b))^(1/2))/(a^4*(a^3*b + a^4)))*((3*a)/4 + b)*(-b*(a
 + b))^(1/2))/(a^3*b + a^4))*((3*a)/4 + b)*(-b*(a + b))^(1/2))/(a^3*b + a^
4) - (((tan(e + f*x)*(16*a*b^4 + 16*b^5 + 5*a^2*b^3))/a^4 - (((4*a^6*b^3 +
 2*a^7*b^2)/a^6 - (tan(e + f*x)*(16*a^6*b^3 + 8*a^7*b^2)*((3*a)/4 + b)*(-b
*(a + b))^(1/2))/(a^4*(a^3*b + a^4)))*((3*a)/4 + b)*(-b*(a + b))^(1/2))/(a
^3*b + a^4))*((3*a)/4 + b)*(-b*(a + b))^(1/2))/(a^3*b + a^4)))*((3*a)/4 +
b)*(-b*(a + b))^(1/2)*2i)/(f*(a^3*b + a^4)) - (atan((b^2*tan(e + f*x))/(4*
(b^2/4 + b^3/a)) + (b^3*tan(e + f*x))/((a*b^2)/4 + b^3))*(a*1i + b*4i)*1i)
/(2*a^3*f) - ((tan(e + f*x)*(a + 2*b))/(2*a^2) + (b*tan(e + f*x)^3)/a^2)/(
f*(a + b + b*tan(e + f*x)^4 + tan(e + f*x)^2*(a + 2*b)))
```

3.50 $\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

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3.50.1 Optimal result

Integrand size = 14, antiderivative size = 92

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{x}{a^2} - \frac{\sqrt{b}(3a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{3/2}f} - \frac{b\tan(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)}$$

output
```
x/a^2-1/2*(3*a+2*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(1/2)/a^2/(a+
b)^(3/2)/f-1/2*b*tan(f*x+e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

3.50.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.68 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.61

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^4(e+fx)\left(2x(a+2b+a\cos(2(e+fx))) + \frac{b(3a+2b)\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e}{2\sqrt{a+b}}\right.\right.}{8a^2\left(a+b\sec^2(e+\right.}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(-2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*(2*x*(a + 2*b + a*Cos[2*(e
+ f*x)]) + (b*(3*a + 2*b)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a +
 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin
[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[2*e] - I*Sin[2*e]))/((a + b)
^(3/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (b*((a + 2*b)*Sin[2*e] - a*Sin[2
*f*x]))/((a + b)*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))))/(8*a^2*(a + b*Se
c[e + f*x]^2)^2)
```

3.50.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.21, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4616, 316, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sec^2(e+fx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{(a+b\sec(e+fx)^2)^2}\,dx$$

$$\downarrow 4616$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^2}\,d\tan(e+fx)}{f}$$

$$\downarrow 316$$

$$\frac{\int \frac{-b\tan^2(e+fx)+2a+b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}\,d\tan(e+fx)}{2a(a+b)} - \frac{b\tan(e+fx)}{2a(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

$$\downarrow 397$$

$$\frac{\frac{2(a+b)\int \frac{1}{\tan^2(e+fx)+1}\,d\tan(e+fx)}{a} - \frac{b(3a+2b)\int \frac{1}{b\tan^2(e+fx)+a+b}\,d\tan(e+fx)}{a}}{2a(a+b)} - \frac{b\tan(e+fx)}{2a(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

$$\downarrow 216$$

$$\frac{\frac{2(a+b)\arctan(\tan(e+fx))}{a}-\frac{b(3a+2b)\int\frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{2a(a+b)}-\frac{b\tan(e+fx)}{2a(a+b)(a+b\tan^2(e+fx)+b)}$$
$$f$$

$$\downarrow 218$$

$$\frac{\frac{2(a+b)\arctan(\tan(e+fx))}{a}-\frac{\sqrt{b}(3a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{2a(a+b)}-\frac{b\tan(e+fx)}{2a(a+b)(a+b\tan^2(e+fx)+b)}$$
$$f$$

input
```
Int[(a + b*Sec[e + f*x]^2)^(-2),x]
```

output
```
(((2*(a + b)*ArcTan[Tan[e + f*x]])/a - (Sqrt[b]*(3*a + 2*b)*ArcTan[(Sqrt[b
]*Tan[e + f*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(2*a*(a + b)) - (b*Tan[e +
f*x])/(2*a*(a + b)*(a + b + b*Tan[e + f*x]^2)))/f
```

3.50.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f  Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

3.50.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{a\tan(fx+e)}{2(a+b)\left(a+b+b\tan(fx+e)^2\right)}+\frac{(3a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}\right)}{a^2}+\frac{\arctan(\tan(fx+e))}{a^2}}{f}$
default	$\frac{-\frac{b\left(\frac{a\tan(fx+e)}{2(a+b)\left(a+b+b\tan(fx+e)^2\right)}+\frac{(3a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}\right)}{a^2}+\frac{\arctan(\tan(fx+e))}{a^2}}{f}$
risch	$\frac{x}{a^2}-\frac{ib\left(a\,e^{2i(fx+e)}+2b\,e^{2i(fx+e)}+a\right)}{a^2(a+b)f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)}+\frac{3\sqrt{-(a+b)b}\,\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-(a+b)b}+a+2b}{a}\right)}{4(a+b)^2fa}+$

input
```
int(1/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-b/a^2*(1/2*a/(a+b)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(3*a+2*b)/(a+
b)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))+1/a^2*arctan(tan(
f*x+e)))
```

3.50.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(80) = 160$.

Time $= 0.31$ (sec) , antiderivative size $= 435$, normalized size of antiderivative $= 4.73$

$$\int \frac{1}{\left(a + b\sec^2(e+fx)\right)^2}\,dx$$

$$= \left[\frac{8\left(a^2+ab\right)fx\cos\left(fx+e\right)^2 - 4\,ab\cos\left(fx+e\right)\sin\left(fx+e\right) + 8\left(ab+b^2\right)fx + \left(\left(3\,a^2+2\,ab\right)\cos\left(fx+e\right)\right.}{8\left(\left(a^4+a^{3}\right.\right.}\right.$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[1/8*(8*(a^2 + a*b)*f*x*cos(f*x + e)^2 - 4*a*b*cos(f*x + e)*sin(f*x + e) +
 8*(a*b + b^2)*f*x + ((3*a^2 + 2*a*b)*cos(f*x + e)^2 + 3*a*b + 2*b^2)*sqrt
(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)
*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*co
s(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a
*b*cos(f*x + e)^2 + b^2)))/((a^4 + a^3*b)*f*cos(f*x + e)^2 + (a^3*b + a^2*
b^2)*f), 1/4*(4*(a^2 + a*b)*f*x*cos(f*x + e)^2 - 2*a*b*cos(f*x + e)*sin(f*
x + e) + 4*(a*b + b^2)*f*x + ((3*a^2 + 2*a*b)*cos(f*x + e)^2 + 3*a*b + 2*b
^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a +
b))/(b*cos(f*x + e)*sin(f*x + e))))/((a^4 + a^3*b)*f*cos(f*x + e)^2 + (a^3
*b + a^2*b^2)*f)]
```

3.50.6 Sympy [F]

$$\int \frac{1}{\left(a + b\sec^2(e+fx)\right)^2}\,dx = \int \frac{1}{\left(a + b\sec^2\left(e+fx\right)\right)^2}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(-2), x)
```

3.50.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.15

$$\int \frac{1}{(a+b\sec^2(e+fx))^2}\,dx$$
$$= -\frac{\frac{b\tan(fx+e)}{a^3+2a^2b+ab^2+(a^2b+ab^2)\tan(fx+e)^2} + \frac{(3ab+2b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^3+a^2b)\sqrt{(a+b)b}} - \frac{2(fx+e)}{a^2}}{2f}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/2*(b*tan(f*x + e)/(a^3 + 2*a^2*b + a*b^2 + (a^2*b + a*b^2)*tan(f*x + e)
^2) + (3*a*b + 2*b^2)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^3 + a^2*b
)*sqrt((a + b)*b)) - 2*(f*x + e)/a^2)/f
```

3.50.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.24

$$\int \frac{1}{(a+b\sec^2(e+fx))^2}\,dx$$
$$= -\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(3ab+2b^2)}{(a^3+a^2b)\sqrt{ab+b^2}} + \frac{b\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)(a^2+ab)} - \frac{2(fx+e)}{a^2}}{2f}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-1/2*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*
b + b^2)))*(3*a*b + 2*b^2)/((a^3 + a^2*b)*sqrt(a*b + b^2)) + b*tan(f*x + e
)/((b*tan(f*x + e)^2 + a + b)*(a^2 + a*b)) - 2*(f*x + e)/a^2)/f
```

3.50.9 Mupad [B] (verification not implemented)

Time = 20.41 (sec) , antiderivative size = 2056, normalized size of antiderivative = 22.35

$$\int \frac{1}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(1/(a + b/cos(e + f*x)^2)^2,x)
```

output
```
atan((((((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)*1i)/(2*(2*a^4*b + a^5 + a^3*b
^2)) - (tan(e + f*x)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/
(8*a^2*(2*a^3*b + a^4 + a^2*b^2)))/(2*a^2) + (tan(e + f*x)*(20*a*b^4 + 8*b
^5 + 13*a^2*b^3))/(4*(2*a^3*b + a^4 + a^2*b^2)))/a^2 - ((((2*a^4*b^4 + 6*a
^5*b^3 + 4*a^6*b^2)*1i)/(2*(2*a^4*b + a^5 + a^3*b^2)) + (tan(e + f*x)*(32*
a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*a^2*(2*a^3*b + a^4 + a
^2*b^2)))/(2*a^2) - (tan(e + f*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3))/(4*(2*a
^3*b + a^4 + a^2*b^2)))/a^2)/((((((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)*1i)/
(2*(2*a^4*b + a^5 + a^3*b^2)) - (tan(e + f*x)*(32*a^4*b^5 + 80*a^5*b^4 + 6
4*a^6*b^3 + 16*a^7*b^2))/(8*a^2*(2*a^3*b + a^4 + a^2*b^2)))*1i)/(2*a^2) +
(tan(e + f*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3)*1i)/(4*(2*a^3*b + a^4 + a^2*
b^2)))/a^2 + (((((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)*1i)/(2*(2*a^4*b + a^5
 + a^3*b^2)) + (tan(e + f*x)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^
7*b^2))/(8*a^2*(2*a^3*b + a^4 + a^2*b^2)))*1i)/(2*a^2) - (tan(e + f*x)*(20
*a*b^4 + 8*b^5 + 13*a^2*b^3)*1i)/(4*(2*a^3*b + a^4 + a^2*b^2)))/a^2 + ((3*
a*b^3)/2 + b^4)/(2*a^4*b + a^5 + a^3*b^2)))/(a^2*f) + (atan((((tan(e + f*
x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3))/(2*(2*a^3*b + a^4 + a^2*b^2)) - ((-b*(
a + b)^3)^(1/2)*((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)/(2*a^4*b + a^5 + a^3*
b^2) - (tan(e + f*x)*(-b*(a + b)^3)^(1/2)*(3*a + 2*b)*(32*a^4*b^5 + 80*a^5
*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*(2*a^3*b + a^4 + a^2*b^2)*(3*a^4*b ...
```

3.51 $\int \frac{\csc^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

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3.51.1 Optimal result

Integrand size = 23, antiderivative size = 91

$$\int \frac{\csc^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{3\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2(a+b)^{5/2}f} - \frac{3\cot(e+fx)}{2(a+b)^2 f} + \frac{\cot(e+fx)}{2(a+b)f\left(a+b+b\tan^2(e+fx)\right)}$$

output
```
-3/2*cot(f*x+e)/(a+b)^2/f-3/2*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(1/
2)/(a+b)^(5/2)/f+1/2*cot(f*x+e)/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

3.51.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.54 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.66

$$\int \frac{\csc^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^4(e+fx)\left(\frac{3b\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(a+2b+a\cos(2(e-$$

$$8(a+b)^2 f\,(a+$$

input
```
Integrate[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*((3*b*ArcTan[(Sec[f*x]*(Cos
[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a
+ b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[2
*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + 2*(a + 2*
b + a*Cos[2*(e + f*x)])*Csc[e]*Csc[e + f*x]*Sin[f*x] + (b*((a + 2*b)*Sin[2
*e] - a*Sin[2*f*x]))/(a*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))))/(8*(a + b)^
2*f*(a + b*Sec[e + f*x]^2)^2)
```

3.51.3 Rubi [A] (verified)

Time $= 0.26$ (sec) , antiderivative size $= 92$, normalized size of antiderivative $= 1.01$, number of steps used $= 6$, number of rules used $= 5$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used $= \{3042, 4620, 253, 264, 218\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sin(e+fx)^2\,(a+b\sec(e+fx)^2)^2}\,dx$$

$$\downarrow 4620$$

$$\frac{\int \frac{\cot^2(e+fx)}{(b\tan^2(e+fx)+a+b)^2}\,d\tan(e+fx)}{f}$$

$$\downarrow 253$$

$$\frac{\frac{3\int \frac{\cot^2(e+fx)}{b\tan^2(e+fx)+a+b}\,d\tan(e+fx)}{2(a+b)} + \frac{\cot(e+fx)}{2(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

$$\downarrow 264$$

$$\frac{3\left(-\frac{b\int \frac{1}{b\tan^2(e+fx)+a+b}\,d\tan(e+fx)}{a+b} - \frac{\cot(e+fx)}{a+b}\right)}{2(a+b)} + \frac{\cot(e+fx)}{2(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

$$\downarrow 218$$

$$\frac{3\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{\cot(e+fx)}{a+b}\right)}{2(a+b)}+\frac{\cot(e+fx)}{2(a+b)(a+b\tan^2(e+fx)+b)}$$
$$f$$

input
```
Int[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((3*(-((Sqrt[b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(3/2))
 - Cot[e + f*x]/(a + b)))/(2*(a + b)) + Cot[e + f*x]/(2*(a + b)*(a + b + b
*Tan[e + f*x]^2)))/f
```

3.51.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.51.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{(a+b)^2}-\frac{1}{(a+b)^2\tan(fx+e)}}{f}$
default	$\frac{-\frac{b\left(\frac{\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{(a+b)^2}-\frac{1}{(a+b)^2\tan(fx+e)}}{f}$
risch	$-\frac{i\left(2a^2e^{4i(fx+e)}+ab\,e^{4i(fx+e)}+2b^2e^{4i(fx+e)}+4a^2e^{2i(fx+e)}+8ab\,e^{2i(fx+e)}-2b^2e^{2i(fx+e)}+2a^2-ab\right)}{a(a+b)^2f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)\left(e^{2i(fx+e)}-1\right)}-\frac{3\sqrt{-(a+b)b}}{}$

input
```
int(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-b/(a+b)^2*(1/2*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+3/2/((a+b)*b)^(1/2)*a
rctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))-1/(a+b)^2/tan(f*x+e))
```

3.51.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(77) = 154.

Time = 0.31 (sec) , antiderivative size = 407, normalized size of antiderivative = 4.47

$$\int \frac{\csc^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$= \left[-\frac{4\left(2\,a-b\right)\cos\left(fx+e\right)^3-3\left(a\cos\left(fx+e\right)^2+b\right)\sqrt{-\frac{b}{a+b}}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\left(3\,ab+4\,b^2\right)\cos(fx+e)}{a^2\,\mathrm{co}}\right.}{8\left(\left(a^3+2\,a^2b+ab^2\right)f\cos\left(fx+e\right)^2+(a\right.}\right.$$

$$\left. -\frac{2\left(2\,a-b\right)\cos\left(fx+e\right)^3-3\left(a\cos\left(fx+e\right)^2+b\right)\sqrt{\frac{b}{a+b}}\arctan\left(\frac{\left((a+2\,b)\cos(fx+e)^2-b\right)\sqrt{\frac{b}{a+b}}}{2\,b\cos(fx+e)\sin(fx+e)}\right)\sin\left(fx+e\right)}{4\left(\left(a^3+2\,a^2b+ab^2\right)f\cos\left(fx+e\right)^2+\left(a^2b+2\,ab^2+b^3\right)f\right)\sin\left(fx+e\right)}\right]$$

input
```
integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[-1/8*(4*(2*a - b)*cos(f*x + e)^3 - 3*(a*cos(f*x + e)^2 + b)*sqrt(-b/(a +
b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x
+ e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e
))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*
x + e)^2 + b^2))*sin(f*x + e) + 12*b*cos(f*x + e))/(((a^3 + 2*a^2*b + a*b^
2)*f*cos(f*x + e)^2 + (a^2*b + 2*a*b^2 + b^3)*f)*sin(f*x + e)), -1/4*(2*(2
*a - b)*cos(f*x + e)^3 - 3*(a*cos(f*x + e)^2 + b)*sqrt(b/(a + b))*arctan(1
/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x
+ e)))*sin(f*x + e) + 6*b*cos(f*x + e))/(((a^3 + 2*a^2*b + a*b^2)*f*cos(f*
x + e)^2 + (a^2*b + 2*a*b^2 + b^3)*f)*sin(f*x + e))]
```

3.51.6 Sympy [F]

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

input
```
integrate(csc(f*x+e)**2/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral(csc(e + f*x)**2/(a + b*sec(e + f*x)**2)**2, x)
```

3.51.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.29

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\frac{3\,b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^2+2\,ab+b^2)\sqrt{(a+b)b}} + \frac{3\,b\tan(fx+e)^2+2\,a+2\,b}{(a^2b+2\,ab^2+b^3)\tan(fx+e)^3+(a^3+3\,a^2b+3\,ab^2+b^3)\tan(fx+e)}}{2\,f}$$

input
```
integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/2*(3*b*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^2 + 2*a*b + b^2)*sqrt
((a + b)*b)) + (3*b*tan(f*x + e)^2 + 2*a + 2*b)/((a^2*b + 2*a*b^2 + b^3)*t
an(f*x + e)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*tan(f*x + e)))/f
```

3.51.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.40

$$\int \frac{\csc^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{\frac{3\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b}{(a^2+2ab+b^2)\sqrt{ab+b^2}}+\frac{3b\tan(fx+e)^2+2a+2b}{\left(b\tan(fx+e)^3+a\tan(fx+e)+b\tan(fx+e)\right)(a^2+2ab+b^2)}}{2f}$$

input
```
integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-1/2*(3*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(
a*b + b^2)))*b/((a^2 + 2*a*b + b^2)*sqrt(a*b + b^2)) + (3*b*tan(f*x + e)^2
 + 2*a + 2*b)/((b*tan(f*x + e)^3 + a*tan(f*x + e) + b*tan(f*x + e))*(a^2 +
 2*a*b + b^2)))/f
```

3.51.9 Mupad [B] (verification not implemented)

Time = 18.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00

$$\int \frac{\csc^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{\frac{1}{a+b}+\frac{3b\tan(e+fx)^2}{2(a+b)^2}}{f\left(b\tan(e+fx)^3+(a+b)\tan(e+fx)\right)} - \frac{3\sqrt{b}\operatorname{atan}\left(\frac{\sqrt{b}\tan(e+fx)\,(a^2+2ab+b^2)}{(a+b)^{5/2}}\right)}{2f(a+b)^{5/2}}$$

input
```
int(1/(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^2),x)
```

output
```
- (1/(a + b) + (3*b*tan(e + f*x)^2)/(2*(a + b)^2))/(f*(b*tan(e + f*x)^3 +
tan(e + f*x)*(a + b))) - (3*b^(1/2)*atan((b^(1/2)*tan(e + f*x)*(2*a*b + a^
2 + b^2))/(a + b)^(5/2)))/(2*f*(a + b)^(5/2))
```

3.52 $\int \frac{\csc^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

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3.52.1 Optimal result

Integrand size = 23, antiderivative size = 123

$$\int \frac{\csc^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{(3a-2b)\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2(a+b)^{7/2}f} - \frac{(a-b)\cot(e+fx)}{(a+b)^3 f} - \frac{\cot^3(e+fx)}{3(a+b)^2 f} - \frac{ab\tan(e+fx)}{2(a+b)^3 f\left(a+b+b\tan^2(e+fx)\right)}$$

output

```
-(a-b)*cot(f*x+e)/(a+b)^3/f-1/3*cot(f*x+e)^3/(a+b)^2/f-1/2*(3*a-2*b)*arcta
n(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(1/2)/(a+b)^(7/2)/f-1/2*a*b*tan(f*x+e)
/(a+b)^3/f/(a+b+b*tan(f*x+e)^2)
```

3.52.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.80 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.46

$$\int \frac{\csc^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^4(e+fx)\left(-2(a+b)(a+2b+a\cos(2(e+fx)))\cot(e)\csc^2(e+fx)+\right.}{}$$

input
```
Integrate[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*(-2*(a + b)*(a + 2*b + a*Co
s[2*(e + f*x)])*Cot[e]*Csc[e + f*x]^2 + (3*(3*a - 2*b)*b*ArcTan[(Sec[f*x]*
(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqr
t[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])*(C
os[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + 4*(a
- 2*b)*(a + 2*b + a*Cos[2*(e + f*x)])*Csc[e]*Csc[e + f*x]*Sin[f*x] + 2*(a
+ b)*(a + 2*b + a*Cos[2*(e + f*x)])*Csc[e]*Csc[e + f*x]^3*Sin[f*x] - 3*a*b
*Sec[2*e]*Sin[2*f*x] + 3*b*(a + 2*b)*Tan[2*e]))/(24*(a + b)^3*f*(a + b*Sec
[e + f*x]^2)^2)
```

3.52.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4620, 361, 25, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sin(e+fx)^4\left(a+b\sec(e+fx)^2\right)^2}dx$$

$$\downarrow 4620$$

$$\frac{\int \frac{\cot^4(e+fx)(\tan^2(e+fx)+1)}{(b\tan^2(e+fx)+a+b)^2}d\tan(e+fx)}{f}$$

$$\downarrow 361$$

$$\frac{-\frac{1}{2}b\int -\frac{\cot^4(e+fx)\left(-\frac{a\tan^4(e+fx)}{(a+b)^3}+\frac{2a\tan^2(e+fx)}{b(a+b)^2}+\frac{2}{b(a+b)}\right)}{b\tan^2(e+fx)+a+b}d\tan(e+fx)-\frac{ab\tan(e+fx)}{2(a+b)^3(a+b\tan^2(e+fx)+b)}}{f}$$

$$\downarrow 25$$

$$\frac{\frac{1}{2}b\int \frac{\cot^4(e+fx)\left(-\frac{a\tan^4(e+fx)}{(a+b)^3}+\frac{2a\tan^2(e+fx)}{b(a+b)^2}+\frac{2}{b(a+b)}\right)}{b\tan^2(e+fx)+a+b}d\tan(e+fx)-\frac{ab\tan(e+fx)}{2(a+b)^3(a+b\tan^2(e+fx)+b)}}{f}$$

$$\downarrow 1584$$

$$\frac{\frac{1}{2}b\int\left(\frac{2\cot^4(e+fx)}{b(a+b)^2}+\frac{2(a-b)\cot^2(e+fx)}{b(a+b)^3}+\frac{2b-3a}{(a+b)^3(b\tan^2(e+fx)+a+b)}\right)d\tan(e+fx)-\frac{ab\tan(e+fx)}{2(a+b)^3(a+b\tan^2(e+fx)+b)}}{f}$$

$$\downarrow 2009$$

$$\frac{\frac{1}{2}b\left(-\frac{(3a-2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{b}(a+b)^{7/2}}-\frac{2\cot^3(e+fx)}{3b(a+b)^2}-\frac{2(a-b)\cot(e+fx)}{b(a+b)^3}\right)-\frac{ab\tan(e+fx)}{2(a+b)^3(a+b\tan^2(e+fx)+b)}}{f}$$

input
```
Int[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((b*(-(((3*a - 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(Sqrt[b]*(
a + b)^(7/2))) - (2*(a - b)*Cot[e + f*x])/(b*(a + b)^3) - (2*Cot[e + f*x]^
3)/(3*b*(a + b)^2)))/2 - (a*b*Tan[e + f*x])/(2*(a + b)^3*(a + b + b*Tan[e
+ f*x]^2)))/f
```

3.52.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.52.4 Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{a\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{(3a-2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{(a+b)^3}-\frac{1}{3(a+b)^2\tan(fx+e)^3}-\frac{a-b}{(a+b)^3\tan(fx+e)}}{f}$
default	$\frac{-\frac{b\left(\frac{a\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{(3a-2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{(a+b)^3}-\frac{1}{3(a+b)^2\tan(fx+e)^3}-\frac{a-b}{(a+b)^3\tan(fx+e)}}{f}$
risch	$\frac{i\left(9ab\,e^{8i(fx+e)}-6b^2e^{8i(fx+e)}+12a^2e^{6i(fx+e)}+18ab\,e^{6i(fx+e)}+66b^2e^{6i(fx+e)}+20a^2e^{4i(fx+e)}+44ab\,e^{4i(fx+e)}-66b^2e^{4i}\right.}{3f(a+b)^3\left(e^{2i(fx+e)}-1\right)^3\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(f}\right.}$

input
```
int(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/(a+b)^3*b*(1/2*a*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(3*a-2*b)/((a
+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))-1/3/(a+b)^2/tan(f*x+e)^
3-(a-b)/(a+b)^3/tan(f*x+e))
```

3.52.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(109) = 218$.

Time $= 0.31$ (sec) , antiderivative size $= 663$, normalized size of antiderivative $= 5.39$

$$\int \frac{\csc^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$= \left[-\frac{4\left(4\,a^2-11\,ab\right)\cos\left(fx+e\right)^5-8\left(3\,a^2-8\,ab+4\,b^2\right)\cos\left(fx+e\right)^3+3\left(\left(3\,a^2-2\,ab\right)\cos\left(fx+e\right)^4-(3\right.}{24\left(\left(a^4+3\,a^3b+\right.\right.} \right.$$

$$\left. -\frac{2\left(4\,a^2-11\,ab\right)\cos\left(fx+e\right)^5-4\left(3\,a^2-8\,ab+4\,b^2\right)\cos\left(fx+e\right)^3-3\left(\left(3\,a^2-2\,ab\right)\cos\left(fx+e\right)^4-(3\right.}{12\left(\left(a^4+3\,a^3b+3\,a^2b^2+ab^3\right)f\cos\left(fx+e\right)^4-(a^4+\right.} \right]$$

input
```
integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[-1/24*(4*(4*a^2 - 11*a*b)*cos(f*x + e)^5 - 8*(3*a^2 - 8*a*b + 4*b^2)*cos(
f*x + e)^3 + 3*((3*a^2 - 2*a*b)*cos(f*x + e)^4 - (3*a^2 - 5*a*b + 2*b^2)*c
os(f*x + e)^2 - 3*a*b + 2*b^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)
*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*b + 2*b
^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x +
e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e)
- 12*(3*a*b - 2*b^2)*cos(f*x + e))/(((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f
*cos(f*x + e)^4 - (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*f*cos(f*x + e)^2 - (a^3*
b + 3*a^2*b^2 + 3*a*b^3 + b^4)*f)*sin(f*x + e)), -1/12*(2*(4*a^2 - 11*a*b)
*cos(f*x + e)^5 - 4*(3*a^2 - 8*a*b + 4*b^2)*cos(f*x + e)^3 - 3*((3*a^2 - 2
*a*b)*cos(f*x + e)^4 - (3*a^2 - 5*a*b + 2*b^2)*cos(f*x + e)^2 - 3*a*b + 2*
b^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a +
 b))/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) - 6*(3*a*b - 2*b^2)*cos(f
*x + e))/(((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^4 - (a^4 + 2
*a^3*b - 2*a*b^3 - b^4)*f*cos(f*x + e)^2 - (a^3*b + 3*a^2*b^2 + 3*a*b^3 +
b^4)*f)*sin(f*x + e))]
```

3.52.6 Sympy [F]

$$\int \frac{\csc^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \int \frac{\csc^4\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^2}\,dx$$

input
```
integrate(csc(f*x+e)**4/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral(csc(e + f*x)**4/(a + b*sec(e + f*x)**2)**2, x)
```

3.52.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.57

$$\int \frac{\csc^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$
$$= -\frac{\frac{3\left(3\,ab-2\,b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\sqrt{(a+b)b}} + \frac{3\left(3\,ab-2\,b^2\right)\tan(fx+e)^4+2\left(3\,a^2+ab-2\,b^2\right)\tan(fx+e)^2+2\,a^2+4\,ab+2\,b^2}{\left(a^3b+3\,a^2b^2+3\,ab^3+b^4\right)\tan(fx+e)^5+\left(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4\right)\tan(fx+e)^3}}{6\,f}$$

input
```
integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/6*(3*(3*a*b - 2*b^2)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*sqrt((a + b)*b)) + (3*(3*a*b - 2*b^2)*tan(f*x + e)^4
 + 2*(3*a^2 + a*b - 2*b^2)*tan(f*x + e)^2 + 2*a^2 + 4*a*b + 2*b^2)/((a^3*b
 + 3*a^2*b^2 + 3*a*b^3 + b^4)*tan(f*x + e)^5 + (a^4 + 4*a^3*b + 6*a^2*b^2
+ 4*a*b^3 + b^4)*tan(f*x + e)^3))/f
```

3.52.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.50

$$\int \frac{\csc^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx =$$
$$-\frac{\frac{3\,ab\tan(fx+e)}{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\left(b\tan(fx+e)^2+a+b\right)} + \frac{3\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)\left(3\,ab-2\,b^2\right)}{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\sqrt{ab+b^2}} + \frac{2\left(3\,a\tan(fx+e)^2-3\,b\tan(f\right.}{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\tan}}{6\,f}$$

input
```
integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-1/6*(3*a*b*tan(f*x + e)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(b*tan(f*x + e)^
2 + a + b)) + 3*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x +
e)/sqrt(a*b + b^2)))*(3*a*b - 2*b^2)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt
(a*b + b^2)) + 2*(3*a*tan(f*x + e)^2 - 3*b*tan(f*x + e)^2 + a + b)/((a^3 +
 3*a^2*b + 3*a*b^2 + b^3)*tan(f*x + e)^3))/f
```

3.52.9 Mupad [B] (verification not implemented)

Time = 19.58 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.15

$$\int \frac{\csc^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{\frac{1}{3\,(a+b)}+\frac{\tan(e+f\,x)^2\,(3\,a-2\,b)}{3\,(a+b)^2}+\frac{b\tan(e+f\,x)^4\,(3\,a-2\,b)}{2\,(a+b)^3}}{f\,\left(b\tan\left(e+f\,x\right)^5+(a+b)\,\tan\left(e+f\,x\right)^3\right)} - \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\tan(e+f\,x)\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}{(a+b)^{7/2}}\right)\,(3\,a-2\,b)}{2\,f\,(a+b)^{7/2}}$$

input
```
int(1/(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^2),x)
```

output
```
- (1/(3*(a + b)) + (tan(e + f*x)^2*(3*a - 2*b))/(3*(a + b)^2) + (b*tan(e +
 f*x)^4*(3*a - 2*b))/(2*(a + b)^3))/(f*(tan(e + f*x)^3*(a + b) + b*tan(e +
 f*x)^5)) - (b^(1/2)*atan((b^(1/2)*tan(e + f*x)*(3*a*b^2 + 3*a^2*b + a^3 +
 b^3))/(a + b)^(7/2))*(3*a - 2*b))/(2*f*(a + b)^(7/2))
```

3.53 $\int \frac{\csc^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

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3.53.1 Optimal result

Integrand size = 23, antiderivative size = 188

$$\begin{aligned}\int \frac{\csc^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = &-\frac{a(3a-4b)\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2(a+b)^{9/2}f}\\ &-\frac{(5a^2-10ab-b^2)\cot(e+fx)}{5(a+b)^4 f}-\frac{(10a+3b)\cot^3(e+fx)}{15(a+b)^3 f}\\ &-\frac{\cot^5(e+fx)}{5(a+b)f\left(a+b+b\tan^2(e+fx)\right)}\\ &-\frac{b(5a^2+2b^2)\tan(e+fx)}{10(a+b)^4 f\left(a+b+b\tan^2(e+fx)\right)}\end{aligned}$$

output

```
-1/5*(5*a^2-10*a*b-b^2)*cot(f*x+e)/(a+b)^4/f-1/15*(10*a+3*b)*cot(f*x+e)^3/
(a+b)^3/f-1/2*a*(3*a-4*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(1/2)/(
a+b)^(9/2)/f-1/5*cot(f*x+e)^5/(a+b)/f/(a+b+b*tan(f*x+e)^2)-1/10*b*(5*a^2+2
*b^2)*tan(f*x+e)/(a+b)^4/f/(a+b+b*tan(f*x+e)^2)
```

3.53.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.24 (sec) , antiderivative size = 777, normalized size of antiderivative = 4.13

$$\int \frac{\csc^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^4(e+fx)\left(\frac{960a(3a-4b)b\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(a+2b}{\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right.}{}$$

input
```
Integrate[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*((960*a*(3*a - 4*b)*b*ArcTa
n[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f
*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e
 + f*x)])*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])
^4]) - Csc[e]*Csc[e + f*x]^5*Sec[2*e]*(10*a*(16*a^2 + 34*a*b + 123*b^2)*Si
n[f*x] - a*(16*a^2 - 223*a*b + 1336*b^2)*Sin[3*f*x] + 240*a^3*Sin[2*e - f*
x] + 640*a^2*b*Sin[2*e - f*x] - 1460*a*b^2*Sin[2*e - f*x] + 240*b^3*Sin[2*
e - f*x] - 240*a^3*Sin[2*e + f*x] - 715*a^2*b*Sin[2*e + f*x] + 860*a*b^2*S
in[2*e + f*x] - 240*b^3*Sin[2*e + f*x] + 160*a^3*Sin[4*e + f*x] + 415*a^2*
b*Sin[4*e + f*x] + 1830*a*b^2*Sin[4*e + f*x] + 165*a^2*b*Sin[2*e + 3*f*x]
- 30*a*b^2*Sin[2*e + 3*f*x] + 120*b^3*Sin[2*e + 3*f*x] - 16*a^3*Sin[4*e +
3*f*x] + 208*a^2*b*Sin[4*e + 3*f*x] - 1036*a*b^2*Sin[4*e + 3*f*x] + 180*a^
2*b*Sin[6*e + 3*f*x] - 330*a*b^2*Sin[6*e + 3*f*x] + 120*b^3*Sin[6*e + 3*f*
x] + 48*a^3*Sin[2*e + 5*f*x] - 268*a^2*b*Sin[2*e + 5*f*x] + 290*a*b^2*Sin[
2*e + 5*f*x] - 24*b^3*Sin[2*e + 5*f*x] + 48*a^3*Sin[6*e + 5*f*x] - 223*a^2
*b*Sin[6*e + 5*f*x] + 230*a*b^2*Sin[6*e + 5*f*x] - 24*b^3*Sin[6*e + 5*f*x]
 - 45*a^2*b*Sin[8*e + 5*f*x] + 60*a*b^2*Sin[8*e + 5*f*x] - 16*a^3*Sin[4*e
+ 7*f*x] + 83*a^2*b*Sin[4*e + 7*f*x] - 6*a*b^2*Sin[4*e + 7*f*x] - 15*a^2*b
*Sin[6*e + 7*f*x] - 16*a^3*Sin[8*e + 7*f*x] + 68*a^2*b*Sin[8*e + 7*f*x] -
6*a*b^2*Sin[8*e + 7*f*x])))/(7680*(a + b)^4*f*(a + b*Sec[e + f*x]^2)^2)
```

3.53.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4620, 365, 361, 25, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sin(e+fx)^6\left(a+b\sec(e+fx)^2\right)^2}dx$$

$$\downarrow 4620$$

$$\frac{\int \frac{\cot^6(e+fx)\left(\tan^2(e+fx)+1\right)^2}{\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{f}$$

$$\downarrow 365$$

$$\frac{\int \frac{\cot^4(e+fx)\left(5(a+b)\tan^2(e+fx)+10a+3b\right)}{\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{5(a+b)} - \frac{\cot^5(e+fx)}{5(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

$$\downarrow 361$$

$$\frac{-\frac{1}{2}b\int -\frac{\cot^4(e+fx)\left(-\frac{\left(5a^2+2b^2\right)\tan^4(e+fx)}{(a+b)^3}+\frac{2\left(5a^2+2b^2\right)\tan^2(e+fx)}{b(a+b)^2}+\frac{2(10a+3b)}{b(a+b)}\right)}{b\tan^2(e+fx)+a+b}d\tan(e+fx)-\frac{b\left(5a^2+2b^2\right)\tan(e+fx)}{2(a+b)^3\left(a+b\tan^2(e+fx)+b\right)}}{5(a+b)} - \frac{\cot^5(e+f}{5(a+b)(a+b\tan^2(}}{f}$$

$$\downarrow 25$$

$$\frac{\frac{1}{2}b\int \frac{\cot^4(e+fx)\left(-\frac{\left(5a^2+2b^2\right)\tan^4(e+fx)}{(a+b)^3}+\frac{2\left(5a^2+2b^2\right)\tan^2(e+fx)}{b(a+b)^2}+\frac{2(10a+3b)}{b(a+b)}\right)}{b\tan^2(e+fx)+a+b}d\tan(e+fx)-\frac{b\left(5a^2+2b^2\right)\tan(e+fx)}{2(a+b)^3\left(a+b\tan^2(e+fx)+b\right)}}{5(a+b)} - \frac{\cot^5(e+fx)}{5(a+b)(a+b\tan^2(e+}}{f}$$

$$\downarrow 1584$$

$$\frac{\frac{1}{2}b\int\left(\frac{2(10a+3b)\cot^4(e+fx)}{b(a+b)^2}-\frac{2\left(-5a^2+10ba+b^2\right)\cot^2(e+fx)}{b(a+b)^3}-\frac{5a(3a-4b)}{(a+b)^3\left(b\tan^2(e+fx)+a+b\right)}\right)d\tan(e+fx)-\frac{b\left(5a^2+2b^2\right)\tan(e+fx)}{2(a+b)^3\left(a+b\tan^2(e+fx)+b\right)}}{5(a+b)}-\frac{\cot^5}{5(a+b)(a+b}$$
$$f$$

$$\downarrow 2009$$

$$\frac{\frac{1}{2}b\left(-\frac{2\left(5a^2-10ab-b^2\right)\cot(e+fx)}{b(a+b)^3}-\frac{5a(3a-4b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{b}(a+b)^{7/2}}-\frac{2(10a+3b)\cot^3(e+fx)}{3b(a+b)^2}\right)-\frac{b\left(5a^2+2b^2\right)\tan(e+fx)}{2(a+b)^3\left(a+b\tan^2(e+fx)+b\right)}}{5(a+b)}-\frac{\cot^5(e+fx)}{5(a+b)(a+b\tan^2(e+}$$
$$f$$

input
```
Int[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(-1/5*Cot[e + f*x]^5/((a + b)*(a + b + b*Tan[e + f*x]^2)) + ((b*((-5*a*(3*
a - 4*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(Sqrt[b]*(a + b)^(7/2
)) - (2*(5*a^2 - 10*a*b - b^2)*Cot[e + f*x])/(b*(a + b)^3) - (2*(10*a + 3*
b)*Cot[e + f*x]^3)/(3*b*(a + b)^2)))/2 - (b*(5*a^2 + 2*b^2)*Tan[e + f*x])/
(2*(a + b)^3*(a + b + b*Tan[e + f*x]^2)))/(5*(a + b)))/f
```

3.53.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.53.4 Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.66

method	result
derivativedivides	$\frac{-\frac{ba\left(\frac{a\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{(3a-4b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{(a+b)^4}-\frac{1}{5(a+b)^2\tan(fx+e)^5}-\frac{a(a-2b)}{(a+b)^4\tan(fx+e)}-\frac{2a}{3(a+b)^3\tan(fx+e)^3}}{f}$
default	$\frac{-\frac{ba\left(\frac{a\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{(3a-4b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{(a+b)^4}-\frac{1}{5(a+b)^2\tan(fx+e)^5}-\frac{a(a-2b)}{(a+b)^4\tan(fx+e)}-\frac{2a}{3(a+b)^3\tan(fx+e)^3}}{f}$
risch	$-\frac{i\left(-45a^2b\,e^{12i(fx+e)}+60a\,b^2e^{12i(fx+e)}+180a^2b\,e^{10i(fx+e)}-330a\,b^2e^{10i(fx+e)}+120b^3e^{10i(fx+e)}+160a^3e^{8i(fx+e)}+4\right.}{}$

input
```
int(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-b*a/(a+b)^4*(1/2*a*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(3*a-4*b)/((a
+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))-1/5/(a+b)^2/tan(f*x+e)^
5-a*(a-2*b)/(a+b)^4/tan(f*x+e)-2/3*a/(a+b)^3/tan(f*x+e)^3)
```

3.53.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(170) = 340$.

Time = 0.32 (sec) , antiderivative size = 987, normalized size of antiderivative = 5.25

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[-1/120*(4*(16*a^3 - 83*a^2*b + 6*a*b^2)*cos(f*x + e)^7 - 4*(40*a^3 - 201*
a^2*b + 68*a*b^2 - 6*b^3)*cos(f*x + e)^5 + 20*(6*a^3 - 29*a^2*b + 28*a*b^2
)*cos(f*x + e)^3 + 15*((3*a^3 - 4*a^2*b)*cos(f*x + e)^6 - (6*a^3 - 11*a^2*
b + 4*a*b^2)*cos(f*x + e)^4 + 3*a^2*b - 4*a*b^2 + (3*a^3 - 10*a^2*b + 8*a*
b^2)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x +
 e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*b + 2*b^2)*cos(f*
x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/
(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) + 60*(3*a^
2*b - 4*a*b^2)*cos(f*x + e))/(((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*
b^4)*f*cos(f*x + e)^6 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4
 - b^5)*f*cos(f*x + e)^4 + (a^5 + 2*a^4*b - 2*a^3*b^2 - 8*a^2*b^3 - 7*a*b^
4 - 2*b^5)*f*cos(f*x + e)^2 + (a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b
^5)*f)*sin(f*x + e)), -1/60*(2*(16*a^3 - 83*a^2*b + 6*a*b^2)*cos(f*x + e)^
7 - 2*(40*a^3 - 201*a^2*b + 68*a*b^2 - 6*b^3)*cos(f*x + e)^5 + 10*(6*a^3 -
 29*a^2*b + 28*a*b^2)*cos(f*x + e)^3 - 15*((3*a^3 - 4*a^2*b)*cos(f*x + e)^
6 - (6*a^3 - 11*a^2*b + 4*a*b^2)*cos(f*x + e)^4 + 3*a^2*b - 4*a*b^2 + (3*a
^3 - 10*a^2*b + 8*a*b^2)*cos(f*x + e)^2)*sqrt(b/(a + b))*arctan(1/2*((a +
2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e)))*si
n(f*x + e) + 30*(3*a^2*b - 4*a*b^2)*cos(f*x + e))/(((a^5 + 4*a^4*b + 6*a^3
*b^2 + 4*a^2*b^3 + a*b^4)*f*cos(f*x + e)^6 - (2*a^5 + 7*a^4*b + 8*a^3*b...
```

3.53.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**6/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Timed out
```

3.53.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.43

$$\int \frac{\csc^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx =$$

$$-\frac{\frac{15\left(3a^2b-4ab^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\left(a^4+4a^3b+6a^2b^2+4ab^3+b^4\right)\sqrt{(a+b)b}}+\frac{15\left(3a^2b-4ab^2\right)\tan(fx+e)^6+10\left(3a^3-a^2b-4ab^2\right)\tan(fx+e)^4+6a^3+18a^2b+18ab^2+6b^3+2\,(1}{\left(a^4b+4a^3b^2+6a^2b^3+4ab^4+b^5\right)\tan(fx+e)^7+\left(a^5+5a^4b+10a^3b^2+10a^2b^3+5a\right.}}{30\,f}$$

input
```
integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/30*(15*(3*a^2*b - 4*a*b^2)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^4
 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*sqrt((a + b)*b)) + (15*(3*a^2*b -
4*a*b^2)*tan(f*x + e)^6 + 10*(3*a^3 - a^2*b - 4*a*b^2)*tan(f*x + e)^4 + 6*
a^3 + 18*a^2*b + 18*a*b^2 + 6*b^3 + 2*(10*a^3 + 23*a^2*b + 16*a*b^2 + 3*b^
3)*tan(f*x + e)^2)/((a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*tan(f*
x + e)^7 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*tan(f
*x + e)^5))/f
```

3.53.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.35

$$\int \frac{\csc^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx =$$

$$-\frac{\frac{15a^2b\tan(fx+e)}{\left(a^4+4a^3b+6a^2b^2+4ab^3+b^4\right)\left(b\tan(fx+e)^2+a+b\right)}+\frac{15\left(3a^2b-4ab^2\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{\left(a^4+4a^3b+6a^2b^2+4ab^3+b^4\right)\sqrt{ab+b^2}}+\frac{2\left(15a^2\tan(fx+\right.}}{30\,f}$$

input
```
integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-1/30*(15*a^2*b*tan(f*x + e)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*
(b*tan(f*x + e)^2 + a + b)) + 15*(3*a^2*b - 4*a*b^2)*(pi*floor((f*x + e)/p
i + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^4 + 4*a^3*b
+ 6*a^2*b^2 + 4*a*b^3 + b^4)*sqrt(a*b + b^2)) + 2*(15*a^2*tan(f*x + e)^4 -
 30*a*b*tan(f*x + e)^4 + 10*a^2*tan(f*x + e)^2 + 10*a*b*tan(f*x + e)^2 + 3
*a^2 + 6*a*b + 3*b^2)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*tan(f*x
 + e)^5))/f
```

3.53.9 Mupad [B] (verification not implemented)

Time = 20.34 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.05

$$\int \frac{\csc^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$
$$= \frac{a\sqrt{b}\,\operatorname{atan}\left(\frac{a\sqrt{b}\tan(e+fx)\,(3a-4b)\,(a^4+4a^3b+6a^2b^2+4ab^3+b^4)}{(a+b)^{9/2}\,(4ab-3a^2)}\right)\,(3a-4b)}{2f\,(a+b)^{9/2}}$$
$$- \frac{\frac{1}{5(a+b)} - \frac{\tan(e+fx)^4\,(4ab-3a^2)}{3(a+b)^3} + \frac{\tan(e+fx)^2\,(10a+3b)}{15(a+b)^2} - \frac{b\tan(e+fx)^6\,(4ab-3a^2)}{2(a+b)^4}}{f\,\left(b\tan(e+fx)^7+(a+b)\,\tan(e+fx)^5\right)}$$

input
```
int(1/(sin(e + f*x)^6*(a + b/cos(e + f*x)^2)^2),x)
```

output
```
(a*b^(1/2)*atan((a*b^(1/2)*tan(e + f*x)*(3*a - 4*b)*(4*a*b^3 + 4*a^3*b + a
^4 + b^4 + 6*a^2*b^2))/((a + b)^(9/2)*(4*a*b - 3*a^2)))*(3*a - 4*b))/(2*f*
(a + b)^(9/2)) - (1/(5*(a + b)) - (tan(e + f*x)^4*(4*a*b - 3*a^2))/(3*(a +
 b)^3) + (tan(e + f*x)^2*(10*a + 3*b))/(15*(a + b)^2) - (b*tan(e + f*x)^6*
(4*a*b - 3*a^2))/(2*(a + b)^4))/(f*(tan(e + f*x)^5*(a + b) + b*tan(e + f*x
)^7))
```

3.54 $\int \frac{\sin^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

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3.54.1 Optimal result

Integrand size = 23, antiderivative size = 214

$$\begin{aligned}\int \frac{\sin^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = {}& \frac{\sqrt{b}(15a^2+70ab+63b^2)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8a^{11/2}f} \\ & - \frac{(3a^2+14ab+13b^2)\cos(e+fx)}{2a^5 f} \\ & + \frac{(a+3b)(3a+5b)\cos^3(e+fx)}{12a^4 bf} \\ & - \frac{\cos^5(e+fx)}{5a^3 f} - \frac{(a+b)^2\cos^7(e+fx)}{4a^2 bf\left(b+a\cos^2(e+fx)\right)^2} \\ & - \frac{b(a+b)(3a+11b)\cos(e+fx)}{8a^5 f\left(b+a\cos^2(e+fx)\right)}\end{aligned}$$

output

```
-1/2*(3*a^2+14*a*b+13*b^2)*cos(f*x+e)/a^5/f+1/12*(a+3*b)*(3*a+5*b)*cos(f*x
+e)^3/a^4/b/f-1/5*cos(f*x+e)^5/a^3/f-1/4*(a+b)^2*cos(f*x+e)^7/a^2/b/f/(b+a
*cos(f*x+e)^2)^2-1/8*b*(a+b)*(3*a+11*b)*cos(f*x+e)/a^5/f/(b+a*cos(f*x+e)^2
)+1/8*(15*a^2+70*a*b+63*b^2)*arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*b^(1/2)/a^
(11/2)/f
```

3.54.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.36 (sec) , antiderivative size = 1641, normalized size of antiderivative = 7.67

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
Integrate[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^6*(-900*a^(11/2)*b^(3/2)*Cos[
e + f*x] - 109000*a^(9/2)*b^(5/2)*Cos[e + f*x] - 936000*a^(7/2)*b^(7/2)*Co
s[e + f*x] - 2803072*a^(5/2)*b^(9/2)*Cos[e + f*x] - 3763200*a^(3/2)*b^(11/
2)*Cos[e + f*x] - 1935360*Sqrt[a]*b^(13/2)*Cos[e + f*x] - 900*a^(11/2)*b^(
3/2)*Cos[e + f*x]*Cos[2*(e + f*x)] + 900*a^(9/2)*b^(3/2)*Cos[e + f*x]*(a +
 2*b + a*Cos[2*(e + f*x)]) + 24000*a^(7/2)*b^(5/2)*Cos[e + f*x]*(a + 2*b +
 a*Cos[2*(e + f*x)]) + 43200*a^(5/2)*b^(7/2)*Cos[e + f*x]*(a + 2*b + a*Cos
[2*(e + f*x)]) + 225*a^5*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] -
I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Co
s[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])
^2 + 115200*a^2*b^3*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin
[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e]
- I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2 +
537600*a*b^4*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]
)*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin
[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2 + 483840*
b^5*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*
Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*T
an[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2 + 225*a^5*ArcTan[(
(-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)...
```

3.54.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4621, 366, 25, 360, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sin(e+fx)^5}{(a+b\sec(e+fx)^2)^3}\,dx$$

$$\downarrow 4621$$

$$-\frac{\int \frac{\cos^6(e+fx)\left(1-\cos^2(e+fx)\right)^2}{\left(a\cos^2(e+fx)+b\right)^3}\,d\cos(e+fx)}{f}$$

$$\downarrow 366$$

$$-\frac{\frac{(a+b)^2\cos^7(e+fx)}{4a^2b\left(a\cos^2(e+fx)+b\right)^2}-\frac{\int -\frac{\cos^6(e+fx)\left(4a^2+4b\cos^2(e+fx)a-7(a+b)^2\right)}{\left(a\cos^2(e+fx)+b\right)^2}\,d\cos(e+fx)}{4a^2b}}{f}$$

$$\downarrow 25$$

$$-\frac{\frac{\int \frac{\cos^6(e+fx)\left(4a^2+4b\cos^2(e+fx)a-7(a+b)^2\right)}{\left(a\cos^2(e+fx)+b\right)^2}\,d\cos(e+fx)}{4a^2b}+\frac{(a+b)^2\cos^7(e+fx)}{4a^2b\left(a\cos^2(e+fx)+b\right)^2}}{f}$$

$$\downarrow 360$$

$$-\frac{\frac{b^2(a+b)(3a+11b)\cos(e+fx)}{2a^3\left(a\cos^2(e+fx)+b\right)}-\frac{\int \frac{-8a^4b\cos^6(e+fx)+2a^3(a+b)(3a+11b)\cos^4(e+fx)-2a^2b(a+b)(3a+11b)\cos^2(e+fx)+ab^2(a+b)(3a+11b)}{a\cos^2(e+fx)+b}\,d\cos(e+fx)}{2a^4}}{4a^2b}+\frac{(a}{4a^2b}}{f}$$

$$\downarrow 2341$$

$$-\frac{\frac{b^2(a+b)(3a+11b)\cos(e+fx)}{2a^3\left(a\cos^2(e+fx)+b\right)}-\frac{\int \left(-8a^3b\cos^4(e+fx)+2a^2(a+3b)(3a+5b)\cos^2(e+fx)-4ab\left(3a^2+14ba+13b^2\right)+\frac{63ab^4+70a^2b^3+15a^3b^2}{a\cos^2(e+fx)+b}\right)d\cos(e+fx)}{2a^4}}{4a^2b}+\frac{(a+}{4a^2b(}}{f}$$

$$\downarrow 2009$$

$$-\frac{\frac{(a+b)^2\cos^7(e+fx)}{4a^2b\left(a\cos^2(e+fx)+b\right)^2}+\frac{\frac{b^2(a+b)(3a+11b)\cos(e+fx)}{2a^3\left(a\cos^2(e+fx)+b\right)}-\frac{-\frac{8}{5}a^3b\cos^5(e+fx)+\sqrt{a}b^{3/2}\left(15a^2+70ab+63b^2\right)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)-4ab\left(3a^2+14ab+13b^2\right)}{2a^4}}{4a^2b}}{f}$$

input
```
Int[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
-((((a + b)^2*Cos[e + f*x]^7)/(4*a^2*b*(b + a*Cos[e + f*x]^2)^2) + ((b^2*(
a + b)*(3*a + 11*b)*Cos[e + f*x])/(2*a^3*(b + a*Cos[e + f*x]^2)) - (Sqrt[a
]*b^(3/2)*(15*a^2 + 70*a*b + 63*b^2)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]
] - 4*a*b*(3*a^2 + 14*a*b + 13*b^2)*Cos[e + f*x] + (2*a^2*(a + 3*b)*(3*a +
 5*b)*Cos[e + f*x]^3)/3 - (8*a^3*b*Cos[e + f*x]^5)/5)/(2*a^4))/(4*a^2*b))/
f)
```

3.54.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

3.54.4 Maple [A] (verified)

Time = 10.37 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{-\frac{\cos(fx+e)^5 a^2}{5}-\frac{2a^2\cos(fx+e)^3}{3}-a\cos(fx+e)^3 b+a^2\cos(fx+e)+6ab\cos(fx+e)+6b^2\cos(fx+e)}{a^5}+\frac{b\left(\frac{\left(-\frac{9}{8}a^3-\frac{13}{4}a^2b-\frac{17}{8}ab^2\right)\cos}{(b-}\right.}{f}$
default	$\frac{-\frac{\cos(fx+e)^5 a^2}{5}-\frac{2a^2\cos(fx+e)^3}{3}-a\cos(fx+e)^3 b+a^2\cos(fx+e)+6ab\cos(fx+e)+6b^2\cos(fx+e)}{a^5}+\frac{b\left(\frac{\left(-\frac{9}{8}a^3-\frac{13}{4}a^2b-\frac{17}{8}ab^2\right)\cos}{(b-}\right.}{f}$
risch	$-\frac{e^{5i(fx+e)}}{160a^3f}+\frac{5e^{3i(fx+e)}}{96a^3f}+\frac{e^{3i(fx+e)}b}{8a^4f}-\frac{5e^{i(fx+e)}}{16a^3f}-\frac{21e^{i(fx+e)}b}{8a^4f}-\frac{3e^{i(fx+e)}b^2}{a^5f}-\frac{5e^{-i(fx+e)}}{16a^3f}-\frac{21e^{-i(}}{8a}$

input

```
int(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output

```
1/f*(-1/a^5*(1/5*cos(f*x+e)^5*a^2-2/3*a^2*cos(f*x+e)^3-a*cos(f*x+e)^3*b+a^
2*cos(f*x+e)+6*a*b*cos(f*x+e)+6*b^2*cos(f*x+e))+b/a^5*(((-9/8*a^3-13/4*a^2
*b-17/8*a*b^2)*cos(f*x+e)^3-1/8*b*(7*a^2+22*a*b+15*b^2)*cos(f*x+e))/(b+a*c
os(f*x+e)^2)^2+1/8*(15*a^2+70*a*b+63*b^2)/(a*b)^(1/2)*arctan(a*cos(f*x+e)/
(a*b)^(1/2))))
```

3.54.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 579, normalized size of antiderivative = 2.71

$$\int \frac{\sin^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$

$$= \left[-\frac{48\,a^4\cos(fx+e)^9 - 16\,(10\,a^4+9\,a^3b)\cos(fx+e)^7 + 16\,(15\,a^4+70\,a^3b+63\,a^2b^2)\cos(fx+e)^5 + 50}{} \right.$$

$$\left. -\frac{24\,a^4\cos(fx+e)^9 - 8\,(10\,a^4+9\,a^3b)\cos(fx+e)^7 + 8\,(15\,a^4+70\,a^3b+63\,a^2b^2)\cos(fx+e)^5 + 25\,(1}{} \right.$$

input
```
integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[-1/240*(48*a^4*cos(f*x + e)^9 - 16*(10*a^4 + 9*a^3*b)*cos(f*x + e)^7 + 16
*(15*a^4 + 70*a^3*b + 63*a^2*b^2)*cos(f*x + e)^5 + 50*(15*a^3*b + 70*a^2*b
^2 + 63*a*b^3)*cos(f*x + e)^3 - 15*((15*a^4 + 70*a^3*b + 63*a^2*b^2)*cos(f
*x + e)^4 + 15*a^2*b^2 + 70*a*b^3 + 63*b^4 + 2*(15*a^3*b + 70*a^2*b^2 + 63
*a*b^3)*cos(f*x + e)^2)*sqrt(-b/a)*log(-(a*cos(f*x + e)^2 + 2*a*sqrt(-b/a)
*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) + 30*(15*a^2*b^2 + 70*a*b^3 + 6
3*b^4)*cos(f*x + e))/(a^7*f*cos(f*x + e)^4 + 2*a^6*b*f*cos(f*x + e)^2 + a^
5*b^2*f), -1/120*(24*a^4*cos(f*x + e)^9 - 8*(10*a^4 + 9*a^3*b)*cos(f*x + e
)^7 + 8*(15*a^4 + 70*a^3*b + 63*a^2*b^2)*cos(f*x + e)^5 + 25*(15*a^3*b + 7
0*a^2*b^2 + 63*a*b^3)*cos(f*x + e)^3 - 15*((15*a^4 + 70*a^3*b + 63*a^2*b^2
)*cos(f*x + e)^4 + 15*a^2*b^2 + 70*a*b^3 + 63*b^4 + 2*(15*a^3*b + 70*a^2*b
^2 + 63*a*b^3)*cos(f*x + e)^2)*sqrt(b/a)*arctan(a*sqrt(b/a)*cos(f*x + e)/b
) + 15*(15*a^2*b^2 + 70*a*b^3 + 63*b^4)*cos(f*x + e))/(a^7*f*cos(f*x + e)^
4 + 2*a^6*b*f*cos(f*x + e)^2 + a^5*b^2*f)]
```

3.54.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**5/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Timed out
```

3.54.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.95

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{15\left(\left(9\,a^3b+26\,a^2b^2+17\,ab^3\right)\cos(fx+e)^3+\left(7\,a^2b^2+22\,ab^3+15\,b^4\right)\cos(fx+e)\right)}{a^7\cos(fx+e)^4+2\,a^6b\cos(fx+e)^2+a^5b^2}-\frac{15\left(15\,a^2b+70\,ab^2+63\,b^3\right)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}a^5}+\frac{8\left(3\,a^2}{120\,f}$$

input
```
integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
-1/120*(15*((9*a^3*b + 26*a^2*b^2 + 17*a*b^3)*cos(f*x + e)^3 + (7*a^2*b^2
+ 22*a*b^3 + 15*b^4)*cos(f*x + e))/(a^7*cos(f*x + e)^4 + 2*a^6*b*cos(f*x +
 e)^2 + a^5*b^2) - 15*(15*a^2*b + 70*a*b^2 + 63*b^3)*arctan(a*cos(f*x + e)
/sqrt(a*b))/(sqrt(a*b)*a^5) + 8*(3*a^2*cos(f*x + e)^5 - 5*(2*a^2 + 3*a*b)*
cos(f*x + e)^3 + 15*(a^2 + 6*a*b + 6*b^2)*cos(f*x + e))/a^5)/f
```

3.54.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 781 vs. 2(194) = 388.

Time = 0.47 (sec) , antiderivative size = 781, normalized size of antiderivative = 3.65

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
-1/120*(15*(15*a^2*b + 70*a*b^2 + 63*b^3)*arctan(-(a*cos(f*x + e) - b)/(sq
rt(a*b)*cos(f*x + e) + sqrt(a*b)))/(sqrt(a*b)*a^5) + 30*(9*a^3*b + 33*a^2*
b^2 + 39*a*b^3 + 15*b^4 + 27*a^3*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) +
 49*a^2*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 23*a*b^3*(cos(f*x + e)
 - 1)/(cos(f*x + e) + 1) - 45*b^4*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) +
27*a^3*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 27*a^2*b^2*(cos(f*x +
 e) - 1)^2/(cos(f*x + e) + 1)^2 - 3*a*b^3*(cos(f*x + e) - 1)^2/(cos(f*x +
e) + 1)^2 + 45*b^4*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 9*a^3*b*(co
s(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 11*a^2*b^2*(cos(f*x + e) - 1)^3/(
cos(f*x + e) + 1)^3 - 13*a*b^3*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 -
 15*b^4*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3)/((a + b + 2*a*(cos(f*x
+ e) - 1)/(cos(f*x + e) + 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) +
 a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos
(f*x + e) + 1)^2)^2*a^5) - 16*(8*a^2 + 75*a*b + 90*b^2 - 40*a^2*(cos(f*x +
 e) - 1)/(cos(f*x + e) + 1) - 330*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1
) - 360*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 80*a^2*(cos(f*x + e) -
 1)^2/(cos(f*x + e) + 1)^2 + 480*a*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) +
1)^2 + 540*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 270*a*b*(cos(f*
x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 360*b^2*(cos(f*x + e) - 1)^3/(cos(f*x
 + e) + 1)^3 + 45*a*b*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 90*b^...
```

3.54.9 Mupad [B] (verification not implemented)

Time = 17.90 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.19

$$
\begin{aligned}
&\int \frac{\sin^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx \\
&= \frac{\cos(e+fx)^3\left(\frac{b}{a^4}+\frac{2}{3a^3}\right)}{f} \\
&\quad - \frac{\left(\frac{9a^3b}{8}+\frac{13a^2b^2}{4}+\frac{17ab^3}{8}\right)\cos(e+fx)^3+\left(\frac{7a^2b^2}{8}+\frac{11ab^3}{4}+\frac{15b^4}{8}\right)\cos(e+fx)}{f\left(a^7\cos(e+fx)^4+2a^6b\cos(e+fx)^2+a^5b^2\right)} \\
&\quad - \frac{\cos(e+fx)^5}{5a^3f} - \frac{\cos(e+fx)\left(\frac{1}{a^3}-\frac{3b^2}{a^5}+\frac{3b\left(\frac{3b}{a^4}+\frac{2}{a^3}\right)}{a}\right)}{f} \\
&\quad + \frac{\sqrt{b}\,\operatorname{atan}\left(\frac{\sqrt{a}\sqrt{b}\cos(e+fx)\left(15a^2+70ab+63b^2\right)}{15a^2b+70ab^2+63b^3}\right)\left(15a^2+70ab+63b^2\right)}{8a^{11/2}f}
\end{aligned}
$$

input
```
int(sin(e + f*x)^5/(a + b/cos(e + f*x)^2)^3,x)
```

output
```
(cos(e + f*x)^3*(b/a^4 + 2/(3*a^3)))/f - (cos(e + f*x)^3*((17*a*b^3)/8 + (
9*a^3*b)/8 + (13*a^2*b^2)/4) + cos(e + f*x)*((11*a*b^3)/4 + (15*b^4)/8 + (
7*a^2*b^2)/8))/(f*(a^5*b^2 + a^7*cos(e + f*x)^4 + 2*a^6*b*cos(e + f*x)^2))
 - cos(e + f*x)^5/(5*a^3*f) - (cos(e + f*x)*(1/a^3 - (3*b^2)/a^5 + (3*b*((
3*b)/a^4 + 2/a^3))/a))/f + (b^(1/2)*atan((a^(1/2)*b^(1/2)*cos(e + f*x)*(70
*a*b + 15*a^2 + 63*b^2))/(70*a*b^2 + 15*a^2*b + 63*b^3))*(70*a*b + 15*a^2
+ 63*b^2))/(8*a^(11/2)*f)
```

3.55 $\int \frac{\sin^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

3.55.1 Optimal result 532
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3.55.9 Mupad [B] (verification not implemented) 539

3.55.1 Optimal result

Integrand size = 23, antiderivative size = 154

$$\int \frac{\sin^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{5\sqrt{b}(3a+7b)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8a^{9/2}f} - \frac{(a+3b)\cos(e+fx)}{a^4 f} + \frac{\cos^3(e+fx)}{3a^3 f} + \frac{b^2(a+b)\cos(e+fx)}{4a^4 f\left(b+a\cos^2(e+fx)\right)^2} - \frac{b(9a+13b)\cos(e+fx)}{8a^4 f\left(b+a\cos^2(e+fx)\right)}$$

output
```
-(a+3*b)*cos(f*x+e)/a^4/f+1/3*cos(f*x+e)^3/a^3/f+1/4*b^2*(a+b)*cos(f*x+e)/
a^4/f/(b+a*cos(f*x+e)^2)^2-1/8*b*(9*a+13*b)*cos(f*x+e)/a^4/f/(b+a*cos(f*x+
e)^2)+5/8*(3*a+7*b)*arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*b^(1/2)/a^(9/2)/f
```

3.55.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 8.74 (sec) , antiderivative size = 1153, normalized size of antiderivative = 7.49

$$\int \frac{\sin^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))^3\sec^6(e+fx)\left(3(9a^4+1920ab^3+4480b^4)\arctan\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))}\right.}{}\right.\right.}{}$$

input
```
Integrate[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])^3*Sec[e + f*x]^6*(3*(9*a^4 + 1920*a*b^3 +
4480*b^4)*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*S
in[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e]
)^2]*Tan[(f*x)/2]))/Sqrt[b]] - (-27*a^(11/2)*Sqrt[b]*Cos[e + f*x] + 162*a^
(9/2)*b^(3/2)*Cos[e + f*x] + 10816*a^(7/2)*b^(5/2)*Cos[e + f*x] + 51552*a^
(5/2)*b^(7/2)*Cos[e + f*x] + 87424*a^(3/2)*b^(9/2)*Cos[e + f*x] + 53760*Sq
rt[a]*b^(11/2)*Cos[e + f*x] - 27*a^(11/2)*Sqrt[b]*Cos[e + f*x]*Cos[2*(e +
f*x)] + 47936*a^(5/2)*b^(7/2)*Cos[e + f*x]*Cos[2*(e + f*x)] + 44800*a^(3/2
)*b^(9/2)*Cos[e + f*x]*Cos[2*(e + f*x)] + 27*a^(9/2)*Sqrt[b]*Cos[e + f*x]*
(a + 2*b + a*Cos[2*(e + f*x)]) - 216*a^(7/2)*b^(3/2)*Cos[e + f*x]*(a + 2*b
 + a*Cos[2*(e + f*x)]) - 3600*a^(5/2)*b^(5/2)*Cos[e + f*x]*(a + 2*b + a*Co
s[2*(e + f*x)]) - 5184*a^(3/2)*b^(7/2)*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e
+ f*x)]) - 27*a^4*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e
])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] -
I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2 - 57
60*a*b^3*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Si
n[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])
^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2 - 13440*b^4*A
rcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(
f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan...
```

3.55.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4621, 360, 25, 2345, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sin(e+fx)^3}{(a+b\sec(e+fx)^2)^3}\,dx$$

$$\downarrow 4621$$

$$-\frac{\int \frac{\cos^6(e+fx)(1-\cos^2(e+fx))}{(a\cos^2(e+fx)+b)^3}\,d\cos(e+fx)}{f}$$

$$\downarrow 360$$

$$-\frac{\int -\frac{-4a^3\cos^6(e+fx)+4a^2(a+b)\cos^4(e+fx)-4ab(a+b)\cos^2(e+fx)+b^2(a+b)}{\left(a\cos^2(e+fx)+b\right)^2}\,d\cos(e+fx)}{4a^4} - \frac{b^2(a+b)\cos(e+fx)}{4a^4(a\cos^2(e+fx)+b)^2}}{f}$$

$$\downarrow 25$$

$$-\frac{\frac{\int \frac{-4a^3\cos^6(e+fx)+4a^2(a+b)\cos^4(e+fx)-4ab(a+b)\cos^2(e+fx)+b^2(a+b)}{\left(a\cos^2(e+fx)+b\right)^2}\,d\cos(e+fx)}{4a^4} - \frac{b^2(a+b)\cos(e+fx)}{4a^4(a\cos^2(e+fx)+b)^2}}{f}$$

$$\downarrow 2345$$

$$-\frac{\frac{\frac{b(9a+13b)\cos(e+fx)}{2\left(a\cos^2(e+fx)+b\right)} - \frac{\int \frac{8a^2b\cos^4(e+fx)-8ab(a+2b)\cos^2(e+fx)+b^2(7a+11b)}{a\cos^2(e+fx)+b}\,d\cos(e+fx)}{2b}}{4a^4} - \frac{b^2(a+b)\cos(e+fx)}{4a^4(a\cos^2(e+fx)+b)^2}}{f}$$

$$\downarrow 1467$$

$$-\frac{\frac{\frac{b(9a+13b)\cos(e+fx)}{2\left(a\cos^2(e+fx)+b\right)} - \frac{\int \left(8ab\cos^2(e+fx)-8b(a+3b)+\frac{5\left(7b^3+3ab^2\right)}{a\cos^2(e+fx)+b}\right)d\cos(e+fx)}{2b}}{4a^4} - \frac{b^2(a+b)\cos(e+fx)}{4a^4(a\cos^2(e+fx)+b)^2}}{f}$$

$$\downarrow 2009$$

$$-\frac{\frac{b(9a+13b)\cos(e+fx)}{2\left(a\cos^2(e+fx)+b\right)}-\frac{\frac{5b^{3/2}(3a+7b)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{\sqrt{a}}+\frac{8}{3}ab\cos^3(e+fx)-8b(a+3b)\cos(e+fx)}{2b}}{4a^4}-\frac{b^2(a+b)\cos(e+fx)}{4a^4(a\cos^2(e+fx)+b)^2}}{f}$$

input
```
Int[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
-((-1/4*(b^2*(a + b)*Cos[e + f*x])/(a^4*(b + a*Cos[e + f*x]^2)^2) + ((b*(9
*a + 13*b)*Cos[e + f*x])/(2*(b + a*Cos[e + f*x]^2)) - ((5*b^(3/2)*(3*a + 7
*b)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/Sqrt[a] - 8*b*(a + 3*b)*Cos[e
+ f*x] + (8*a*b*Cos[e + f*x]^3)/3)/(2*b))/(4*a^4))/f)
```

3.55.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

3.55.4 Maple [A] (verified)

Time = 5.74 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{\frac{a\cos(fx+e)^3}{3}-\cos(fx+e)a-3b\cos(fx+e)}{a^4}+\frac{b\left(\frac{\left(-\frac{9}{8}a^2-\frac{13}{8}ab\right)\cos(fx+e)^3-\frac{b(7a+11b)\cos(fx+e)}{8}}{\left(b+a\cos(fx+e)^2\right)^2}+\frac{5(3a+7b)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right.}{8\sqrt{ab}}}{a^4}}{f}$
default	$\frac{\frac{a\cos(fx+e)^3}{3}-\cos(fx+e)a-3b\cos(fx+e)}{a^4}+\frac{b\left(\frac{\left(-\frac{9}{8}a^2-\frac{13}{8}ab\right)\cos(fx+e)^3-\frac{b(7a+11b)\cos(fx+e)}{8}}{\left(b+a\cos(fx+e)^2\right)^2}+\frac{5(3a+7b)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right.}{8\sqrt{ab}}}{a^4}}{f}$
risch	$\frac{e^{3i(fx+e)}}{24a^3f}-\frac{3e^{i(fx+e)}}{8a^3f}-\frac{3e^{i(fx+e)}b}{2a^4f}-\frac{3e^{-i(fx+e)}}{8a^3f}-\frac{3e^{-i(fx+e)}b}{2a^4f}+\frac{e^{-3i(fx+e)}}{24a^3f}-\frac{b(9a^2e^{7i(fx+e)}+13abe^{7i}}{}$

input
```
int(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/a^4*(1/3*a*cos(f*x+e)^3-cos(f*x+e)*a-3*b*cos(f*x+e))+b/a^4*(((-9/8*
a^2-13/8*a*b)*cos(f*x+e)^3-1/8*b*(7*a+11*b)*cos(f*x+e))/(b+a*cos(f*x+e)^2)
^2+5/8*(3*a+7*b)/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1/2))))
```

3.55.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.85

$$\int \frac{\sin^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$

$$= \left[\frac{16\,a^3\cos(fx+e)^7 - 16\,(3\,a^3+7\,a^2b)\cos(fx+e)^5 - 50\,(3\,a^2b+7\,ab^2)\cos(fx+e)^3 + 15\,\left((3\,a^3+7\,a^2b\right.}{48\,\left(a^6 f\,\mathrm{co}\right.}\right.$$

input
```
integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[1/48*(16*a^3*cos(f*x + e)^7 - 16*(3*a^3 + 7*a^2*b)*cos(f*x + e)^5 - 50*(3
*a^2*b + 7*a*b^2)*cos(f*x + e)^3 + 15*((3*a^3 + 7*a^2*b)*cos(f*x + e)^4 +
3*a*b^2 + 7*b^3 + 2*(3*a^2*b + 7*a*b^2)*cos(f*x + e)^2)*sqrt(-b/a)*log(-(a
*cos(f*x + e)^2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b))
 - 30*(3*a*b^2 + 7*b^3)*cos(f*x + e))/(a^6*f*cos(f*x + e)^4 + 2*a^5*b*f*co
s(f*x + e)^2 + a^4*b^2*f), 1/24*(8*a^3*cos(f*x + e)^7 - 8*(3*a^3 + 7*a^2*b
)*cos(f*x + e)^5 - 25*(3*a^2*b + 7*a*b^2)*cos(f*x + e)^3 + 15*((3*a^3 + 7*
a^2*b)*cos(f*x + e)^4 + 3*a*b^2 + 7*b^3 + 2*(3*a^2*b + 7*a*b^2)*cos(f*x +
e)^2)*sqrt(b/a)*arctan(a*sqrt(b/a)*cos(f*x + e)/b) - 15*(3*a*b^2 + 7*b^3)*
cos(f*x + e))/(a^6*f*cos(f*x + e)^4 + 2*a^5*b*f*cos(f*x + e)^2 + a^4*b^2*f
)]
```

3.55.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**3/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Timed out
```

3.55.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.97

$$\int \frac{\sin^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{\frac{3\left(\left(9a^2b+13ab^2\right)\cos(fx+e)^3+\left(7ab^2+11b^3\right)\cos(fx+e)\right)}{a^6\cos(fx+e)^4+2a^5b\cos(fx+e)^2+a^4b^2} - \frac{15\left(3ab+7b^2\right)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}a^4} - \frac{8\left(a\cos(fx+e)^3-3\left(a+3b\right)\cos(fx+e)\right.}{a^4}}{24f}$$

input
```
integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
-1/24*(3*((9*a^2*b + 13*a*b^2)*cos(f*x + e)^3 + (7*a*b^2 + 11*b^3)*cos(f*x
 + e))/(a^6*cos(f*x + e)^4 + 2*a^5*b*cos(f*x + e)^2 + a^4*b^2) - 15*(3*a*b
 + 7*b^2)*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^4) - 8*(a*cos(f*x
+ e)^3 - 3*(a + 3*b)*cos(f*x + e))/a^4)/f
```

3.55.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.13

$$\begin{aligned}
&\int \frac{\sin^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx \\
&= \frac{5\left(3ab+7b^2\right)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{8\sqrt{ab}a^4f} \\
&\quad - \frac{\frac{9a^2b\cos(fx+e)^3}{f}+\frac{13ab^2\cos(fx+e)^3}{f}+\frac{7ab^2\cos(fx+e)}{f}+\frac{11b^3\cos(fx+e)}{f}}{8\left(a\cos\left(fx+e\right)^2+b\right)^2a^4} \\
&\quad + \frac{a^6f^{17}\cos\left(fx+e\right)^3-3a^6f^{17}\cos\left(fx+e\right)-9a^5bf^{17}\cos\left(fx+e\right)}{3a^9f^{18}}
\end{aligned}$$

input
```
integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
5/8*(3*a*b + 7*b^2)*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^4*f) - 1
/8*(9*a^2*b*cos(f*x + e)^3/f + 13*a*b^2*cos(f*x + e)^3/f + 7*a*b^2*cos(f*x
 + e)/f + 11*b^3*cos(f*x + e)/f)/((a*cos(f*x + e)^2 + b)^2*a^4) + 1/3*(a^6
*f^17*cos(f*x + e)^3 - 3*a^6*f^17*cos(f*x + e) - 9*a^5*b*f^17*cos(f*x + e)
)/(a^9*f^18)
```

3.55.9 Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.12

$$\int \frac{\sin^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\cos(e+fx)^3}{3a^3 f} - \frac{\left(\frac{9a^2b}{8}+\frac{13ab^2}{8}\right)\cos(e+fx)^3 + \left(\frac{11b^3}{8}+\frac{7ab^2}{8}\right)\cos(e+fx)}{f\left(a^6\cos(e+fx)^4 + 2a^5 b\cos(e+fx)^2 + a^4b^2\right)} - \frac{\cos(e+fx)\left(\frac{3b}{a^4}+\frac{1}{a^3}\right)}{f} + \frac{5\sqrt{b}\,\operatorname{atan}\left(\frac{\sqrt{a}\sqrt{b}\,\cos(e+fx)\,(3a+7b)}{7b^2+3ab}\right)(3a+7b)}{8a^{9/2} f}$$

input
```
int(sin(e + f*x)^3/(a + b/cos(e + f*x)^2)^3,x)
```

output
```
cos(e + f*x)^3/(3*a^3*f) - (cos(e + f*x)^3*((13*a*b^2)/8 + (9*a^2*b)/8) +
cos(e + f*x)*((7*a*b^2)/8 + (11*b^3)/8))/(f*(a^4*b^2 + a^6*cos(e + f*x)^4
+ 2*a^5*b*cos(e + f*x)^2)) - (cos(e + f*x)*((3*b)/a^4 + 1/a^3))/f + (5*b^(
1/2)*atan((a^(1/2)*b^(1/2)*cos(e + f*x)*(3*a + 7*b))/(3*a*b + 7*b^2))*(3*a
 + 7*b))/(8*a^(9/2)*f)
```

3.56 $\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

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3.56.1 Optimal result

Integrand size = 21, antiderivative size = 116

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{15\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8a^{7/2}f} - \frac{15\cos(e+fx)}{8a^3 f} + \frac{\cos^5(e+fx)}{4af\left(b+a\cos^2(e+fx)\right)^2} + \frac{5\cos^3(e+fx)}{8a^2 f\left(b+a\cos^2(e+fx)\right)}$$

output
```
-15/8*cos(f*x+e)/a^3/f+1/4*cos(f*x+e)^5/a/f/(b+a*cos(f*x+e)^2)^2+5/8*cos(f
*x+e)^3/a^2/f/(b+a*cos(f*x+e)^2)+15/8*arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*b
^(1/2)/a^(7/2)/f
```

3.56.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.52 (sec) , antiderivative size = 656, normalized size of antiderivative = 5.66

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))^3\sec^6(e+fx)\left(15(a^3+64b^3)\arctan\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)}{\sqrt{}}\right.\right.}{}$$

input
```
Integrate[Sin[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])^3*Sec[e + f*x]^6*(15*(a^3 + 64*b^3)*ArcTan
[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/
2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2
]))/Sqrt[b]] + 15*(a^3 + 64*b^3)*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(C
os[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*
Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]] + (Sqrt[a]*(24*a^4*Sqr
t[b]*Cos[e + f*x] - 24*a^3*b^(3/2)*Cos[e + f*x] - 144*a^2*b^(5/2)*Cos[e +
f*x] + 512*b^(9/2)*Cos[e + f*x] - 72*a^3*b^(3/2)*Cos[e + f*x]*Cos[2*(e + f
*x)] - 24*a^3*Sqrt[b]*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)]) + 72*a^2
*b^(3/2)*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)]) - 1152*b^(7/2)*Cos[e
+ f*x]*(a + 2*b + a*Cos[2*(e + f*x)]) - 15*a^(5/2)*ArcTan[(Sqrt[a] - Sqrt[
a + b]*Tan[(e + f*x)/2])/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2 - 15*a^
(5/2)*ArcTan[(Sqrt[a] + Sqrt[a + b]*Tan[(e + f*x)/2])/Sqrt[b]]*(a + 2*b +
a*Cos[2*(e + f*x)])^2 - 512*b^(5/2)*Cos[e]*Cos[f*x]*(a + 2*b + a*Cos[2*(e
+ f*x)])^2 + 512*b^(5/2)*(a + 2*b + a*Cos[2*(e + f*x)])^2*Sin[e]*Sin[f*x]
+ 6*a^4*Sqrt[b]*Csc[e + f*x]*Sin[4*(e + f*x)]))/(a + 2*b + a*Cos[2*(e + f*
x)])^2))/(4096*a^(7/2)*b^(5/2)*f*(a + b*Sec[e + f*x]^2)^3)
```

3.56.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4621, 252, 252, 262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\sin(e+fx)}{(a+b\sec(e+fx)^2)^3}\,dx$$
$$\downarrow 4621$$
$$-\frac{\int \frac{\cos^6(e+fx)}{(a\cos^2(e+fx)+b)^3}\,d\cos(e+fx)}{f}$$
$$\downarrow 252$$

$$-\frac{\frac{5\int \frac{\cos^4(e+fx)}{\left(a\cos^2(e+fx)+b\right)^2}d\cos(e+fx)}{4a}-\frac{\cos^5(e+fx)}{4a(a\cos^2(e+fx)+b)^2}}{f}$$

$$\downarrow 252$$

$$-\frac{5\left(\frac{3\int \frac{\cos^2(e+fx)}{a\cos^2(e+fx)+b}d\cos(e+fx)}{2a}-\frac{\cos^3(e+fx)}{2a\left(a\cos^2(e+fx)+b\right)}\right)}{4a}-\frac{\cos^5(e+fx)}{4a(a\cos^2(e+fx)+b)^2}}{f}$$

$$\downarrow 262$$

$$-\frac{5\left(\frac{3\left(\frac{\cos(e+fx)}{a}-\frac{b\int \frac{1}{a\cos^2(e+fx)+b}d\cos(e+fx)}{a}\right)}{2a}-\frac{\cos^3(e+fx)}{2a\left(a\cos^2(e+fx)+b\right)}\right)}{4a}-\frac{\cos^5(e+fx)}{4a(a\cos^2(e+fx)+b)^2}}{f}$$

$$\downarrow 218$$

$$-\frac{5\left(\frac{3\left(\frac{\cos(e+fx)}{a}-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{a^{3/2}}\right)}{2a}-\frac{\cos^3(e+fx)}{2a\left(a\cos^2(e+fx)+b\right)}\right)}{4a}-\frac{\cos^5(e+fx)}{4a(a\cos^2(e+fx)+b)^2}}{f}$$

input
```
Int[Sin[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
-((-1/4*Cos[e + f*x]^5/(a*(b + a*Cos[e + f*x]^2)^2) + (5*((3*(-((Sqrt[b]*A
rcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/a^(3/2)) + Cos[e + f*x]/a))/(2*a) -
 Cos[e + f*x]^3/(2*a*(b + a*Cos[e + f*x]^2))))/(4*a))/f)
```

3.56.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

3.56.4 Maple [A] (verified)

Time = 2.95 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.72

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{7b\sec(fx+e)^3}{8}+\frac{9\sec(fx+e)a}{8}}{\left(a+b\sec(fx+e)^2\right)^2}+\frac{15\arctan\left(\frac{\sec(fx+e)b}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^3}-\frac{1}{a^3\sec(fx+e)}}{f}$
default	$\frac{-\frac{b\left(\frac{7b\sec(fx+e)^3}{8}+\frac{9\sec(fx+e)a}{8}}{\left(a+b\sec(fx+e)^2\right)^2}+\frac{15\arctan\left(\frac{\sec(fx+e)b}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^3}-\frac{1}{a^3\sec(fx+e)}}{f}$
risch	$-\frac{e^{i(fx+e)}}{2a^3f}-\frac{e^{-i(fx+e)}}{2a^3f}-\frac{b(9a\,e^{7i(fx+e)}+27a\,e^{5i(fx+e)}+28b\,e^{5i(fx+e)}+27a\,e^{3i(fx+e)}+28b\,e^{3i(fx+e)}+9a\,e^{i(fx+e)}}{4a^3\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)^2f}$

input
```
int(sin(f*x+e)/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/a^3*b*((7/8*b*sec(f*x+e)^3+9/8*sec(f*x+e)*a)/(a+b*sec(f*x+e)^2)^2+
15/8/(a*b)^(1/2)*arctan(sec(f*x+e)*b/(a*b)^(1/2)))-1/a^3/sec(f*x+e))
```

3.56.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.58

$$\int \frac{\sin(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$

$$= \left[-\frac{16\,a^2\cos(fx+e)^5+50\,ab\cos(fx+e)^3+30\,b^2\cos(fx+e)-15\left(a^2\cos(fx+e)^4+2\,ab\cos(fx+e)\right.}{16\left(a^5f\cos(fx+e)^4+2\,a^4bf\cos(fx+e)^2+a^3b^2f\right.} \right.$$

$$\left. -\frac{8\,a^2\cos(fx+e)^5+25\,ab\cos(fx+e)^3+15\,b^2\cos(fx+e)-15\left(a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2\right.}{8\left(a^5f\cos(fx+e)^4+2\,a^4bf\cos(fx+e)^2+a^3b^2f\right)} \right.$$

input
```
integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[-1/16*(16*a^2*cos(f*x + e)^5 + 50*a*b*cos(f*x + e)^3 + 30*b^2*cos(f*x + e
) - 15*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*sqrt(-b/a)*log(-(
a*cos(f*x + e)^2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)
))/(a^5*f*cos(f*x + e)^4 + 2*a^4*b*f*cos(f*x + e)^2 + a^3*b^2*f), -1/8*(8*
a^2*cos(f*x + e)^5 + 25*a*b*cos(f*x + e)^3 + 15*b^2*cos(f*x + e) - 15*(a^2
*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*sqrt(b/a)*arctan(a*sqrt(b/a)
*cos(f*x + e)/b))/(a^5*f*cos(f*x + e)^4 + 2*a^4*b*f*cos(f*x + e)^2 + a^3*b
^2*f)]
```

3.56.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Timed out
```

3.56.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.89

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{9\,ab\cos(fx+e)^3+7\,b^2\cos(fx+e)}{a^5\cos(fx+e)^4+2\,a^4b\cos(fx+e)^2+a^3b^2} - \frac{15\,b\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{\sqrt{ab}a^3} + \frac{8\cos(fx+e)}{a^3}}{8\,f}$$

input
```
integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
-1/8*((9*a*b*cos(f*x + e)^3 + 7*b^2*cos(f*x + e))/(a^5*cos(f*x + e)^4 + 2*
a^4*b*cos(f*x + e)^2 + a^3*b^2) - 15*b*arctan(a*cos(f*x + e)/sqrt(a*b))/(s
qrt(a*b)*a^3) + 8*cos(f*x + e)/a^3)/f
```

3.56.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.79

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{15\,b\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{8\sqrt{ab}a^3 f} - \frac{\cos(fx+e)}{a^3 f} - \frac{\frac{9\,ab\cos(fx+e)^3}{f} + \frac{7\,b^2\cos(fx+e)}{f}}{8\left(a\cos(fx+e)^2+b\right)^2 a^3}$$

input
```
integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
15/8*b*arctan(a*cos(f*x + e)/sqrt(a*b))/(sqrt(a*b)*a^3*f) - cos(f*x + e)/(
a^3*f) - 1/8*(9*a*b*cos(f*x + e)^3/f + 7*b^2*cos(f*x + e)/f)/((a*cos(f*x +
 e)^2 + b)^2*a^3)
```

3.56.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.91

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{15\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\cos(e+f\,x)}{\sqrt{b}}\right)}{8\,a^{7/2}\,f} - \frac{\frac{7\,b^2\,\cos(e+f\,x)}{8} + \frac{9\,a\,b\cos(e+f\,x)^3}{8}}{f\,\left(a^5\cos(e+f\,x)^4 + 2\,a^4\,b\cos(e+f\,x)^2 + a^3\,b^2\right)} - \frac{\cos(e+f\,x)}{a^3\,f}$$

input
```
int(sin(e + f*x)/(a + b/cos(e + f*x)^2)^3,x)
```

output
```
(15*b^(1/2)*atan((a^(1/2)*cos(e + f*x))/b^(1/2)))/(8*a^(7/2)*f) - ((7*b^2*
cos(e + f*x))/8 + (9*a*b*cos(e + f*x)^3)/8)/(f*(a^3*b^2 + a^5*cos(e + f*x)
^4 + 2*a^4*b*cos(e + f*x)^2)) - cos(e + f*x)/(a^3*f)
```

3.57 $\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

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3.57.1 Optimal result

Integrand size = 21, antiderivative size = 154

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\sqrt{b}(15a^2+10ab+3b^2)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8a^{5/2}(a+b)^3 f} - \frac{\operatorname{arctanh}(\cos(e+fx))}{(a+b)^3 f} - \frac{b\cos^3(e+fx)}{4a(a+b)f\left(b+a\cos^2(e+fx)\right)^2} - \frac{b(7a+3b)\cos(e+fx)}{8a^2(a+b)^2 f\left(b+a\cos^2(e+fx)\right)}$$

output
```
-arctanh(cos(f*x+e))/(a+b)^3/f-1/4*b*cos(f*x+e)^3/a/(a+b)/f/(b+a*cos(f*x+e
)^2)^2-1/8*b*(7*a+3*b)*cos(f*x+e)/a^2/(a+b)^2/f/(b+a*cos(f*x+e)^2)+1/8*(15
*a^2+10*a*b+3*b^2)*arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*b^(1/2)/a^(5/2)/(a+b
)^3/f
```

3.57.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.22 (sec) , antiderivative size = 447, normalized size of antiderivative = 2.90

$$\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^5(e+fx)\left(\frac{8b^2(a+b)^2}{a^2} - \frac{2b(a+b)(9a+5b)(a+2b+a\cos(2(e+fx)))}{a^2} + \frac{\sqrt{b}(15a^2+10ab+3b^2)\arc}{}\right.}{}$$

input
```
Integrate[Csc[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^5*((8*b^2*(a + b)^2)/a^2 - (2
*b*(a + b)*(9*a + 5*b)*(a + 2*b + a*Cos[2*(e + f*x)]))/a^2 + (Sqrt[b]*(15*
a^2 + 10*a*b + 3*b^2)*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*S
in[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e
] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2*
Sec[e + f*x])/a^(5/2) + (Sqrt[b]*(15*a^2 + 10*a*b + 3*b^2)*ArcTan[((-Sqrt[
a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[
e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[
b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2*Sec[e + f*x])/a^(5/2) - 8*(a + 2*b +
a*Cos[2*(e + f*x)])^2*Log[Cos[(e + f*x)/2]]*Sec[e + f*x] + 8*(a + 2*b + a*
Cos[2*(e + f*x)])^2*Log[Sin[(e + f*x)/2]]*Sec[e + f*x]))/(64*(a + b)^3*f*(
a + b*Sec[e + f*x]^2)^3)
```

3.57.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4621, 372, 440, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sin(e+fx)\left(a+b\sec(e+fx)^2\right)^3}dx$$

$$\downarrow 4621$$

$$-\frac{\int \frac{\cos^6(e+fx)}{(1-\cos^2(e+fx))(a\cos^2(e+fx)+b)^3}\,d\cos(e+fx)}{f}$$

$$\downarrow 372$$

$$-\frac{\frac{b\cos^3(e+fx)}{4a(a+b)(a\cos^2(e+fx)+b)^2}-\frac{\int \frac{\cos^2(e+fx)\left(3b-(4a+3b)\cos^2(e+fx)\right)}{\left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)^2}\,d\cos(e+fx)}{4a(a+b)}}{f}$$

$$\downarrow 440$$

$$-\frac{\frac{b\cos^3(e+fx)}{4a(a+b)(a\cos^2(e+fx)+b)^2}-\frac{\frac{\int \frac{b(7a+3b)-\left(8a^2+7ba+3b^2\right)\cos^2(e+fx)}{\left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)}\,d\cos(e+fx)}{2a(a+b)}-\frac{b(7a+3b)\cos(e+fx)}{2a(a+b)\left(a\cos^2(e+fx)+b\right)}}{4a(a+b)}}{f}$$

$$\downarrow 397$$

$$-\frac{\frac{b\cos^3(e+fx)}{4a(a+b)(a\cos^2(e+fx)+b)^2}-\frac{\frac{b\left(15a^2+10ab+3b^2\right)\int \frac{1}{a\cos^2(e+fx)+b}\,d\cos(e+fx)}{a+b}-8a^2\int \frac{1}{1-\cos^2(e+fx)}\,d\cos(e+fx)}{2a(a+b)}-\frac{b(7a+3b)\cos(e+fx)}{2a(a+b)\left(a\cos^2(e+fx)+b\right)}}{4a(a+b)}}{f}$$

$$\downarrow 218$$

$$-\frac{\frac{b\cos^3(e+fx)}{4a(a+b)(a\cos^2(e+fx)+b)^2}-\frac{\frac{\sqrt{b}\left(15a^2+10ab+3b^2\right)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{\sqrt{a}(a+b)}-8a^2\int \frac{1}{1-\cos^2(e+fx)}\,d\cos(e+fx)}{2a(a+b)}-\frac{b(7a+3b)\cos(e+fx)}{2a(a+b)\left(a\cos^2(e+fx)+b\right)}}{4a(a+b)}}{f}$$

$$\downarrow 219$$

$$-\frac{\frac{b\cos^3(e+fx)}{4a(a+b)(a\cos^2(e+fx)+b)^2}-\frac{\frac{\sqrt{b}\left(15a^2+10ab+3b^2\right)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{\sqrt{a}(a+b)}-\frac{8a^2\operatorname{arctanh}(\cos(e+fx))}{a+b}}{2a(a+b)}-\frac{b(7a+3b)\cos(e+fx)}{2a(a+b)\left(a\cos^2(e+fx)+b\right)}}{4a(a+b)}}{f}$$

input
```
Int[Csc[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

3.57. $\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

output
```
-(((b*Cos[e + f*x]^3)/(4*a*(a + b)*(b + a*Cos[e + f*x]^2)^2) - (((Sqrt[b]*
(15*a^2 + 10*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]])/(Sqrt[a]
*(a + b)) - (8*a^2*ArcTanh[Cos[e + f*x]])/(a + b))/(2*a*(a + b)) - (b*(7*a
 + 3*b)*Cos[e + f*x])/(2*a*(a + b)*(b + a*Cos[e + f*x]^2)))/(4*a*(a + b)))
/f)
```

3.57.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

3.57.4 Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{\ln(-1+\cos(fx+e))}{2(a+b)^3}+\frac{b\left(\frac{-\frac{\left(9a^2+14ab+5b^2\right)\cos(fx+e)^3}{8a}-\frac{b\left(7a^2+10ab+3b^2\right)\cos(fx+e)}{8a^2}}{\left(b+a\cos(fx+e)^2\right)^2}+\frac{\left(15a^2+10ab+3b^2\right)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{8a^2\sqrt{ab}}\right)}{(a+b)^3}}{f}$
default	$\frac{\ln(-1+\cos(fx+e))}{2(a+b)^3}+\frac{b\left(\frac{-\frac{\left(9a^2+14ab+5b^2\right)\cos(fx+e)^3}{8a}-\frac{b\left(7a^2+10ab+3b^2\right)\cos(fx+e)}{8a^2}}{\left(b+a\cos(fx+e)^2\right)^2}+\frac{\left(15a^2+10ab+3b^2\right)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{8a^2\sqrt{ab}}\right)}{(a+b)^3}}{f}$
risch	$-\frac{b\left(9a^2e^{7i(fx+e)}+5abe^{7i(fx+e)}+27a^2e^{5i(fx+e)}+43abe^{5i(fx+e)}+12b^2e^{5i(fx+e)}+27a^2e^{3i(fx+e)}+43abe^{3i(fx+e)}+12b^2\right.}{4a^2(a+b)^2f\left(ae^{4i(fx+e)}+2ae^{2i(fx+e)}+4be^{2i(fx+e)}+a\right)^2}$

input
```
int(csc(f*x+e)/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/2/(a+b)^3*ln(-1+cos(f*x+e))+b/(a+b)^3*((-1/8*(9*a^2+14*a*b+5*b^2)/a
*cos(f*x+e)^3-1/8*b*(7*a^2+10*a*b+3*b^2)/a^2*cos(f*x+e))/(b+a*cos(f*x+e)^2
)^2+1/8*(15*a^2+10*a*b+3*b^2)/a^2/(a*b)^(1/2)*arctan(a*cos(f*x+e)/(a*b)^(1
/2)))-1/2/(a+b)^3*ln(1+cos(f*x+e)))
```

3.57.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(140) = 280$.

Time = 0.40 (sec) , antiderivative size = 779, normalized size of antiderivative = 5.06

$$\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$

$$= \left[-\frac{2\left(9\,a^3b+14\,a^2b^2+5\,ab^3\right)\cos\left(fx+e\right)^3-\left(\left(15\,a^4+10\,a^3b+3\,a^2b^2\right)\cos\left(fx+e\right)^4+15\,a^2b^2+10\,ab^3+\right.}{} \right.$$

$$\left. -\frac{\left(9\,a^3b+14\,a^2b^2+5\,ab^3\right)\cos\left(fx+e\right)^3-\left(\left(15\,a^4+10\,a^3b+3\,a^2b^2\right)\cos\left(fx+e\right)^4+15\,a^2b^2+10\,ab^3+3\right.}{} \right.$$

input
```
integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[-1/16*(2*(9*a^3*b + 14*a^2*b^2 + 5*a*b^3)*cos(f*x + e)^3 - ((15*a^4 + 10*
a^3*b + 3*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2 + 10*a*b^3 + 3*b^4 + 2*(15*
a^3*b + 10*a^2*b^2 + 3*a*b^3)*cos(f*x + e)^2)*sqrt(-b/a)*log(-(a*cos(f*x +
 e)^2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) + 2*(7*a^
2*b^2 + 10*a*b^3 + 3*b^4)*cos(f*x + e) + 8*(a^4*cos(f*x + e)^4 + 2*a^3*b*c
os(f*x + e)^2 + a^2*b^2)*log(1/2*cos(f*x + e) + 1/2) - 8*(a^4*cos(f*x + e)
^4 + 2*a^3*b*cos(f*x + e)^2 + a^2*b^2)*log(-1/2*cos(f*x + e) + 1/2))/((a^7
 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^4 + 2*(a^6*b + 3*a^5*b^2
+ 3*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^2 + (a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4
 + a^2*b^5)*f), -1/8*((9*a^3*b + 14*a^2*b^2 + 5*a*b^3)*cos(f*x + e)^3 - ((
15*a^4 + 10*a^3*b + 3*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2 + 10*a*b^3 + 3*
b^4 + 2*(15*a^3*b + 10*a^2*b^2 + 3*a*b^3)*cos(f*x + e)^2)*sqrt(b/a)*arctan
(a*sqrt(b/a)*cos(f*x + e)/b) + (7*a^2*b^2 + 10*a*b^3 + 3*b^4)*cos(f*x + e)
 + 4*(a^4*cos(f*x + e)^4 + 2*a^3*b*cos(f*x + e)^2 + a^2*b^2)*log(1/2*cos(f
*x + e) + 1/2) - 4*(a^4*cos(f*x + e)^4 + 2*a^3*b*cos(f*x + e)^2 + a^2*b^2)
*log(-1/2*cos(f*x + e) + 1/2))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*f*co
s(f*x + e)^4 + 2*(a^6*b + 3*a^5*b^2 + 3*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^
2 + (a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^5)*f)]
```

3.57.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Timed out
```

3.57.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.69

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \frac{(15a^2b+10ab^2+3b^3)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{(a^5+3a^4b+3a^3b^2+a^2b^3)\sqrt{ab}} - \frac{(9a^2b+5ab^2)\cos(fx+e)^3+(7ab^2+3b^3)\cos(fx+e)}{a^4b^2+2a^3b^3+a^2b^4+(a^6+2a^5b+a^4b^2)\cos(fx+e)^4+2(a^5b+2a^4b^2+a^3b^3)\cos(fx+e)^2} - \frac{4\log(c}{a^3+3}$$
$$\overline{\qquad\qquad 8f}$$

input
```
integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
1/8*((15*a^2*b + 10*a*b^2 + 3*b^3)*arctan(a*cos(f*x + e)/sqrt(a*b))/((a^5
+ 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sqrt(a*b)) - ((9*a^2*b + 5*a*b^2)*cos(f*x
 + e)^3 + (7*a*b^2 + 3*b^3)*cos(f*x + e))/(a^4*b^2 + 2*a^3*b^3 + a^2*b^4 +
 (a^6 + 2*a^5*b + a^4*b^2)*cos(f*x + e)^4 + 2*(a^5*b + 2*a^4*b^2 + a^3*b^3
)*cos(f*x + e)^2) - 4*log(cos(f*x + e) + 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3
) + 4*log(cos(f*x + e) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3))/f
```

3.57.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 592 vs. 2(140) = 280.

Time = 0.41 (sec) , antiderivative size = 592, normalized size of antiderivative = 3.84

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx =$$
$$\frac{(15a^2b+10ab^2+3b^3)\arctan\left(-\frac{a\cos(fx+e)-b}{\sqrt{ab}\cos(fx+e)+\sqrt{ab}}\right)}{(a^5+3a^4b+3a^3b^2+a^2b^3)\sqrt{ab}} - \frac{4\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a^3+3a^2b+3ab^2+b^3} + \frac{2\left(9a^3b+21a^2b^2+15ab^3+3b^4+\frac{27a^3b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{1}{}\right.}{}$$
$$-\frac{}{}$$

input
```
integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
-1/8*((15*a^2*b + 10*a*b^2 + 3*b^3)*arctan(-(a*cos(f*x + e) - b)/(sqrt(a*b
)*cos(f*x + e) + sqrt(a*b)))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sqrt(a
*b)) - 4*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1))/(a^3 + 3*a^2*b
+ 3*a*b^2 + b^3) + 2*(9*a^3*b + 21*a^2*b^2 + 15*a*b^3 + 3*b^4 + 27*a^3*b*(
cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 13*a^2*b^2*(cos(f*x + e) - 1)/(cos(
f*x + e) + 1) - 23*a*b^3*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 9*b^4*(co
s(f*x + e) - 1)/(cos(f*x + e) + 1) + 27*a^3*b*(cos(f*x + e) - 1)^2/(cos(f*
x + e) + 1)^2 - 9*a^2*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 21*a
*b^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 9*b^4*(cos(f*x + e) - 1)^
2/(cos(f*x + e) + 1)^2 + 9*a^3*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3
 - a^2*b^2*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 13*a*b^3*(cos(f*x +
 e) - 1)^3/(cos(f*x + e) + 1)^3 - 3*b^4*(cos(f*x + e) - 1)^3/(cos(f*x + e)
 + 1)^3)/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*(a + b + 2*a*(cos(f*x + e)
 - 1)/(cos(f*x + e) + 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(
cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x
 + e) + 1)^2)^2))/f
```

3.57.9 Mupad [B] (verification not implemented)

Time = 21.90 (sec) , antiderivative size = 3557, normalized size of antiderivative = 23.10

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(e + f*x)*(a + b/cos(e + f*x)^2)^3),x)
```

output

```
(atan((((-a^5*b)^(1/2)*((cos(e + f*x)*(60*a*b^5 + 64*a^6 + 9*b^6 + 190*a^2
*b^4 + 300*a^3*b^3 + 225*a^4*b^2))/(32*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^
3 + 6*a^5*b^2)) + (((224*a^10*b + 96*a^3*b^8 + 800*a^4*b^7 + 2784*a^5*b^6
+ 5280*a^6*b^5 + 5920*a^7*b^4 + 3936*a^8*b^3 + 1440*a^9*b^2)/(64*(6*a^8*b
+ a^9 + a^3*b^6 + 6*a^4*b^5 + 15*a^5*b^4 + 20*a^6*b^3 + 15*a^7*b^2)) - (co
s(e + f*x)*(-a^5*b)^(1/2)*(10*a*b + 15*a^2 + 3*b^2)*(1280*a^11*b + 256*a^1
2 - 256*a^5*b^7 - 1280*a^6*b^6 - 2304*a^7*b^5 - 1280*a^8*b^4 + 1280*a^9*b^
3 + 2304*a^10*b^2))/(512*(3*a^7*b + a^8 + a^5*b^3 + 3*a^6*b^2)*(4*a^6*b +
a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)))*(-a^5*b)^(1/2)*(10*a*b + 15*a^2 +
 3*b^2))/(16*(3*a^7*b + a^8 + a^5*b^3 + 3*a^6*b^2)))*(10*a*b + 15*a^2 + 3*
b^2)*1i)/(16*(3*a^7*b + a^8 + a^5*b^3 + 3*a^6*b^2)) + ((-a^5*b)^(1/2)*((co
s(e + f*x)*(60*a*b^5 + 64*a^6 + 9*b^6 + 190*a^2*b^4 + 300*a^3*b^3 + 225*a^
4*b^2))/(32*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)) - (((224*a^
10*b + 96*a^3*b^8 + 800*a^4*b^7 + 2784*a^5*b^6 + 5280*a^6*b^5 + 5920*a^7*b
^4 + 3936*a^8*b^3 + 1440*a^9*b^2)/(64*(6*a^8*b + a^9 + a^3*b^6 + 6*a^4*b^5
 + 15*a^5*b^4 + 20*a^6*b^3 + 15*a^7*b^2)) + (cos(e + f*x)*(-a^5*b)^(1/2)*(
10*a*b + 15*a^2 + 3*b^2)*(1280*a^11*b + 256*a^12 - 256*a^5*b^7 - 1280*a^6*
b^6 - 2304*a^7*b^5 - 1280*a^8*b^4 + 1280*a^9*b^3 + 2304*a^10*b^2))/(512*(3
*a^7*b + a^8 + a^5*b^3 + 3*a^6*b^2)*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 +
 6*a^5*b^2)))*(-a^5*b)^(1/2)*(10*a*b + 15*a^2 + 3*b^2))/(16*(3*a^7*b + ...
```

3.58 $\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

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3.58.1 Optimal result

Integrand size = 23, antiderivative size = 213

$$\begin{aligned}\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = &\frac{\sqrt{b}(15a^2-10ab-b^2)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8a^{3/2}(a+b)^4 f}\\ &-\frac{(a-5b)\operatorname{arctanh}(\cos(e+fx))}{2(a+b)^4 f}\\ &-\frac{(2a-b)b\cos(e+fx)}{4a(a+b)^2 f\left(b+a\cos^2(e+fx)\right)^2}\\ &+\frac{(4a^2-9ab-b^2)\cos(e+fx)}{8a(a+b)^3 f\left(b+a\cos^2(e+fx)\right)}\\ &-\frac{\cos(e+fx)\cot^2(e+fx)}{2(a+b)f\left(b+a\cos^2(e+fx)\right)^2}\end{aligned}$$

output

```
-1/2*(a-5*b)*arctanh(cos(f*x+e))/(a+b)^4/f-1/4*(2*a-b)*b*cos(f*x+e)/a/(a+b
)^2/f/(b+a*cos(f*x+e)^2)^2+1/8*(4*a^2-9*a*b-b^2)*cos(f*x+e)/a/(a+b)^3/f/(b
+a*cos(f*x+e)^2)-1/2*cos(f*x+e)*cot(f*x+e)^2/(a+b)/f/(b+a*cos(f*x+e)^2)^2+
1/8*(15*a^2-10*a*b-b^2)*arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*b^(1/2)/a^(3/2)
/(a+b)^4/f
```

3.58.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.36 (sec) , antiderivative size = 532, normalized size of antiderivative = 2.50

$$\int \frac{\csc^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^5(e+fx)\left(\frac{8b^2(a+b)^2}{a} - \frac{2b(a+b)(9a+b)(a+2b+a\cos(2(e+fx)))}{a} - \frac{\sqrt{b}(-15a^2+10ab+b^2)\arc}{}\right.}{}$$

input
```
Integrate[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^5*((8*b^2*(a + b)^2)/a - (2*b
*(a + b)*(9*a + b)*(a + 2*b + a*Cos[2*(e + f*x)]))/a - (Sqrt[b]*(-15*a^2 +
 10*a*b + b^2)*ArcTan[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^
2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*S
in[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2*Sec[e +
 f*x])/a^(3/2) - (Sqrt[b]*(-15*a^2 + 10*a*b + b^2)*ArcTan[((-Sqrt[a] + I*S
qrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt
[a] + Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a +
 2*b + a*Cos[2*(e + f*x)])^2*Sec[e + f*x])/a^(3/2) - (a + b)*(a + 2*b + a*
Cos[2*(e + f*x)])^2*Csc[(e + f*x)/2]^2*Sec[e + f*x] - 4*(a - 5*b)*(a + 2*b
 + a*Cos[2*(e + f*x)])^2*Log[Cos[(e + f*x)/2]]*Sec[e + f*x] + 4*(a - 5*b)*
(a + 2*b + a*Cos[2*(e + f*x)])^2*Log[Sin[(e + f*x)/2]]*Sec[e + f*x] + (a +
 b)*(a + 2*b + a*Cos[2*(e + f*x)])^2*Sec[(e + f*x)/2]^2*Sec[e + f*x]))/(64
*(a + b)^4*f*(a + b*Sec[e + f*x]^2)^3)
```

3.58.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.11, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 4621, 372, 440, 27, 402, 25, 27, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sin(e+fx)^3\,(a+b\sec(e+fx)^2)^3}dx$$

$$\downarrow 4621$$

$$-\frac{\int \frac{\cos^6(e+fx)}{(1-\cos^2(e+fx))^2(a\cos^2(e+fx)+b)^3}d\cos(e+fx)}{f}$$

$$\downarrow 372$$

$$-\frac{\frac{\cos^3(e+fx)}{2(a+b)(1-\cos^2(e+fx))(a\cos^2(e+fx)+b)^2}-\frac{\int \frac{\cos^2(e+fx)\left(3b-(a-2b)\cos^2(e+fx)\right)}{\left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)^3}d\cos(e+fx)}{2(a+b)}}{f}$$

$$\downarrow 440$$

$$-\frac{\frac{\cos^3(e+fx)}{2(a+b)(1-\cos^2(e+fx))(a\cos^2(e+fx)+b)^2}-\frac{\frac{\int \frac{2\left((2a-b)b-\left(2a^2-8ba-b^2\right)\cos^2(e+fx)\right)}{\left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)^2}d\cos(e+fx)}{4a(a+b)}-\frac{b(2a-b)\cos(e+fx)}{2a(a+b)\left(a\cos^2(e+fx)+b\right)^2}}{2(a+b)}}{f}$$

$$\downarrow 27$$

$$-\frac{\frac{\cos^3(e+fx)}{2(a+b)(1-\cos^2(e+fx))(a\cos^2(e+fx)+b)^2}-\frac{\frac{\int \frac{(2a-b)b-\left(2a^2-8ba-b^2\right)\cos^2(e+fx)}{\left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)^2}d\cos(e+fx)}{2a(a+b)}-\frac{b(2a-b)\cos(e+fx)}{2a(a+b)\left(a\cos^2(e+fx)+b\right)^2}}{2(a+b)}}{f}$$

$$\downarrow 402$$

$$-\frac{\frac{\cos^3(e+fx)}{2(a+b)(1-\cos^2(e+fx))(a\cos^2(e+fx)+b)^2}-\frac{\frac{\left(4a^2-9ab-b^2\right)\cos(e+fx)}{2(a+b)\left(a\cos^2(e+fx)+b\right)}-\frac{\int -\frac{b\left((11a-b)b-\left(4a^2-9ba-b^2\right)\cos^2(e+fx)\right)}{\left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)}d\cos(e+fx)}{2b(a+b)}}{2a(a+b)}-\frac{b(2a-b)\cos}{2a(a+b)\left(a\cos^2(}}{2(a+b)}}{f}$$

$$\downarrow 25$$

$$-\frac{\cos^3(e+fx)}{2(a+b)(1-\cos^2(e+fx))(a\cos^2(e+fx)+b)^2}-\frac{\dfrac{\int\frac{b\left((11a-b)b-\left(4a^2-9ba-b^2\right)\cos^2(e+fx)\right)}{\left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)}d\cos(e+fx)}{2b(a+b)}+\frac{\left(4a^2-9ab-b^2\right)\cos(e+fx)}{2(a+b)\left(a\cos^2(e+fx)+b\right)}}{2a(a+b)}-\frac{b(2a-b)\cos(e}{2a(a+b)\left(a\cos^2(e+}}{2(a+b)}$$

$$f$$

$$\downarrow 27$$

$$-\frac{\cos^3(e+fx)}{2(a+b)(1-\cos^2(e+fx))(a\cos^2(e+fx)+b)^2}-\frac{\dfrac{\int\frac{(11a-b)b-\left(4a^2-9ba-b^2\right)\cos^2(e+fx)}{\left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)}d\cos(e+fx)}{2(a+b)}+\frac{\left(4a^2-9ab-b^2\right)\cos(e+fx)}{2(a+b)\left(a\cos^2(e+fx)+b\right)}}{2a(a+b)}-\frac{b(2a-b)\cos(e+fx}{2a(a+b)\left(a\cos^2(e+fx}}{2(a+b)}$$

$$f$$

$$\downarrow 397$$

$$-\frac{\cos^3(e+fx)}{2(a+b)(1-\cos^2(e+fx))(a\cos^2(e+fx)+b)^2}-\frac{\dfrac{\frac{b\left(15a^2-10ab-b^2\right)\int\frac{1}{a\cos^2(e+fx)+b}d\cos(e+fx)}{a+b}-\frac{4a(a-5b)\int\frac{1}{1-\cos^2(e+fx)}d\cos(e+fx)}{a+b}}{2(a+b)}+\frac{\left(4a^2-9ab-b}{2(a+b)\left(a\cos}}{2a(a+b)}}{2(a+b)}$$

$$f$$

$$\downarrow 218$$

$$-\frac{\cos^3(e+fx)}{2(a+b)(1-\cos^2(e+fx))(a\cos^2(e+fx)+b)^2}-\frac{\dfrac{\frac{\sqrt{b}\left(15a^2-10ab-b^2\right)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{\sqrt{a}(a+b)}-\frac{4a(a-5b)\int\frac{1}{1-\cos^2(e+fx)}d\cos(e+fx)}{a+b}}{2(a+b)}+\frac{\left(4a^2-9ab-b^2\right)\cos}{2(a+b)\left(a\cos^2(e+}}{2a(a+b)}}{2(a+b)}$$

$$f$$

$$\downarrow 219$$

$$-\frac{\cos^3(e+fx)}{2(a+b)(1-\cos^2(e+fx))(a\cos^2(e+fx)+b)^2}-\frac{\dfrac{\frac{\sqrt{b}\left(15a^2-10ab-b^2\right)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{\sqrt{a}(a+b)}-\frac{4a(a-5b)\operatorname{arctanh}(\cos(e+fx))}{a+b}}{2(a+b)}+\frac{\left(4a^2-9ab-b^2\right)\cos(e+}{2(a+b)\left(a\cos^2(e+fx)+}}{2a(a+b)}}{2(a+b)}$$

$$f$$

input
```
Int[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

3.58. $\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

output
```
-((Cos[e + f*x]^3/(2*(a + b)*(1 - Cos[e + f*x]^2)*(b + a*Cos[e + f*x]^2)^2
) - (-1/2*((2*a - b)*b*Cos[e + f*x])/(a*(a + b)*(b + a*Cos[e + f*x]^2)^2)
+ (((Sqrt[b]*(15*a^2 - 10*a*b - b^2)*ArcTan[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]
])/(Sqrt[a]*(a + b)) - (4*a*(a - 5*b)*ArcTanh[Cos[e + f*x]])/(a + b))/(2*(
a + b)) + ((4*a^2 - 9*a*b - b^2)*Cos[e + f*x])/(2*(a + b)*(b + a*Cos[e + f
*x]^2)))/(2*a*(a + b)))/(2*(a + b)))/f)
```

3.58.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

3.58.4 Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{1}{4(a+b)^3(-1+\cos(fx+e))}+\frac{(a-5b)\ln(-1+\cos(fx+e))}{4(a+b)^4}+\frac{b\left(\frac{\left(-\frac{9}{8}a^2-\frac{5}{4}ab-\frac{1}{8}b^2\right)\cos(fx+e)^3-\frac{b\left(7a^2+6ab-b^2\right)\cos(fx+e)}{8a}}{\left(b+a\cos(fx+e)^2\right)^2}+\frac{\left(15a^2-$
default	$\frac{1}{4(a+b)^3(-1+\cos(fx+e))}+\frac{(a-5b)\ln(-1+\cos(fx+e))}{4(a+b)^4}+\frac{b\left(\frac{\left(-\frac{9}{8}a^2-\frac{5}{4}ab-\frac{1}{8}b^2\right)\cos(fx+e)^3-\frac{b\left(7a^2+6ab-b^2\right)\cos(fx+e)}{8a}}{\left(b+a\cos(fx+e)^2\right)^2}+\frac{\left(15a^2-$
risch	$-\frac{-4a^3e^{11i(fx+e)}+9a^2be^{11i(fx+e)}+ab^2e^{11i(fx+e)}-20a^3e^{9i(fx+e)}-23a^2be^{9i(fx+e)}+29ab^2e^{9i(fx+e)}-4b^3e^{9i(fx+e)}-$

input
```
int(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/4/(a+b)^3/(-1+cos(f*x+e))+1/4*(a-5*b)/(a+b)^4*ln(-1+cos(f*x+e))+b/(
a+b)^4*(((-9/8*a^2-5/4*a*b-1/8*b^2)*cos(f*x+e)^3-1/8*b*(7*a^2+6*a*b-b^2)/a
*cos(f*x+e))/(b+a*cos(f*x+e)^2)^2+1/8*(15*a^2-10*a*b-b^2)/a/(a*b)^(1/2)*ar
ctan(a*cos(f*x+e)/(a*b)^(1/2)))+1/4/(a+b)^3/(1+cos(f*x+e))+1/4/(a+b)^4*(-a
+5*b)*ln(1+cos(f*x+e)))
```

3.58.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 649 vs. $2(195) = 390$.

Time $= 0.43$ (sec) , antiderivative size $= 1332$, normalized size of antiderivative $= 6.25$

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output

```
[1/16*(2*(4*a^4 - 5*a^3*b - 10*a^2*b^2 - a*b^3)*cos(f*x + e)^5 + 2*(17*a^3
*b + 11*a^2*b^2 - 5*a*b^3 + b^4)*cos(f*x + e)^3 - ((15*a^4 - 10*a^3*b - a^
2*b^2)*cos(f*x + e)^6 - (15*a^4 - 40*a^3*b + 19*a^2*b^2 + 2*a*b^3)*cos(f*x
 + e)^4 - 15*a^2*b^2 + 10*a*b^3 + b^4 - (30*a^3*b - 35*a^2*b^2 + 8*a*b^3 +
 b^4)*cos(f*x + e)^2)*sqrt(-b/a)*log((a*cos(f*x + e)^2 - 2*a*sqrt(-b/a)*co
s(f*x + e) - b)/(a*cos(f*x + e)^2 + b)) + 2*(11*a^2*b^2 + 10*a*b^3 - b^4)*
cos(f*x + e) - 4*((a^4 - 5*a^3*b)*cos(f*x + e)^6 - (a^4 - 7*a^3*b + 10*a^2
*b^2)*cos(f*x + e)^4 - a^2*b^2 + 5*a*b^3 - (2*a^3*b - 11*a^2*b^2 + 5*a*b^3
)*cos(f*x + e)^2)*log(1/2*cos(f*x + e) + 1/2) + 4*((a^4 - 5*a^3*b)*cos(f*x
 + e)^6 - (a^4 - 7*a^3*b + 10*a^2*b^2)*cos(f*x + e)^4 - a^2*b^2 + 5*a*b^3
 - (2*a^3*b - 11*a^2*b^2 + 5*a*b^3)*cos(f*x + e)^2)*log(-1/2*cos(f*x + e) +
 1/2))/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^6
 - (a^7 + 2*a^6*b - 2*a^5*b^2 - 8*a^4*b^3 - 7*a^3*b^4 - 2*a^2*b^5)*f*cos(f
*x + e)^4 - (2*a^6*b + 7*a^5*b^2 + 8*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - a*b
^6)*f*cos(f*x + e)^2 - (a^5*b^2 + 4*a^4*b^3 + 6*a^3*b^4 + 4*a^2*b^5 + a*b^
6)*f), 1/8*((4*a^4 - 5*a^3*b - 10*a^2*b^2 - a*b^3)*cos(f*x + e)^5 + (17*a^
3*b + 11*a^2*b^2 - 5*a*b^3 + b^4)*cos(f*x + e)^3 + ((15*a^4 - 10*a^3*b - a
^2*b^2)*cos(f*x + e)^6 - (15*a^4 - 40*a^3*b + 19*a^2*b^2 + 2*a*b^3)*cos(f*
x + e)^4 - 15*a^2*b^2 + 10*a*b^3 + b^4 - (30*a^3*b - 35*a^2*b^2 + 8*a*b^3
+ b^4)*cos(f*x + e)^2)*sqrt(b/a)*arctan(a*sqrt(b/a)*cos(f*x + e)/b) + (...
```

3.58.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

input

```
integrate(csc(f*x+e)**3/(a+b*sec(f*x+e)**2)**3,x)
```

output

```
Timed out
```

3.58.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 399 vs. $2(195) = 390$.

Time = 0.27 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.87

$$\int \frac{\csc^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx =$$

$$-\frac{\frac{2\,(a-5\,b)\log(\cos(fx+e)+1)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}-\frac{2\,(a-5\,b)\log(\cos(fx+e)-1)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}-\frac{\left(15\,a^2b-10\,ab^2-b^3\right)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{\left(a^5+4\,a^4b+6\,a^3b^2+4\,a^2b^3+ab^4\right)\sqrt{ab}}-\frac{}{\left(a^6+3\,a^5b+3\,a^4b^2+a^3b^3\right)\cos(}}{8\,f}$$

input
```
integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
-1/8*(2*(a - 5*b)*log(cos(f*x + e) + 1)/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b
^3 + b^4) - 2*(a - 5*b)*log(cos(f*x + e) - 1)/(a^4 + 4*a^3*b + 6*a^2*b^2 +
 4*a*b^3 + b^4) - (15*a^2*b - 10*a*b^2 - b^3)*arctan(a*cos(f*x + e)/sqrt(a
*b))/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*sqrt(a*b)) - ((4*a^3
 - 9*a^2*b - a*b^2)*cos(f*x + e)^5 + (17*a^2*b - 6*a*b^2 + b^3)*cos(f*x +
e)^3 + (11*a*b^2 - b^3)*cos(f*x + e))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^
3)*cos(f*x + e)^6 - a^4*b^2 - 3*a^3*b^3 - 3*a^2*b^4 - a*b^5 - (a^6 + a^5*b
 - 3*a^4*b^2 - 5*a^3*b^3 - 2*a^2*b^4)*cos(f*x + e)^4 - (2*a^5*b + 5*a^4*b^
2 + 3*a^3*b^3 - a^2*b^4 - a*b^5)*cos(f*x + e)^2))/f
```

3.58.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 750 vs. $2(195) = 390$.

Time = 0.52 (sec) , antiderivative size = 750, normalized size of antiderivative = 3.52

$$\int \frac{\csc^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$

$$=\frac{\frac{2\,(a-5\,b)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}-\frac{\left(15\,a^2b-10\,ab^2-b^3\right)\arctan\left(-\frac{a\cos(fx+e)-b}{\sqrt{ab}\cos(fx+e)+\sqrt{ab}}\right)}{\left(a^5+4\,a^4b+6\,a^3b^2+4\,a^2b^3+ab^4\right)\sqrt{ab}}+\frac{\left(a+b-\frac{2\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{10\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}\right)(\cos(fx-}{\left(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4\right)(\cos(fx+e)-1)}$$

input
```
integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
1/8*(2*(a - 5*b)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1))/(a^4 +
4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - (15*a^2*b - 10*a*b^2 - b^3)*arctan(
-(a*cos(f*x + e) - b)/(sqrt(a*b)*cos(f*x + e) + sqrt(a*b)))/((a^5 + 4*a^4*
b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*sqrt(a*b)) + (a + b - 2*a*(cos(f*x + e)
 - 1)/(cos(f*x + e) + 1) + 10*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1))*(co
s(f*x + e) + 1)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(cos(f*x + e)
 - 1)) - (cos(f*x + e) - 1)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(cos(f*x + e)
 + 1)) - 2*(9*a^3*b + 17*a^2*b^2 + 7*a*b^3 - b^4 + 27*a^3*b*(cos(f*x + e)
- 1)/(cos(f*x + e) + 1) + a^2*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) -
23*a*b^3*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 3*b^4*(cos(f*x + e) - 1)/
(cos(f*x + e) + 1) + 27*a^3*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 -
21*a^2*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 29*a*b^3*(cos(f*x +
 e) - 1)^2/(cos(f*x + e) + 1)^2 - 3*b^4*(cos(f*x + e) - 1)^2/(cos(f*x + e)
 + 1)^2 + 9*a^3*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 5*a^2*b^2*(c
os(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 13*a*b^3*(cos(f*x + e) - 1)^3/(c
os(f*x + e) + 1)^3 + b^4*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3)/((a^5
+ 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*(a + b + 2*a*(cos(f*x + e) - 1)
/(cos(f*x + e) + 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f
*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e)
 + 1)^2)^2))/f
```

3.58.9 Mupad [B] (verification not implemented)

Time = 19.87 (sec) , antiderivative size = 2728, normalized size of antiderivative = 12.81

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^3),x)
```

output

```
- ((cos(e + f*x)^3*(17*a^2*b - 6*a*b^2 + b^3))/(8*(a*b^3 + 3*a^3*b + a^4 +
 3*a^2*b^2)) - (cos(e + f*x)^5*(9*a*b - 4*a^2 + b^2))/(8*(3*a*b^2 + 3*a^2*
b + a^3 + b^3)) + (b^2*cos(e + f*x)*(11*a - b))/(8*(a*b^3 + 3*a^3*b + a^4
+ 3*a^2*b^2)))/(f*(b^2 - cos(e + f*x)^4*(2*a*b - a^2) + cos(e + f*x)^2*(2*
a*b - b^2) - a^2*cos(e + f*x)^6)) - (log(cos(e + f*x) - 1)*((3*b)/(2*(a +
b)^4) - 1/(4*(a + b)^3)))/f - (log(cos(e + f*x) + 1)*(a - 5*b))/(4*f*(a +
b)^4) - (atan(((((cos(e + f*x)*(20*a*b^5 - 160*a^5*b + 16*a^6 + b^6 + 70*a
^2*b^4 - 300*a^3*b^3 + 625*a^4*b^2))/(32*(a*b^6 + 6*a^6*b + a^7 + 6*a^2*b^
5 + 15*a^3*b^4 + 20*a^4*b^3 + 15*a^5*b^2)) + ((-a^3*b)^(1/2)*(((11*a^11*b)
/2 - (a^2*b^10)/2 + (3*a^3*b^9)/2 + 30*a^4*b^8 + 126*a^5*b^7 + 273*a^6*b^6
 + 357*a^7*b^5 + 294*a^8*b^4 + 150*a^9*b^3 + (87*a^10*b^2)/2)/(a*b^9 + 9*a
^9*b + a^10 + 9*a^2*b^8 + 36*a^3*b^7 + 84*a^4*b^6 + 126*a^5*b^5 + 126*a^6*
b^4 + 84*a^7*b^3 + 36*a^8*b^2) - (cos(e + f*x)*(-a^3*b)^(1/2)*(10*a*b - 15
*a^2 + b^2)*(1792*a^11*b + 256*a^12 - 256*a^3*b^9 - 1792*a^4*b^8 - 5120*a^
5*b^7 - 7168*a^6*b^6 - 3584*a^7*b^5 + 3584*a^8*b^4 + 7168*a^9*b^3 + 5120*a
^10*b^2))/(512*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)*(a*b^6 +
6*a^6*b + a^7 + 6*a^2*b^5 + 15*a^3*b^4 + 20*a^4*b^3 + 15*a^5*b^2)))*(10*a*
b - 15*a^2 + b^2))/(16*(4*a^6*b + a^7 + a^3*b^4 + 4*a^4*b^3 + 6*a^5*b^2)))
*(-a^3*b)^(1/2)*(10*a*b - 15*a^2 + b^2)*1i)/(16*(4*a^6*b + a^7 + a^3*b^4 +
 4*a^4*b^3 + 6*a^5*b^2)) + (((cos(e + f*x)*(20*a*b^5 - 160*a^5*b + 16*a...
```

3.59 $\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

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3.59.1 Optimal result

Integrand size = 23, antiderivative size = 257

$$\begin{aligned}\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = {}& \frac{3\sqrt{b}(5a^2-10ab+b^2)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{8\sqrt{a}(a+b)^5 f}\\ &-\frac{3(a^2-10ab+5b^2)\operatorname{arctanh}(\cos(e+fx))}{8(a+b)^5 f}\\ &+\frac{(a^2-9ab+2b^2)\cos(e+fx)}{8(a+b)^3 f\left(b+a\cos^2(e+fx)\right)^2}\\ &+\frac{3(a^2-6ab+b^2)\cos(e+fx)}{8(a+b)^4 f\left(b+a\cos^2(e+fx)\right)}\\ &-\frac{(a-7b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2 f\left(b+a\cos^2(e+fx)\right)^2}\\ &-\frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\left(b+a\cos^2(e+fx)\right)^2}\end{aligned}$$

output

```
-3/8*(a^2-10*a*b+5*b^2)*arctanh(cos(f*x+e))/(a+b)^5/f+1/8*(a^2-9*a*b+2*b^2
)*cos(f*x+e)/(a+b)^3/f/(b+a*cos(f*x+e)^2)^2+3/8*(a^2-6*a*b+b^2)*cos(f*x+e)
/(a+b)^4/f/(b+a*cos(f*x+e)^2)-1/8*(a-7*b)*cot(f*x+e)*csc(f*x+e)/(a+b)^2/f/
(b+a*cos(f*x+e)^2)^2-1/4*cot(f*x+e)^3*csc(f*x+e)/(a+b)/f/(b+a*cos(f*x+e)^2
)^2+3/8*(5*a^2-10*a*b+b^2)*arctan(cos(f*x+e)*a^(1/2)/b^(1/2))*b^(1/2)/(a+b
)^5/f/a^(1/2)
```

3.59.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.32 (sec) , antiderivative size = 549, normalized size of antiderivative = 2.14

$$\int \frac{\csc^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\left(\frac{48\sqrt{b}\left(5a^2-10ab+b^2\right)\arctan\left(\frac{\left(-\sqrt{a}-i\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}\right)\sin(e)\tan\left(\frac{fx}{2}\right)+\cos(e)\left(\sqrt{a}-\sqrt{a+b}\sqrt{(\cos(e)}\right.\right.\right.}{\sqrt{b}}}{\sqrt{a}}$$

input
```
Integrate[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*((48*Sqrt[b]*(5*a^2 - 10*a*b + b^2)*ArcTan
[((-Sqrt[a] - I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2])*Sin[e]*Tan[(f*x)/
2] + Cos[e]*(Sqrt[a] - Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Tan[(f*x)/2
]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2)/Sqrt[a] + (48*Sqrt[b]*(5*a^
2 - 10*a*b + b^2)*ArcTan[((-Sqrt[a] + I*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e
])^2])*Sin[e]*Tan[(f*x)/2] + Cos[e]*(Sqrt[a] + Sqrt[a + b]*Sqrt[(Cos[e] -
I*Sin[e])^2]*Tan[(f*x)/2]))/Sqrt[b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2)/Sqr
t[a] - 2*(a + b)*(30*a^3 + 112*a^2*b + 182*a*b^2 - 140*b^3 + (35*a^3 + 78*
a^2*b - 93*a*b^2 + 224*b^3)*Cos[2*(e + f*x)] + 2*(a^3 - 8*a^2*b + 53*a*b^2
 - 10*b^3)*Cos[4*(e + f*x)] - 3*a^3*Cos[6*(e + f*x)] + 18*a^2*b*Cos[6*(e +
 f*x)] - 3*a*b^2*Cos[6*(e + f*x)])*Cot[e + f*x]*Csc[e + f*x]^3 - 48*(a^2 -
 10*a*b + 5*b^2)*(a + 2*b + a*Cos[2*(e + f*x)])^2*Log[Cos[(e + f*x)/2]] +
48*(a^2 - 10*a*b + 5*b^2)*(a + 2*b + a*Cos[2*(e + f*x)])^2*Log[Sin[(e + f*
x)/2]])*Sec[e + f*x]^6)/(1024*(a + b)^5*f*(a + b*Sec[e + f*x]^2)^3)
```

3.59.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.11, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 4621, 372, 440, 402, 27, 402, 27, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sin(e+fx)^5\,(a+b\sec(e+fx)^2)^3}dx$$

$$\downarrow 4621$$

$$-\frac{\int \frac{\cos^6(e+fx)}{(1-\cos^2(e+fx))^3(a\cos^2(e+fx)+b)^3}d\cos(e+fx)}{f}$$

$$\downarrow 372$$

$$-\frac{\frac{\cos^3(e+fx)}{4(a+b)(1-\cos^2(e+fx))^2(a\cos^2(e+fx)+b)^2}-\frac{\int \frac{\cos^2(e+fx)\left(3b-(a-4b)\cos^2(e+fx)\right)}{\left(1-\cos^2(e+fx)\right)^2\left(a\cos^2(e+fx)+b\right)^3}d\cos(e+fx)}{4(a+b)}}{f}$$

$$\downarrow 440$$

$$-\frac{\frac{\cos^3(e+fx)}{4(a+b)(1-\cos^2(e+fx))^2(a\cos^2(e+fx)+b)^2}-\frac{\frac{\int \frac{(a-7b)b-\left(3a^2-29ba+8b^2\right)\cos^2(e+fx)}{\left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)^3}d\cos(e+fx)}{2(a+b)}-\frac{(a-7b)\cos(e+fx)}{2(a+b)\left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)^2}}{4(a+b)}}{f}$$

$$\downarrow 402$$

$$-\frac{\frac{\cos^3(e+fx)}{4(a+b)(1-\cos^2(e+fx))^2(a\cos^2(e+fx)+b)^2}-\frac{\frac{\left(a^2-9ab+2b^2\right)\cos(e+fx)}{(a+b)\left(a\cos^2(e+fx)+b\right)^2}-\frac{\int -\frac{12b\left((a-3b)b-\left(a^2-9ba+2b^2\right)\cos^2(e+fx)\right)}{\left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)^2}d\cos(e+fx)}{4b(a+b)}}{2(a+b)}-\frac{(}{2(a+b)\left(1-\cos\right.}}{4(a+b)}}{f}$$

$$\downarrow 27$$

$$-\frac{\frac{\cos^3(e+fx)}{4(a+b)(1-\cos^2(e+fx))^2(a\cos^2(e+fx)+b)^2}-\frac{\frac{3\int \frac{(a-3b)b-\left(a^2-9ba+2b^2\right)\cos^2(e+fx)}{\left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)^2}d\cos(e+fx)}{a+b}+\frac{\left(a^2-9ab+2b^2\right)\cos(e+fx)}{(a+b)\left(a\cos^2(e+fx)+b\right)^2}}{2(a+b)}-\frac{(a-7b)}{2(a+b)\left(1-\cos^2(e+f\right.}}{4(a+b)}}{f}$$

$$\downarrow 402$$

$$-\frac{\cos^3(e+fx)}{4(a+b)(1-\cos^2(e+fx))^2(a\cos^2(e+fx)+b)^2}-\frac{3\left(\frac{\left(a^2-6ab+b^2\right)\cos(e+fx)}{(a+b)\left(a\cos^2(e+fx)+b\right)}-\frac{\int-\frac{2b\left(4(a-b)b-\left(a^2-6ba+b^2\right)\cos^2(e+fx)\right)}{\left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)}d\cos(e+fx)}{2b(a+b)}\right)}{a+b}+\frac{\left(a^2-9ab+2b}{(a+b)\left(a\cos^2}}{2(a+b)}}{4(a+b)}}{f}$$

$$\downarrow 27$$

$$-\frac{\cos^3(e+fx)}{4(a+b)(1-\cos^2(e+fx))^2(a\cos^2(e+fx)+b)^2}-\frac{3\left(\frac{\int\frac{4(a-b)b-\left(a^2-6ba+b^2\right)\cos^2(e+fx)}{\left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)}d\cos(e+fx)}{a+b}+\frac{\left(a^2-6ab+b^2\right)\cos(e+fx)}{(a+b)\left(a\cos^2(e+fx)+b\right)}\right)}{a+b}+\frac{\left(a^2-9ab+2b^2\right)\cos}{(a+b)\left(a\cos^2(e+fx}}{2(a+b)}}{4(a+b)}}{f}$$

$$\downarrow 397$$

$$-\frac{\cos^3(e+fx)}{4(a+b)(1-\cos^2(e+fx))^2(a\cos^2(e+fx)+b)^2}-\frac{3\left(\frac{b\left(5a^2-10ab+b^2\right)\int\frac{1}{a\cos^2(e+fx)+b}d\cos(e+fx)}{a+b}-\frac{\left(a^2-10ab+5b^2\right)\int\frac{1}{1-\cos^2(e+fx)}d\cos(e+fx)}{a+b}+\frac{(}{(}\right)}{a+b}+\frac{}{2(a+b)}}{4(a+}}{f}$$

$$\downarrow 218$$

$$-\frac{\cos^3(e+fx)}{4(a+b)(1-\cos^2(e+fx))^2(a\cos^2(e+fx)+b)^2}-\frac{3\left(\frac{\sqrt{b}\left(5a^2-10ab+b^2\right)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{\sqrt{a}(a+b)}-\frac{\left(a^2-10ab+5b^2\right)\int\frac{1}{1-\cos^2(e+fx)}d\cos(e+fx)}{a+b}+\frac{(a^2-6}{(a+b)}\right)}{a+b}+\frac{}{2(a+b)}}{4(a+b)}}{f}$$

$$\downarrow 219$$

$$-\frac{\cos^3(e+fx)}{4(a+b)(1-\cos^2(e+fx))^2(a\cos^2(e+fx)+b)^2}-\frac{3\left(\frac{\frac{\sqrt{b}\left(5a^2-10ab+b^2\right)\arctan\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)}{\sqrt{a}(a+b)}-\frac{\left(a^2-10ab+5b^2\right)\operatorname{arctanh}(\cos(e+fx))}{a+b}}{a+b}+\frac{\left(a^2-6ab+b^2\right)\cos(e+fx)}{(a+b)\left(a\cos^2(e+fx)+b\right)}\right)}{2(a+b)}}{4(a+b)}}{f}$$

input
```
Int[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
-((Cos[e + f*x]^3/(4*(a + b)*(1 - Cos[e + f*x]^2)^2*(b + a*Cos[e + f*x]^2)
^2) - (-1/2*((a - 7*b)*Cos[e + f*x])/((a + b)*(1 - Cos[e + f*x]^2)*(b + a*
Cos[e + f*x]^2)^2) + (((a^2 - 9*a*b + 2*b^2)*Cos[e + f*x])/((a + b)*(b + a
*Cos[e + f*x]^2)^2) + (3*(((Sqrt[b]*(5*a^2 - 10*a*b + b^2)*ArcTan[(Sqrt[a]
*Cos[e + f*x])/Sqrt[b]])/(Sqrt[a]*(a + b)) - ((a^2 - 10*a*b + 5*b^2)*ArcTa
nh[Cos[e + f*x]])/(a + b))/(a + b) + ((a^2 - 6*a*b + b^2)*Cos[e + f*x])/((
a + b)*(b + a*Cos[e + f*x]^2))))/(a + b))/(2*(a + b)))/(4*(a + b)))/f)
```

3.59.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4621
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*x)^n)^p/(ff*x)^(n*p)),
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/
2] && IntegerQ[n] && IntegerQ[p]
```

3.59.4 Maple [A] (verified)

Time = 2.31 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.01

method	result
derivativedivides	$-\frac{1}{16(a+b)^3(-1+\cos(fx+e))^2}-\frac{-3a+9b}{16(a+b)^4(-1+\cos(fx+e))}+\frac{\left(3a^2-30ab+15b^2\right)\ln(-1+\cos(fx+e))}{16(a+b)^5}+\frac{b\left(\frac{\left(-\frac{9}{8}a^3-\frac{3}{4}a^2b+\frac{3}{8}ab^2\right)\cos(}{(b+}\right.}{}$
default	$-\frac{1}{16(a+b)^3(-1+\cos(fx+e))^2}-\frac{-3a+9b}{16(a+b)^4(-1+\cos(fx+e))}+\frac{\left(3a^2-30ab+15b^2\right)\ln(-1+\cos(fx+e))}{16(a+b)^5}+\frac{b\left(\frac{\left(-\frac{9}{8}a^3-\frac{3}{4}a^2b+\frac{3}{8}ab^2\right)\cos(}{(b+}\right.}{}$
risch	Expression too large to display

input
```
int(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/16/(a+b)^3/(-1+cos(f*x+e))^2-1/16*(-3*a+9*b)/(a+b)^4/(-1+cos(f*x+e
))+1/16/(a+b)^5*(3*a^2-30*a*b+15*b^2)*ln(-1+cos(f*x+e))+b/(a+b)^5*(((-9/8*
a^3-3/4*a^2*b+3/8*a*b^2)*cos(f*x+e)^3-1/8*b*(7*a^2+2*a*b-5*b^2)*cos(f*x+e)
)/(b+a*cos(f*x+e)^2)^2+3/8*(5*a^2-10*a*b+b^2)/(a*b)^(1/2)*arctan(a*cos(f*x
+e)/(a*b)^(1/2)))+1/16/(a+b)^3/(1+cos(f*x+e))^2-1/16*(-3*a+9*b)/(a+b)^4/(1
+cos(f*x+e))+1/16/(a+b)^5*(-3*a^2+30*a*b-15*b^2)*ln(1+cos(f*x+e)))
```

3.59.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 901 vs. 2(237) = 474.

Time = 0.47 (sec) , antiderivative size = 1833, normalized size of antiderivative = 7.13

$$\int \frac{\csc^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output

```
[1/16*(6*(a^4 - 5*a^3*b - 5*a^2*b^2 + a*b^3)*cos(f*x + e)^7 - 2*(5*a^4 - 2
6*a^3*b + 26*a*b^3 - 5*b^4)*cos(f*x + e)^5 - 2*(19*a^3*b - 15*a^2*b^2 - 15
*a*b^3 + 19*b^4)*cos(f*x + e)^3 + 3*((5*a^4 - 10*a^3*b + a^2*b^2)*cos(f*x
+ e)^8 - 2*(5*a^4 - 15*a^3*b + 11*a^2*b^2 - a*b^3)*cos(f*x + e)^6 + (5*a^4
 - 30*a^3*b + 46*a^2*b^2 - 14*a*b^3 + b^4)*cos(f*x + e)^4 + 5*a^2*b^2 - 10
*a*b^3 + b^4 + 2*(5*a^3*b - 15*a^2*b^2 + 11*a*b^3 - b^4)*cos(f*x + e)^2)*s
qrt(-b/a)*log(-(a*cos(f*x + e)^2 + 2*a*sqrt(-b/a)*cos(f*x + e) - b)/(a*cos
(f*x + e)^2 + b)) - 24*(a^2*b^2 - b^4)*cos(f*x + e) - 3*((a^4 - 10*a^3*b +
 5*a^2*b^2)*cos(f*x + e)^8 - 2*(a^4 - 11*a^3*b + 15*a^2*b^2 - 5*a*b^3)*cos
(f*x + e)^6 + (a^4 - 14*a^3*b + 46*a^2*b^2 - 30*a*b^3 + 5*b^4)*cos(f*x + e
)^4 + a^2*b^2 - 10*a*b^3 + 5*b^4 + 2*(a^3*b - 11*a^2*b^2 + 15*a*b^3 - 5*b^
4)*cos(f*x + e)^2)*log(1/2*cos(f*x + e) + 1/2) + 3*((a^4 - 10*a^3*b + 5*a^
2*b^2)*cos(f*x + e)^8 - 2*(a^4 - 11*a^3*b + 15*a^2*b^2 - 5*a*b^3)*cos(f*x
+ e)^6 + (a^4 - 14*a^3*b + 46*a^2*b^2 - 30*a*b^3 + 5*b^4)*cos(f*x + e)^4 +
 a^2*b^2 - 10*a*b^3 + 5*b^4 + 2*(a^3*b - 11*a^2*b^2 + 15*a*b^3 - 5*b^4)*co
s(f*x + e)^2)*log(-1/2*cos(f*x + e) + 1/2))/((a^7 + 5*a^6*b + 10*a^5*b^2 +
 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^8 - 2*(a^7 + 4*a^6*b + 5
*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*f*cos(f*x + e)^6 + (a^7 + a^6*b
- 9*a^5*b^2 - 25*a^4*b^3 - 25*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*f*cos(f*x
 + e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 - 5*a^2*b^5 - 4*a*b^6 - b^7)...
```

3.59.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

input

```
integrate(csc(f*x+e)**5/(a+b*sec(f*x+e)**2)**3,x)
```

output

```
Timed out
```

3.59.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 528 vs. $2(237) = 474$.

Time = 0.28 (sec) , antiderivative size = 528, normalized size of antiderivative = 2.05

$$\int \frac{\csc^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx =$$

$$-\frac{3\left(a^2-10ab+5b^2\right)\log(\cos(fx+e)+1)}{a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5} - \frac{3\left(a^2-10ab+5b^2\right)\log(\cos(fx+e)-1)}{a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5} - \frac{6\left(5a^2b-10ab^2+b^3\right)\arctan\left(\frac{a\cos(fx+e)}{\sqrt{ab}}\right)}{\left(a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5\right)\sqrt{ab}} - \frac{}{(a^6+4}$$

input
```
integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
-1/16*(3*(a^2 - 10*a*b + 5*b^2)*log(cos(f*x + e) + 1)/(a^5 + 5*a^4*b + 10*
a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5) - 3*(a^2 - 10*a*b + 5*b^2)*log(cos(f
*x + e) - 1)/(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5) - 6
*(5*a^2*b - 10*a*b^2 + b^3)*arctan(a*cos(f*x + e)/sqrt(a*b))/((a^5 + 5*a^4
*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*sqrt(a*b)) - 2*(3*(a^3 - 6*a
^2*b + a*b^2)*cos(f*x + e)^7 - (5*a^3 - 31*a^2*b + 31*a*b^2 - 5*b^3)*cos(f
*x + e)^5 - (19*a^2*b - 34*a*b^2 + 19*b^3)*cos(f*x + e)^3 - 12*(a*b^2 - b^
3)*cos(f*x + e))/((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*cos(f*
x + e)^8 - 2*(a^6 + 3*a^5*b + 2*a^4*b^2 - 2*a^3*b^3 - 3*a^2*b^4 - a*b^5)*c
os(f*x + e)^6 + a^4*b^2 + 4*a^3*b^3 + 6*a^2*b^4 + 4*a*b^5 + b^6 + (a^6 - 9
*a^4*b^2 - 16*a^3*b^3 - 9*a^2*b^4 + b^6)*cos(f*x + e)^4 + 2*(a^5*b + 3*a^4
*b^2 + 2*a^3*b^3 - 2*a^2*b^4 - 3*a*b^5 - b^6)*cos(f*x + e)^2))/f
```

3.59.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1316 vs. $2(237) = 474$.

Time = 0.46 (sec) , antiderivative size = 1316, normalized size of antiderivative = 5.12

$$\int \frac{\csc^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
1/64*(12*(a^2 - 10*a*b + 5*b^2)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e
) + 1))/(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5) - 24*(5*
a^2*b - 10*a*b^2 + b^3)*arctan(-(a*cos(f*x + e) - b)/(sqrt(a*b)*cos(f*x +
e) + sqrt(a*b)))/((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5
)*sqrt(a*b)) - (8*a^3*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 24*a*b^2*(co
s(f*x + e) - 1)/(cos(f*x + e) + 1) - 16*b^3*(cos(f*x + e) - 1)/(cos(f*x +
e) + 1) - a^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 3*a^2*b*(cos(f*x
 + e) - 1)^2/(cos(f*x + e) + 1)^2 - 3*a*b^2*(cos(f*x + e) - 1)^2/(cos(f*x
+ e) + 1)^2 - b^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/(a^6 + 6*a^5*
b + 15*a^4*b^2 + 20*a^3*b^3 + 15*a^2*b^4 + 6*a*b^5 + b^6) - (a^4 + 4*a^3*b
 + 6*a^2*b^2 + 4*a*b^3 + b^4 - 4*a^4*(cos(f*x + e) - 1)/(cos(f*x + e) + 1)
 + 24*a^2*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 32*a*b^3*(cos(f*x +
e) - 1)/(cos(f*x + e) + 1) + 12*b^4*(cos(f*x + e) - 1)/(cos(f*x + e) + 1)
- 20*a^4*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 136*a^3*b*(cos(f*x +
e) - 1)^2/(cos(f*x + e) + 1)^2 + 224*a^2*b^2*(cos(f*x + e) - 1)^2/(cos(f*x
 + e) + 1)^2 - 40*a*b^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 108*b^
4*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 20*a^4*(cos(f*x + e) - 1)^3/
(cos(f*x + e) + 1)^3 + 280*a^3*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3
 - 64*a^2*b^2*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 152*a*b^3*(cos(f
*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 212*b^4*(cos(f*x + e) - 1)^3/(cos...
```

3.59.9 Mupad [B] (verification not implemented)

Time = 23.78 (sec) , antiderivative size = 5613, normalized size of antiderivative = 21.84

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^3),x)
```

output

```
(atan(((((cos(e + f*x)*(9*a*b^6 - 180*a^6*b + 9*a^7 - 180*a^2*b^5 + 1215*a
^3*b^4 - 1800*a^4*b^3 + 1215*a^5*b^2))/(32*(8*a*b^7 + 8*a^7*b + a^8 + b^8
+ 28*a^2*b^6 + 56*a^3*b^5 + 70*a^4*b^4 + 56*a^5*b^3 + 28*a^6*b^2)) + (3*(-
a*b)^(1/2)*((6*a^13*b - 6*a^2*b^12 - 54*a^3*b^11 - 210*a^4*b^10 - 450*a^5*
b^9 - 540*a^6*b^8 - 252*a^7*b^7 + 252*a^8*b^6 + 540*a^9*b^5 + 450*a^10*b^4
 + 210*a^11*b^3 + 54*a^12*b^2)/(12*a*b^11 + 12*a^11*b + a^12 + b^12 + 66*a
^2*b^10 + 220*a^3*b^9 + 495*a^4*b^8 + 792*a^5*b^7 + 924*a^6*b^6 + 792*a^7*
b^5 + 495*a^8*b^4 + 220*a^9*b^3 + 66*a^10*b^2) - (3*cos(e + f*x)*(-a*b)^(1
/2)*(5*a^2 - 10*a*b + b^2)*(2304*a^12*b + 256*a^13 - 256*a^2*b^11 - 2304*a
^3*b^10 - 8960*a^4*b^9 - 19200*a^5*b^8 - 23040*a^6*b^7 - 10752*a^7*b^6 + 1
0752*a^8*b^5 + 23040*a^9*b^4 + 19200*a^10*b^3 + 8960*a^11*b^2))/(512*(a*b^
5 + 5*a^5*b + a^6 + 5*a^2*b^4 + 10*a^3*b^3 + 10*a^4*b^2)*(8*a*b^7 + 8*a^7*
b + a^8 + b^8 + 28*a^2*b^6 + 56*a^3*b^5 + 70*a^4*b^4 + 56*a^5*b^3 + 28*a^6
*b^2)))*(5*a^2 - 10*a*b + b^2))/(16*(a*b^5 + 5*a^5*b + a^6 + 5*a^2*b^4 + 1
0*a^3*b^3 + 10*a^4*b^2)))*(-a*b)^(1/2)*(5*a^2 - 10*a*b + b^2)*3i)/(16*(a*b
^5 + 5*a^5*b + a^6 + 5*a^2*b^4 + 10*a^3*b^3 + 10*a^4*b^2)) + (((cos(e + f*
x)*(9*a*b^6 - 180*a^6*b + 9*a^7 - 180*a^2*b^5 + 1215*a^3*b^4 - 1800*a^4*b^
3 + 1215*a^5*b^2))/(32*(8*a*b^7 + 8*a^7*b + a^8 + b^8 + 28*a^2*b^6 + 56*a^
3*b^5 + 70*a^4*b^4 + 56*a^5*b^3 + 28*a^6*b^2)) - (3*(-a*b)^(1/2)*((6*a^13*
b - 6*a^2*b^12 - 54*a^3*b^11 - 210*a^4*b^10 - 450*a^5*b^9 - 540*a^6*b^8...
```

3.60 $\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

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3.60.1 Optimal result

Integrand size = 23, antiderivative size = 314

$$\begin{aligned}\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx &= \frac{5(a+2b)\left(a^2+16ab+16b^2\right)x}{16a^6} \\ &- \frac{5\sqrt{b}\sqrt{a+b}(a+4b)(3a+4b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^6 f} \\ &- \frac{\left(33a^2+110ab+80b^2\right)\cos(e+fx)\sin(e+fx)}{48a^3 f\left(a+b+b\tan^2(e+fx)\right)^2} \\ &+ \frac{(9a+10b)\cos^3(e+fx)\sin(e+fx)}{24a^2 f\left(a+b+b\tan^2(e+fx)\right)^2} \\ &+ \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)^2} \\ &- \frac{5b\left(9a^2+32ab+24b^2\right)\tan(e+fx)}{48a^4 f\left(a+b+b\tan^2(e+fx)\right)^2} \\ &- \frac{5b\left(5a^2+20ab+16b^2\right)\tan(e+fx)}{16a^5 f\left(a+b+b\tan^2(e+fx)\right)}\end{aligned}$$

output
```
5/16*(a+2*b)*(a^2+16*a*b+16*b^2)*x/a^6-5/8*(a+4*b)*(3*a+4*b)*arctan(b^(1/2
)*tan(f*x+e)/(a+b)^(1/2))*b^(1/2)*(a+b)^(1/2)/a^6/f-1/48*(33*a^2+110*a*b+8
0*b^2)*cos(f*x+e)*sin(f*x+e)/a^3/f/(a+b+b*tan(f*x+e)^2)^2+1/24*(9*a+10*b)*
cos(f*x+e)^3*sin(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)^2+1/6*cos(f*x+e)^3*sin(
f*x+e)^3/a/f/(a+b+b*tan(f*x+e)^2)^2-5/48*b*(9*a^2+32*a*b+24*b^2)*tan(f*x+e
)/a^4/f/(a+b+b*tan(f*x+e)^2)^2-5/16*b*(5*a^2+20*a*b+16*b^2)*tan(f*x+e)/a^5
/f/(a+b+b*tan(f*x+e)^2)
```

3.60.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 17.42 (sec) , antiderivative size = 1639, normalized size of antiderivative = 5.22

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
Integrate[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
(5*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(((3*a^2 + 8*a*b + 8*b^
2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) - (a*Sqrt[b]*
(3*a^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)
])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2)))/(65536*b^(5/2)*f*(a + b*
Sec[e + f*x]^2)^3) - (15*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(
(-6*a^2*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) +
a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e
] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (a*Sec[2*e]
*((-9*a^4 - 16*a^3*b + 48*a^2*b^2 + 128*a*b^3 + 64*b^4)*Sin[2*f*x] + a*(-3
*a^3 + 2*a^2*b + 24*a*b^2 + 16*b^3)*Sin[2*(e + 2*f*x)] + (3*a^4 - 64*a^2*b
^2 - 128*a*b^3 - 64*b^4)*Sin[4*e + 2*f*x]) + (9*a^5 + 18*a^4*b - 64*a^3*b^
2 - 256*a^2*b^3 - 320*a*b^4 - 128*b^5)*Tan[2*e])/(a^2*(a + 2*b + a*Cos[2*(
e + f*x)])^2)))/(262144*b^2*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^3) + (3*(a
+ 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(-1536*(a + 2*b)*x - (3*(a^6
- 8*a^5*b + 120*a^4*b^2 + 1280*a^3*b^3 + 3200*a^2*b^4 + 3072*a*b^5 + 1024*
b^6)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*S
in[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] -
 I*Sin[2*e]))/(b^2*(a + b)^(5/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (4*(a^
4 + 32*a^3*b + 160*a^2*b^2 + 256*a*b^3 + 128*b^4)*Sec[2*e]*((a + 2*b)*Sin[
2*e] - a*Sin[2*f*x]))/(b*(a + b)*f*(a + 2*b + a*Cos[2*(e + f*x)])^2) + ...
```

3.60.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.10, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 4620, 372, 440, 402, 27, 402, 27, 402, 27, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sin(e+fx)^6}{(a+b\sec(e+fx)^2)^3}\,dx$$

$$\downarrow 4620$$

$$\frac{\int \frac{\tan^6(e+fx)}{(\tan^2(e+fx)+1)^4(b\tan^2(e+fx)+a+b)^3}\,d\tan(e+fx)}{f}$$

$$\downarrow 372$$

$$\frac{\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)^2} - \frac{\int \frac{\tan^2(e+fx)\left(3(a+b)-(6a+7b)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)^3\left(b\tan^2(e+fx)+a+b\right)^3}\,d\tan(e+fx)}{6a}}{f}$$

$$\downarrow 440$$

$$\frac{\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)^2} - \frac{\frac{\int \frac{(a+b)(9a+10b)-\left(24a^2+91ba+70b^2\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^3}\,d\tan(e+fx)}{4a} - \frac{(9a+10b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\left(a+b\tan^2(e+fx)+b\right)^2}}{6a}}{f}$$

$$\downarrow 402$$

$$\frac{\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)^2} - \frac{\frac{\int \frac{5\left((a+b)\left(3a^2+18ba+16b^2\right)-b\left(33a^2+110ba+80b^2\right)\tan^2(e+fx)\right.}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^3}}{2a} - \frac{\left(33a^2+110ab+80b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^2}}{4a}}{6a}}{f}$$

$$\downarrow 27$$

$$\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)^2} - \frac{\frac{\left(33a^2+110ab+80b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^2} - \frac{5\int \frac{(a+b)\left(3a^2+18ba+16b^2\right)-b\left(33a^2+110ba+80b^2\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^3}}{2a}}{4a}}{6a}$$

$$\Big\downarrow 402$$

$$\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)^2} - \frac{\frac{\left(33a^2+110ab+80b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^2} - \frac{5\left(\int \frac{12(a+b)\left((a+b)\left(a^2+8ba+8b^2\right)-b\left(9a^2+32ba+24b^2\right)\tan^2(e\right.}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}\right.}{2a}}{4a}}{6a}$$

$$\Big\downarrow 27$$

$$\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)^2} - \frac{\frac{\left(33a^2+110ab+80b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^2} - \frac{5\left(\frac{3\int \frac{(a+b)\left(a^2+8ba+8b^2\right)-b\left(9a^2+32ba+24b^2\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}dt}{a}\right.}{2a}}{4a}}{6a}$$

$$\Big\downarrow 402$$

$$\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)^2} - \frac{\frac{\left(33a^2+110ab+80b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^2} - \frac{5\left(\frac{3\left(\int \frac{2(a+b)\left((a+b)\left(a^2+12ba+16b^2\right)-b\left(5a^2+20ba+16b^2\right)\ta\right.}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}\right.}{2a(a+b)}\right.}{a}\right.}{2a}}{4a}}{6a}$$

$$\Big\downarrow 27$$

$$\frac{\tan^3(e+fx)}{6a\left(\tan^2(e+fx)+1\right)^3\left(a+b\tan^2(e+fx)+b\right)^2}-\frac{\frac{\left(33a^2+110ab+80b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^2}-\frac{5\left(\frac{3\left(\int\frac{(a+b)\left(a^2+12ba+16b^2\right)-b\left(5a^2+20ba+16b^2\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}}{a}\right)}{4a}}{f}$$

$$\downarrow 397$$

$$\frac{\tan^3(e+fx)}{6a\left(\tan^2(e+fx)+1\right)^3\left(a+b\tan^2(e+fx)+b\right)^2}-\frac{\frac{\left(33a^2+110ab+80b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^2}-\frac{5\left(\frac{3\left(\frac{(a+2b)\left(a^2+16ab+16b^2\right)\int\frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}-\right.\right.}{a}\right)}{f}$$

$$\downarrow 216$$

$$\frac{\tan^3(e+fx)}{6a\left(\tan^2(e+fx)+1\right)^3\left(a+b\tan^2(e+fx)+b\right)^2}-\frac{\frac{\left(33a^2+110ab+80b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^2}-\frac{5\left(\frac{3\left(\frac{(a+2b)\left(a^2+16ab+16b^2\right)\arctan(\tan(e+fx))}{a}-\frac{2b(a+b)(a+}{a}\right)\right.}{f}$$

$$\downarrow 218$$

$$\frac{\tan^3(e+fx)}{6a\left(\tan^2(e+fx)+1\right)^3\left(a+b\tan^2(e+fx)+b\right)^2}-\frac{\frac{\left(33a^2+110ab+80b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^2}-\frac{5\left(\frac{3\left(\frac{(a+2b)\left(a^2+16ab+16b^2\right)\arctan(\tan(e+fx))}{a}-\frac{2\sqrt{b}\sqrt{a+b}(}{a}\right)\right.}{4}}{f}$$

3.60. $\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

input
```
Int[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
(Tan[e + f*x]^3/(6*a*(1 + Tan[e + f*x]^2)^3*(a + b + b*Tan[e + f*x]^2)^2)
- (-1/4*((9*a + 10*b)*Tan[e + f*x])/(a*(1 + Tan[e + f*x]^2)^2*(a + b + b*T
an[e + f*x]^2)^2) + (((33*a^2 + 110*a*b + 80*b^2)*Tan[e + f*x])/(2*a*(1 +
Tan[e + f*x]^2)*(a + b + b*Tan[e + f*x]^2)^2) - (5*(-((b*(9*a^2 + 32*a*b +
 24*b^2)*Tan[e + f*x])/(a*(a + b + b*Tan[e + f*x]^2)^2)) + (3*((((a + 2*b)
*(a^2 + 16*a*b + 16*b^2)*ArcTan[Tan[e + f*x]])/a - (2*Sqrt[b]*Sqrt[a + b]*
(a + 4*b)*(3*a + 4*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/a)/a - (
b*(5*a^2 + 20*a*b + 16*b^2)*Tan[e + f*x])/(a*(a + b + b*Tan[e + f*x]^2))))
/a))/(2*a))/(4*a))/(6*a))/f
```

3.60.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.60.4 Maple [A] (verified)

Time = 11.70 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.79

method	result
derivativedivides	$\frac{\left(-\frac{27}{8}a^2b-3ab^2-\frac{11}{16}a^3\right)\tan(fx+e)^5+\left(-6a^2b-6ab^2-\frac{5}{6}a^3\right)\tan(fx+e)^3+\left(-\frac{5}{16}a^3-\frac{21}{8}a^2b-3ab^2\right)\tan(fx+e)}{\left(1+\tan(fx+e)^2\right)^3}+\frac{5\left(a^3+18a^2b+48a$ a^6
default	$\frac{\left(-\frac{27}{8}a^2b-3ab^2-\frac{11}{16}a^3\right)\tan(fx+e)^5+\left(-6a^2b-6ab^2-\frac{5}{6}a^3\right)\tan(fx+e)^3+\left(-\frac{5}{16}a^3-\frac{21}{8}a^2b-3ab^2\right)\tan(fx+e)}{\left(1+\tan(fx+e)^2\right)^3}+\frac{5\left(a^3+18a^2b+48a$ a^6
risch	$\frac{5x}{16a^3}+\frac{45xb}{8a^4}+\frac{15xb^2}{a^5}+\frac{10xb^3}{a^6}+\frac{15ie^{2i(fx+e)}}{128a^3f}+\frac{3ie^{-4i(fx+e)}b}{64a^4f}+\frac{3ie^{2i(fx+e)}b}{4a^4f}+\frac{3ie^{2i(fx+e)}b^2}{4a^5f}-\frac{ie^{-6i(f}$ $384a$

input
```
int(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/a^6*(((-27/8*a^2*b-3*a*b^2-11/16*a^3)*tan(f*x+e)^5+(-6*a^2*b-6*a*b^
2-5/6*a^3)*tan(f*x+e)^3+(-5/16*a^3-21/8*a^2*b-3*a*b^2)*tan(f*x+e))/(1+tan(
f*x+e)^2)^3+5/16*(a^3+18*a^2*b+48*a*b^2+32*b^3)*arctan(tan(f*x+e)))-(a+b)*
b/a^6*(((7/8*a^2*b+2*a*b^2)*tan(f*x+e)^3+1/8*a*(9*a^2+25*a*b+16*b^2)*tan(f
*x+e))/(a+b+b*tan(f*x+e)^2)^2+5/8*(3*a^2+16*a*b+16*b^2)/((a+b)*b)^(1/2)*ar
ctan(b*tan(f*x+e)/((a+b)*b)^(1/2))))
```

3.60.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 930, normalized size of antiderivative = 2.96

$$\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$

$$= \left[\frac{30\left(a^5+18a^4b+48a^3b^2+32a^2b^3\right)fx\cos(fx+e)^4+60\left(a^4b+18a^3b^2+48a^2b^3+32ab^4\right)fx\cos(fx+\right.}{\left.\right.}$$

input
```
integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output

```
[1/96*(30*(a^5 + 18*a^4*b + 48*a^3*b^2 + 32*a^2*b^3)*f*x*cos(f*x + e)^4 +
60*(a^4*b + 18*a^3*b^2 + 48*a^2*b^3 + 32*a*b^4)*f*x*cos(f*x + e)^2 + 30*(a
^3*b^2 + 18*a^2*b^3 + 48*a*b^4 + 32*b^5)*f*x + 15*((3*a^4 + 16*a^3*b + 16*
a^2*b^2)*cos(f*x + e)^4 + 3*a^2*b^2 + 16*a*b^3 + 16*b^4 + 2*(3*a^3*b + 16*
a^2*b^2 + 16*a*b^3)*cos(f*x + e)^2)*sqrt(-a*b - b^2)*log(((a^2 + 8*a*b + 8
*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a + 2*b)*cos
(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b - b^2)*sin(f*x + e) + b^2)/(a^2*co
s(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) - 2*(8*a^5*cos(f*x + e)^9 - 2*
(13*a^5 + 10*a^4*b)*cos(f*x + e)^7 + (33*a^5 + 110*a^4*b + 80*a^3*b^2)*cos
(f*x + e)^5 + 20*(6*a^4*b + 23*a^3*b^2 + 18*a^2*b^3)*cos(f*x + e)^3 + 15*(
5*a^3*b^2 + 20*a^2*b^3 + 16*a*b^4)*cos(f*x + e))*sin(f*x + e))/(a^8*f*cos(
f*x + e)^4 + 2*a^7*b*f*cos(f*x + e)^2 + a^6*b^2*f), 1/48*(15*(a^5 + 18*a^4
*b + 48*a^3*b^2 + 32*a^2*b^3)*f*x*cos(f*x + e)^4 + 30*(a^4*b + 18*a^3*b^2
+ 48*a^2*b^3 + 32*a*b^4)*f*x*cos(f*x + e)^2 + 15*(a^3*b^2 + 18*a^2*b^3 + 4
8*a*b^4 + 32*b^5)*f*x + 15*((3*a^4 + 16*a^3*b + 16*a^2*b^2)*cos(f*x + e)^4
 + 3*a^2*b^2 + 16*a*b^3 + 16*b^4 + 2*(3*a^3*b + 16*a^2*b^2 + 16*a*b^3)*cos
(f*x + e)^2)*sqrt(a*b + b^2)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/(sq
rt(a*b + b^2)*cos(f*x + e)*sin(f*x + e))) - (8*a^5*cos(f*x + e)^9 - 2*(13*
a^5 + 10*a^4*b)*cos(f*x + e)^7 + (33*a^5 + 110*a^4*b + 80*a^3*b^2)*cos(f*x
 + e)^5 + 20*(6*a^4*b + 23*a^3*b^2 + 18*a^2*b^3)*cos(f*x + e)^3 + 15*(5...
```

3.60.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

input

```
integrate(sin(f*x+e)**6/(a+b*sec(f*x+e)**2)**3,x)
```

output

```
Timed out
```

3.60.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.33

$$\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx =$$

$$-\frac{15\left(5a^2b^2+20ab^3+16b^4\right)\tan(fx+e)^9+40\left(3a^3b+19a^2b^2+39ab^3+24b^4\right)\tan(fx+e)^7+\left(33a^4+470a^3b+1910a^2b^2+2880ab^3+1440b^4\right)\tan(f}{a^5b^2\tan(fx+e)^{10}+\left(2a^6b+5a^5b^2\right)\tan(fx+e)^8+a^7+2a^6b+a^5b^2+\left(a^7+8a^6b+10a^5b^2\right)\tan(fx+e)^6+}$$

input
```
integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
-1/48*((15*(5*a^2*b^2 + 20*a*b^3 + 16*b^4)*tan(f*x + e)^9 + 40*(3*a^3*b +
19*a^2*b^2 + 39*a*b^3 + 24*b^4)*tan(f*x + e)^7 + (33*a^4 + 470*a^3*b + 191
0*a^2*b^2 + 2880*a*b^3 + 1440*b^4)*tan(f*x + e)^5 + 40*(a^4 + 14*a^3*b + 4
6*a^2*b^2 + 57*a*b^3 + 24*b^4)*tan(f*x + e)^3 + 15*(a^4 + 14*a^3*b + 41*a^
2*b^2 + 44*a*b^3 + 16*b^4)*tan(f*x + e))/(a^5*b^2*tan(f*x + e)^10 + (2*a^6
*b + 5*a^5*b^2)*tan(f*x + e)^8 + a^7 + 2*a^6*b + a^5*b^2 + (a^7 + 8*a^6*b
+ 10*a^5*b^2)*tan(f*x + e)^6 + (3*a^7 + 12*a^6*b + 10*a^5*b^2)*tan(f*x + e
)^4 + (3*a^7 + 8*a^6*b + 5*a^5*b^2)*tan(f*x + e)^2) - 15*(a^3 + 18*a^2*b +
 48*a*b^2 + 32*b^3)*(f*x + e)/a^6 + 30*(3*a^3*b + 19*a^2*b^2 + 32*a*b^3 +
16*b^4)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^6))/f
```

3.60.8 Giac [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.12

$$\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$

$$=\frac{15\left(a^3+18a^2b+48ab^2+32b^3\right)(fx+e)}{a^6}-\frac{30\left(3a^3b+19a^2b^2+32ab^3+16b^4\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{\sqrt{ab+b^2}a^6}-\frac{6\left(7a^2b^2\tan(fx+}{}$$

input
```
integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
1/48*(15*(a^3 + 18*a^2*b + 48*a*b^2 + 32*b^3)*(f*x + e)/a^6 - 30*(3*a^3*b
+ 19*a^2*b^2 + 32*a*b^3 + 16*b^4)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + a
rctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/(sqrt(a*b + b^2)*a^6) - 6*(7*a^2*b^
2*tan(f*x + e)^3 + 23*a*b^3*tan(f*x + e)^3 + 16*b^4*tan(f*x + e)^3 + 9*a^3
*b*tan(f*x + e) + 34*a^2*b^2*tan(f*x + e) + 41*a*b^3*tan(f*x + e) + 16*b^4
*tan(f*x + e))/((b*tan(f*x + e)^2 + a + b)^2*a^5) - (33*a^2*tan(f*x + e)^5
 + 162*a*b*tan(f*x + e)^5 + 144*b^2*tan(f*x + e)^5 + 40*a^2*tan(f*x + e)^3
 + 288*a*b*tan(f*x + e)^3 + 288*b^2*tan(f*x + e)^3 + 15*a^2*tan(f*x + e) +
 126*a*b*tan(f*x + e) + 144*b^2*tan(f*x + e))/((tan(f*x + e)^2 + 1)^3*a^5)
)/f
```

3.60.9 Mupad [B] (verification not implemented)

Time = 22.33 (sec) , antiderivative size = 2117, normalized size of antiderivative = 6.74

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(sin(e + f*x)^6/(a + b/cos(e + f*x)^2)^3,x)
```

output
```
(5*atan(((5*((tan(e + f*x)*(179200*a*b^8 + 51200*b^9 + 249600*a^2*b^7 + 17
6000*a^3*b^6 + 65800*a^4*b^5 + 12300*a^5*b^4 + 925*a^6*b^3))/(128*a^10) -
(((20*a^12*b^5 + 35*a^13*b^4 + (65*a^14*b^3)/4 + (5*a^15*b^2)/4)/a^15 - (t
an(e + f*x)*(2048*a^12*b^3 + 1024*a^13*b^2)*(a + 2*b)*(16*a*b + a^2 + 16*b
^2)*5i)/(4096*a^16))*(a + 2*b)*(16*a*b + a^2 + 16*b^2)*5i)/(32*a^6))*(a +
2*b)*(16*a*b + a^2 + 16*b^2))/(32*a^6) + (5*((tan(e + f*x)*(179200*a*b^8 +
 51200*b^9 + 249600*a^2*b^7 + 176000*a^3*b^6 + 65800*a^4*b^5 + 12300*a^5*b
^4 + 925*a^6*b^3))/(128*a^10) + (((20*a^12*b^5 + 35*a^13*b^4 + (65*a^14*b^
3)/4 + (5*a^15*b^2)/4)/a^15 + (tan(e + f*x)*(2048*a^12*b^3 + 1024*a^13*b^2
)*(a + 2*b)*(16*a*b + a^2 + 16*b^2)*5i)/(4096*a^16))*(a + 2*b)*(16*a*b + a
^2 + 16*b^2)*5i)/(32*a^6))*(a + 2*b)*(16*a*b + a^2 + 16*b^2))/(32*a^6))/((
4750*a*b^10 + 1000*b^11 + (18875*a^2*b^9)/2 + (40625*a^3*b^8)/4 + (204875*
a^4*b^7)/32 + (305125*a^5*b^6)/128 + (256125*a^6*b^5)/512 + (53125*a^7*b^4
)/1024 + (1875*a^8*b^3)/1024)/a^15 - (((tan(e + f*x)*(179200*a*b^8 + 51200
*b^9 + 249600*a^2*b^7 + 176000*a^3*b^6 + 65800*a^4*b^5 + 12300*a^5*b^4 + 9
25*a^6*b^3))/(128*a^10) - (((20*a^12*b^5 + 35*a^13*b^4 + (65*a^14*b^3)/4 +
 (5*a^15*b^2)/4)/a^15 - (tan(e + f*x)*(2048*a^12*b^3 + 1024*a^13*b^2)*(a +
 2*b)*(16*a*b + a^2 + 16*b^2)*5i)/(4096*a^16))*(a + 2*b)*(16*a*b + a^2 + 1
6*b^2)*5i)/(32*a^6))*(a + 2*b)*(16*a*b + a^2 + 16*b^2)*5i)/(32*a^6) + (((t
an(e + f*x)*(179200*a*b^8 + 51200*b^9 + 249600*a^2*b^7 + 176000*a^3*b^6...
```

3.61 $\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

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3.61.1 Optimal result

Integrand size = 23, antiderivative size = 238

$$\begin{aligned}\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = {}& \frac{3(a^2+12ab+16b^2)\,x}{8a^5} \\ & - \frac{3\sqrt{b}(5a^2+20ab+16b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^5\sqrt{a+b}f} \\ & - \frac{(5a+8b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^2} \\ & + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^2} \\ & - \frac{b(7a+12b)\tan(e+fx)}{8a^3f\left(a+b+b\tan^2(e+fx)\right)^2} \\ & - \frac{3b(a+2b)\tan(e+fx)}{2a^4f\left(a+b+b\tan^2(e+fx)\right)}\end{aligned}$$

output

```
3/8*(a^2+12*a*b+16*b^2)*x/a^5-3/8*(5*a^2+20*a*b+16*b^2)*arctan(b^(1/2)*tan
(f*x+e)/(a+b)^(1/2))*b^(1/2)/a^5/f/(a+b)^(1/2)-1/8*(5*a+8*b)*cos(f*x+e)*si
n(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)^2+1/4*cos(f*x+e)^3*sin(f*x+e)/a/f/(a+b
+b*tan(f*x+e)^2)^2-1/8*b*(7*a+12*b)*tan(f*x+e)/a^3/f/(a+b+b*tan(f*x+e)^2)^
2-3/2*b*(a+2*b)*tan(f*x+e)/a^4/f/(a+b+b*tan(f*x+e)^2)
```

3.61.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 21.95 (sec) , antiderivative size = 2469, normalized size of antiderivative = 10.37

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
(3*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(((3*a^2 + 8*a*b + 8*b^
2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) - (a*Sqrt[b]*
(3*a^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)
])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2)))/(16384*b^(5/2)*f*(a + b*
Sec[e + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*((-3
*a*(a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) + (
Sqrt[b]*(3*a^3 + 14*a^2*b + 24*a*b^2 + 16*b^3 + a*(3*a^2 + 4*a*b + 4*b^2)*
Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x
)])^2)))/(16384*b^(5/2)*f*(a + b*Sec[e + f*x]^2)^3) - (3*(a + 2*b + a*Cos[
2*e + 2*f*x])^3*Sec[e + f*x]^6*((2*(3*a^5 - 10*a^4*b + 80*a^3*b^2 + 480*a^
2*b^3 + 640*a*b^4 + 256*b^5)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((
a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*
Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[
e])^4]) + (Sec[2*e]*(256*b^2*(a + b)^2*(3*a^2 + 8*a*b + 8*b^2)*f*x*Cos[2*e
] + 512*a*b^2*(a + b)^2*(a + 2*b)*f*x*Cos[2*f*x] + 128*a^4*b^2*f*x*Cos[2*(
e + 2*f*x)] + 256*a^3*b^3*f*x*Cos[2*(e + 2*f*x)] + 128*a^2*b^4*f*x*Cos[2*(
e + 2*f*x)] + 512*a^4*b^2*f*x*Cos[4*e + 2*f*x] + 2048*a^3*b^3*f*x*Cos[4*e
+ 2*f*x] + 2560*a^2*b^4*f*x*Cos[4*e + 2*f*x] + 1024*a*b^5*f*x*Cos[4*e + 2*
f*x] + 128*a^4*b^2*f*x*Cos[6*e + 4*f*x] + 256*a^3*b^3*f*x*Cos[6*e + 4*f*x]
 + 128*a^2*b^4*f*x*Cos[6*e + 4*f*x] - 9*a^6*Sin[2*e] + 12*a^5*b*Sin[2*e...
```

3.61.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.10, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 4620, 372, 402, 402, 27, 402, 27, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sin(e+fx)^4}{(a+b\sec(e+fx)^2)^3}\,dx$$

$$\downarrow 4620$$

$$\frac{\int \frac{\tan^4(e+fx)}{(\tan^2(e+fx)+1)^3(b\tan^2(e+fx)+a+b)^3}\,d\tan(e+fx)}{f}$$

$$\downarrow 372$$

$$\frac{\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)^2} - \frac{\int \frac{-\left((4a+7b)\tan^2(e+fx)\right)+a+b}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^3}\,d\tan(e+fx)}{4a}}{f}$$

$$\downarrow 402$$

$$\frac{\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)^2} - \frac{\frac{(5a+8b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^2} - \frac{\int \frac{(a+b)(3a+8b)-5b(5a+8b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^3}\,d\tan(e+fx)}{2a}}{4a}}{f}$$

$$\downarrow 402$$

$$\frac{\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)^2} - \frac{\frac{(5a+8b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^2} - \frac{\int \frac{12(a+b)\left((a+b)(a+4b)-b(7a+12b)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}\,d\tan(e+fx)}{4a(a+b)}}{2a}}{4a}}{f}$$

$$\downarrow 27$$

$$\frac{\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)^2} - \frac{\frac{(5a+8b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^2} - \frac{3\int \frac{(a+b)(a+4b)-b(7a+12b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}\,d\tan(e+fx)}{a}}{4a}}{f}$$

$$\downarrow 402$$

$$\frac{\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\left(a+b\tan^2(e+fx)+b\right)^2}-\frac{\frac{(5a+8b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^2}-\frac{3\left(\int\frac{2(a+b)\left(a^2+8ba+8b^2-4b(a+2b)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2a(a+b)}}{4a}}{f}$$

$$\downarrow 27$$

$$\frac{\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\left(a+b\tan^2(e+fx)+b\right)^2}-\frac{\frac{(5a+8b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^2}-\frac{3\left(\int\frac{a^2+8ba+8b^2-4b(a+2b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{a}-\frac{4b(}{a(a+}\right)}{a}}{4a}}{f}$$

$$\downarrow 397$$

$$\frac{\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\left(a+b\tan^2(e+fx)+b\right)^2}-\frac{\frac{(5a+8b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^2}-\frac{3\left(\frac{\left(a^2+12ab+16b^2\right)\int\frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}-\frac{b\left(5a^2+20a\right.}{a}\right)}{a}}{4a}}{f}$$

$$\downarrow 216$$

$$\frac{\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\left(a+b\tan^2(e+fx)+b\right)^2}-\frac{\frac{(5a+8b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^2}-\frac{3\left(\frac{\left(a^2+12ab+16b^2\right)\arctan(\tan(e+fx))}{a}-\frac{b\left(5a^2+20ab+16b^2\right)\int}{a}\right)}{a}}{4a}}{f}$$

$$\downarrow 218$$

$$\frac{\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\left(a+b\tan^2(e+fx)+b\right)^2}-\frac{\frac{(5a+8b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^2}-\frac{3\left(\frac{\left(a^2+12ab+16b^2\right)\arctan(\tan(e+fx))}{a}-\frac{\sqrt{b}\left(5a^2+20ab+16b^2\right)}{a}\right)}{a}}{4a}}{f}$$

3.61. $\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

input
```
Int[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
(Tan[e + f*x]/(4*a*(1 + Tan[e + f*x]^2)^2*(a + b + b*Tan[e + f*x]^2)^2) -
(((5*a + 8*b)*Tan[e + f*x])/(2*a*(1 + Tan[e + f*x]^2)*(a + b + b*Tan[e + f
*x]^2)^2) - (-((b*(7*a + 12*b)*Tan[e + f*x])/(a*(a + b + b*Tan[e + f*x]^2)
^2)) + (3*((((a^2 + 12*a*b + 16*b^2)*ArcTan[Tan[e + f*x]])/a - (Sqrt[b]*(5
*a^2 + 20*a*b + 16*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*Sqr
t[a + b]))/a - (4*b*(a + 2*b)*Tan[e + f*x])/(a*(a + b + b*Tan[e + f*x]^2))
))/a)/(2*a))/(4*a))/f
```

3.61.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.61.4 Maple [A] (verified)

Time = 6.37 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{\frac{\left(-\frac{3}{2}ab-\frac{5}{8}a^2\right)\tan(fx+e)^3+\left(-\frac{3}{8}a^2-\frac{3}{2}ab\right)\tan(fx+e)}{\left(1+\tan(fx+e)^2\right)^2}+\frac{3\left(a^2+12ab+16b^2\right)\arctan(\tan(fx+e))}{8}}{a^5}-\frac{b\left(\frac{\left(\frac{7}{8}a^2b+\frac{3}{2}ab^2\right)\tan(fx+e)^3+\frac{3}{}\right.}{\left(a+b+b\tan(\right.}}{f}$
default	$\frac{\frac{\left(-\frac{3}{2}ab-\frac{5}{8}a^2\right)\tan(fx+e)^3+\left(-\frac{3}{8}a^2-\frac{3}{2}ab\right)\tan(fx+e)}{\left(1+\tan(fx+e)^2\right)^2}+\frac{3\left(a^2+12ab+16b^2\right)\arctan(\tan(fx+e))}{8}}{a^5}-\frac{b\left(\frac{\left(\frac{7}{8}a^2b+\frac{3}{2}ab^2\right)\tan(fx+e)^3+\frac{3}{}\right.}{\left(a+b+b\tan(\right.}}{f}$
risch	$\frac{3x}{8a^3}+\frac{9xb}{2a^4}+\frac{6xb^2}{a^5}-\frac{ib\left(9a^3e^{6i(fx+e)}+36a^2be^{6i(fx+e)}+32ab^2e^{6i(fx+e)}+27a^3e^{4i(fx+e)}+114a^2be^{4i(fx+e)}+184a\right.}{4a^5f\left(ae^{4i(fx+e)}+2ae^{2i}\right.}$

input
```
int(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/a^5*(((-3/2*a*b-5/8*a^2)*tan(f*x+e)^3+(-3/8*a^2-3/2*a*b)*tan(f*x+e)
)/(1+tan(f*x+e)^2)^2+3/8*(a^2+12*a*b+16*b^2)*arctan(tan(f*x+e)))-b/a^5*(((
7/8*a^2*b+3/2*a*b^2)*tan(f*x+e)^3+3/8*a*(3*a^2+7*a*b+4*b^2)*tan(f*x+e))/(a
+b+b*tan(f*x+e)^2)^2+3/8*(5*a^2+20*a*b+16*b^2)/((a+b)*b)^(1/2)*arctan(b*ta
n(f*x+e)/((a+b)*b)^(1/2))))
```

3.61.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 803, normalized size of antiderivative = 3.37

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \left[\frac{12\,(a^4+12\,a^3b+16\,a^2b^2)fx\cos(fx+e)^4+24\,(a^3b+12\,a^2b^2+16\,ab^3)fx\cos(fx+e)^2+12\,(a^2b^2+12\right.$$

input
```
integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[1/32*(12*(a^4 + 12*a^3*b + 16*a^2*b^2)*f*x*cos(f*x + e)^4 + 24*(a^3*b + 1
2*a^2*b^2 + 16*a*b^3)*f*x*cos(f*x + e)^2 + 12*(a^2*b^2 + 12*a*b^3 + 16*b^4
)*f*x + 3*((5*a^4 + 20*a^3*b + 16*a^2*b^2)*cos(f*x + e)^4 + 5*a^2*b^2 + 20
*a*b^3 + 16*b^4 + 2*(5*a^3*b + 20*a^2*b^2 + 16*a*b^3)*cos(f*x + e)^2)*sqrt
(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)
*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*co
s(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a
*b*cos(f*x + e)^2 + b^2)) + 4*(2*a^4*cos(f*x + e)^7 - (5*a^4 + 8*a^3*b)*co
s(f*x + e)^5 - (19*a^3*b + 36*a^2*b^2)*cos(f*x + e)^3 - 12*(a^2*b^2 + 2*a*
b^3)*cos(f*x + e))*sin(f*x + e))/(a^7*f*cos(f*x + e)^4 + 2*a^6*b*f*cos(f*x
 + e)^2 + a^5*b^2*f), 1/16*(6*(a^4 + 12*a^3*b + 16*a^2*b^2)*f*x*cos(f*x +
e)^4 + 12*(a^3*b + 12*a^2*b^2 + 16*a*b^3)*f*x*cos(f*x + e)^2 + 6*(a^2*b^2
+ 12*a*b^3 + 16*b^4)*f*x + 3*((5*a^4 + 20*a^3*b + 16*a^2*b^2)*cos(f*x + e)
^4 + 5*a^2*b^2 + 20*a*b^3 + 16*b^4 + 2*(5*a^3*b + 20*a^2*b^2 + 16*a*b^3)*c
os(f*x + e)^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*s
qrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e))) + 2*(2*a^4*cos(f*x + e)^7 -
(5*a^4 + 8*a^3*b)*cos(f*x + e)^5 - (19*a^3*b + 36*a^2*b^2)*cos(f*x + e)^3
- 12*(a^2*b^2 + 2*a*b^3)*cos(f*x + e))*sin(f*x + e))/(a^7*f*cos(f*x + e)^4
 + 2*a^6*b*f*cos(f*x + e)^2 + a^5*b^2*f)]
```

3.61.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**4/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Timed out
```

3.61.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.26

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx =$$

$$-\frac{\frac{12\,(ab^2+2\,b^3)\tan(fx+e)^7+(19\,a^2b+72\,ab^2+72\,b^3)\tan(fx+e)^5+(5\,a^3+46\,a^2b+108\,ab^2+72\,b^3)\tan(fx+e)^3+3\,(a^3+9\,a^2b+16\,ab^2+8\,b^3)\tan(}{a^4b^2\tan(fx+e)^8+2\,(a^5b+2\,a^4b^2)\tan(fx+e)^6+a^6+2\,a^5b+a^4b^2+(a^6+6\,a^5b+6\,a^4b^2)\tan(fx+e)^4+2\,(a^6+3\,a^5b+2\,a^4b^2)\tan(fx+e)^2}}{8\,f}$$

input
```
integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
-1/8*((12*(a*b^2 + 2*b^3)*tan(f*x + e)^7 + (19*a^2*b + 72*a*b^2 + 72*b^3)*
tan(f*x + e)^5 + (5*a^3 + 46*a^2*b + 108*a*b^2 + 72*b^3)*tan(f*x + e)^3 +
3*(a^3 + 9*a^2*b + 16*a*b^2 + 8*b^3)*tan(f*x + e))/(a^4*b^2*tan(f*x + e)^8
 + 2*(a^5*b + 2*a^4*b^2)*tan(f*x + e)^6 + a^6 + 2*a^5*b + a^4*b^2 + (a^6 +
 6*a^5*b + 6*a^4*b^2)*tan(f*x + e)^4 + 2*(a^6 + 3*a^5*b + 2*a^4*b^2)*tan(f
*x + e)^2) - 3*(a^2 + 12*a*b + 16*b^2)*(f*x + e)/a^5 + 3*(5*a^2*b + 20*a*b
^2 + 16*b^3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^5))
/f
```

3.61.8 Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.29

$$\int \frac{\sin^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$

$$= \frac{3\,(a^2+12\,ab+16\,b^2)(fx+e)}{a^5} - \frac{3\,(5\,a^2b+20\,ab^2+16\,b^3)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{\sqrt{ab+b^2}\,a^5} - \frac{12\,ab^2\tan(fx+e)^7+24\,b^3\tan(fx+e)^7-}{}$$

input
```
integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
1/8*(3*(a^2 + 12*a*b + 16*b^2)*(f*x + e)/a^5 - 3*(5*a^2*b + 20*a*b^2 + 16*
b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
 + b^2)))/(sqrt(a*b + b^2)*a^5) - (12*a*b^2*tan(f*x + e)^7 + 24*b^3*tan(f*
x + e)^7 + 19*a^2*b*tan(f*x + e)^5 + 72*a*b^2*tan(f*x + e)^5 + 72*b^3*tan(
f*x + e)^5 + 5*a^3*tan(f*x + e)^3 + 46*a^2*b*tan(f*x + e)^3 + 108*a*b^2*ta
n(f*x + e)^3 + 72*b^3*tan(f*x + e)^3 + 3*a^3*tan(f*x + e) + 27*a^2*b*tan(f
*x + e) + 48*a*b^2*tan(f*x + e) + 24*b^3*tan(f*x + e))/((b*tan(f*x + e)^4
+ a*tan(f*x + e)^2 + 2*b*tan(f*x + e)^2 + a + b)^2*a^4))/f
```

3.61.9 Mupad [B] (verification not implemented)

Time = 21.11 (sec) , antiderivative size = 1317, normalized size of antiderivative = 5.53

$$\int \frac{\sin^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(sin(e + f*x)^4/(a + b/cos(e + f*x)^2)^3,x)
```

output

```
(atan(((((tan(e + f*x)*(4608*a*b^6 + 2304*b^7 + 3312*a^2*b^5 + 1008*a^3*b^
4 + 117*a^4*b^3))/(16*a^8) - (3*((12*a^10*b^4 + 12*a^11*b^3 + (3*a^12*b^2)
/2)/a^12 - (3*tan(e + f*x)*(256*a^10*b^3 + 128*a^11*b^2)*(-b*(a + b))^(1/2
)*(20*a*b + 5*a^2 + 16*b^2))/(256*a^8*(a^5*b + a^6)))*(-b*(a + b))^(1/2)*(
20*a*b + 5*a^2 + 16*b^2))/(16*(a^5*b + a^6)))*(-b*(a + b))^(1/2)*(20*a*b +
 5*a^2 + 16*b^2)*3i)/(16*(a^5*b + a^6)) + (((tan(e + f*x)*(4608*a*b^6 + 23
04*b^7 + 3312*a^2*b^5 + 1008*a^3*b^4 + 117*a^4*b^3))/(16*a^8) + (3*((12*a^
10*b^4 + 12*a^11*b^3 + (3*a^12*b^2)/2)/a^12 + (3*tan(e + f*x)*(256*a^10*b^
3 + 128*a^11*b^2)*(-b*(a + b))^(1/2)*(20*a*b + 5*a^2 + 16*b^2))/(256*a^8*(
a^5*b + a^6)))*(-b*(a + b))^(1/2)*(20*a*b + 5*a^2 + 16*b^2))/(16*(a^5*b +
a^6)))*(-b*(a + b))^(1/2)*(20*a*b + 5*a^2 + 16*b^2)*3i)/(16*(a^5*b + a^6))
)/((540*a*b^7 + 216*b^8 + (999*a^2*b^6)/2 + (837*a^3*b^5)/4 + (1215*a^4*b^
4)/32 + (135*a^5*b^3)/64)/a^12 - (3*((tan(e + f*x)*(4608*a*b^6 + 2304*b^7
+ 3312*a^2*b^5 + 1008*a^3*b^4 + 117*a^4*b^3))/(16*a^8) - (3*((12*a^10*b^4
+ 12*a^11*b^3 + (3*a^12*b^2)/2)/a^12 - (3*tan(e + f*x)*(256*a^10*b^3 + 128
*a^11*b^2)*(-b*(a + b))^(1/2)*(20*a*b + 5*a^2 + 16*b^2))/(256*a^8*(a^5*b +
 a^6)))*(-b*(a + b))^(1/2)*(20*a*b + 5*a^2 + 16*b^2))/(16*(a^5*b + a^6)))*
(-b*(a + b))^(1/2)*(20*a*b + 5*a^2 + 16*b^2))/(16*(a^5*b + a^6)) + (3*((ta
n(e + f*x)*(4608*a*b^6 + 2304*b^7 + 3312*a^2*b^5 + 1008*a^3*b^4 + 117*a^4*
b^3))/(16*a^8) + (3*((12*a^10*b^4 + 12*a^11*b^3 + (3*a^12*b^2)/2)/a^12 ...
```

3.62 $\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

3.62.1 Optimal result 599
3.62.2 Mathematica [C] (warning: unable to verify) 600
3.62.3 Rubi [A] (verified) 600
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3.62.1 Optimal result

Integrand size = 23, antiderivative size = 184

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{(a+6b)x}{2a^4} - \frac{\sqrt{b}(15a^2+40ab+24b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^4(a+b)^{3/2}f} - \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{3b\tan(e+fx)}{4a^2f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(11a+12b)\tan(e+fx)}{8a^3(a+b)f\left(a+b+b\tan^2(e+fx)\right)}$$

output

```
1/2*(a+6*b)*x/a^4-1/8*(15*a^2+40*a*b+24*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+
b)^(1/2))*b^(1/2)/a^4/(a+b)^(3/2)/f-1/2*cos(f*x+e)*sin(f*x+e)/a/f/(a+b+b*t
an(f*x+e)^2)^2-3/4*b*tan(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)^2-1/8*b*(11*a+1
2*b)*tan(f*x+e)/a^3/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

3.62.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 15.36 (sec) , antiderivative size = 1915, normalized size of antiderivative = 10.41

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
Integrate[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
(5*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(((3*a^2 + 8*a*b + 8*b^
2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) - (a*Sqrt[b]*
(3*a^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)
])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2)))/(8192*b^(5/2)*f*(a + b*S
ec[e + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*((-3*
a*(a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) + (S
qrt[b]*(3*a^3 + 14*a^2*b + 24*a*b^2 + 16*b^3 + a*(3*a^2 + 4*a*b + 4*b^2)*C
os[2*(e + f*x)])*Sin[2*(e + f*x)])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)
])^2)))/(2048*b^(5/2)*f*(a + b*Sec[e + f*x]^2)^3) - ((a + 2*b + a*Cos[2*e
+ 2*f*x])^3*Sec[e + f*x]^6*((2*(3*a^5 - 10*a^4*b + 80*a^3*b^2 + 480*a^2*b^
3 + 640*a*b^4 + 256*b^5)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a +
2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[
e])^4])]*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^
4]) + (Sec[2*e]*(256*b^2*(a + b)^2*(3*a^2 + 8*a*b + 8*b^2)*f*x*Cos[2*e] +
512*a*b^2*(a + b)^2*(a + 2*b)*f*x*Cos[2*f*x] + 128*a^4*b^2*f*x*Cos[2*(e +
2*f*x)] + 256*a^3*b^3*f*x*Cos[2*(e + 2*f*x)] + 128*a^2*b^4*f*x*Cos[2*(e +
2*f*x)] + 512*a^4*b^2*f*x*Cos[4*e + 2*f*x] + 2048*a^3*b^3*f*x*Cos[4*e + 2*
f*x] + 2560*a^2*b^4*f*x*Cos[4*e + 2*f*x] + 1024*a*b^5*f*x*Cos[4*e + 2*f*x]
 + 128*a^4*b^2*f*x*Cos[6*e + 4*f*x] + 256*a^3*b^3*f*x*Cos[6*e + 4*f*x] + 1
28*a^2*b^4*f*x*Cos[6*e + 4*f*x] - 9*a^6*Sin[2*e] + 12*a^5*b*Sin[2*e] + ...
```

3.62.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.17, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4620, 373, 402, 27, 402, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sin(e+fx)^2}{(a+b\sec(e+fx)^2)^3}\,dx$$

$$\downarrow 4620$$

$$\frac{\int \frac{\tan^2(e+fx)}{(\tan^2(e+fx)+1)^2(b\tan^2(e+fx)+a+b)^3}\,d\tan(e+fx)}{f}$$

$$\downarrow 373$$

$$\frac{\int \frac{-5b\tan^2(e+fx)+a+b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^3}\,d\tan(e+fx)}{2a} - \frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\downarrow 402$$

$$\frac{\frac{\int \frac{2(a+b)\left(-9b\tan^2(e+fx)+2a+3b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}\,d\tan(e+fx)}{4a(a+b)} - \frac{3b\tan(e+fx)}{2a\left(a+b\tan^2(e+fx)+b\right)^2}}{2a} - \frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\downarrow 27$$

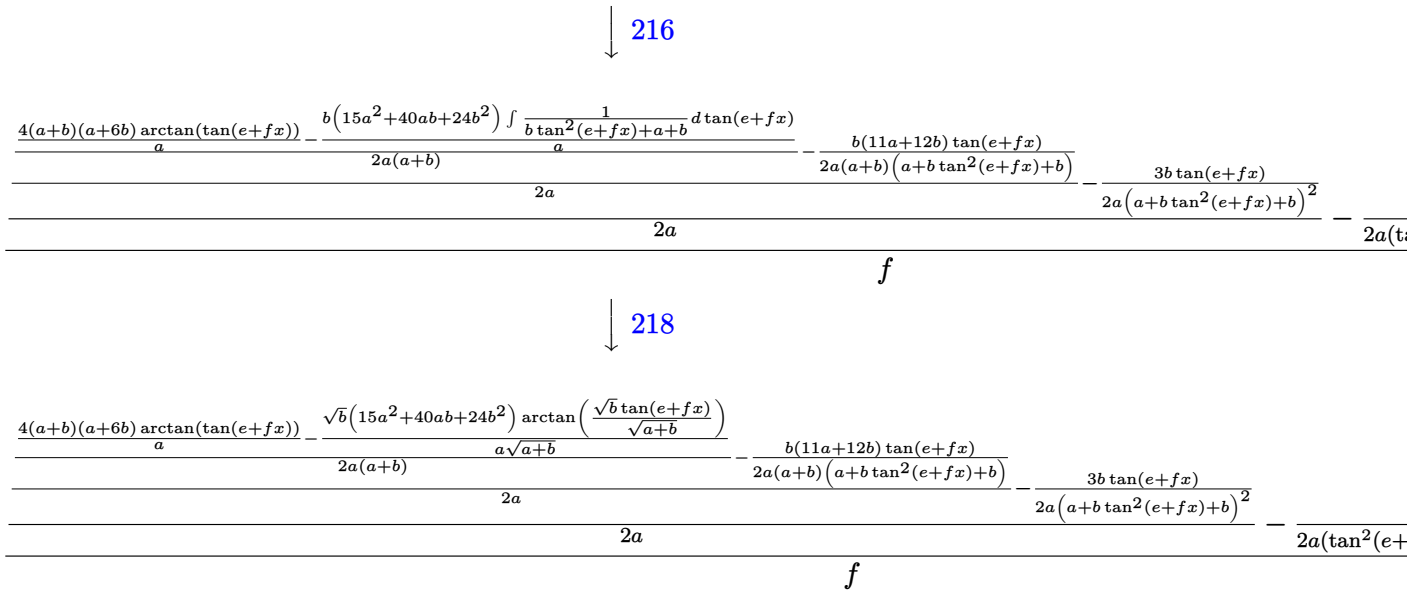
$$\frac{\frac{\int \frac{-9b\tan^2(e+fx)+2a+3b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}\,d\tan(e+fx)}{2a} - \frac{3b\tan(e+fx)}{2a\left(a+b\tan^2(e+fx)+b\right)^2}}{2a} - \frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\downarrow 402$$

$$\frac{\frac{\int \frac{4a^2+17ba+12b^2-b(11a+12b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}\,d\tan(e+fx)}{2a(a+b)} - \frac{b(11a+12b)\tan(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{2a} - \frac{\frac{3b\tan(e+fx)}{2a\left(a+b\tan^2(e+fx)+b\right)^2} - \frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)^2}}{2a}}{f}$$

$$\downarrow 397$$

$$\frac{\frac{4(a+b)(a+6b)\int \frac{1}{\tan^2(e+fx)+1}\,d\tan(e+fx)}{a} - \frac{b\left(15a^2+40ab+24b^2\right)\int \frac{1}{b\tan^2(e+fx)+a+b}\,d\tan(e+fx)}{a}}{2a(a+b)} - \frac{b(11a+12b)\tan(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{2a} - \frac{\frac{3b\tan(e+fx)}{2a\left(a+b\tan^2(e+fx)+b\right)^2} - \frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)^2}}{2a}}{f}$$



input
```
Int[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
(-1/2*Tan[e + f*x]/(a*(1 + Tan[e + f*x]^2)*(a + b + b*Tan[e + f*x]^2)^2) +
 ((-3*b*Tan[e + f*x])/(2*a*(a + b + b*Tan[e + f*x]^2)^2) + (((4*(a + b)*(a
 + 6*b)*ArcTan[Tan[e + f*x]])/a - (Sqrt[b]*(15*a^2 + 40*a*b + 24*b^2)*ArcT
an[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(2*a*(a + b)) - (
b*(11*a + 12*b)*Tan[e + f*x])/(2*a*(a + b)*(a + b + b*Tan[e + f*x]^2)))/(2
*a))/(2*a))/f
```

3.62.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.62.4 Maple [A] (verified)

Time = 4.21 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{-\frac{a\tan(fx+e)}{2\left(1+\tan(fx+e)^2\right)}+\frac{(a+6b)\arctan(\tan(fx+e))}{2}}{a^4}-\frac{b\left(\frac{\frac{ab(7a+8b)\tan(fx+e)^3}{8a+8b}+\frac{a(9a+8b)\tan(fx+e)}{8}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(15a^2+40ab+24b^2\right)\arctan($ f
default	$\frac{-\frac{a\tan(fx+e)}{2\left(1+\tan(fx+e)^2\right)}+\frac{(a+6b)\arctan(\tan(fx+e))}{2}}{a^4}-\frac{b\left(\frac{\frac{ab(7a+8b)\tan(fx+e)^3}{8a+8b}+\frac{a(9a+8b)\tan(fx+e)}{8}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(15a^2+40ab+24b^2\right)\arctan($ f
risch	$\frac{x}{2a^3}+\frac{3xb}{a^4}+\frac{ie^{2i(fx+e)}}{8a^3f}-\frac{ie^{-2i(fx+e)}}{8a^3f}-\frac{ib\left(9a^3e^{6i(fx+e)}+32a^2be^{6i(fx+e)}+24ab^2e^{6i(fx+e)}+27a^3e^{4i(fx+e)}+10\right.}{4a^4(a+}$

input
```
int(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/a^4*(-1/2*a*tan(f*x+e)/(1+tan(f*x+e)^2)+1/2*(a+6*b)*arctan(tan(f*x+
e)))-1/a^4*b*((1/8*a*b*(7*a+8*b)/(a+b)*tan(f*x+e)^3+1/8*a*(9*a+8*b)*tan(f*
x+e))/(a+b+b*tan(f*x+e)^2)^2+1/8*(15*a^2+40*a*b+24*b^2)/(a+b)/((a+b)*b)^(1
/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))))
```

3.62.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 364 vs. 2(166) = 332.

Time = 0.33 (sec) , antiderivative size = 815, normalized size of antiderivative = 4.43

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$= \left[\frac{16\left(a^4+7a^3b+6a^2b^2\right)fx\cos(fx+e)^4+32\left(a^3b+7a^2b^2+6ab^3\right)fx\cos(fx+e)^2+16\left(a^2b^2+7ab^3+\right.}{}\right.$$

input
```
integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[1/32*(16*(a^4 + 7*a^3*b + 6*a^2*b^2)*f*x*cos(f*x + e)^4 + 32*(a^3*b + 7*a
^2*b^2 + 6*a*b^3)*f*x*cos(f*x + e)^2 + 16*(a^2*b^2 + 7*a*b^3 + 6*b^4)*f*x
+ ((15*a^4 + 40*a^3*b + 24*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2 + 40*a*b^3
 + 24*b^4 + 2*(15*a^3*b + 40*a^2*b^2 + 24*a*b^3)*cos(f*x + e)^2)*sqrt(-b/(
a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(
f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x
 + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*co
s(f*x + e)^2 + b^2)) - 4*(4*(a^4 + a^3*b)*cos(f*x + e)^5 + (17*a^3*b + 18*
a^2*b^2)*cos(f*x + e)^3 + (11*a^2*b^2 + 12*a*b^3)*cos(f*x + e))*sin(f*x +
e))/((a^7 + a^6*b)*f*cos(f*x + e)^4 + 2*(a^6*b + a^5*b^2)*f*cos(f*x + e)^2
 + (a^5*b^2 + a^4*b^3)*f), 1/16*(8*(a^4 + 7*a^3*b + 6*a^2*b^2)*f*x*cos(f*x
 + e)^4 + 16*(a^3*b + 7*a^2*b^2 + 6*a*b^3)*f*x*cos(f*x + e)^2 + 8*(a^2*b^2
 + 7*a*b^3 + 6*b^4)*f*x + ((15*a^4 + 40*a^3*b + 24*a^2*b^2)*cos(f*x + e)^4
 + 15*a^2*b^2 + 40*a*b^3 + 24*b^4 + 2*(15*a^3*b + 40*a^2*b^2 + 24*a*b^3)*c
os(f*x + e)^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*s
qrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e))) - 2*(4*(a^4 + a^3*b)*cos(f*x
 + e)^5 + (17*a^3*b + 18*a^2*b^2)*cos(f*x + e)^3 + (11*a^2*b^2 + 12*a*b^3)
*cos(f*x + e))*sin(f*x + e))/((a^7 + a^6*b)*f*cos(f*x + e)^4 + 2*(a^6*b +
a^5*b^2)*f*cos(f*x + e)^2 + (a^5*b^2 + a^4*b^3)*f)]
```

3.62.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**2/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Timed out
```

3.62.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.48

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx =$$

$$-\frac{\frac{\left(15\,a^2b+40\,ab^2+24\,b^3\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\left(a^5+a^4b\right)\sqrt{(a+b)b}}+\frac{\left(11\,ab^2+12\,b^3\right)\tan(fx+e)^5+\left(17\,a^2b+40\,ab^2+24\,b^3\right)\tan(fx+e)^3+\left(4\,a^3+21\,a^2b+\right.}{\left(a^4b^2+a^3b^3\right)\tan(fx+e)^6+a^6+3\,a^5b+3\,a^4b^2+a^3b^3+\left(2\,a^5b+5\,a^4b^2+3\,a^3b^3\right)\tan(fx+e)^4+(a}}{8\,f}$$

input
```
integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
-1/8*((15*a^2*b + 40*a*b^2 + 24*b^3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b)
)/((a^5 + a^4*b)*sqrt((a + b)*b)) + ((11*a*b^2 + 12*b^3)*tan(f*x + e)^5 +
(17*a^2*b + 40*a*b^2 + 24*b^3)*tan(f*x + e)^3 + (4*a^3 + 21*a^2*b + 29*a*b
^2 + 12*b^3)*tan(f*x + e))/((a^4*b^2 + a^3*b^3)*tan(f*x + e)^6 + a^6 + 3*a
^5*b + 3*a^4*b^2 + a^3*b^3 + (2*a^5*b + 5*a^4*b^2 + 3*a^3*b^3)*tan(f*x + e
)^4 + (a^6 + 5*a^5*b + 7*a^4*b^2 + 3*a^3*b^3)*tan(f*x + e)^2) - 4*(f*x + e
)*(a + 6*b)/a^4)/f
```

3.62.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.13

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{\frac{\left(15\,a^2b+40\,ab^2+24\,b^3\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{\left(a^5+a^4b\right)\sqrt{ab+b^2}}+\frac{7\,ab^2\tan(fx+e)^3+8\,b^3\tan(fx+e)^3+9\,a^2b\tan(fx+e)+17\,ab^2\tan(f}{\left(a^4+a^3b\right)\left(b\tan(fx+e)^2+a+b\right)^2}}{8\,f}$$

input
```
integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
-1/8*((15*a^2*b + 40*a*b^2 + 24*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b)
+ arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^5 + a^4*b)*sqrt(a*b + b^2))
+ (7*a*b^2*tan(f*x + e)^3 + 8*b^3*tan(f*x + e)^3 + 9*a^2*b*tan(f*x + e) +
17*a*b^2*tan(f*x + e) + 8*b^3*tan(f*x + e))/((a^4 + a^3*b)*(b*tan(f*x + e)
^2 + a + b)^2) - 4*(f*x + e)*(a + 6*b)/a^4 + 4*tan(f*x + e)/((tan(f*x + e)
^2 + 1)*a^3))/f
```

3.62.9 Mupad [B] (verification not implemented)

Time = 22.32 (sec) , antiderivative size = 2628, normalized size of antiderivative = 14.28

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(sin(e + f*x)^2/(a + b/cos(e + f*x)^2)^3,x)
```

output

```
(atan(((((tan(e + f*x)*(3264*a*b^6 + 1152*b^7 + 3296*a^2*b^5 + 1424*a^3*b^
4 + 241*a^4*b^3))/(32*(2*a^7*b + a^8 + a^6*b^2)) - (((6*a^8*b^5 + (29*a^9*
b^4)/2 + (21*a^10*b^3)/2 + 2*a^11*b^2)/(2*a^10*b + a^11 + a^9*b^2) - (tan(
e + f*x)*(a*1i + b*6i)*(512*a^8*b^5 + 1280*a^9*b^4 + 1024*a^10*b^3 + 256*a
^11*b^2))/(128*a^4*(2*a^7*b + a^8 + a^6*b^2)))*(a*1i + b*6i))/(4*a^4))*(a*
1i + b*6i)*1i)/(4*a^4) + (((tan(e + f*x)*(3264*a*b^6 + 1152*b^7 + 3296*a^2
*b^5 + 1424*a^3*b^4 + 241*a^4*b^3))/(32*(2*a^7*b + a^8 + a^6*b^2)) + (((6*
a^8*b^5 + (29*a^9*b^4)/2 + (21*a^10*b^3)/2 + 2*a^11*b^2)/(2*a^10*b + a^11
 + a^9*b^2) + (tan(e + f*x)*(a*1i + b*6i)*(512*a^8*b^5 + 1280*a^9*b^4 + 102
4*a^10*b^3 + 256*a^11*b^2))/(128*a^4*(2*a^7*b + a^8 + a^6*b^2)))*(a*1i + b
*6i))/(4*a^4))*(a*1i + b*6i)*1i)/(4*a^4))/(((297*a*b^6)/4 + 27*b^7 + (279*
a^2*b^5)/4 + (805*a^3*b^4)/32 + (165*a^4*b^3)/64)/(2*a^10*b + a^11 + a^9*b
^2) - (((tan(e + f*x)*(3264*a*b^6 + 1152*b^7 + 3296*a^2*b^5 + 1424*a^3*b^4
 + 241*a^4*b^3))/(32*(2*a^7*b + a^8 + a^6*b^2)) - (((6*a^8*b^5 + (29*a^9*b
^4)/2 + (21*a^10*b^3)/2 + 2*a^11*b^2)/(2*a^10*b + a^11 + a^9*b^2) - (tan(e
 + f*x)*(a*1i + b*6i)*(512*a^8*b^5 + 1280*a^9*b^4 + 1024*a^10*b^3 + 256*a^
11*b^2))/(128*a^4*(2*a^7*b + a^8 + a^6*b^2)))*(a*1i + b*6i))/(4*a^4))*(a*1
i + b*6i))/(4*a^4) + (((tan(e + f*x)*(3264*a*b^6 + 1152*b^7 + 3296*a^2*b^5
 + 1424*a^3*b^4 + 241*a^4*b^3))/(32*(2*a^7*b + a^8 + a^6*b^2)) + (((6*a^8*
b^5 + (29*a^9*b^4)/2 + (21*a^10*b^3)/2 + 2*a^11*b^2)/(2*a^10*b + a^11 +...
```

3.63 $\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

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3.63.1 Optimal result

Integrand size = 14, antiderivative size = 144

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{x}{a^3} - \frac{\sqrt{b}(15a^2+20ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{5/2}f} - \frac{b\tan(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(7a+4b)\tan(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}$$

output
```
x/a^3-1/8*(15*a^2+20*a*b+8*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(
1/2)/a^3/(a+b)^(5/2)/f-1/4*b*tan(f*x+e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2-1
/8*b*(7*a+4*b)*tan(f*x+e)/a^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)
```

3.63.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.39 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.31

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^6(e+fx)\left(8x(a+2b+a\cos(2(e+fx)))^2 + \frac{b(15a^2+20ab+8b^2)\arctan\left(\frac{\sec(fx)(\cos}{}\right.}{}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(-3),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^6*(8*x*(a + 2*b + a*Cos[2*(e
+ f*x)])^2 + (b*(15*a^2 + 20*a*b + 8*b^2)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*S
in[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b
*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])^2*(Cos[2*e] - I*S
in[2*e]))/((a + b)^(5/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) - (4*b^2*((a + 2
*b)*Sin[2*e] - a*Sin[2*f*x]))/((a + b)*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e
])) + (b*(a + 2*b + a*Cos[2*(e + f*x)])*((9*a^2 + 28*a*b + 16*b^2)*Sin[2*e
] - 3*a*(3*a + 2*b)*Sin[2*f*x]))/((a + b)^2*f*(Cos[e] - Sin[e])*(Cos[e] +
Sin[e])))))/(64*a^3*(a + b*Sec[e + f*x]^2)^3)
```

3.63.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.22, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4616, 316, 402, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sec^2(e+fx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{(a+b\sec(e+fx)^2)^3}\,dx$$

$$\downarrow 4616$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^3}\,d\tan(e+fx)}{f}$$

$$\downarrow 316$$

$$\frac{\int \frac{-3b\tan^2(e+fx)+4a+b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}\,d\tan(e+fx)}{4a(a+b)} - \frac{b\tan(e+fx)}{4a(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\downarrow 402$$

$$\frac{\frac{\int \frac{8a^2+9ba+4b^2-b(7a+4b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2a(a+b)}-\frac{b(7a+4b)\tan(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{4a(a+b)}-\frac{b\tan(e+fx)}{4a(a+b)(a+b\tan^2(e+fx)+b)^2}$$
$$f$$
$$\downarrow 397$$
$$\frac{\frac{8(a+b)^2\int\frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}-\frac{b\left(15a^2+20ab+8b^2\right)\int\frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{2a(a+b)}-\frac{b(7a+4b)\tan(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{4a(a+b)}-\frac{b\tan(e+fx)}{4a(a+b)(a+b\tan^2(e+fx)-}$$
$$f$$
$$\downarrow 216$$
$$\frac{\frac{8(a+b)^2\arctan(\tan(e+fx))}{a}-\frac{b\left(15a^2+20ab+8b^2\right)\int\frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{2a(a+b)}-\frac{b(7a+4b)\tan(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{4a(a+b)}-\frac{b\tan(e+fx)}{4a(a+b)(a+b\tan^2(e+fx)+b)^2}$$
$$f$$
$$\downarrow 218$$
$$\frac{\frac{8(a+b)^2\arctan(\tan(e+fx))}{a}-\frac{\sqrt{b}\left(15a^2+20ab+8b^2\right)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{2a(a+b)}-\frac{b(7a+4b)\tan(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{4a(a+b)}-\frac{b\tan(e+fx)}{4a(a+b)(a+b\tan^2(e+fx)+b)^2}$$
$$f$$

input
```
Int[(a + b*Sec[e + f*x]^2)^(-3),x]
```

output
```
(-1/4*(b*Tan[e + f*x])/(a*(a + b)*(a + b + b*Tan[e + f*x]^2)^2) + (((8*(a
+ b)^2*ArcTan[Tan[e + f*x]])/a - (Sqrt[b]*(15*a^2 + 20*a*b + 8*b^2)*ArcTan
[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(2*a*(a + b)) - (b*
(7*a + 4*b)*Tan[e + f*x])/(2*a*(a + b)*(a + b + b*Tan[e + f*x]^2)))/(4*a*(
a + b)))/f
```

3.63.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

3.63.4 Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{\frac{\arctan(\tan(fx+e))}{a^3}-\frac{b\left(\frac{\frac{ab(7a+4b)\tan(fx+e)^3}{8a^2+16ab+8b^2}+\frac{(9a+4b)a\tan(fx+e)}{8a+8b}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(15a^2+20ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\left(a^2+2ab+b^2\right)\sqrt{(a+b)b}}\right)}{a^3}}{f}$
default	$\frac{\frac{\arctan(\tan(fx+e))}{a^3}-\frac{b\left(\frac{\frac{ab(7a+4b)\tan(fx+e)^3}{8a^2+16ab+8b^2}+\frac{(9a+4b)a\tan(fx+e)}{8a+8b}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(15a^2+20ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\left(a^2+2ab+b^2\right)\sqrt{(a+b)b}}\right)}{a^3}}{f}$
risch	$\frac{x}{a^3}-\frac{ib\left(9a^3e^{6i(fx+e)}+28a^2be^{6i(fx+e)}+16ab^2e^{6i(fx+e)}+27a^3e^{4i(fx+e)}+90a^2be^{4i(fx+e)}+120ab^2e^{4i(fx+e)}+48b^3e^{4i}\right.}{4a^3(a+b)^2f\left(ae^{4i(fx+e)}+2ae^{2i(fx+e)}+4be^{2i(fx+\right.}$

input
```
int(1/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/a^3*arctan(tan(f*x+e))-b/a^3*((1/8*a*b*(7*a+4*b)/(a^2+2*a*b+b^2)*ta
n(f*x+e)^3+1/8*(9*a+4*b)*a/(a+b)*tan(f*x+e))/(a+b+b*tan(f*x+e)^2)^2+1/8*(1
5*a^2+20*a*b+8*b^2)/(a^2+2*a*b+b^2)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((
a+b)*b)^(1/2))))
```

3.63.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. 2(130) = 260.

Time = 0.33 (sec) , antiderivative size = 819, normalized size of antiderivative = 5.69

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$

$$=\left[\frac{32\left(a^4+2a^3b+a^2b^2\right)fx\cos(fx+e)^4+64\left(a^3b+2a^2b^2+ab^3\right)fx\cos(fx+e)^2+32\left(a^2b^2+2ab^3+b^4\right)}{}\right.$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[1/32*(32*(a^4 + 2*a^3*b + a^2*b^2)*f*x*cos(f*x + e)^4 + 64*(a^3*b + 2*a^2
*b^2 + a*b^3)*f*x*cos(f*x + e)^2 + 32*(a^2*b^2 + 2*a*b^3 + b^4)*f*x + ((15
*a^4 + 20*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2 + 20*a*b^3 + 8*b^
4 + 2*(15*a^3*b + 20*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*sqrt(-b/(a + b))*l
og(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^
2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sq
rt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e
)^2 + b^2)) - 4*(3*(3*a^3*b + 2*a^2*b^2)*cos(f*x + e)^3 + (7*a^2*b^2 + 4*a
*b^3)*cos(f*x + e))*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*x + e
)^4 + 2*(a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*a^4*
b^3 + a^3*b^4)*f), 1/16*(16*(a^4 + 2*a^3*b + a^2*b^2)*f*x*cos(f*x + e)^4 +
 32*(a^3*b + 2*a^2*b^2 + a*b^3)*f*x*cos(f*x + e)^2 + 16*(a^2*b^2 + 2*a*b^3
 + b^4)*f*x + ((15*a^4 + 20*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2
 + 20*a*b^3 + 8*b^4 + 2*(15*a^3*b + 20*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*
sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/
(b*cos(f*x + e)*sin(f*x + e))) - 2*(3*(3*a^3*b + 2*a^2*b^2)*cos(f*x + e)^3
 + (7*a^2*b^2 + 4*a*b^3)*cos(f*x + e))*sin(f*x + e))/((a^7 + 2*a^6*b + a^5
*b^2)*f*cos(f*x + e)^4 + 2*(a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2
+ (a^5*b^2 + 2*a^4*b^3 + a^3*b^4)*f)]
```

3.63.6 Sympy [F]

$$\int \frac{1}{(a+b\sec^2(e+fx))^3}\,dx = \int \frac{1}{(a+b\sec^2(e+fx))^3}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(-3), x)
```

3.63.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.60

$$\int \frac{1}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{(15\,a^2b+20\,ab^2+8\,b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^5+2\,a^4b+a^3b^2)\sqrt{(a+b)b}} + \frac{(7\,ab^2+4\,b^3)\tan(fx+e)^3+(9\,a^2b+13\,ab^2+4\,b^3)\tan(fx+e)}{a^6+4\,a^5b+6\,a^4b^2+4\,a^3b^3+a^2b^4+(a^4b^2+2\,a^3b^3+a^2b^4)\tan(fx+e)^4+2\,(a^5b+3\,a^4b^2+3\,a^3b^3+a}}{8\,f}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
-1/8*((15*a^2*b + 20*a*b^2 + 8*b^3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))
/((a^5 + 2*a^4*b + a^3*b^2)*sqrt((a + b)*b)) + ((7*a*b^2 + 4*b^3)*tan(f*x
+ e)^3 + (9*a^2*b + 13*a*b^2 + 4*b^3)*tan(f*x + e))/(a^6 + 4*a^5*b + 6*a^4
*b^2 + 4*a^3*b^3 + a^2*b^4 + (a^4*b^2 + 2*a^3*b^3 + a^2*b^4)*tan(f*x + e)^
4 + 2*(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*tan(f*x + e)^2) - 8*(f*x +
 e)/a^3)/f
```

3.63.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.36

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{\frac{\left(15a^2b+20ab^2+8b^3\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{\left(a^5+2a^4b+a^3b^2\right)\sqrt{ab+b^2}} + \frac{7ab^2\tan(fx+e)^3+4b^3\tan(fx+e)^3+9a^2b\tan(fx+e)+13ab^2\tan(fx}{\left(a^4+2a^3b+a^2b^2\right)\left(b\tan(fx+e)^2+a+b\right)^2}}{8f}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
-1/8*((15*a^2*b + 20*a*b^2 + 8*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) +
 arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^5 + 2*a^4*b + a^3*b^2)*sqrt(a
*b + b^2)) + (7*a*b^2*tan(f*x + e)^3 + 4*b^3*tan(f*x + e)^3 + 9*a^2*b*tan(
f*x + e) + 13*a*b^2*tan(f*x + e) + 4*b^3*tan(f*x + e))/((a^4 + 2*a^3*b + a
^2*b^2)*(b*tan(f*x + e)^2 + a + b)^2) - 8*(f*x + e)/a^3)/f
```

3.63.9 Mupad [B] (verification not implemented)

Time = 22.74 (sec) , antiderivative size = 3271, normalized size of antiderivative = 22.72

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(a + b/cos(e + f*x)^2)^3,x)
```

output

```
atan(((((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^1
0*b^2)*1i)/(2*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)) - (tan(e
 + f*x)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a
^10*b^3 + 256*a^11*b^2))/(128*a^3*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6
*a^6*b^2)))/(2*a^3) + (tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 +
856*a^3*b^4 + 289*a^4*b^3))/(64*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a
^6*b^2)))/a^3 - ((((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)
/2 + 4*a^10*b^2)*1i)/(2*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)
) + (tan(e + f*x)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^
4 + 1536*a^10*b^3 + 256*a^11*b^2))/(128*a^3*(4*a^7*b + a^8 + a^4*b^4 + 4*a
^5*b^3 + 6*a^6*b^2)))/(2*a^3) - (tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*
a^2*b^5 + 856*a^3*b^4 + 289*a^4*b^3))/(64*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5
*b^3 + 6*a^6*b^2)))/a^3)/(((17*a*b^5)/4 + b^6 + (25*a^2*b^4)/4 + (105*a^3*
b^3)/32)/(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2) + (((((2*a^6*b
^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10*b^2)*1i)/(2*(4*
a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)) - (tan(e + f*x)*(512*a^6*
b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a^10*b^3 + 256*a^1
1*b^2))/(128*a^3*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*1i)/(
2*a^3) + (tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a^3*b^4 +
 289*a^4*b^3)*1i)/(64*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)...
```

3.64 $\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

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3.64.1 Optimal result

Integrand size = 23, antiderivative size = 124

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{15\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8(a+b)^{7/2}f} - \frac{15\cot(e+fx)}{8(a+b)^3 f} + \frac{\cot(e+fx)}{4(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{5\cot(e+fx)}{8(a+b)^2 f\left(a+b+b\tan^2(e+fx)\right)}$$

output
```
-15/8*cot(f*x+e)/(a+b)^3/f-15/8*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(
1/2)/(a+b)^(7/2)/f+1/4*cot(f*x+e)/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2+5/8*cot(f
*x+e)/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)
```

3.64.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.88 (sec) , antiderivative size = 749, normalized size of antiderivative = 6.04

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^6(e+fx)\left(\frac{120b\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(a+2b+a\cos(2$$

input
```
Integrate[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^6*((120*b*ArcTan[(Sec[f*x]*(C
os[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[
a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])^2*(C
os[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (Csc[
e]*Csc[e + f*x]*Sec[2*e]*((-32*a^4 - 64*a^3*b + 22*a^2*b^2 + 80*a*b^3 + 16
*b^4)*Sin[f*x] + 2*a*(16*a^3 + 23*a^2*b - 27*a*b^2 - 4*b^3)*Sin[3*f*x] - 4
8*a^4*Sin[2*e - f*x] - 128*a^3*b*Sin[2*e - f*x] - 106*a^2*b^2*Sin[2*e - f*
x] + 80*a*b^3*Sin[2*e - f*x] + 16*b^4*Sin[2*e - f*x] + 48*a^4*Sin[2*e + f*
x] + 146*a^3*b*Sin[2*e + f*x] + 182*a^2*b^2*Sin[2*e + f*x] + 80*a*b^3*Sin[
2*e + f*x] + 16*b^4*Sin[2*e + f*x] - 32*a^4*Sin[4*e + f*x] - 82*a^3*b*Sin[
4*e + f*x] - 54*a^2*b^2*Sin[4*e + f*x] - 80*a*b^3*Sin[4*e + f*x] - 16*b^4*
Sin[4*e + f*x] - 8*a^4*Sin[2*e + 3*f*x] + 18*a^3*b*Sin[2*e + 3*f*x] + 54*a
^2*b^2*Sin[2*e + 3*f*x] + 8*a*b^3*Sin[2*e + 3*f*x] + 32*a^4*Sin[4*e + 3*f*
x] + 73*a^3*b*Sin[4*e + 3*f*x] + 24*a^2*b^2*Sin[4*e + 3*f*x] + 8*a*b^3*Sin
[4*e + 3*f*x] - 8*a^4*Sin[6*e + 3*f*x] - 9*a^3*b*Sin[6*e + 3*f*x] - 24*a^2
*b^2*Sin[6*e + 3*f*x] - 8*a*b^3*Sin[6*e + 3*f*x] + 8*a^4*Sin[2*e + 5*f*x]
- 9*a^3*b*Sin[2*e + 5*f*x] - 2*a^2*b^2*Sin[2*e + 5*f*x] + 9*a^3*b*Sin[4*e
+ 5*f*x] + 2*a^2*b^2*Sin[4*e + 5*f*x] + 8*a^4*Sin[6*e + 5*f*x]))/a^2))/(51
2*(a + b)^3*f*(a + b*Sec[e + f*x]^2)^3)
```

3.64.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4620, 253, 253, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sin(e+fx)^2\,(a+b\sec(e+fx)^2)^3}\,dx$$

$$\downarrow 4620$$

$$\frac{\int \frac{\cot^2(e+fx)}{(b\tan^2(e+fx)+a+b)^3}\,d\tan(e+fx)}{f}$$

$$\downarrow 253$$

$$\frac{5\int \frac{\cot^2(e+fx)}{\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{4(a+b)} + \frac{\cot(e+fx)}{4(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\downarrow 253$$

$$\frac{5\left(\frac{3\int \frac{\cot^2(e+fx)}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{2(a+b)} + \frac{\cot(e+fx)}{2(a+b)\left(a+b\tan^2(e+fx)+b\right)}\right)}{4(a+b)} + \frac{\cot(e+fx)}{4(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\downarrow 264$$

$$\frac{5\left(\frac{3\left(-\frac{b\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a+b} - \frac{\cot(e+fx)}{a+b}\right)}{2(a+b)} + \frac{\cot(e+fx)}{2(a+b)\left(a+b\tan^2(e+fx)+b\right)}\right)}{4(a+b)} + \frac{\cot(e+fx)}{4(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\downarrow 218$$

$$\frac{5\left(\frac{3\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} - \frac{\cot(e+fx)}{a+b}\right)}{2(a+b)} + \frac{\cot(e+fx)}{2(a+b)\left(a+b\tan^2(e+fx)+b\right)}\right)}{4(a+b)} + \frac{\cot(e+fx)}{4(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

input
```
Int[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
(Cot[e + f*x]/(4*(a + b)*(a + b + b*Tan[e + f*x]^2)^2) + (5*((3*(-((Sqrt[b
]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(3/2)) - Cot[e + f*x
]/(a + b)))/(2*(a + b)) + Cot[e + f*x]/(2*(a + b)*(a + b + b*Tan[e + f*x]^
2))))/(4*(a + b)))/f
```

3.64.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.64.4 Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{7b\tan(fx+e)^3}{8}+\left(\frac{9a}{8}+\frac{9b}{8}\right)\tan(fx+e)}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{15\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\sqrt{(a+b)b}}\right)}{(a+b)^3}-\frac{1}{(a+b)^3\tan(fx+e)}}{f}$
default	$\frac{-\frac{b\left(\frac{7b\tan(fx+e)^3}{8}+\left(\frac{9a}{8}+\frac{9b}{8}\right)\tan(fx+e)}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{15\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\sqrt{(a+b)b}}\right)}{(a+b)^3}-\frac{1}{(a+b)^3\tan(fx+e)}}{f}$
risch	$-\frac{i\left(8a^4e^{8i(fx+e)}+9a^3be^{8i(fx+e)}+24a^2b^2e^{8i(fx+e)}+8ab^3e^{8i(fx+e)}+32a^4e^{6i(fx+e)}+82a^3be^{6i(fx+e)}+54a^2b^2e^{6i(fx+e}\right.$

input
```
int(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-b/(a+b)^3*((7/8*b*tan(f*x+e)^3+(9/8*a+9/8*b)*tan(f*x+e))/(a+b+b*tan(
f*x+e)^2)^2+15/8/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))-1/(
a+b)^3/tan(f*x+e))
```

3.64.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(108) = 216$.

Time $= 0.31$ (sec) , antiderivative size $= 615$, normalized size of antiderivative $= 4.96$

$$\int \frac{\csc^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$

$$= \left[-\frac{4\left(8a^2-9ab-2b^2\right)\cos(fx+e)^5+20\left(5ab-b^2\right)\cos(fx+e)^3-15\left(a^2\cos(fx+e)^4+2ab\cos(fx+\right.}{32\left(\left(a^5+3a^4b+3a^3b^2+a^2b^3\right)f\cos(fx+\right.}\right.$$

$$\left. -\frac{2\left(8a^2-9ab-2b^2\right)\cos(fx+e)^5+10\left(5ab-b^2\right)\cos(fx+e)^3-15\left(a^2\cos(fx+e)^4+2ab\cos(fx+\right.}{16\left(\left(a^5+3a^4b+3a^3b^2+a^2b^3\right)f\cos(fx+e)^4+2\left(a^4b+3a^3b^2+3a^2b^3+ab^4\right)\right.}\right]$$

input
```
integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[-1/32*(4*(8*a^2 - 9*a*b - 2*b^2)*cos(f*x + e)^5 + 20*(5*a*b - b^2)*cos(f*
x + e)^3 - 15*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*sqrt(-b/(a
 + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f
*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x
+ e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos
(f*x + e)^2 + b^2))*sin(f*x + e) + 60*b^2*cos(f*x + e))/(((a^5 + 3*a^4*b +
 3*a^3*b^2 + a^2*b^3)*f*cos(f*x + e)^4 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3
+ a*b^4)*f*cos(f*x + e)^2 + (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*f)*sin(f
*x + e)), -1/16*(2*(8*a^2 - 9*a*b - 2*b^2)*cos(f*x + e)^5 + 10*(5*a*b - b^
2)*cos(f*x + e)^3 - 15*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*s
qrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(
b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) + 30*b^2*cos(f*x + e))/(((a^5 +
 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*f*cos(f*x + e)^4 + 2*(a^4*b + 3*a^3*b^2 +
3*a^2*b^3 + a*b^4)*f*cos(f*x + e)^2 + (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5
)*f)*sin(f*x + e))]
```

3.64.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**2/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Timed out
```

3.64.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(108) = 216$.

Time = 0.27 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.77

$$\int \frac{\csc^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx =$$

$$-\frac{15\,b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\sqrt{(a+b)b}} + \frac{15\,b^2\tan(fx+e)^4+25\,(ab+b^2)\tan(fx+e)^2+8\,a^2+16\,ab+8\,b^2}{\left(a^3b^2+3\,a^2b^3+3\,ab^4+b^5\right)\tan(fx+e)^5+2\,\left(a^4b+4\,a^3b^2+6\,a^2b^3+4\,ab^4+b^5\right)\tan(fx+e)^3+(a^5+5\,a^4b+10\,a^3b}$$

$$8\,f$$

input
```
integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
-1/8*(15*b*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^3 + 3*a^2*b + 3*a*b^
2 + b^3)*sqrt((a + b)*b)) + (15*b^2*tan(f*x + e)^4 + 25*(a*b + b^2)*tan(f*
x + e)^2 + 8*a^2 + 16*a*b + 8*b^2)/((a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*
tan(f*x + e)^5 + 2*(a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*tan(f*x
 + e)^3 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*tan(f*
x + e)))/f
```

3.64.8 Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.43

$$\int \frac{\csc^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{\frac{15\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b}{(a^3+3a^2b+3ab^2+b^3)\sqrt{ab+b^2}}+\frac{7b^2\tan(fx+e)^3+9ab\tan(fx+e)+9b^2\tan(fx+e)}{(a^3+3a^2b+3ab^2+b^3)\left(b\tan(fx+e)^2+a+b\right)^2}+\frac{8}{(a^3+3a^2b+3ab^2+b^3)\tan(fx+e)}}{8f}$$

input
```
integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
-1/8*(15*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt
(a*b + b^2)))*b/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*b + b^2)) + (7*b^2
*tan(f*x + e)^3 + 9*a*b*tan(f*x + e) + 9*b^2*tan(f*x + e))/((a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*(b*tan(f*x + e)^2 + a + b)^2) + 8/((a^3 + 3*a^2*b + 3*a*
b^2 + b^3)*tan(f*x + e)))/f
```

3.64.9 Mupad [B] (verification not implemented)

Time = 19.49 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.18

$$\int \frac{\csc^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{\frac{1}{a+b}+\frac{25\,b\tan(e+fx)^2}{8\,(a+b)^2}+\frac{15\,b^2\tan(e+fx)^4}{8\,(a+b)^3}}{f\,\left(\tan(e+fx)^3\,(2b^2+2ab)+\tan(e+fx)\,(a^2+2ab+b^2)+b^2\tan(e+fx)^5\right)} - \frac{15\sqrt{b}\,\operatorname{atan}\left(\frac{\sqrt{b}\tan(e+fx)\,(a^3+3a^2b+3ab^2+b^3)}{(a+b)^{7/2}}\right)}{8f\,(a+b)^{7/2}}$$

input
```
int(1/(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^3),x)
```

output
```
- (1/(a + b) + (25*b*tan(e + f*x)^2)/(8*(a + b)^2) + (15*b^2*tan(e + f*x)^
4)/(8*(a + b)^3))/(f*(tan(e + f*x)^3*(2*a*b + 2*b^2) + tan(e + f*x)*(2*a*b
 + a^2 + b^2) + b^2*tan(e + f*x)^5)) - (15*b^(1/2)*atan((b^(1/2)*tan(e + f
*x)*(3*a*b^2 + 3*a^2*b + a^3 + b^3))/(a + b)^(7/2)))/(8*f*(a + b)^(7/2))
```

3.65 $\int \frac{\csc^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

3.65.1 Optimal result . . . 624
3.65.2 Mathematica [C] (warning: unable to verify) . . . 624
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3.65.1 Optimal result

Integrand size = 23, antiderivative size = 164

$$\int \frac{\csc^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{5(3a-4b)\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8(a+b)^{9/2}f} - \frac{(a-2b)\cot(e+fx)}{(a+b)^4 f} - \frac{\cot^3(e+fx)}{3(a+b)^3 f} - \frac{ab\tan(e+fx)}{4(a+b)^3 f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{(7a-4b)b\tan(e+fx)}{8(a+b)^4 f\left(a+b+b\tan^2(e+fx)\right)}$$

output

```
-(a-2*b)*cot(f*x+e)/(a+b)^4/f-1/3*cot(f*x+e)^3/(a+b)^3/f-5/8*(3*a-4*b)*arc
tan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(1/2)/(a+b)^(9/2)/f-1/4*a*b*tan(f*x+
e)/(a+b)^3/f/(a+b+b*tan(f*x+e)^2)^2-1/8*(7*a-4*b)*b*tan(f*x+e)/(a+b)^4/f/(
a+b+b*tan(f*x+e)^2)
```

3.65.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.03 (sec) , antiderivative size = 994, normalized size of antiderivative = 6.06

$$\int \frac{\csc^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^6(e+fx)\left(\frac{480(3a-4b)b\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(a+2b-}{\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right.}{}$$

input
```
Integrate[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^6*((480*(3*a - 4*b)*b*ArcTan[
(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x
]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e +
 f*x)])^2*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])
^4]) - (Csc[e]*Csc[e + f*x]^3*Sec[2*e]*(4*(44*a^4 + 122*a^3*b + 63*a^2*b^2
 + 126*a*b^3 + 36*b^4)*Sin[f*x] + (-96*a^4 - 71*a^3*b + 344*a^2*b^2 - 1208
*a*b^3 + 48*b^4)*Sin[3*f*x] + 224*a^4*Sin[2*e - f*x] + 576*a^3*b*Sin[2*e -
 f*x] + 124*a^2*b^2*Sin[2*e - f*x] - 2184*a*b^3*Sin[2*e - f*x] + 144*b^4*S
in[2*e - f*x] - 224*a^4*Sin[2*e + f*x] - 657*a^3*b*Sin[2*e + f*x] - 538*a^
2*b^2*Sin[2*e + f*x] + 984*a*b^3*Sin[2*e + f*x] + 144*b^4*Sin[2*e + f*x] +
 176*a^4*Sin[4*e + f*x] + 569*a^3*b*Sin[4*e + f*x] + 666*a^2*b^2*Sin[4*e +
 f*x] + 1704*a*b^3*Sin[4*e + f*x] - 144*b^4*Sin[4*e + f*x] + 48*a^4*Sin[2*
e + 3*f*x] + 111*a^3*b*Sin[2*e + 3*f*x] + 360*a^2*b^2*Sin[2*e + 3*f*x] + 3
12*a*b^3*Sin[2*e + 3*f*x] - 48*b^4*Sin[2*e + 3*f*x] - 96*a^4*Sin[4*e + 3*f
*x] - 152*a^3*b*Sin[4*e + 3*f*x] + 146*a^2*b^2*Sin[4*e + 3*f*x] - 728*a*b^
3*Sin[4*e + 3*f*x] - 48*b^4*Sin[4*e + 3*f*x] + 48*a^4*Sin[6*e + 3*f*x] + 1
92*a^3*b*Sin[6*e + 3*f*x] + 558*a^2*b^2*Sin[6*e + 3*f*x] - 168*a*b^3*Sin[6
*e + 3*f*x] + 48*b^4*Sin[6*e + 3*f*x] + 16*a^4*Sin[2*e + 5*f*x] - 598*a^2*
b^2*Sin[2*e + 5*f*x] + 48*a*b^3*Sin[2*e + 5*f*x] + 72*a^3*b*Sin[4*e + 5*f*
x] + 150*a^2*b^2*Sin[4*e + 5*f*x] - 48*a*b^3*Sin[4*e + 5*f*x] + 16*a^4*...
```

3.65.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4620, 361, 25, 1582, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sin(e+fx)^4\,(a+b\sec(e+fx)^2)^3}dx$$

$$\downarrow 4620$$

$$\frac{\int \frac{\cot^4(e+fx)(\tan^2(e+fx)+1)}{(b\tan^2(e+fx)+a+b)^3}d\tan(e+fx)}{f}$$

$$\downarrow 361$$

$$\frac{-\frac{1}{4}b\int -\frac{\cot^4(e+fx)\left(-\frac{3a\tan^4(e+fx)}{(a+b)^3}+\frac{4a\tan^2(e+fx)}{b(a+b)^2}+\frac{4}{b(a+b)}\right)}{(b\tan^2(e+fx)+a+b)^2}d\tan(e+fx)-\frac{ab\tan(e+fx)}{4(a+b)^3(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\downarrow 25$$

$$\frac{\frac{1}{4}b\int \frac{\cot^4(e+fx)\left(-\frac{3a\tan^4(e+fx)}{(a+b)^3}+\frac{4a\tan^2(e+fx)}{b(a+b)^2}+\frac{4}{b(a+b)}\right)}{(b\tan^2(e+fx)+a+b)^2}d\tan(e+fx)-\frac{ab\tan(e+fx)}{4(a+b)^3(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\downarrow 1582$$

$$\frac{\frac{1}{4}b\left(\frac{\int \frac{\cot^4(e+fx)\left(-\frac{(7a-4b)b^2\tan^4(e+fx)}{a+b}+8(a-b)b\tan^2(e+fx)+8b(a+b)\right)}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{2b^2(a+b)^3}-\frac{(7a-4b)\tan(e+fx)}{2(a+b)^4(a+b\tan^2(e+fx)+b)}\right)-\frac{ab\tan(e+f}{4(a+b)^3(a+b\tan^2(e}}{f}$$

$$\downarrow 1584$$

$$\frac{\frac{1}{4}b\left(\frac{\int\left(8b\cot^4(e+fx)+\frac{8(a-2b)b\cot^2(e+fx)}{a+b}+\frac{5b^2(4b-3a)}{(a+b)\left(b\tan^2(e+fx)+a+b\right)}\right)d\tan(e+fx)}{2b^2(a+b)^3}-\frac{(7a-4b)\tan(e+fx)}{2(a+b)^4(a+b\tan^2(e+fx)+b)}\right)-\frac{ab\tan(e+}{4(a+b)^3(a+b\tan^2($$
$$f$$

$$\downarrow 2009$$

$$\frac{\frac{1}{4}b\left(\frac{-\frac{5b^{3/2}(3a-4b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{8b(a-2b)\cot(e+fx)}{a+b}-\frac{8}{3}b\cot^3(e+fx)}{2b^2(a+b)^3}-\frac{(7a-4b)\tan(e+fx)}{2(a+b)^4(a+b\tan^2(e+fx)+b)}\right)-\frac{ab\tan(e+fx)}{4(a+b)^3(a+b\tan^2(e+fx)}$$
$$f$$

input
```
Int[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
(-1/4*(a*b*Tan[e + f*x])/((a + b)^3*(a + b + b*Tan[e + f*x]^2)^2) + (b*(((
-5*(3*a - 4*b)*b^(3/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)
^(3/2) - (8*(a - 2*b)*b*Cot[e + f*x])/(a + b) - (8*b*Cot[e + f*x]^3)/3)/(2
*b^2*(a + b)^3) - ((7*a - 4*b)*Tan[e + f*x])/(2*(a + b)^4*(a + b + b*Tan[e
 + f*x]^2))))/4)/f
```

3.65.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1582
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1))   Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e
*x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.65.4 Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{-b\left(\frac{\left(\frac{7}{8}ab-\frac{1}{2}b^2\right)\tan(fx+e)^3+\left(\frac{9}{8}a^2+\frac{5}{8}ab-\frac{1}{2}b^2\right)\tan(fx+e)}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{5(3a-4b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\sqrt{(a+b)b}}\right)}{(a+b)^4}-\frac{1}{3(a+b)^3\tan(fx+e)^3}-\frac{a-}{(a+b)^4 t}}{f}$
default	$\frac{-b\left(\frac{\left(\frac{7}{8}ab-\frac{1}{2}b^2\right)\tan(fx+e)^3+\left(\frac{9}{8}a^2+\frac{5}{8}ab-\frac{1}{2}b^2\right)\tan(fx+e)}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{5(3a-4b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\sqrt{(a+b)b}}\right)}{(a+b)^4}-\frac{1}{3(a+b)^3\tan(fx+e)^3}-\frac{a-}{(a+b)^4 t}}{f}$
risch	$-\frac{i\left(-569a^3b\,e^{8i(fx+e)}-666a^2b^2e^{8i(fx+e)}-1704a\,b^3e^{8i(fx+e)}-176a^4e^{8i(fx+e)}-144b^4e^{6i(fx+e)}-576a^3b\,e^{6i(fx+e)}-12\right.}{}$

input
```
int(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/(a+b)^4*b*(((7/8*a*b-1/2*b^2)*tan(f*x+e)^3+(9/8*a^2+5/8*a*b-1/2*b^
2)*tan(f*x+e))/(a+b+b*tan(f*x+e)^2)^2+5/8*(3*a-4*b)/((a+b)*b)^(1/2)*arctan
(b*tan(f*x+e)/((a+b)*b)^(1/2)))-1/3/(a+b)^3/tan(f*x+e)^3-(a-2*b)/(a+b)^4/t
an(f*x+e))
```

3.65.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(148) = 296$.

Time $= 0.33$ (sec) , antiderivative size $= 1009$, normalized size of antiderivative $= 6.15$

$$\int \frac{\csc^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[-1/96*(4*(16*a^3 - 83*a^2*b + 6*a*b^2)*cos(f*x + e)^7 - 4*(24*a^3 - 134*a
^2*b + 145*a*b^2 - 12*b^3)*cos(f*x + e)^5 - 20*(15*a^2*b - 32*a*b^2 + 16*b
^3)*cos(f*x + e)^3 + 15*((3*a^3 - 4*a^2*b)*cos(f*x + e)^6 - (3*a^3 - 10*a^
2*b + 8*a*b^2)*cos(f*x + e)^4 - 3*a*b^2 + 4*b^3 - (6*a^2*b - 11*a*b^2 + 4*
b^3)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x +
 e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*b + 2*b^2)*cos(f*
x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/
(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) - 60*(3*a*
b^2 - 4*b^3)*cos(f*x + e))/(((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*
b^4)*f*cos(f*x + e)^6 - (a^6 + 2*a^5*b - 2*a^4*b^2 - 8*a^3*b^3 - 7*a^2*b^4
 - 2*a*b^5)*f*cos(f*x + e)^4 - (2*a^5*b + 7*a^4*b^2 + 8*a^3*b^3 + 2*a^2*b^
4 - 2*a*b^5 - b^6)*f*cos(f*x + e)^2 - (a^4*b^2 + 4*a^3*b^3 + 6*a^2*b^4 + 4
*a*b^5 + b^6)*f)*sin(f*x + e)), -1/48*(2*(16*a^3 - 83*a^2*b + 6*a*b^2)*cos
(f*x + e)^7 - 2*(24*a^3 - 134*a^2*b + 145*a*b^2 - 12*b^3)*cos(f*x + e)^5 -
 10*(15*a^2*b - 32*a*b^2 + 16*b^3)*cos(f*x + e)^3 - 15*((3*a^3 - 4*a^2*b)*
cos(f*x + e)^6 - (3*a^3 - 10*a^2*b + 8*a*b^2)*cos(f*x + e)^4 - 3*a*b^2 + 4
*b^3 - (6*a^2*b - 11*a*b^2 + 4*b^3)*cos(f*x + e)^2)*sqrt(b/(a + b))*arctan
(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*
x + e)))*sin(f*x + e) - 30*(3*a*b^2 - 4*b^3)*cos(f*x + e))/(((a^6 + 4*a^5*
b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*f*cos(f*x + e)^6 - (a^6 + 2*a^5*b ...
```

3.65.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**4/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Timed out
```

3.65.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(148) = 296$.

Time = 0.27 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.97

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{15\,(3\,ab-4\,b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)\sqrt{(a+b)b}} + \frac{15\,(3\,ab^2-4\,b^3)\tan(fx+e)^6+25\,(3\,a^2b-ab^2-4\,b^3)\tan(fx+e)^4+8\,a^3+24\,a^2b+2}{(a^4b^2+4\,a^3b^3+6\,a^2b^4+4\,ab^5+b^6)\tan(fx+e)^7+2\,(a^5b+5\,a^4b^2+10\,a^3b^3+10\,a^2b^4+5\,ab^5+b^6)\tan(f}}{24\,f}$$

input
```
integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
-1/24*(15*(3*a*b - 4*b^2)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^4 + 4
*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*sqrt((a + b)*b)) + (15*(3*a*b^2 - 4*b^
3)*tan(f*x + e)^6 + 25*(3*a^2*b - a*b^2 - 4*b^3)*tan(f*x + e)^4 + 8*a^3 +
24*a^2*b + 24*a*b^2 + 8*b^3 + 8*(3*a^3 + 2*a^2*b - 5*a*b^2 - 4*b^3)*tan(f*
x + e)^2)/((a^4*b^2 + 4*a^3*b^3 + 6*a^2*b^4 + 4*a*b^5 + b^6)*tan(f*x + e)^
7 + 2*(a^5*b + 5*a^4*b^2 + 10*a^3*b^3 + 10*a^2*b^4 + 5*a*b^5 + b^6)*tan(f*
x + e)^5 + (a^6 + 6*a^5*b + 15*a^4*b^2 + 20*a^3*b^3 + 15*a^2*b^4 + 6*a*b^5
 + b^6)*tan(f*x + e)^3))/f
```

3.65.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.61

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{15\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(3\,ab-4\,b^2)}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)\sqrt{ab+b^2}} + \frac{3\left(7\,ab^2\tan(fx+e)^3-4\,b^3\tan(fx+e)^3+9\,a^2b\tan(fx+e)+5\,ab^2\tan(fx+e)-\right.}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)\left(b\tan(fx+e)^2+a+b\right)^2}}{24\,f}$$

input
```
integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
-1/24*(15*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqr
t(a*b + b^2)))*(3*a*b - 4*b^2)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4
)*sqrt(a*b + b^2)) + 3*(7*a*b^2*tan(f*x + e)^3 - 4*b^3*tan(f*x + e)^3 + 9*
a^2*b*tan(f*x + e) + 5*a*b^2*tan(f*x + e) - 4*b^3*tan(f*x + e))/((a^4 + 4*
a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(b*tan(f*x + e)^2 + a + b)^2) + 8*(3*a*
tan(f*x + e)^2 - 6*b*tan(f*x + e)^2 + a + b)/((a^4 + 4*a^3*b + 6*a^2*b^2 +
 4*a*b^3 + b^4)*tan(f*x + e)^3))/f
```

3.65.9 Mupad [B] (verification not implemented)

Time = 20.64 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.26

$$\int \frac{\csc^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$
$$= -\frac{\frac{1}{3\,(a+b)} + \frac{25\tan(e+f\,x)^4\,(3\,a\,b-4\,b^2)}{24\,(a+b)^3} + \frac{\tan(e+f\,x)^2\,(3\,a-4\,b)}{3\,(a+b)^2} + \frac{5\tan(e+f\,x)^6\,(3\,a\,b^2-4\,b^3)}{8\,(a+b)^4}}{f\,\left(\tan\left(e+f\,x\right)^3\,\left(a^2+2\,a\,b+b^2\right)+\tan\left(e+f\,x\right)^5\,\left(2\,b^2+2\,a\,b\right)+b^2\tan\left(e+f\,x\right)^7\right)}$$
$$- \frac{5\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\tan(e+f\,x)\,\left(a^4+4\,a^3\,b+6\,a^2\,b^2+4\,a\,b^3+b^4\right)}{(a+b)^{9/2}}\right)\,\left(3\,a-4\,b\right)}{8\,f\,\left(a+b\right)^{9/2}}$$

input
```
int(1/(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^3),x)
```

output
```
- (1/(3*(a + b)) + (25*tan(e + f*x)^4*(3*a*b - 4*b^2))/(24*(a + b)^3) + (t
an(e + f*x)^2*(3*a - 4*b))/(3*(a + b)^2) + (5*tan(e + f*x)^6*(3*a*b^2 - 4*
b^3))/(8*(a + b)^4))/(f*(tan(e + f*x)^3*(2*a*b + a^2 + b^2) + tan(e + f*x)
^5*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^7)) - (5*b^(1/2)*atan((b^(1/2)*tan(e
 + f*x)*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2))/(a + b)^(9/2))*(3*a -
 4*b))/(8*f*(a + b)^(9/2))
```

3.66 $\int \frac{\csc^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

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3.66.1 Optimal result

Integrand size = 23, antiderivative size = 242

$$\begin{aligned}\int \frac{\csc^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = &-\frac{\sqrt{b}(15a^2-40ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8(a+b)^{11/2}f}\\ &-\frac{(5a^2-20ab+2b^2)\cot(e+fx)}{5(a+b)^5 f}-\frac{(10a+b)\cot^3(e+fx)}{15(a+b)^4 f}\\ &-\frac{\cot^5(e+fx)}{5(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2}\\ &-\frac{b(5a^2+4b^2)\tan(e+fx)}{20(a+b)^4 f\left(a+b+b\tan^2(e+fx)\right)^2}\\ &-\frac{b(35a^2-40ab+24b^2)\tan(e+fx)}{40(a+b)^5 f\left(a+b+b\tan^2(e+fx)\right)}\end{aligned}$$

output

```
-1/5*(5*a^2-20*a*b+2*b^2)*cot(f*x+e)/(a+b)^5/f-1/15*(10*a+b)*cot(f*x+e)^3/
(a+b)^4/f-1/8*(15*a^2-40*a*b+8*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))
*b^(1/2)/(a+b)^(11/2)/f-1/5*cot(f*x+e)^5/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2-1/
20*b*(5*a^2+4*b^2)*tan(f*x+e)/(a+b)^4/f/(a+b+b*tan(f*x+e)^2)^2-1/40*b*(35*
a^2-40*a*b+24*b^2)*tan(f*x+e)/(a+b)^5/f/(a+b+b*tan(f*x+e)^2)
```

3.66.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.57 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.98

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^6(e+fx)\left(-8(4a-11b)(a+b)(a+2b+a\cos(2(e+fx)))^2\cot(e)\csc^2(e\right.}{}$$

input
```
Integrate[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^6*(-8*(4*a - 11*b)*(a + b)*(a
 + 2*b + a*Cos[2*(e + f*x)])^2*Cot[e]*Csc[e + f*x]^2 - 24*(a + b)^2*(a + 2
*b + a*Cos[2*(e + f*x)])^2*Cot[e]*Csc[e + f*x]^4 + (15*b*(15*a^2 - 40*a*b
+ 8*b^2)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) +
 a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*
b + a*Cos[2*(e + f*x)])^2*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Co
s[e] - I*Sin[e])^4]) + 8*(8*a^2 - 59*a*b + 23*b^2)*(a + 2*b + a*Cos[2*(e +
 f*x)])^2*Csc[e]*Csc[e + f*x]*Sin[f*x] + 8*(4*a - 11*b)*(a + b)*(a + 2*b +
 a*Cos[2*(e + f*x)])^2*Csc[e]*Csc[e + f*x]^3*Sin[f*x] + 24*(a + b)^2*(a +
2*b + a*Cos[2*(e + f*x)])^2*Csc[e]*Csc[e + f*x]^5*Sin[f*x] - 60*b^2*(a + b
)*Sec[2*e]*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]) + 15*b*(a + 2*b + a*Cos[2*(
e + f*x)])*Sec[2*e]*((9*a^2 + 16*a*b - 8*b^2)*Sin[2*e] + 3*a*(-3*a + 2*b)*
Sin[2*f*x])))/(960*(a + b)^5*f*(a + b*Sec[e + f*x]^2)^3)
```

3.66.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4620, 365, 361, 25, 1582, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sin(e+fx)^6\,(a+b\sec(e+fx)^2)^3}\,dx$$

$$\downarrow 4620$$

$$\frac{\int \frac{\cot^6(e+fx)\left(\tan^2(e+fx)+1\right)^2}{\left(b\tan^2(e+fx)+a+b\right)^3}\,d\tan(e+fx)}{f}$$

$$\downarrow 365$$

$$\frac{\int \frac{\cot^4(e+fx)\left(5(a+b)\tan^2(e+fx)+10a+b\right)}{\left(b\tan^2(e+fx)+a+b\right)^3}\,d\tan(e+fx)}{5(a+b)} - \frac{\cot^5(e+fx)}{5(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\downarrow 361$$

$$\frac{-\frac{1}{4}b\int -\frac{\cot^4(e+fx)\left(-\frac{3\left(5a^2+4b^2\right)\tan^4(e+fx)}{(a+b)^3}+\frac{4\left(5a^2+4b^2\right)\tan^2(e+fx)}{b(a+b)^2}+\frac{4(10a+b)}{b(a+b)}\right)}{\left(b\tan^2(e+fx)+a+b\right)^2}\,d\tan(e+fx) - \frac{b\left(5a^2+4b^2\right)\tan(e+fx)}{4(a+b)^3\left(a+b\tan^2(e+fx)+b\right)^2}}{5(a+b)} - \frac{\cot^5(e+}{5(a+b)(a+b\tan^2}}{f}$$

$$\downarrow 25$$

$$\frac{\frac{1}{4}b\int \frac{\cot^4(e+fx)\left(-\frac{3\left(5a^2+4b^2\right)\tan^4(e+fx)}{(a+b)^3}+\frac{4\left(5a^2+4b^2\right)\tan^2(e+fx)}{b(a+b)^2}+\frac{4(10a+b)}{b(a+b)}\right)}{\left(b\tan^2(e+fx)+a+b\right)^2}\,d\tan(e+fx) - \frac{b\left(5a^2+4b^2\right)\tan(e+fx)}{4(a+b)^3\left(a+b\tan^2(e+fx)+b\right)^2}}{5(a+b)} - \frac{\cot^5(e+fx)}{5(a+b)(a+b\tan^2(e+}}{f}$$

$$\downarrow 1582$$

$$\frac{\frac{1}{4}b\left(\frac{\int \frac{\cot^4(e+fx)\left(-\frac{b^2\left(35a^2-40ba+24b^2\right)\tan^4(e+fx)}{a+b}+8b\left(5a^2-10ba+3b^2\right)\tan^2(e+fx)+8b(a+b)(10a+b)\right)}{b\tan^2(e+fx)+a+b}\,d\tan(e+fx)}{2b^2(a+b)^3} - \frac{\left(35a^2-40ab+24b^2\right)\tan(e+fx)}{2(a+b)^4\left(a+b\tan^2(e+fx)+b\right)}\right) -}{5(a+b)}}{f}$$

$$\downarrow 1584$$

$$\frac{\frac{1}{4}b\left(\frac{\int\left(\frac{8b(10a+b)\cot^4(e+fx)+\frac{8b\left(5a^2-20ba+2b^2\right)\cot^2(e+fx)}{a+b}-\frac{5b^2\left(15a^2-40ba+8b^2\right)}{(a+b)\left(b\tan^2(e+fx)+a+b\right)}}{2b^2(a+b)^3}\right)d\tan(e+fx)-\frac{\left(35a^2-40ab+24b^2\right)\tan(e+fx)}{2(a+b)^4\left(a+b\tan^2(e+fx)+b\right)}}{5(a+b)}\right)-\frac{b\left(5a^2+4b^2}{4(a+b)^3\left(a+b\,t}}{f}$$

$$\downarrow 2009$$

$$\frac{\frac{1}{4}b\left(\frac{-\frac{5b^{3/2}\left(15a^2-40ab+8b^2\right)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{8b\left(5a^2-20ab+2b^2\right)\cot(e+fx)}{a+b}-\frac{8}{3}b(10a+b)\cot^3(e+fx)}{2b^2(a+b)^3}-\frac{\left(35a^2-40ab+24b^2\right)\tan(e+fx)}{2(a+b)^4\left(a+b\tan^2(e+fx)+b\right)}\right)-\frac{b\left(5a^2}{4(a+b)^3\left(}}{5(a+b)}}{f}$$

input
```
Int[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
(-1/5*Cot[e + f*x]^5/((a + b)*(a + b + b*Tan[e + f*x]^2)^2) + (-1/4*(b*(5*
a^2 + 4*b^2)*Tan[e + f*x])/((a + b)^3*(a + b + b*Tan[e + f*x]^2)^2) + (b*(
((-5*b^(3/2)*(15*a^2 - 40*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[
a + b]])/(a + b)^(3/2) - (8*b*(5*a^2 - 20*a*b + 2*b^2)*Cot[e + f*x])/(a +
b) - (8*b*(10*a + b)*Cot[e + f*x]^3)/3)/(2*b^2*(a + b)^3) - ((35*a^2 - 40*
a*b + 24*b^2)*Tan[e + f*x])/(2*(a + b)^4*(a + b + b*Tan[e + f*x]^2))))/4)/
(5*(a + b)))/f
```

3.66.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 1582
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1))   Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e
*x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.66.4 Maple [A] (verified)

Time = 2.55 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.74

method	result
derivativedivides	$\frac{b\left(\frac{\left(\frac{7}{8}a^2b-ab^2\right)\tan(fx+e)^3+\frac{a\left(9a^2+ab-8b^2\right)\tan(fx+e)}{8}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(15a^2-40ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\sqrt{(a+b)b}}\right)}{(a+b)^5}-\frac{1}{5(a+b)^3\tan(fx+e)^5}}{f}$
default	$\frac{b\left(\frac{\left(\frac{7}{8}a^2b-ab^2\right)\tan(fx+e)^3+\frac{a\left(9a^2+ab-8b^2\right)\tan(fx+e)}{8}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(15a^2-40ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\sqrt{(a+b)b}}\right)}{(a+b)^5}-\frac{1}{5(a+b)^3\tan(fx+e)^5}}{f}$
risch	Expression too large to display

input
```
int(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-b/(a+b)^5*(((7/8*a^2*b-a*b^2)*tan(f*x+e)^3+1/8*a*(9*a^2+a*b-8*b^2)*t
an(f*x+e))/(a+b+b*tan(f*x+e)^2)^2+1/8*(15*a^2-40*a*b+8*b^2)/((a+b)*b)^(1/2
)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))-1/5/(a+b)^3/tan(f*x+e)^5-1/3*(2*a-
b)/(a+b)^4/tan(f*x+e)^3-(a^2-4*a*b+b^2)/(a+b)^5/tan(f*x+e))
```

3.66.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 668 vs. 2(222) = 444.

Time = 0.36 (sec) , antiderivative size = 1423, normalized size of antiderivative = 5.88

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[-1/480*(4*(64*a^4 - 607*a^3*b + 274*a^2*b^2)*cos(f*x + e)^9 - 4*(160*a^4
- 1533*a^3*b + 1599*a^2*b^2 - 488*a*b^3)*cos(f*x + e)^7 + 4*(120*a^4 - 120
5*a^3*b + 2769*a^2*b^2 - 1392*a*b^3 + 184*b^4)*cos(f*x + e)^5 + 20*(75*a^3
*b - 305*a^2*b^2 + 320*a*b^3 - 56*b^4)*cos(f*x + e)^3 - 15*((15*a^4 - 40*a
^3*b + 8*a^2*b^2)*cos(f*x + e)^8 - 2*(15*a^4 - 55*a^3*b + 48*a^2*b^2 - 8*a
*b^3)*cos(f*x + e)^6 + (15*a^4 - 100*a^3*b + 183*a^2*b^2 - 72*a*b^3 + 8*b^
4)*cos(f*x + e)^4 + 15*a^2*b^2 - 40*a*b^3 + 8*b^4 + 2*(15*a^3*b - 55*a^2*b
^2 + 48*a*b^3 - 8*b^4)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b
+ 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a
*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*si
n(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f
*x + e) + 60*(15*a^2*b^2 - 40*a*b^3 + 8*b^4)*cos(f*x + e))/(((a^7 + 5*a^6*
b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^8 - 2*(a
^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*f*cos(f*x + e)^6
 + (a^7 + a^6*b - 9*a^5*b^2 - 25*a^4*b^3 - 25*a^3*b^4 - 9*a^2*b^5 + a*b^6
+ b^7)*f*cos(f*x + e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 - 5*a^2*b^5 - 4
*a*b^6 - b^7)*f*cos(f*x + e)^2 + (a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^
2*b^5 + 5*a*b^6 + b^7)*f)*sin(f*x + e)), -1/240*(2*(64*a^4 - 607*a^3*b + 2
74*a^2*b^2)*cos(f*x + e)^9 - 2*(160*a^4 - 1533*a^3*b + 1599*a^2*b^2 - 488*
a*b^3)*cos(f*x + e)^7 + 2*(120*a^4 - 1205*a^3*b + 2769*a^2*b^2 - 1392*a...
```

3.66.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**6/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Timed out
```

3.66.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.79

$$\int \frac{\csc^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx =$$

$$-\frac{15\left(15a^2b-40ab^2+8b^3\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\left(a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5\right)\sqrt{(a+b)b}} + \frac{15\left(15a^2b^2-40ab^3+8b^4\right)\tan(fx+e)^8+25\left(15a^3b-25a^2b^2-32ab^3+8b^4\right)\tan(fx+e)^6-}{\left(a^5b^2+5a^4b^3+10a^3b^4+10a^2b^5+5ab^6+b^7\right)\tan(fx+e)^9+2\left(a^6b+6a^5\right.}$$

1

input
```
integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
-1/120*(15*(15*a^2*b - 40*a*b^2 + 8*b^3)*arctan(b*tan(f*x + e)/sqrt((a + b
)*b))/((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*sqrt((a +
 b)*b)) + (15*(15*a^2*b^2 - 40*a*b^3 + 8*b^4)*tan(f*x + e)^8 + 25*(15*a^3*
b - 25*a^2*b^2 - 32*a*b^3 + 8*b^4)*tan(f*x + e)^6 + 8*(15*a^4 - 10*a^3*b -
 57*a^2*b^2 - 24*a*b^3 + 8*b^4)*tan(f*x + e)^4 + 24*a^4 + 96*a^3*b + 144*a
^2*b^2 + 96*a*b^3 + 24*b^4 + 8*(10*a^4 + 31*a^3*b + 33*a^2*b^2 + 13*a*b^3
 + b^4)*tan(f*x + e)^2)/((a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5
*a*b^6 + b^7)*tan(f*x + e)^9 + 2*(a^6*b + 6*a^5*b^2 + 15*a^4*b^3 + 20*a^3*
b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)*tan(f*x + e)^7 + (a^7 + 7*a^6*b + 21*a^5
*b^2 + 35*a^4*b^3 + 35*a^3*b^4 + 21*a^2*b^5 + 7*a*b^6 + b^7)*tan(f*x + e)^
5))/f
```

3.66.8 Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.52

$$\int \frac{\csc^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx =$$

$$-\frac{15\left(15a^2b-40ab^2+8b^3\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{\left(a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5\right)\sqrt{ab+b^2}} + \frac{15\left(7a^2b^2\tan(fx+e)^3-8ab^3\tan(fx+e)^3+9a^3b\tan(fx+e)+a^2b^2\right.}{\left(a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5\right)\left(b\tan(f\right.}$$

input
```
integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
-1/120*(15*(15*a^2*b - 40*a*b^2 + 8*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn
(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^5 + 5*a^4*b + 10*a^3*b^2
 + 10*a^2*b^3 + 5*a*b^4 + b^5)*sqrt(a*b + b^2)) + 15*(7*a^2*b^2*tan(f*x +
e)^3 - 8*a*b^3*tan(f*x + e)^3 + 9*a^3*b*tan(f*x + e) + a^2*b^2*tan(f*x + e
) - 8*a*b^3*tan(f*x + e))/((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*
b^4 + b^5)*(b*tan(f*x + e)^2 + a + b)^2) + 8*(15*a^2*tan(f*x + e)^4 - 60*a
*b*tan(f*x + e)^4 + 15*b^2*tan(f*x + e)^4 + 10*a^2*tan(f*x + e)^2 + 5*a*b*
tan(f*x + e)^2 - 5*b^2*tan(f*x + e)^2 + 3*a^2 + 6*a*b + 3*b^2)/((a^5 + 5*a
^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*tan(f*x + e)^5))/f
```

3.66.9 Mupad [B] (verification not implemented)

Time = 21.39 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.10

$$\int \frac{\csc^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx =$$

$$-\frac{\frac{1}{5\,(a+b)}+\frac{\tan(e+f\,x)^2\,(10\,a+b)}{15\,(a+b)^2}+\frac{5\tan(e+f\,x)^6\,(15\,a^2\,b-40\,a\,b^2+8\,b^3)}{24\,(a+b)^4}+\frac{\tan(e+f\,x)^4\,(15\,a^2-40\,a\,b+8\,b^2)}{15\,(a+b)^3}+\frac{\tan(e+f\,x)^8\,(15\,a^2\,b^2-}{8\,(a+b)^5}}{f\,\left(\tan\left(e+f\,x\right)^5\,\left(a^2+2\,a\,b+b^2\right)+\tan\left(e+f\,x\right)^7\,\left(2\,b^2+2\,a\,b\right)+b^2\tan\left(e+f\,x\right)^9\right)}$$

$$-\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\tan(e+f\,x)\,\left(a^5+5\,a^4\,b+10\,a^3\,b^2+10\,a^2\,b^3+5\,a\,b^4+b^5\right)}{(a+b)^{11/2}}\right)\,\left(15\,a^2-40\,a\,b+8\,b^2\right)}{8\,f\,(a+b)^{11/2}}$$

input
```
int(1/(sin(e + f*x)^6*(a + b/cos(e + f*x)^2)^3),x)
```

output
```
- (1/(5*(a + b)) + (tan(e + f*x)^2*(10*a + b))/(15*(a + b)^2) + (5*tan(e +
 f*x)^6*(15*a^2*b - 40*a*b^2 + 8*b^3))/(24*(a + b)^4) + (tan(e + f*x)^4*(1
5*a^2 - 40*a*b + 8*b^2))/(15*(a + b)^3) + (tan(e + f*x)^8*(8*b^4 - 40*a*b^
3 + 15*a^2*b^2))/(8*(a + b)^5))/(f*(tan(e + f*x)^5*(2*a*b + a^2 + b^2) + t
an(e + f*x)^7*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^9)) - (b^(1/2)*atan((b^(1
/2)*tan(e + f*x)*(5*a*b^4 + 5*a^4*b + a^5 + b^5 + 10*a^2*b^3 + 10*a^3*b^2)
)/(a + b)^(11/2))*(15*a^2 - 40*a*b + 8*b^2))/(8*f*(a + b)^(11/2))
```

3.67 $\int \sqrt{a+b\sec^2(e+fx)}\sin^5(e+fx)\,dx$

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3.67.1 Optimal result

Integrand size = 25, antiderivative size = 139

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^5(e+fx)\,dx = \frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f} - \frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{2(5a+b)\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{15a^2f} - \frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{5af}$$

output

```
2/15*(5*a+b)*cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2)/a^2/f-1/5*cos(f*x+e)^5*
(a+b*sec(f*x+e)^2)^(3/2)/a/f+arctanh(sec(f*x+e)*b^(1/2)/(a+b*sec(f*x+e)^2)
^(1/2))*b^(1/2)/f-cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/f
```

3.67.2 Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.09

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^5(e+fx)\,dx = -\frac{\cos(e+fx)\left(-2\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b+a\cos^2(e+fx)}}{\sqrt{b}}\right)+2\sqrt{b+a\cos^2(e+fx)}-\frac{2(2a+b)\left(b+a\cos^2(e+fx)\right)^{3/2}}{3a^2}+\frac{2(b+a}{\sqrt{2}f\sqrt{a+2b+a\cos(2e+2fx)}}$$

input
```
Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^5,x]
```

output
```
-((Cos[e + f*x]*(-2*Sqrt[b]*ArcTanh[Sqrt[b + a*Cos[e + f*x]^2]/Sqrt[b]] +
2*Sqrt[b + a*Cos[e + f*x]^2] - (2*(2*a + b)*(b + a*Cos[e + f*x]^2)^(3/2))/
(3*a^2) + (2*(b + a*Cos[e + f*x]^2)^(5/2))/(5*a^2))*Sqrt[a + b*Sec[e + f*x
]^2])/(Sqrt[2]*f*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]))
```

3.67.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4622, 365, 25, 358, 247, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \sin(e+fx)^5\sqrt{a+b\sec(e+fx)^2}dx$$

$$\downarrow 4622$$

$$\frac{\int \cos^6(e+fx)\left(1-\sec^2(e+fx)\right)^2\sqrt{b\sec^2(e+fx)+a}d\sec(e+fx)}{f}$$

$$\downarrow 365$$

$$\frac{\frac{\int -\cos^4(e+fx)(2(5a+b)-5a\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}d\sec(e+fx)}{5a} - \frac{\cos^5(e+fx)(a+b\sec^2(e+fx))^{3/2}}{5a}}{f}$$

$$\downarrow 25$$

$$\frac{-\frac{\int \cos^4(e+fx)(2(5a+b)-5a\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}d\sec(e+fx)}{5a} - \frac{\cos^5(e+fx)(a+b\sec^2(e+fx))^{3/2}}{5a}}{f}$$

$$\downarrow 358$$

$$\frac{-\frac{-5a\int \cos^2(e+fx)\sqrt{b\sec^2(e+fx)+a}d\sec(e+fx)-\frac{2(5a+b)\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3a}}{5a} - \frac{\cos^5(e+fx)(a+b\sec^2(e+fx))^{3/2}}{5a}}{f}$$

$$\downarrow 247$$

$$\frac{-\frac{-5a\left(b\int\frac{1}{\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)-\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\right)}{5a}-\frac{2(5a+b)\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3a}}{5a}-\frac{\cos^5(e+fx)(a+b\sec^2(e+fx)}{5a}}{f}$$

$$\downarrow 224$$

$$\frac{-\frac{-5a\left(b\int\frac{1}{1-\frac{b\sec^2(e+fx)}{b\sec^2(e+fx)+a}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}-\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\right)}{5a}-\frac{2(5a+b)\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3a}}{5a}-\frac{\cos^5(e+fx)(a+b\sec}{5a}}{f}$$

$$\downarrow 219$$

$$\frac{-\frac{-5a\left(\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)-\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\right)}{5a}-\frac{2(5a+b)\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3a}}{5a}-\frac{\cos^5(e+fx)(a+b\sec^2(e+f}{5a}}{f}$$

input
```
Int[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^5,x]
```

output
```
(-1/5*(Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2))/a - ((-2*(5*a + b)*Cos
[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2))/(3*a) - 5*a*(Sqrt[b]*ArcTanh[(Sq
rt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]] - Cos[e + f*x]*Sqrt[a + b*
Sec[e + f*x]^2]))/(5*a))/f
```

3.67.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 358
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_
Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] + S
imp[d/e^2   Int[(e*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b*c - a*d, 0] && EqQ[Simplify[m + 2*p + 3], 0] && NeQ[m,
 -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

3.67.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 571 vs. $2(123) = 246$.

Time $= 3.31$ (sec) , antiderivative size $= 572$, normalized size of antiderivative $= 4.12$

method	result
default	$\frac{\left(-3\cos(fx+e)^5\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,a^2-3\cos(fx+e)^4\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,a^2+10\cos(fx+e)^3\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,a^2-\cos(fx+e)^3\sqrt{\frac{b+a\cos}{(1+\cos}}\right.$

input
```
int(sin(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/15/f/a^2*(-3*cos(f*x+e)^5*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^
2-3*cos(f*x+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2+10*cos(f*
x+e)^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2-cos(f*x+e)^3*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b+10*cos(f*x+e)^2*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*a^2-cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*a*b+15*b^(1/2)*ln(-4*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)-4*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*sec(f*
x+e)-4*sec(f*x+e)*b)*a^2-15*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*a^2+10*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b
+2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2*cos(f*x+e)-15*a^2*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+10*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*a*b+2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2)*(a+b*se
c(f*x+e)^2)^(1/2)*cos(f*x+e)/(1+cos(f*x+e))/((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)
```

3.67.5 Fricas [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.12

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^5(e+fx)\,dx$$

$$= \left[\frac{15\,a^2\sqrt{b}\log\left(\frac{a\cos(fx+e)^2+2\sqrt{b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+2\,b}{\cos(fx+e)^2}\right)-2\left(3\,a^2\cos(fx+e)^5-(10\,a^2-ab)\cos(fx+e\right.\right.$$

$$\left.\left.\frac{}{30\,a^2 f}\right.\right.$$

$$-\frac{15\,a^2\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{b}\right)+\left(3\,a^2\cos(fx+e)^5-(10\,a^2-ab)\cos(fx+e)^3+(15\right.}{15\,a^2 f}$$

input
```
integrate(sin(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/30*(15*a^2*sqrt(b)*log((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2) - 2*(3*a^2*c
os(f*x + e)^5 - (10*a^2 - a*b)*cos(f*x + e)^3 + (15*a^2 - 10*a*b - 2*b^2)*
cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a^2*f), -1/15*
(15*a^2*sqrt(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^
2)*cos(f*x + e)/b) + (3*a^2*cos(f*x + e)^5 - (10*a^2 - a*b)*cos(f*x + e)^3
 + (15*a^2 - 10*a*b - 2*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2))/(a^2*f)]
```

3.67.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^5(e+fx)\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**5*(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Timed out
```

3.67.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.23

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^5(e+fx)\,dx$$

$$= \frac{\frac{20\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3}{a} - 30\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e) - 15\sqrt{b}\log\left(\frac{\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)-\sqrt{b}}{\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)+\sqrt{b}}\right) - \frac{2\left(3}{30\,f}$$

input
```
integrate(sin(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/30*(20*(a + b/cos(f*x + e)^2)^(3/2)*cos(f*x + e)^3/a - 30*sqrt(a + b/cos
(f*x + e)^2)*cos(f*x + e) - 15*sqrt(b)*log((sqrt(a + b/cos(f*x + e)^2)*cos
(f*x + e) - sqrt(b))/(sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e) + sqrt(b)))
- 2*(3*(a + b/cos(f*x + e)^2)^(5/2)*cos(f*x + e)^5 - 5*(a + b/cos(f*x + e)
^2)^(3/2)*b*cos(f*x + e)^3)/a^2)/f
```

3.67.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1281 vs. 2(123) = 246.

Time = 0.65 (sec) , antiderivative size = 1281, normalized size of antiderivative = 9.22

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^5(e+fx)\,dx = \text{Too large to display}$$

input
```
integrate(sin(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
-2/15*(15*b*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1
/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2
+ 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) - sqrt(a + b))/sqrt(-b))/sqrt(-b) -
2*(15*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4
+ b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x
+ 1/2*e)^2 + a + b))^9*b + 165*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(
a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/
2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^8*sqrt(a + b)*b - 20*(sqrt(a
 + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f
*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 +
a + b))^7*(16*a^2 + 5*a*b - 27*b^2) + 20*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)
^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/
2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^6*(32*a^2 - 83*a*b
 + 33*b^2)*sqrt(a + b) + 2*(416*a^3 + 625*a^2*b - 1230*a*b^2 - 15*b^3)*(sq
rt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1
/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^
2 + a + b))^5 - 10*(256*a^3 - 391*a^2*b + 90*a*b^2 + 81*b^3)*(sqrt(a + b)*
tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1
/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)
)^4*sqrt(a + b) + 20*(16*a^4 - 161*a^3*b + 157*a^2*b^2 + 45*a*b^3 - 33*...
```

3.67.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^5(e+fx)\,dx = \int \sin(e+fx)^5\sqrt{a+\frac{b}{\cos(e+fx)^2}}\,dx$$

input
```
int(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.68 $\int \sqrt{a+b\sec^2(e+fx)}\sin^3(e+fx)\,dx$

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3.68.1 Optimal result

Integrand size = 25, antiderivative size = 100

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^3(e+fx)\,dx = \frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f} - \frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3af}$$

output
```
1/3*cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2)/a/f+arctanh(sec(f*x+e)*b^(1/2)/(
a+b*sec(f*x+e)^2)^(1/2))*b^(1/2)/f-cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/f
```

3.68.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.20

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^3(e+fx)\,dx = \frac{\sqrt{2}\cos(e+fx)\left(3a\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b+a\cos^2(e+fx)}}{\sqrt{b}}\right)+\sqrt{b+a\cos^2(e+fx)}(-3a+b+a\cos^2(e+fx))\right)\sqrt{a-}}{3af\sqrt{a+2b+a\cos(2(e+fx))}}$$

input
```
Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^3,x]
```

output
```
(Sqrt[2]*Cos[e + f*x]*(3*a*Sqrt[b]*ArcTanh[Sqrt[b + a*Cos[e + f*x]^2]/Sqrt
[b]] + Sqrt[b + a*Cos[e + f*x]^2]*(-3*a + b + a*Cos[e + f*x]^2))*Sqrt[a +
b*Sec[e + f*x]^2])/(3*a*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]])
```

3.68.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4622, 25, 358, 247, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \sin(e+fx)^3\sqrt{a+b\sec(e+fx)^2}dx$$

$$\downarrow 4622$$

$$\frac{\int -\cos^4(e+fx)\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}d\sec(e+fx)}{f}$$

$$\downarrow 25$$

$$-\frac{\int \cos^4(e+fx)\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}d\sec(e+fx)}{f}$$

$$\downarrow 358$$

$$\frac{\int \cos^2(e+fx)\sqrt{b\sec^2(e+fx)+a}d\sec(e+fx)+\frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3a}}{f}$$

$$\downarrow 247$$

$$\frac{b\int \frac{1}{\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)+\frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3a}-\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f}$$

$$\downarrow 224$$

$$\frac{b\int \frac{1}{1-\frac{b\sec^2(e+fx)}{b\sec^2(e+fx)+a}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}+\frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3a}-\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f}$$

$$\downarrow 219$$

$$\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)+\frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3a}-\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f}$$

input
```
Int[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^3,x]
```

output
```
(Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]] - Cos[
e + f*x]*Sqrt[a + b*Sec[e + f*x]^2] + (Cos[e + f*x]^3*(a + b*Sec[e + f*x]^
2)^(3/2))/(3*a))/f
```

3.68.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 358
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_
Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] + S
imp[d/e^2   Int[(e*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b*c - a*d, 0] && EqQ[Simplify[m + 2*p + 3], 0] && NeQ[m,
 -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

3.68.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 339 vs. $2(88) = 176$.

Time = 0.38 (sec) , antiderivative size = 340, normalized size of antiderivative = 3.40

method	result
default	$\frac{\left(\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)^3a+\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)^2a+3\ln\left(-4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sec(fx+e)\right.\right.}{3fa}$

input

```
int(sin(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/3/f/a*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)^3*a+((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)^2*a+3*ln(-4*b^(1/2)*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*sec(f*x+e)-4*sec(f*x+e)*b)*b^(1/2)*a-3*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*cos(f*x+e)*b-3*a*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b)*(a+b*sec(f*x+e)^2)^(1/2)*cos(
f*x+e)/(1+cos(f*x+e))/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
```

3.68.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.32

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^3(e+fx)\,dx$$

$$= \left[\frac{3\,a\sqrt{b}\log\left(\frac{a\cos(fx+e)^2+2\sqrt{b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+2b}{\cos(fx+e)^2}\right)+2\left(a\cos(fx+e)^3-(3a-b)\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{6af}\right.$$

$$\left.-\frac{3\,a\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{b}\right)-\left(a\cos(fx+e)^3-(3a-b)\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{3af}\right]$$

input
```
integrate(sin(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*(3*a*sqrt(b)*log((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2) + 2*(a*cos(f*x +
 e)^3 - (3*a - b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
))/(a*f), -1/3*(3*a*sqrt(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)*cos(f*x + e)/b) - (a*cos(f*x + e)^3 - (3*a - b)*cos(f*x + e
))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a*f)]
```

3.68.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^3(e+fx)\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**3*(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Timed out
```

3.68.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.16

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^3(e+fx)\,dx$$
$$= \frac{\frac{2\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3}{a} - 6\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e) - 3\sqrt{b}\log\left(\frac{\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)-\sqrt{b}}{\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)+\sqrt{b}}\right)}{6f}$$

input
```
integrate(sin(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/6*(2*(a + b/cos(f*x + e)^2)^(3/2)*cos(f*x + e)^3/a - 6*sqrt(a + b/cos(f*
x + e)^2)*cos(f*x + e) - 3*sqrt(b)*log((sqrt(a + b/cos(f*x + e)^2)*cos(f*x
 + e) - sqrt(b))/(sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e) + sqrt(b))))/f
```

3.68.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 805 vs. 2(88) = 176.

Time = 0.47 (sec) , antiderivative size = 805, normalized size of antiderivative = 8.05

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^3(e+fx)\,dx = \text{Too large to display}$$

input
```
integrate(sin(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output

```
-2/3*(3*b*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2
*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 +
2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) - sqrt(a + b))/sqrt(-b))/sqrt(-b) - 2*
(3*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b
*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1
/2*e)^2 + a + b))^5*b - 3*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan
(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^
2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^4*(4*a - 3*b)*sqrt(a + b) + 2*(sq
rt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1
/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^
2 + a + b))^3*(8*a^2 - 9*a*b + 3*b^2) + 6*(sqrt(a + b)*tan(1/2*f*x + 1/2*e
)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1
/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2*(4*a^2 - a*b -
b^2)*sqrt(a + b) - 3*(16*a^3 - 5*a^2*b - 2*a*b^2 + 3*b^3)*(sqrt(a + b)*tan
(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*
e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)) +
 (20*a^3 - 19*a^2*b + 6*a*b^2 - 3*b^3)*sqrt(a + b))/((sqrt(a + b)*tan(1/2*
f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4
- 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2 + 2*
(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b...
```

3.68.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^3(e+fx)\,dx = \int \sin(e+fx)^3\sqrt{a+\frac{b}{\cos(e+fx)^2}}\,dx$$

input

```
int(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2),x)
```

output

```
int(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.69 $\int \sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\,dx$

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3.69.1 Optimal result

Integrand size = 23, antiderivative size = 66

$$\int \sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\,dx = \frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f} - \frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f}$$

output
```
arctanh(sec(f*x+e)*b^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))*b^(1/2)/f-cos(f*x+e)*
(a+b*sec(f*x+e)^2)^(1/2)/f
```

3.69.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.50

$$\int \sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\,dx = -\frac{\sqrt{2}\cos(e+fx)\left(-\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b+a\cos^2(e+fx)}}{\sqrt{b}}\right)+\sqrt{b+a\cos^2(e+fx)}\right)\sqrt{a+b\sec^2(e+fx)}}{f\sqrt{a+2b+a\cos(2e+2fx)}}$$

input
```
Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x],x]
```

output
```
-((Sqrt[2]*Cos[e + f*x]*(-(Sqrt[b]*ArcTanh[Sqrt[b + a*Cos[e + f*x]^2]/Sqrt
[b]]) + Sqrt[b + a*Cos[e + f*x]^2])*Sqrt[a + b*Sec[e + f*x]^2])/(f*Sqrt[a
+ 2*b + a*Cos[2*e + 2*f*x]]))
```

3.69.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4622, 247, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \sin(e+fx)\sqrt{a+b\sec(e+fx)^2}dx$$

$$\downarrow 4622$$

$$\frac{\int \cos^2(e+fx)\sqrt{b\sec^2(e+fx)+a}d\sec(e+fx)}{f}$$

$$\downarrow 247$$

$$\frac{b\int \frac{1}{\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx) - \cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f}$$

$$\downarrow 224$$

$$\frac{b\int \frac{1}{1-\frac{b\sec^2(e+fx)}{b\sec^2(e+fx)+a}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a}} - \cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f}$$

$$\downarrow 219$$

$$\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right) - \cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f}$$

input
```
Int[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x],x]
```

output
```
(Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]] - Cos[
e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/f
```

3.69.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

3.69.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.41

method	result	size
derivativedivides	$-\frac{\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}}{fa\sec(fx+e)}+\frac{b\sec(fx+e)\sqrt{a+b\sec(fx+e)^2}}{fa}+\frac{\sqrt{b}\ln\left(\sqrt{b}\sec(fx+e)+\sqrt{a+b\sec(fx+e)^2}\right)}{f}$	93
default	$-\frac{\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}}{fa\sec(fx+e)}+\frac{b\sec(fx+e)\sqrt{a+b\sec(fx+e)^2}}{fa}+\frac{\sqrt{b}\ln\left(\sqrt{b}\sec(fx+e)+\sqrt{a+b\sec(fx+e)^2}\right)}{f}$	93

input
```
int(sin(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f/a/sec(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2)+1/f*b/a*sec(f*x+e)*(a+b*sec(f*x
+e)^2)^(1/2)+1/f*b^(1/2)*ln(b^(1/2)*sec(f*x+e)+(a+b*sec(f*x+e)^2)^(1/2))
```

3.69.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.76

$$\int \sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\,dx$$
$$= \left[-\frac{2\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e) - \sqrt{b}\log\left(\frac{a\cos(fx+e)^2+2\sqrt{b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+2b}{\cos(fx+e)^2}\right)}{2f}, \right.$$
$$\left. -\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{b}\right) + \sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{f} \right]$$

input
```
integrate(sin(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*(2*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) - sqrt(b
)*log((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2))/f, -(sqrt(-b)*arctan(sqrt(-b)*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b) + sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e))/f]
```

3.69.6 Sympy [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\,dx = \int \sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\,dx$$

input
```
integrate(sin(f*x+e)*(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*sin(e + f*x), x)
```

3.69.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.33

$$\int \sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\,dx$$
$$= -\frac{2\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)+\sqrt{b}\log\left(\frac{\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)-\sqrt{b}}{\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)+\sqrt{b}}\right)}{2f}$$

input
```
integrate(sin(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*(2*sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e) + sqrt(b)*log((sqrt(a + b/
cos(f*x + e)^2)*cos(f*x + e) - sqrt(b))/(sqrt(a + b/cos(f*x + e)^2)*cos(f*
x + e) + sqrt(b))))/f
```

3.69.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.83

$$\int \sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\,dx$$
$$= -\frac{\left(\frac{b\arctan\left(\frac{\sqrt{a\cos(fx+e)^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}}+\sqrt{a\cos(fx+e)^2+b}\right)\operatorname{sgn}(\cos(fx+e))}{f}$$

input
```
integrate(sin(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
-(b*arctan(sqrt(a*cos(f*x + e)^2 + b)/sqrt(-b))/sqrt(-b) + sqrt(a*cos(f*x
+ e)^2 + b))*sgn(cos(f*x + e))/f
```

3.69.9 Mupad [B] (verification not implemented)

Time = 18.36 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.32

$$\int \sqrt{a+b\sec^2(e+fx)}\sin(e+fx)\,dx = -\frac{\cos(e+f\,x)\,\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}{f} - \frac{\sqrt{b}\,\operatorname{asin}\left(\frac{\sqrt{b}\,1\mathrm{i}}{\sqrt{a}\,\cos(e+f\,x)}\right)\,\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}\,1\mathrm{i}}{\sqrt{a}\,f\,\sqrt{\frac{b}{a\cos(e+f\,x)^2}+1}}$$

input
```
int(sin(e + f*x)*(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
- (cos(e + f*x)*(a + b/cos(e + f*x)^2)^(1/2))/f - (b^(1/2)*asin((b^(1/2)*1
i)/(a^(1/2)*cos(e + f*x)))*(a + b/cos(e + f*x)^2)^(1/2)*1i)/(a^(1/2)*f*(b/
(a*cos(e + f*x)^2) + 1)^(1/2))
```

3.70 $\int \csc(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

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3.70.1 Optimal result

Integrand size = 23, antiderivative size = 82

$$\int \csc(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f} - \frac{\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}$$

output
```
arctanh(sec(f*x+e)*b^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))*b^(1/2)/f-arctanh(sec
(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))*(a+b)^(1/2)/f
```

3.70.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.45

$$\int \csc(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{2}\left(\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{a+b-a\sin^2(e+fx)}}{\sqrt{b}}\right) - \sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b-a\sin^2(e+fx)}}{\sqrt{a+b}}\right)\right)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f\sqrt{a+2b+a\cos(2e+2fx)}}$$

input
```
Integrate[Csc[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Sqrt[2]*(Sqrt[b]*ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[b]] - Sqrt[a
 + b]*ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[a + b]])*Cos[e + f*x]*Sq
rt[a + b*Sec[e + f*x]^2])/(f*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]])
```

3.70.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4622, 25, 301, 224, 219, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sqrt{a+b\sec(e+fx)^2}}{\sin(e+fx)}dx$$

$$\downarrow 4622$$

$$\frac{\int -\frac{\sqrt{b\sec^2(e+fx)+a}}{1-\sec^2(e+fx)}d\sec(e+fx)}{f}$$

$$\downarrow 25$$

$$-\frac{\int \frac{\sqrt{b\sec^2(e+fx)+a}}{1-\sec^2(e+fx)}d\sec(e+fx)}{f}$$

$$\downarrow 301$$

$$\frac{b\int \frac{1}{\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx) - (a+b)\int \frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{f}$$

$$\downarrow 224$$

$$\frac{b\int \frac{1}{1-\frac{b\sec^2(e+fx)}{b\sec^2(e+fx)+a}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a}} - (a+b)\int \frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{f}$$

$$\downarrow 219$$

$$\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right) - (a+b)\int \frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{f}$$

$$\downarrow 291$$

$$\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)-(a+b)\int\frac{1}{1-\frac{(a+b)\sec^2(e+fx)}{b\sec^2(e+fx)+a}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}}{f}$$

$$\downarrow 219$$

$$\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)-\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f}$$

input
```
Int[Csc[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]] - Sqrt
[a + b]*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/f
```

3.70.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

3.70.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 541 vs. $2(70) = 140$.

Time = 0.56 (sec) , antiderivative size = 542, normalized size of antiderivative = 6.61

method	result
default	$\frac{\sqrt{a+b\sec(fx+e)^2}\left(2\sqrt{b}\,\ln\left(-4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\sec(fx+e)-4\sec(fx+e)b\right)\sqrt{a+b}-\ln\left(\frac{2\sqrt{\frac{b+a\cos(fx+}{(1+\cos(fx+}\right.\right.$

input
```
int(csc(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f/(a+b)^(1/2)*(a+b*sec(f*x+e)^2)^(1/2)*(2*b^(1/2)*ln(-4*b^(1/2)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*sec(f*x+e)-4*sec(f*x+e)*b)*(a+b)^(1/2)-ln(2/(a+b)^(1/2)*
(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f
*x+e)))*a-b*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(
1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))-ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*a-ln(-4*(((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*b)*co
s(f*x+e)/(1+cos(f*x+e))/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
```

3.70.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 496, normalized size of antiderivative = 6.05

$$\int \csc(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$= \left[\frac{\sqrt{a+b}\log\left(\frac{2\left(a\cos(fx+e)^2-2\sqrt{a+b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+a+2b\right)}{\cos(fx+e)^2-1}\right)+\sqrt{b}\log\left(\frac{a\cos(fx+e)^2+2\sqrt{b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos}{\cos(fx+e)^2}\right.}{2f}\right.$$

$$\left. -\frac{2\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{b}\right)-\sqrt{a+b}\log\left(\frac{2\left(a\cos(fx+e)^2-2\sqrt{a+b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+a+2\right.}{\cos(fx+e)^2-1}\right.}{2f} \right]$$

input
```
integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a + b)*log(2*(a*cos(f*x + e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x
+ e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1))
+ sqrt(b)*log((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2))/f, 1/2*(2*sqrt(-a - b)*
arctan(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x +
e)/(a + b)) + sqrt(b)*log((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2))/f, -1/2*(2*
sqrt(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f
*x + e)/b) - sqrt(a + b)*log(2*(a*cos(f*x + e)^2 - 2*sqrt(a + b)*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2
 - 1)))/f, (sqrt(-a - b)*arctan(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)*cos(f*x + e)/(a + b)) - sqrt(-b)*arctan(sqrt(-b)*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b))/f]
```

3.70.6 Sympy [F]

$$\int \csc(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{a+b\sec^2(e+fx)}\csc(e+fx)\,dx$$

input
```
integrate(csc(f*x+e)*(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*csc(e + f*x), x)
```

3.70.7 Maxima [F]

$$\int \csc(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\csc(fx+e)\,dx$$

input
```
integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*csc(f*x + e), x)
```

3.70.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(70) = 140$.

Time = 0.70 (sec) , antiderivative size = 405, normalized size of antiderivative = 4.94

$$\int \csc(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx =$$

$$-\left(\frac{4\,b\arctan\left(-\frac{\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a+b}-\sqrt{a+b}}{2\sqrt{-b}}\right)}{\sqrt{-b}}+\sqrt{a+\right.$$

input
```
integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*(4*b*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2
*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 +
2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) - sqrt(a + b))/sqrt(-b))/sqrt(-b) + sq
rt(a + b)*log(abs(-sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 + sqrt(a*tan(1/2*f*x
 + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*
tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))) - sqrt(a + b)*log(abs(-sqr
t(a + b)*tan(1/2*f*x + 1/2*e)^2 + sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/
2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2
 + a + b) - sqrt(a + b))) - sqrt(a + b)*log(abs((sqrt(a + b)*tan(1/2*f*x +
 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a
*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*(a + b) - s
qrt(a + b)*(a - b))))*sgn(cos(f*x + e))/f
```

3.70.9 Mupad [F(-1)]

Timed out.

$$\int \csc(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(e+fx)^2}}}{\sin(e+fx)}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x),x)
```

output
```
int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x), x)
```

3.71 $\int \csc^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

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3.71.1 Optimal result

Integrand size = 25, antiderivative size = 124

$$\int \csc^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f} - \frac{(a+2b)\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2\sqrt{a+b}f} - \frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}$$

output

```
arctanh(sec(f*x+e)*b^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))*b^(1/2)/f-1/2*(a+2*b)
*arctanh(sec(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))/f/(a+b)^(1/2)-1/
2*cot(f*x+e)*csc(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/f
```

3.71.2 Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.31

$$\int \csc^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(2\sqrt{b}(a+b)\operatorname{arctanh}\left(\frac{\sqrt{a+b-a\sin^2(e+fx)}}{\sqrt{b}}\right) - \sqrt{a+b}(a+2b)\operatorname{arctanh}\left(\frac{\sqrt{a+b-}}{}\right.\right.}{\sqrt{2}(a+b)f\sqrt{a+2b+a\cos(2(e+fx))}}$$

input
```
Integrate[Csc[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*(2*Sqrt[b]*(a + b)*ArcTanh[Sqrt[a
 + b - a*Sin[e + f*x]^2]/Sqrt[b]] - Sqrt[a + b]*(a + 2*b)*ArcTanh[Sqrt[a +
 b - a*Sin[e + f*x]^2]/Sqrt[a + b]] - (a + b)*Csc[e + f*x]^2*Sqrt[a + b -
a*Sin[e + f*x]^2]))/(Sqrt[2]*(a + b)*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]])
```

3.71.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4622, 369, 398, 224, 219, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sqrt{a+b\sec(e+fx)^2}}{\sin(e+fx)^3}dx$$

$$\downarrow 4622$$

$$\frac{\int \frac{\sec^2(e+fx)\sqrt{b\sec^2(e+fx)+a}}{\left(1-\sec^2(e+fx)\right)^2}d\sec(e+fx)}{f}$$

$$\downarrow 369$$

$$\frac{\frac{\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2\left(1-\sec^2(e+fx)\right)} - \frac{1}{2}\int \frac{2b\sec^2(e+fx)+a}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{f}$$

$$\downarrow 398$$

$$\frac{\frac{1}{2}\left(2b\int \frac{1}{\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx) - (a+2b)\int \frac{1}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)\right) + \frac{\sec(e+fx)\sqrt{a+b\sec^2(e+}}{2\left(1-\sec^2(e+fx)\right)}}{f}$$

$$\downarrow 224$$

$$\frac{\frac{1}{2}\left(2b\int\frac{1}{1-\frac{b\sec^2(e+fx)}{b\sec^2(e+fx)+a}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}-(a+2b)\int\frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)\right)+\frac{\sec(e+fx)\sqrt{a+b\sec}}{2(1-\sec^2(e+f}}{f}$$

$$\downarrow 219$$

$$\frac{\frac{1}{2}\left(2\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)-(a+2b)\int\frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)\right)+\frac{\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(1-\sec^2(e+fx))}}{f}$$

$$\downarrow 291$$

$$\frac{\frac{1}{2}\left(2\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)-(a+2b)\int\frac{1}{1-\frac{(a+b)\sec^2(e+fx)}{b\sec^2(e+fx)+a}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}\right)+\frac{\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(1-\sec^2(e+fx))}}{f}$$

$$\downarrow 219$$

$$\frac{\frac{1}{2}\left(2\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)-\frac{(a+2b)\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{\sqrt{a+b}}\right)+\frac{\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(1-\sec^2(e+fx))}}{f}$$

input
```
Int[Csc[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
((2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]] - (
(a + 2*b)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/
Sqrt[a + b])/2 + (Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(2*(1 - Sec[e +
 f*x]^2)))/f
```

3.71.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

3.71.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2128 vs. $2(106) = 212$.

Time = 0.44 (sec) , antiderivative size = 2129, normalized size of antiderivative = 17.17

method	result	size
default	Expression too large to display	2129

input
```
int(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/f/(a+b)^(5/2)*(4*(a+b)^(3/2)*cos(f*x+e)*ln(-4*b^(1/2)*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)-4*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*sec(f*x+e)-4*sec(f*x+e)*b)*b^(3/2)+4*(a+b)^(3/2)*cos(f*x+e)*ln(-4
*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*b^(1/2)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*sec(f*x+e)-4*sec(f*x+e)*b)*b^(1/2)*a-4*(a
+b)^(3/2)*ln(-4*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*b^(1
/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*sec(f*x+e)-4*sec(f*x+e)*b)
*b^(3/2)-4*(a+b)^(3/2)*ln(-4*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)-4*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*sec(f*x+e)-4*
sec(f*x+e)*b)*b^(1/2)*a+2*(a+b)^(3/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*a+2*(a+b)^(3/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b-cos(
f*x+e)*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)
^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-
cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^3-4*cos(f*x+e)*ln(2/(a+b)^(1/2)*(((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*
a^2*b-5*cos(f*x+e)*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a*b^2-2*cos(f*x+e)*ln(2/(a+b)^
(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+...
```

3.71.5 Fricas [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 867, normalized size of antiderivative = 6.99

$$\int \csc^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(2*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) +
 ((a + 2*b)*cos(f*x + e)^2 - a - 2*b)*sqrt(a + b)*log(2*(a*cos(f*x + e)^2
- 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) +
 a + 2*b)/(cos(f*x + e)^2 - 1)) + 2*((a + b)*cos(f*x + e)^2 - a - b)*sqrt(
b)*log((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2))/((a + b)*f*cos(f*x + e)^2 - (a
 + b)*f), 1/2*(((a + 2*b)*cos(f*x + e)^2 - a - 2*b)*sqrt(-a - b)*arctan(sq
rt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a + b
)) + (a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + ((
a + b)*cos(f*x + e)^2 - a - b)*sqrt(b)*log((a*cos(f*x + e)^2 + 2*sqrt(b)*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e
)^2))/((a + b)*f*cos(f*x + e)^2 - (a + b)*f), -1/4*(4*((a + b)*cos(f*x + e
)^2 - a - b)*sqrt(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)*cos(f*x + e)/b) - 2*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*cos(f*x + e) - ((a + 2*b)*cos(f*x + e)^2 - a - 2*b)*sqrt(a + b)*log(
2*(a*cos(f*x + e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1)))/((a + b)*f*cos(f*x +
e)^2 - (a + b)*f), 1/2*(((a + 2*b)*cos(f*x + e)^2 - a - 2*b)*sqrt(-a - b)*
arctan(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x +
e)/(a + b)) - 2*((a + b)*cos(f*x + e)^2 - a - b)*sqrt(-b)*arctan(sqrt(-...
```

3.71.6 Sympy [F]

$$\int \csc^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{a+b\sec^2(e+fx)}\csc^3(e+fx)\,dx$$

input
```
integrate(csc(f*x+e)**3*(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*csc(e + f*x)**3, x)
```

3.71.7 Maxima [F]

$$\int \csc^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\csc(fx+e)^3\,dx$$

input
```
integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*csc(f*x + e)^3, x)
```

3.71.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 578 vs. $2(106) = 212$.

Time $= 0.82$ (sec) , antiderivative size $= 578$, normalized size of antiderivative $= 4.66$

$$\int \csc^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$= \left(\frac{4\,(a+2\,b)\arctan\left(-\frac{\sqrt{a+b}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-\sqrt{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-2\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+2\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+a+b}}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}} - \frac{16\,b\arctan\right.$$

input
```
integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
1/8*(4*(a + 2*b)*arctan(-(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(
1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2
 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))/sqrt(-a - b))/sqrt(-a - b) - 16*b*
arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2
*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/
2*f*x + 1/2*e)^2 + a + b) - sqrt(a + b))/sqrt(-b))/sqrt(-b) + 2*(a + 2*b)*
log(abs(-(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)
^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f
*x + 1/2*e)^2 + a + b))*(a + b) + sqrt(a + b)*(a - b)))/sqrt(a + b) - sqrt
(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1
/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) - 2*((sqrt(a + b)*tan(1/2*f*
x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 -
2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*(a - b)
- (a + b)^(3/2))/((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x
 + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*
tan(1/2*f*x + 1/2*e)^2 + a + b))^2 - a - b))*sgn(cos(f*x + e))/f
```

3.71.9 Mupad [F(-1)]

Timed out.

$$\int \csc^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(e+fx)^2}}}{\sin(e+fx)^3}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x)^3,x)
```

output
```
int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x)^3, x)
```

3.72 $\int \csc^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

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3.72.1 Optimal result

Integrand size = 25, antiderivative size = 183

$$\begin{aligned}
&\int \csc^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx \\
&= \frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f} - \frac{(3a^2+12ab+8b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8(a+b)^{3/2}f} \\
&\quad - \frac{(3a+4b)\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)f} \\
&\quad - \frac{\cot(e+fx)\csc^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f}
\end{aligned}$$

output

```
-1/8*(3*a^2+12*a*b+8*b^2)*arctanh(sec(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2
)^(1/2))/(a+b)^(3/2)/f+arctanh(sec(f*x+e)*b^(1/2)/(a+b*sec(f*x+e)^2)^(1/2)
)*b^(1/2)/f-1/8*(3*a+4*b)*cot(f*x+e)*csc(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/(
a+b)/f-1/4*cot(f*x+e)*csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2)/f
```

3.72.2 Mathematica [A] (verified)

Time = 1.72 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.14

$$\int \csc^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{\cos(e+fx)\left(8\sqrt{b}(a+b)^2\operatorname{arctanh}\left(\frac{\sqrt{a+b-a\sin^2(e+fx)}}{\sqrt{b}}\right)-\sqrt{a+b}(3a^2+12ab+8b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+b-a\sin^2(e+f}}{\sqrt{a+b}}\right.\right.}{4\sqrt{2}(a+b)^2 f\sqrt{a+2b+a\cos(}$$

input
```
Integrate[Csc[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Cos[e + f*x]*(8*Sqrt[b]*(a + b)^2*ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/
Sqrt[b]] - Sqrt[a + b]*(3*a^2 + 12*a*b + 8*b^2)*ArcTanh[Sqrt[a + b - a*Sin
[e + f*x]^2]/Sqrt[a + b]] + ((a + b)*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(-
7*a - 8*b + (3*a + 4*b)*Cos[2*(e + f*x)])*Csc[e + f*x]^4)/(2*Sqrt[2]))*Sqr
t[a + b*Sec[e + f*x]^2])/(4*Sqrt[2]*(a + b)^2*f*Sqrt[a + 2*b + a*Cos[2*(e
+ f*x)]])
```

3.72.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.13, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 4622, 25, 369, 440, 25, 398, 224, 219, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\sqrt{a+b\sec(e+fx)^2}}{\sin(e+fx)^5}dx$$
$$\downarrow 4622$$
$$\frac{\int -\frac{\sec^4(e+fx)\sqrt{b\sec^2(e+fx)+a}}{(1-\sec^2(e+fx))^3}d\sec(e+fx)}{f}$$
$$\downarrow 25$$

$$-\frac{\int \frac{\sec^4(e+fx)\sqrt{b\sec^2(e+fx)+a}}{(1-\sec^2(e+fx))^3}d\sec(e+fx)}{f}$$

$$\downarrow 369$$

$$\frac{\frac{1}{4}\int \frac{\sec^2(e+fx)(4b\sec^2(e+fx)+3a)}{(1-\sec^2(e+fx))^2\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)-\frac{\sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4(1-\sec^2(e+fx))^2}}{f}$$

$$\downarrow 440$$

$$\frac{\frac{1}{4}\left(\frac{\int -\frac{8b(a+b)\sec^2(e+fx)+a(3a+4b)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{2(a+b)}+\frac{(3a+4b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)(1-\sec^2(e+fx))}\right)-\frac{\sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4(1-\sec^2(e+fx))^2}}{f}$$

$$\downarrow 25$$

$$\frac{\frac{1}{4}\left(\frac{(3a+4b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)(1-\sec^2(e+fx))}-\frac{\int \frac{8b(a+b)\sec^2(e+fx)+a(3a+4b)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{2(a+b)}\right)-\frac{\sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4(1-\sec^2(e+fx))^2}}{f}$$

$$\downarrow 398$$

$$\frac{\frac{1}{4}\left(\frac{(3a+4b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)(1-\sec^2(e+fx))}-\frac{\left(3a^2+12ab+8b^2\right)\int \frac{1}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)-8b(a+b)\int \frac{1}{\sqrt{b\sec^2(e+fx)+a}}d\sec(e}{2(a+b)}\right)}{f}$$

$$\downarrow 224$$

$$\frac{\frac{1}{4}\left(\frac{(3a+4b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)(1-\sec^2(e+fx))}-\frac{\left(3a^2+12ab+8b^2\right)\int \frac{1}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)-8b(a+b)\int \frac{1}{1-\frac{b\sec^2(e+fx)}{b\sec^2(e+fx)+a}}d\frac{}{\sqrt{b\,s}}}{2(a+b)}\right)}{f}$$

$$\downarrow 219$$

$$\frac{\frac{1}{4}\left(\frac{(3a+4b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)(1-\sec^2(e+fx))}-\frac{\left(3a^2+12ab+8b^2\right)\int \frac{1}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)-8\sqrt{b}(a+b)\text{arctanh}\left(\frac{\sqrt{b}\sec(e+}{\sqrt{a+b\sec^2(e}}\right.\right.}{2(a+b)}\right)}{f}$$

$$\downarrow 291$$

$$\frac{\frac{1}{4}\left(\frac{(3a+4b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)(1-\sec^2(e+fx))}-\frac{(3a^2+12ab+8b^2)\int\frac{1}{1-\frac{(a+b)\sec^2(e+fx)}{b\sec^2(e+fx)+a}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}-8\sqrt{b}(a+b)\operatorname{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2(a+b)}\right)}{f}$$

$$\downarrow 219$$

$$\frac{\frac{1}{4}\left(\frac{(3a+4b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)(1-\sec^2(e+fx))}-\frac{\frac{\left(3a^2+12ab+8b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{\sqrt{a+b}}-8\sqrt{b}(a+b)\operatorname{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2(a+b)}\right)}{f}-\text{sec}$$

input
```
Int[Csc[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(-1/4*(Sec[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2])/(1 - Sec[e + f*x]^2)^2 +
 (-1/2*(-8*Sqrt[b]*(a + b)*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e
 + f*x]^2]] + ((3*a^2 + 12*a*b + 8*b^2)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])
/Sqrt[a + b*Sec[e + f*x]^2]])/Sqrt[a + b])/(a + b) + ((3*a + 4*b)*Sec[e +
f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(2*(a + b)*(1 - Sec[e + f*x]^2)))/4)/f
```

3.72.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

3.72.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 4317 vs. $2(161) = 322$.

Time $= 0.51$ (sec) , antiderivative size $= 4318$, normalized size of antiderivative $= 23.60$

method	result	size
default	Expression too large to display	4318

input
```
int(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/64/f/(a+b)^(9/2)*((a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*cs
c(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+
e)^2+a+b)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^2)^(1/2)*((1-cos(f*x+e))^2*csc
(f*x+e)^2-1)*(-128*b^(3/2)*ln(4*(b*(1-cos(f*x+e))^2*csc(f*x+e)^2+b^(1/2)*(
a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos
(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)+b)/((
1-cos(f*x+e))^2*csc(f*x+e)^2-1))*(a+b)^(5/2)*a*(1-cos(f*x+e))^4*csc(f*x+e)
^4-18*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a
*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2
)*(a+b)^(5/2)*a*b*(1-cos(f*x+e))^6*csc(f*x+e)^6-64*b^(1/2)*ln(4*(b*(1-cos(
f*x+e))^2*csc(f*x+e)^2+b^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f
*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))
^2*csc(f*x+e)^2+a+b)^(1/2)+b)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))*(a+b)^(5/
2)*a^2*(1-cos(f*x+e))^4*csc(f*x+e)^4+14*a*b*(a*(1-cos(f*x+e))^4*csc(f*x+e)
^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(
1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(5/2)*(1-cos(f*x+e))^4*csc(f
*x+e)^4+212*ln((a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x
+e)^2+(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a
*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2
)*(a+b)^(1/2)-a+b)/(a+b)^(1/2))*a^3*b^2*(1-cos(f*x+e))^4*csc(f*x+e)^4+2...
```

3.72.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(161) = 322$.

Time $= 0.66$ (sec) , antiderivative size $= 1476$, normalized size of antiderivative $= 8.07$

$$\int \csc^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(((3*a^2 + 12*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a^2 + 12*a*b + 8*b^
2)*cos(f*x + e)^2 + 3*a^2 + 12*a*b + 8*b^2)*sqrt(a + b)*log(2*(a*cos(f*x +
 e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x
+ e) + a + 2*b)/(cos(f*x + e)^2 - 1)) + 8*((a^2 + 2*a*b + b^2)*cos(f*x + e
)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(b)*lo
g((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)*cos(f*x + e) + 2*b)/cos(f*x + e)^2) + 2*((3*a^2 + 7*a*b + 4*b^2)*cos(f*x
 + e)^3 - (5*a^2 + 11*a*b + 6*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^4 - 2*(a^2 + 2*a*b
 + b^2)*f*cos(f*x + e)^2 + (a^2 + 2*a*b + b^2)*f), 1/8*(((3*a^2 + 12*a*b +
 8*b^2)*cos(f*x + e)^4 - 2*(3*a^2 + 12*a*b + 8*b^2)*cos(f*x + e)^2 + 3*a^2
 + 12*a*b + 8*b^2)*sqrt(-a - b)*arctan(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a + b)) + 4*((a^2 + 2*a*b + b^2)*cos(f
*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt
(b)*log((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2) + ((3*a^2 + 7*a*b + 4*b^2)*cos
(f*x + e)^3 - (5*a^2 + 11*a*b + 6*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^4 - 2*(a^2 + 2
*a*b + b^2)*f*cos(f*x + e)^2 + (a^2 + 2*a*b + b^2)*f), -1/16*(16*((a^2 + 2
*a*b + b^2)*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f*x + e)^2 + a^2...
```

3.72.6 Sympy [F]

$$\int \csc^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{a+b\sec^2(e+fx)}\csc^5(e+fx)\,dx$$

input
```
integrate(csc(f*x+e)**5*(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*csc(e + f*x)**5, x)
```

3.72.7 Maxima [F]

$$\int \csc^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\csc(fx+e)^5\,dx$$

input
```
integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*csc(f*x + e)^5, x)
```

3.72.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 867 vs. $2(161) = 322$.

Time $= 1.21$ (sec) , antiderivative size $= 867$, normalized size of antiderivative $= 4.74$

$$\int \csc^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/64*(sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(
1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)*(tan(1/2*f*x + 1/
2*e)^2 + (9*a + 11*b)/(a + b)) + 128*b*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*
x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 -
2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) - sqrt(a
+ b))/sqrt(-b))/sqrt(-b) - 8*(3*a^2 + 12*a*b + 8*b^2)*arctan(-(sqrt(a + b)
*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x +
1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b
))/sqrt(-a - b))/((a + b)*sqrt(-a - b)) - 4*(3*a^2 + 12*a*b + 8*b^2)*log(a
bs(-(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 +
b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x +
1/2*e)^2 + a + b))*(a + b) + sqrt(a + b)*(a - b)))/(a + b)^(3/2) + 4*(2*(s
qrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(
1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)
^2 + a + b))^3*(2*a^2 - 3*b^2) - (sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqr
t(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x +
1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2*(3*a^2 + 10*a*b + 7*b^2)
*sqrt(a + b) - 2*(3*a^3 + 3*a^2*b - 2*a*b^2 - 2*b^3)*(sqrt(a + b)*tan(1/2*
f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4
- 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)) + 5...
```

3.72.9 Mupad [F(-1)]

Timed out.

$$\int \csc^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(e+fx)^2}}}{\sin(e+fx)^5}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x)^5,x)
```

output
```
int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x)^5, x)
```

3.73 $\int \sqrt{a+b\sec^2(e+fx)}\sin^6(e+fx)\,dx$

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3.73.1 Optimal result

Integrand size = 25, antiderivative size = 240

$$\begin{aligned}
&\int \sqrt{a+b\sec^2(e+fx)}\sin^6(e+fx)\,dx \\
&= \frac{(5a^3-15a^2b-5ab^2-b^3)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{5/2}f} + \frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} \\
&\quad - \frac{(a-b)(5a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16a^2f} \\
&\quad - \frac{(5a-b)\cos(e+fx)\sin^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24af} \\
&\quad - \frac{\cos(e+fx)\sin^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6f}
\end{aligned}$$

output

```
1/16*(5*a^3-15*a^2*b-5*a*b^2-b^3)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x
+e)^2)^(1/2))/a^(5/2)/f+arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1
/2))*b^(1/2)/f-1/16*(a-b)*(5*a+b)*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^
2)^(1/2)/a^2/f-1/24*(5*a-b)*cos(f*x+e)*sin(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(
1/2)/a/f-1/6*cos(f*x+e)*sin(f*x+e)^5*(a+b+b*tan(f*x+e)^2)^(1/2)/f
```

3.73.2 Mathematica [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^6(e+fx)\,dx = \int \sqrt{a+b\sec^2(e+fx)}\sin^6(e+fx)\,dx$$

input
```
Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^6,x]
```

output
```
Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^6, x]
```

3.73.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.12, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 4620, 369, 440, 27, 440, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \sin(e+fx)^6\sqrt{a+b\sec(e+fx)^2}dx$$

$$\downarrow 4620$$

$$\frac{\int \frac{\tan^6(e+fx)\sqrt{b\tan^2(e+fx)+a+b}}{(\tan^2(e+fx)+1)^4}d\tan(e+fx)}{f}$$

$$\downarrow 369$$

$$\frac{\frac{1}{6}\int \frac{\tan^4(e+fx)(6b\tan^2(e+fx)+5(a+b))}{(\tan^2(e+fx)+1)^3\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx) - \frac{\tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{6(\tan^2(e+fx)+1)^3}}{f}$$

$$\downarrow 440$$

$$\frac{\frac{1}{6}\left(\frac{\int \frac{3\tan^2(e+fx)\left(8ab\tan^2(e+fx)+(5a-b)(a+b)\right)}{\left(\tan^2(e+fx)+1\right)^2\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{4a} - \frac{(5a-b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4a(\tan^2(e+fx)+1)^2}\right) - \frac{\tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)+}}{6(\tan^2(e+fx)+1)^3}}{f}$$

$$\downarrow 27$$

$$\frac{\frac{1}{6}\left(\frac{3\int \frac{\tan^2(e+fx)\left(8ab\tan^2(e+fx)+(5a-b)(a+b)\right)}{\left(\tan^2(e+fx)+1\right)^2\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{4a}-\frac{(5a-b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4a(\tan^2(e+fx)+1)^2}\right)-\frac{\tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)+}}{6(\tan^2(e+fx)+1)^3}}{f}$$

$$\downarrow 440$$

$$\frac{\frac{1}{6}\left(\frac{3\left(\frac{\int \frac{16a^2b\tan^2(e+fx)+(5a+b)\left(a^2-b^2\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2a}-\frac{(a-b)(5a+b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a\left(\tan^2(e+fx)+1\right)}\right)}{4a}-\frac{(5a-b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4a(\tan^2(e+fx)+1)^2}\right)}{f}$$

$$\downarrow 398$$

$$\frac{\frac{1}{6}\left(\frac{3\left(\frac{16a^2b\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-\left(16a^2b-(5a+b)\left(a^2-b^2\right)\right)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2a}-\frac{(a-b)(5a+b)\tan(e+fx)\sqrt{a-}}{2a\left(\tan^2(e+fx)\right.}\right)}{4a}\right)}{f}$$

$$\downarrow 224$$

$$\frac{\frac{1}{6}\left(\frac{3\left(\frac{16a^2b\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}-\left(16a^2b-(5a+b)\left(a^2-b^2\right)\right)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2a}-\frac{(a-b)(5a+b)\tan}{2a}\right)}{4a}\right)}{f}$$

$$\downarrow 219$$

$$\frac{1}{6}\left(\frac{3\left(\frac{16a^2\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)-\left(16a^2b-(5a+b)\left(a^2-b^2\right)\right)\int\frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2a}-\frac{(a-b)(5a+b)\tan(e+fx)\sqrt{}}{2a\left(\tan^2(e+f\right.}\right.}{4a}\right.}{f}$$

$$\downarrow 291$$

$$\frac{1}{6}\left(\frac{3\left(\frac{16a^2\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)-\left(16a^2b-(5a+b)\left(a^2-b^2\right)\right)\int\frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{2a}-\frac{(a-b)(5a+b)\tan(e+fx)\sqrt{a+b}}{2a\left(\tan^2(e+fx)+\right.}\right.}{4a}\right.}{f}$$

$$\downarrow 216$$

$$\frac{1}{6}\left(\frac{3\left(\frac{16a^2\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)-\frac{\left(16a^2b-(5a+b)\left(a^2-b^2\right)\right)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}}}{2a}-\frac{(a-b)(5a+b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a\left(\tan^2(e+fx)+1\right)}\right.}{4a}\right.}{f}$$

input
```
Int[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^6,x]
```

output
```
(-1/6*(Tan[e + f*x]^5*Sqrt[a + b + b*Tan[e + f*x]^2])/(1 + Tan[e + f*x]^2)
^3 + (-1/4*((5*a - b)*Tan[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(a*(1
 + Tan[e + f*x]^2)^2) + (3*((-(((16*a^2*b - (5*a + b)*(a^2 - b^2))*ArcTan[
(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/Sqrt[a]) + 16*a^2*
Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(2
*a) - ((a - b)*(5*a + b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*a
*(1 + Tan[e + f*x]^2))))/(4*a))/6)/f
```

3.73.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.73.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1179 vs. $2(214) = 428$.

Time = 12.64 (sec) , antiderivative size = 1180, normalized size of antiderivative = 4.92

method	result	size
default	Expression too large to display	1180

input
```
int(sin(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/48/f/a^2/(-a)^(1/2)*(8*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*a^2*cos(f*x+e)^5*sin(f*x+e)+8*(-a)^(1/2)*sin(f*x+e)*cos(f*x+e)^4*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2-26*(-a)^(1/2)*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*cos(f*x+e)^3*sin(f*x+e)+2*(-a)^(1/2)*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b*cos(f*x+e)^3*sin(f*x+e)-26*(-
a)^(1/2)*sin(f*x+e)*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*a^2+2*(-a)^(1/2)*sin(f*x+e)*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*a*b+33*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*a^2*cos(f*x+e)*sin(f*x+e)-14*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*a*b*cos(f*x+e)*sin(f*x+e)-3*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*b^2*cos(f*x+e)*sin(f*x+e)+33*(-a)^(1/2)*sin(f*x+e)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2-14*(-a)^(1/2)*sin(f*x+e)*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b-3*(-a)^(1/2)*sin(f*x+e)*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2-24*(-a)^(1/2)*b^(1/2)*ln(4*(-((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-
b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-a-b)/(sin(f*x+e)-1))*a
^2-24*(-a)^(1/2)*b^(1/2)*ln(-4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)+a+b)/(sin(f*x+e)+1))*a^2-15*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/...
```

3.73.5 Fricas [A] (verification not implemented)

Time = 2.89 (sec) , antiderivative size = 1715, normalized size of antiderivative = 7.15

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^6(e+fx)\,dx = \text{Too large to display}$$

input
```
integrate(sin(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/384*(96*a^3*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b -
b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*
x + e)^4) + 3*(5*a^3 - 15*a^2*b - 5*a*b^2 - b^3)*sqrt(-a)*log(128*a^4*cos(
f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a
^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32
*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x +
e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*co
s(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 8*(8*a^3*cos(f*x +
 e)^5 - 2*(13*a^3 - a^2*b)*cos(f*x + e)^3 + (33*a^3 - 14*a^2*b - 3*a*b^2)*
cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a
^3*f), 1/384*(192*a^3*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*c
os(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*co
s(f*x + e)^2 + b^2)*sin(f*x + e))) + 3*(5*a^3 - 15*a^2*b - 5*a*b^2 - b^3)*
sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 3
2*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*
b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)
^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3
 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)...
```

3.73.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^6(e+fx)\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**6*(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Timed out
```

3.73.7 Maxima [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^6(e+fx)\,dx = \int \sqrt{b\sec(fx+e)^2+a}\sin(fx+e)^6\,dx$$

input
```
integrate(sin(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*sin(f*x + e)^6, x)
```

3.73.8 Giac [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^6(e+fx)\,dx = \int \sqrt{b\sec(fx+e)^2+a}\sin(fx+e)^6\,dx$$

input
```
integrate(sin(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*sin(f*x + e)^6, x)
```

3.73.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^6(e+fx)\,dx = \int \sin(e+fx)^6\sqrt{a+\frac{b}{\cos(e+fx)^2}}\,dx$$

input
```
int(sin(e + f*x)^6*(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(sin(e + f*x)^6*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.74 $\int \sqrt{a+b\sec^2(e+fx)}\sin^4(e+fx)\,dx$

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3.74.4 Maple [B] (warning: unable to verify) 700
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3.74.1 Optimal result

Integrand size = 25, antiderivative size = 181

$$\begin{aligned}
&\int \sqrt{a+b\sec^2(e+fx)}\sin^4(e+fx)\,dx \\
&= \frac{(3a^2-6ab-b^2)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{3/2}f} + \frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} \\
&\quad - \frac{(3a-b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8af} \\
&\quad - \frac{\cos(e+fx)\sin^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4f}
\end{aligned}$$

output

```
1/8*(3*a^2-6*a*b-b^2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2)
)/a^(3/2)/f+arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2)
/f-1/8*(3*a-b)*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/a/f-1/4*co
s(f*x+e)*sin(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(1/2)/f
```

3.74.2 Mathematica [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^4(e+fx)\,dx = \int \sqrt{a+b\sec^2(e+fx)}\sin^4(e+fx)\,dx$$

input
```
Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^4,x]
```

output
```
Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^4, x]
```

3.74.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4620, 369, 440, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \sin(e+fx)^4\sqrt{a+b\sec(e+fx)^2}dx$$

$$\downarrow 4620$$

$$\frac{\int \frac{\tan^4(e+fx)\sqrt{b\tan^2(e+fx)+a+b}}{(\tan^2(e+fx)+1)^3}d\tan(e+fx)}{f}$$

$$\downarrow 369$$

$$\frac{\frac{1}{4}\int \frac{\tan^2(e+fx)(4b\tan^2(e+fx)+3(a+b))}{(\tan^2(e+fx)+1)^2\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx) - \frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4(\tan^2(e+fx)+1)^2}}{f}$$

$$\downarrow 440$$

$$\frac{\frac{1}{4}\left(\frac{\int \frac{8ab\tan^2(e+fx)+(3a-b)(a+b)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2a} - \frac{(3a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a(\tan^2(e+fx)+1)}\right) - \frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4(\tan^2(e+fx)+1)^2}}{f}$$

$$\downarrow 398$$

$$\frac{\frac{1}{4}\left(\frac{\left(3a^2-6ab-b^2\right)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+8ab\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2a}-\frac{(3a-b)\tan(e+fx)\sqrt{a+b\tan^2(e-}}{2a(\tan^2(e+fx)+1)}\right.}{f}$$

$$\downarrow 224$$

$$\frac{\frac{1}{4}\left(\frac{\left(3a^2-6ab-b^2\right)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+8ab\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{2a}-\frac{(3a-b)\tan(e+fx)\sqrt{a+}}{2a(\tan^2(e+f}\right.}{f}$$

$$\downarrow 219$$

$$\frac{\frac{1}{4}\left(\frac{\left(3a^2-6ab-b^2\right)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+8a\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2a}-\frac{(3a-b)\tan(e+fx)\sqrt{a+b\tan^2(e}}{2a(\tan^2(e+fx)+1)}\right.}{f}$$

$$\downarrow 291$$

$$\frac{\frac{1}{4}\left(\frac{\left(3a^2-6ab-b^2\right)\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}+8a\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2a}-\frac{(3a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)}}{2a(\tan^2(e+fx)+1)}\right.}{f}$$

$$\downarrow 216$$

$$\frac{\frac{1}{4}\left(\frac{\left(3a^2-6ab-b^2\right)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}}+8a\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2a}-\frac{(3a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a(\tan^2(e+fx)+1)}\right)-\frac{\tan^3(e}{f}$$

input
```
Int[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^4,x]
```

3.74. $\int \sqrt{a+b\sec^2(e+fx)}\sin^4(e+fx)\,dx$

output
```
(-1/4*(Tan[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(1 + Tan[e + f*x]^2)
^2 + ((((3*a^2 - 6*a*b - b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b
*Tan[e + f*x]^2]])/Sqrt[a] + 8*a*Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sq
rt[a + b + b*Tan[e + f*x]^2]])/(2*a) - ((3*a - b)*Tan[e + f*x]*Sqrt[a + b
+ b*Tan[e + f*x]^2])/(2*a*(1 + Tan[e + f*x]^2)))/4)/f
```

3.74.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.74.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 790 vs. $2(159) = 318$.

Time = 9.84 (sec) , antiderivative size = 791, normalized size of antiderivative = 4.37

method	result
default	$\left(2\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{-a}\,a\cos(fx+e)^3\sin(fx+e)+2\cos(fx+e)^2\sin(fx+e)\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{-a}\,a-5\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{-a}\,a\cos$

input
```
int(sin(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/f/a/(-a)^(1/2)*(2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1/
2)*a*cos(f*x+e)^3*sin(f*x+e)+2*cos(f*x+e)^2*sin(f*x+e)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*a-5*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*(-a)^(1/2)*a*cos(f*x+e)*sin(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*(-a)^(1/2)*b*cos(f*x+e)*sin(f*x+e)+4*b^(1/2)*ln(4*(((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*(-a)^(1
/2)*a+4*b^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)
*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)
*a+a+b)/(sin(f*x+e)-1))*(-a)^(1/2)*a-5*sin(f*x+e)*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*(-a)^(1/2)*a+sin(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*(-a)^(1/2)*b+3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)-4*sin(f*x+e)*a)*a^2-6*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)-4*sin(f*x+e)*a)*a*b-ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)-4*sin(f*x+e)*a)*b^2)*(a+b*sec(f*x+e)^2)^(1/2)*cos(f*x+e)/(1+cos
(f*x+e))/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
```

3.74.5 Fricas [A] (verification not implemented)

Time = 1.09 (sec) , antiderivative size = 1565, normalized size of antiderivative = 8.65

$$\int \sqrt{a + b\sec^2(e+fx)}\sin^4(e+fx)\,dx = \text{Too large to display}$$

input
```
integrate(sin(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/64*(16*a^2*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b
^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x
 + e)^4) + (3*a^2 - 6*a*b - b^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256
*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x
+ e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b
+ 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 -
 a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (
a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)
^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(2*a^2*cos(f*x + e)^3 - (5*a^2 -
 a*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x +
e))/(a^2*f), 1/64*(32*a^2*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2
*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*
b*cos(f*x + e)^2 + b^2)*sin(f*x + e))) + (3*a^2 - 6*a*b - b^2)*sqrt(-a)*lo
g(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 -
14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*
b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*
a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b
 + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))
*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8...
```

3.74.6 Sympy [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^4(e+fx)\,dx = \int \sqrt{a+b\sec^2(e+fx)}\sin^4(e+fx)\,dx$$

input
```
integrate(sin(f*x+e)**4*(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*sin(e + f*x)**4, x)
```

3.74.7 Maxima [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^4(e+fx)\,dx = \int \sqrt{b\sec(fx+e)^2+a}\sin(fx+e)^4\,dx$$

input
```
integrate(sin(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*sin(f*x + e)^4, x)
```

3.74.8 Giac [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^4(e+fx)\,dx = \int \sqrt{b\sec(fx+e)^2+a}\sin(fx+e)^4\,dx$$

input
```
integrate(sin(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*sin(f*x + e)^4, x)
```

3.74.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^4(e+fx)\,dx = \int \sin(e+fx)^4\sqrt{a+\frac{b}{\cos(e+fx)^2}}\,dx$$

input
```
int(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.75 $\int \sqrt{a+b\sec^2(e+fx)}\sin^2(e+fx)\,dx$

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3.75.1 Optimal result

Integrand size = 25, antiderivative size = 123

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^2(e+fx)\,dx = \frac{(a-b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2\sqrt{a}f} + \frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f}$$

output

```
1/2*(a-b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/a^(1/2)+
arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f-1/2*cos(f
*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/f
```

3.75.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.29 (sec) , antiderivative size = 432, normalized size of antiderivative = 3.51

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^2(e+fx)\,dx$$

$$= \frac{e^{-i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos(e+fx)\left(i\left(-1+e^{2i(e+fx)}\right)+\frac{2e^{2i(e+fx)}\left(2afx-2bfx-i(a-b)\log\left(a+2b+ae^{2i(e+fx)}+2\sqrt{b}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\right)+i(a-b)\log\left(a+ae^{2i(e+fx)}+2be^{2i(e+fx)}+\sqrt{a}\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\right)-4\sqrt{a}\sqrt{b}\log\left(\frac{\left(-\sqrt{b}\left(-1+e^{2i(e+fx)}\right)+i\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\right)f}{2be^{2i(e+fx)}+b}\right)}{\sqrt{a}\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}}\right)\sqrt{a+b\sec^2(e+fx)}}{4\sqrt{2}e^{i(e+fx)}f\sqrt{a+2b+a\cos(2e+2fx)}}$$

input
```
Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^2,x]
```

output
```
(Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f*x))]*Cos[e + f
*x]*(I*(-1 + E^((2*I)*(e + f*x))) + (2*E^((2*I)*(e + f*x))*(2*a*f*x - 2*b*
f*x - I*(a - b)*Log[a + 2*b + a*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^(
(2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] + I*(a - b)*Log[a + a*E
^((2*I)*(e + f*x)) + 2*b*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(
e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] - 4*Sqrt[a]*Sqrt[b]*Log[((-(Sq
rt[b]*(-1 + E^((2*I)*(e + f*x)))) + I*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1
+ E^((2*I)*(e + f*x)))^2])*f)/(2*b*(1 + E^((2*I)*(e + f*x))))]))/(Sqrt[a]*
Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]))*Sqrt[a + b
*Sec[e + f*x]^2])/(4*Sqrt[2]*E^(I*(e + f*x))*f*Sqrt[a + 2*b + a*Cos[2*e +
2*f*x]])
```

3.75.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4620, 369, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \sin(e+fx)^2\sqrt{a+b\sec(e+fx)^2}\,dx$$

$$\downarrow 4620$$

$$\frac{\int \frac{\tan^2(e+fx)\sqrt{b\tan^2(e+fx)+a+b}}{(\tan^2(e+fx)+1)^2}d\tan(e+fx)}{f}$$

$$\downarrow 369$$

$$\frac{\frac{1}{2}\int \frac{2b\tan^2(e+fx)+a+b}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx) - \frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2(\tan^2(e+fx)+1)}}{f}$$

$$\downarrow 398$$

$$\frac{\frac{1}{2}\left(2b\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx) + (a-b)\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)\right) - \frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2(\tan^2(e+fx)+1)}}{f}$$

$$\downarrow 224$$

$$\frac{\frac{1}{2}\left((a-b)\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx) + 2b\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}\right) - \frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2(\tan^2(e+fx)+1)}}{f}$$

$$\downarrow 219$$

$$\frac{\frac{1}{2}\left((a-b)\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx) + 2\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)\right) - \frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2(\tan^2(e+fx)+1)}}{f}$$

$$\downarrow 291$$

$$\frac{\frac{1}{2}\left((a-b)\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}} + 2\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)\right) - \frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2(\tan^2(e+fx)+1)}}{f}$$

$$\downarrow 216$$

$$\frac{\frac{1}{2}\left(\frac{(a-b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}} + 2\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)\right) - \frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2(\tan^2(e+fx)+1)}}{f}$$

input
```
Int[Sqrt[a + b*Sec[e + f*x]^2]*Sin[e + f*x]^2,x]
```

3.75. $\int \sqrt{a+b\sec^2(e+fx)}\sin^2(e+fx)\,dx$

output
```
((((a - b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/
Sqrt[a] + 2*Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e +
f*x]^2]])/2 - (Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*(1 + Tan[e
+ f*x]^2)))/f
```

3.75.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.75.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 515 vs. $2(105) = 210$.

Time = 7.38 (sec) , antiderivative size = 516, normalized size of antiderivative = 4.20

method	result
default	$-\left(\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{-a}\cos(fx+e)\sin(fx+e)+\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sin(fx+e)-\sqrt{b}\ln\left(\frac{4\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{b}\cos(fx+e)-4\sin$ $\sin(fx$

input
```
int(sin(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/f/(-a)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*
cos(f*x+e)*sin(f*x+e)+(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*sin(f*x+e)-b^(1/2)*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^
(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f
*x+e)*a-a-b)/(sin(f*x+e)+1))*(-a)^(1/2)-b^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*(-a)^(1/2)-ln(4*(-
a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/
2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a+ln(4*(-a)
^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*b)*(a+b*sec(f
*x+e)^2)^(1/2)*cos(f*x+e)/(1+cos(f*x+e))/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)
```

3.75.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(105) = 210$.

Time = 0.61 (sec) , antiderivative size = 1417, normalized size of antiderivative = 11.52

$$\int \sqrt{a + b\sec^2(e+fx)}\sin^2(e+fx)\,dx = \text{Too large to display}$$

input
```
integrate(sin(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/16*(8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f
*x + e) - sqrt(-a)*(a - b)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*
cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 -
28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a
*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x
 + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b +
 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)*sin(f*x + e)) - 4*a*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)
^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*
x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) +
 8*b^2)/cos(f*x + e)^4))/(a*f), -1/16*(8*a*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)*cos(f*x + e)*sin(f*x + e) - 8*a*sqrt(-b)*arctan(-1/2*((a - b)
*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e))) - sqrt(-a)*(a -
b)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a
^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 -
28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8
*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*
a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x
+ e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)...
```

3.75.6 Sympy [F]

$$\int \sqrt{a + b\sec^2(e+fx)}\sin^2(e+fx)\,dx = \int \sqrt{a + b\sec^2(e+fx)}\sin^2(e+fx)\,dx$$

input
```
integrate(sin(f*x+e)**2*(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*sin(e + f*x)**2, x)
```

3.75.7 Maxima [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^2(e+fx)\,dx = \int \sqrt{b\sec(fx+e)^2+a}\sin(fx+e)^2\,dx$$

input
```
integrate(sin(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*sin(f*x + e)^2, x)
```

3.75.8 Giac [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^2(e+fx)\,dx = \int \sqrt{b\sec(fx+e)^2+a}\sin(fx+e)^2\,dx$$

input
```
integrate(sin(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*sin(f*x + e)^2, x)
```

3.75.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec^2(e+fx)}\sin^2(e+fx)\,dx = \int \sin(e+fx)^2\sqrt{a+\frac{b}{\cos(e+fx)^2}}\,dx$$

input
```
int(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.76 $\int \sqrt{a+b\sec^2(e+fx)}\,dx$

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3.76.1 Optimal result

Integrand size = 16, antiderivative size = 79

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}$$

output
```
arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*a^(1/2)/f+arctanh(b^
(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f
```

3.76.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.89 (sec) , antiderivative size = 284, normalized size of antiderivative = 3.59

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx = -\frac{i\left(1+e^{2i(e+fx)}\right)\left(2\sqrt{b}\arctan\left(\frac{\sqrt{b}\left(-1+e^{2i(e+fx)}\right)}{\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}}\right)+\sqrt{a}\operatorname{arctanh}\left(\frac{a+2b+ae^{2i(e+fx)}}{\sqrt{a}\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}}\right)-\right.}{2\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}f}$$

input
```
Integrate[Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
((-1/2*I)*(1 + E^((2*I)*(e + f*x)))*(2*Sqrt[b]*ArcTan[(Sqrt[b]*(-1 + E^((2
*I)*(e + f*x))))/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x))
)^2]] + Sqrt[a]*ArcTanh[(a + 2*b + a*E^((2*I)*(e + f*x)))/(Sqrt[a]*Sqrt[4*
b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])] - Sqrt[a]*ArcTanh
[(a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*I)*(e + f*x)))/(Sqrt[a]*Sqrt[4*b*E
^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])])*Sqrt[a + b*Sec[e +
f*x]^2])/(Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)
```

3.76.3 Rubi [A] (verified)

Time $= 0.23$ (sec) , antiderivative size $= 77$, normalized size of antiderivative $= 0.97$, number of steps used $= 8$, number of rules used $= 7$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used $= \{3042, 4616, 301, 224, 219, 291, 216\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a + b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \sqrt{a + b\sec(e+fx)^2}dx$$

$$\downarrow 4616$$

$$\frac{\int \frac{\sqrt{b\tan^2(e+fx)+a+b}}{\tan^2(e+fx)+1} d\tan(e+fx)}{f}$$

$$\downarrow 301$$

$$\frac{b\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}} d\tan(e+fx) + a\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}} d\tan(e+fx)}{f}$$

$$\downarrow 224$$

$$\frac{a\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}} d\tan(e+fx) + b\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}} d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{f}$$

$$\downarrow 219$$

$$\frac{a\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}} d\tan(e+fx) + \sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}$$

$$\downarrow 291$$

$$\frac{a\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}+\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}$$

$$\downarrow 216$$

$$\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)+\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}$$

input
```
Int[Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]] + S
qrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
```

3.76.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

3.76.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(67) = 134$.

Time = 6.39 (sec) , antiderivative size = 353, normalized size of antiderivative = 4.47

method	result
default	$\frac{\sqrt{a+b\sec(fx+e)^2}\left(\sqrt{b}\ln\left(\frac{-4\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{b}\cos(fx+e)-4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}+4\sin(fx+e)a-4a-4b}{\sin(fx+e)-1}\right)\sqrt{-a}+\sqrt{b}\ln\left(-\frac{4\left(-\sqrt{\frac{b+}{(1}\right.\right.\right.$

input
```
int((a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f/(-a)^(1/2)*(a+b*sec(f*x+e)^2)^(1/2)*(b^(1/2)*ln(4*(-((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-a-b)/(sin(f*x+e)-1))*(-a)^(1/2)+b^(1
/2)*ln(-4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)
+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+a+b)/(si
n(f*x+e)+1))*(-a)^(1/2)+2*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)-4*sin(f*x+e)*a)*a)*cos(f*x+e)/(1+cos(f*x+e))/((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)
```

3.76.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(67) = 134$.

Time $= 0.48$ (sec) , antiderivative size $= 1227$, normalized size of antiderivative $= 15.53$

$$\int \sqrt{a + b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)
^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 7
0*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*
x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*
(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b
^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin
(f*x + e)) + 2*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b -
b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*
x + e)^4))/f, 1/8*(4*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*co
s(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos
(f*x + e)^2 + b^2)*sin(f*x + e))) + sqrt(-a)*log(128*a^4*cos(f*x + e)^8 -
256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f
*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3
*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^
3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3
- (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)))/f, -1/4*(sqrt(a)*arctan(1/4*(8*a
^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos
(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*...
```

3.76.6 Sympy [F]

$$\int \sqrt{a + b\sec^2(e+fx)}\,dx = \int \sqrt{a + b\sec^2(e+fx)}\,dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2), x)
```

3.76.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.76 (sec) , antiderivative size = 3227, normalized size of antiderivative = 40.85

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*(2*sqrt(a)*b^(3/2)*arctan2(a*sin(2*f*x + 2*e) + (a^2*cos(4*f*x + 4*e)
^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 +
 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^
2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*c
os(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*sin(1/2*
arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4
*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)), a*cos(2*f*x + 2*e) + (a^2*cos(4*
f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x
 + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*
a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x
 + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a
)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos
(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) + a + 2*b) + a^(3/2)*sq
rt(b)*arctan2(2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2
 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin
(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2
 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos
(2*f*x + 2*e))^(1/4)*sqrt(a)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2
*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) +
a)), 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*...
```

3.76.8 Giac [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec\left(fx+e\right)^2+a}\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a), x)
```

3.76.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a + b \sec^2(e + fx)}\, dx = \int \sqrt{a + \frac{b}{\cos(e + f x)^2}}\, dx$$

input
```
int((a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int((a + b/cos(e + f*x)^2)^(1/2), x)
```

3.77 $\int \csc^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

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3.77.1 Optimal result

Integrand size = 25, antiderivative size = 68

$$\int \csc^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f}$$

output
```
arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f-cot(f*x+e
)*(a+b+b*tan(f*x+e)^2)^(1/2)/f
```

3.77.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.90

$$\int \csc^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = -\frac{\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{b\sin^2(e+fx)}{a+b-a\sin^2(e+fx)}\right)\sqrt{a+b\sec^2(e+fx)}}{f}$$

input
```
Integrate[Csc[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
-((Cot[e + f*x]*Hypergeometric2F1[-1/2, 1, 1/2, (b*Sin[e + f*x]^2)/(a + b
- a*Sin[e + f*x]^2)]*Sqrt[a + b*Sec[e + f*x]^2])/f)
```

3.77.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4620, 247, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sqrt{a+b\sec(e+fx)^2}}{\sin(e+fx)^2}dx$$

$$\downarrow 4620$$

$$\frac{\int \cot^2(e+fx)\sqrt{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{f}$$

$$\downarrow 247$$

$$\frac{b\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx) - \cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

$$\downarrow 224$$

$$\frac{b\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}} - \cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

$$\downarrow 219$$

$$\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right) - \cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

input
```
Int[Csc[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]] -
Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/f
```

3.77.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.77.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 325 vs. $2(60) = 120$.

Time = 5.34 (sec) , antiderivative size = 326, normalized size of antiderivative = 4.79

method	result
default	$-\left(\left(-\sqrt{b}\ln\left(\frac{4\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{b}\cos(fx+e)-4\sin(fx+e)a+4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4a-4b}{\sin(fx+e)+1}\right)\right)\sin(fx+e)-\sqrt{b}\ln\left(-\frac{4\left(\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\right.\right.\right.$ … $2f(1+$

input
```
int(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/f*(-b^(1/2)*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)
*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)
*a-a-b)/(sin(f*x+e)+1))*sin(f*x+e)-b^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*sin(f*x+e)+2*cos(f*x+e)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+2*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2))*(a+b*sec(f*x+e)^2)^(1/2)/(1+cos(f*x+e))/((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*cot(f*x+e)
```

3.77.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(60) = 120$.

Time $= 0.31$ (sec) , antiderivative size $= 306$, normalized size of antiderivative $= 4.50$

$$\int \csc^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$= \left[\frac{\sqrt{b}\log\left(\frac{\left(a^2-6ab+b^2\right)\cos(fx+e)^4+8\left(ab-b^2\right)\cos(fx+e)^2+4\left((a-b)\cos(fx+e)^3+2b\cos(fx+e)\right)\sqrt{b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+e)+8b^2}{\cos(fx+e)^4}\right)}{4f\sin(fx+e)}\right.$$

input
```
integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(
f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a
*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)
*sin(f*x + e) - 4*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)
)/(f*sin(f*x + e)), 1/2*(sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*
b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b
*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*sin(f*x + e) - 2*sqrt((a*cos(f*x + e
)^2 + b)/cos(f*x + e)^2)*cos(f*x + e))/(f*sin(f*x + e))]
```

3.77.6 Sympy [F]

$$\int \csc^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{a+b\sec^2(e+fx)}\csc^2(e+fx)\,dx$$

input
```
integrate(csc(f*x+e)**2*(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*csc(e + f*x)**2, x)
```

3.77.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.74

$$\int \csc^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{b}\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right) - \frac{\sqrt{b\tan(fx+e)^2+a+b}}{\tan(fx+e)}}{f}$$

input
```
integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
(sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b)) - sqrt(b*tan(f*x + e)^2 +
 a + b)/tan(f*x + e))/f
```

3.77.8 Giac [F]

$$\int \csc^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\csc(fx+e)^2\,dx$$

input
```
integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*csc(f*x + e)^2, x)
```

3.77.9 Mupad [F(-1)]

Timed out.

$$\int \csc^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(e+fx)^2}}}{\sin(e+fx)^2}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x)^2,x)
```

output
```
int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x)^2, x)
```

3.78 $\int \csc^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

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3.78.1 Optimal result

Integrand size = 25, antiderivative size = 105

$$\int \csc^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} - \frac{\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{3(a+b)f}$$

output
```
arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f-cot(f*x+e
)*(a+b+b*tan(f*x+e)^2)^(1/2)/f-1/3*cot(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(3/2)
/(a+b)/f
```

3.78.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.76 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.71

$$\int \csc^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \sqrt{2}\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)\left(-\frac{4b\,\text{Hypergeometric2F1}\left(2,2,\frac{3}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)}{3f\sqrt{a+2b+a\c}\right.$$

input
```
Integrate[Csc[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
-1/3*(Sqrt[2]*Cot[e + f*x]*Csc[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2]*(1 -
(a*Sin[e + f*x]^2)/(a + b))*((4*b*Hypergeometric2F1[2, 2, 3/2, -((b*Tan[e
+ f*x]^2)/(a + b))]*Sec[e + f*x]^2*Sqrt[(a + b*Sec[e + f*x]^2)/(a + b)]*(a
 + b - a*Sin[e + f*x]^2)^2*Tan[e + f*x]^2)/(a + b)^2 + (a + b + 2*a*Sin[e
+ f*x]^2)*(Sqrt[(a + b*Sec[e + f*x]^2)/(a + b)] + ArcSin[Sqrt[-((b*Tan[e +
 f*x]^2)/(a + b))]]*Sqrt[-((b*Tan[e + f*x]^2)/(a + b))]))/(f*Sqrt[a + 2*b
 + a*Cos[2*e + 2*f*x]]*Sqrt[(a + b*Sec[e + f*x]^2)/(a + b)]*Sqrt[a + b - a
*Sin[e + f*x]^2])
```

3.78.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 4620, 358, 247, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sqrt{a+b\sec(e+fx)^2}}{\sin(e+fx)^4}dx$$

$$\downarrow 4620$$

$$\frac{\int \cot^4(e+fx)\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{f}$$

$$\downarrow 358$$

$$\frac{\int \cot^2(e+fx)\sqrt{b\tan^2(e+fx)+a+b}d\tan(e+fx)-\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{3(a+b)}}{f}$$

$$\downarrow 247$$

$$\frac{b\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-\frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{3(a+b)}-\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

$$\downarrow 224$$

$$\frac{b\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}} - \frac{\cot^3(e+fx)(a+b\tan^2(e+fx)+b)^{3/2}}{3(a+b)} - \cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

$$\downarrow 219$$

$$\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right) - \frac{\cot^3(e+fx)(a+b\tan^2(e+fx)+b)^{3/2}}{3(a+b)} - \cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

input
```
Int[Csc[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]] -
Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2] - (Cot[e + f*x]^3*(a + b + b*T
an[e + f*x]^2)^(3/2))/(3*(a + b)))/f
```

3.78.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 358
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_
Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] + S
imp[d/e^2   Int[(e*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b*c - a*d, 0] && EqQ[Simplify[m + 2*p + 3], 0] && NeQ[m,
 -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.78.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1036 vs. $2(93) = 186$.

Time = 7.32 (sec) , antiderivative size = 1037, normalized size of antiderivative = 9.88

method	result	size
default	Expression too large to display	1037

input
```
int(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-1/6/f/(a+b)*(3*sin(f*x+e)*b^(3/2)*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*cos(f*x+e)+3*sin(f*x+e)*b^(3/2)*
ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1
/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x
+e)-1))*cos(f*x+e)-3*sin(f*x+e)*b^(3/2)*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))-3*sin(f*x+e)*b^(3/2)*ln(-4*
(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1)
)+3*sin(f*x+e)*b^(1/2)*a*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-si
n(f*x+e)*a-a-b)/(sin(f*x+e)+1))*cos(f*x+e)+3*sin(f*x+e)*b^(1/2)*a*ln(-4*((
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*
cos(f*x+e)-3*sin(f*x+e)*b^(1/2)*a*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))-3*sin(f*x+e)*b^(1/2)*a*ln(-4*(((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))...
```

3.78.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(93) = 186$.

Time $= 0.47$ (sec) , antiderivative size $= 436$, normalized size of antiderivative $= 4.15$

$$\int \csc^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$= \left[\frac{3\left((a+b)\cos(fx+e)^2 - a - b\right)\sqrt{b}\log\left(\frac{(a^2-6ab+b^2)\cos(fx+e)^4+8(ab-b^2)\cos(fx+e)^2+4\left((a-b)\cos(fx+e)^3+2b\cos(fx+e)\right.\right.}{\cos(fx+e)^4}\right.}{12\left((a+b)f\cos(fx\right.} \right.$$

input

```
integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(3*((a + b)*cos(f*x + e)^2 - a - b)*sqrt(b)*log(((a^2 - 6*a*b + b^2)
*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3
 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*s
in(f*x + e) + 8*b^2)/cos(f*x + e)^4)*sin(f*x + e) - 4*((2*a + 3*b)*cos(f*x
 + e)^3 - (3*a + 4*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2))/(((a + b)*f*cos(f*x + e)^2 - (a + b)*f)*sin(f*x + e)), 1/6*(3*((a +
 b)*cos(f*x + e)^2 - a - b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 +
 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((
a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*sin(f*x + e) - 2*((2*a + 3*b)*cos
(f*x + e)^3 - (3*a + 4*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2))/(((a + b)*f*cos(f*x + e)^2 - (a + b)*f)*sin(f*x + e))]
```

3.78.6 Sympy [F]

$$\int \csc^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{a+b\sec^2(e+fx)}\csc^4(e+fx)\,dx$$

input
```
integrate(csc(f*x+e)**4*(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*csc(e + f*x)**4, x)
```

3.78.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.78

$$\int \csc^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{3\sqrt{b}\,\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right) - \frac{3\sqrt{b\tan(fx+e)^2+a+b}}{\tan(fx+e)} - \frac{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}}{(a+b)\tan(fx+e)^3}}{3f}$$

input
```
integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*(3*sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b)) - 3*sqrt(b*tan(f*x
+ e)^2 + a + b)/tan(f*x + e) - (b*tan(f*x + e)^2 + a + b)^(3/2)/((a + b)*t
an(f*x + e)^3))/f
```

3.78.8 Giac [F]

$$\int \csc^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\csc(fx+e)^4\,dx$$

input
```
integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*csc(f*x + e)^4, x)
```

3.78.9 Mupad [F(-1)]

Timed out.

$$\int \csc^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(e+fx)^2}}}{\sin(e+fx)^4}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x)^4,x)
```

output
```
int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x)^4, x)
```

3.79 $\int \csc^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

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3.79.1 Optimal result

Integrand size = 25, antiderivative size = 149

$$\begin{aligned}
&\int \csc^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx \\
&= \frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} \\
&\quad - \frac{2(5a+4b)\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{15(a+b)^2 f} \\
&\quad - \frac{\cot^5(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{5(a+b)f}
\end{aligned}$$

output

```
arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f-cot(f*x+e
)*(a+b+b*tan(f*x+e)^2)^(1/2)/f-2/15*(5*a+4*b)*cot(f*x+e)^3*(a+b+b*tan(f*x+
e)^2)^(3/2)/(a+b)^2/f-1/5*cot(f*x+e)^5*(a+b+b*tan(f*x+e)^2)^(3/2)/(a+b)/f
```

3.79.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 8.73 (sec) , antiderivative size = 941, normalized size of antiderivative = 6.32

$$\int \csc^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$= \frac{\sqrt{2}\csc^5(e+fx)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)\left(-3(a+b)\cos^2(e+fx)-4a\cos^2(e+\right.}{\left.\right.}$$

input
```
Integrate[Csc[e + f*x]^6*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Sqrt[2]*Csc[e + f*x]^5*Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*(1 - (a*Si
n[e + f*x]^2)/(a + b))*(-3*(a + b)*Cos[e + f*x]^2 - 4*a*Cos[e + f*x]^2*Sin
[e + f*x]^2 - 16*b*Hypergeometric2F1[2, 2, 3/2, -((b*Tan[e + f*x]^2)/(a +
b))]*Sin[e + f*x]^2 + 8*b*HypergeometricPFQ[{2, 2, 2}, {1, 3/2}, -((b*Tan[
e + f*x]^2)/(a + b))]*Sin[e + f*x]^2 - (8*a^2*Cos[e + f*x]^2*Sin[e + f*x]^
4)/(a + b) - (8*a*b*Hypergeometric2F1[2, 2, 3/2, -((b*Tan[e + f*x]^2)/(a +
 b))]*Sin[e + f*x]^4)/(a + b) - (16*a*b*HypergeometricPFQ[{2, 2, 2}, {1, 3
/2}, -((b*Tan[e + f*x]^2)/(a + b))]*Sin[e + f*x]^4)/(a + b) + (24*a^2*b*Hy
pergeometric2F1[2, 2, 3/2, -((b*Tan[e + f*x]^2)/(a + b))]*Sin[e + f*x]^6)/
(a + b)^2 + (8*a^2*b*HypergeometricPFQ[{2, 2, 2}, {1, 3/2}, -((b*Tan[e + f
*x]^2)/(a + b))]*Sin[e + f*x]^6)/(a + b)^2 - (16*b^2*Hypergeometric2F1[2,
2, 3/2, -((b*Tan[e + f*x]^2)/(a + b))]*Sin[e + f*x]^2*Tan[e + f*x]^2)/(a +
 b) + (8*b^2*HypergeometricPFQ[{2, 2, 2}, {1, 3/2}, -((b*Tan[e + f*x]^2)/(
a + b))]*Sin[e + f*x]^2*Tan[e + f*x]^2)/(a + b) - (8*a*b^2*Hypergeometric2
F1[2, 2, 3/2, -((b*Tan[e + f*x]^2)/(a + b))]*Sin[e + f*x]^4*Tan[e + f*x]^2
)/(a + b)^2 - (16*a*b^2*HypergeometricPFQ[{2, 2, 2}, {1, 3/2}, -((b*Tan[e
+ f*x]^2)/(a + b))]*Sin[e + f*x]^4*Tan[e + f*x]^2)/(a + b)^2 + (24*a^2*b^2
*Hypergeometric2F1[2, 2, 3/2, -((b*Tan[e + f*x]^2)/(a + b))]*Sin[e + f*x]^
6*Tan[e + f*x]^2)/(a + b)^3 + (8*a^2*b^2*HypergeometricPFQ[{2, 2, 2}, {1,
3/2}, -((b*Tan[e + f*x]^2)/(a + b))]*Sin[e + f*x]^6*Tan[e + f*x]^2)/(a ...
```

3.79.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4620, 365, 358, 247, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sqrt{a+b\sec(e+fx)^2}}{\sin(e+fx)^6}dx$$

$$\downarrow 4620$$

$$\frac{\int \cot^6(e+fx)\left(\tan^2(e+fx)+1\right)^2\sqrt{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{f}$$

$$\downarrow 365$$

$$\frac{\frac{\int \cot^4(e+fx)\sqrt{b\tan^2(e+fx)+a+b}(5(a+b)\tan^2(e+fx)+2(5a+4b))d\tan(e+fx)}{5(a+b)} - \frac{\cot^5(e+fx)(a+b\tan^2(e+fx)+b)^{3/2}}{5(a+b)}}{f}$$

$$\downarrow 358$$

$$\frac{\frac{5(a+b)\int \cot^2(e+fx)\sqrt{b\tan^2(e+fx)+a+b}d\tan(e+fx) - \frac{2(5a+4b)\cot^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{3(a+b)}}{5(a+b)} - \frac{\cot^5(e+fx)(a+b\tan^2(e+fx)+b)^{3/2}}{5(a+b)}}{f}$$

$$\downarrow 247$$

$$\frac{\frac{5(a+b)\left(b\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx) - \cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right) - \frac{2(5a+4b)\cot^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{3(a+b)}}{5(a+b)} - \frac{\cot^5(e+fx)(a+b}{5(}}{f}$$

$$\downarrow 224$$

$$\frac{\frac{5(a+b)\left(b\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}} - \cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right) - \frac{2(5a+4b)\cot^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{3(a+b)}}{5(a+b)} - \frac{\cot^5(e+}{}}{f}$$

$$\downarrow 219$$

$$\frac{\frac{5(a+b)\left(\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)-\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)-\frac{2(5a+4b)\cot^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{3(a+b)}}{5(a+b)} - \frac{\cot^5(e+fx)(a+}{5}}{f}$$

input
```
Int[Csc[e + f*x]^6*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(-1/5*(Cot[e + f*x]^5*(a + b + b*Tan[e + f*x]^2)^(3/2))/(a + b) + ((-2*(5*
a + 4*b)*Cot[e + f*x]^3*(a + b + b*Tan[e + f*x]^2)^(3/2))/(3*(a + b)) + 5*
(a + b)*(Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x
]^2]] - Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2]))/(5*(a + b)))/f
```

3.79.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 358
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_
Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] + S
imp[d/e^2   Int[(e*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b*c - a*d, 0] && EqQ[Simplify[m + 2*p + 3], 0] && NeQ[m,
 -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.79.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1679 vs. $2(133) = 266$.

Time = 7.18 (sec) , antiderivative size = 1680, normalized size of antiderivative = 11.28

method	result	size
default	Expression too large to display	1680

input
```
int(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-1/30/f/(a+b)^2*(15*sin(f*x+e)^3*b^(5/2)*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*cos(f*x+e)+15*sin(f*x+e)^3
*b^(5/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*
x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)
/(sin(f*x+e)-1))*cos(f*x+e)+30*sin(f*x+e)^3*b^(3/2)*a*ln(4*(((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*cos(f*x+e)+30
*sin(f*x+e)^3*b^(3/2)*a*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-si
n(f*x+e)*a+a+b)/(sin(f*x+e)-1))*cos(f*x+e)-15*sin(f*x+e)^3*b^(5/2)*ln(4*((
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))-
15*sin(f*x+e)^3*b^(5/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-si
n(f*x+e)*a+a+b)/(sin(f*x+e)-1))+15*sin(f*x+e)^3*b^(1/2)*a^2*ln(4*(((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*cos(f*x
+e)+15*sin(f*x+e)^3*b^(1/2)*a^2*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^...
```

3.79.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 312 vs. $2(133) = 266$.

Time = 1.39 (sec) , antiderivative size = 656, normalized size of antiderivative = 4.40

$$\int \csc^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$= \left[\frac{15\left((a^2+2ab+b^2)\cos(fx+e)^4 - 2\left(a^2+2ab+b^2\right)\cos(fx+e)^2 + a^2+2ab+b^2\right)\sqrt{b}\log\left(\frac{(a^2-6ab+b^2}{}\right.\right.$$

input

```
integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/60*(15*((a^2 + 2*a*b + b^2)*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(
f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x +
 e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos
(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e
) + 8*b^2)/cos(f*x + e)^4)*sin(f*x + e) - 4*((8*a^2 + 25*a*b + 15*b^2)*cos
(f*x + e)^5 - (20*a^2 + 59*a*b + 35*b^2)*cos(f*x + e)^3 + (15*a^2 + 40*a*b
 + 23*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a
^2 + 2*a*b + b^2)*f*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*f*cos(f*x + e)^
2 + (a^2 + 2*a*b + b^2)*f)*sin(f*x + e)), 1/30*(15*((a^2 + 2*a*b + b^2)*co
s(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*s
qrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin
(f*x + e)))*sin(f*x + e) - 2*((8*a^2 + 25*a*b + 15*b^2)*cos(f*x + e)^5 - (
20*a^2 + 59*a*b + 35*b^2)*cos(f*x + e)^3 + (15*a^2 + 40*a*b + 23*b^2)*cos(
f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^2 + 2*a*b + b^
2)*f*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2 + (a^2 + 2*a*
b + b^2)*f)*sin(f*x + e))]
```

3.79.6 Sympy [F(-1)]

Timed out.

$$\int \csc^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**6*(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Timed out
```

3.79.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.96

$$\int \csc^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$= \frac{15\sqrt{b}\,\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right) - \frac{15\sqrt{b\tan(fx+e)^2+a+b}}{\tan(fx+e)} - \frac{10\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}}{(a+b)\tan(fx+e)^3} + \frac{2\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}b}{(a+b)^2\tan(fx+e)^3} - \frac{3\left(b\tan(fx+e}{(a+b)\tan(}}{15\,f}$$

input
```
integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/15*(15*sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b)) - 15*sqrt(b*tan(f
*x + e)^2 + a + b)/tan(f*x + e) - 10*(b*tan(f*x + e)^2 + a + b)^(3/2)/((a
+ b)*tan(f*x + e)^3) + 2*(b*tan(f*x + e)^2 + a + b)^(3/2)*b/((a + b)^2*tan
(f*x + e)^3) - 3*(b*tan(f*x + e)^2 + a + b)^(3/2)/((a + b)*tan(f*x + e)^5)
)/f
```

3.79.8 Giac [F]

$$\int \csc^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec\left(fx+e\right)^2+a}\csc\left(fx+e\right)^6\,dx$$

input
```
integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*csc(f*x + e)^6, x)
```

3.79.9 Mupad [F(-1)]

Timed out.

$$\int \csc^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(e+fx)^2}}}{\sin\left(e+fx\right)^6}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x)^6,x)
```

output
```
int((a + b/cos(e + f*x)^2)^(1/2)/sin(e + f*x)^6, x)
```

3.80 $\int (a+b\sec^2(e+fx))^{3/2}\sin^5(e+fx)\,dx$

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3.80.1 Optimal result

Integrand size = 25, antiderivative size = 196

$$\int (a+b\sec^2(e+fx))^{3/2}\sin^5(e+fx)\,dx = \frac{(3a-4b)\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f} + \frac{(3a-4b)b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2af} - \frac{(3a-4b)\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3af} + \frac{2\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{3af} - \frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{5af}$$

output

```
-1/3*(3*a-4*b)*cos(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2)/a/f+2/3*cos(f*x+e)^3*(a
+b*sec(f*x+e)^2)^(5/2)/a/f-1/5*cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(5/2)/a/f+1
/2*(3*a-4*b)*arctanh(sec(f*x+e)*b^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))*b^(1/2)/
f+1/2*(3*a-4*b)*b*sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/a/f
```

3.80.2 Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.96

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^5(e+fx)\,dx = \frac{\sqrt{2}\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\left(-\frac{6b(a+b-a\sin^2(e+fx))^{5/2}}{a}+15\sec^2(e+fx)\left(a+b-a\sin^2\right.\right.}{15bf(a}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^5,x]
```

output
```
(Sqrt[2]*Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2)*((-6*b*(a + b - a*Sin
[e + f*x]^2)^(5/2))/a + 15*Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)^(5/2)
 - 5*(3*a - 4*b)*(-3*b^(3/2)*ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[b
]] + Sqrt[a + b - a*Sin[e + f*x]^2]*(a + 4*b - a*Sin[e + f*x]^2))))/(15*b*
f*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2))
```

3.80.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.89, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4622, 365, 27, 359, 247, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx$$

$$\downarrow 3042$$

$$\int \sin(e+fx)^5\left(a+b\sec(e+fx)^2\right)^{3/2}dx$$

$$\downarrow 4622$$

$$\frac{\int \cos^6(e+fx)\left(1-\sec^2(e+fx)\right)^2\left(b\sec^2(e+fx)+a\right)^{3/2}d\sec(e+fx)}{f}$$

$$\downarrow 365$$

$$\frac{\int -5a\cos^4(e+fx)\left(2-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{3/2}d\sec(e+fx)}{5a} - \frac{\cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{5a}}{f}$$

$$\downarrow 27$$

$$\frac{-\int \cos^4(e+fx)\left(2-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{3/2}d\sec(e+fx)-\frac{\cos^5(e+fx)(a+b\sec^2(e+fx))^{5/2}}{5a}}{f}$$

$$\downarrow 359$$

$$\frac{\frac{(3a-4b)\int \cos^2(e+fx)(b\sec^2(e+fx)+a)^{3/2}d\sec(e+fx)}{3a}-\frac{\cos^5(e+fx)(a+b\sec^2(e+fx))^{5/2}}{5a}+\frac{2\cos^3(e+fx)(a+b\sec^2(e+fx))^{5/2}}{3a}}{f}$$

$$\downarrow 247$$

$$\frac{\frac{(3a-4b)\left(3b\int \sqrt{b\sec^2(e+fx)+a}d\sec(e+fx)-\cos(e+fx)(a+b\sec^2(e+fx))^{3/2}\right)}{3a}-\frac{\cos^5(e+fx)(a+b\sec^2(e+fx))^{5/2}}{5a}+\frac{2\cos^3(e+fx)(a+b\sec^2(}{f}$$

$$\downarrow 211$$

$$\frac{\frac{(3a-4b)\left(3b\left(\frac{1}{2}a\int \frac{1}{\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)+\frac{1}{2}\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\right)-\cos(e+fx)(a+b\sec^2(e+fx))^{3/2}\right)}{3a}-\frac{\cos^5(e+fx)(a+b\sec^2(}{f}$$

$$\downarrow 224$$

$$\frac{\frac{(3a-4b)\left(3b\left(\frac{1}{2}a\int \frac{1}{1-\frac{b\sec^2(e+fx)}{b\sec^2(e+fx)+a}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}+\frac{1}{2}\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\right)-\cos(e+fx)(a+b\sec^2(e+fx))^{3/2}\right)}{3a}-\frac{\cos^5(e+fx)(a-}{f}$$

$$\downarrow 219$$

$$\frac{\frac{(3a-4b)\left(3b\left(\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2\sqrt{b}}+\frac{1}{2}\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\right)-\cos(e+fx)(a+b\sec^2(e+fx))^{3/2}\right)}{3a}-\frac{\cos^5(e+fx)(a+b\sec^2(e-}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^5,x]
```

output
```
((2*Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(5/2))/(3*a) - (Cos[e + f*x]^5*(
a + b*Sec[e + f*x]^2)^(5/2))/(5*a) + ((3*a - 4*b)*(-(Cos[e + f*x]*(a + b*S
ec[e + f*x]^2)^(3/2)) + 3*b*((a*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*
Sec[e + f*x]^2]])/(2*Sqrt[b]) + (Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/
2)))/(3*a))/f
```

3.80.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

3.80.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 801 vs. $2(172) = 344$.

Time = 7.03 (sec) , antiderivative size = 802, normalized size of antiderivative = 4.09

method	result
default	$-\frac{\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}\left(6\cos(fx+e)^8\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,a^2b+6\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,a^2b\cos(fx+e)^7+60\cos(fx+e)^3b^{\frac{5}{2}}\ln\left(-4\sqrt{b}\sqrt{\frac{b+a}{(1+}\right.\right.$

input
```
int((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^5,x,method=_RETURNVERBOSE)
```

output

```
-1/30/f/a/b*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)/(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))*(6*cos(f*x+e)^8*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*a^2*b+6*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*a^2*b*cos(f*x+e)^7+60*cos(f*x+e)^3*b^(5/2)*ln(-4*b^(1/2)*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*sec(f*x+e)-4*sec(f*x+e)*b)*a-20*cos(f*x+e)^6*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*a^2*b+12*cos(f*x+e)^6*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*a*b^2-45*cos(f*x+e)^3*b^(3/2)*ln(-4*b^(1/2)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*sec(f*x+e)-4*sec(f*x+e)*b)*a^2-20*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*a^2*b*cos(f*x+e)^5+12*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*a*b^2*cos(f*x+e)^5+30*cos(f*x+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*a^2*b-80*cos(f*x+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*a*b^2+6*cos(f*x+e)^4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^3
+30*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)^3*a^2*b-80*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)^3*b^2*a+6*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^3*cos(f*x+e)^3-15*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*cos(f*x+e)^2*b^2*a-15*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*cos(f*x+e)*a*b^2)
```

3.80.5 Fricas [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.79

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^5(e+fx)\,dx = \left[\begin{array}{l} -\dfrac{15\left(3a^2-4ab\right)\sqrt{b}\cos\left(fx+e\right)\log\left(\dfrac{a\cos(fx+e)^2-2\sqrt{b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+2b}{\cos(fx+e)^2}\right)+2\left(6a^2\cos\left(f\right.\right.}{60af} \\ \\ -\dfrac{15\left(3a^2-4ab\right)\sqrt{-b}\arctan\left(\dfrac{\sqrt{-b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{b}\right)\cos\left(fx+e\right)+\left(6a^2\cos\left(fx+e\right)^6-4\left(5a^2-3ab\right.\right.}{30af\cos\left(fx+e\right)} \end{array}\right.$$

input

```
integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^5,x, algorithm="fricas")
```

output
```
[-1/60*(15*(3*a^2 - 4*a*b)*sqrt(b)*cos(f*x + e)*log((a*cos(f*x + e)^2 - 2*
sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/co
s(f*x + e)^2) + 2*(6*a^2*cos(f*x + e)^6 - 4*(5*a^2 - 3*a*b)*cos(f*x + e)^4
 + 2*(15*a^2 - 40*a*b + 3*b^2)*cos(f*x + e)^2 - 15*a*b)*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2))/(a*f*cos(f*x + e)), -1/30*(15*(3*a^2 - 4*a*b)*s
qrt(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*
x + e)/b)*cos(f*x + e) + (6*a^2*cos(f*x + e)^6 - 4*(5*a^2 - 3*a*b)*cos(f*x
 + e)^4 + 2*(15*a^2 - 40*a*b + 3*b^2)*cos(f*x + e)^2 - 15*a*b)*sqrt((a*cos
(f*x + e)^2 + b)/cos(f*x + e)^2))/(a*f*cos(f*x + e))]
```

3.80.6 Sympy [F(-1)]

Timed out.

$$\int (a + b\sec^2(e + fx))^{3/2} \sin^5(e + fx)\, dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e)**2)**(3/2)*sin(f*x+e)**5,x)
```

output
```
Timed out
```

3.80.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.41

$$\int (a + b\sec^2(e + fx))^{3/2} \sin^5(e + fx)\, dx =$$

$$-\frac{\frac{12\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{5}{2}}\cos(fx+e)^5}{a} - 40\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3 + 60\sqrt{a+\frac{b}{\cos(fx+e)^2}}a\cos(fx+e) - 120\sqrt{}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^5,x, algorithm="maxima")
```

output
```
-1/60*(12*(a + b/cos(f*x + e)^2)^(5/2)*cos(f*x + e)^5/a - 40*(a + b/cos(f*
x + e)^2)^(3/2)*cos(f*x + e)^3 + 60*sqrt(a + b/cos(f*x + e)^2)*a*cos(f*x +
 e) - 120*sqrt(a + b/cos(f*x + e)^2)*b*cos(f*x + e) - 30*sqrt(a + b/cos(f*
x + e)^2)*a*b*cos(f*x + e)/((a + b/cos(f*x + e)^2)*cos(f*x + e)^2 - b) + 4
5*a*sqrt(b)*log((sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e) - sqrt(b))/(sqrt(
a + b/cos(f*x + e)^2)*cos(f*x + e) + sqrt(b))) - 60*b^(3/2)*log((sqrt(a +
b/cos(f*x + e)^2)*cos(f*x + e) - sqrt(b))/(sqrt(a + b/cos(f*x + e)^2)*cos(
f*x + e) + sqrt(b))))/f
```

3.80.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1952 vs. $2(172) = 344$.

Time $= 1.95$ (sec) , antiderivative size $= 1952$, normalized size of antiderivative $= 9.96$

$$\int \left(a + b\sec^2(e + fx)\right)^{3/2} \sin^5(e + fx)\, dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^5,x, algorithm="giac")
```

output
```
-1/15*(15*(3*a*b - 4*b^2)*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2
- sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f
*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) - sqrt(a + b))/sqrt(-b
))*sgn(cos(f*x + e))/sqrt(-b) + 30*((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 -
sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x
 + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^3*(a*b + 2*b^2)*sgn(cos
(f*x + e)) - (sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/
2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1
/2*f*x + 1/2*e)^2 + a + b))^2*(3*a*b - 2*b^2)*sqrt(a + b)*sgn(cos(f*x + e)
) + (3*a^2*b - 3*a*b^2 - 2*b^3)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt
(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1
/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*sgn(cos(f*x + e)) - (a^2*b
- a*b^2 + 2*b^3)*sqrt(a + b)*sgn(cos(f*x + e)))/((sqrt(a + b)*tan(1/2*f*x
+ 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*
a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2 - 2*(sqr
t(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/
2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2
 + a + b))*sqrt(a + b) + a - 3*b)^2 - 4*(15*(sqrt(a + b)*tan(1/2*f*x + 1/2
*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan
(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^9*(2*a*b - b...
```

3.80.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\sec^2(e+fx))^{3/2}\sin^5(e+fx)\,dx = \int \sin(e+fx)^5\left(a+\frac{b}{\cos(e+fx)^2}\right)^{3/2}dx$$

input
```
int(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.81 $\int (a+b\sec^2(e+fx))^{3/2}\sin^3(e+fx)\,dx$

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3.81.1 Optimal result

Integrand size = 25, antiderivative size = 162

$$\int (a+b\sec^2(e+fx))^{3/2}\sin^3(e+fx)\,dx = \frac{(3a-2b)\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f} + \frac{(3a-2b)b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2af} - \frac{(3a-2b)\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3af} + \frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{3af}$$

output

```
-1/3*(3*a-2*b)*cos(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2)/a/f+1/3*cos(f*x+e)^3*(a
+b*sec(f*x+e)^2)^(5/2)/a/f+1/2*(3*a-2*b)*arctanh(sec(f*x+e)*b^(1/2)/(a+b*s
ec(f*x+e)^2)^(1/2))*b^(1/2)/f+1/2*(3*a-2*b)*b*sec(f*x+e)*(a+b*sec(f*x+e)^2
)^(1/2)/a/f
```

3.81.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.01

$$\int (a+b\sec^2(e+fx))^{3/2}\sin^3(e+fx)\,dx = \frac{\sqrt{2}\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\left(3\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)^{5/2} - (3a-2b)\left(\right.\right.}{3bf(a+2b+a\cos(2$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^3,x]
```

output
```
(Sqrt[2]*Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2)*(3*Sec[e + f*x]^2*(a
+ b - a*Sin[e + f*x]^2)^(5/2) - (3*a - 2*b)*(-3*b^(3/2)*ArcTanh[Sqrt[a + b
 - a*Sin[e + f*x]^2]/Sqrt[b]] + Sqrt[a + b - a*Sin[e + f*x]^2]*(a + 4*b -
a*Sin[e + f*x]^2))))/(3*b*f*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2))
```

3.81.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.89, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4622, 25, 359, 247, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx$$

$$\downarrow 3042$$

$$\int \sin(e+fx)^3\left(a+b\sec(e+fx)^2\right)^{3/2}dx$$

$$\downarrow 4622$$

$$\frac{\int -\cos^4(e+fx)\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{3/2}d\sec(e+fx)}{f}$$

$$\downarrow 25$$

$$-\frac{\int \cos^4(e+fx)\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{3/2}d\sec(e+fx)}{f}$$

$$\downarrow 359$$

$$\frac{\frac{(3a-2b)\int \cos^2(e+fx)\left(b\sec^2(e+fx)+a\right)^{3/2}d\sec(e+fx)}{3a}+\frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{3a}}{f}$$

$$\downarrow 247$$

$$\frac{\frac{(3a-2b)\left(3b\int \sqrt{b\sec^2(e+fx)+a}d\sec(e+fx)-\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\right)}{3a}+\frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{5/2}}{3a}}{f}$$

$$\downarrow 211$$

$$\frac{(3a-2b)\left(3b\left(\frac{1}{2}a\int\frac{1}{\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)+\frac{1}{2}\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\right)-\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\right)}{3a}+\frac{\cos^3(e+fx)(a+b\sec^2(}{3a}$$
$$f$$

$$\downarrow 224$$

$$\frac{(3a-2b)\left(3b\left(\frac{1}{2}a\int\frac{1}{1-\frac{b\sec^2(e+fx)}{b\sec^2(e+fx)+a}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}+\frac{1}{2}\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\right)-\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\right)}{3a}+\frac{\cos^3(e+fx)(a-}{3a}$$
$$f$$

$$\downarrow 219$$

$$\frac{(3a-2b)\left(3b\left(\frac{{}_a\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2\sqrt{b}}+\frac{1}{2}\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\right)-\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\right)}{3a}+\frac{\cos^3(e+fx)(a+b\sec^2(e-}{3a}$$
$$f$$

input
```
Int[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^3,x]
```

output
```
((Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(5/2))/(3*a) + ((3*a - 2*b)*(-(Cos
[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2)) + 3*b*((a*ArcTanh[(Sqrt[b]*Sec[e +
 f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/(2*Sqrt[b]) + (Sec[e + f*x]*Sqrt[a + b
*Sec[e + f*x]^2])/2)))/(3*a))/f
```

3.81.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

3.81.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 551 vs. $2(142) = 284$.

Time = 6.93 (sec) , antiderivative size = 552, normalized size of antiderivative = 3.41

method	result
default	$-\frac{\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}\left(6\ln\left(-4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sec(fx+e)-4\sec(fx+e)b\right)\cos(fx+e)^3 b^{\frac{5}{2}}-2\cos(fx-$

input
```
int((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/6/f/b*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)/(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))*(6*ln(-4*b^(1/2)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)-4*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*sec(f*x+e)-4*sec(f*x+e)*b)*cos(f*x+e)^3*b^(5/2)-2*cos(f*x+e)^6*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b-9*ln(-4*b^(1/2)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)-4*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*sec(f*x+e)-4*sec(f*x+e)*b)*cos(f*x+e)^3*b^(3/2)*a-2*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b*cos(f*x+e)^5+6*cos(f*x+e)^4*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b-8*cos(f*x+e)^4*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*b^2+6*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*a*b-8*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2*cos(f*x+e)
^3-3*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2-3*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2*cos(f*x+e))
```

3.81.5 Fricas [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.72

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^3(e+fx)\,dx = \left[-\frac{3\,(3\,a-2\,b)\sqrt{b}\cos\left(fx+e\right)\log\left(\frac{a\cos(fx+e)^2-2\sqrt{b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+2\,b}{\cos(fx+e)^2}\right)-2\left(2\,a\cos\left(fx+\right.\right.\right.}{12\,f\cos\left(fx+e\right)}$$

$$-\frac{3\,(3\,a-2\,b)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{b}\right)\cos\left(fx+e\right)-\left(2\,a\cos\left(fx+e\right)^4-2\,(3\,a-4\,b)\cos\left(\right.\right.}{6\,f\cos\left(fx+e\right)}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^3,x, algorithm="fricas")
```

output
```
[-1/12*(3*(3*a - 2*b)*sqrt(b)*cos(f*x + e)*log((a*cos(f*x + e)^2 - 2*sqrt(
b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x
 + e)^2) - 2*(2*a*cos(f*x + e)^4 - 2*(3*a - 4*b)*cos(f*x + e)^2 + 3*b)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e)), -1/6*(3*(3*a -
 2*b)*sqrt(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
*cos(f*x + e)/b)*cos(f*x + e) - (2*a*cos(f*x + e)^4 - 2*(3*a - 4*b)*cos(f*
x + e)^2 + 3*b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x +
e))]
```

3.81.6 Sympy [F(-1)]

Timed out.

$$\int (a+b\sec^2(e+fx))^{3/2}\sin^3(e+fx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e)**2)**(3/2)*sin(f*x+e)**3,x)
```

output
```
Timed out
```

3.81.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.54

$$\int (a+b\sec^2(e+fx))^{3/2}\sin^3(e+fx)\,dx = \frac{4\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3 - 12\sqrt{a+\frac{b}{\cos(fx+e)^2}}a\cos(fx+e) + 12\sqrt{a+\frac{b}{\cos(fx+e)^2}}b\cos(f\ldots}{}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^3,x, algorithm="maxima")
```

output
```
1/12*(4*(a + b/cos(f*x + e)^2)^(3/2)*cos(f*x + e)^3 - 12*sqrt(a + b/cos(f*
x + e)^2)*a*cos(f*x + e) + 12*sqrt(a + b/cos(f*x + e)^2)*b*cos(f*x + e) +
6*sqrt(a + b/cos(f*x + e)^2)*a*b*cos(f*x + e)/((a + b/cos(f*x + e)^2)*cos(
f*x + e)^2 - b) - 9*a*sqrt(b)*log((sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e)
 - sqrt(b))/(sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e) + sqrt(b))) + 6*b^(3/
2)*log((sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e) - sqrt(b))/(sqrt(a + b/cos
(f*x + e)^2)*cos(f*x + e) + sqrt(b))))/f
```

3.81.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1388 vs. 2(142) = 284.

Time = 1.24 (sec) , antiderivative size = 1388, normalized size of antiderivative = 8.57

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^3(e+fx)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^3,x, algorithm="giac")
```

output
```
-1/3*(3*(3*a*b - 2*b^2)*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 -
sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x
 + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) - sqrt(a + b))/sqrt(-b))
*sgn(cos(f*x + e))/sqrt(-b) + 6*((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqr
t(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x +
1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^3*(a*b + 2*b^2)*sgn(cos(f*
x + e)) - (sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e
)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*
f*x + 1/2*e)^2 + a + b))^2*(3*a*b - 2*b^2)*sqrt(a + b)*sgn(cos(f*x + e)) +
 (3*a^2*b - 3*a*b^2 - 2*b^3)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*
tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*
e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*sgn(cos(f*x + e)) - (a^2*b - a
*b^2 + 2*b^3)*sqrt(a + b)*sgn(cos(f*x + e)))/((sqrt(a + b)*tan(1/2*f*x + 1
/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*t
an(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2 - 2*(sqrt(a
 + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f
*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 +
a + b))*sqrt(a + b) + a - 3*b)^2 - 8*(3*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^
2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2
*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^5*a*b*sgn(cos(f*...
```

3.81.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^3(e+fx)\,dx = \int \sin\left(e+f\,x\right)^3\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}dx$$

input
```
int(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.82 $\int (a+b\sec^2(e+fx))^{3/2}\sin(e+fx)\,dx$

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3.82.1 Optimal result

Integrand size = 23, antiderivative size = 100

$$\int (a+b\sec^2(e+fx))^{3/2}\sin(e+fx)\,dx = \frac{3a\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f} + \frac{3b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f} - \frac{\cos(e+fx)\,(a+b\sec^2(e+fx))^{3/2}}{f}$$

output

```
-cos(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2)/f+3/2*a*arctanh(sec(f*x+e)*b^(1/2)/(a
+b*sec(f*x+e)^2)^(1/2))*b^(1/2)/f+3/2*b*sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2
)/f
```

3.82.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.48 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.73

$$\int (a+b\sec^2(e+fx))^{3/2}\sin(e+fx)\,dx = -\frac{a\cos(e+fx)(a+2b+a\cos(2(e+fx)))^2\,\text{Hypergeometric2F1}\left(2,\frac{5}{2},\frac{7}{2},1+\frac{a\cos^2(e+fx)}{b}\right)\sqrt{a+b\sec^2(e+f}}{20b^2f}$$

input

```
Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x],x]
```

output
```
-1/20*(a*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)])^2*Hypergeometric2F1[2
, 5/2, 7/2, 1 + (a*Cos[e + f*x]^2)/b]*Sqrt[a + b*Sec[e + f*x]^2])/(b^2*f)
```

3.82.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4622, 247, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx$$

$$\downarrow 3042$$

$$\int \sin(e+fx)\left(a+b\sec(e+fx)^2\right)^{3/2}dx$$

$$\downarrow 4622$$

$$\frac{\int \cos^2(e+fx)\left(b\sec^2(e+fx)+a\right)^{3/2}d\sec(e+fx)}{f}$$

$$\downarrow 247$$

$$\frac{3b\int \sqrt{b\sec^2(e+fx)+a}d\sec(e+fx)-\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{f}$$

$$\downarrow 211$$

$$\frac{3b\left(\frac{1}{2}a\int \frac{1}{\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)+\frac{1}{2}\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\right)-\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{f}$$

$$\downarrow 224$$

$$\frac{3b\left(\frac{1}{2}a\int \frac{1}{1-\frac{b\sec^2(e+fx)}{b\sec^2(e+fx)+a}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}+\frac{1}{2}\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\right)-\cos(e+fx)\left(a+b\sec^2(e+fx)\right)}{f}$$

$$\downarrow 219$$

$$\frac{3b\left(\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2\sqrt{b}}+\frac{1}{2}\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\right)-\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x],x]
```

output
```
(-(Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2)) + 3*b*((a*ArcTanh[(Sqrt[b]*S
ec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/(2*Sqrt[b]) + (Sec[e + f*x]*Sqrt
[a + b*Sec[e + f*x]^2])/2))/f
```

3.82.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

3.82.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.21

method	result
derivativedivides	$-\frac{\left(a+b\sec(fx+e)^2\right)^{\frac{5}{2}}}{fa\sec(fx+e)}+\frac{b\sec(fx+e)\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}}{fa}+\frac{3b\sec(fx+e)\sqrt{a+b\sec(fx+e)^2}}{2f}+\frac{3\sqrt{b}a\ln\left(\sqrt{b}\sec($
default	$-\frac{\left(a+b\sec(fx+e)^2\right)^{\frac{5}{2}}}{fa\sec(fx+e)}+\frac{b\sec(fx+e)\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}}{fa}+\frac{3b\sec(fx+e)\sqrt{a+b\sec(fx+e)^2}}{2f}+\frac{3\sqrt{b}a\ln\left(\sqrt{b}\sec($

input

```
int((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e),x,method=_RETURNVERBOSE)
```

output

```
-1/f/a/sec(f*x+e)*(a+b*sec(f*x+e)^2)^(5/2)+1/f*b/a*sec(f*x+e)*(a+b*sec(f*x
+e)^2)^(3/2)+3/2*b*sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/f+3/2/f*b^(1/2)*a*l
n(b^(1/2)*sec(f*x+e)+(a+b*sec(f*x+e)^2)^(1/2))
```

3.82.5 Fricas [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.31

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin(e+fx)\,dx = \left[\frac{3\,a\sqrt{b}\cos\left(fx+e\right)\log\left(\frac{a\cos(fx+e)^2+2\sqrt{b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+2b}{\cos(fx+e)^2}\right)-2\left(2\,a\cos\left(fx+e\right)^2-b\right)\sqrt{}}{4\,f\cos\left(fx+e\right)}\right.$$

$$\left.-\frac{3\,a\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{b}\right)\cos\left(fx+e\right)+\left(2\,a\cos\left(fx+e\right)^2-b\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{2\,f\cos\left(fx+e\right)}\right]$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e),x, algorithm="fricas")
```

output
```
[1/4*(3*a*sqrt(b)*cos(f*x + e)*log((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2) - 2
*(2*a*cos(f*x + e)^2 - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*
cos(f*x + e)), -1/2*(3*a*sqrt(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)*cos(f*x + e)/b)*cos(f*x + e) + (2*a*cos(f*x + e)^2 - b
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e))]
```

3.82.6 Sympy [F]

$$\int (a + b\sec^2(e + fx))^{3/2} \sin(e + fx)\, dx = \int (a + b\sec^2(e + fx))^{\frac{3}{2}} \sin(e + fx)\, dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**(3/2)*sin(f*x+e),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(3/2)*sin(e + f*x), x)
```

3.82.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.42

$$\int (a + b\sec^2(e + fx))^{3/2} \sin(e + fx)\, dx =$$
$$-\frac{4\sqrt{a + \frac{b}{\cos(fx+e)^2}}\, a\cos(fx + e) - \frac{2\sqrt{a+\frac{b}{\cos(fx+e)^2}}\, ab\cos(fx+e)}{\left(a+\frac{b}{\cos(fx+e)^2}\right)\cos(fx+e)^2 - b} + 3\, a\sqrt{b}\log\left(\frac{\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e) - \sqrt{b}}{\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e) + \sqrt{b}}\right)}{4\, f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e),x, algorithm="maxima")
```

output
```
-1/4*(4*sqrt(a + b/cos(f*x + e)^2)*a*cos(f*x + e) - 2*sqrt(a + b/cos(f*x +
 e)^2)*a*b*cos(f*x + e)/((a + b/cos(f*x + e)^2)*cos(f*x + e)^2 - b) + 3*a*
sqrt(b)*log((sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e) - sqrt(b))/(sqrt(a +
b/cos(f*x + e)^2)*cos(f*x + e) + sqrt(b))))/f
```

3.82.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.87

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin(e+fx)\,dx = -\frac{\left(\frac{3\,b\arctan\left(\frac{\sqrt{a\cos(fx+e)^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}}+2\sqrt{a\cos\left(fx+e\right)^2+b}-\frac{\sqrt{a\cos(fx+e)^2+b}\,b}{a\cos(fx+e)^2}\right)a\,\mathrm{sgn}(\cos\left(fx+e\right))}{2\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e),x, algorithm="giac")
```

output
```
-1/2*(3*b*arctan(sqrt(a*cos(f*x + e)^2 + b)/sqrt(-b))/sqrt(-b) + 2*sqrt(a*
cos(f*x + e)^2 + b) - sqrt(a*cos(f*x + e)^2 + b)*b/(a*cos(f*x + e)^2))*a*s
gn(cos(f*x + e))/f
```

3.82.9 Mupad [B] (verification not implemented)

Time = 20.41 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.61

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin(e+fx)\,dx = -\frac{\cos\left(e+f\,x\right)\,\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{b}{a\cos(e+f\,x)^2}\right)}{f\left(\frac{b}{a\cos(e+f\,x)^2}+1\right)^{3/2}}$$

input
```
int(sin(e + f*x)*(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
-(cos(e + f*x)*(a + b/cos(e + f*x)^2)^(3/2)*hypergeom([-3/2, -1/2], 1/2, -
b/(a*cos(e + f*x)^2)))/(f*(b/(a*cos(e + f*x)^2) + 1)^(3/2))
```

3.83 $\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

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3.83.2 Mathematica [A] (verified) . . . 761
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3.83.1 Optimal result

Integrand size = 23, antiderivative size = 122

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\sqrt{b}(3a+2b)\operatorname{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f} - \frac{(a+b)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f} + \frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}$$

output

```
-(a+b)^(3/2)*arctanh(sec(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))/f+1/
2*(3*a+2*b)*arctanh(sec(f*x+e)*b^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))*b^(1/2)/f
+1/2*b*sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/f
```

3.83.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.40

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\left(2\sqrt{b}(3a+2b)\operatorname{arctanh}\left(\frac{\sqrt{a+b-a\sin^2(e+fx)}}{\sqrt{b}}\right)\cos^2(e+fx) - 4(a+b)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+b-a\sin^2(e+f}}{\sqrt{a+b}}\right.\right.}{2\sqrt{2}f\sqrt{a+2b+a\c}}$$

input

```
Integrate[Csc[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
((2*Sqrt[b]*(3*a + 2*b)*ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[b]]*Co
s[e + f*x]^2 - 4*(a + b)^(3/2)*ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt
[a + b]]*Cos[e + f*x]^2 + Sqrt[2]*b*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]])*Se
c[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(2*Sqrt[2]*f*Sqrt[a + 2*b + a*Cos[2
*(e + f*x)]])
```

3.83.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 4622, 25, 318, 25, 398, 224, 219, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^{3/2}}{\sin(e+fx)}dx$$

$$\downarrow 4622$$

$$\frac{\int -\frac{\left(b\sec^2(e+fx)+a\right)^{3/2}}{1-\sec^2(e+fx)}d\sec(e+fx)}{f}$$

$$\downarrow 25$$

$$-\frac{\int \frac{\left(b\sec^2(e+fx)+a\right)^{3/2}}{1-\sec^2(e+fx)}d\sec(e+fx)}{f}$$

$$\downarrow 318$$

$$\frac{\frac{1}{2}\int -\frac{b(3a+2b)\sec^2(e+fx)+a(2a+b)}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)+\frac{1}{2}b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f}$$

$$\downarrow 25$$

$$\frac{\frac{1}{2}b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}-\frac{1}{2}\int \frac{b(3a+2b)\sec^2(e+fx)+a(2a+b)}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{f}$$

$$\downarrow 398$$

$$\frac{\frac{1}{2}\left(b(3a+2b)\int \frac{1}{\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)-2(a+b)^2\int \frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)\right)+\frac{1}{2}b\sec(e+}{f}$$

$$\downarrow 224$$

$$\frac{\frac{1}{2}\left(b(3a+2b)\int \frac{1}{1-\frac{b\sec^2(e+fx)}{b\sec^2(e+fx)+a}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}-2(a+b)^2\int \frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)\right)+\frac{1}{2}b\sec(}{f}$$

$$\downarrow 219$$

$$\frac{\frac{1}{2}\left(\sqrt{b}(3a+2b)\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)-2(a+b)^2\int \frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)\right)+\frac{1}{2}b\sec(e+fx}{f}$$

$$\downarrow 291$$

$$\frac{\frac{1}{2}\left(\sqrt{b}(3a+2b)\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)-2(a+b)^2\int \frac{1}{1-\frac{(a+b)\sec^2(e+fx)}{b\sec^2(e+fx)+a}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}\right)+\frac{1}{2}b\sec(e+fx)\sqrt{a+b}}{f}$$

$$\downarrow 219$$

$$\frac{\frac{1}{2}\left(\sqrt{b}(3a+2b)\text{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)-2(a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)\right)+\frac{1}{2}b\sec(e+fx)\sqrt{a+b\sec^2(e}}{f}$$

input
```
Int[Csc[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
((Sqrt[b]*(3*a + 2*b)*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*
x]^2]] - 2*(a + b)^(3/2)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec
[e + f*x]^2]])/2 + (b*Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/2)/f
```

3.83.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

3.83.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1436 vs. 2(104) = 208.

Time = 6.76 (sec) , antiderivative size = 1437, normalized size of antiderivative = 11.78

method	result	size
default	Expression too large to display	1437

input
```
int(csc(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f/b/(a+b)^(5/2)*(a+b*sec(f*x+e)^2)^(3/2)/(1+cos(f*x+e))/(b+a*cos(f*x+e
)^2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(2*cos(f*x+e)^3*b^(5/2)*(
a+b)^(5/2)*ln(-4*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*b^(
1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*sec(f*x+e)-4*sec(f*x+e)*b
)+3*cos(f*x+e)^3*b^(3/2)*(a+b)^(5/2)*ln(-4*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)-4*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*sec(f*x+e)-4*sec(f*x+e)*b)*a+cos(f*x+e)^2*(a+b)^(5/2)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*b^2-cos(f*x+e)^3*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^4*b-4
*cos(f*x+e)^3*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)
^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^3*b^2-6*cos(f*x+e)^3*ln(2/(a+b)^(
1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+
cos(f*x+e)))*a^2*b^3-4*cos(f*x+e)^3*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a*b^4-cos(f*x
+e)^3*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^
(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2...
```

3.83.5 Fricas [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 715, normalized size of antiderivative = 5.86

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \left[\begin{array}{l} \dfrac{2(a+b)^{\frac{3}{2}}\cos(fx+e)\log\left(\dfrac{2\left(a\cos(fx+e)^2-2\sqrt{a+b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+a+2b\right)}{\cos(fx+e)^2-1}\right)+(3a+2b)\cdots}{4f\cos(fx+\cdots} \\ \\ -\dfrac{(3a+2b)\sqrt{-b}\arctan\left(\dfrac{\sqrt{-b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{b}\right)\cos(fx+e)-(a+b)^{\frac{3}{2}}\cos(fx+e)\log\left(\dfrac{2\left(a\cos(fx+e)^2\right.\right.}{2f\cos(fx+e)} \end{array}\right.$$

input
```
integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(2*(a + b)^(3/2)*cos(f*x + e)*log(2*(a*cos(f*x + e)^2 - 2*sqrt(a + b)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + 2*b)/(cos(
f*x + e)^2 - 1)) + (3*a + 2*b)*sqrt(b)*cos(f*x + e)*log((a*cos(f*x + e)^2
+ 2*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b
)/cos(f*x + e)^2) + 2*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*co
s(f*x + e)), 1/4*(4*(a + b)*sqrt(-a - b)*arctan(sqrt(-a - b)*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a + b))*cos(f*x + e) + (3*a +
 2*b)*sqrt(b)*cos(f*x + e)*log((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2) + 2*b*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e)), -1/2*((3*a +
 2*b)*sqrt(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
*cos(f*x + e)/b)*cos(f*x + e) - (a + b)^(3/2)*cos(f*x + e)*log(2*(a*cos(f*
x + e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f
*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1)) - b*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2))/(f*cos(f*x + e)), 1/2*(2*(a + b)*sqrt(-a - b)*arctan(sqrt(
-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a + b))*
cos(f*x + e) - (3*a + 2*b)*sqrt(-b)*arctan(sqrt(-b)*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)*cos(f*x + e)/b)*cos(f*x + e) + b*sqrt((a*cos(f*x + e
)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e))]
```

3.83.6 Sympy [F]

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(a+b\sec^2(e+fx)\right)^{\frac{3}{2}}\csc(e+fx)\,dx$$

input
```
integrate(csc(f*x+e)*(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(3/2)*csc(e + f*x), x)
```

3.83.7 Maxima [F]

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}\csc(fx+e)\,dx$$

input
```
integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*csc(f*x + e), x)
```

3.83.8 Giac [F(-2)]

Exception generated.

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.83.9 Mupad [F(-1)]

Timed out.

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}{\sin\left(e+f\,x\right)}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x),x)
```

output
```
int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x), x)
```

3.84 $\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

3.84.1 Optimal result . . . 769
3.84.2 Mathematica [A] (verified) . . . 769
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3.84.8 Giac [F(-2)] . . . 776
3.84.9 Mupad [F(-1)] . . . 776

3.84.1 Optimal result

Integrand size = 25, antiderivative size = 161

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\sqrt{b}(3a+4b)\operatorname{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f} - \frac{\sqrt{a+b}(a+4b)\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f} + \frac{b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f} - \frac{\cot(e+fx)\csc(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{2f}$$

output

```
-1/2*cot(f*x+e)*csc(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2)/f+1/2*(3*a+4*b)*arctan
h(sec(f*x+e)*b^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))*b^(1/2)/f-1/2*(a+4*b)*arcta
nh(sec(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))*(a+b)^(1/2)/f+b*sec(f*
x+e)*(a+b*sec(f*x+e)^2)^(1/2)/f
```

3.84.2 Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.25

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = -\frac{\csc^2(e+fx)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(\sqrt{2}\sqrt{a+2b+a\cos(2(e+fx))}(a+(a+2b)\cos(2(e+fx)\right.}{4\sqrt{2}f\sqrt{}}$$

input
```
Integrate[Csc[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
-1/4*(Csc[e + f*x]^2*Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*(Sqrt[2]*Sqrt
[a + 2*b + a*Cos[2*(e + f*x)]]*(a + (a + 2*b)*Cos[2*(e + f*x)]) - Sqrt[b]*
(3*a + 4*b)*ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[b]]*Sin[2*(e + f*x
)]^2 + Sqrt[a + b]*(a + 4*b)*ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[a
 + b]]*Sin[2*(e + f*x)]^2))/(Sqrt[2]*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]])
```

3.84.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4622, 369, 403, 27, 398, 224, 219, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^{3/2}}{\sin(e+fx)^3}dx$$

$$\downarrow 4622$$

$$\frac{\int \frac{\sec^2(e+fx)\left(b\sec^2(e+fx)+a\right)^{3/2}}{\left(1-\sec^2(e+fx)\right)^2}d\sec(e+fx)}{f}$$

$$\downarrow 369$$

$$\frac{\frac{\sec(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{2\left(1-\sec^2(e+fx)\right)}-\frac{1}{2}\int \frac{\sqrt{b\sec^2(e+fx)+a}\left(4b\sec^2(e+fx)+a\right)}{1-\sec^2(e+fx)}d\sec(e+fx)}{f}$$

$$\downarrow 403$$

$$\frac{\frac{1}{2}\left(\frac{1}{2}\int -\frac{2\left(b(3a+4b)\sec^2(e+fx)+a(a+2b)\right)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)+2b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\right)+\frac{\sec(e+fx)\left(a+b\sec^2(e+fx)\right)}{2\left(1-\sec^2(e+fx)\right)}}{f}$$

$$\downarrow 27$$

$$\frac{\frac{1}{2}\left(2b\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}-\int\frac{b(3a+4b)\sec^2(e+fx)+a(a+2b)}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)\right)+\frac{\sec(e+fx)(a+b\sec^2(e+fx))^{3/2}}{2(1-\sec^2(e+fx))}}{f}$$

$$\downarrow 398$$

$$\frac{\frac{1}{2}\left(b(3a+4b)\int\frac{1}{\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)-(a+b)(a+4b)\int\frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)+2b\sec($$

$$\downarrow 224$$

$$\frac{\frac{1}{2}\left(-(a+b)(a+4b)\int\frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)+b(3a+4b)\int\frac{1}{1-\frac{b\sec^2(e+fx)}{b\sec^2(e+fx)+a}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}+2\right.}{f}$$

$$\downarrow 219$$

$$\frac{\frac{1}{2}\left(-(a+b)(a+4b)\int\frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)+\sqrt{b}(3a+4b)\operatorname{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)+2b\sec(e\right.}{f}$$

$$\downarrow 291$$

$$\frac{\frac{1}{2}\left(-(a+b)(a+4b)\int\frac{1}{1-\frac{(a+b)\sec^2(e+fx)}{b\sec^2(e+fx)+a}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}+\sqrt{b}(3a+4b)\operatorname{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)+2b\sec(e+fx)\sqrt{\right.}}{f}$$

$$\downarrow 219$$

$$\frac{\frac{1}{2}\left(\sqrt{b}(3a+4b)\operatorname{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)-\sqrt{a+b}(a+4b)\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)+2b\sec(e+fx)\sqrt{a+b\sec^2\right.}}{f}$$

input
```
Int[Csc[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
((Sec[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2))/(2*(1 - Sec[e + f*x]^2)) + (S
qrt[b]*(3*a + 4*b)*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^
2]] - Sqrt[a + b]*(a + 4*b)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*
Sec[e + f*x]^2]] + 2*b*Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/2)/f
```

3.84.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

3.84.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3469 vs. $2(139) = 278$.

Time = 6.92 (sec) , antiderivative size = 3470, normalized size of antiderivative = 21.55

method	result	size
default	Expression too large to display	3470

input
```
int(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/f/b/(a+b)^(7/2)*(14*ln(-4*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)-4*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*sec(f*x+e)-
4*sec(f*x+e)*b)*cos(f*x+e)^3*(a+b)^(5/2)*b^(5/2)*a+6*ln(-4*b^(1/2)*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*sec(f*x+e)-4*sec(f*x+e)*b)*cos(f*x+e)^3*(a+b)^(5/2)*b^(3/
2)*a^2-14*ln(-4*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*b^(1
/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*sec(f*x+e)-4*sec(f*x+e)*b)
*cos(f*x+e)^2*(a+b)^(5/2)*b^(5/2)*a-6*ln(-4*b^(1/2)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)-4*b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*sec(f*x+e)-4*sec(f*x+e)*b)*cos(f*x+e)^2*(a+b)^(5/2)*b^(3/2)*a^2+2*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)^2*(a+b)^(5/2)*a^2*b+6*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)^2*(a+b)^(5/2)*a*b^2-2*
(a+b)^(5/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^3-4*ln(2/(a+b)^(
1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+
cos(f*x+e)))*cos(f*x+e)^3*b^6-4*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*cos(f*x+e)^3*b^6+4*ln(2/(a+
b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+
e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a...
```

3.84.5 Fricas [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 984, normalized size of antiderivative = 6.11

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(((a + 4*b)*cos(f*x + e)^3 - (a + 4*b)*cos(f*x + e))*sqrt(a + b)*log(
2*(a*cos(f*x + e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1)) + ((3*a + 4*b)*cos(f*x
 + e)^3 - (3*a + 4*b)*cos(f*x + e))*sqrt(b)*log((a*cos(f*x + e)^2 + 2*sqrt
(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*
x + e)^2) + 2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2))/(f*cos(f*x + e)^3 - f*cos(f*x + e)), 1/4*(2*((a + 4*b)*cos
(f*x + e)^3 - (a + 4*b)*cos(f*x + e))*sqrt(-a - b)*arctan(sqrt(-a - b)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a + b)) + ((3*a + 4
*b)*cos(f*x + e)^3 - (3*a + 4*b)*cos(f*x + e))*sqrt(b)*log((a*cos(f*x + e)
^2 + 2*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) +
2*b)/cos(f*x + e)^2) + 2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e)^3 - f*cos(f*x + e)), -1/4*(2*((
3*a + 4*b)*cos(f*x + e)^3 - (3*a + 4*b)*cos(f*x + e))*sqrt(-b)*arctan(sqrt
(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/b) - ((a + 4
*b)*cos(f*x + e)^3 - (a + 4*b)*cos(f*x + e))*sqrt(a + b)*log(2*(a*cos(f*x
+ e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x
 + e) + a + 2*b)/(cos(f*x + e)^2 - 1)) - 2*((a + 2*b)*cos(f*x + e)^2 - b)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e)^3 - f*cos(f*x
 + e)), 1/2*(((a + 4*b)*cos(f*x + e)^3 - (a + 4*b)*cos(f*x + e))*sqrt(-...
```

3.84.6 Sympy [F(-1)]

Timed out.

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**3*(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

3.84.7 Maxima [F]

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^3\,dx$$

input
```
integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^3, x)
```

3.84.8 Giac [F(-2)]

Exception generated.

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[%%%{16384,[5,4]%%%},0]:[1,0,%%%{-1,[1,0]%%%}+%%%{-1,[0
,1]%%%}]%
```

3.84.9 Mupad [F(-1)]

Timed out.

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}{\sin\left(e+f\,x\right)^3}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x)^3,x)
```

output
```
int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x)^3, x)
```

3.85 $\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

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3.85.1 Optimal result

Integrand size = 25, antiderivative size = 218

$$\begin{aligned}\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx &= \frac{3\sqrt{b}(a+2b)\operatorname{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2f}\\ &-\frac{3(a^2+8ab+8b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8\sqrt{a+b}f}\\ &+\frac{3(a+4b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f}\\ &-\frac{3(a+2b)\csc^2(e+fx)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f}\\ &-\frac{\cot(e+fx)\csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{4f}\end{aligned}$$

output

```
-1/4*cot(f*x+e)*csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2)/f+3/2*(a+2*b)*arctan
h(sec(f*x+e)*b^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))*b^(1/2)/f-3/8*(a^2+8*a*b+8*
b^2)*arctanh(sec(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))/f/(a+b)^(1/2
)+3/8*(a+4*b)*sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/f-3/8*(a+2*b)*csc(f*x+e)
^2*sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/f
```

3.85.2 Mathematica [A] (verified)

Time = 2.60 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.20

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx =$$

$$-\frac{(b+a\cos^2(e+fx))\left(-12b^{3/2}(a^2+3ab+2b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+b-a\sin^2(e+fx)}}{\sqrt{b}}\right)\cos^2(e+fx)+3b\sqrt{a+b}(a^2+8\right.}{}$$

input
```
Integrate[Csc[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
-1/2*((b + a*Cos[e + f*x]^2)*(-12*b^(3/2)*(a^2 + 3*a*b + 2*b^2)*ArcTanh[Sq
rt[a + b - a*Sin[e + f*x]^2]/Sqrt[b]]*Cos[e + f*x]^2 + 3*b*Sqrt[a + b]*(a^
2 + 8*a*b + 8*b^2)*ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[a + b]]*Cos
[e + f*x]^2 + (b*(a + b)*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(11*a + 4*b +
8*(a + 3*b)*Cos[2*(e + f*x)] - 3*(a + 4*b)*Cos[4*(e + f*x)])*Csc[e + f*x]^
4)/(8*Sqrt[2]))*Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(Sqrt[2]*b*(a + b
)*f*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2))
```

3.85.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.04, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 4622, 25, 369, 27, 439, 25, 444, 27, 398, 224, 219, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^{3/2}}{\sin(e+fx)^5}dx$$

$$\downarrow 4622$$

$$\frac{\int -\frac{\sec^4(e+fx)\left(b\sec^2(e+fx)+a\right)^{3/2}}{\left(1-\sec^2(e+fx)\right)^3}d\sec(e+fx)}{f}$$

$$\downarrow 25$$

$$-\frac{\int \frac{\sec^4(e+fx)\left(b\sec^2(e+fx)+a\right)^{3/2}}{\left(1-\sec^2(e+fx)\right)^3}d\sec(e+fx)}{f}$$

$$\downarrow 369$$

$$\frac{\frac{1}{4}\int \frac{3\sec^2(e+fx)\sqrt{b\sec^2(e+fx)+a}\left(2b\sec^2(e+fx)+a\right)}{\left(1-\sec^2(e+fx)\right)^2}d\sec(e+fx)-\frac{\sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{4\left(1-\sec^2(e+fx)\right)^2}}{f}$$

$$\downarrow 27$$

$$\frac{\frac{3}{4}\int \frac{\sec^2(e+fx)\sqrt{b\sec^2(e+fx)+a}\left(2b\sec^2(e+fx)+a\right)}{\left(1-\sec^2(e+fx)\right)^2}d\sec(e+fx)-\frac{\sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}}{4\left(1-\sec^2(e+fx)\right)^2}}{f}$$

$$\downarrow 439$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\int -\frac{\sec^2(e+fx)\left(2b(a+4b)\sec^2(e+fx)+a(a+6b)\right)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)+\frac{(a+2b)\sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2\left(1-\sec^2(e+fx)\right)}\right)-\frac{\sec^3(e+fx)\left(a+b\sec^2(e+\right.}{f}$$

$$\downarrow 25$$

$$\frac{\frac{3}{4}\left(\frac{(a+2b)\sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2\left(1-\sec^2(e+fx)\right)}-\frac{1}{2}\int \frac{\sec^2(e+fx)\left(2b(a+4b)\sec^2(e+fx)+a(a+6b)\right)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)\right)-\frac{\sec^3(e+fx)\left(a+b\sec^2(e+f\right.}{f}$$

$$\downarrow 444$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left((a+4b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}-\frac{\int \frac{2b\left(4b(a+2b)\sec^2(e+fx)+a(a+4b)\right)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{2b}\right)+\frac{(a+2b)\sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2\left(1-\sec^2(e+fx)\right)}\right)}{f}$$

$$\downarrow 27$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left((a+4b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}-\int \frac{4b(a+2b)\sec^2(e+fx)+a(a+4b)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)\right)+\frac{(a+2b)\sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2\left(1-\sec^2(e+fx)\right)}\right)}{f}$$

$$\downarrow 398$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(-\left(a^2+8ab+8b^2\right)\int \frac{1}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)+4b(a+2b)\int \frac{1}{\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)+\right.\right.}{f}$$

$$\downarrow 224$$

3.85. $\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(-\left(a^2+8ab+8b^2\right)\int\frac{1}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)+4b(a+2b)\int\frac{1}{1-\frac{b\sec^2(e+fx)}{b\sec^2(e+fx)+a}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}\right.\right.}{f}$$

$$\downarrow 219$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(-\left(a^2+8ab+8b^2\right)\int\frac{1}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)+4\sqrt{b}(a+2b)\operatorname{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)+(a\right.\right.}{f}$$

$$\downarrow 291$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(-\left(a^2+8ab+8b^2\right)\int\frac{1}{1-\frac{(a+b)\sec^2(e+fx)}{b\sec^2(e+fx)+a}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}+4\sqrt{b}(a+2b)\operatorname{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)+(a+4b)\,se\right.\right.}{f}$$

$$\downarrow 219$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(-\frac{\left(a^2+8ab+8b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{\sqrt{a+b}}+4\sqrt{b}(a+2b)\operatorname{arctanh}\left(\frac{\sqrt{b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)+(a+4b)\sec(e+fx)\sqrt{a+\right.\right.}{f}$$

input
```
Int[Csc[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(-1/4*(Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2))/(1 - Sec[e + f*x]^2)^2
 + (3*(((a + 2*b)*Sec[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2])/(2*(1 - Sec[e
 + f*x]^2)) + (4*Sqrt[b]*(a + 2*b)*ArcTanh[(Sqrt[b]*Sec[e + f*x])/Sqrt[a +
 b*Sec[e + f*x]^2]] - ((a^2 + 8*a*b + 8*b^2)*ArcTanh[(Sqrt[a + b]*Sec[e +
f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/Sqrt[a + b] + (a + 4*b)*Sec[e + f*x]*Sq
rt[a + b*Sec[e + f*x]^2])/2))/4)/f
```

3.85.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 439
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
 + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*b*g*(p + 1))), x] + Simp[1/(2*a*b*(p
+ 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*b*e*(
p + 1) + (b*e - a*f)*(m + 1)) + d*(2*b*e*(p + 1) + (b*e - a*f)*(m + 2*q + 1
))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && LtQ[p, -1] && G
tQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

3.85.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 5130 vs. $2(190) = 380$.

Time = 6.61 (sec) , antiderivative size = 5131, normalized size of antiderivative = 23.54

method	result	size
default	Expression too large to display	5131

input
```
int(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.85.5 Fricas [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 1511, normalized size of antiderivative = 6.93

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/16*(3*((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^5 - 2*(a^2 + 8*a*b + 8*b^2)*c
os(f*x + e)^3 + (a^2 + 8*a*b + 8*b^2)*cos(f*x + e))*sqrt(a + b)*log(2*(a*c
os(f*x + e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*
cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1)) + 12*((a^2 + 3*a*b + 2*b^2)*
cos(f*x + e)^5 - 2*(a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 + (a^2 + 3*a*b + 2
*b^2)*cos(f*x + e))*sqrt(b)*log((a*cos(f*x + e)^2 + 2*sqrt(b)*sqrt((a*cos(
f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + 2*b)/cos(f*x + e)^2) + 2*(3
*(a^2 + 5*a*b + 4*b^2)*cos(f*x + e)^4 - (5*a^2 + 23*a*b + 18*b^2)*cos(f*x
+ e)^2 + 4*a*b + 4*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a +
 b)*f*cos(f*x + e)^5 - 2*(a + b)*f*cos(f*x + e)^3 + (a + b)*f*cos(f*x + e)
), 1/8*(3*((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^5 - 2*(a^2 + 8*a*b + 8*b^2)*
cos(f*x + e)^3 + (a^2 + 8*a*b + 8*b^2)*cos(f*x + e))*sqrt(-a - b)*arctan(s
qrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a +
b)) + 6*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^5 - 2*(a^2 + 3*a*b + 2*b^2)*co
s(f*x + e)^3 + (a^2 + 3*a*b + 2*b^2)*cos(f*x + e))*sqrt(b)*log((a*cos(f*x
+ e)^2 + 2*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e
) + 2*b)/cos(f*x + e)^2) + (3*(a^2 + 5*a*b + 4*b^2)*cos(f*x + e)^4 - (5*a^
2 + 23*a*b + 18*b^2)*cos(f*x + e)^2 + 4*a*b + 4*b^2)*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2))/((a + b)*f*cos(f*x + e)^5 - 2*(a + b)*f*cos(f*x +
e)^3 + (a + b)*f*cos(f*x + e)), -1/16*(24*((a^2 + 3*a*b + 2*b^2)*cos(f*...
```

3.85.6 Sympy [F(-1)]

Timed out.

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**5*(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

3.85.7 Maxima [F]

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^5\,dx$$

input
```
integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^5, x)
```

3.85.8 Giac [F(-2)]

Exception generated.

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{262144,[6,6]%%%},[12]%%%}+%%%{%%{[%%%{1572864,[6,6]%%%
},0]:[1,0
```

3.85.9 Mupad [F(-1)]

Timed out.

$$\int \csc^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}{\sin\left(e+f\,x\right)^5}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x)^5,x)
```

output
```
int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x)^5, x)
```

3.86 $\int (a+b\sec^2(e+fx))^{3/2}\sin^6(e+fx)\,dx$

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3.86.1 Optimal result

Integrand size = 25, antiderivative size = 298

$$\int (a+b\sec^2(e+fx))^{3/2}\sin^6(e+fx)\,dx = \frac{(5a^3-45a^2b+15ab^2+b^3)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{3/2}f} + \frac{(3a-5b)\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f} - \frac{(5a^2-26ab+b^2)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16af} + \frac{(5a^2-40ab+3b^2)\sin^2(e+fx)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48af} + \frac{(5a-3b)\sin^4(e+fx)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24f} - \frac{\cos(e+fx)\sin^5(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{6f}$$

output

```
1/16*(5*a^3-45*a^2*b+15*a*b^2+b^3)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*
x+e)^2)^(1/2))/a^(3/2)/f+1/2*(3*a-5*b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*t
an(f*x+e)^2)^(1/2))*b^(1/2)/f-1/16*(5*a^2-26*a*b+b^2)*(a+b+b*tan(f*x+e)^2)
^(1/2)*tan(f*x+e)/a/f+1/48*(5*a^2-40*a*b+3*b^2)*sin(f*x+e)^2*(a+b+b*tan(f*
x+e)^2)^(1/2)*tan(f*x+e)/a/f+1/24*(5*a-3*b)*sin(f*x+e)^4*(a+b+b*tan(f*x+e)
^2)^(1/2)*tan(f*x+e)/f-1/6*cos(f*x+e)*sin(f*x+e)^5*(a+b+b*tan(f*x+e)^2)^(3
/2)/f
```

3.86.2 Mathematica [F]

$$\int (a+b\sec^2(e+fx))^{3/2}\sin^6(e+fx)\,dx = \int (a+b\sec^2(e+fx))^{3/2}\sin^6(e+fx)\,dx$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^6,x]
```

output
```
Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^6, x]
```

3.86.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.04, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 4620, 369, 439, 440, 27, 444, 27, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx$$

$$\downarrow 3042$$

$$\int \sin(e+fx)^6\left(a+b\sec(e+fx)^2\right)^{3/2}dx$$

$$\downarrow 4620$$

$$\frac{\int \frac{\tan^6(e+fx)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}{\left(\tan^2(e+fx)+1\right)^4}d\tan(e+fx)}{f}$$

$$\downarrow 369$$

$$\frac{\frac{1}{6}\int \frac{\tan^4(e+fx)\sqrt{b\tan^2(e+fx)+a+b}\left(8b\tan^2(e+fx)+5(a+b)\right)}{\left(\tan^2(e+fx)+1\right)^3}d\tan(e+fx) - \frac{\tan^5(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{6\left(\tan^2(e+fx)+1\right)^3}}{f}$$

$$\downarrow 439$$

$$\frac{\frac{1}{6}\left(\frac{(5a-3b)\tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4\left(\tan^2(e+fx)+1\right)^2} - \frac{1}{4}\int \frac{\tan^4(e+fx)\left(2(5a-19b)b\tan^2(e+fx)+5(a-7b)(a+b)\right)}{\left(\tan^2(e+fx)+1\right)^2\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)\right) - \frac{\tan^5(e+fx)(}{6(ta}}{f}$$

$$\downarrow 440$$

$$\frac{\frac{1}{6}\left(\frac{1}{4}\left(\frac{\left(5a^2-40ab+3b^2\right)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a\left(\tan^2(e+fx)+1\right)}-\frac{\int \frac{3\tan^2(e+fx)\left(2b\left(5a^2-26ba+b^2\right)\tan^2(e+fx)+(a+b)\left(5a^2-40ba+3b^2\right)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2a}\right)\right)}{f}$$

$$\downarrow 27$$

$$\frac{\frac{1}{6}\left(\frac{1}{4}\left(\frac{\left(5a^2-40ab+3b^2\right)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a\left(\tan^2(e+fx)+1\right)}-\frac{3\int \frac{\tan^2(e+fx)\left(2b\left(5a^2-26ba+b^2\right)\tan^2(e+fx)+(a+b)\left(5a^2-40ba+3b^2\right)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2a}\right)\right)}{f}$$

$$\downarrow 444$$

$$\frac{\frac{1}{6}\left(\frac{1}{4}\left(\frac{\left(5a^2-40ab+3b^2\right)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a\left(\tan^2(e+fx)+1\right)}-\frac{3\left(\left(5a^2-26ab+b^2\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}-\frac{\int \frac{2b\left(8a(3a-5b)b\tan^2(e+fx)+\right.}{\left(\tan^2(e+fx)+1\right)\sqrt{b}}}{2a}\right)\right)}{f}$$

$$\downarrow 27$$

$$\frac{\frac{1}{6}\left(\frac{1}{4}\left(\frac{\left(5a^2-40ab+3b^2\right)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a\left(\tan^2(e+fx)+1\right)}-\frac{3\left(\left(5a^2-26ab+b^2\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}-\int \frac{8a(3a-5b)b\tan^2(e+fx)+(a}{\left(\tan^2(e+fx)+1\right)\sqrt{b\,ta}}\right)}{2a}\right)\right)}{f}$$

$$\downarrow 398$$

$$\frac{\frac{1}{6}\left(\frac{1}{4}\left(\frac{\left(5a^2-40ab+3b^2\right)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a\left(\tan^2(e+fx)+1\right)}-\frac{3\left(-\left(5a^3-45a^2b+15ab^2+b^3\right)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-\right.}{2a}\right)\right)}{f}$$

$$\downarrow 224$$

$$\frac{1}{6}\left(\frac{1}{4}\left(\frac{\left(5a^2-40ab+3b^2\right)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a(\tan^2(e+fx)+1)} - \frac{3\left(-\left(5a^3-45a^2b+15ab^2+b^3\right)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-\right.\right.\right.$$

$$\downarrow 219$$

$$\frac{1}{6}\left(\frac{1}{4}\left(\frac{\left(5a^2-40ab+3b^2\right)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a(\tan^2(e+fx)+1)} - \frac{3\left(-\left(5a^3-45a^2b+15ab^2+b^3\right)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+\right.\right.\right.$$

$$\downarrow 291$$

$$\frac{1}{6}\left(\frac{1}{4}\left(\frac{\left(5a^2-40ab+3b^2\right)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a(\tan^2(e+fx)+1)} - \frac{3\left(-\left(5a^3-45a^2b+15ab^2+b^3\right)\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}+\left(5a^2-\right.\right.\right.\right.$$

$$\downarrow 216$$

$$\frac{1}{6}\left(\frac{1}{4}\left(\frac{\left(5a^2-40ab+3b^2\right)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a(\tan^2(e+fx)+1)} - \frac{3\left(\left(5a^2-26ab+b^2\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}-\frac{\left(5a^3-45a^2b+15ab^2+b^3\right)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}\right)}{\sqrt{a}}\right)}{2a}\right.\right.\right.$$

input
```
Int[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^6,x]
```

output
```
(-1/6*(Tan[e + f*x]^5*(a + b + b*Tan[e + f*x]^2)^(3/2))/(1 + Tan[e + f*x]^
2)^3 + (((5*a - 3*b)*Tan[e + f*x]^5*Sqrt[a + b + b*Tan[e + f*x]^2])/(4*(1
+ Tan[e + f*x]^2)^2) + (((5*a^2 - 40*a*b + 3*b^2)*Tan[e + f*x]^3*Sqrt[a +
b + b*Tan[e + f*x]^2])/(2*a*(1 + Tan[e + f*x]^2)) - (3*(-(((5*a^3 - 45*a^2
*b + 15*a*b^2 + b^3)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e +
f*x]^2]])/Sqrt[a]) - 8*a*(3*a - 5*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x]
)/Sqrt[a + b + b*Tan[e + f*x]^2]] + (5*a^2 - 26*a*b + b^2)*Tan[e + f*x]*Sq
rt[a + b + b*Tan[e + f*x]^2]))/(2*a))/4)/6)/f
```

3.86.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 439
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
 + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*b*g*(p + 1))), x] + Simp[1/(2*a*b*(p
+ 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*b*e*(
p + 1) + (b*e - a*f)*(m + 1)) + d*(2*b*e*(p + 1) + (b*e - a*f)*(m + 2*q + 1
))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && LtQ[p, -1] && G
tQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.86.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1601 vs. 2(266) = 532.

Time = 16.25 (sec) , antiderivative size = 1602, normalized size of antiderivative = 5.38

method	result	size
default	Expression too large to display	1602

input
```
int((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^6,x,method=_RETURNVERBOSE)
```

output
```
-1/48/f/b/(-a)^(1/2)/a*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)/(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))*(8*cos(f*x+e)^8*sin(f*
x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*a^2*b+8*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*a^2*b*cos(f*x+e)^7*sin(f*
x+e)-26*cos(f*x+e)^6*sin(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*(-a)^(1/2)*a^2*b+14*cos(f*x+e)^6*sin(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*(-a)^(1/2)*a*b^2+60*cos(f*x+e)^3*(-a)^(1/2)*b^(5/2)*ln(4*(
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))
*a+60*cos(f*x+e)^3*(-a)^(1/2)*b^(5/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*a-26*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*a^2*b*cos(f*x+e)^5*sin(f*x+e)+14*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*a*b^2*cos(f*x+e)^5*sin(f*x+e
)-36*cos(f*x+e)^3*(-a)^(1/2)*b^(3/2)*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*a^2-36*cos(f*x+e)^3*(-a)^(1/2)
*b^(3/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*
x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)
/(sin(f*x+e)-1))*a^2+33*cos(f*x+e)^4*sin(f*x+e)*((b+a*cos(f*x+e)^2)/(1+...
```

3.86.5 Fricas [A] (verification not implemented)

Time = 10.53 (sec) , antiderivative size = 1855, normalized size of antiderivative = 6.22

$$\int \left(a + b\sec^2(e+fx)\right)^{3/2} \sin^6(e+fx)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^6,x, algorithm="fricas")
```

output

```
[-1/384*(3*(5*a^3 - 45*a^2*b + 15*a*b^2 + b^3)*sqrt(-a)*cos(f*x + e)*log(1
28*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*
a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3
 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3
*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b +
5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sq
rt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 48*(3*a
^3 - 5*a^2*b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4
 + 8*(a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x
+ e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8
*b^2)/cos(f*x + e)^4) + 8*(8*a^3*cos(f*x + e)^6 - 2*(13*a^3 - 7*a^2*b)*cos
(f*x + e)^4 - 24*a^2*b + (33*a^3 - 68*a^2*b + 3*a*b^2)*cos(f*x + e)^2)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^2*f*cos(f*x + e)
), 1/384*(96*(3*a^3 - 5*a^2*b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^
3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e) - 3*(5*a^3 - 45*a
^2*b + 15*a*b^2 + b^3)*sqrt(-a)*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 -
256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f
*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3
*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*...
```

3.86.6 Sympy [F(-1)]

Timed out.

$$\int (a + b\sec^2(e + fx))^{3/2} \sin^6(e + fx)\, dx = \text{Timed out}$$

input

```
integrate((a+b*sec(f*x+e)**2)**(3/2)*sin(f*x+e)**6,x)
```

output

```
Timed out
```

3.86.7 Maxima [F]

$$\int (a+b\sec^2(e+fx))^{3/2}\sin^6(e+fx)\,dx = \int (b\sec(fx+e)^2+a)^{\frac{3}{2}}\sin(fx+e)^6\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^6,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^6, x)
```

3.86.8 Giac [F]

$$\int (a+b\sec^2(e+fx))^{3/2}\sin^6(e+fx)\,dx = \int (b\sec(fx+e)^2+a)^{\frac{3}{2}}\sin(fx+e)^6\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^6,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^6, x)
```

3.86.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\sec^2(e+fx))^{3/2}\sin^6(e+fx)\,dx = \int \sin(e+fx)^6\left(a+\frac{b}{\cos(e+fx)^2}\right)^{3/2}dx$$

input
```
int(sin(e + f*x)^6*(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(sin(e + f*x)^6*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.87 $\int (a+b\sec^2(e+fx))^{3/2}\sin^4(e+fx)\,dx$

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3.87.1 Optimal result

Integrand size = 25, antiderivative size = 217

$$\int (a + b\sec^2(e+fx))^{3/2}\sin^4(e+fx)\,dx = \frac{3(a^2-6ab+b^2)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8\sqrt{a}f} + \frac{3(a-b)\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f} - \frac{3(a-3b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} + \frac{3(a-b)\sin^2(e+fx)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} - \frac{\cos(e+fx)\sin^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{4f}$$

output

```
3/8*(a^2-6*a*b+b^2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/
f/a^(1/2)+3/2*(a-b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))
*b^(1/2)/f-3/8*(a-3*b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f+3/8*(a-b)*s
in(f*x+e)^2*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f-1/4*cos(f*x+e)*sin(f*x
+e)^3*(a+b+b*tan(f*x+e)^2)^(3/2)/f
```

3.87.2 Mathematica [A] (verified)

Time = 3.57 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.04

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^4(e+fx)\,dx = \frac{3\left(\left(a^2-6ab+b^2\right)\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)+4\sqrt{a}(a-b)\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)\right)\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\tan(e+fx)}{2\sqrt{2}\sqrt{a}f(a+2b+a\cos(2(e+fx)))^{3/2}} + \frac{(-7a+26b+(-6a+10b)\cos(2(e+fx))+a\cos(4(e+fx)))\sqrt{a+b\sec^2(e+fx)}\tan(e+fx)}{32f}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^4,x]
```

output
```
(3*((a^2 - 6*a*b + b^2)*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e
 + f*x]^2]] + 4*Sqrt[a]*(a - b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqr
t[a + b - a*Sin[e + f*x]^2]])*Cos[e + f*x]*(b + a*Cos[e + f*x]^2)*Sqrt[a +
 b*Sec[e + f*x]^2])/(2*Sqrt[2]*Sqrt[a]*f*(a + 2*b + a*Cos[2*(e + f*x)])^(3
/2)) + ((-7*a + 26*b + (-6*a + 10*b)*Cos[2*(e + f*x)] + a*Cos[4*(e + f*x)]
)*Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x])/(32*f)
```

3.87.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.02, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 4620, 369, 27, 439, 444, 27, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx$$

$$\downarrow 3042$$

$$\int \sin(e+fx)^4\left(a+b\sec(e+fx)^2\right)^{3/2}dx$$

$$\downarrow 4620$$

$$\frac{\int \frac{\tan^4(e+fx)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}{\left(\tan^2(e+fx)+1\right)^3}d\tan(e+fx)}{f}$$

$$\downarrow 369$$

$$\frac{\frac{1}{4}\int \frac{3\tan^2(e+fx)\sqrt{b\tan^2(e+fx)+a+b}\left(2b\tan^2(e+fx)+a+b\right)}{\left(\tan^2(e+fx)+1\right)^2}d\tan(e+fx)-\frac{\tan^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4\left(\tan^2(e+fx)+1\right)^2}}{f}$$

$$\downarrow 27$$

$$\frac{\frac{3}{4}\int \frac{\tan^2(e+fx)\sqrt{b\tan^2(e+fx)+a+b}\left(2b\tan^2(e+fx)+a+b\right)}{\left(\tan^2(e+fx)+1\right)^2}d\tan(e+fx)-\frac{\tan^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4\left(\tan^2(e+fx)+1\right)^2}}{f}$$

$$\downarrow 439$$

$$\frac{\frac{3}{4}\left(\frac{(a-b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2\left(\tan^2(e+fx)+1\right)}-\frac{1}{2}\int \frac{\tan^2(e+fx)\left(2(a-3b)b\tan^2(e+fx)+(a-5b)(a+b)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)\right)-\frac{\tan^3(e+fx)(a+b\,t}{4(\tan^2(e+}}{f}$$

$$\downarrow 444$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{\int \frac{2b\left(4(a-b)b\tan^2(e+fx)+(a-3b)(a+b)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2b}-(a-3b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)\right)+\frac{(a-b)\tan^3(e+fx}{2(\tan^2}}{f}$$

$$\downarrow 27$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\int \frac{4(a-b)b\tan^2(e+fx)+(a-3b)(a+b)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-(a-3b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)\right)+\frac{(a-b)\tan^3(}{2}}{f}$$

$$\downarrow 398$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\left(a^2-6ab+b^2\right)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+4b(a-b)\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)\right.\right.}{}$$

$$\downarrow 224$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\left(a^2-6ab+b^2\right)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+4b(a-b)\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)}}\right.\right.}{}$$

$$\downarrow 219$$

3.87. $\int (a+b\sec^2(e+fx))^{3/2}\sin^4(e+fx)\,dx$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\left(a^2-6ab+b^2\right)\int\frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+4\sqrt{b}(a-b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)-(\right.\right.}{f}$$

$$\downarrow 291$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\left(a^2-6ab+b^2\right)\int\frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}+4\sqrt{b}(a-b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)-(a-3b)\right.\right.}{f}$$

$$\downarrow 216$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\left(\frac{\left(a^2-6ab+b^2\right)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}}+4\sqrt{b}(a-b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)-(a-3b)\tan(e+fx)\sqrt{a+}\right.\right.}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^4,x]
```

output
```
(-1/4*(Tan[e + f*x]^3*(a + b + b*Tan[e + f*x]^2)^(3/2))/(1 + Tan[e + f*x]^
2)^2 + (3*(((a - b)*Tan[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*(1 +
 Tan[e + f*x]^2)) + (((a^2 - 6*a*b + b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sq
rt[a + b + b*Tan[e + f*x]^2]])/Sqrt[a] + 4*(a - b)*Sqrt[b]*ArcTanh[(Sqrt[b
]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]] - (a - 3*b)*Tan[e + f*x]*S
qrt[a + b + b*Tan[e + f*x]^2])/2))/4)/f
```

3.87.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 439
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
 + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*b*g*(p + 1))), x] + Simp[1/(2*a*b*(p
+ 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*b*e*(
p + 1) + (b*e - a*f)*(m + 1)) + d*(2*b*e*(p + 1) + (b*e - a*f)*(m + 2*q + 1
))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && LtQ[p, -1] && G
tQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.87.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1185 vs. $2(189) = 378$.

Time = 16.91 (sec) , antiderivative size = 1186, normalized size of antiderivative = 5.47

method	result	size
default	Expression too large to display	1186

input
```
int((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^4,x,method=_RETURNVERBOSE)
```

output

```
1/8/f/(-a)^(1/2)/b*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)/(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))*(2*cos(f*x+e)^6*sin(f*x+e)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*a*b-6*ln(4*(((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*cos(f
*x+e)^3*(-a)^(1/2)*b^(5/2)-6*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*cos(f*x+e)^3*(-a)^(1/2)*b^(5/2)+2*cos
(f*x+e)^5*sin(f*x+e)*a*b*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-a)^
(1/2)+6*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+
e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(
sin(f*x+e)+1))*cos(f*x+e)^3*(-a)^(1/2)*b^(3/2)*a+6*ln(-4*(((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*cos(f*x+e)^3*(-
a)^(1/2)*b^(3/2)*a-5*cos(f*x+e)^4*sin(f*x+e)*a*b*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*(-a)^(1/2)+5*sin(f*x+e)*cos(f*x+e)^4*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*b^2-5*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*a*b*cos(f*x+e)^3*sin(f*x+e)+5*sin(f*x+e)*cos(f*x+e
)^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*b^2+3*ln(4*(-a)
^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1...
```

3.87.5 Fricas [A] (verification not implemented)

Time = 3.35 (sec) , antiderivative size = 1667, normalized size of antiderivative = 7.68

$$\int \left(a + b\sec^2(e+fx)\right)^{3/2} \sin^4(e+fx)\,dx = \text{Too large to display}$$

input

```
integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^4,x, algorithm="fricas")
```

output

```
[-1/64*(3*(a^2 - 6*a*b + b^2)*sqrt(-a)*cos(f*x + e)*log(128*a^4*cos(f*x +
e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2
)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4
- 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 -
 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x
+ e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 24*(a^2 - a*b)*sqrt(b)*c
os(f*x + e)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*
x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) -
 8*(2*a^2*cos(f*x + e)^4 - 5*(a^2 - a*b)*cos(f*x + e)^2 + 4*a*b)*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a*f*cos(f*x + e)), 1/64*
(48*(a^2 - a*b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x
 + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x
+ e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e) - 3*(a^2 - 6*a*b + b^2)*sqrt(-a)
*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^
6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70
*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x
 + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(
5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 -...
```

3.87.6 Sympy [F(-1)]

Timed out.

$$\int (a + b\sec^2(e + fx))^{3/2} \sin^4(e + fx)\, dx = \text{Timed out}$$

input

```
integrate((a+b*sec(f*x+e)**2)**(3/2)*sin(f*x+e)**4,x)
```

output

```
Timed out
```

3.87.7 Maxima [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^4(e+fx)\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sin\left(fx+e\right)^4\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^4,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^4, x)
```

3.87.8 Giac [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^4(e+fx)\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sin\left(fx+e\right)^4\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^4,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^4, x)
```

3.87.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^4(e+fx)\,dx = \int \sin\left(e+f\,x\right)^4\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}dx$$

input
```
int(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.88 $\int (a+b\sec^2(e+fx))^{3/2}\sin^2(e+fx)\,dx$

3.88.1 Optimal result . . . 803
3.88.2 Mathematica [C] (warning: unable to verify) . . . 803
3.88.3 Rubi [A] (verified) . . . 804
3.88.4 Maple [B] (warning: unable to verify) . . . 807
3.88.5 Fricas [B] (verification not implemented) . . . 808
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3.88.8 Giac [F] . . . 810
3.88.9 Mupad [F(-1)] . . . 810

3.88.1 Optimal result

Integrand size = 25, antiderivative size = 161

$$\int (a+b\sec^2(e+fx))^{3/2}\sin^2(e+fx)\,dx = \frac{\sqrt{a}(a-3b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f} + \frac{(3a-b)\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f} + \frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f} - \frac{\cos(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{2f}$$

output

```
1/2*(a-3*b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*a^(1/2)/
f+1/2*(3*a-b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/
2)/f+b*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f-1/2*cos(f*x+e)*sin(f*x+e)*(
a+b+b*tan(f*x+e)^2)^(3/2)/f
```

3.88.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.78 (sec) , antiderivative size = 493, normalized size of antiderivative = 3.06

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^2(e+fx)\,dx = \frac{e^{-i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos^3(e+fx)\left(\frac{i\left(-1+e^{2i(e+fx)}\right)\left(-4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2\right)}{\left(1+e^{2i(e+fx)}\right)^2}\right.}{}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^2,x]
```

output
```
(Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f*x))]*Cos[e + f
*x]^3*((I*(-1 + E^((2*I)*(e + f*x)))*(-4*b*E^((2*I)*(e + f*x)) + a*(1 + E^
((2*I)*(e + f*x)))^2))/(1 + E^((2*I)*(e + f*x)))^2 + (2*E^((2*I)*(e + f*x)
)*(2*Sqrt[a]*(a - 3*b)*f*x - I*Sqrt[a]*(a - 3*b)*Log[a + 2*b + a*E^((2*I)*
(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f
*x)))^2]] + I*Sqrt[a]*(a - 3*b)*Log[a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*
I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e
+ f*x)))^2]] + 2*Sqrt[b]*(-3*a + b)*Log[((Sqrt[b]*(-1 + E^((2*I)*(e + f*x)
)) - I*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*f)/(
b*(-3*a + b)*(1 + E^((2*I)*(e + f*x))))])/Sqrt[4*b*E^((2*I)*(e + f*x)) +
a*(1 + E^((2*I)*(e + f*x)))^2])*(a + b*Sec[e + f*x]^2)^(3/2))/(2*Sqrt[2]*E
^(I*(e + f*x))*f*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2))
```

3.88.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4620, 369, 403, 27, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx$$

$$\downarrow \text{3042}$$

$$\int \sin(e+fx)^2\left(a+b\sec(e+fx)^2\right)^{3/2}dx$$

$$\downarrow 4620$$

$$\frac{\int \frac{\tan^2(e+fx)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}{\left(\tan^2(e+fx)+1\right)^2}d\tan(e+fx)}{f}$$

$$\downarrow 369$$

$$\frac{\frac{1}{2}\int \frac{\sqrt{b\tan^2(e+fx)+a+b}\left(4b\tan^2(e+fx)+a+b\right)}{\tan^2(e+fx)+1}d\tan(e+fx)-\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{2\left(\tan^2(e+fx)+1\right)}}{f}$$

$$\downarrow 403$$

$$\frac{\frac{1}{2}\left(\frac{1}{2}\int \frac{2\left(a^2-b^2+(3a-b)b\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+2b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)-\frac{\tan(e+fx)(a+b\tan^2}{2(\tan^2(e+f}}{f}$$

$$\downarrow 27$$

$$\frac{\frac{1}{2}\left(\int \frac{a^2-b^2+(3a-b)b\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+2b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)-\frac{\tan(e+fx)(a+b\tan^2(e}{2(\tan^2(e+fx)}}{f}$$

$$\downarrow 398$$

$$\frac{\frac{1}{2}\left(b(3a-b)\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+a(a-3b)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+2b\tan(\right.}{f}$$

$$\downarrow 224$$

$$\frac{\frac{1}{2}\left(a(a-3b)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+b(3a-b)\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}+2b\right.}{f}$$

$$\downarrow 219$$

$$\frac{\frac{1}{2}\left(a(a-3b)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+\sqrt{b}(3a-b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)+2b\tan(e+\right.}{f}$$

$$\downarrow 291$$

$$\frac{\frac{1}{2}\left(a(a-3b)\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}+\sqrt{b}(3a-b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)+2b\tan(e+fx)\sqrt{a}\right.}{f}$$

↓ 216

$$\frac{\frac{1}{2}\left(\sqrt{a}(a-3b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)+\sqrt{b}(3a-b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)+2b\tan(e+fx)\sqrt{a+b\tan^2}\right.}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^(3/2)*Sin[e + f*x]^2,x]
```

output
```
(-1/2*(Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/2))/(1 + Tan[e + f*x]^2)
 + (Sqrt[a]*(a - 3*b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e +
 f*x]^2]] + (3*a - b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b +
b*Tan[e + f*x]^2]] + 2*b*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/2)/f
```

3.88.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.88.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 899 vs. $2(139) = 278$.

Time = 11.54 (sec) , antiderivative size = 900, normalized size of antiderivative = 5.59

method	result	size
default	Expression too large to display	900

input
```
int((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4/f/b/(-a)^(1/2)*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)/(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))*(cos(f*x+e)^3*b^(5/2)*ln(
4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x
+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-a-b)/(sin(f*x+e)
-1))*(-a)^(1/2)+cos(f*x+e)^3*b^(5/2)*ln(-4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)+a+b)/(sin(f*x+e)+1))*(-a)^(1/2)-3*cos(f*x+e)^3*b^
(3/2)*ln(4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e
)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-a-b)/(s
in(f*x+e)-1))*(-a)^(1/2)*a-3*cos(f*x+e)^3*b^(3/2)*ln(-4*(-((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+a+b)/(sin(f*x+e)+1))*(-a)^(1/2)*a+2*
cos(f*x+e)^4*sin(f*x+e)*a*b*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-
a)^(1/2)+2*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b*cos(
f*x+e)^3*sin(f*x+e)-2*sin(f*x+e)*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*(-a)^(1/2)*b^2-2*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)-4*sin(f*x+e)*a)*cos(f*x+e)^3*a^2*b+6*ln(4*(-a)^(1/2)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*cos(f*x+e)^3*a*b^2-2*(-a)...
```

3.88.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(139) = 278$.

Time = 1.18 (sec) , antiderivative size = 1535, normalized size of antiderivative = 9.53

$$\int \left(a + b\sec^2(e+fx)\right)^{3/2} \sin^2(e+fx)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^2,x, algorithm="fricas")
```

output

```
[-1/16*(sqrt(-a)*(a - 3*b)*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(
a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x +
e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b +
7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a
^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^
3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 2*(3*a - b)*sqrt(b)*cos(f*x + e)*log
(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 - 4*((
a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 8*(a*cos(f*x +
 e)^2 - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(f*co
s(f*x + e)), 1/16*(4*(3*a - b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^
3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e) - sqrt(-a)*(a - 3
*b)*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x +
e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b +
 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(
f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 +
2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 -
 b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2...
```

3.88.6 Sympy [F(-1)]

Timed out.

$$\int (a + b\sec^2(e + fx))^{3/2} \sin^2(e + fx)\, dx = \text{Timed out}$$

input

```
integrate((a+b*sec(f*x+e)**2)**(3/2)*sin(f*x+e)**2,x)
```

output

```
Timed out
```

3.88.7 Maxima [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^2(e+fx)\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sin\left(fx+e\right)^2\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^2, x)
```

3.88.8 Giac [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^2(e+fx)\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sin\left(fx+e\right)^2\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^2, x)
```

3.88.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\sin^2(e+fx)\,dx = \int \sin\left(e+f\,x\right)^2\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}dx$$

input
```
int(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.89 $\int (a+b\sec^2(e+fx))^{3/2}\,dx$

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3.89.1 Optimal result

Integrand size = 16, antiderivative size = 118

$$\int (a+b\sec^2(e+fx))^{3/2}\,dx = \frac{a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{\sqrt{b}(3a+b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f} + \frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f}$$

output

```
a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f+1/2*(3*a+b
)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f+1/2*b*(
a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f
```

3.89.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.46 (sec) , antiderivative size = 527, normalized size of antiderivative = 4.47

$$\int (a+b\sec^2(e+fx))^{3/2}\,dx = \frac{\sqrt{2}e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos^3(e+fx)\left(-\frac{ib\left(-1+e^{2i(e+fx)}\right)}{\left(1+e^{2i(e+fx)}\right)^2}+\frac{2a^{3/2}fx-ia^{3/2}\log(\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[2]*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*
I)*(e + f*x))]*Cos[e + f*x]^3*(((-I)*b*(-1 + E^((2*I)*(e + f*x))))/(1 + E^
((2*I)*(e + f*x)))^2 + (2*a^(3/2)*f*x - I*a^(3/2)*Log[a + 2*b + a*E^((2*I)
*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e +
f*x)))^2]] + I*a^(3/2)*Log[a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*I)*(e + f
*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^
2]] - 3*a*Sqrt[b]*Log[(-2*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*f + (2*I)*Sqr
t[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)/(b*(3*a + b)
*(1 + E^((2*I)*(e + f*x))))] - b^(3/2)*Log[(-2*Sqrt[b]*(-1 + E^((2*I)*(e +
 f*x)))*f + (2*I)*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)
))^2]*f)/(b*(3*a + b)*(1 + E^((2*I)*(e + f*x))))])/Sqrt[4*b*E^((2*I)*(e +
f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*(a + b*Sec[e + f*x]^2)^(3/2))/(f*(
a + 2*b + a*Cos[2*e + 2*f*x])^(3/2))
```

3.89.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4616, 318, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a + b\sec^2(e+fx)\right)^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int \left(a + b\sec(e+fx)^2\right)^{3/2} dx$$

$$\downarrow 4616$$

$$\frac{\int \frac{\left(b\tan^2(e+fx)+a+b\right)^{3/2}}{\tan^2(e+fx)+1} d\tan(e+fx)}{f}$$

$$\downarrow 318$$

$$\frac{\frac{1}{2}\int \frac{b(3a+b)\tan^2(e+fx)+(a+b)(2a+b)}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}} d\tan(e+fx) + \frac{1}{2}b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

$$\downarrow 398$$

$$\frac{\frac{1}{2}\left(2a^2\int\frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+b(3a+b)\int\frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)\right)+\frac{1}{2}b\tan(e+}{f}$$

$$\downarrow 224$$

$$\frac{\frac{1}{2}\left(2a^2\int\frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+b(3a+b)\int\frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}\right)+\frac{1}{2}b\tan}{f}$$

$$\downarrow 219$$

$$\frac{\frac{1}{2}\left(2a^2\int\frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+\sqrt{b}(3a+b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)\right)+\frac{1}{2}b\tan(e+fx)}{f}$$

$$\downarrow 291$$

$$\frac{\frac{1}{2}\left(2a^2\int\frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}+\sqrt{b}(3a+b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)\right)+\frac{1}{2}b\tan(e+fx)\sqrt{a+b}}{f}$$

$$\downarrow 216$$

$$\frac{\frac{1}{2}\left(2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)+\sqrt{b}(3a+b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)\right)+\frac{1}{2}b\tan(e+fx)\sqrt{a+b\tan^2(e+}}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
((2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]
+ Sqrt[b]*(3*a + b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e +
f*x]^2]])/2 + (b*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/2)/f
```

3.89.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

3.89.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 708 vs. $2(100) = 200$.

Time = 9.88 (sec) , antiderivative size = 709, normalized size of antiderivative = 6.01

method	result
default	$\frac{\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}\left(\cos(fx+e)^3 b^{\frac{5}{2}}\ln\left(\frac{-4\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{b}\cos(fx+e)-4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}+4\sin(fx+e)a-4a-4b}{\sin(fx+e)-1}\right)\sqrt{-a}+\cos(f\right.}{}$

input

```
int((a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
1/4/f/(-a)^(1/2)/b*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)/(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))*(cos(f*x+e)^3*b^(5/2)*ln(4
*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+
e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-a-b)/(sin(f*x+e)-
1))*(-a)^(1/2)+cos(f*x+e)^3*b^(5/2)*ln(-4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)+a+b)/(sin(f*x+e)+1))*(-a)^(1/2)+3*cos(f*x+e)^3*b^(
3/2)*ln(4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)
+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-a-b)/(si
n(f*x+e)-1))*(-a)^(1/2)*a+3*cos(f*x+e)^3*b^(3/2)*ln(-4*(-((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+a+b)/(sin(f*x+e)+1))*(-a)^(1/2)*a+2*s
in(f*x+e)*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1
/2)*b^2+4*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(
f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+
e)*a)*cos(f*x+e)^3*a^2*b+2*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*b^2*cos(f*x+e)*sin(f*x+e))
```

3.89.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(100) = 200$.

Time = 0.61 (sec) , antiderivative size = 1457, normalized size of antiderivative = 12.35

$$\int \left(a + b\sec^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(sqrt(-a)*a*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*
b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4
 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2
- a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(
f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*
b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)*sin(f*x + e)) + (3*a + b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a
*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f
*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 4*b*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)*sin(f*x + e))/(f*cos(f*x + e)), 1/8*(2*(3*a + b)*sqr
t(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f
*x + e)))*cos(f*x + e) + sqrt(-a)*a*cos(f*x + e)*log(128*a^4*cos(f*x + e)^
8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*c
os(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7
*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24
*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e
)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 4*b*sqrt((a*cos(f*x + e)...
```

3.89.6 Sympy [F]

$$\int \left(a + b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(a + b\sec^2(e+fx)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(3/2), x)
```

3.89.7 Maxima [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.89.8 Giac [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.89.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int((a + b/cos(e + f*x)^2)^(3/2), x)
```

3.90 $\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

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3.90.1 Optimal result

Integrand size = 25, antiderivative size = 105

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{3\sqrt{b}(a+b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f} + \frac{3b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f} - \frac{\cot(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{f}$$

output
```
3/2*(a+b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f
+3/2*b*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f-cot(f*x+e)*(a+b+b*tan(f*x+e
)^2)^(3/2)/f
```

3.90.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.61

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = -\frac{(a+b)\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},2,\frac{1}{2},\frac{b\sin^2(e+fx)}{a+b-a\sin^2(e+fx)}\right)\sqrt{a+b\sec^2(e+fx)}}{f}$$

input
```
Integrate[Csc[e + f*x]^2*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
-(((a + b)*Cot[e + f*x]*Hypergeometric2F1[-1/2, 2, 1/2, (b*Sin[e + f*x]^2)
/(a + b - a*Sin[e + f*x]^2)]*Sqrt[a + b*Sec[e + f*x]^2])/f)
```

3.90.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 4620, 247, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^{3/2}}{\sin(e+fx)^2}dx$$

$$\downarrow 4620$$

$$\frac{\int \cot^2(e+fx)\left(b\tan^2(e+fx)+a+b\right)^{3/2}d\tan(e+fx)}{f}$$

$$\downarrow 247$$

$$\frac{3b\int \sqrt{b\tan^2(e+fx)+a+b}d\tan(e+fx)-\cot(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{f}$$

$$\downarrow 211$$

$$\frac{3b\left(\frac{1}{2}(a+b)\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)-\cot(e+fx)\left(a+b\tan^{2}\right.}{f}$$

$$\downarrow 224$$

$$\frac{3b\left(\frac{1}{2}(a+b)\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)-\cot(e+fx)\left(a+\right.}{f}$$

$$\downarrow 219$$

$$\frac{3b\left(\frac{(a+b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2\sqrt{b}}+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)-\cot(e+fx)\left(a+b\tan^2(e+fx)\right.}{f}$$

input
```
Int[Csc[e + f*x]^2*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(-(Cot[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/2)) + 3*b*(((a + b)*ArcTanh[
(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(2*Sqrt[b]) + (Tan
[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/2))/f
```

3.90.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.90.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 740 vs. $2(91) = 182$.

Time = 10.17 (sec) , antiderivative size = 741, normalized size of antiderivative = 7.06

method	result
default	$\frac{\left(3b^{\frac{5}{2}}\ln\left(\frac{-4\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{b}\cos(fx+e)-4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}+4\sin(fx+e)a-4a-4b}{\sin(fx+e)-1}\right)\cos(fx+e)^2\sin(fx+e)+3b^{\frac{5}{2}}\ln\left(-\frac{4\left(-\sqrt{\frac{b-}{(1}\right.\right.}$

input

```
int(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
1/4/f/b*(3*b^(5/2)*ln(4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1
/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)-a-b)/(sin(f*x+e)-1))*cos(f*x+e)^2*sin(f*x+e)+3*b^(5/2)*ln(-4*(-((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(
1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+a+b)/(sin(f*x+e)+1))*cos(
f*x+e)^2*sin(f*x+e)+3*b^(3/2)*ln(4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)-a-b)/(sin(f*x+e)-1))*a*cos(f*x+e)^2*sin(f*x+e)+3*b^(3/2)*
ln(-4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin
(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+a+b)/(sin(f*
x+e)+1))*a*cos(f*x+e)^2*sin(f*x+e)-4*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*a*b-6*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2*
cos(f*x+e)^3-4*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*
b-6*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2+2*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2*cos(f*x+e)+2*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*b^2)*(a+b*sec(f*x+e)^2)^(3/2)/(b+a*cos(f*x+e)^2)/(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)/(1+cos(f*x+e))*cot(f*x+e)
```

3.90.5 Fricas [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 370, normalized size of antiderivative = 3.52

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \left[\frac{3\,(a+b)\sqrt{b}\cos\left(fx+e\right)\log\left(\frac{(a^2-6\,ab+b^2)\cos(fx+e)^4+8\,(ab-b^2)\cos(fx+e)^2+4\left((a-b)\cos(fx+e)^3+2\,b\cos(f\right.\right.}{\cos(fx+e)^4}}{8\,f\cos\left(fx-\right.}\right.$$

input
```
integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(3*(a + b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)
^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*
x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) +
 8*b^2)/cos(f*x + e)^4)*sin(f*x + e) - 4*((2*a + 3*b)*cos(f*x + e)^2 - b)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x + e)*sin(f*x + e))
, 1/4*(3*(a + b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*
x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x
 + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e)*sin(f*x + e) - 2*((2*a + 3*b)*c
os(f*x + e)^2 - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(f*cos(f*x
 + e)*sin(f*x + e))]
```

3.90.6 Sympy [F(-1)]

Timed out.

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**2*(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

3.90.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.93

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{3\,a\sqrt{b}\,\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)+3\,b^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)+3\sqrt{b\tan(fx+e)^2+a+b}\,b\tan(fx+$$

input
```
integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/2*(3*a*sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b)) + 3*b^(3/2)*arcsi
nh(b*tan(f*x + e)/sqrt((a + b)*b)) + 3*sqrt(b*tan(f*x + e)^2 + a + b)*b*ta
n(f*x + e) - 2*(b*tan(f*x + e)^2 + a + b)^(3/2)/tan(f*x + e))/f
```

3.90.8 Giac [F]

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}\csc(fx+e)^2\,dx$$

input
```
integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^2, x)
```

3.90.9 Mupad [F(-1)]

Timed out.

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \frac{\left(a+\frac{b}{\cos(e+fx)^2}\right)^{3/2}}{\sin(e+fx)^2}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x)^2,x)
```

output
```
int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x)^2, x)
```

3.91 $\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

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3.91.1 Optimal result

Integrand size = 25, antiderivative size = 172

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\sqrt{b}(3a+5b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f} + \frac{b(3a+5b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2(a+b)f} - \frac{(3a+5b)\cot(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{3(a+b)f} - \frac{\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{3(a+b)f}$$

output

```
1/2*(3*a+5*b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/
2)/f+1/2*b*(3*a+5*b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/(a+b)/f-1/3*(3*
a+5*b)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(3/2)/(a+b)/f-1/3*cot(f*x+e)^3*(a+b
+b*tan(f*x+e)^2)^(5/2)/(a+b)/f
```

3.91.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.69 (sec) , antiderivative size = 369, normalized size of antiderivative = 2.15

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\sqrt{2}e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos^3(e+fx)\left(-\frac{i\left(4a\left(1+e^{2i(e+fx)}\right)^2\left(1-4e^{2i(e+fx)}+e^{4i(e+\right.\right.}{(-1}\right.}$$

input
```
Integrate[Csc[e + f*x]^4*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[2]*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*
I)*(e + f*x))]*Cos[e + f*x]^3*(((-I)*(4*a*(1 + E^((2*I)*(e + f*x)))^2*(1 -
 4*E^((2*I)*(e + f*x)) + E^((4*I)*(e + f*x))) + b*(15 - 20*E^((2*I)*(e + f
*x)) - 22*E^((4*I)*(e + f*x)) - 20*E^((6*I)*(e + f*x)) + 15*E^((8*I)*(e +
f*x)))))/((-1 + E^((2*I)*(e + f*x)))^3*(1 + E^((2*I)*(e + f*x)))^2) - (3*S
qrt[b]*(3*a + 5*b)*Log[(-4*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*f + (4*I)*Sq
rt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)/(1 + E^((2*
I)*(e + f*x)))])/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x))
)^2])*(a + b*Sec[e + f*x]^2)^(3/2))/(3*f*(a + 2*b + a*Cos[2*e + 2*f*x])^(3
/2))
```

3.91.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.90, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4620, 359, 247, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^{3/2}}{\sin(e+fx)^4}dx$$

$$\downarrow 4620$$

$$\frac{\int \cot^4(e+fx)\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}d\tan(e+fx)}{f}$$

$$\downarrow 359$$

$$\frac{\frac{(3a+5b)\int \cot^2(e+fx)\left(b\tan^2(e+fx)+a+b\right)^{3/2}d\tan(e+fx)}{3(a+b)} - \frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{3(a+b)}}{f}$$

$$\downarrow 247$$

$$\frac{\frac{(3a+5b)\left(3b\int \sqrt{b\tan^2(e+fx)+a+b}d\tan(e+fx)-\cot(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}\right)}{3(a+b)} - \frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{3(a+b)}}{f}$$

$$\downarrow 211$$

$$\frac{\frac{(3a+5b)\left(3b\left(\frac{1}{2}(a+b)\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)-\cot(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}\right)}{3(a+b)} - \frac{\cot^3(e+}{}}{f}$$

$$\downarrow 224$$

$$\frac{\frac{(3a+5b)\left(3b\left(\frac{1}{2}(a+b)\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)-\cot(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}\right)}{3(a+b)} - }{f}$$

$$\downarrow 219$$

$$\frac{\frac{(3a+5b)\left(3b\left(\frac{(a+b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2\sqrt{b}}+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)-\cot(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}\right)}{3(a+b)} - \frac{\cot^3(e+fx}{}}{f}$$

input
```
Int[Csc[e + f*x]^4*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

3.91. $\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

output
```
(-1/3*(Cot[e + f*x]^3*(a + b + b*Tan[e + f*x]^2)^(5/2))/(a + b) + ((3*a +
5*b)*(-(Cot[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/2)) + 3*b*(((a + b)*Arc
Tanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(2*Sqrt[b]) +
 (Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/2)))/(3*(a + b)))/f
```

3.91.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.91.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1140 vs. $2(152) = 304$.

Time = 10.65 (sec) , antiderivative size = 1141, normalized size of antiderivative = 6.63

method	result	size
default	Expression too large to display	1141

input
```
int(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12/f/b*(15*cos(f*x+e)^3*sin(f*x+e)*b^(5/2)*ln(4*(((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))+15*cos(f*x+e)^3*sin(f
*x+e)*b^(5/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*c
os(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a
+a+b)/(sin(f*x+e)-1))-15*cos(f*x+e)^2*sin(f*x+e)*b^(5/2)*ln(4*(((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))-15*cos(f*x
+e)^2*sin(f*x+e)*b^(5/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-s
in(f*x+e)*a+a+b)/(sin(f*x+e)-1))+9*cos(f*x+e)^3*sin(f*x+e)*b^(3/2)*a*ln(4*
(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1)
)+9*cos(f*x+e)^3*sin(f*x+e)*b^(3/2)*a*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))-9*cos(f*x+e)^2*sin(f*x+e)*b^
(3/2)*a*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+
e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(
sin(f*x+e)+1))-9*cos(f*x+e)^2*sin(f*x+e)*b^(3/2)*a*ln(-4*(((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)...
```

3.91.5 Fricas [A] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 472, normalized size of antiderivative = 2.74

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \left[\frac{3\left(\left(3\,a+5\,b\right)\cos\left(fx+e\right)^3-\left(3\,a+5\,b\right)\cos\left(fx+e\right)\right)\sqrt{b}\log\left(\frac{\left(a^2-6\,ab+b^2\right)\cos(fx+e)^4+8\left(ab-b^2\right)\cos(fx+e)^2+4\left(\left(a-b\right)\cos(fx+e)^3+2\,b\cos(fx+e)\right)\sqrt{b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+e)+8\,b^2}{\cos(fx+e)^4}\right)\sin(fx+e)-4\left(\left(4\,a+15\,b\right)\cos(fx+e)^4-2\left(3\,a+10\,b\right)\cos(fx+e)^2+3\,b\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{24\left(f\cos(fx+e)^3-f\cos(fx+e)\right)\sin(fx+e)}\right.$$

input
```
integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/24*(3*((3*a + 5*b)*cos(f*x + e)^3 - (3*a + 5*b)*cos(f*x + e))*sqrt(b)*l
og(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*
((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)*sin(f*x + e) -
 4*((4*a + 15*b)*cos(f*x + e)^4 - 2*(3*a + 10*b)*cos(f*x + e)^2 + 3*b)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((f*cos(f*x + e)^3 - f*cos(f*x +
 e))*sin(f*x + e)), 1/12*(3*((3*a + 5*b)*cos(f*x + e)^3 - (3*a + 5*b)*cos(
f*x + e))*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))
*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2
 + b^2)*sin(f*x + e)))*sin(f*x + e) - 2*((4*a + 15*b)*cos(f*x + e)^4 - 2*(
3*a + 10*b)*cos(f*x + e)^2 + 3*b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2))/((f*cos(f*x + e)^3 - f*cos(f*x + e))*sin(f*x + e))]
```

3.91.6 Sympy [F(-1)]

Timed out.

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**4*(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

3.91.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.41

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{9\,a\sqrt{b}\,\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right) + \frac{6\,ab^{\frac{3}{2}}\,\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a+b} + 9\,b^{\frac{3}{2}}\,\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right) + \frac{6\,b^{\frac{5}{2}}\,\operatorname{arsinh}\left(\frac{b\tan(fx}{\sqrt{(a+b}}\right.}{a+b}$$

input
```
integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/6*(9*a*sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b)) + 6*a*b^(3/2)*arc
sinh(b*tan(f*x + e)/sqrt((a + b)*b))/(a + b) + 9*b^(3/2)*arcsinh(b*tan(f*x
 + e)/sqrt((a + b)*b)) + 6*b^(5/2)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))
/(a + b) + 9*sqrt(b*tan(f*x + e)^2 + a + b)*b*tan(f*x + e) + 6*sqrt(b*tan(
f*x + e)^2 + a + b)*b^2*tan(f*x + e)/(a + b) - 6*(b*tan(f*x + e)^2 + a + b
)^(3/2)/tan(f*x + e) - 4*(b*tan(f*x + e)^2 + a + b)^(3/2)*b/((a + b)*tan(f
*x + e)) - 2*(b*tan(f*x + e)^2 + a + b)^(5/2)/((a + b)*tan(f*x + e)^3))/f
```

3.91.8 Giac [F]

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^4\,dx$$

input
```
integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^4, x)
```

3.91.9 Mupad [F(-1)]

Timed out.

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \frac{\left(a+\frac{b}{\cos(e+fx)^2}\right)^{3/2}}{\sin\left(e+fx\right)^4}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x)^4,x)
```

output
```
int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x)^4, x)
```

3.92 $\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

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3.92.1 Optimal result

Integrand size = 25, antiderivative size = 209

$$\begin{aligned}\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx &= \frac{\sqrt{b}(3a+7b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f}\\ &+\frac{b(3a+7b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2(a+b)f}\\ &-\frac{(3a+7b)\cot(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{3(a+b)f}\\ &-\frac{2\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{3(a+b)f}\\ &-\frac{\cot^5(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{5(a+b)f}\end{aligned}$$

output

```
1/2*(3*a+7*b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/
2)/f+1/2*b*(3*a+7*b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/(a+b)/f-1/3*(3*
a+7*b)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(3/2)/(a+b)/f-2/3*cot(f*x+e)^3*(a+b
+b*tan(f*x+e)^2)^(5/2)/(a+b)/f-1/5*cot(f*x+e)^5*(a+b+b*tan(f*x+e)^2)^(5/2)
/(a+b)/f
```

3.92.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.26 (sec) , antiderivative size = 512, normalized size of antiderivative = 2.45

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\sqrt{2}e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos^3(e+fx)\left(-\frac{i\left(16a^2\left(1+e^{2i(e+fx)}\right)^2\left(1-6e^{2i(e+fx)}+16e\right.\right.\right.}{}$$

input
```
Integrate[Csc[e + f*x]^6*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[2]*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*
I)*(e + f*x))]*Cos[e + f*x]^3*(((-I)*(16*a^2*(1 + E^((2*I)*(e + f*x)))^2*(
1 - 6*E^((2*I)*(e + f*x)) + 16*E^((4*I)*(e + f*x)) - 6*E^((6*I)*(e + f*x))
 + E^((8*I)*(e + f*x))) + b^2*(105 - 350*E^((2*I)*(e + f*x)) + 231*E^((4*I
)*(e + f*x)) + 412*E^((6*I)*(e + f*x)) + 231*E^((8*I)*(e + f*x)) - 350*E^(
(10*I)*(e + f*x)) + 105*E^((12*I)*(e + f*x))) + a*b*(115 - 402*E^((2*I)*(e
 + f*x)) + 317*E^((4*I)*(e + f*x)) + 708*E^((6*I)*(e + f*x)) + 317*E^((8*I
)*(e + f*x)) - 402*E^((10*I)*(e + f*x)) + 115*E^((12*I)*(e + f*x)))))/((a
+ b)*(-1 + E^((2*I)*(e + f*x)))^5*(1 + E^((2*I)*(e + f*x)))^2) - (15*Sqrt[
b]*(3*a + 7*b)*Log[(-4*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*f + (4*I)*Sqrt[4
*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)/(1 + E^((2*I)*(
e + f*x)))])/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]
)*(a + b*Sec[e + f*x]^2)^(3/2))/(15*f*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2)
)
```

3.92.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.90, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4620, 365, 27, 359, 247, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^{3/2}}{\sin(e+fx)^6}dx$$

$$\downarrow 4620$$

$$\frac{\int \cot^6(e+fx)\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^{3/2}d\tan(e+fx)}{f}$$

$$\downarrow 365$$

$$\frac{\frac{\int 5(a+b)\cot^4(e+fx)(\tan^2(e+fx)+2)(b\tan^2(e+fx)+a+b)^{3/2}d\tan(e+fx)}{5(a+b)}-\frac{\cot^5(e+fx)(a+b\tan^2(e+fx)+b)^{5/2}}{5(a+b)}}{f}$$

$$\downarrow 27$$

$$\frac{\int \cot^4(e+fx)\left(\tan^2(e+fx)+2\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}d\tan(e+fx)-\frac{\cot^5(e+fx)(a+b\tan^2(e+fx)+b)^{5/2}}{5(a+b)}}{f}$$

$$\downarrow 359$$

$$\frac{\frac{(3a+7b)\int \cot^2(e+fx)(b\tan^2(e+fx)+a+b)^{3/2}d\tan(e+fx)}{3(a+b)}-\frac{\cot^5(e+fx)(a+b\tan^2(e+fx)+b)^{5/2}}{5(a+b)}-\frac{2\cot^3(e+fx)(a+b\tan^2(e+fx)+b)^{5/2}}{3(a+b)}}{f}$$

$$\downarrow 247$$

$$\frac{\frac{(3a+7b)\left(3b\int \sqrt{b\tan^2(e+fx)+a+b}d\tan(e+fx)-\cot(e+fx)(a+b\tan^2(e+fx)+b)^{3/2}\right)}{3(a+b)}-\frac{\cot^5(e+fx)(a+b\tan^2(e+fx)+b)^{5/2}}{5(a+b)}-\frac{2\cot^3(e+fx)}{}}{f}$$

$$\downarrow 211$$

$$\frac{\frac{(3a+7b)\left(3b\left(\frac{1}{2}(a+b)\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)-\cot(e+fx)(a+b\tan^2(e+fx)+b)^{3/2}\right)}{3(a+b)}-\frac{\cot^5(e+}{}}{f}$$

$$\downarrow 224$$

$$\frac{\frac{(3a+7b)\left(3b\left(\frac{1}{2}(a+b)\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)-\cot(e+fx)(a+b\tan^2(e+fx)+b)^{3/2}\right)}{3(a+b)}-}{f}$$

3.92. $\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

$$\downarrow 219$$

$$\frac{(3a+7b)\left(3b\left(\frac{(a+b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2\sqrt{b}}+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)-\cot(e+fx)(a+b\tan^2(e+fx)+b)^{3/2}\right)}{3(a+b)}-\frac{\cot^5(e+fx)}{f}$$

input
```
Int[Csc[e + f*x]^6*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
((-2*Cot[e + f*x]^3*(a + b + b*Tan[e + f*x]^2)^(5/2))/(3*(a + b)) - (Cot[e
 + f*x]^5*(a + b + b*Tan[e + f*x]^2)^(5/2))/(5*(a + b)) + ((3*a + 7*b)*(-(
Cot[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/2)) + 3*b*(((a + b)*ArcTanh[(Sq
rt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(2*Sqrt[b]) + (Tan[e
+ f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/2)))/(3*(a + b)))/f
```

3.92.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.92.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1850 vs. $2(185) = 370$.

Time = 11.45 (sec) , antiderivative size = 1851, normalized size of antiderivative = 8.86

method	result	size
default	Expression too large to display	1851

input
```
int(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/60/f/(a+b)/b*(105*cos(f*x+e)^3*sin(f*x+e)^3*b^(7/2)*ln(4*(((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))+105*cos(f*x+
e)^3*sin(f*x+e)^3*b^(7/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-
sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))-105*cos(f*x+e)^2*sin(f*x+e)^3*b^(7/2)*ln
(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)
+1))-105*cos(f*x+e)^2*sin(f*x+e)^3*b^(7/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))+150*cos(f*x+e)^3*sin(f*
x+e)^3*b^(5/2)*a*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)
*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)
*a-a-b)/(sin(f*x+e)+1))+150*cos(f*x+e)^3*sin(f*x+e)^3*b^(5/2)*a*ln(-4*(((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))-15
0*cos(f*x+e)^2*sin(f*x+e)^3*b^(5/2)*a*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))-150*cos(f*x+e)^2*sin(f*x+e)^3
*b^(5/2)*a*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*c...
```

3.92.5 Fricas [A] (verification not implemented)

Time = 6.34 (sec) , antiderivative size = 682, normalized size of antiderivative = 3.26

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \left[\frac{15\left(\left(3\,a^2+10\,ab+7\,b^2\right)\cos\left(fx+e\right)^5-2\left(3\,a^2+10\,ab+7\,b^2\right)\cos\left(fx+e\right)^3+\left(3\,a^2+10\,ab\right.\right.\right.}{}$$

input
```
integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/120*(15*((3*a^2 + 10*a*b + 7*b^2)*cos(f*x + e)^5 - 2*(3*a^2 + 10*a*b +
7*b^2)*cos(f*x + e)^3 + (3*a^2 + 10*a*b + 7*b^2)*cos(f*x + e))*sqrt(b)*log
(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((
a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)*sin(f*x + e) - 4
*((16*a^2 + 115*a*b + 105*b^2)*cos(f*x + e)^6 - (40*a^2 + 273*a*b + 245*b^
2)*cos(f*x + e)^4 + (30*a^2 + 185*a*b + 161*b^2)*cos(f*x + e)^2 - 15*a*b -
 15*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a + b)*f*cos(f*x
+ e)^5 - 2*(a + b)*f*cos(f*x + e)^3 + (a + b)*f*cos(f*x + e))*sin(f*x + e)
), 1/60*(15*((3*a^2 + 10*a*b + 7*b^2)*cos(f*x + e)^5 - 2*(3*a^2 + 10*a*b +
 7*b^2)*cos(f*x + e)^3 + (3*a^2 + 10*a*b + 7*b^2)*cos(f*x + e))*sqrt(-b)*a
rctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)
))*sin(f*x + e) - 2*((16*a^2 + 115*a*b + 105*b^2)*cos(f*x + e)^6 - (40*a^2
 + 273*a*b + 245*b^2)*cos(f*x + e)^4 + (30*a^2 + 185*a*b + 161*b^2)*cos(f*
x + e)^2 - 15*a*b - 15*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(
((a + b)*f*cos(f*x + e)^5 - 2*(a + b)*f*cos(f*x + e)^3 + (a + b)*f*cos(f*x
 + e))*sin(f*x + e))]
```

3.92.6 Sympy [F(-1)]

Timed out.

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**6*(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

3.92.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.31

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{45\,a\sqrt{b}\,\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right) + \frac{60\,ab^{\frac{3}{2}}\,\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a+b} + 45\,b^{\frac{3}{2}}\,\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right) + \frac{60\,b^{\frac{5}{2}}\,\operatorname{arsinh}\left(\frac{b}{a+b}\right.}{a+b}$$

input
```
integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/30*(45*a*sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b)) + 60*a*b^(3/2)*
arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/(a + b) + 45*b^(3/2)*arcsinh(b*tan
(f*x + e)/sqrt((a + b)*b)) + 60*b^(5/2)*arcsinh(b*tan(f*x + e)/sqrt((a + b
)*b))/(a + b) + 45*sqrt(b*tan(f*x + e)^2 + a + b)*b*tan(f*x + e) + 60*sqrt
(b*tan(f*x + e)^2 + a + b)*b^2*tan(f*x + e)/(a + b) - 30*(b*tan(f*x + e)^2
 + a + b)^(3/2)/tan(f*x + e) - 40*(b*tan(f*x + e)^2 + a + b)^(3/2)*b/((a +
 b)*tan(f*x + e)) - 20*(b*tan(f*x + e)^2 + a + b)^(5/2)/((a + b)*tan(f*x +
 e)^3) - 6*(b*tan(f*x + e)^2 + a + b)^(5/2)/((a + b)*tan(f*x + e)^5))/f
```

3.92.8 Giac [F]

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^6\,dx$$

input
```
integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^6, x)
```

3.92.9 Mupad [F(-1)]

Timed out.

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}{\sin\left(e+f\,x\right)^6}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x)^6,x)
```

output
```
int((a + b/cos(e + f*x)^2)^(3/2)/sin(e + f*x)^6, x)
```

3.93 $\int \frac{\sin^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

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3.93.1 Optimal result

Integrand size = 25, antiderivative size = 123

$$\int \frac{\sin^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{(15a^2+20ab+8b^2)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{15a^3f} + \frac{2(5a+2b)\cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{15a^2f} - \frac{\cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}}{5af}$$

output

```
-1/15*(15*a^2+20*a*b+8*b^2)*cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/a^3/f+2/15
*(5*a+2*b)*cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2)/a^2/f-1/5*cos(f*x+e)^5*(a
+b*sec(f*x+e)^2)^(1/2)/a/f
```

3.93.2 Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.76

$$\int \frac{\sin^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))\,(89a^2+144ab+64b^2-4a(7a+4b)\cos(2(e+fx))+3a^2\cos(4(e+fx)))\,s}{240a^3f\sqrt{a+b\sec^2(e+fx)}}$$

input
```
Integrate[Sin[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
-1/240*((a + 2*b + a*Cos[2*(e + f*x)])*(89*a^2 + 144*a*b + 64*b^2 - 4*a*(7
*a + 4*b)*Cos[2*(e + f*x)] + 3*a^2*Cos[4*(e + f*x)])*Sec[e + f*x])/(a^3*f*
Sqrt[a + b*Sec[e + f*x]^2])
```

3.93.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 4622, 365, 25, 359, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sin(e+fx)^5}{\sqrt{a+b\sec(e+fx)^2}}\,dx$$

$$\downarrow 4622$$

$$\frac{\int \frac{\cos^6(e+fx)\left(1-\sec^2(e+fx)\right)^2}{\sqrt{b\sec^2(e+fx)+a}}\,d\sec(e+fx)}{f}$$

$$\downarrow 365$$

$$\frac{\int -\frac{\cos^4(e+fx)\left(2(5a+2b)-5a\sec^2(e+fx)\right)}{\sqrt{b\sec^2(e+fx)+a}}\,d\sec(e+fx)}{5a} - \frac{\cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}}{5a}}{f}$$

$$\downarrow 25$$

$$\frac{-\frac{\int \frac{\cos^4(e+fx)\left(2(5a+2b)-5a\sec^2(e+fx)\right)}{\sqrt{b\sec^2(e+fx)+a}}\,d\sec(e+fx)}{5a} - \frac{\cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}}{5a}}{f}$$

$$\downarrow 359$$

$$\frac{-\frac{\left(15a^2+20ab+8b^2\right)\int \frac{\cos^2(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}\,d\sec(e+fx)}{3a} - \frac{2(5a+2b)\cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{3a}}{5a} - \frac{\cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}}{5a}}{f}$$

$$\downarrow 242$$

$$\frac{-\frac{\left(15a^2+20ab+8b^2\right)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{3a^2}-\frac{2(5a+2b)\cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{3a}}{5a}-\frac{\cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}}{5a}}{f}$$

input
```
Int[Sin[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(-1/5*(Cos[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2])/a - (((15*a^2 + 20*a*b +
 8*b^2)*Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(3*a^2) - (2*(5*a + 2*b)*
Cos[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2])/(3*a))/(5*a))/f
```

3.93.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

3.93.4 Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.76

method	result	size
default	$-\frac{\left(b+a\cos(fx+e)^2\right)\left(3\cos(fx+e)^4a^2-10\cos(fx+e)^2a^2-4\cos(fx+e)^2ab+15a^2+20ab+8b^2\right)\sec(fx+e)}{15fa^3\sqrt{a+b\sec(fx+e)^2}}$	94

input
```
int(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15/f/a^3*(b+a*cos(f*x+e)^2)*(3*cos(f*x+e)^4*a^2-10*cos(f*x+e)^2*a^2-4*c
os(f*x+e)^2*a*b+15*a^2+20*a*b+8*b^2)/(a+b*sec(f*x+e)^2)^(1/2)*sec(f*x+e)
```

3.93.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.71

$$\int \frac{\sin^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\left(3\,a^2\cos(fx+e)^5 - 2\,(5\,a^2+2\,ab)\cos(fx+e)^3 + (15\,a^2+20\,ab+8\,b^2)\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{15\,a^3 f}$$

input
```
integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/15*(3*a^2*cos(f*x + e)^5 - 2*(5*a^2 + 2*a*b)*cos(f*x + e)^3 + (15*a^2 +
 20*a*b + 8*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
/(a^3*f)
```

3.93.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**5/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Timed out
```

3.93.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.32

$$\int \frac{\sin^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\frac{15\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{a} - \frac{10\left(\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3 - 3\sqrt{a+\frac{b}{\cos(fx+e)^2}}b\cos(fx+e)\right)}{a^2} + \frac{3\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{5}{2}}\cos(fx+e)^5}{}}{15\,f}$$

input
```
integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/15*(15*sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e)/a - 10*((a + b/cos(f*x +
 e)^2)^(3/2)*cos(f*x + e)^3 - 3*sqrt(a + b/cos(f*x + e)^2)*b*cos(f*x + e))
/a^2 + (3*(a + b/cos(f*x + e)^2)^(5/2)*cos(f*x + e)^5 - 10*(a + b/cos(f*x
+ e)^2)^(3/2)*b*cos(f*x + e)^3 + 15*sqrt(a + b/cos(f*x + e)^2)*b^2*cos(f*x
 + e))/a^3)/f
```

3.93.8 Giac [F]

$$\int \frac{\sin^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sin(fx+e)^5}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.93.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sin(e+fx)^5}{\sqrt{a+\frac{b}{\cos(e+fx)^2}}}\,dx$$

input
```
int(sin(e + f*x)^5/(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(sin(e + f*x)^5/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.94 $\int \frac{\sin^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

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3.94.1 Optimal result

Integrand size = 25, antiderivative size = 74

$$\int \frac{\sin^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{(3a+2b)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{3a^2 f} + \frac{\cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{3af}$$

output
```
-1/3*(3*a+2*b)*cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/a^2/f+1/3*cos(f*x+e)^3*
(a+b*sec(f*x+e)^2)^(1/2)/a/f
```

3.94.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int \frac{\sin^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{(-5a-4b+a\cos(2(e+fx)))(a+2b+a\cos(2(e+fx)))\sec(e+fx)}{12a^2 f\sqrt{a+b\sec^2(e+fx)}}$$

input
```
Integrate[Sin[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
((-5*a - 4*b + a*Cos[2*(e + f*x)])*(a + 2*b + a*Cos[2*(e + f*x)])*Sec[e +
f*x])/(12*a^2*f*Sqrt[a + b*Sec[e + f*x]^2])
```

3.94.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4622, 25, 359, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sin(e+fx)^3}{\sqrt{a+b\sec(e+fx)^2}}\,dx$$

$$\downarrow 4622$$

$$\frac{\int -\frac{\cos^4(e+fx)(1-\sec^2(e+fx))}{\sqrt{b\sec^2(e+fx)+a}}\,d\sec(e+fx)}{f}$$

$$\downarrow 25$$

$$-\frac{\int \frac{\cos^4(e+fx)(1-\sec^2(e+fx))}{\sqrt{b\sec^2(e+fx)+a}}\,d\sec(e+fx)}{f}$$

$$\downarrow 359$$

$$\frac{\frac{(3a+2b)\int \frac{\cos^2(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}\,d\sec(e+fx)}{3a} + \frac{\cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{3a}}{f}$$

$$\downarrow 242$$

$$\frac{\frac{\cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}}{3a} - \frac{(3a+2b)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{3a^2}}{f}$$

input
```
Int[Sin[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(-1/3*((3*a + 2*b)*Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/a^2 + (Cos[e +
 f*x]^3*Sqrt[a + b*Sec[e + f*x]^2])/(3*a))/f
```

3.94.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

3.94.4 Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{\left(b+a\cos(fx+e)^2\right)\left(a\cos(fx+e)^2-3a-2b\right)\sec(fx+e)}{3fa^2\sqrt{a+b\sec(fx+e)^2}}$	58

input
```
int(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/f/a^2*(b+a*cos(f*x+e)^2)*(a*cos(f*x+e)^2-3*a-2*b)/(a+b*sec(f*x+e)^2)^(
1/2)*sec(f*x+e)
```

3.94.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.77

$$\int \frac{\sin^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\left(a\cos(fx+e)^3-(3\,a+2\,b)\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{3\,a^2 f}$$

input
```
integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(a*cos(f*x + e)^3 - (3*a + 2*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)/(a^2*f)
```

3.94.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**3/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Timed out
```

3.94.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.12

$$\int \frac{\sin^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$
$$= -\frac{\frac{3\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{a} - \frac{\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3 - 3\sqrt{a+\frac{b}{\cos(fx+e)^2}}\,b\cos(fx+e)}{a^2}}{3\,f}$$

input
```
integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*(3*sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e)/a - ((a + b/cos(f*x + e)^2
)^(3/2)*cos(f*x + e)^3 - 3*sqrt(a + b/cos(f*x + e)^2)*b*cos(f*x + e))/a^2)
/f
```

3.94.8 Giac [F]

$$\int \frac{\sin^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sin(fx+e)^3}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.94.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sin(e+fx)^3}{\sqrt{a+\frac{b}{\cos(e+fx)^2}}}\,dx$$

input
```
int(sin(e + f*x)^3/(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(sin(e + f*x)^3/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.95 $\int \frac{\sin(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

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3.95.1 Optimal result

Integrand size = 23, antiderivative size = 30

$$\int \frac{\sin(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{af}$$

output
```
-cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/a/f
```

3.95.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.60

$$\int \frac{\sin(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{(a+2b+a\cos(2e+2fx))\sec(e+fx)}{2af\sqrt{a+b\sec^2(e+fx)}}$$

input
```
Integrate[Sin[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
-1/2*((a + 2*b + a*Cos[2*e + 2*f*x])*Sec[e + f*x])/(a*f*Sqrt[a + b*Sec[e +
 f*x]^2])
```

3.95.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4622, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\sin(e+fx)}{\sqrt{a+b\sec(e+fx)^2}}\,dx$$
$$\downarrow 4622$$
$$\frac{\int \frac{\cos^2(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}\,d\sec(e+fx)}{f}$$
$$\downarrow 242$$
$$-\frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{af}$$

input
```
Int[Sin[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
-((Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(a*f))
```

3.95.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

3.95.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

method	result	size
derivativedivides	$-\frac{\sqrt{a+b\sec(fx+e)^2}}{fa\sec(fx+e)}$	31
default	$-\frac{\sqrt{a+b\sec(fx+e)^2}}{fa\sec(fx+e)}$	31

input
```
int(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f/a/sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)
```

3.95.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.23

$$\int \frac{\sin(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{af}$$

input
```
integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a*f)
```

3.95.6 Sympy [F]

$$\int \frac{\sin(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sin(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
integrate(sin(f*x+e)/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sin(e + f*x)/sqrt(a + b*sec(e + f*x)**2), x)
```

3.95.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{\sin(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{af}$$

input
```
integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e)/(a*f)
```

3.95.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \frac{\sin(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\sqrt{a\cos(fx+e)^2+b}}{af\operatorname{sgn}(\cos(fx+e))}$$

input
```
integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
-sqrt(a*cos(f*x + e)^2 + b)/(a*f*sgn(cos(f*x + e)))
```

3.95.9 Mupad [B] (verification not implemented)

Time = 17.83 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.53

$$\int \frac{\sin(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\cos(e+fx)\,\sqrt{\frac{a+2b+a\cos(2e+2fx)}{\cos(2e+2fx)+1}}}{af}$$

input
```
int(sin(e + f*x)/(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
-(cos(e + f*x)*((a + 2*b + a*cos(2*e + 2*f*x))/(cos(2*e + 2*f*x) + 1))^(1/
2))/(a*f)
```

3.96 $\int \frac{\csc(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

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3.96.1 Optimal result

Integrand size = 23, antiderivative size = 43

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{\sqrt{a+b}f}$$

output
```
-arctanh(sec(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))/f/(a+b)^(1/2)
```

3.96.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.00

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b-a\sin^2(e+fx)}}{\sqrt{a+b}}\right)\sqrt{a+2b+a\cos(2e+2fx)}\sec(e+fx)}{\sqrt{2}\sqrt{a+b}f\sqrt{a+b\sec^2(e+fx)}}$$

input
```
Integrate[Csc[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
-((ArcTanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[a + b]]*Sqrt[a + 2*b + a*Co
s[2*e + 2*f*x]]*Sec[e + f*x])/(Sqrt[2]*Sqrt[a + b]*f*Sqrt[a + b*Sec[e + f*
x]^2]))
```

3.96.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4622, 25, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sin(e+fx)\sqrt{a+b\sec(e+fx)^2}}dx$$

$$\downarrow 4622$$

$$\frac{\int -\frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{f}$$

$$\downarrow 25$$

$$-\frac{\int \frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{f}$$

$$\downarrow 291$$

$$-\frac{\int \frac{1}{1-\frac{(a+b)\sec^2(e+fx)}{b\sec^2(e+fx)+a}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}}{f}$$

$$\downarrow 219$$

$$-\frac{\arctanh\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{f\sqrt{a+b}}$$

input
```
Int[Csc[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
-(ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]]/(Sqrt[a +
 b]*f))
```

3.96.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

3.96.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(37) = 74$.

Time = 1.22 (sec) , antiderivative size = 247, normalized size of antiderivative = 5.74

method	result
default	$-\frac{\left(\ln\left(\frac{2\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{a+b}\cos(fx+e)+2\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{a+b}-2\cos(fx+e)a+2b}{\sqrt{a+b}(1+\cos(fx+e))}\right)+\ln\left(-\frac{4\left(\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{a+b}\cos(fx+e)+\right.\right.}{-1+\cos}\right.}{2f\sqrt{a+b}\sqrt{a+b\sec(fx+e)^2}}$

input
```
int(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/f/(a+b)^(1/2)*(ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))+ln(-4*(((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e))))*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2)*(sec(f*x+e)+1)
```

3.96.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 140, normalized size of antiderivative = 3.26

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$
$$= \left[\frac{\log\left(\frac{2\left(a\cos(fx+e)^2-2\sqrt{a+b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)+a+2b\right)}{\cos(fx+e)^2-1}\right)}{2\sqrt{a+b}f}, \frac{\sqrt{-a-b}\arctan\left(\frac{\sqrt{-a-b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{a+b}\right)}{(a+b)f}\right]$$

input
```
integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log(2*(a*cos(f*x + e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1))/(sqrt(a + b)*
f), sqrt(-a - b)*arctan(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2)*cos(f*x + e)/(a + b))/((a + b)*f)]
```

3.96.6 Sympy [F]

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\csc(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
integrate(csc(f*x+e)/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(csc(e + f*x)/sqrt(a + b*sec(e + f*x)**2), x)
```

3.96.7 Maxima [F]

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\csc(fx+e)}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(csc(f*x + e)/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.96.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(37) = 74$.

Time $= 0.63$ (sec) , antiderivative size $= 305$, normalized size of antiderivative $= 7.09$

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx =$$
$$-\frac{\log\left(\left|-\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a+b}+\sqrt{a+b}\right|\right)}{\sqrt{a+b}} - \frac{\log($$

input
```
integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*(log(abs(-sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 + sqrt(a*tan(1/2*f*x + 1
/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(
1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b)))/sqrt(a + b) - log(abs(-sqrt(a
+ b)*tan(1/2*f*x + 1/2*e)^2 + sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*
x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a
 + b) - sqrt(a + b)))/sqrt(a + b) - log(abs(-(sqrt(a + b)*tan(1/2*f*x + 1/
2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*ta
n(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*(a + b) + sqrt
(a + b)*(a - b)))/sqrt(a + b))/(f*sgn(cos(f*x + e)))
```

3.96.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{1}{\sin(e+f\,x)\,\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

input
```
int(1/(sin(e + f*x)*(a + b/cos(e + f*x)^2)^(1/2)),x)
```

output
```
int(1/(sin(e + f*x)*(a + b/cos(e + f*x)^2)^(1/2)), x)
```

3.97 $\int \frac{\csc^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

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3.97.1 Optimal result

Integrand size = 25, antiderivative size = 87

$$\int \frac{\csc^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{a\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2(a+b)^{3/2}f} - \frac{\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)f}$$

output

```
-1/2*a*arctanh(sec(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))/(a+b)^(3/2
)/f-1/2*cot(f*x+e)*csc(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/(a+b)/f
```

3.97.2 Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.61

$$\int \frac{\csc^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{a\sqrt{a+2b+a\cos(2e+2fx)}\sec(e+fx)\sqrt{a+b-a\sin^2(e+fx)}\left(\frac{(a+b)\csc^2(e+fx)}{a}+\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{a\sin^2(e}{a+}\right.}}{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}\right.}{2\sqrt{2}(a+b)^2 f\sqrt{a+b\sec^2(e+fx)}}$$

input
```
Integrate[Csc[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
-1/2*(a*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]*Sec[e + f*x]*Sqrt[a + b - a*Sin
[e + f*x]^2]*(((a + b)*Csc[e + f*x]^2)/a + ArcTanh[Sqrt[1 - (a*Sin[e + f*x
]^2)/(a + b)]]/Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]))/(Sqrt[2]*(a + b)^2*f
*Sqrt[a + b*Sec[e + f*x]^2])
```

3.97.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 4622, 373, 27, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sin(e+fx)^3\sqrt{a+b\sec(e+fx)^2}}dx$$

$$\downarrow 4622$$

$$\frac{\int \frac{\sec^2(e+fx)}{(1-\sec^2(e+fx))^2\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{f}$$

$$\downarrow 373$$

$$\frac{\frac{\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)(1-\sec^2(e+fx))} - \frac{\int \frac{a}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{2(a+b)}}{f}$$

$$\downarrow 27$$

$$\frac{\frac{\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)(1-\sec^2(e+fx))} - \frac{a\int \frac{1}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{2(a+b)}}{f}$$

$$\downarrow 291$$

$$\frac{\frac{\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)(1-\sec^2(e+fx))} - \frac{a\int \frac{1}{1-\frac{(a+b)\sec^2(e+fx)}{b\sec^2(e+fx)+a}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}}{2(a+b)}}{f}$$

$$\downarrow 219$$

$$\frac{\frac{\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)(1-\sec^2(e+fx))} - \frac{a\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2(a+b)^{3/2}}}{f}$$

input
```
Int[Csc[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(-1/2*(a*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/(
a + b)^(3/2) + (Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(2*(a + b)*(1 - S
ec[e + f*x]^2)))/f
```

3.97.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

3.97.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1214 vs. 2(75) = 150.

Time = 1.28 (sec) , antiderivative size = 1215, normalized size of antiderivative = 13.97

method	result	size
default	Expression too large to display	1215

input
```
int(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/f/(a+b)^(5/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(
f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)
^2+a+b)^(1/2)/((a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x
+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+
a+b)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^2)^(1/2)/((1-cos(f*x+e))^2*csc(f*x+
e)^2-1)/(1-cos(f*x+e))^2*((a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e)
)^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*cs
c(f*x+e)^2+a+b)^(1/2)*(a+b)^(3/2)*(1-cos(f*x+e))^2+2*ln((a*(1-cos(f*x+e))^
2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+(a*(1-cos(f*x+e))^4*csc(f*x
+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*
b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(1/2)-a+b)/(a+b)^(1/2))*a
^2*(1-cos(f*x+e))^2+2*ln((a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))
^2*csc(f*x+e)^2+(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*
x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2
+a+b)^(1/2)*(a+b)^(1/2)-a+b)/(a+b)^(1/2))*(1-cos(f*x+e))^2*a*b+2*ln(2/(1-c
os(f*x+e))^2*(-a*(1-cos(f*x+e))^2+b*(1-cos(f*x+e))^2+(a*(1-cos(f*x+e))^4*c
sc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e
)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(1/2)*sin(f*x+e)^2+
a*sin(f*x+e)^2+b*sin(f*x+e)^2))*a^2*(1-cos(f*x+e))^2+2*ln(2/(1-cos(f*x+e))
^2*(-a*(1-cos(f*x+e))^2+b*(1-cos(f*x+e))^2+(a*(1-cos(f*x+e))^4*csc(f*x+...
```

3.97.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 305, normalized size of antiderivative = 3.51

$$\int \frac{\csc^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \left[\frac{2\,(a+b)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e) + \left(a\cos(fx+e)^2 - a\right)\sqrt{a+b}\log\left(\frac{2\left(a\cos(fx+e)^2 - 2\sqrt{a+b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e) + 2a + b\right)}{\cos(fx+e)^2 - 1}\right)}{4\left((a^2+2ab+b^2)f\cos(fx+e)^2 - (a^2+2ab+b^2)f\right)}\right.$$

input
```
integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(2*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) +
 (a*cos(f*x + e)^2 - a)*sqrt(a + b)*log(2*(a*cos(f*x + e)^2 - 2*sqrt(a + b
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + 2*b)/(cos
(f*x + e)^2 - 1)))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2 - (a^2 + 2*a*b +
b^2)*f), 1/2*((a*cos(f*x + e)^2 - a)*sqrt(-a - b)*arctan(sqrt(-a - b)*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/(a + b)) + (a + b)*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e))/((a^2 + 2*a*b + b^
2)*f*cos(f*x + e)^2 - (a^2 + 2*a*b + b^2)*f)]
```

3.97.6 Sympy [F]

$$\int \frac{\csc^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\csc^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
integrate(csc(f*x+e)**3/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(csc(e + f*x)**3/sqrt(a + b*sec(e + f*x)**2), x)
```

3.97.7 Maxima [F]

$$\int \frac{\csc^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\csc\left(fx+e\right)^3}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(csc(f*x + e)^3/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.97.8 Giac [F]

$$\int \frac{\csc^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\csc\left(fx+e\right)^3}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.97.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{1}{\sin\left(e+f\,x\right)^3\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

input
```
int(1/(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2)),x)
```

output
```
int(1/(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2)), x)
```

3.98 $\int \frac{\csc^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

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3.98.1 Optimal result

Integrand size = 25, antiderivative size = 138

$$\int \frac{\csc^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{3a^2\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8(a+b)^{5/2}f} - \frac{(5a+2b)\cot(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)^2f} - \frac{\cot^3(e+fx)\csc(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4(a+b)f}$$

output
```
-3/8*a^2*arctanh(sec(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))/(a+b)^(5
/2)/f-1/8*(5*a+2*b)*cot(f*x+e)*csc(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/(a+b)^2
/f-1/4*cot(f*x+e)^3*csc(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2)/(a+b)/f
```

3.98.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.15 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.57

$$\int \frac{\csc^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{a^2(a+2b+a\cos(2(e+fx)))\,\text{Hypergeometric2F1}\left(\frac{1}{2},3,\frac{3}{2},1-\frac{a\sin^2(e+fx)}{a+b}\right)\sec(e+fx)}{2(a+b)^3f\sqrt{a+b\sec^2(e+fx)}}$$

input
```
Integrate[Csc[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
-1/2*(a^2*(a + 2*b + a*Cos[2*(e + f*x)])*Hypergeometric2F1[1/2, 3, 3/2, 1
- (a*Sin[e + f*x]^2)/(a + b)]*Sec[e + f*x])/((a + b)^3*f*Sqrt[a + b*Sec[e
+ f*x]^2])
```

3.98.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4622, 25, 372, 402, 27, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sin(e+fx)^5\sqrt{a+b\sec(e+fx)^2}}dx$$

$$\downarrow 4622$$

$$\frac{\int -\frac{\sec^4(e+fx)}{(1-\sec^2(e+fx))^3\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{f}$$

$$\downarrow 25$$

$$-\frac{\int \frac{\sec^4(e+fx)}{(1-\sec^2(e+fx))^3\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{f}$$

$$\downarrow 372$$

$$\frac{\int \frac{2(2a+b)\sec^2(e+fx)+a}{\left(1-\sec^2(e+fx)\right)^2\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{4(a+b)} - \frac{\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4(a+b)(1-\sec^2(e+fx))^2}}{f}$$

$$\downarrow 402$$

$$\frac{\frac{\int -\frac{3a^2}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{2(a+b)} + \frac{(5a+2b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)\left(1-\sec^2(e+fx)\right)}}{4(a+b)} - \frac{\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4(a+b)(1-\sec^2(e+fx))^2}}{f}$$

$$\downarrow 27$$

$$\frac{\frac{(5a+2b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)\left(1-\sec^2(e+fx)\right)}-\frac{3a^2\int\frac{1}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{2(a+b)}}{4(a+b)}-\frac{\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4(a+b)(1-\sec^2(e+fx))^2}$$
$$f$$

$$\downarrow 291$$

$$\frac{\frac{(5a+2b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)\left(1-\sec^2(e+fx)\right)}-\frac{3a^2\int\frac{1}{1-\frac{(a+b)\sec^2(e+fx)}{b\sec^2(e+fx)+a}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}}{2(a+b)}}{4(a+b)}-\frac{\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4(a+b)(1-\sec^2(e+fx))^2}$$
$$f$$

$$\downarrow 219$$

$$\frac{\frac{(5a+2b)\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)\left(1-\sec^2(e+fx)\right)}-\frac{3a^2\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2(a+b)^{3/2}}}{4(a+b)}-\frac{\sec(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4(a+b)(1-\sec^2(e+fx))^2}$$
$$f$$

input
```
Int[Csc[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(-1/4*(Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/((a + b)*(1 - Sec[e + f*x]
^2)^2) + ((-3*a^2*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*
x]^2]])/(2*(a + b)^(3/2)) + ((5*a + 2*b)*Sec[e + f*x]*Sqrt[a + b*Sec[e + f
*x]^2])/(2*(a + b)*(1 - Sec[e + f*x]^2)))/(4*(a + b)))/f
```

3.98.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

3.98.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1759 vs. $2(122) = 244$.

Time = 1.14 (sec) , antiderivative size = 1760, normalized size of antiderivative = 12.75

method	result	size
default	Expression too large to display	1760

input
```
int(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/16/f/(a+b)^(9/2)*(6*(a+b)^(5/2)*cos(f*x+e)^4*a^2-3*sin(f*x+e)^4*ln(-4*((
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*
x+e)))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^4*cos(f*x+e)-6*sin(f*
x+e)^4*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(
f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*
a+b)/(-1+cos(f*x+e)))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^3*b*co
s(f*x+e)-3*sin(f*x+e)^4*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^
(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*a^2*b^2*cos(f*x+e)-3*sin(f*x+e)^4*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^4*cos(f*x+e)-6*sin(f*x+e)^4*ln(2/(
a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*
x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+
b)/(1+cos(f*x+e)))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^3*b*cos(f
*x+e)-3*sin(f*x+e)^4*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*((b+a*cos(f*x+e)^2)/(1+co...
```

3.98.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 491, normalized size of antiderivative = 3.56

$$\int \frac{\csc^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \left[\frac{3\left(a^2\cos(fx+e)^4 - 2\,a^2\cos(fx+e)^2 + a^2\right)\sqrt{a+b}\log\left(\frac{2\left(a\cos(fx+e)^2 - 2\sqrt{a+b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e) + a + 2\,b\right.}{\cos(fx+e)^2-1}\right.}{16\left(\left(a^3+3\,a^2b+3\,ab^2+b^3\right)f\cos(fx+e)^4 - 2\left(a^3+3\,a^2b+3\,ab^2+\right.\right.}\right.$$

input
```
integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(3*(a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x + e)^2 + a^2)*sqrt(a + b)*log
(2*(a*cos(f*x + e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1)) + 2*(3*(a^2 + a*b)*co
s(f*x + e)^3 - (5*a^2 + 7*a*b + 2*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*cos(f*x + e)^4
 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*cos(f*x + e)^2 + (a^3 + 3*a^2*b + 3*
a*b^2 + b^3)*f), 1/8*(3*(a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x + e)^2 + a^2)*
sqrt(-a - b)*arctan(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^
2)*cos(f*x + e)/(a + b)) + (3*(a^2 + a*b)*cos(f*x + e)^3 - (5*a^2 + 7*a*b
 + 2*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^3
 + 3*a^2*b + 3*a*b^2 + b^3)*f*cos(f*x + e)^4 - 2*(a^3 + 3*a^2*b + 3*a*b^2 +
 b^3)*f*cos(f*x + e)^2 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f)]
```

3.98.6 Sympy [F]

$$\int \frac{\csc^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\csc^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
integrate(csc(f*x+e)**5/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(csc(e + f*x)**5/sqrt(a + b*sec(e + f*x)**2), x)
```

3.98.7 Maxima [F]

$$\int \frac{\csc^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\csc(fx+e)^5}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(csc(f*x + e)^5/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.98.8 Giac [F]

$$\int \frac{\csc^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\csc\left(fx+e\right)^5}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.98.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{1}{\sin\left(e+f\,x\right)^5\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

input
```
int(1/(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2)),x)
```

output
```
int(1/(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2)), x)
```

3.99 $\int \frac{\sin^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

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3.99.1 Optimal result

Integrand size = 25, antiderivative size = 193

$$
\begin{aligned}
&\int \frac{\sin^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx \\
&= \frac{5(a+b)^3 \arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{7/2}f} \\
&\quad - \frac{(33a^2+40ab+15b^2)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48a^3 f} \\
&\quad + \frac{(9a+5b)\cos^3(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24a^2 f} \\
&\quad + \frac{\cos^3(e+fx)\sin^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6af}
\end{aligned}
$$

output

```
5/16*(a+b)^3*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(7/2)
/f-1/48*(33*a^2+40*a*b+15*b^2)*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^
(1/2)/a^3/f+1/24*(9*a+5*b)*cos(f*x+e)^3*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1
/2)/a^2/f+1/6*cos(f*x+e)^3*sin(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(1/2)/a/f
```

3.99.2 Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.84

$$\int \frac{\sin^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$
$$= \frac{\sqrt{a+2b+a\cos(2(e+fx))}\sec(e+fx)\left(15(a+b)^3\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)-\sqrt{a}\sin(e+fx)\sqrt{a+b}\right.}{48\sqrt{2}a^{7/2}f\sqrt{a+b\sec^2(e+fx}}$$

input
```
Integrate[Sin[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*Sec[e + f*x]*(15*(a + b)^3*ArcTan[(Sqr
t[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]] - Sqrt[a]*Sin[e + f*x]*
Sqrt[a + b - a*Sin[e + f*x]^2]*(15*(a + b)^2 + 10*a*(a + b)*Sin[e + f*x]^2
 + 8*a^2*Sin[e + f*x]^4)))/(48*Sqrt[2]*a^(7/2)*f*Sqrt[a + b*Sec[e + f*x]^2
])
```

3.99.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4620, 372, 440, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\sin(e+fx)^6}{\sqrt{a+b\sec(e+fx)^2}}\,dx$$
$$\downarrow 4620$$
$$\frac{\int \frac{\tan^6(e+fx)}{(\tan^2(e+fx)+1)^4\sqrt{b\tan^2(e+fx)+a+b}}\,d\tan(e+fx)}{f}$$
$$\downarrow 372$$

$$\frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{6a(\tan^2(e+fx)+1)^3}-\frac{\int\frac{\tan^2(e+fx)\left(3(a+b)-2(3a+b)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)^3\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{6a}$$
$$f$$

$$\downarrow 440$$

$$\frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{6a(\tan^2(e+fx)+1)^3}-\frac{\frac{\int\frac{(a+b)(9a+5b)-2\left(12a^2+13ba+5b^2\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)^2\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{4a}-\frac{(9a+5b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4a\left(\tan^2(e+fx)+1\right)^2}}{6a}}{f}$$

$$\downarrow 402$$

$$\frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{6a(\tan^2(e+fx)+1)^3}-\frac{\frac{\left(33a^2+40ab+15b^2\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a\left(\tan^2(e+fx)+1\right)}-\frac{\int\frac{15(a+b)^3}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2a}}{4a}-\frac{(9a+5b)\,t}{6a}}{f}$$

$$\downarrow 27$$

$$\frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{6a(\tan^2(e+fx)+1)^3}-\frac{\frac{\left(33a^2+40ab+15b^2\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a\left(\tan^2(e+fx)+1\right)}-\frac{15(a+b)^3\int\frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2a}}{4a}-\frac{}{6a}}{f}$$

$$\downarrow 291$$

$$\frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{6a(\tan^2(e+fx)+1)^3}-\frac{\frac{\left(33a^2+40ab+15b^2\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a\left(\tan^2(e+fx)+1\right)}-\frac{15(a+b)^3\int\frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{2a}}{4a}-\frac{(9a+}{6a}}{f}$$

$$\downarrow 216$$

$$\frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{6a(\tan^2(e+fx)+1)^3}-\frac{\frac{\left(33a^2+40ab+15b^2\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a\left(\tan^2(e+fx)+1\right)}-\frac{15(a+b)^3\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2a^{3/2}}}{4a}-\frac{(9a+5b)\tan(e+fx)\sqrt{}}{4a\left(\tan^2(e-\right.}{6a}}{f}$$

3.99. $\int\frac{\sin^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}dx$

input
```
Int[Sin[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
((Tan[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(6*a*(1 + Tan[e + f*x]^2)
^3) - (-1/4*((9*a + 5*b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(a*(
1 + Tan[e + f*x]^2)^2) + ((-15*(a + b)^3*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqr
t[a + b + b*Tan[e + f*x]^2]])/(2*a^(3/2)) + ((33*a^2 + 40*a*b + 15*b^2)*Ta
n[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*a*(1 + Tan[e + f*x]^2)))/(4*
a))/(6*a))/f
```

3.99.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.99.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1135 vs. $2(173) = 346$.

Time = 6.76 (sec) , antiderivative size = 1136, normalized size of antiderivative = 5.89

method	result	size
default	Expression too large to display	1136

input
```
int(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/48/f/a^3/(-a)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2)*(-8*cos(f*x+e)^5*sin(f*x+e)
*(-a)^(1/2)*a^3+26*cos(f*x+e)^3*sin(f*x+e)*(-a)^(1/2)*a^3+2*cos(f*x+e)^3*s
in(f*x+e)*(-a)^(1/2)*a^2*b-33*cos(f*x+e)*sin(f*x+e)*(-a)^(1/2)*a^3-14*cos(
f*x+e)*sin(f*x+e)*(-a)^(1/2)*a^2*b-5*cos(f*x+e)*sin(f*x+e)*(-a)^(1/2)*a*b^
2+15*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^3+45*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)-4*sin(f*x+e)*a)*a^2*b+45*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(
4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)
^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a*b^2+1
5*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*b^3+15*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4
*sin(f*x+e)*a)*a^3*sec(f*x+e)+45*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+
4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a...
```

3.99.5 Fricas [A] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 639, normalized size of antiderivative = 3.31

$$\int \frac{\sin^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \left[-\frac{15\,(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{-a}\log\left(128\,a^4\cos\left(fx+e\right)^8-256\,(a^4-a^3b)\cos\left(fx+e\right)^6+32\,(5\,a^4-1\right.}{}\right.$$

$$-\frac{15\,(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{a}\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^5-8\,(a^2-ab)\cos(fx+e)^3+(a^2-6\,ab+b^2)\cos(fx+e)\right)\sqrt{a}\sqrt{\frac{a\cos(fx+e}{\cos(fx+}}}{4\left(2\,a^3\cos(fx+e)^4-a^2b+ab^2-(a^3-3\,a^2b)\cos(fx+e)^2\right)\sin(fx+e)}\right.}{}$$

input
```
integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/384*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(-a)*log(128*a^4*cos(f*x +
 e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^
2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4
 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7
- 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x
 + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(8*a^3*cos(f*x + e)^5
 - 2*(13*a^3 + 5*a^2*b)*cos(f*x + e)^3 + (33*a^3 + 40*a^2*b + 15*a*b^2)*co
s(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^4
*f), -1/192*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a)*arctan(1/4*(8*a^2*
cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*
x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*
x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))
+ 4*(8*a^3*cos(f*x + e)^5 - 2*(13*a^3 + 5*a^2*b)*cos(f*x + e)^3 + (33*a^3
+ 40*a^2*b + 15*a*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2)*sin(f*x + e))/(a^4*f)]
```

3.99.6 Sympy [F]

$$\int \frac{\sin^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sin^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
integrate(sin(f*x+e)**6/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sin(e + f*x)**6/sqrt(a + b*sec(e + f*x)**2), x)
```

3.99.7 Maxima [F]

$$\int \frac{\sin^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sin(fx+e)^6}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sin(f*x + e)^6/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.99.8 Giac [F]

$$\int \frac{\sin^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sin\left(fx+e\right)^6}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.99.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sin\left(e+f\,x\right)^6}{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

input
```
int(sin(e + f*x)^6/(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(sin(e + f*x)^6/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.100 $\int \frac{\sin^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

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3.100.7 Maxima [F] . . . 888
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3.100.9 Mupad [F(-1)] . . . 889

3.100.1 Optimal result

Integrand size = 25, antiderivative size = 135

$$\int \frac{\sin^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{3(a+b)^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{5/2}f} - \frac{(5a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8a^2f} + \frac{\cos^3(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4af}$$

output
```
3/8*(a+b)^2*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/
f-1/8*(5*a+3*b)*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/a^2/f+1/4
*cos(f*x+e)^3*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/a/f
```

3.100.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.07

$$\int \frac{\sin^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\sqrt{a+2b+a\cos(2(e+fx))}\sec(e+fx)\left(3(a+b)^2\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right) - \sqrt{a}\sin(e+fx)\sqrt{a+b-}\right.}{8\sqrt{2}a^{5/2}f\sqrt{a+b\sec^2(e+fx)}}$$

input
```
Integrate[Sin[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*Sec[e + f*x]*(3*(a + b)^2*ArcTan[(Sqrt
[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]] - Sqrt[a]*Sin[e + f*x]*S
qrt[a + b - a*Sin[e + f*x]^2]*(3*(a + b) + 2*a*Sin[e + f*x]^2)))/(8*Sqrt[2
]*a^(5/2)*f*Sqrt[a + b*Sec[e + f*x]^2])
```

3.100.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4620, 372, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sin(e+fx)^4}{\sqrt{a+b\sec(e+fx)^2}}dx$$

$$\downarrow 4620$$

$$\frac{\int \frac{\tan^4(e+fx)}{(\tan^2(e+fx)+1)^3\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{f}$$

$$\downarrow 372$$

$$\frac{\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4a(\tan^2(e+fx)+1)^2} - \frac{\int \frac{-2(2a+b)\tan^2(e+fx)+a+b}{(\tan^2(e+fx)+1)^2\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{4a}}{f}$$

$$\downarrow 402$$

$$\frac{\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4a(\tan^2(e+fx)+1)^2} - \frac{\frac{(5a+3b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a\left(\tan^2(e+fx)+1\right)} - \frac{\int \frac{3(a+b)^2}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2a}}{4a}}{f}$$

$$\downarrow 27$$

$$\frac{\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4a(\tan^2(e+fx)+1)^2}-\frac{\frac{(5a+3b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a\left(\tan^2(e+fx)+1\right)}-\frac{3(a+b)^2\int\frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2a}}{4a}}{f}$$

$$\downarrow 291$$

$$\frac{\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4a(\tan^2(e+fx)+1)^2}-\frac{\frac{(5a+3b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a\left(\tan^2(e+fx)+1\right)}-\frac{3(a+b)^2\int\frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{2a}}{4a}}{f}$$

$$\downarrow 216$$

$$\frac{\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4a(\tan^2(e+fx)+1)^2}-\frac{\frac{(5a+3b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a\left(\tan^2(e+fx)+1\right)}-\frac{3(a+b)^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2a^{3/2}}}{4a}}{f}$$

input
```
Int[Sin[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
((Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(4*a*(1 + Tan[e + f*x]^2)^2
) - ((-3*(a + b)^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*
x]^2]])/(2*a^(3/2)) + ((5*a + 3*b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x
]^2])/(2*a*(1 + Tan[e + f*x]^2)))/(4*a))/f
```

3.100.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.100.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 808 vs. $2(119) = 238$.

Time $= 5.56$ (sec) , antiderivative size $= 809$, normalized size of antiderivative $= 5.99$

method	result
default	$\frac{2\sin(fx+e)\cos(fx+e)^3\sqrt{-a}\,a^2-5\sin(fx+e)\cos(fx+e)\sqrt{-a}\,a^2-\cos(fx+e)\sqrt{-a}\,\sin(fx+e)ab+3\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\ln\left(4\sqrt{-a}\sqrt{\frac{b+}{(1+}\right.$

input
```
int(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/f/a^2/(-a)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2)*(2*sin(f*x+e)*cos(f*x+e)^3*(
-a)^(1/2)*a^2-5*sin(f*x+e)*cos(f*x+e)*(-a)^(1/2)*a^2-cos(f*x+e)*(-a)^(1/2)
*sin(f*x+e)*a*b+3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1
/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^2+6*ln(4*(-a)^
(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b+3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*b^2-5*(-a)^(1/2)*a*b*tan(f*x+e)-3*(-a)^(1/2)*b^2*tan(f*x+e)+3*sec(f*x+e)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^2+6*ln(4*(-a)^(1/2)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*a*b*sec(f*x+e)+3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2*
sec(f*x+e))
```

3.100.5 Fricas [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 565, normalized size of antiderivative = 4.19

$$\int \frac{\sin^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \left[-\frac{3\,(a^2+2\,ab+b^2)\sqrt{-a}\log\left(128\,a^4\cos\left(fx+e\right)^8-256\,(a^4-a^3b)\cos\left(fx+e\right)^6+32\,(5\,a^4-14\,a^3b+5\right.}{} \right.$$

$$\left. -\frac{3\,(a^2+2\,ab+b^2)\sqrt{a}\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^5-8\,(a^2-ab)\cos(fx+e)^3+(a^2-6\,ab+b^2)\cos(fx+e)\right)\sqrt{a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\left(2\,a^3\cos(fx+e)^4-a^2b+ab^2-(a^3-3\,a^2b)\cos(fx+e)^2\right)\sin(fx+e)}\right)-4}{32\,a^3f} \right]$$

input
```
integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/64*(3*(a^2 + 2*a*b + b^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a
^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e
)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7
*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^
2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3
 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)*sin(f*x + e)) - 8*(2*a^2*cos(f*x + e)^3 - (5*a^2 + 3*
a*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e
))/(a^3*f), -1/32*(3*(a^2 + 2*a*b + b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x
 + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))
*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^
4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(2*
a^2*cos(f*x + e)^3 - (5*a^2 + 3*a*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)*sin(f*x + e))/(a^3*f)]
```

3.100.6 Sympy [F]

$$\int \frac{\sin^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sin^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
integrate(sin(f*x+e)**4/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sin(e + f*x)**4/sqrt(a + b*sec(e + f*x)**2), x)
```

3.100.7 Maxima [F]

$$\int \frac{\sin^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sin(fx+e)^4}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sin(f*x + e)^4/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.100.8 Giac [F]

$$\int \frac{\sin^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sin\left(fx+e\right)^4}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.100.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sin\left(e+f\,x\right)^4}{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

input
```
int(sin(e + f*x)^4/(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(sin(e + f*x)^4/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.101 $\int \frac{\sin^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

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3.101.1 Optimal result

Integrand size = 25, antiderivative size = 85

$$\int \frac{\sin^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{(a+b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2a^{3/2}f} - \frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2af}$$

output
```
1/2*(a+b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f-
1/2*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/a/f
```

3.101.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.47

$$\int \frac{\sin^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\sqrt{a+2b+a\cos(2(e+fx))}\sec(e+fx)\left((a+b)\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right) - \sqrt{a}\sin(e+fx)\sqrt{a+b-a}\right.}{2\sqrt{2}a^{3/2}f\sqrt{a+b\sec^2(e+fx)}}$$

input
```
Integrate[Sin[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*Sec[e + f*x]*((a + b)*ArcTan[(Sqrt[a]*
Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]] - Sqrt[a]*Sin[e + f*x]*Sqrt[
a + b - a*Sin[e + f*x]^2]))/(2*Sqrt[2]*a^(3/2)*f*Sqrt[a + b*Sec[e + f*x]^2
])
```

3.101.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 4620, 373, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sin(e+fx)^2}{\sqrt{a+b\sec(e+fx)^2}}\,dx$$

$$\downarrow 4620$$

$$\frac{\int \frac{\tan^2(e+fx)}{(\tan^2(e+fx)+1)^2\sqrt{b\tan^2(e+fx)+a+b}}\,d\tan(e+fx)}{f}$$

$$\downarrow 373$$

$$\frac{\int \frac{\frac{a+b}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}\,d\tan(e+fx)}{2a} - \frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a(\tan^2(e+fx)+1)}}{f}$$

$$\downarrow 27$$

$$\frac{(a+b)\int \frac{\frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}\,d\tan(e+fx)}{2a} - \frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a(\tan^2(e+fx)+1)}}{f}$$

$$\downarrow 291$$

$$\frac{(a+b)\int \frac{\frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}\,d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{2a} - \frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a(\tan^2(e+fx)+1)}}{f}$$

$$\downarrow 216$$

$$\frac{\frac{(a+b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2a^{3/2}}-\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a(\tan^2(e+fx)+1)}}{f}$$

input
```
Int[Sin[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(((a + b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(
2*a^(3/2)) - (Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*a*(1 + Tan[e
 + f*x]^2)))/f
```

3.101.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.101.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 508 vs. $2(73) = 146$.

Time = 4.49 (sec) , antiderivative size = 509, normalized size of antiderivative = 5.99

method	result
default	$-\frac{\sin(fx+e)\cos(fx+e)\sqrt{-a}\,a-\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\ln\left(4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\cos(fx+e)+4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a\right.$

input

```
int(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-1/2/f/a/(-a)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2)*(sin(f*x+e)*cos(f*x+e)*(-a)^(
1/2)*a-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a-((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4
*sin(f*x+e)*a)*b+(-a)^(1/2)*b*tan(f*x+e)-sec(f*x+e)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)-4*sin(f*x+e)*a)*a-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^
(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*b*sec(f*x+e))
```

3.101.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(73) = 146$.

Time $= 0.41$ (sec) , antiderivative size $= 497$, normalized size of antiderivative $= 5.85$

$$\int \frac{\sin^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \left[-\frac{8\,a\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)\sin(fx+e) + \sqrt{-a}(a+b)\log\left(128\,a^4\cos(fx+e)^8 - 256\,(a^4-a^3b)\,\mathrm{co}\right.}{\left.}\right.$$

$$\left. -\frac{4\,a\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)\sin(fx+e) + (a+b)\sqrt{a}\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^5-8\,(a^2-ab)\cos(fx+e)^3+(a^2-6\,a\right.}{4\left(2\,a^3\cos(fx+e)^4-a^2b+ab^2-(a^3-3\,a\right.}\right.}{8\,a^2 f} \right]$$

input
```
integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/16*(8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f
*x + e) + sqrt(-a)*(a + b)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*
cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 -
28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a
*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x
 + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b +
 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)*sin(f*x + e)))/(a^2*f), -1/8*(4*a*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)*cos(f*x + e)*sin(f*x + e) + (a + b)*sqrt(a)*arctan(1/4*(8*a^2
*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f
*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f
*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))
)/(a^2*f)]
```

3.101.6 Sympy [F]

$$\int \frac{\sin^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sin^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
integrate(sin(f*x+e)**2/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sin(e + f*x)**2/sqrt(a + b*sec(e + f*x)**2), x)
```

3.101.7 Maxima [F]

$$\int \frac{\sin^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sin(fx+e)^2}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sin(f*x + e)^2/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.101.8 Giac [F]

$$\int \frac{\sin^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sin(fx+e)^2}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.101.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sin(e+f\,x)^2}{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

input
```
int(sin(e + f*x)^2/(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(sin(e + f*x)^2/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.102 $\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

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3.102.1 Optimal result

Integrand size = 16, antiderivative size = 39

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}f}$$

output
```
arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/a^(1/2)
```

3.102.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 87 vs. 2(39) = 78.

Time = 0.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.23

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)\sqrt{a+2b+a\cos(2e+2fx)}\sec(e+fx)}{\sqrt{2}\sqrt{a}f\sqrt{a+b\sec^2(e+fx)}}$$

input
```
Integrate[1/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*Sqrt[a + 2*
b + a*Cos[2*e + 2*f*x]]*Sec[e + f*x])/(Sqrt[2]*Sqrt[a]*f*Sqrt[a + b*Sec[e
+ f*x]^2])
```

3.102.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 4616, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)^2}}dx$$

$$\downarrow 4616$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{f}$$

$$\downarrow 291$$

$$\frac{\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{f}$$

$$\downarrow 216$$

$$\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}f}$$

input
```
Int[1/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[a]*f)
```

3.102.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

3.102.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(33) = 66$.

Time = 2.73 (sec) , antiderivative size = 138, normalized size of antiderivative = 3.54

method	result	size
default	$\frac{\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\ln\left(4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)+4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a\right)(\sec(fx+e)+1)}{f\sqrt{-a}\sqrt{a+b\sec(fx+e)^2}}$	138

input
```
int(1/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f/(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)/(a+b*sec(f*x+e)^2)^
(1/2)*(sec(f*x+e)+1)
```

3.102.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(33) = 66$.

Time = 0.39 (sec) , antiderivative size = 408, normalized size of antiderivative = 10.46

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \left[-\frac{\sqrt{-a}\log\left(128\,a^4\cos(fx+e)^8 - 256\,(a^4-a^3b)\cos(fx+e)^6 + 32\,(5\,a^4-14\,a^3b+5\,a^2b^2)\cos(fx+e)\right.}{} \right.$$

$$\left. \left. -\frac{\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^5-8\,(a^2-ab)\cos(fx+e)^3+(a^2-6\,ab+b^2)\cos(fx+e)\right)\sqrt{a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\left(2\,a^3\cos(fx+e)^4-a^2b+ab^2-(a^3-3\,a^2b)\cos(fx+e)^2\right)\sin(fx+e)}\right)}{4\sqrt{a}f} \right]$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)
^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 7
0*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*
x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*
(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b
^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin
(f*x + e))/(a*f), -1/4*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*co
s(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 -
 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))/(sqrt(a)*f)]
```

3.102.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*sec(e + f*x)**2), x)
```

3.102.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 992 vs. $2(33) = 66$.

Time = 0.43 (sec) , antiderivative size = 992, normalized size of antiderivative = 25.44

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*(arctan2(2*a*sin(2*f*x + 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*
f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b
)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x +
2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e)
 + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*sin(1/2*arctan2(a*sin(4
*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*
b)*cos(2*f*x + 2*e) + a)), 2*a*cos(2*f*x + 2*e) + 2*(a^2*cos(4*f*x + 4*e)^
2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 +
4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2
)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*co
s(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*cos(1/2*a
rctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*
e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) + 2*a + 4*b) - arctan2(2*(a^2*cos(
4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*
x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 +
4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f
*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt
(a)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*c
os(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)), 2*(a^2*cos(4*f*x + 4
*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*...
```

3.102.8 Giac [F]

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.102.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{\cos(e+fx)^2}}}\,dx$$

input
```
int(1/(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(1/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.103 $\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

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3.103.1 Optimal result

Integrand size = 25, antiderivative size = 33

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{(a+b)f}$$

output
```
-cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)/f
```

3.103.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.67

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))\csc(e+fx)\sec(e+fx)}{2(a+b)f\sqrt{a+b\sec^2(e+fx)}}$$

input
```
Integrate[Csc[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
-1/2*((a + 2*b + a*Cos[2*(e + f*x)])*Csc[e + f*x]*Sec[e + f*x])/((a + b)*f
*Sqrt[a + b*Sec[e + f*x]^2])
```

3.103.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3042, 4620, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sin(e+fx)^2\sqrt{a+b\sec(e+fx)^2}}dx$$

$$\downarrow 4620$$

$$\frac{\int \frac{\cot^2(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{f}$$

$$\downarrow 242$$

$$-\frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f(a+b)}$$

input
```
Int[Csc[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
-((Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/((a + b)*f))
```

3.103.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.103.4 Maple [A] (verified)

Time = 2.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.45

method	result	size
default	$-\frac{\cot(fx+e)a+b\sec(fx+e)\csc(fx+e)}{f(a+b)\sqrt{a+b\sec(fx+e)^2}}$	48

input
```
int(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f/(a+b)/(a+b*sec(f*x+e)^2)^(1/2)*(cot(f*x+e)*a+b*sec(f*x+e)*csc(f*x+e))
```

3.103.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.42

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)}{(a+b)f\sin(fx+e)}$$

input
```
integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)/((a + b)*f*sin(f
*x + e))
```

3.103.6 Sympy [F]

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
integrate(csc(f*x+e)**2/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(csc(e + f*x)**2/sqrt(a + b*sec(e + f*x)**2), x)
```

3.103.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\sqrt{b\tan(fx+e)^2+a+b}}{(a+b)f\tan(fx+e)}$$

input
```
integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(b*tan(f*x + e)^2 + a + b)/((a + b)*f*tan(f*x + e))
```

3.103.8 Giac [F]

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\csc(fx+e)^2}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.103.9 Mupad [B] (verification not implemented)

Time = 18.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.24

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{(2\sin(2e+2fx)+\sin(4e+4fx))\sqrt{\frac{a+2b+a\cos(2e+2fx)}{\cos(2e+2fx)+1}}}{2f\sin(2e+2fx)^2\,(a+b)}$$

input
```
int(1/(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^(1/2)),x)
```

output
```
-((2*sin(2*e + 2*f*x) + sin(4*e + 4*f*x))*((a + 2*b + a*cos(2*e + 2*f*x))/
(cos(2*e + 2*f*x) + 1))^(1/2))/(2*f*sin(2*e + 2*f*x)^2*(a + b))
```

3.104 $\int \frac{\csc^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

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3.104.2 Mathematica [A] (verified) . . . 908
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3.104.6 Sympy [F] . . . 911
3.104.7 Maxima [A] (verification not implemented) . . . 911
3.104.8 Giac [F] . . . 912
3.104.9 Mupad [B] (verification not implemented) . . . 912

3.104.1 Optimal result

Integrand size = 25, antiderivative size = 78

$$\int \frac{\csc^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{(3a+b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)^2 f} - \frac{\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)f}$$

output
```
-1/3*(3*a+b)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)^2/f-1/3*cot(f*x+e
)^3*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)/f
```

3.104.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.95

$$\int \frac{\csc^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{(-2a-b+a\cos(2(e+fx)))(a+2b+a\cos(2(e+fx)))\csc^3(e+fx)\sec(e+fx)}{6(a+b)^2 f\sqrt{a+b\sec^2(e+fx)}}$$

input
```
Integrate[Csc[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
((-2*a - b + a*Cos[2*(e + f*x)])*(a + 2*b + a*Cos[2*(e + f*x)])*Csc[e + f*
x]^3*Sec[e + f*x])/(6*(a + b)^2*f*Sqrt[a + b*Sec[e + f*x]^2])
```

3.104.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 4620, 359, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sin(e+fx)^4\sqrt{a+b\sec(e+fx)^2}}dx$$

$$\downarrow 4620$$

$$\frac{\int \frac{\cot^4(e+fx)(\tan^2(e+fx)+1)}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{f}$$

$$\downarrow 359$$

$$\frac{\frac{(3a+b)\int \frac{\cot^2(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{3(a+b)} - \frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)}}{f}$$

$$\downarrow 242$$

$$\frac{-\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)} - \frac{(3a+b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)^2}}{f}$$

input
```
Int[Csc[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(-1/3*((3*a + b)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(a + b)^2 -
(Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*(a + b)))/f
```

3.104.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.104.4 Maple [A] (verified)

Time = 3.64 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.99

method	result	size
default	$\frac{\left(b+a\cos(fx+e)^2\right)\left(2a\cos(fx+e)^2-3a-b\right)\sec(fx+e)\csc(fx+e)^3}{3f(a^2+2ab+b^2)\sqrt{a+b\sec(fx+e)^2}}$	77

input
```
int(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/f/(a^2+2*a*b+b^2)*(b+a*cos(f*x+e)^2)*(2*a*cos(f*x+e)^2-3*a-b)/(a+b*sec
(f*x+e)^2)^(1/2)*sec(f*x+e)*csc(f*x+e)^3
```

3.104.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.23

$$\int \frac{\csc^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\left(2\,a\cos\left(fx+e\right)^3-\left(3\,a+b\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{3\left(\left(a^2+2\,ab+b^2\right)f\cos\left(fx+e\right)^2-\left(a^2+2\,ab+b^2\right)f\right)\sin\left(fx+e\right)}$$

input
```
integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(2*a*cos(f*x + e)^3 - (3*a + b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)/(((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2 - (a^2 + 2*a*b
 + b^2)*f)*sin(f*x + e))
```

3.104.6 Sympy [F]

$$\int \frac{\csc^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\csc^4\left(e+fx\right)}{\sqrt{a+b\sec^2\left(e+fx\right)}}\,dx$$

input
```
integrate(csc(f*x+e)**4/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(csc(e + f*x)**4/sqrt(a + b*sec(e + f*x)**2), x)
```

3.104.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.23

$$\int \frac{\csc^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\frac{3\sqrt{b\tan(fx+e)^2+a+b}}{(a+b)\tan(fx+e)}-\frac{2\sqrt{b\tan(fx+e)^2+a+b}b}{(a+b)^2\tan(fx+e)}+\frac{\sqrt{b\tan(fx+e)^2+a+b}}{(a+b)\tan(fx+e)^3}}{3\,f}$$

input
```
integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*(3*sqrt(b*tan(f*x + e)^2 + a + b)/((a + b)*tan(f*x + e)) - 2*sqrt(b*t
an(f*x + e)^2 + a + b)*b/((a + b)^2*tan(f*x + e)) + sqrt(b*tan(f*x + e)^2
+ a + b)/((a + b)*tan(f*x + e)^3))/f
```

3.104.8 Giac [F]

$$\int \frac{\csc^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\csc\left(fx+e\right)^4}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.104.9 Mupad [B] (verification not implemented)

Time = 22.95 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.58

$$\int \frac{\csc^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{2\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\sqrt{a+\frac{b}{\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}\right)^2}}\left(a\,1\mathrm{i}-a\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,4\mathrm{i}+a\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,1\mathrm{i}-b\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,2\mathrm{i}\right)}{3\,f\,(a+b)^2\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-1\right)^3}$$

input
```
int(1/(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^(1/2)),x)
```

output
```
-(2*(exp(e*2i + f*x*2i) + 1)*(a + b/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f
*x*1i)/2)^2)^(1/2)*(a*1i - a*exp(e*2i + f*x*2i)*4i + a*exp(e*4i + f*x*4i)*
1i - b*exp(e*2i + f*x*2i)*2i))/(3*f*(a + b)^2*(exp(e*2i + f*x*2i) - 1)^3)
```

3.105 $\int \frac{\csc^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

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3.105.2 Mathematica [A] (verified) . . . 913
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3.105.4 Maple [A] (verified) . . . 916
3.105.5 Fricas [A] (verification not implemented) . . . 916
3.105.6 Sympy [F] . . . 917
3.105.7 Maxima [A] (verification not implemented) . . . 917
3.105.8 Giac [F] . . . 917
3.105.9 Mupad [B] (verification not implemented) . . . 918

3.105.1 Optimal result

Integrand size = 25, antiderivative size = 132

$$\int \frac{\csc^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{(15a^2+10ab+3b^2)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^3 f} - \frac{2(5a+3b)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^2 f} - \frac{\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5(a+b)f}$$

output
```
-1/15*(15*a^2+10*a*b+3*b^2)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)^3/
f-2/15*(5*a+3*b)*cot(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)^2/f-1/5*cot
(f*x+e)^5*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)/f
```

3.105.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.76

$$\int \frac{\csc^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))\,(8a^2+8ab+3b^2-2a(3a+b)\cos(2(e+fx))+a^2\cos(4(e+fx)))\csc^5(e+}{30(a+b)^3 f\sqrt{a+b\sec^2(e+fx)}}$$

input
```
Integrate[Csc[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
-1/30*((a + 2*b + a*Cos[2*(e + f*x)])*(8*a^2 + 8*a*b + 3*b^2 - 2*a*(3*a +
b)*Cos[2*(e + f*x)] + a^2*Cos[4*(e + f*x)])*Csc[e + f*x]^5*Sec[e + f*x])/(
(a + b)^3*f*Sqrt[a + b*Sec[e + f*x]^2])
```

3.105.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4620, 365, 359, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sin(e+fx)^6\sqrt{a+b\sec(e+fx)^2}}dx$$

$$\downarrow 4620$$

$$\frac{\int \frac{\cot^6(e+fx)\left(\tan^2(e+fx)+1\right)^2}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{f}$$

$$\downarrow 365$$

$$\frac{\int \frac{\cot^4(e+fx)\left(5(a+b)\tan^2(e+fx)+2(5a+3b)\right)}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{5(a+b)} - \frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{5(a+b)}}{f}$$

$$\downarrow 359$$

$$\frac{\frac{\left(15a^2+10ab+3b^2\right)\int \frac{\cot^2(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{3(a+b)} - \frac{2(5a+3b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)}}{5(a+b)} - \frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{5(a+b)}}{f}$$

$$\downarrow 242$$

$$\frac{-\frac{\left(15a^2+10ab+3b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)^2}-\frac{2(5a+3b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)}}{5(a+b)}-\frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{5(a+b)}}{f}$$

input
```
Int[Csc[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(-1/5*(Cot[e + f*x]^5*Sqrt[a + b + b*Tan[e + f*x]^2])/(a + b) + (-1/3*((15
*a^2 + 10*a*b + 3*b^2)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(a + b
)^2 - (2*(5*a + 3*b)*Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(3*(a
+ b)))/(5*(a + b)))/f
```

3.105.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.105.4 Maple [A] (verified)

Time = 5.02 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{\left(b+a\cos(fx+e)^2\right)\left(8\cos(fx+e)^4a^2-20\cos(fx+e)^2a^2-4\cos(fx+e)^2ab+15a^2+10ab+3b^2\right)\sec(fx+e)\csc(fx+e)^5}{15f(a^3+3a^2b+3ab^2+b^3)\sqrt{a+b\sec(fx+e)^2}}$	120

input
```
int(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15/f/(a^3+3*a^2*b+3*a*b^2+b^3)*(b+a*cos(f*x+e)^2)*(8*cos(f*x+e)^4*a^2-2
0*cos(f*x+e)^2*a^2-4*cos(f*x+e)^2*a*b+15*a^2+10*a*b+3*b^2)/(a+b*sec(f*x+e)
^2)^(1/2)*sec(f*x+e)*csc(f*x+e)^5
```

3.105.5 Fricas [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.30

$$\int \frac{\csc^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx =$$
$$-\frac{\left(8\,a^2\cos(fx+e)^5-4\,(5\,a^2+ab)\cos(fx+e)^3+(15\,a^2+10\,ab+3\,b^2)\cos(fx+e)\right)\sqrt{}}{15\left((a^3+3\,a^2b+3\,ab^2+b^3)f\cos(fx+e)^4-2\,(a^3+3\,a^2b+3\,ab^2+b^3)f\cos(fx+e)^2+(a^3+3\,a^2b+\right.}$$

input
```
integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/15*(8*a^2*cos(f*x + e)^5 - 4*(5*a^2 + a*b)*cos(f*x + e)^3 + (15*a^2 + 1
0*a*b + 3*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(
((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*cos(f*x + e)^4 - 2*(a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*f*cos(f*x + e)^2 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f)*sin(f*x
+ e))
```

3.105.6 Sympy [F]

$$\int \frac{\csc^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\csc^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
integrate(csc(f*x+e)**6/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(csc(e + f*x)**6/sqrt(a + b*sec(e + f*x)**2), x)
```

3.105.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.45

$$\int \frac{\csc^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx =$$
$$-\frac{\frac{15\sqrt{b\tan(fx+e)^2+a+b}}{(a+b)\tan(fx+e)} - \frac{20\sqrt{b\tan(fx+e)^2+a+b}b}{(a+b)^2\tan(fx+e)} + \frac{8\sqrt{b\tan(fx+e)^2+a+b}b^2}{(a+b)^3\tan(fx+e)} + \frac{10\sqrt{b\tan(fx+e)^2+a+b}}{(a+b)\tan(fx+e)^3} - \frac{4\sqrt{b\tan(fx+e)^2+a+b}}{(a+b)^2\tan(fx+e)^3}}{15\,f}$$

input
```
integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/15*(15*sqrt(b*tan(f*x + e)^2 + a + b)/((a + b)*tan(f*x + e)) - 20*sqrt(
b*tan(f*x + e)^2 + a + b)*b/((a + b)^2*tan(f*x + e)) + 8*sqrt(b*tan(f*x +
e)^2 + a + b)*b^2/((a + b)^3*tan(f*x + e)) + 10*sqrt(b*tan(f*x + e)^2 + a
+ b)/((a + b)*tan(f*x + e)^3) - 4*sqrt(b*tan(f*x + e)^2 + a + b)*b/((a + b
)^2*tan(f*x + e)^3) + 3*sqrt(b*tan(f*x + e)^2 + a + b)/((a + b)*tan(f*x +
e)^5))/f
```

3.105.8 Giac [F]

$$\int \frac{\csc^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\csc\left(fx+e\right)^6}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.105.9 Mupad [B] (verification not implemented)

Time = 30.64 (sec) , antiderivative size = 723, normalized size of antiderivative = 5.48

$$\int \frac{\csc^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(e + f*x)^6*(a + b/cos(e + f*x)^2)^(1/2)),x)
```

output
```
(((32*a + 16*b)/(5*f*(6*a + 6*b)*(a*1i + b*1i)) + (32*a + 80*b)/(5*f*(6*a
+ 6*b)*(a*1i + b*1i)))*(a + b/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)
/2)^2)^(1/2)*(2*exp(e*2i + f*x*2i) + exp(e*4i + f*x*4i) + 1))/((exp(e*2i +
 f*x*2i) - 1)^3*(exp(e*2i + f*x*2i) + 1)) - ((a + b/(exp(- e*1i - f*x*1i)/
2 + exp(e*1i + f*x*1i)/2)^2)^(1/2)*((a*(2*a + b)*32i)/(15*f*(a*1i + b*1i)^
2*(8*a + 8*b)) + (a*(2*a + 3*b)*32i)/(15*f*(a*1i + b*1i)^2*(8*a + 8*b)))*(
2*exp(e*2i + f*x*2i) + exp(e*4i + f*x*4i) + 1))/((exp(e*2i + f*x*2i) - 1)^
2*(exp(e*2i + f*x*2i) + 1)) + ((a + b/(exp(- e*1i - f*x*1i)/2 + exp(e*1i +
 f*x*1i)/2)^2)^(1/2)*(2*exp(e*2i + f*x*2i) + exp(e*4i + f*x*4i) + 1)*((96*
a + 32*b)/(5*f*(a*1i + b*1i)*(16*a + 16*b)) + (160*a + 160*b)/(5*f*(a*1i +
 b*1i)*(16*a + 16*b)) + (256*a + 320*b)/(5*f*(a*1i + b*1i)*(16*a + 16*b)))
)/((exp(e*2i + f*x*2i) - 1)^4*(exp(e*2i + f*x*2i) + 1)) - ((a + b/(exp(- e
*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2)^2)^(1/2)*(2*exp(e*2i + f*x*2i) + e
xp(e*4i + f*x*4i) + 1)*32i)/(f*(exp(e*2i + f*x*2i) - 1)^5*(exp(e*2i + f*x*
2i) + 1)*(10*a + 10*b)) - (8*a^2*(a + b/(exp(- e*1i - f*x*1i)/2 + exp(e*1i
 + f*x*1i)/2)^2)^(1/2)*(2*exp(e*2i + f*x*2i) + exp(e*4i + f*x*4i) + 1))/(1
5*f*(exp(e*2i + f*x*2i) - 1)*(exp(e*2i + f*x*2i) + 1)*(a*1i + b*1i)^3)
```

3.106 $\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

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3.106.1 Optimal result

Integrand size = 25, antiderivative size = 171

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{(15a^2+40ab+24b^2)\cos(e+fx)}{15a^3f\sqrt{a+b\sec^2(e+fx)}} + \frac{2(5a+3b)\cos^3(e+fx)}{15a^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cos^5(e+fx)}{5af\sqrt{a+b\sec^2(e+fx)}} - \frac{2b(15a^2+40ab+24b^2)\sec(e+fx)}{15a^4f\sqrt{a+b\sec^2(e+fx)}}$$

output

```
-1/15*(15*a^2+40*a*b+24*b^2)*cos(f*x+e)/a^3/f/(a+b*sec(f*x+e)^2)^(1/2)+2/1
5*(5*a+3*b)*cos(f*x+e)^3/a^2/f/(a+b*sec(f*x+e)^2)^(1/2)-1/5*cos(f*x+e)^5/a
/f/(a+b*sec(f*x+e)^2)^(1/2)-2/15*b*(15*a^2+40*a*b+24*b^2)*sec(f*x+e)/a^4/f
/(a+b*sec(f*x+e)^2)^(1/2)
```

3.106.2 Mathematica [A] (verified)

Time = 6.91 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.76

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))\,(150a^3+1528a^2b+2944ab^2+1536b^3+a(125a^2+544ab+384b^2)\cos(2(e+$$

input
```
Integrate[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
-1/960*((a + 2*b + a*Cos[2*(e + f*x)])*(150*a^3 + 1528*a^2*b + 2944*a*b^2
+ 1536*b^3 + a*(125*a^2 + 544*a*b + 384*b^2)*Cos[2*(e + f*x)] - 2*a^2*(11*
a + 12*b)*Cos[4*(e + f*x)] + 3*a^3*Cos[6*(e + f*x)])*Sec[e + f*x]^3)/(a^4*
f*(a + b*Sec[e + f*x]^2)^(3/2))
```

3.106.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4622, 365, 25, 359, 245, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sin(e+fx)^5}{(a+b\sec(e+fx)^2)^{3/2}}\,dx$$

$$\downarrow 4622$$

$$\frac{\int \frac{\cos^6(e+fx)\left(1-\sec^2(e+fx)\right)^2}{(b\sec^2(e+fx)+a)^{3/2}}\,d\sec(e+fx)}{f}$$

$$\downarrow 365$$

$$\frac{\int -\frac{\cos^4(e+fx)\left(2(5a+3b)-5a\sec^2(e+fx)\right)}{\left(b\sec^2(e+fx)+a\right)^{3/2}}\,d\sec(e+fx)}{5a} - \frac{\cos^5(e+fx)}{5a\sqrt{a+b\sec^2(e+fx)}}}{f}$$

$$\downarrow 25$$

$$\frac{-\frac{\int \frac{\cos^4(e+fx)\left(2(5a+3b)-5a\sec^2(e+fx)\right)}{\left(b\sec^2(e+fx)+a\right)^{3/2}}\,d\sec(e+fx)}{5a} - \frac{\cos^5(e+fx)}{5a\sqrt{a+b\sec^2(e+fx)}}}{f}$$

$$\downarrow 359$$

$$\frac{-\frac{\left(15a^2+40ab+24b^2\right)\int \frac{\cos^2(e+fx)}{\left(b\sec^2(e+fx)+a\right)^{3/2}}\,d\sec(e+fx)}{3a}-\frac{2(5a+3b)\cos^3(e+fx)}{3a\sqrt{a+b\sec^2(e+fx)}}}{5a}-\frac{\cos^5(e+fx)}{5a\sqrt{a+b\sec^2(e+fx)}}}{f}$$

$$\downarrow 245$$

$$\frac{-\frac{\left(15a^2+40ab+24b^2\right)\left(-\frac{2b\int \frac{1}{\left(b\sec^2(e+fx)+a\right)^{3/2}}\,d\sec(e+fx)}{a}-\frac{\cos(e+fx)}{a\sqrt{a+b\sec^2(e+fx)}}\right)}{3a}-\frac{2(5a+3b)\cos^3(e+fx)}{3a\sqrt{a+b\sec^2(e+fx)}}}{5a}-\frac{\cos^5(e+fx)}{5a\sqrt{a+b\sec^2(e+fx)}}}{f}$$

$$\downarrow 208$$

$$\frac{-\frac{\left(15a^2+40ab+24b^2\right)\left(-\frac{2b\sec(e+fx)}{a^2\sqrt{a+b\sec^2(e+fx)}}-\frac{\cos(e+fx)}{a\sqrt{a+b\sec^2(e+fx)}}\right)}{3a}-\frac{2(5a+3b)\cos^3(e+fx)}{3a\sqrt{a+b\sec^2(e+fx)}}}{5a}-\frac{\cos^5(e+fx)}{5a\sqrt{a+b\sec^2(e+fx)}}}{f}$$

input
```
Int[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(-1/5*Cos[e + f*x]^5/(a*Sqrt[a + b*Sec[e + f*x]^2]) - ((-2*(5*a + 3*b)*Cos
[e + f*x]^3)/(3*a*Sqrt[a + b*Sec[e + f*x]^2]) - ((15*a^2 + 40*a*b + 24*b^2
)*(-(Cos[e + f*x]/(a*Sqrt[a + b*Sec[e + f*x]^2])) - (2*b*Sec[e + f*x])/(a^
2*Sqrt[a + b*Sec[e + f*x]^2])))/(3*a))/(5*a))/f
```

3.106.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

3.106.4 Maple [A] (verified)

Time = 5.04 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.99

method	result
default	$-\frac{(a+b)^6 a^2 \left(b+a\cos(fx+e)^2\right)\left(3a^3\cos(fx+e)^6-10\cos(fx+e)^4 a^3-6\cos(fx+e)^4 a^2 b+15\cos(fx+e)^2 a^3+40\cos(fx+e)^2 a^2 b+24\cos\right.}{15 f\left(\sqrt{-ab}-a\right)^6\left(\sqrt{-ab}+a\right)^6\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}}$

input
```
int(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15/f*(a+b)^6*a^2/((-a*b)^(1/2)-a)^6/((-a*b)^(1/2)+a)^6*(b+a*cos(f*x+e)^
2)*(3*a^3*cos(f*x+e)^6-10*cos(f*x+e)^4*a^3-6*cos(f*x+e)^4*a^2*b+15*cos(f*x
+e)^2*a^3+40*cos(f*x+e)^2*a^2*b+24*cos(f*x+e)^2*a*b^2+30*a^2*b+80*a*b^2+48
*b^3)/(a+b*sec(f*x+e)^2)^(3/2)*sec(f*x+e)^3
```

3.106.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.80

$$\int \frac{\sin^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx =$$

$$-\frac{\left(3\,a^3\cos(fx+e)^7-2\,(5\,a^3+3\,a^2b)\cos(fx+e)^5+(15\,a^3+40\,a^2b+24\,ab^2)\cos(fx+e)^3+2\,(15\,a^2b+4\right.}{15\left(a^5f\cos(fx+e)^2+a^4bf\right)}$$

input
```
integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/15*(3*a^3*cos(f*x + e)^7 - 2*(5*a^3 + 3*a^2*b)*cos(f*x + e)^5 + (15*a^3
 + 40*a^2*b + 24*a*b^2)*cos(f*x + e)^3 + 2*(15*a^2*b + 40*a*b^2 + 24*b^3)*
cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(a^5*f*cos(f*x +
 e)^2 + a^4*b*f)
```

3.106.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**5/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

3.106.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.46

$$\int \frac{\sin^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx =$$

$$-\frac{\frac{15\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{a^2}-\frac{10\left(\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3-6\sqrt{a+\frac{b}{\cos(fx+e)^2}}\,b\cos(fx+e)\right)}{a^3}+\frac{\frac{15\,b}{\sqrt{a+\frac{b}{\cos(fx+e)^2}}}a^2\cos(fx+e)}{a^3}+\frac{}{}}{}$$

input
```
integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/15*(15*sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e)/a^2 - 10*((a + b/cos(f*x
 + e)^2)^(3/2)*cos(f*x + e)^3 - 6*sqrt(a + b/cos(f*x + e)^2)*b*cos(f*x + e
))/a^3 + 15*b/(sqrt(a + b/cos(f*x + e)^2)*a^2*cos(f*x + e)) + 30*b^2/(sqrt
(a + b/cos(f*x + e)^2)*a^3*cos(f*x + e)) + 15*b^3/(sqrt(a + b/cos(f*x + e)
^2)*a^4*cos(f*x + e)) + 3*((a + b/cos(f*x + e)^2)^(5/2)*cos(f*x + e)^5 - 5
*(a + b/cos(f*x + e)^2)^(3/2)*b*cos(f*x + e)^3 + 15*sqrt(a + b/cos(f*x + e
)^2)*b^2*cos(f*x + e))/a^4)/f
```

3.106.8 Giac [F]

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sin(fx+e)^5}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.106.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sin(e+fx)^5}{\left(a+\frac{b}{\cos(e+fx)^2}\right)^{3/2}}\,dx$$

input
```
int(sin(e + f*x)^5/(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(sin(e + f*x)^5/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.107 $\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

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3.107.2 Mathematica [A] (verified) . . . 925
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3.107.1 Optimal result

Integrand size = 25, antiderivative size = 114

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{(3a+4b)\cos(e+fx)}{3a^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{\cos^3(e+fx)}{3af\sqrt{a+b\sec^2(e+fx)}} - \frac{2b(3a+4b)\sec(e+fx)}{3a^3f\sqrt{a+b\sec^2(e+fx)}}$$

output
```
-1/3*(3*a+4*b)*cos(f*x+e)/a^2/f/(a+b*sec(f*x+e)^2)^(1/2)+1/3*cos(f*x+e)^3/
a/f/(a+b*sec(f*x+e)^2)^(1/2)-2/3*b*(3*a+4*b)*sec(f*x+e)/a^3/f/(a+b*sec(f*x
+e)^2)^(1/2)
```

3.107.2 Mathematica [A] (verified)

Time = 2.61 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.82

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))\,(9a^2+64ab+64b^2+8a(a+2b)\cos(2(e+fx))-a^2\cos(4(e+fx)))\sec^3(e+}{48a^3f\,(a+b\sec^2(e+fx))^{3/2}}$$

input
```
Integrate[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
-1/48*((a + 2*b + a*Cos[2*(e + f*x)])*(9*a^2 + 64*a*b + 64*b^2 + 8*a*(a +
2*b)*Cos[2*(e + f*x)] - a^2*Cos[4*(e + f*x)])*Sec[e + f*x]^3)/(a^3*f*(a +
b*Sec[e + f*x]^2)^(3/2))
```

3.107.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 4622, 25, 359, 245, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sin(e+fx)^3}{(a+b\sec(e+fx)^2)^{3/2}}\,dx$$

$$\downarrow 4622$$

$$\frac{\int -\frac{\cos^4(e+fx)(1-\sec^2(e+fx))}{(b\sec^2(e+fx)+a)^{3/2}}\,d\sec(e+fx)}{f}$$

$$\downarrow 25$$

$$-\frac{\int \frac{\cos^4(e+fx)(1-\sec^2(e+fx))}{(b\sec^2(e+fx)+a)^{3/2}}\,d\sec(e+fx)}{f}$$

$$\downarrow 359$$

$$\frac{\frac{(3a+4b)\int \frac{\cos^2(e+fx)}{\left(b\sec^2(e+fx)+a\right)^{3/2}}\,d\sec(e+fx)}{3a}+\frac{\cos^3(e+fx)}{3a\sqrt{a+b\sec^2(e+fx)}}}{f}$$

$$\downarrow 245$$

$$\frac{\frac{(3a+4b)\left(-\frac{2b\int \frac{1}{\left(b\sec^2(e+fx)+a\right)^{3/2}}\,d\sec(e+fx)}{a}-\frac{\cos(e+fx)}{a\sqrt{a+b\sec^2(e+fx)}}\right)}{3a}+\frac{\cos^3(e+fx)}{3a\sqrt{a+b\sec^2(e+fx)}}}{f}$$

$$\downarrow 208$$

$$\frac{\frac{(3a+4b)\left(-\frac{2b\sec(e+fx)}{a^2\sqrt{a+b\sec^2(e+fx)}}-\frac{\cos(e+fx)}{a\sqrt{a+b\sec^2(e+fx)}}\right)}{3a}+\frac{\cos^3(e+fx)}{3a\sqrt{a+b\sec^2(e+fx)}}}{f}$$

input
```
Int[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Cos[e + f*x]^3/(3*a*Sqrt[a + b*Sec[e + f*x]^2]) + ((3*a + 4*b)*(-(Cos[e +
 f*x]/(a*Sqrt[a + b*Sec[e + f*x]^2])) - (2*b*Sec[e + f*x])/(a^2*Sqrt[a + b
*Sec[e + f*x]^2])))/(3*a))/f
```

3.107.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

3.107.4 Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.01

method	result	size
default	$\frac{a(a+b)^4\left(b+a\cos(fx+e)^2\right)\left(\cos(fx+e)^4a^2-3\cos(fx+e)^2a^2-4\cos(fx+e)^2ab-6ab-8b^2\right)\sec(fx+e)^3}{3f\left(\sqrt{-ab}-a\right)^4\left(\sqrt{-ab}+a\right)^4\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}}$	115

input
```
int(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/f*a/((-a*b)^(1/2)-a)^4/((-a*b)^(1/2)+a)^4*(a+b)^4*(b+a*cos(f*x+e)^2)*(
cos(f*x+e)^4*a^2-3*cos(f*x+e)^2*a^2-4*cos(f*x+e)^2*a*b-6*a*b-8*b^2)/(a+b*s
ec(f*x+e)^2)^(3/2)*sec(f*x+e)^3
```

3.107.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.86

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\left(a^2\cos(fx+e)^5-(3\,a^2+4\,ab)\cos(fx+e)^3-2\,(3\,ab+4\,b^2)\cos(fx+e)\right)}{3\left(a^4f\cos(fx+e)^2+a^3bf\right)}$$

input
```
integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(a^2*cos(f*x + e)^5 - (3*a^2 + 4*a*b)*cos(f*x + e)^3 - 2*(3*a*b + 4*b^
2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(a^4*f*cos(f*
x + e)^2 + a^3*b*f)
```

3.107.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**3/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

3.107.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.24

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx =$$
$$-\frac{\frac{3\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{a^2} - \frac{\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3 - 6\sqrt{a+\frac{b}{\cos(fx+e)^2}}b\cos(fx+e)}{a^3} + \frac{3b}{\sqrt{a+\frac{b}{\cos(fx+e)^2}}a^2\cos(fx+e)} + \frac{}{\sqrt{a+\frac{}{\cos($$
$$3f$$

input
```
integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/3*(3*sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e)/a^2 - ((a + b/cos(f*x + e)
^2)^(3/2)*cos(f*x + e)^3 - 6*sqrt(a + b/cos(f*x + e)^2)*b*cos(f*x + e))/a^
3 + 3*b/(sqrt(a + b/cos(f*x + e)^2)*a^2*cos(f*x + e)) + 3*b^2/(sqrt(a + b/
cos(f*x + e)^2)*a^3*cos(f*x + e)))/f
```

3.107.8 Giac [F]

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sin(fx+e)^3}{\left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.107.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sin(e+fx)^3}{\left(a+\frac{b}{\cos(e+fx)^2}\right)^{3/2}}\,dx$$

input
```
int(sin(e + f*x)^3/(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(sin(e + f*x)^3/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.108 $\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

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3.108.3 Rubi [A] (verified) . . . 932
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3.108.5 Fricas [A] (verification not implemented) . . . 934
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3.108.7 Maxima [A] (verification not implemented) . . . 934
3.108.8 Giac [A] (verification not implemented) . . . 935
3.108.9 Mupad [B] (verification not implemented) . . . 935

3.108.1 Optimal result

Integrand size = 23, antiderivative size = 62

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{\cos(e+fx)}{af\sqrt{a+b\sec^2(e+fx)}} - \frac{2b\sec(e+fx)}{a^2f\sqrt{a+b\sec^2(e+fx)}}$$

output
```
-cos(f*x+e)/a/f/(a+b*sec(f*x+e)^2)^(1/2)-2*b*sec(f*x+e)/a^2/f/(a+b*sec(f*x
+e)^2)^(1/2)
```

3.108.2 Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.03

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))(a+4b+a\cos(2(e+fx)))\sec^3(e+fx)}{4a^2f\,(a+b\sec^2(e+fx))^{3/2}}$$

input
```
Integrate[Sin[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
-1/4*((a + 2*b + a*Cos[2*(e + f*x)])*(a + 4*b + a*Cos[2*(e + f*x)])*Sec[e
+ f*x]^3)/(a^2*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

3.108.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4622, 245, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sin(e+fx)}{(a+b\sec(e+fx)^2)^{3/2}}\,dx$$

$$\downarrow 4622$$

$$\frac{\int \frac{\cos^2(e+fx)}{(b\sec^2(e+fx)+a)^{3/2}}\,d\sec(e+fx)}{f}$$

$$\downarrow 245$$

$$\frac{-\frac{2b\int \frac{1}{\left(b\sec^2(e+fx)+a\right)^{3/2}}\,d\sec(e+fx)}{a} - \frac{\cos(e+fx)}{a\sqrt{a+b\sec^2(e+fx)}}}{f}$$

$$\downarrow 208$$

$$\frac{-\frac{2b\sec(e+fx)}{a^2\sqrt{a+b\sec^2(e+fx)}} - \frac{\cos(e+fx)}{a\sqrt{a+b\sec^2(e+fx)}}}{f}$$

input
```
Int[Sin[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(-(Cos[e + f*x]/(a*Sqrt[a + b*Sec[e + f*x]^2])) - (2*b*Sec[e + f*x])/(a^2*
Sqrt[a + b*Sec[e + f*x]^2]))/f
```

3.108.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

3.108.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$\frac{-\frac{1}{a\sec(fx+e)\sqrt{a+b\sec(fx+e)^2}}-\frac{2b\sec(fx+e)}{a^2\sqrt{a+b\sec(fx+e)^2}}}{f}$	59
default	$\frac{-\frac{1}{a\sec(fx+e)\sqrt{a+b\sec(fx+e)^2}}-\frac{2b\sec(fx+e)}{a^2\sqrt{a+b\sec(fx+e)^2}}}{f}$	59

input
```
int(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/a/sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2)-2*b/a^2*sec(f*x+e)/(a+b*sec(
f*x+e)^2)^(1/2))
```

3.108.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.08

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{\left(a\cos(fx+e)^3+2\,b\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{a^3 f\cos(fx+e)^2+a^2bf}$$

input
```
integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
-(a*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)/(a^3*f*cos(f*x + e)^2 + a^2*b*f)
```

3.108.6 Sympy [F]

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(sin(e + f*x)/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.108.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{\frac{\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{a^2}+\frac{b}{\sqrt{a+\frac{b}{\cos(fx+e)^2}}a^2\cos(fx+e)}}{f}$$

input
```
integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
-(sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e)/a^2 + b/(sqrt(a + b/cos(f*x + e)
^2)*a^2*cos(f*x + e)))/f
```

3.108.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.77

$$\int \frac{\sin(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = -\frac{\sqrt{a\cos\left(fx+e\right)^2+b}+\frac{b}{\sqrt{a\cos(fx+e)^2+b}}}{a^2 f \operatorname{sgn}\left(\cos\left(fx+e\right)\right)}$$

input
```
integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
-(sqrt(a*cos(f*x + e)^2 + b) + b/sqrt(a*cos(f*x + e)^2 + b))/(a^2*f*sgn(co
s(f*x + e)))
```

3.108.9 Mupad [B] (verification not implemented)

Time = 24.73 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.50

$$\int \frac{\sin(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx =$$
$$-\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\sqrt{a+\frac{b}{\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}\right)^2}}\left(a+2\,a\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+a\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}+8\,b\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\right)}{2\,a^2\,f\,\left(a+2\,a\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+a\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}+4\,b\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\right)}$$

input
```
int(sin(e + f*x)/(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
-(exp(- e*1i - f*x*1i)*(exp(e*2i + f*x*2i) + 1)*(a + b/(exp(- e*1i - f*x*1
i)/2 + exp(e*1i + f*x*1i)/2)^2)^(1/2)*(a + 2*a*exp(e*2i + f*x*2i) + a*exp(
e*4i + f*x*4i) + 8*b*exp(e*2i + f*x*2i)))/(2*a^2*f*(a + 2*a*exp(e*2i + f*x
*2i) + a*exp(e*4i + f*x*4i) + 4*b*exp(e*2i + f*x*2i)))
```

3.109 $\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

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3.109.1 Optimal result

Integrand size = 23, antiderivative size = 80

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{(a+b)^{3/2}f} - \frac{b\sec(e+fx)}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}}$$

output
```
-arctanh(sec(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))/(a+b)^(3/2)/f-b*
sec(f*x+e)/a/(a+b)/f/(a+b*sec(f*x+e)^2)^(1/2)
```

3.109.2 Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.41

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))\sec^3(e+fx)\left(b\sqrt{a+b}+a\operatorname{arctanh}\left(\frac{\sqrt{a+b-a\sin^2(e+fx)}}{\sqrt{a+b}}\right)\right)\sqrt{a+b-a\sin^2(e+fx)}}{2a(a+b)^{3/2}f\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

input
```
Integrate[Csc[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
-1/2*((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(b*Sqrt[a + b] + a*Arc
Tanh[Sqrt[a + b - a*Sin[e + f*x]^2]/Sqrt[a + b]]*Sqrt[a + b - a*Sin[e + f*
x]^2]))/(a*(a + b)^(3/2)*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

3.109.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4622, 25, 296, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sin(e+fx)\,(a+b\sec(e+fx)^2)^{3/2}}dx$$

$$\downarrow 4622$$

$$\frac{\int -\frac{1}{(1-\sec^2(e+fx))(b\sec^2(e+fx)+a)^{3/2}}d\sec(e+fx)}{f}$$

$$\downarrow 25$$

$$-\frac{\int \frac{1}{(1-\sec^2(e+fx))(b\sec^2(e+fx)+a)^{3/2}}d\sec(e+fx)}{f}$$

$$\downarrow 296$$

$$\frac{-\frac{\int \frac{1}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{a+b} - \frac{b\sec(e+fx)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{f}$$

$$\downarrow 291$$

$$\frac{-\frac{\int \frac{1}{1-\frac{(a+b)\sec^2(e+fx)}{b\sec^2(e+fx)+a}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}}{a+b} - \frac{b\sec(e+fx)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{f}$$

$$\downarrow 219$$

$$\frac{-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{(a+b)^{3/2}} - \frac{b\sec(e+fx)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{f}$$

input
```
Int[Csc[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(-(ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]]/(a + b)^
(3/2)) - (b*Sec[e + f*x])/(a*(a + b)*Sqrt[a + b*Sec[e + f*x]^2]))/f
```

3.109.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

3.109.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1060 vs. 2(72) = 144.

Time = 1.18 (sec) , antiderivative size = 1061, normalized size of antiderivative = 13.26

method	result	size
default	Expression too large to display	1061

input
```
int(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/f/(a+b)^(5/2)/a*(b+a*cos(f*x+e)^2)*(cos(f*x+e)*ln(2/(a+b)^(1/2)*(((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)
))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2+cos(f*x+e)*ln(2/(a+b)^(
1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+
cos(f*x+e)))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b+cos(f*x+e)*ln
(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+
cos(f*x+e)))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2+cos(f*x+e)*ln
(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+
cos(f*x+e)))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b+2*(a+b)^(3/2)
*b+ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/
2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(
f*x+e)*a+b)/(1+cos(f*x+e)))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^
2+ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2
)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f
*x+e)*a+b)/(1+cos(f*x+e)))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b
+ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x...
```

3.109.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(72) = 144$.

Time = 0.33 (sec) , antiderivative size = 344, normalized size of antiderivative = 4.30

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \left[-\frac{2\,(ab+b^2)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e) - \left(a^2\cos(fx+e)^2+ab\right)\sqrt{a+b}\,\mathrm{l}}{2\left((a^4+2\,a^3b+a^2b^2)f\cos(fx+e)^2+(\right.}\right.$$

input
```
integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/2*(2*(a*b + b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x +
 e) - (a^2*cos(f*x + e)^2 + a*b)*sqrt(a + b)*log(2*(a*cos(f*x + e)^2 - 2*s
qrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a +
2*b)/(cos(f*x + e)^2 - 1)))/((a^4 + 2*a^3*b + a^2*b^2)*f*cos(f*x + e)^2 +
(a^3*b + 2*a^2*b^2 + a*b^3)*f), ((a^2*cos(f*x + e)^2 + a*b)*sqrt(-a - b)*a
rctan(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e
)/(a + b)) - (a*b + b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f
*x + e))/((a^4 + 2*a^3*b + a^2*b^2)*f*cos(f*x + e)^2 + (a^3*b + 2*a^2*b^2
+ a*b^3)*f)]
```

3.109.6 Sympy [F]

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(csc(e + f*x)/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.109.7 Maxima [F]

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\csc(fx+e)}{\left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(csc(f*x + e)/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.109.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 503 vs. $2(72) = 144$.

Time = 0.86 (sec) , antiderivative size = 503, normalized size of antiderivative = 6.29

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{2\left(\frac{\left(ab^2+b^3\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2}{a^3b\operatorname{sgn}(\cos(fx+e))+2a^2b^2\operatorname{sgn}(\cos(fx+e))+ab^3\operatorname{sgn}(\cos(fx+e))}+\frac{ab^2+b^3}{a^3b\operatorname{sgn}(\cos(fx+e))+2a^2b^2\operatorname{sgn}(\cos(fx+e))+ab^3\operatorname{sgn}(\cos(fx+e))}\right)}{\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a+b}} - \frac{\log\left(\left|-\sqrt{a+b}\tan(\right.\right.}{}$$

input
```
integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/2*(2*((a*b^2 + b^3)*tan(1/2*f*x + 1/2*e)^2/(a^3*b*sgn(cos(f*x + e)) + 2
*a^2*b^2*sgn(cos(f*x + e)) + a*b^3*sgn(cos(f*x + e))) + (a*b^2 + b^3)/(a^3
*b*sgn(cos(f*x + e)) + 2*a^2*b^2*sgn(cos(f*x + e)) + a*b^3*sgn(cos(f*x + e
))))/sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/
2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) - log(abs(-sqrt(a +
 b)*tan(1/2*f*x + 1/2*e)^2 + sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x
 + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a
+ b) - sqrt(a + b)))/((a + b)^(3/2)*sgn(cos(f*x + e))) - log(abs((sqrt(a +
 b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x
 + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a
+ b))*sqrt(a + b) - a + b))/((a + b)^(3/2)*sgn(cos(f*x + e))) + log(abs((s
qrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(
1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)
^2 + a + b))*sqrt(a + b) - a - b))/((a + b)^(3/2)*sgn(cos(f*x + e))))/f
```

3.109.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{1}{\sin(e+fx)\left(a+\frac{b}{\cos(e+fx)^2}\right)^{3/2}}\,dx$$

input
```
int(1/(sin(e + f*x)*(a + b/cos(e + f*x)^2)^(3/2)),x)
```

output
```
int(1/(sin(e + f*x)*(a + b/cos(e + f*x)^2)^(3/2)), x)
```

3.110 $\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

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3.110.1 Optimal result

Integrand size = 25, antiderivative size = 126

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{(a-2b)\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2(a+b)^{5/2}f} - \frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{3b\sec(e+fx)}{2(a+b)^2f\sqrt{a+b\sec^2(e+fx)}}$$

output

```
-1/2*(a-2*b)*arctanh(sec(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))/(a+b
)^(5/2)/f-1/2*cot(f*x+e)*csc(f*x+e)/(a+b)/f/(a+b*sec(f*x+e)^2)^(1/2)-3/2*b
*sec(f*x+e)/(a+b)^2/f/(a+b*sec(f*x+e)^2)^(1/2)
```

3.110.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.77

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))\left((a+b)\csc^2(e+fx) - (a-2b)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1-\frac{a\sin^2(e+fx)}{a+b}\right.\right.}{4(a+b)^2f\,(a+b\sec^2(e+fx))^{3/2}}$$

input
```
Integrate[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
-1/4*((a + 2*b + a*Cos[2*(e + f*x)])*((a + b)*Csc[e + f*x]^2 - (a - 2*b)*H
ypergeometric2F1[-1/2, 1, 1/2, 1 - (a*Sin[e + f*x]^2)/(a + b)])*Sec[e + f*
x]^3)/((a + b)^2*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

3.110.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4622, 373, 402, 25, 27, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sin(e+fx)^3\,(a+b\sec(e+fx)^2)^{3/2}}dx$$

$$\downarrow 4622$$

$$\frac{\int \frac{\sec^2(e+fx)}{(1-\sec^2(e+fx))^2(b\sec^2(e+fx)+a)^{3/2}}d\sec(e+fx)}{f}$$

$$\downarrow 373$$

$$\frac{\frac{\sec(e+fx)}{2(a+b)(1-\sec^2(e+fx))\sqrt{a+b\sec^2(e+fx)}} - \frac{\int \frac{a-2b\sec^2(e+fx)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec(e+fx)}{2(a+b)}}{f}$$

$$\downarrow 402$$

$$\frac{\frac{\sec(e+fx)}{2(a+b)(1-\sec^2(e+fx))\sqrt{a+b\sec^2(e+fx)}} - \frac{\frac{3b\sec(e+fx)}{(a+b)\sqrt{a+b\sec^2(e+fx)}} - \frac{\int -\frac{a(a-2b)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{a(a+b)}}{2(a+b)}}{f}$$

$$\downarrow 25$$

$$\frac{\frac{\sec(e+fx)}{2(a+b)(1-\sec^2(e+fx))\sqrt{a+b\sec^2(e+fx)}}-\frac{\frac{\int\frac{a(a-2b)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{a(a+b)}+\frac{3b\sec(e+fx)}{(a+b)\sqrt{a+b\sec^2(e+fx)}}}{2(a+b)}}{f}$$

$$\downarrow 27$$

$$\frac{\frac{\sec(e+fx)}{2(a+b)(1-\sec^2(e+fx))\sqrt{a+b\sec^2(e+fx)}}-\frac{\frac{(a-2b)\int\frac{1}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{a+b}+\frac{3b\sec(e+fx)}{(a+b)\sqrt{a+b\sec^2(e+fx)}}}{2(a+b)}}{f}$$

$$\downarrow 291$$

$$\frac{\frac{\sec(e+fx)}{2(a+b)(1-\sec^2(e+fx))\sqrt{a+b\sec^2(e+fx)}}-\frac{\frac{(a-2b)\int\frac{1}{1-\frac{(a+b)\sec^2(e+fx)}{b\sec^2(e+fx)+a}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}}{a+b}+\frac{3b\sec(e+fx)}{(a+b)\sqrt{a+b\sec^2(e+fx)}}}{2(a+b)}}{f}$$

$$\downarrow 219$$

$$\frac{\frac{\sec(e+fx)}{2(a+b)(1-\sec^2(e+fx))\sqrt{a+b\sec^2(e+fx)}}-\frac{\frac{(a-2b)\text{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{(a+b)^{3/2}}+\frac{3b\sec(e+fx)}{(a+b)\sqrt{a+b\sec^2(e+fx)}}}{2(a+b)}}{f}$$

input
```
Int[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Sec[e + f*x]/(2*(a + b)*(1 - Sec[e + f*x]^2)*Sqrt[a + b*Sec[e + f*x]^2])
- (((a - 2*b)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2
]])/(a + b)^(3/2) + (3*b*Sec[e + f*x])/((a + b)*Sqrt[a + b*Sec[e + f*x]^2]
))/(2*(a + b)))/f
```

3.110.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

3.110.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1713 vs. 2(110) = 220.

Time = 1.09 (sec) , antiderivative size = 1714, normalized size of antiderivative = 13.60

method	result	size
default	Expression too large to display	1714

input
```
int(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/f/(a+b)^(9/2)*(b+a*cos(f*x+e)^2)*(2*cos(f*x+e)^2*(a+b)^(5/2)*a-4*cos(
f*x+e)^2*(a+b)^(5/2)*b+cos(f*x+e)*sin(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a
+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^3-3*cos(f*x+e)*sin(f*x+e)^2*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a*b^2-2
*cos(f*x+e)*sin(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(2/
(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f
*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a
+b)/(1+cos(f*x+e)))*b^3+cos(f*x+e)*sin(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)
^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+
cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*a^3-3*cos(f*x+e)*sin(f*x+e)^2*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*a*b^2-2*cos(f*x+e)*sin(f*x
+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(...
```

3.110.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(110) = 220$.

Time = 0.38 (sec) , antiderivative size = 547, normalized size of antiderivative = 4.34

$$\int \frac{\csc^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \left[-\frac{\left((a^2-2\,ab)\cos(fx+e)^4-(a^2-3\,ab+2\,b^2)\cos(fx+e)^2-ab+2\,b^2\right)}{4\left((a^4+3\,a^3b+3\,a^2b^2+a\right.}\right.$$

input
```
integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*(((a^2 - 2*a*b)*cos(f*x + e)^4 - (a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^
2 - a*b + 2*b^2)*sqrt(a + b)*log(2*(a*cos(f*x + e)^2 + 2*sqrt(a + b)*sqrt(
(a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*x +
e)^2 - 1)) - 2*((a^2 - a*b - 2*b^2)*cos(f*x + e)^3 + 3*(a*b + b^2)*cos(f*x
 + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^4 + 3*a^3*b + 3*a^
2*b^2 + a*b^3)*f*cos(f*x + e)^4 - (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*f*cos(f*
x + e)^2 - (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*f), 1/2*(((a^2 - 2*a*b)*cos
(f*x + e)^4 - (a^2 - 3*a*b + 2*b^2)*cos(f*x + e)^2 - a*b + 2*b^2)*sqrt(-a
- b)*arctan(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f
*x + e)/(a + b)) + ((a^2 - a*b - 2*b^2)*cos(f*x + e)^3 + 3*(a*b + b^2)*cos
(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^4 + 3*a^3*b +
3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^4 - (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*f*co
s(f*x + e)^2 - (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*f)]
```

3.110.6 Sympy [F]

$$\int \frac{\csc^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\csc^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)**3/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(csc(e + f*x)**3/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.110.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.110.8 Giac [F]

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\csc\left(fx+e\right)^3}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.110.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{1}{\sin\left(e+f\,x\right)^3\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

input
```
int(1/(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2)),x)
```

output
```
int(1/(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2)), x)
```

3.111 $\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

3.111.1 Optimal result . . . 950
3.111.2 Mathematica [C] (verified) . . . 950
3.111.3 Rubi [A] (verified) . . . 951
3.111.4 Maple [B] (warning: unable to verify) . . . 954
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3.111.7 Maxima [F(-1)] . . . 957
3.111.8 Giac [F] . . . 957
3.111.9 Mupad [F(-1)] . . . 957

3.111.1 Optimal result

Integrand size = 25, antiderivative size = 177

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{3a(a-4b)\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8(a+b)^{7/2}f} - \frac{5a\cot(e+fx)\csc(e+fx)}{8(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\sqrt{a+b\sec^2(e+fx)}} - \frac{(13a-2b)b\sec(e+fx)}{8(a+b)^3f\sqrt{a+b\sec^2(e+fx)}}$$

output

```
-3/8*a*(a-4*b)*arctanh(sec(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))/(a
+b)^(7/2)/f-5/8*a*cot(f*x+e)*csc(f*x+e)/(a+b)^2/f/(a+b*sec(f*x+e)^2)^(1/2)
-1/4*cot(f*x+e)^3*csc(f*x+e)/(a+b)/f/(a+b*sec(f*x+e)^2)^(1/2)-1/8*(13*a-2*
b)*b*sec(f*x+e)/(a+b)^3/f/(a+b*sec(f*x+e)^2)^(1/2)
```

3.111.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.38 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.56

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))\left((a+b)^2\csc^4(e+fx) - a(a-4b)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},2,\frac{1}{2},1-\frac{a\sin^2(e+}{a+b}\right.\right.}{8(a+b)^3f\,(a+b\sec^2(e+fx))^{3/2}}$$

input
```
Integrate[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
-1/8*((a + 2*b + a*Cos[2*(e + f*x)])*((a + b)^2*Csc[e + f*x]^4 - a*(a - 4*
b)*Hypergeometric2F1[-1/2, 2, 1/2, 1 - (a*Sin[e + f*x]^2)/(a + b)])*Sec[e
+ f*x]^3)/((a + b)^3*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

3.111.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.13, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 4622, 25, 372, 27, 402, 25, 402, 27, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sin(e+fx)^5\,(a+b\sec(e+fx)^2)^{3/2}}dx$$

$$\downarrow 4622$$

$$\frac{\int -\frac{\sec^4(e+fx)}{(1-\sec^2(e+fx))^3(b\sec^2(e+fx)+a)^{3/2}}d\sec(e+fx)}{f}$$

$$\downarrow 25$$

$$-\frac{\int \frac{\sec^4(e+fx)}{(1-\sec^2(e+fx))^3(b\sec^2(e+fx)+a)^{3/2}}d\sec(e+fx)}{f}$$

$$\downarrow 372$$

$$\frac{\int \frac{a\left(4\sec^2(e+fx)+1\right)}{\left(1-\sec^2(e+fx)\right)^2\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec(e+fx)}{4(a+b)} - \frac{\sec(e+fx)}{4(a+b)(1-\sec^2(e+fx))^2\sqrt{a+b\sec^2(e+fx)}}}{f}$$

$$\downarrow 27$$

$$\frac{a\int \frac{4\sec^2(e+fx)+1}{\left(1-\sec^2(e+fx)\right)^2\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec(e+fx)}{4(a+b)} - \frac{\sec(e+fx)}{4(a+b)(1-\sec^2(e+fx))^2\sqrt{a+b\sec^2(e+fx)}}}{f}$$

$$\downarrow 402$$

$$\frac{a\left(\frac{\int -\frac{-10b\sec^2(e+fx)+3a-2b}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec(e+fx)}{2(a+b)}+\frac{5\sec(e+fx)}{2(a+b)\left(1-\sec^2(e+fx)\right)\sqrt{a+b\sec^2(e+fx)}}\right)}{4(a+b)}-\frac{\sec(e+fx)}{4(a+b)(1-\sec^2(e+fx))^2\sqrt{a+b\sec^2(e+fx)}}}{f}$$

$$\downarrow 25$$

$$\frac{a\left(\frac{5\sec(e+fx)}{2(a+b)\left(1-\sec^2(e+fx)\right)\sqrt{a+b\sec^2(e+fx)}}-\frac{\int \frac{-10b\sec^2(e+fx)+3a-2b}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec(e+fx)}{2(a+b)}\right)}{4(a+b)}-\frac{\sec(e+fx)}{4(a+b)(1-\sec^2(e+fx))^2\sqrt{a+b\sec^2(e+fx)}}}{f}$$

$$\downarrow 402$$

$$\frac{a\left(\frac{5\sec(e+fx)}{2(a+b)\left(1-\sec^2(e+fx)\right)\sqrt{a+b\sec^2(e+fx)}}-\frac{\frac{b(13a-2b)\sec(e+fx)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}-\frac{\int -\frac{3a(a-4b)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{a(a+b)}}{2(a+b)}\right)}{4(a+b)}-\frac{\sec(e+f}{4(a+b)(1-\sec^2(e+fx))^2}}{f}$$

$$\downarrow 27$$

$$\frac{a\left(\frac{5\sec(e+fx)}{2(a+b)\left(1-\sec^2(e+fx)\right)\sqrt{a+b\sec^2(e+fx)}}-\frac{\frac{3(a-4b)\int \frac{1}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{a+b}+\frac{b(13a-2b)\sec(e+fx)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{2(a+b)}\right)}{4(a+b)}-\frac{\text{se}}{4(a+b)(1-\sec^2(e+}}{f}$$

$$\downarrow 291$$

$$\frac{a\left(\frac{5\sec(e+fx)}{2(a+b)\left(1-\sec^2(e+fx)\right)\sqrt{a+b\sec^2(e+fx)}}-\frac{\frac{3(a-4b)\int \frac{1}{1-\frac{(a+b)\sec^2(e+fx)}{b\sec^2(e+fx)+a}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}}{a+b}+\frac{b(13a-2b)\sec(e+fx)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{2(a+b)}\right)}{4(a+b)}-\frac{\sec(e+}{4(a+b)(1-\sec^2(e+fx))}}{f}$$

$$\downarrow 219$$

3.111. $\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

$$\frac{a\left(\frac{5\sec(e+fx)}{2(a+b)\left(1-\sec^2(e+fx)\right)\sqrt{a+b\sec^2(e+fx)}}-\frac{\frac{3(a-4b)\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{(a+b)^{3/2}}+\frac{b(13a-2b)\sec(e+fx)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{2(a+b)}\right)}{4(a+b)}-\frac{\sec(e+fx)}{4(a+b)(1-\sec^2(e+fx))^2\sqrt{a+b\sec^2(e+fx)}}$$
$$f$$

input
```
Int[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(-1/4*Sec[e + f*x]/((a + b)*(1 - Sec[e + f*x]^2)^2*Sqrt[a + b*Sec[e + f*x]
^2]) + (a*((5*Sec[e + f*x])/(2*(a + b)*(1 - Sec[e + f*x]^2)*Sqrt[a + b*Sec
[e + f*x]^2]) - ((3*(a - 4*b)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a +
b*Sec[e + f*x]^2]])/(a + b)^(3/2) + ((13*a - 2*b)*b*Sec[e + f*x])/(a*(a +
b)*Sqrt[a + b*Sec[e + f*x]^2]))/(2*(a + b))))/(4*(a + b)))/f
```

3.111.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

3.111.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 3611 vs. $2(157) = 314$.

Time = 1.23 (sec) , antiderivative size = 3612, normalized size of antiderivative = 20.41

method	result	size
default	Expression too large to display	3612

input
```
int(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
1/64/f/(a+b)^(13/2)*(2*a*b*(a+b)^(7/2)+(a+b)^(7/2)*a^2+(a+b)^(7/2)*b^2+7*a
^2*(1-cos(f*x+e))^2*(a+b)^(7/2)*csc(f*x+e)^2-3*b^2*(1-cos(f*x+e))^2*(a+b)^
(7/2)*csc(f*x+e)^2+(a+b)^(7/2)*a^2*(1-cos(f*x+e))^10*csc(f*x+e)^10+(a+b)^(
7/2)*b^2*(1-cos(f*x+e))^10*csc(f*x+e)^10-12*ln(2/(1-cos(f*x+e))^2*(-a*(1-c
os(f*x+e))^2+b*(1-cos(f*x+e))^2+(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(
f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e)
)^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(1/2)*sin(f*x+e)^2+a*sin(f*x+e)^2+b*sin(
f*x+e)^2))*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^
4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)
^(1/2)*a^4*b*(1-cos(f*x+e))^4*csc(f*x+e)^4-108*ln(2/(1-cos(f*x+e))^2*(-a*(
1-cos(f*x+e))^2+b*(1-cos(f*x+e))^2+(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-c
os(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x
+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(1/2)*sin(f*x+e)^2+a*sin(f*x+e)^2+b*s
in(f*x+e)^2))*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+
e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a
+b)^(1/2)*a^3*b^2*(1-cos(f*x+e))^4*csc(f*x+e)^4-132*ln(2/(1-cos(f*x+e))^2*
(-a*(1-cos(f*x+e))^2+b*(1-cos(f*x+e))^2+(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b
*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-co
s(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(1/2)*sin(f*x+e)^2+a*sin(f*x+e)^
2+b*sin(f*x+e)^2))*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*...
```

3.111.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(157) = 314$.

Time $= 0.42$ (sec) , antiderivative size $= 873$, normalized size of antiderivative $= 4.93$

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \left[-\frac{3\left((a^3-4\,a^2b)\cos(fx+e)^6-(2\,a^3-9\,a^2b+4\,ab^2)\cos(fx+e)^4+a^2b\right.}{16\left((a^5+4\,a^4b+6\,a^{3}\right.}\right.$$

input

```
integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/16*(3*((a^3 - 4*a^2*b)*cos(f*x + e)^6 - (2*a^3 - 9*a^2*b + 4*a*b^2)*co
s(f*x + e)^4 + a^2*b - 4*a*b^2 + (a^3 - 6*a^2*b + 8*a*b^2)*cos(f*x + e)^2)
*sqrt(a + b)*log(2*(a*cos(f*x + e)^2 + 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1)) - 2*(
3*(a^3 - 3*a^2*b - 4*a*b^2)*cos(f*x + e)^5 - (5*a^3 - 16*a^2*b - 17*a*b^2
+ 4*b^3)*cos(f*x + e)^3 - (13*a^2*b + 11*a*b^2 - 2*b^3)*cos(f*x + e))*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a
^2*b^3 + a*b^4)*f*cos(f*x + e)^6 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 + 2*a^2*b^
3 - 2*a*b^4 - b^5)*f*cos(f*x + e)^4 + (a^5 + 2*a^4*b - 2*a^3*b^2 - 8*a^2*b
^3 - 7*a*b^4 - 2*b^5)*f*cos(f*x + e)^2 + (a^4*b + 4*a^3*b^2 + 6*a^2*b^3 +
4*a*b^4 + b^5)*f), 1/8*(3*((a^3 - 4*a^2*b)*cos(f*x + e)^6 - (2*a^3 - 9*a^2
*b + 4*a*b^2)*cos(f*x + e)^4 + a^2*b - 4*a*b^2 + (a^3 - 6*a^2*b + 8*a*b^2)
*cos(f*x + e)^2)*sqrt(-a - b)*arctan(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)*cos(f*x + e)/(a + b)) + (3*(a^3 - 3*a^2*b - 4*a*b^2)*c
os(f*x + e)^5 - (5*a^3 - 16*a^2*b - 17*a*b^2 + 4*b^3)*cos(f*x + e)^3 - (13
*a^2*b + 11*a*b^2 - 2*b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2))/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*f*cos(f*x + e
)^6 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 - b^5)*f*cos(f*x
+ e)^4 + (a^5 + 2*a^4*b - 2*a^3*b^2 - 8*a^2*b^3 - 7*a*b^4 - 2*b^5)*f*cos(f
*x + e)^2 + (a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*f)]
```

3.111.6 Sympy [F]

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)**5/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(csc(e + f*x)**5/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.111.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.111.8 Giac [F]

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\csc\left(fx+e\right)^5}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.111.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{1}{\sin\left(e+f\,x\right)^5\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

input
```
int(1/(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2)),x)
```

output
```
int(1/(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2)), x)
```

3.112 $\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

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3.112.1 Optimal result

Integrand size = 25, antiderivative size = 242

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{5(a+b)^2(a+7b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{9/2}f} - \frac{(a+b)(33a+35b)\cos(e+fx)\sin(e+fx)}{48a^3f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(9a+7b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{b(81a^2+190ab+105b^2)\tan(e+fx)}{48a^4f\sqrt{a+b+b\tan^2(e+fx)}}$$

output

```
5/16*(a+b)^2*(a+7*b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))
/a^(9/2)/f-1/48*(a+b)*(33*a+35*b)*cos(f*x+e)*sin(f*x+e)/a^3/f/(a+b+b*tan(f
*x+e)^2)^(1/2)+1/24*(9*a+7*b)*cos(f*x+e)^3*sin(f*x+e)/a^2/f/(a+b+b*tan(f*x
+e)^2)^(1/2)+1/6*cos(f*x+e)^3*sin(f*x+e)^3/a/f/(a+b+b*tan(f*x+e)^2)^(1/2)-
1/48*b*(81*a^2+190*a*b+105*b^2)*tan(f*x+e)/a^4/f/(a+b+b*tan(f*x+e)^2)^(1/2
)
```

3.112.2 Mathematica [A] (verified)

Time = 6.30 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.06

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{(a+2b+a\cos(2(e+fx)))\sec^3(e+fx)\left(120(a+b)^2(a+7b)\arcsin\left(\frac{\sqrt{a}\sin}{\sqrt{a}}\right.\right.}{}$$

input
```
Integrate[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(120*(a + b)^2*(a + 7*b)*Ar
cSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*(a + 2*b + a*Cos[2*(e + f*x)]) -
2*Sqrt[2]*Sqrt[a]*Sqrt[a + b]*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]
*(37*a^3 + 439*a^2*b + 830*a*b^2 + 420*b^3 + a*(29*a^2 + 108*a*b + 70*b^2)
*Cos[2*(e + f*x)] - 7*a^2*(a + b)*Cos[4*(e + f*x)] + a^3*Cos[6*(e + f*x)])
*Sin[e + f*x]))/(1536*a^(9/2)*Sqrt[a + b]*f*(a + b*Sec[e + f*x]^2)^(3/2)*S
qrt[(a + b - a*Sin[e + f*x]^2)/(a + b)])
```

3.112.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4620, 372, 440, 27, 402, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\sin(e+fx)^6}{(a+b\sec(e+fx)^2)^{3/2}}\,dx$$
$$\downarrow 4620$$
$$\frac{\int \frac{\tan^6(e+fx)}{(\tan^2(e+fx)+1)^4(b\tan^2(e+fx)+a+b)^{3/2}}\,d\tan(e+fx)}{f}$$
$$\downarrow 372$$

$$\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3\sqrt{a+b\tan^2(e+fx)+b}} - \frac{\int \frac{\tan^2(e+fx)\left(2(b-3(a+b))\tan^2(e+fx)+3(a+b)\right)}{\left(\tan^2(e+fx)+1\right)^3\left(b\tan^2(e+fx)+a+b\right)^{3/2}}\,d\tan(e+fx)}{6a}}{f}$$

$$\downarrow 440$$

$$\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3\sqrt{a+b\tan^2(e+fx)+b}} - \frac{\frac{\int \frac{(a+b)\left(-4(6a+7b)\tan^2(e+fx)+9a+7b\right)}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^{3/2}}\,d\tan(e+fx)}{4a} - \frac{(9a+7b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\sqrt{a+b\tan^2(e+fx)+b}}}{6a}}{f}$$

$$\downarrow 27$$

$$\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3\sqrt{a+b\tan^2(e+fx)+b}} - \frac{\frac{(a+b)\int \frac{-4(6a+7b)\tan^2(e+fx)+9a+7b}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^{3/2}}\,d\tan(e+fx)}{4a} - \frac{(9a+7b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\sqrt{a+b\tan^2(e+fx)+b}}}{6a}}{f}$$

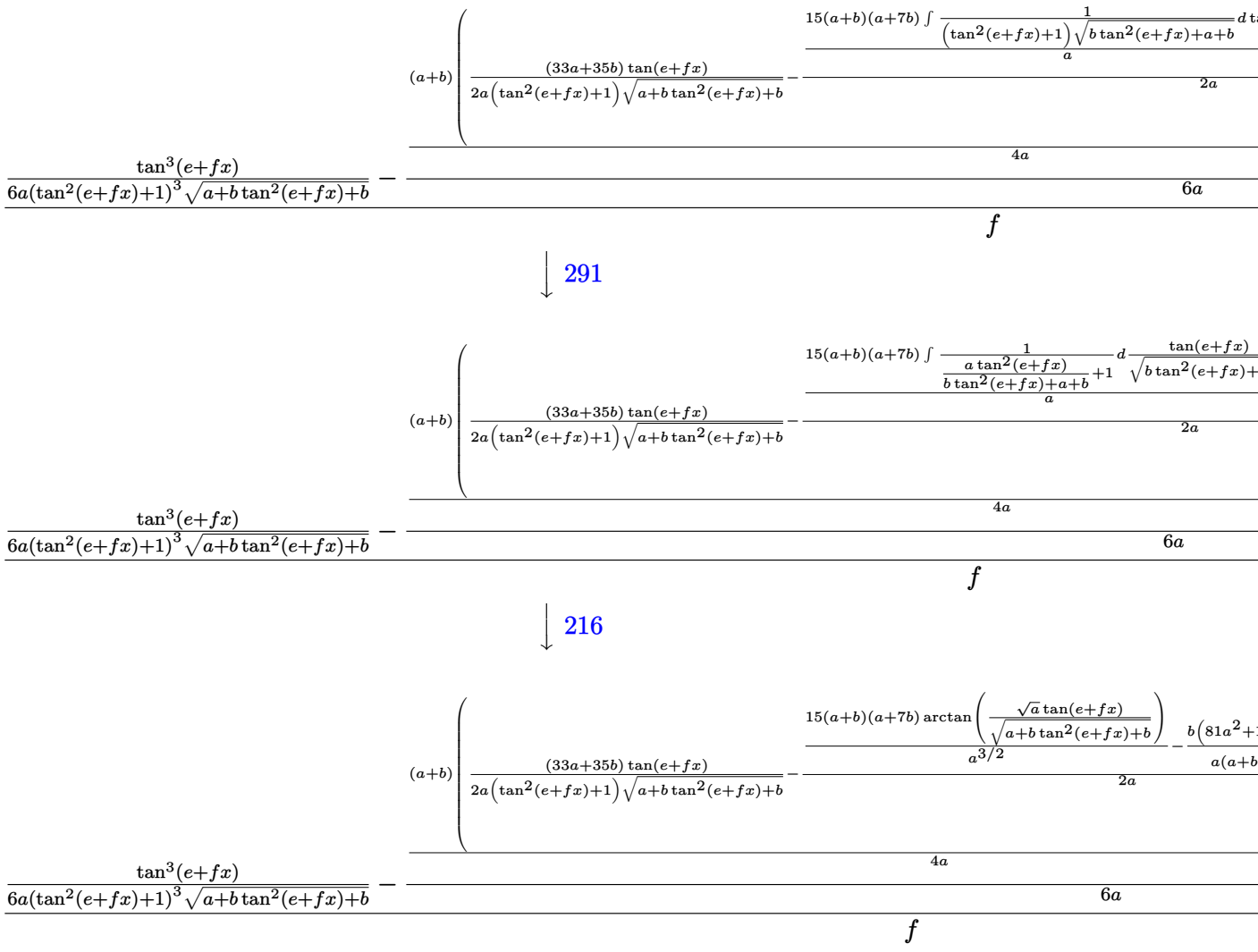
$$\downarrow 402$$

$$\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3\sqrt{a+b\tan^2(e+fx)+b}} - \frac{(a+b)\left(\frac{(33a+35b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)+b}} - \frac{\int \frac{15a^2+54ba+35b^2-2b(33a+35b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}\,d\tan(e+fx)}{2a}\right)}{4a}}{6a}}{f}$$

$$\downarrow 402$$

$$\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3\sqrt{a+b\tan^2(e+fx)+b}} - \frac{(a+b)\left(\frac{(33a+35b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)+b}} - \frac{\int \frac{15(a+b)^2(a+7b)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}\,d\tan(e+fx)}{a(a+b)}\right)}{4a}}{6a}}{f}$$

$$\downarrow 27$$



input
```
Int[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Tan[e + f*x]^3/(6*a*(1 + Tan[e + f*x]^2)^3*Sqrt[a + b + b*Tan[e + f*x]^2]
) - (-1/4*((9*a + 7*b)*Tan[e + f*x])/(a*(1 + Tan[e + f*x]^2)^2*Sqrt[a + b
+ b*Tan[e + f*x]^2]) + ((a + b)*(((33*a + 35*b)*Tan[e + f*x])/(2*a*(1 + Ta
n[e + f*x]^2)*Sqrt[a + b + b*Tan[e + f*x]^2]) - ((15*(a + b)*(a + 7*b)*Arc
Tan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/a^(3/2) - (b*(
81*a^2 + 190*a*b + 105*b^2)*Tan[e + f*x])/(a*(a + b)*Sqrt[a + b + b*Tan[e
+ f*x]^2]))/(2*a)))/(4*a))/(6*a))/f
```

3.112. $\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

3.112.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.112.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1161 vs. $2(218) = 436$.

Time = 9.33 (sec) , antiderivative size = 1162, normalized size of antiderivative = 4.80

method	result	size
default	Expression too large to display	1162

input
```
int(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
-1/48/f/a^4/(-a)^(1/2)*(b+a*cos(f*x+e)^2)*(8*(-a)^(1/2)*a^3*cos(f*x+e)^6*s
in(f*x+e)-26*(-a)^(1/2)*a^3*cos(f*x+e)^4*sin(f*x+e)-14*(-a)^(1/2)*a^2*b*co
s(f*x+e)^4*sin(f*x+e)+33*(-a)^(1/2)*a^3*cos(f*x+e)^2*sin(f*x+e)+68*(-a)^(1
/2)*a^2*b*cos(f*x+e)^2*sin(f*x+e)+35*(-a)^(1/2)*a*b^2*cos(f*x+e)^2*sin(f*x
+e)-15*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x
+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*
a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^3*cos(f*x+e)-135*ln(4*(-a
)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2
)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*b*cos(f*x+e)-225*ln(4*(-a)^(1/2)*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*a*b^2*cos(f*x+e)-105*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*b^3*cos(f*x+e)+81*(-a)^(1/2)*a^2*b*sin(f*x+e)+190*(-a)^(1/2)*a*b^2*s
in(f*x+e)+105*(-a)^(1/2)*b^3*sin(f*x+e)-15*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*c
os(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f
*x+e)*a)*a^3-135*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^...
```

3.112.5 Fricas [A] (verification not implemented)

Time = 5.91 (sec) , antiderivative size = 813, normalized size of antiderivative = 3.36

$$\int \frac{\sin^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \left[-\frac{15\left(a^3b+9\,a^2b^2+15\,ab^3+7\,b^4+\left(a^4+9\,a^3b+15\,a^2b^2+7\,ab^3\right)\cos\left(fx+\right.\right.}{}\right.$$

$$\left. -\frac{15\left(a^3b+9\,a^2b^2+15\,ab^3+7\,b^4+\left(a^4+9\,a^3b+15\,a^2b^2+7\,ab^3\right)\cos\left(fx+e\right)^2\right)\sqrt{a}\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^5-8\right.}{4\left(2\,a^3\,c\right.}\right.}{}\right]$$

input

```
integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/384*(15*(a^3*b + 9*a^2*b^2 + 15*a*b^3 + 7*b^4 + (a^4 + 9*a^3*b + 15*a^
2*b^2 + 7*a*b^3)*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256
*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x
+ e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b
+ 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 -
 a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (
a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)
^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(8*a^4*cos(f*x + e)^7 - 2*(13*a^
4 + 7*a^3*b)*cos(f*x + e)^5 + (33*a^4 + 68*a^3*b + 35*a^2*b^2)*cos(f*x + e
)^3 + (81*a^3*b + 190*a^2*b^2 + 105*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^6*f*cos(f*x + e)^2 + a^5*b*f),
 -1/192*(15*(a^3*b + 9*a^2*b^2 + 15*a*b^3 + 7*b^4 + (a^4 + 9*a^3*b + 15*a^
2*b^2 + 7*a*b^3)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5
- 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*
b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) + 4*(8*a^4*cos(
f*x + e)^7 - 2*(13*a^4 + 7*a^3*b)*cos(f*x + e)^5 + (33*a^4 + 68*a^3*b + 35
*a^2*b^2)*cos(f*x + e)^3 + (81*a^3*b + 190*a^2*b^2 + 105*a*b^3)*cos(f*x +
e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^6*f*cos(f
*x + e)^2 + a^5*b*f)]
```

3.112.6 Sympy [F]

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)**6/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(sin(e + f*x)**6/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.112.7 Maxima [F]

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sin(fx+e)^6}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sin(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.112.8 Giac [F]

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sin(fx+e)^6}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.112.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sin(e+fx)^6}{\left(a+\frac{b}{\cos(e+fx)^2}\right)^{3/2}}\,dx$$

input
```
int(sin(e + f*x)^6/(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(sin(e + f*x)^6/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.113 $\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

3.113.1 Optimal result . . . 967
3.113.2 Mathematica [A] (verified) . . . 967
3.113.3 Rubi [A] (verified) . . . 968
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3.113.1 Optimal result

Integrand size = 25, antiderivative size = 175

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{3(a+b)(a+5b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{7/2}f} - \frac{5(a+b)\cos(e+fx)\sin(e+fx)}{8a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{b(13a+15b)\tan(e+fx)}{8a^3f\sqrt{a+b+b\tan^2(e+fx)}}$$

output

```
3/8*(a+b)*(a+5*b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^
(7/2)/f-5/8*(a+b)*cos(f*x+e)*sin(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1
/4*cos(f*x+e)^3*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/8*b*(13*a+15*b
)*tan(f*x+e)/a^3/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

3.113.2 Mathematica [A] (verified)

Time = 3.33 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.31

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{(a+2b+a\cos(2(e+fx)))\sec^3(e+fx)\left(24(a^2+6ab+5b^2)\arcsin\left(\frac{\sqrt{a}\sin(e}{\sqrt{a-}}\right.\right.$$

input
```
Integrate[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(24*(a^2 + 6*a*b + 5*b^2)*A
rcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*(a + 2*b + a*Cos[2*(e + f*x)]) -
 2*Sqrt[2]*Sqrt[a]*Sqrt[a + b]*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)
]*(7*a^2 + 62*a*b + 60*b^2 + 2*a*(3*a + 5*b)*Cos[2*(e + f*x)] - a^2*Cos[4*
(e + f*x)])*Sin[e + f*x]))/(256*a^(7/2)*Sqrt[a + b]*f*(a + b*Sec[e + f*x]^
2)^(3/2)*Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)])
```

3.113.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4620, 372, 402, 27, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sin(e+fx)^4}{(a+b\sec(e+fx)^2)^{3/2}}\,dx$$

$$\downarrow 4620$$

$$\frac{\int \frac{\tan^4(e+fx)}{(\tan^2(e+fx)+1)^3(b\tan^2(e+fx)+a+b)^{3/2}}\,d\tan(e+fx)}{f}$$

$$\downarrow 372$$

$$\frac{\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2\sqrt{a+b\tan^2(e+fx)+b}} - \frac{\int \frac{-4(a+b)\tan^2(e+fx)+a+b}{(\tan^2(e+fx)+1)^2(b\tan^2(e+fx)+a+b)^{3/2}}\,d\tan(e+fx)}{4a}}{f}$$

$$\downarrow 402$$

$$\frac{\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2\sqrt{a+b\tan^2(e+fx)+b}} - \frac{\frac{5(a+b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)+b}} - \frac{\int \frac{(a+b)\left(-10b\tan^2(e+fx)+3a+5b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}\,d\tan(e+fx)}{2a}}{4a}}{f}$$

$$\downarrow 27$$

$$\frac{\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\sqrt{a+b\tan^2(e+fx)+b}} - \frac{\frac{5(a+b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)+b}} - \frac{(a+b)\int \frac{-10b\tan^2(e+fx)+3a+5b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}\,d\tan(e+fx)}{2a}}{4a}$$

$$f$$

$$\downarrow 402$$

$$\frac{\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\sqrt{a+b\tan^2(e+fx)+b}} - \frac{\frac{5(a+b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)+b}} - \frac{(a+b)\left(\int \frac{\frac{3(a+b)(a+5b)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}\,d\tan(e+fx)}{a(a+b)} - \frac{b}{a(a}\right.}{2a}}{4a}$$

$$f$$

$$\downarrow 27$$

$$\frac{\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\sqrt{a+b\tan^2(e+fx)+b}} - \frac{\frac{5(a+b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)+b}} - \frac{(a+b)\left(\frac{3(a+5b)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}\,d\tan(e+f}{a}\right.}{2a}}{4a}$$

$$f$$

$$\downarrow 291$$

$$\frac{\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\sqrt{a+b\tan^2(e+fx)+b}} - \frac{\frac{5(a+b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)+b}} - \frac{(a+b)\left(\frac{3(a+5b)\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}\,d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{a}\right.}{2a}}{4a}$$

$$f$$

$$\downarrow 216$$

$$\frac{\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\sqrt{a+b\tan^2(e+fx)+b}} - \frac{\frac{5(a+b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)+b}} - \frac{(a+b)\left(\frac{3(a+5b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}} - \frac{b(13a+15b)\,ta}{a(a+b)\sqrt{a+b\tan}}\right.}{2a}}{4a}$$

$$f$$

input
```
Int[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

3.113. $\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

output
```
(Tan[e + f*x]/(4*a*(1 + Tan[e + f*x]^2)^2*Sqrt[a + b + b*Tan[e + f*x]^2])
- ((5*(a + b)*Tan[e + f*x])/(2*a*(1 + Tan[e + f*x]^2)*Sqrt[a + b + b*Tan[e
 + f*x]^2]) - ((a + b)*((3*(a + 5*b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a
+ b + b*Tan[e + f*x]^2]])/a^(3/2) - (b*(13*a + 15*b)*Tan[e + f*x])/(a*(a +
 b)*Sqrt[a + b + b*Tan[e + f*x]^2])))/(2*a))/(4*a))/f
```

3.113.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.113.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 832 vs. $2(155) = 310$.

Time = 7.29 (sec) , antiderivative size = 833, normalized size of antiderivative = 4.76

method	result
default	$\frac{\left(b+a\cos(fx+e)^2\right)\left(2\cos(fx+e)^4\sin(fx+e)\sqrt{-a}\,a^2-5\cos(fx+e)^2\sin(fx+e)\sqrt{-a}\,a^2-5\sqrt{-a}\,ab\cos(fx+e)^2\sin(fx+e)+3\cos(fx+$

input

```
int(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
1/8/f/a^3/(-a)^(1/2)*(b+a*cos(f*x+e)^2)*(2*cos(f*x+e)^4*sin(f*x+e)*(-a)^(1
/2)*a^2-5*cos(f*x+e)^2*sin(f*x+e)*(-a)^(1/2)*a^2-5*(-a)^(1/2)*a*b*cos(f*x+
e)^2*sin(f*x+e)+3*cos(f*x+e)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2+1
8*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4
*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b*cos(f*x+e)+15*ln(4*(-a)^(1/2
)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*b^2*cos(f*x+e)-13*(-a)^(1/2)*a*b*sin(f*x+e)-15*(
-a)^(1/2)*b^2*sin(f*x+e)+3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(
4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)
^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^2+18*
ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(
-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b+15*ln(4*(-a)^(1/2)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*b^2)/(a+b*sec(f*x+e)^2)^(3/2)*sec(f*x+e)^3
```

3.113.5 Fricas [A] (verification not implemented)

Time = 1.83 (sec) , antiderivative size = 703, normalized size of antiderivative = 4.02

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \left[-\frac{3\left(a^2b+6\,ab^2+5\,b^3+\left(a^3+6\,a^2b+5\,ab^2\right)\cos\left(fx+e\right)^2\right)\sqrt{-a}\log\left(128\,a\right.\right.$$

$$\left. -\frac{3\left(a^2b+6\,ab^2+5\,b^3+\left(a^3+6\,a^2b+5\,ab^2\right)\cos\left(fx+e\right)^2\right)\sqrt{a}\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^5-8\left(a^2-ab\right)\cos(fx+e)^3+(a^2-\right.\right.\right.}{4\left(2\,a^3\cos(fx+e)^4-a^2b+ab^2-(a^3-}$$

input
```
integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/64*(3*(a^2*b + 6*a*b^2 + 5*b^3 + (a^3 + 6*a^2*b + 5*a*b^2)*cos(f*x + e
)^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^
6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70
*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x
 + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(
5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^
3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(
f*x + e)) - 8*(2*a^3*cos(f*x + e)^5 - 5*(a^3 + a^2*b)*cos(f*x + e)^3 - (13
*a^2*b + 15*a*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^
2)*sin(f*x + e))/(a^5*f*cos(f*x + e)^2 + a^4*b*f), -1/32*(3*(a^2*b + 6*a*b
^2 + 5*b^3 + (a^3 + 6*a^2*b + 5*a*b^2)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*
(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)
*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3
*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x
+ e))) - 4*(2*a^3*cos(f*x + e)^5 - 5*(a^3 + a^2*b)*cos(f*x + e)^3 - (13*a^
2*b + 15*a*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*
sin(f*x + e))/(a^5*f*cos(f*x + e)^2 + a^4*b*f)]
```

3.113.6 Sympy [F]

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)**4/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(sin(e + f*x)**4/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.113.7 Maxima [F]

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sin(fx+e)^4}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sin(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.113.8 Giac [F]

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sin(fx+e)^4}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.113.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sin(e+fx)^4}{\left(a+\frac{b}{\cos(e+fx)^2}\right)^{3/2}}\,dx$$

input
```
int(sin(e + f*x)^4/(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(sin(e + f*x)^4/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.114 $\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

3.114.1 Optimal result . . . 975
3.114.2 Mathematica [A] (verified) . . . 975
3.114.3 Rubi [A] (verified) . . . 976
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3.114.5 Fricas [B] (verification not implemented) . . . 979
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3.114.7 Maxima [F] . . . 980
3.114.8 Giac [F] . . . 981
3.114.9 Mupad [F(-1)] . . . 981

3.114.1 Optimal result

Integrand size = 25, antiderivative size = 121

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{(a+3b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2a^{5/2}f} - \frac{\cos(e+fx)\sin(e+fx)}{2af\sqrt{a+b+b\tan^2(e+fx)}} - \frac{3b\tan(e+fx)}{2a^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

output
```
1/2*(a+3*b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/
f-1/2*cos(f*x+e)*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)^(1/2)-3/2*b*tan(f*x+e
)/a^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

3.114.2 Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.57

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{(a+2b+a\cos(2(e+fx)))\sec^3(e+fx)\left(4(a+3b)\arcsin\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)(a+\right.}{32a^{5/2}\sqrt{a+b}f}$$

input
```
Integrate[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(4*(a + 3*b)*ArcSin[(Sqrt[a
]*Sin[e + f*x])/Sqrt[a + b]]*(a + 2*b + a*Cos[2*(e + f*x)]) - 2*Sqrt[2]*Sq
rt[a]*Sqrt[a + b]*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*(a + 6*b +
a*Cos[2*(e + f*x)])*Sin[e + f*x]))/(32*a^(5/2)*Sqrt[a + b]*f*(a + b*Sec[e
+ f*x]^2)^(3/2)*Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)])
```

3.114.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4620, 373, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sin(e+fx)^2}{(a+b\sec(e+fx)^2)^{3/2}}\,dx$$

$$\downarrow 4620$$

$$\frac{\int \frac{\tan^2(e+fx)}{(\tan^2(e+fx)+1)^2(b\tan^2(e+fx)+a+b)^{3/2}}\,d\tan(e+fx)}{f}$$

$$\downarrow 373$$

$$\frac{\int \frac{-2b\tan^2(e+fx)+a+b}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^{3/2}}\,d\tan(e+fx)}{2a} - \frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\downarrow 402$$

$$\frac{\frac{\int \frac{(a+b)(a+3b)}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}\,d\tan(e+fx)}{a(a+b)} - \frac{3b\tan(e+fx)}{a\sqrt{a+b\tan^2(e+fx)+b}}}{2a} - \frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\downarrow 27$$

$$\frac{\frac{(a+3b)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a}-\frac{3b\tan(e+fx)}{a\sqrt{a+b\tan^2(e+fx)+b}}}{2a}-\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\downarrow 291$$

$$\frac{\frac{(a+3b)\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{a}-\frac{3b\tan(e+fx)}{a\sqrt{a+b\tan^2(e+fx)+b}}}{2a}-\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\downarrow 216$$

$$\frac{\frac{(a+3b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}}-\frac{3b\tan(e+fx)}{a\sqrt{a+b\tan^2(e+fx)+b}}}{2a}-\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

input
```
Int[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(-1/2*Tan[e + f*x]/(a*(1 + Tan[e + f*x]^2)*Sqrt[a + b + b*Tan[e + f*x]^2])
 + (((a + 3*b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2
]])/a^(3/2) - (3*b*Tan[e + f*x])/(a*Sqrt[a + b + b*Tan[e + f*x]^2]))/(2*a)
)/f
```

3.114.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.114.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 531 vs. $2(105) = 210$.

Time = 5.31 (sec) , antiderivative size = 532, normalized size of antiderivative = 4.40

method	result
default	$-\frac{\left(b+a\cos(fx+e)^2\right)\left(\cos(fx+e)^2\sin(fx+e)\sqrt{-a}\,a-\cos(fx+e)\ln\left(4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)+4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-\right.\right.}{}$

input
```
int(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
-1/2/f/a^2/(-a)^(1/2)*(b+a*cos(f*x+e)^2)*(cos(f*x+e)^2*sin(f*x+e)*(-a)^(1/
2)*a-cos(f*x+e)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*si
n(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a-3*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)-4*sin(f*x+e)*a)*b*cos(f*x+e)+3*(-a)^(1/2)*b*sin(f*x+e)-((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a-3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e
)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)
*b)/(a+b*sec(f*x+e)^2)^(3/2)*sec(f*x+e)^3
```

3.114.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(105) = 210$.

Time $= 0.67$ (sec) , antiderivative size $= 607$, normalized size of antiderivative $= 5.02$

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \left[-\frac{\left((a^2+3\,ab)\cos\left(fx+e\right)^2+ab+3\,b^2\right)\sqrt{-a}\log\left(128\,a^4\cos\left(fx+e\right)^8-2\right.}{}\right.$$

$$\left. -\frac{\left((a^2+3\,ab)\cos\left(fx+e\right)^2+ab+3\,b^2\right)\sqrt{a}\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^5-8\left(a^2-ab\right)\cos(fx+e)^3+\left(a^2-6\,ab+b^2\right)\cos(fx+e)\right)\sqrt{a}\sqrt{}}{4\left(2\,a^3\cos(fx+e)^4-a^2b+ab^2-\left(a^3-3\,a^2b\right)\cos(fx+e)^2\right)\sin(fx}\right.}{8\left(a^4f\cos\left(fx+e\right)^2+a^3b\right.}$$

input

```
integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/16*(((a^2 + 3*a*b)*cos(f*x + e)^2 + a*b + 3*b^2)*sqrt(-a)*log(128*a^4*
cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b +
 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4
- 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*
x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2
)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(a^2*cos(f*x
 + e)^3 + 3*a*b*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*
sin(f*x + e))/(a^4*f*cos(f*x + e)^2 + a^3*b*f), -1/8*(((a^2 + 3*a*b)*cos(f
*x + e)^2 + a*b + 3*b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2
 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a
*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^
2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) + 4*(a^2*cos(f*x + e)^3
 + 3*a*b*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x
 + e))/(a^4*f*cos(f*x + e)^2 + a^3*b*f)]
```

3.114.6 Sympy [F]

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)**2/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(sin(e + f*x)**2/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.114.7 Maxima [F]

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sin(fx+e)^2}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sin(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.114.8 Giac [F]

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sin\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.114.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sin\left(e+f\,x\right)^2}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

input
```
int(sin(e + f*x)^2/(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(sin(e + f*x)^2/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.115 $\int \frac{1}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

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3.115.1 Optimal result

Integrand size = 16, antiderivative size = 77

$$\int \frac{1}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f} - \frac{b\tan(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

output
```
arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f-b*tan(f*x+
e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

3.115.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 168 vs. 2(77) = 154.

Time = 1.05 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.18

$$\int \frac{1}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{(a+2b+a\cos(2(e+fx)))\sec^3(e+fx)\left(\sqrt{a+b}\arcsin\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)(a+\right.}{4a^{3/2}(a+b)f\,(a+b\sec^2(e+fx$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(-3/2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(Sqrt[a + b]*ArcSin[(Sqrt[a
]*Sin[e + f*x])/Sqrt[a + b]]*(a + 2*b + a*Cos[2*(e + f*x)]) - Sqrt[2]*Sqrt
[a]*b*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*Sin[e + f*x]))/(4*a^(3/
2)*(a + b)*f*(a + b*Sec[e + f*x]^2)^(3/2)*Sqrt[(a + b - a*Sin[e + f*x]^2)/
(a + b)])
```

3.115.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3042, 4616, 296, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{(a+b\sec(e+fx)^2)^{3/2}}\,dx$$

$$\downarrow 4616$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^{3/2}}\,d\tan(e+fx)}{f}$$

$$\downarrow 296$$

$$\frac{\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}\,d\tan(e+fx)}{a} - \frac{b\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\downarrow 291$$

$$\frac{\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}\,d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{a} - \frac{b\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\downarrow 216$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}} - \frac{b\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^(-3/2),x]
```

output
```
(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/a^(3/2) - (
b*Tan[e + f*x])/(a*(a + b)*Sqrt[a + b + b*Tan[e + f*x]^2]))/f
```

3.115.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

3.115.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 514 vs. $2(69) = 138$.

Time = 3.69 (sec) , antiderivative size = 515, normalized size of antiderivative = 6.69

method	result
default	$-\frac{\left(b+a\cos(fx+e)^2\right)\left(-\cos(fx+e)\ln\left(4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)+4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a\right)\sqrt{\frac{b+a\cos(fx-}{(1+\cos(fx+}$

input
```
int(1/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f/(a+b)/a/(-a)^(1/2)*(b+a*cos(f*x+e)^2)*(-cos(f*x+e)*ln(4*(-a)^(1/2)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*a-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(
-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1
/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*b*cos(f*x+
e)+(-a)^(1/2)*b*sin(f*x+e)-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(
4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)
^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a-((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*b)/(a+b*sec(f*x+e)^2)^(3/2)*sec(f*x+e)^
3
```

3.115.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(69) = 138$.

Time = 0.43 (sec) , antiderivative size = 601, normalized size of antiderivative = 7.81

$$\int \frac{1}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \left[-\frac{8\,ab\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)\sin(fx+e) + \left((a^2+ab)\cos(fx+e)^2 + \right.\right.$$

$$\left. -\frac{4\,ab\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)\sin(fx+e) + \left((a^2+ab)\cos(fx+e)^2 + ab + b^2\right)\sqrt{a}\arctan\left(\frac{\left(8\,a^2\cos(fx+e\right.}{4\left(2\right.}\right.}{4\left((a^4+a^3b)f\cos(fx+e)^2 + (a^3b+a^2b^2)f\right)}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(8*a*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(
f*x + e) + ((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)*sqrt(-a)*log(128*a^4*c
os(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b +
5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 -
 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x
 + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)
*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)))/((a^4 + a^3*b)*f
*cos(f*x + e)^2 + (a^3*b + a^2*b^2)*f), -1/4*(4*a*b*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x + e) + ((a^2 + a*b)*cos(f*x + e
)^2 + a*b + b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*
cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3
 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))))/((a^4 + a^3*b)*f*cos(f*x + e)^
2 + (a^3*b + a^2*b^2)*f)]
```

3.115.6 Sympy [F]

$$\int \frac{1}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{1}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(-3/2), x)
```

3.115.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2055 vs. 2(69) = 138.

Time = 0.54 (sec) , antiderivative size = 2055, normalized size of antiderivative = 26.69

$$\int \frac{1}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/2*(2*a*b*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2
*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))*sin(2*f*x + 2
*e) - 2*(a^2 + a*b)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2
*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^3 - 2
*(a*b*cos(2*f*x + 2*e) + (a^2 + a*b)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) +
2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x +
 2*e) + a))^2 - a^2 - 2*a*b)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2
*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) +
a)) - (a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b +
4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2
*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 +
 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2
*e))^(1/4)*(((a + b)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(
2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 +
(a + b)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e),
 a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2)*arctan2(2*a*si
n(2*f*x + 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a
^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*
sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a
^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*...
```

3.115.8 Giac [F]

$$\int \frac{1}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{1}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(-3/2), x)
```

3.115.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(e+fx)^2}\right)^{3/2}}\,dx$$

input
```
int(1/(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(1/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.116 $\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

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3.116.1 Optimal result

Integrand size = 25, antiderivative size = 68

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{\cot(e+fx)}{(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{2b\tan(e+fx)}{(a+b)^2 f\sqrt{a+b+b\tan^2(e+fx)}}$$

output
```
-cot(f*x+e)/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)-2*b*tan(f*x+e)/(a+b)^2/f/(a
+b+b*tan(f*x+e)^2)^(1/2)
```

3.116.2 Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.12

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))(a+3b+(a-b)\cos(2(e+fx)))\csc(e+fx)\sec^3(e+fx)}{4(a+b)^2 f\,(a+b\sec^2(e+fx))^{3/2}}$$

input
```
Integrate[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
-1/4*((a + 2*b + a*Cos[2*(e + f*x)])*(a + 3*b + (a - b)*Cos[2*(e + f*x)])*
Csc[e + f*x]*Sec[e + f*x]^3)/((a + b)^2*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

3.116.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 4620, 245, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sin(e+fx)^2\,(a+b\sec(e+fx)^2)^{3/2}}dx$$

$$\downarrow 4620$$

$$\frac{\int \frac{\cot^2(e+fx)}{(b\tan^2(e+fx)+a+b)^{3/2}}d\tan(e+fx)}{f}$$

$$\downarrow 245$$

$$\frac{-\frac{2b\int \frac{1}{\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{a+b} - \frac{\cot(e+fx)}{(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\downarrow 208$$

$$\frac{-\frac{2b\tan(e+fx)}{(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}} - \frac{\cot(e+fx)}{(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

input
```
Int[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(-(Cot[e + f*x]/((a + b)*Sqrt[a + b + b*Tan[e + f*x]^2])) - (2*b*Tan[e + f
*x])/((a + b)^2*Sqrt[a + b + b*Tan[e + f*x]^2]))/f
```

3.116.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.116.4 Maple [A] (verified)

Time = 3.99 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.24

method	result	size
default	$-\frac{\left(b+a\cos(fx+e)^2\right)\left(a\cos(fx+e)^2-\cos(fx+e)^2b+2b\right)\sec(fx+e)^3\csc(fx+e)}{f(a^2+2ab+b^2)\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}}$	84

input
```
int(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f/(a^2+2*a*b+b^2)*(b+a*cos(f*x+e)^2)*(a*cos(f*x+e)^2-cos(f*x+e)^2*b+2*b
)/(a+b*sec(f*x+e)^2)^(3/2)*sec(f*x+e)^3*csc(f*x+e)
```

3.116.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.50

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{\left((a-b)\cos(fx+e)^3+2\,b\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{\left((a^3+2\,a^2b+ab^2)f\cos(fx+e)^2+(a^2b+2\,ab^2+b^3)f\right)\sin(fx+e)}$$

input
```
integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
-((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)/(((a^3 + 2*a^2*b + a*b^2)*f*cos(f*x + e)^2 + (a^2*b + 2*a*b
^2 + b^3)*f)*sin(f*x + e))
```

3.116.6 Sympy [F]

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)**2/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(csc(e + f*x)**2/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.116.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.94

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{\frac{2\,b\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+b(a+b)^2}}+\frac{1}{\sqrt{b\tan(fx+e)^2+a+b(a+b)\tan(fx+e)}}}{f}$$

input
```
integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
-(2*b*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^2) + 1/(sqrt(b*
tan(f*x + e)^2 + a + b)*(a + b)*tan(f*x + e)))/f
```

3.116.8 Giac [F]

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\csc\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.116.9 Mupad [B] (verification not implemented)

Time = 26.10 (sec) , antiderivative size = 2151, normalized size of antiderivative = 31.63

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^(3/2)),x)
```

output
```
-((a + b/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2)^2)^(1/2)*(2*exp(e
*2i + f*x*2i) + exp(e*4i + f*x*4i) + 1)*(exp(e*4i + f*x*4i)*(((a + 4*b)*((
(a + 4*b)*((((a^2*(a + 3*b)*(a*(a - b) - (a + 3*b)^2))/(a*b + a^2) + (a*(a
 + 3*b)^2*(a*(a + 3*b) - a*(a + 4*b)))/(a*b + a^2))*1i)/(8*f*(a*b^2 + a^2*
b)*(a + 3*b)) - (a^3*(a + 3*b)*3i)/(8*f*(a*b + a^2)*(a*b^2 + a^2*b)) + (a^
2*(a + 3*b)*(a + 4*b)*1i)/(8*f*(a*b + a^2)*(a*b^2 + a^2*b))))/a - (((a + 3
*b)^3 - ((a + 3*b)*(a*(a - b) - (a + 3*b)^2)*(a*(a + 3*b) - a*(a + 4*b)))/
(a*b + a^2))*1i)/(8*f*(a*b^2 + a^2*b)*(a + 3*b)) - (((a^2*(a + 3*b)*(a*(a
 - b) - (a + 3*b)^2))/(a*b + a^2) + (a*(a + 3*b)^2*(a*(a + 3*b) - a*(a + 4*
b)))/(a*b + a^2))*3i)/(8*f*(a*b^2 + a^2*b)*(a + 3*b)) + (a^3*(a + 3*b)*3i)
/(8*f*(a*b + a^2)*(a*b^2 + a^2*b)) + (a^2*(a + 3*b)*(a + 4*b)*1i)/(8*f*(a*
b + a^2)*(a*b^2 + a^2*b))))/a + ((a + 4*b)*((((a^2*(a + 3*b)*(a*(a - b) -
(a + 3*b)^2))/(a*b + a^2) + (a*(a + 3*b)^2*(a*(a + 3*b) - a*(a + 4*b)))/(a
*b + a^2))*1i)/(8*f*(a*b^2 + a^2*b)*(a + 3*b)) - (a^3*(a + 3*b)*3i)/(8*f*(
a*b + a^2)*(a*b^2 + a^2*b)) + (a^2*(a + 3*b)*(a + 4*b)*1i)/(8*f*(a*b + a^2
)*(a*b^2 + a^2*b))))/a + (((a + 3*b)^3 - ((a + 3*b)*(a*(a - b) - (a + 3*b)
^2)*(a*(a + 3*b) - a*(a + 4*b)))/(a*b + a^2))*3i)/(8*f*(a*b^2 + a^2*b)*(a
+ 3*b)) + (((a^2*(a + 3*b)*(a*(a - b) - (a + 3*b)^2))/(a*b + a^2) + (a*(a
+ 3*b)^2*(a*(a + 3*b) - a*(a + 4*b)))/(a*b + a^2))*3i)/(8*f*(a*b^2 + a^2*b
)*(a + 3*b)) - (a^3*(a + 3*b)*1i)/(4*f*(a*b + a^2)*(a*b^2 + a^2*b))) + ...
```

3.117 $\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

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3.117.1 Optimal result

Integrand size = 25, antiderivative size = 123

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{(3a-b)\cot(e+fx)}{3(a+b)^2 f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\cot^3(e+fx)}{3(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{2(3a-b)b\tan(e+fx)}{3(a+b)^3 f\sqrt{a+b+b\tan^2(e+fx)}}$$

output
```
-1/3*(3*a-b)*cot(f*x+e)/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/3*cot(f*x+e
)^3/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)-2/3*(3*a-b)*b*tan(f*x+e)/(a+b)^3/f/
(a+b+b*tan(f*x+e)^2)^(1/2)
```

3.117.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.83

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))\left(-2a(a-3b)+(a^2-2ab-3b^2)\csc^2(e+fx)+(a+b)^2\csc^4(e+fx)\right)\sec^2(e}{6(a+b)^3 f\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

input
```
Integrate[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
-1/6*((a + 2*b + a*Cos[2*(e + f*x)])*(-2*a*(a - 3*b) + (a^2 - 2*a*b - 3*b^
2)*Csc[e + f*x]^2 + (a + b)^2*Csc[e + f*x]^4)*Sec[e + f*x]^2*Tan[e + f*x])
/((a + b)^3*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

3.117.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4620, 359, 245, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sin(e+fx)^4\,(a+b\sec(e+fx)^2)^{3/2}}dx$$

$$\downarrow 4620$$

$$\frac{\int \frac{\cot^4(e+fx)(\tan^2(e+fx)+1)}{(b\tan^2(e+fx)+a+b)^{3/2}}d\tan(e+fx)}{f}$$

$$\downarrow 359$$

$$\frac{\frac{(3a-b)\int \frac{\cot^2(e+fx)}{\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{3(a+b)} - \frac{\cot^3(e+fx)}{3(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\downarrow 245$$

$$\frac{\frac{(3a-b)\left(-\frac{2b\int \frac{1}{\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{a+b} - \frac{\cot(e+fx)}{(a+b)\sqrt{a+b\tan^2(e+fx)+b}}\right)}{3(a+b)} - \frac{\cot^3(e+fx)}{3(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\downarrow 208$$

$$\frac{\frac{(3a-b)\left(-\frac{2b\tan(e+fx)}{(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}} - \frac{\cot(e+fx)}{(a+b)\sqrt{a+b\tan^2(e+fx)+b}}\right)}{3(a+b)} - \frac{\cot^3(e+fx)}{3(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

input
```
Int[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(-1/3*Cot[e + f*x]^3/((a + b)*Sqrt[a + b + b*Tan[e + f*x]^2]) + ((3*a - b)
*(-(Cot[e + f*x]/((a + b)*Sqrt[a + b + b*Tan[e + f*x]^2])) - (2*b*Tan[e +
f*x])/((a + b)^2*Sqrt[a + b + b*Tan[e + f*x]^2])))/(3*(a + b)))/f
```

3.117.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.117.4 Maple [A] (verified)

Time = 4.53 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.15

method	result
default	$\frac{\left(b+a\cos(fx+e)^2\right)\left(2\cos(fx+e)^4a^2-6\cos(fx+e)^4ab-3\cos(fx+e)^2a^2+10\cos(fx+e)^2ab-3\cos(fx+e)^2b^2-6ab+2b^2\right)\sec(fx+e)^3}{3f(a^3+3a^2b+3ab^2+b^3)\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}}$

input
```
int(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/f/(a^3+3*a^2*b+3*a*b^2+b^3)*(b+a*cos(f*x+e)^2)*(2*cos(f*x+e)^4*a^2-6*c
os(f*x+e)^4*a*b-3*cos(f*x+e)^2*a^2+10*cos(f*x+e)^2*a*b-3*cos(f*x+e)^2*b^2-
6*a*b+2*b^2)/(a+b*sec(f*x+e)^2)^(3/2)*sec(f*x+e)^3*csc(f*x+e)^3
```

3.117.5 Fricas [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.54

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx =$$

$$-\frac{\left(2\left(a^2-3ab\right)\cos\left(fx+e\right)^5-\left(3a^2-10ab+3b^2\right)\cos\left(fx+e\right)^3-2\left(3ab-b^2\right)\cos\left(fx+e\right)\right)\sqrt{}}{3\left(\left(a^4+3a^3b+3a^2b^2+ab^3\right)f\cos\left(fx+e\right)^4-\left(a^4+2a^3b-2ab^3-b^4\right)f\cos\left(fx+e\right)^2-\left(a^3b+3a^2b^2+\right.}$$

input
```
integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/3*(2*(a^2 - 3*a*b)*cos(f*x + e)^5 - (3*a^2 - 10*a*b + 3*b^2)*cos(f*x +
e)^3 - 2*(3*a*b - b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2)/(((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^4 - (a^4 + 2*a
^3*b - 2*a*b^3 - b^4)*f*cos(f*x + e)^2 - (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^
4)*f)*sin(f*x + e))
```

3.117.6 Sympy [F]

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)**4/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(csc(e + f*x)**4/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.117.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.27

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx =$$
$$-\frac{\frac{6\,b\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+b(a+b)^2}} - \frac{8\,b^2\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+b(a+b)^3}} + \frac{3}{\sqrt{b\tan(fx+e)^2+a+b(a+b)}\tan(fx+e)} - \frac{4\,b}{\sqrt{b\tan(fx+e)^2+a+b(a+b)^2}\tan(f}}{3\,f}$$

input
```
integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/3*(6*b*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^2) - 8*b^2*
tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^3) + 3/(sqrt(b*tan(f*
x + e)^2 + a + b)*(a + b)*tan(f*x + e)) - 4*b/(sqrt(b*tan(f*x + e)^2 + a +
 b)*(a + b)^2*tan(f*x + e)) + 1/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)*ta
n(f*x + e)^3))/f
```

3.117.8 Giac [F]

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\csc\left(fx+e\right)^4}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.117.9 Mupad [B] (verification not implemented)

Time = 40.46 (sec) , antiderivative size = 124682, normalized size of antiderivative = 1013.67

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^(3/2)),x)
```

output
```
(a^2*(a + b/(((cos(2*f*x) - sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + (
(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2)*128
i)/(3*(32*a^4*f + 32*b^4*f + 192*a^2*b^2*f + 128*a*b^3*f + 128*a^3*b*f - 3
2*a^4*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 32*a^4*f*(
cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 32*a^4*f*(cos(6*f*x
) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 32*b^4*f*(cos(2*f*x) + sin(2
*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 32*b^4*f*(cos(4*f*x) + sin(4*f*x)*1i)
*(cos(4*e) + sin(4*e)*1i) + 32*b^4*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e
) + sin(6*e)*1i) - 128*a*b^3*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + si
n(2*e)*1i) - 128*a^3*b*f*(cos(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)
*1i) - 128*a*b^3*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) -
 128*a^3*b*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 128*a
*b^3*f*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) + 128*a^3*b*f
*(cos(6*f*x) + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i) - 192*a^2*b^2*f*(co
s(2*f*x) + sin(2*f*x)*1i)*(cos(2*e) + sin(2*e)*1i) - 192*a^2*b^2*f*(cos(4*
f*x) + sin(4*f*x)*1i)*(cos(4*e) + sin(4*e)*1i) + 192*a^2*b^2*f*(cos(6*f*x)
 + sin(6*f*x)*1i)*(cos(6*e) + sin(6*e)*1i))) - (5*a*(a + b/(((cos(2*f*x) -
 sin(2*f*x)*1i)*(cos(2*e) - sin(2*e)*1i))/4 + ((cos(2*f*x) + sin(2*f*x)*1i
)*(cos(2*e) + sin(2*e)*1i))/4 + 1/2))^(1/2))/(3*(a^3*f*1i + b^3*f*1i + a*b
^2*f*3i + a^2*b*f*3i - a^3*f*(cos(4*f*x) + sin(4*f*x)*1i)*(cos(4*e) + s...
```

3.118 $\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

3.118.1 Optimal result 1000
3.118.2 Mathematica [A] (verified) 1000
3.118.3 Rubi [A] (verified) 1001
3.118.4 Maple [A] (verified) 1003
3.118.5 Fricas [A] (verification not implemented) 1004
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3.118.7 Maxima [A] (verification not implemented) 1005
3.118.8 Giac [F] 1005
3.118.9 Mupad [F(-1)] 1006

3.118.1 Optimal result

Integrand size = 25, antiderivative size = 183

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{(15a^2-10ab-b^2)\cot(e+fx)}{15(a+b)^3 f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{2(5a+2b)\cot^3(e+fx)}{15(a+b)^2 f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{\cot^5(e+fx)}{5(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{2b(15a^2-10ab-b^2)\tan(e+fx)}{15(a+b)^4 f\sqrt{a+b+b\tan^2(e+fx)}}$$

output

```
-1/15*(15*a^2-10*a*b-b^2)*cot(f*x+e)/(a+b)^3/f/(a+b+b*tan(f*x+e)^2)^(1/2)-
2/15*(5*a+2*b)*cot(f*x+e)^3/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/5*cot(f
*x+e)^5/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)-2/15*b*(15*a^2-10*a*b-b^2)*tan(
f*x+e)/(a+b)^4/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

3.118.2 Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.69

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))\left(-8a^2(a-5b)+4a(a^2-4ab-5b^2)\csc^2(e+fx)+(a-5b)(a+b)^2\csc^4(e+\right.}{30(a+b)^4 f\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

input
```
Integrate[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
-1/30*((a + 2*b + a*Cos[2*(e + f*x)])*(-8*a^2*(a - 5*b) + 4*a*(a^2 - 4*a*b
 - 5*b^2)*Csc[e + f*x]^2 + (a - 5*b)*(a + b)^2*Csc[e + f*x]^4 + 3*(a + b)^
3*Csc[e + f*x]^6)*Sec[e + f*x]^2*Tan[e + f*x])/((a + b)^4*f*(a + b*Sec[e +
 f*x]^2)^(3/2))
```

3.118.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 4620, 365, 359, 245, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sin(e+fx)^6\,(a+b\sec(e+fx)^2)^{3/2}}dx$$

$$\downarrow 4620$$

$$\frac{\int \frac{\cot^6(e+fx)\left(\tan^2(e+fx)+1\right)^2}{\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{f}$$

$$\downarrow 365$$

$$\frac{\int \frac{\cot^4(e+fx)\left(5(a+b)\tan^2(e+fx)+2(5a+2b)\right)}{\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{5(a+b)} - \frac{\cot^5(e+fx)}{5(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\downarrow 359$$

$$\frac{\frac{\left(15a^2-10ab-b^2\right)\int \frac{\cot^2(e+fx)}{\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{3(a+b)} - \frac{2(5a+2b)\cot^3(e+fx)}{3(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{5(a+b)} - \frac{\cot^5(e+fx)}{5(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\downarrow 245$$

$$\frac{\left(15a^2-10ab-b^2\right)\left(-\frac{2b\int\frac{1}{\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{a+b}-\frac{\cot(e+fx)}{(a+b)\sqrt{a+b\tan^2(e+fx)+b}}\right)}{3(a+b)}-\frac{2(5a+2b)\cot^3(e+fx)}{3(a+b)\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\cot^5(e+fx)}{5(a+b)\sqrt{a+b\tan^2(e+fx)+}}$$
$$f$$

$$\downarrow 208$$

$$\frac{\left(15a^2-10ab-b^2\right)\left(-\frac{2b\tan(e+fx)}{(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\cot(e+fx)}{(a+b)\sqrt{a+b\tan^2(e+fx)+b}}\right)}{3(a+b)}-\frac{2(5a+2b)\cot^3(e+fx)}{3(a+b)\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\cot^5(e+fx)}{5(a+b)\sqrt{a+b\tan^2(e+fx)+b}}$$
$$f$$

input
```
Int[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(-1/5*Cot[e + f*x]^5/((a + b)*Sqrt[a + b + b*Tan[e + f*x]^2]) + ((-2*(5*a
+ 2*b)*Cot[e + f*x]^3)/(3*(a + b)*Sqrt[a + b + b*Tan[e + f*x]^2]) + ((15*a
^2 - 10*a*b - b^2)*(-(Cot[e + f*x]/((a + b)*Sqrt[a + b + b*Tan[e + f*x]^2]
)) - (2*b*Tan[e + f*x])/((a + b)^2*Sqrt[a + b + b*Tan[e + f*x]^2])))/(3*(a
 + b)))/(5*(a + b)))/f
```

3.118.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.118.4 Maple [A] (verified)

Time = 7.08 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.19

method	result
default	$-\frac{\left(b+a\cos(fx+e)^2\right)\left(8a^3\cos(fx+e)^6-40\cos(fx+e)^6a^2b-20\cos(fx+e)^4a^3+104\cos(fx+e)^4a^2b-20\cos(fx+e)^4ab^2+15\cos(fx+$ $15f(a^4+4a^3b+6a^2b^2+4ab^3+b^4)\left(a+$

input
```
int(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15/f/(a^4+4*a^3*b+6*a^2*b^2+4*a*b^3+b^4)*(b+a*cos(f*x+e)^2)*(8*a^3*cos(
f*x+e)^6-40*cos(f*x+e)^6*a^2*b-20*cos(f*x+e)^4*a^3+104*cos(f*x+e)^4*a^2*b-
20*cos(f*x+e)^4*a*b^2+15*cos(f*x+e)^2*a^3-85*cos(f*x+e)^2*a^2*b+49*cos(f*x
+e)^2*a*b^2+5*cos(f*x+e)^2*b^3+30*a^2*b-20*a*b^2-2*b^3)/(a+b*sec(f*x+e)^2)
^(3/2)*sec(f*x+e)^3*csc(f*x+e)^5
```

3.118.5 Fricas [A] (verification not implemented)

Time = 2.34 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.72

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx =$$

$$-\frac{\left(8\left(a^3-5\,a^2b\right)\cos\left(fx+e\right)^7-4\left(5\,a^3-26\,a^2b+5\,ab^2\right)\cos\left(fx+e\right)^5+\left(15\,a\right.\right.}{15\left(\left(a^5+4\,a^4b+6\,a^3b^2+4\,a^2b^3+ab^4\right)f\cos\left(fx+e\right)^6-\left(2\,a^5+7\,a^4b+8\,a^3b^2+2\,a^2b^3-2\,ab^4-b^5\right)f\cos\right.}$$

input
```
integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/15*(8*(a^3 - 5*a^2*b)*cos(f*x + e)^7 - 4*(5*a^3 - 26*a^2*b + 5*a*b^2)*c
os(f*x + e)^5 + (15*a^3 - 85*a^2*b + 49*a*b^2 + 5*b^3)*cos(f*x + e)^3 + 2*
(15*a^2*b - 10*a*b^2 - b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)/(((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*f*cos(f*x +
e)^6 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 - b^5)*f*cos(f*x
 + e)^4 + (a^5 + 2*a^4*b - 2*a^3*b^2 - 8*a^2*b^3 - 7*a*b^4 - 2*b^5)*f*cos(
f*x + e)^2 + (a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*f)*sin(f*x +
e))
```

3.118.6 Sympy [F]

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\csc^6\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)**6/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(csc(e + f*x)**6/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.118.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.54

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx =$$

$$-\frac{\frac{30\,b\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+b(a+b)^2}}-\frac{80\,b^2\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+b(a+b)^3}}+\frac{48\,b^3\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+b(a+b)^4}}+\frac{15}{\sqrt{b\tan(fx+e)^2+a+b(a+b)\tan(fx+e)}}-$$

input
```
integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/15*(30*b*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^2) - 80*b
^2*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^3) + 48*b^3*tan(f*
x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^4) + 15/(sqrt(b*tan(f*x + e
)^2 + a + b)*(a + b)*tan(f*x + e)) - 40*b/(sqrt(b*tan(f*x + e)^2 + a + b)*
(a + b)^2*tan(f*x + e)) + 24*b^2/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^3
*tan(f*x + e)) + 10/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)*tan(f*x + e)^3
) - 6*b/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^2*tan(f*x + e)^3) + 3/(sqr
t(b*tan(f*x + e)^2 + a + b)*(a + b)*tan(f*x + e)^5))/f
```

3.118.8 Giac [F]

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\csc\left(fx+e\right)^6}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.118.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Hanged}$$

input
```
int(1/(sin(e + f*x)^6*(a + b/cos(e + f*x)^2)^(3/2)),x)
```

output
```
\text{Hanged}
```

3.119 $\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

3.119.1 Optimal result 1007
3.119.2 Mathematica [A] (verified) 1008
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3.119.8 Giac [F] 1013
3.119.9 Mupad [F(-1)] 1013

3.119.1 Optimal result

Integrand size = 25, antiderivative size = 219

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{(5a^2+20ab+16b^2)\cos(e+fx)}{5a^3f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{2(5a+4b)\cos^3(e+fx)}{15a^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cos^5(e+fx)}{5af\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{4b(5a^2+20ab+16b^2)\sec(e+fx)}{15a^4f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{8b(5a^2+20ab+16b^2)\sec(e+fx)}{15a^5f\sqrt{a+b\sec^2(e+fx)}}$$

output

```
-1/5*(5*a^2+20*a*b+16*b^2)*cos(f*x+e)/a^3/f/(a+b*sec(f*x+e)^2)^(3/2)+2/15*
(5*a+4*b)*cos(f*x+e)^3/a^2/f/(a+b*sec(f*x+e)^2)^(3/2)-1/5*cos(f*x+e)^5/a/f
/(a+b*sec(f*x+e)^2)^(3/2)-4/15*b*(5*a^2+20*a*b+16*b^2)*sec(f*x+e)/a^4/f/(a
+b*sec(f*x+e)^2)^(3/2)-8/15*b*(5*a^2+20*a*b+16*b^2)*sec(f*x+e)/a^5/f/(a+b*
sec(f*x+e)^2)^(1/2)
```

3.119.2 Mathematica [A] (verified)

Time = 3.11 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.83

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx =$$
$$-\frac{(a+2b+a\cos(2(e+fx)))\,(425a^4+6400a^3b+22784a^2b^2+32768ab^3+16384b^4+48a(11a^3+150a^2b+$$

input
```
Integrate[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
-1/3840*((a + 2*b + a*Cos[2*(e + f*x)])*(425*a^4 + 6400*a^3*b + 22784*a^2*
b^2 + 32768*a*b^3 + 16384*b^4 + 48*a*(11*a^3 + 150*a^2*b + 384*a*b^2 + 256
*b^3)*Cos[2*(e + f*x)] + 12*a^2*(7*a^2 + 64*a*b + 64*b^2)*Cos[4*(e + f*x)]
 - 16*a^4*Cos[6*(e + f*x)] - 32*a^3*b*Cos[6*(e + f*x)] + 3*a^4*Cos[8*(e +
f*x)])*Sec[e + f*x]^5)/(a^5*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

3.119.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.89, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4622, 365, 25, 359, 245, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\sin(e+fx)^5}{(a+b\sec(e+fx)^2)^{5/2}}\,dx$$
$$\downarrow 4622$$
$$\frac{\int \frac{\cos^6(e+fx)\left(1-\sec^2(e+fx)\right)^2}{(b\sec^2(e+fx)+a)^{5/2}}\,d\sec(e+fx)}{f}$$
$$\downarrow 365$$

$$\frac{\int -\frac{\cos^4(e+fx)\left(2(5a+4b)-5a\sec^2(e+fx)\right)}{\left(b\sec^2(e+fx)+a\right)^{5/2}}d\sec(e+fx)}{5a}-\frac{\cos^5(e+fx)}{5a(a+b\sec^2(e+fx))^{3/2}}}{f}$$

$$\downarrow 25$$

$$\frac{-\frac{\int \frac{\cos^4(e+fx)\left(2(5a+4b)-5a\sec^2(e+fx)\right)}{\left(b\sec^2(e+fx)+a\right)^{5/2}}d\sec(e+fx)}{5a}-\frac{\cos^5(e+fx)}{5a(a+b\sec^2(e+fx))^{3/2}}}{f}$$

$$\downarrow 359$$

$$\frac{-\frac{\left(5a^2+20ab+16b^2\right)\int \frac{\cos^2(e+fx)}{\left(b\sec^2(e+fx)+a\right)^{5/2}}d\sec(e+fx)}{a}-\frac{2(5a+4b)\cos^3(e+fx)}{3a\left(a+b\sec^2(e+fx)\right)^{3/2}}}{5a}-\frac{\cos^5(e+fx)}{5a(a+b\sec^2(e+fx))^{3/2}}}{f}$$

$$\downarrow 245$$

$$\frac{-\frac{\left(5a^2+20ab+16b^2\right)\left(-\frac{4b\int \frac{1}{\left(b\sec^2(e+fx)+a\right)^{5/2}}d\sec(e+fx)}{a}-\frac{\cos(e+fx)}{a\left(a+b\sec^2(e+fx)\right)^{3/2}}\right)}{a}-\frac{2(5a+4b)\cos^3(e+fx)}{3a\left(a+b\sec^2(e+fx)\right)^{3/2}}}{5a}-\frac{\cos^5(e+fx)}{5a(a+b\sec^2(e+fx))^{3/2}}}{f}$$

$$\downarrow 209$$

$$\frac{-\frac{\left(5a^2+20ab+16b^2\right)\left(-\frac{4b\left(\frac{2\int \frac{1}{\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec(e+fx)}{3a}+\frac{\sec(e+fx)}{3a\left(a+b\sec^2(e+fx)\right)^{3/2}}\right)}{a}-\frac{\cos(e+fx)}{a\left(a+b\sec^2(e+fx)\right)^{3/2}}\right)}{a}-\frac{2(5a+4b)\cos^3(e+fx)}{3a\left(a+b\sec^2(e+fx)\right)^{3/2}}}{5a}-\frac{\cos^5(e+fx)}{5a}}{f}$$

$$\downarrow 208$$

$$\frac{-\frac{\left(5a^2+20ab+16b^2\right)\left(-\frac{4b\left(\frac{2\sec(e+fx)}{3a^2\sqrt{a+b\sec^2(e+fx)}}+\frac{\sec(e+fx)}{3a\left(a+b\sec^2(e+fx)\right)^{3/2}}\right)}{a}-\frac{\cos(e+fx)}{a\left(a+b\sec^2(e+fx)\right)^{3/2}}\right)}{a}-\frac{2(5a+4b)\cos^3(e+fx)}{3a\left(a+b\sec^2(e+fx)\right)^{3/2}}}{5a}-\frac{\cos^5(e+fx)}{5a(a+b\sec^2(e+f}}{f}$$

3.119. $\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

input
```
Int[Sin[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(-1/5*Cos[e + f*x]^5/(a*(a + b*Sec[e + f*x]^2)^(3/2)) - ((-2*(5*a + 4*b)*C
os[e + f*x]^3)/(3*a*(a + b*Sec[e + f*x]^2)^(3/2)) - ((5*a^2 + 20*a*b + 16*
b^2)*(-(Cos[e + f*x]/(a*(a + b*Sec[e + f*x]^2)^(3/2))) - (4*b*(Sec[e + f*x
]/(3*a*(a + b*Sec[e + f*x]^2)^(3/2)) + (2*Sec[e + f*x])/(3*a^2*Sqrt[a + b*
Sec[e + f*x]^2])))/a))/a)/(5*a))/f
```

3.119.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

3.119.4 Maple [A] (verified)

Time = 5.67 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.99

method	result
default	$\frac{a^2\left(b+a\cos(fx+e)^2\right)(a+b)^7\left(3\cos(fx+e)^8a^4-10a^4\cos(fx+e)^6-8a^3b\cos(fx+e)^6+15a^4\cos(fx+e)^4+60\cos(fx+e)^4a^3b+48a^2b^2c\right.}{15f(\sqrt{-ab}+a)^7(\sqrt{-ab}-a)^7\left(a+b\right.}$

input
```
int(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/15/f*a^2/((-a*b)^(1/2)+a)^7/((-a*b)^(1/2)-a)^7*(b+a*cos(f*x+e)^2)*(a+b)^
7*(3*cos(f*x+e)^8*a^4-10*a^4*cos(f*x+e)^6-8*a^3*b*cos(f*x+e)^6+15*a^4*cos(
f*x+e)^4+60*cos(f*x+e)^4*a^3*b+48*a^2*b^2*cos(f*x+e)^4+60*cos(f*x+e)^2*a^3
*b+240*a^2*b^2*cos(f*x+e)^2+192*a*b^3*cos(f*x+e)^2+40*a^2*b^2+160*a*b^3+12
8*b^4)/(a+b*sec(f*x+e)^2)^(5/2)*sec(f*x+e)^5
```

3.119.5 Fricas [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.86

$$\int \frac{\sin^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx =$$

$$-\frac{\left(3\,a^4\cos\left(fx+e\right)^9-2\left(5\,a^4+4\,a^3b\right)\cos\left(fx+e\right)^7+3\left(5\,a^4+20\,a^3b+16\,a^2b^2\right)\cos\left(fx+e\right)^5+12\left(5\,a^3b+\right.\right.}{15\left(a^7f\cos\left(fx+e\right)^4+2\,a^6bf\cos\left(\right.\right.}$$

input
```
integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/15*(3*a^4*cos(f*x + e)^9 - 2*(5*a^4 + 4*a^3*b)*cos(f*x + e)^7 + 3*(5*a^
4 + 20*a^3*b + 16*a^2*b^2)*cos(f*x + e)^5 + 12*(5*a^3*b + 20*a^2*b^2 + 16*
a*b^3)*cos(f*x + e)^3 + 8*(5*a^2*b^2 + 20*a*b^3 + 16*b^4)*cos(f*x + e))*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(a^7*f*cos(f*x + e)^4 + 2*a^6*b*
f*cos(f*x + e)^2 + a^5*b^2*f)
```

3.119.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**5/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Timed out
```

3.119.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.53

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx =$$

$$\frac{15\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{a^3} - \frac{10\left(\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3 - 9\sqrt{a+\frac{b}{\cos(fx+e)^2}}\,b\cos(fx+e)\right)}{a^4} + \frac{3\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{5}{2}}\cos(fx+e)^5 - 20}{}$$

$$-$$

input
```
integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/15*(15*sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e)/a^3 - 10*((a + b/cos(f*x
 + e)^2)^(3/2)*cos(f*x + e)^3 - 9*sqrt(a + b/cos(f*x + e)^2)*b*cos(f*x + e
))/a^4 + (3*(a + b/cos(f*x + e)^2)^(5/2)*cos(f*x + e)^5 - 20*(a + b/cos(f*
x + e)^2)^(3/2)*b*cos(f*x + e)^3 + 90*sqrt(a + b/cos(f*x + e)^2)*b^2*cos(f
*x + e))/a^5 + 5*(6*(a + b/cos(f*x + e)^2)*b*cos(f*x + e)^2 - b^2)/((a + b
/cos(f*x + e)^2)^(3/2)*a^3*cos(f*x + e)^3) + 10*(9*(a + b/cos(f*x + e)^2)*
b^2*cos(f*x + e)^2 - b^3)/((a + b/cos(f*x + e)^2)^(3/2)*a^4*cos(f*x + e)^3
) + 5*(12*(a + b/cos(f*x + e)^2)*b^3*cos(f*x + e)^2 - b^4)/((a + b/cos(f*x
 + e)^2)^(3/2)*a^5*cos(f*x + e)^3))/f
```

3.119.8 Giac [F]

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sin(fx+e)^5}{(b\sec(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.119.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sin(e+fx)^5}{\left(a+\frac{b}{\cos(e+fx)^2}\right)^{5/2}}\,dx$$

input
```
int(sin(e + f*x)^5/(a + b/cos(e + f*x)^2)^(5/2),x)
```

output
```
int(sin(e + f*x)^5/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.120 $\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

3.120.1 Optimal result 1014
3.120.2 Mathematica [A] (verified) 1014
3.120.3 Rubi [A] (verified) 1015
3.120.4 Maple [A] (verified) 1017
3.120.5 Fricas [A] (verification not implemented) 1018
3.120.6 Sympy [F(-1)] 1018
3.120.7 Maxima [A] (verification not implemented) 1018
3.120.8 Giac [F] 1019
3.120.9 Mupad [F(-1)] 1019

3.120.1 Optimal result

Integrand size = 25, antiderivative size = 146

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{(a+2b)\cos(e+fx)}{a^2 f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)}{3af\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{4b(a+2b)\sec(e+fx)}{3a^3 f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{8b(a+2b)\sec(e+fx)}{3a^4 f\sqrt{a+b\sec^2(e+fx)}}$$

output
```
-(a+2*b)*cos(f*x+e)/a^2/f/(a+b*sec(f*x+e)^2)^(3/2)+1/3*cos(f*x+e)^3/a/f/(a
+b*sec(f*x+e)^2)^(3/2)-4/3*b*(a+2*b)*sec(f*x+e)/a^3/f/(a+b*sec(f*x+e)^2)^(
3/2)-8/3*b*(a+2*b)*sec(f*x+e)/a^4/f/(a+b*sec(f*x+e)^2)^(1/2)
```

3.120.2 Mathematica [A] (verified)

Time = 2.19 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.88

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))\left(26a^3+264a^2b+640ab^2+512b^3+3a(11a^2+96ab+128b^2)\cos(2(e+fx))+\right.}{192a^4 f\left(a+b\sec^2(e+fx)\right)^{5/2}}$$

input
```
Integrate[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
-1/192*((a + 2*b + a*Cos[2*(e + f*x)])*(26*a^3 + 264*a^2*b + 640*a*b^2 + 5
12*b^3 + 3*a*(11*a^2 + 96*a*b + 128*b^2)*Cos[2*(e + f*x)] + 6*a^2*(a + 4*b
)*Cos[4*(e + f*x)] - a^3*Cos[6*(e + f*x)])*Sec[e + f*x]^5)/(a^4*f*(a + b*S
ec[e + f*x]^2)^(5/2))
```

3.120.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4622, 25, 359, 245, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sin(e+fx)^3}{(a+b\sec(e+fx)^2)^{5/2}}\,dx$$

$$\downarrow 4622$$

$$\frac{\int -\frac{\cos^4(e+fx)(1-\sec^2(e+fx))}{(b\sec^2(e+fx)+a)^{5/2}}\,d\sec(e+fx)}{f}$$

$$\downarrow 25$$

$$-\frac{\int \frac{\cos^4(e+fx)(1-\sec^2(e+fx))}{(b\sec^2(e+fx)+a)^{5/2}}\,d\sec(e+fx)}{f}$$

$$\downarrow 359$$

$$\frac{\frac{(a+2b)\int \frac{\cos^2(e+fx)}{\left(b\sec^2(e+fx)+a\right)^{5/2}}\,d\sec(e+fx)}{a} + \frac{\cos^3(e+fx)}{3a(a+b\sec^2(e+fx))^{3/2}}}{f}$$

$$\downarrow 245$$

$$\frac{\frac{(a+2b)\left(-\frac{4b\int \frac{1}{\left(b\sec^2(e+fx)+a\right)^{5/2}}\,d\sec(e+fx)}{a} - \frac{\cos(e+fx)}{a\left(a+b\sec^2(e+fx)\right)^{3/2}}\right)}{a} + \frac{\cos^3(e+fx)}{3a(a+b\sec^2(e+fx))^{3/2}}}{f}$$

$$\downarrow 209$$

$$\frac{(a+2b)\left(-\frac{4b\left(\frac{2\int \frac{1}{\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec(e+fx)}{3a}+\frac{\sec(e+fx)}{3a\left(a+b\sec^2(e+fx)\right)^{3/2}}\right)}{a}-\frac{\cos(e+fx)}{a\left(a+b\sec^2(e+fx)\right)^{3/2}}\right)}{a}+\frac{\cos^3(e+fx)}{3a(a+b\sec^2(e+fx))^{3/2}}}{f}$$

$$\downarrow 208$$

$$\frac{(a+2b)\left(-\frac{4b\left(\frac{2\sec(e+fx)}{3a^2\sqrt{a+b\sec^2(e+fx)}}+\frac{\sec(e+fx)}{3a\left(a+b\sec^2(e+fx)\right)^{3/2}}\right)}{a}-\frac{\cos(e+fx)}{a\left(a+b\sec^2(e+fx)\right)^{3/2}}\right)}{a}+\frac{\cos^3(e+fx)}{3a(a+b\sec^2(e+fx))^{3/2}}}{f}$$

input
```
Int[Sin[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(Cos[e + f*x]^3/(3*a*(a + b*Sec[e + f*x]^2)^(3/2)) + ((a + 2*b)*(-(Cos[e +
 f*x]/(a*(a + b*Sec[e + f*x]^2)^(3/2))) - (4*b*(Sec[e + f*x]/(3*a*(a + b*S
ec[e + f*x]^2)^(3/2)) + (2*Sec[e + f*x])/(3*a^2*Sqrt[a + b*Sec[e + f*x]^2]
)))/a))/a)/f
```

3.120.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

3.120.4 Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.01

method	result
default	$-\frac{a\left(b+a\cos(fx+e)^2\right)(a+b)^5\left(a^3\cos(fx+e)^6-3\cos(fx+e)^4a^3-6\cos(fx+e)^4a^2b-12\cos(fx+e)^2a^2b-24\cos(fx+e)^2ab^2-8ab^2-1\right.}{3f\left(\sqrt{-ab}+a\right)^5\left(\sqrt{-ab}-a\right)^5\left(a+b\sec(fx+e)^2\right)^{\frac{5}{2}}}$

input
```
int(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/f*a/((-a*b)^(1/2)+a)^5/((-a*b)^(1/2)-a)^5*(b+a*cos(f*x+e)^2)*(a+b)^5*
(a^3*cos(f*x+e)^6-3*cos(f*x+e)^4*a^3-6*cos(f*x+e)^4*a^2*b-12*cos(f*x+e)^2*
a^2*b-24*cos(f*x+e)^2*a*b^2-8*a*b^2-16*b^3)/(a+b*sec(f*x+e)^2)^(5/2)*sec(f
*x+e)^5
```

3.120.5 Fricas [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.95

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{(a^3\cos(fx+e)^7 - 3(a^3+2a^2b)\cos(fx+e)^5 - 12(a^2b+2ab^2)\cos(fx+$$
$$3\left(a^6 f\cos(fx+e)^4 + 2a^5 bf\cos(fx+e\right.$$

input
```
integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(a^3*cos(f*x + e)^7 - 3*(a^3 + 2*a^2*b)*cos(f*x + e)^5 - 12*(a^2*b + 2
*a*b^2)*cos(f*x + e)^3 - 8*(a*b^2 + 2*b^3)*cos(f*x + e))*sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)/(a^6*f*cos(f*x + e)^4 + 2*a^5*b*f*cos(f*x + e)^
2 + a^4*b^2*f)
```

3.120.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**3/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Timed out
```

3.120.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.34

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx =$$
$$-\frac{\frac{3\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{a^3} - \frac{\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}\cos(fx+e)^3 - 9\sqrt{a+\frac{b}{\cos(fx+e)^2}}b\cos(fx+e)}{a^4} + \frac{6\left(a+\frac{b}{\cos(fx+e)^2}\right)b\cos(fx+e)^2 - b^2}{\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}a^3\cos(fx+e)^3} + \frac{9\left(}{}}{3f}$$

input
```
integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*(3*sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e)/a^3 - ((a + b/cos(f*x + e)
^2)^(3/2)*cos(f*x + e)^3 - 9*sqrt(a + b/cos(f*x + e)^2)*b*cos(f*x + e))/a^
4 + (6*(a + b/cos(f*x + e)^2)*b*cos(f*x + e)^2 - b^2)/((a + b/cos(f*x + e)
^2)^(3/2)*a^3*cos(f*x + e)^3) + (9*(a + b/cos(f*x + e)^2)*b^2*cos(f*x + e)
^2 - b^3)/((a + b/cos(f*x + e)^2)^(3/2)*a^4*cos(f*x + e)^3))/f
```

3.120.8 Giac [F]

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sin\left(fx+e\right)^3}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.120.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sin\left(e+f\,x\right)^3}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

input
```
int(sin(e + f*x)^3/(a + b/cos(e + f*x)^2)^(5/2),x)
```

output
```
int(sin(e + f*x)^3/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.121 $\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

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3.121.1 Optimal result

Integrand size = 23, antiderivative size = 97

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{\cos(e+fx)}{af\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{4b\sec(e+fx)}{3a^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{8b\sec(e+fx)}{3a^3f\sqrt{a+b\sec^2(e+fx)}}$$

output
```
-cos(f*x+e)/a/f/(a+b*sec(f*x+e)^2)^(3/2)-4/3*b*sec(f*x+e)/a^2/f/(a+b*sec(f
*x+e)^2)^(3/2)-8/3*b*sec(f*x+e)/a^3/f/(a+b*sec(f*x+e)^2)^(1/2)
```

3.121.2 Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.91

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))\left((3a+8b)^2+12a(a+4b)\cos(2(e+fx))+3a^2\cos(4(e+fx))\right)\sec^5(e+fx)}{48a^3f\left(a+b\sec^2(e+fx)\right)^{5/2}}$$

input
```
Integrate[Sin[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
-1/48*((a + 2*b + a*Cos[2*(e + f*x)])*((3*a + 8*b)^2 + 12*a*(a + 4*b)*Cos[
2*(e + f*x)] + 3*a^2*Cos[4*(e + f*x)])*Sec[e + f*x]^5)/(a^3*f*(a + b*Sec[e
 + f*x]^2)^(5/2))
```

3.121.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4622, 245, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sin(e+fx)}{(a+b\sec(e+fx)^2)^{5/2}}\,dx$$

$$\downarrow 4622$$

$$\frac{\int \frac{\cos^2(e+fx)}{(b\sec^2(e+fx)+a)^{5/2}}\,d\sec(e+fx)}{f}$$

$$\downarrow 245$$

$$\frac{-\frac{4b\int \frac{1}{\left(b\sec^2(e+fx)+a\right)^{5/2}}\,d\sec(e+fx)}{a} - \frac{\cos(e+fx)}{a(a+b\sec^2(e+fx))^{3/2}}}{f}$$

$$\downarrow 209$$

$$\frac{-\frac{4b\left(\frac{2\int \frac{1}{\left(b\sec^2(e+fx)+a\right)^{3/2}}\,d\sec(e+fx)}{3a} + \frac{\sec(e+fx)}{3a\left(a+b\sec^2(e+fx)\right)^{3/2}}\right)}{a} - \frac{\cos(e+fx)}{a(a+b\sec^2(e+fx))^{3/2}}}{f}$$

$$\downarrow 208$$

$$\frac{-\frac{4b\left(\frac{2\sec(e+fx)}{3a^2\sqrt{a+b\sec^2(e+fx)}} + \frac{\sec(e+fx)}{3a\left(a+b\sec^2(e+fx)\right)^{3/2}}\right)}{a} - \frac{\cos(e+fx)}{a(a+b\sec^2(e+fx))^{3/2}}}{f}$$

input
```
Int[Sin[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(-(Cos[e + f*x]/(a*(a + b*Sec[e + f*x]^2)^(3/2))) - (4*b*(Sec[e + f*x]/(3*
a*(a + b*Sec[e + f*x]^2)^(3/2)) + (2*Sec[e + f*x])/(3*a^2*Sqrt[a + b*Sec[e
 + f*x]^2])))/a)/f
```

3.121.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

3.121.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{-\frac{1}{a\sec(fx+e)\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}}-\frac{4b\left(\frac{\sec(fx+e)}{3a\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}}+\frac{2\sec(fx+e)}{3a^2\sqrt{a+b\sec(fx+e)^2}}\right)}{a}}{f}$	90
default	$\frac{-\frac{1}{a\sec(fx+e)\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}}-\frac{4b\left(\frac{\sec(fx+e)}{3a\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}}+\frac{2\sec(fx+e)}{3a^2\sqrt{a+b\sec(fx+e)^2}}\right)}{a}}{f}$	90

input
```
int(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/a/sec(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2)-4*b/a*(1/3*sec(f*x+e)/a/(a+b
*sec(f*x+e)^2)^(3/2)+2/3/a^2*sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2)))
```

3.121.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.04

$$\int \frac{\sin(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = -\frac{\left(3\,a^2\cos\left(fx+e\right)^5+12\,ab\cos\left(fx+e\right)^3+8\,b^2\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{3\left(a^5 f\cos\left(fx+e\right)^4+2\,a^4bf\cos\left(fx+e\right)^2+a^3b^2f\right)}$$

input
```
integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*(3*a^2*cos(f*x + e)^5 + 12*a*b*cos(f*x + e)^3 + 8*b^2*cos(f*x + e))*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(a^5*f*cos(f*x + e)^4 + 2*a^4*b
*f*cos(f*x + e)^2 + a^3*b^2*f)
```

3.121.6 Sympy [F]

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral(sin(e + f*x)/(a + b*sec(e + f*x)**2)**(5/2), x)
```

3.121.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.89

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{\frac{3\sqrt{a+\frac{b}{\cos(fx+e)^2}}\cos(fx+e)}{a^3} + \frac{6\left(a+\frac{b}{\cos(fx+e)^2}\right)b\cos(fx+e)^2-b^2}{\left(a+\frac{b}{\cos(fx+e)^2}\right)^{\frac{3}{2}}a^3\cos(fx+e)^3}}{3f}$$

input
```
integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*(3*sqrt(a + b/cos(f*x + e)^2)*cos(f*x + e)/a^3 + (6*(a + b/cos(f*x +
e)^2)*b*cos(f*x + e)^2 - b^2)/((a + b/cos(f*x + e)^2)^(3/2)*a^3*cos(f*x +
e)^3))/f
```

3.121.8 Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.72

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{3\sqrt{a\cos(fx+e)^2+b} + \frac{6\left(a\cos(fx+e)^2+b\right)b-b^2}{\left(a\cos(fx+e)^2+b\right)^{\frac{3}{2}}}}{3\,a^3 f\,\mathrm{sgn}(\cos(fx+e))}$$

input
```
integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
-1/3*(3*sqrt(a*cos(f*x + e)^2 + b) + (6*(a*cos(f*x + e)^2 + b)*b - b^2)/(a
*cos(f*x + e)^2 + b)^(3/2))/(a^3*f*sgn(cos(f*x + e)))
```

3.121.9 Mupad [B] (verification not implemented)

Time = 30.61 (sec) , antiderivative size = 26927, normalized size of antiderivative = 277.60

$$\int \frac{\sin(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
int(sin(e + f*x)/(a + b/cos(e + f*x)^2)^(5/2),x)
```

output
```
((a + b/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2)^2)^(1/2)*(exp(e*3i
 + f*x*3i)*(((2*a + 4*b)*(((2*a + 4*b)*((((32*a*b^2 + 30*a^2*b + 3*a^3)/(4
8*a^3*b*f*(a + b)) - ((2*a + 4*b)*(8*a*b^2 + 8*a^2*b + a^3))/(48*a^4*b*f*(
a + b)))*(2*a + 4*b))/a + (8*a*b^2 + 8*a^2*b + a^3)/(48*a^3*b*f*(a + b)) -
 (16*a*b + a^2 + 20*b^2)/(24*a^2*b*f*(a + b))))/a - (a + 6*b)/(24*a*b*f*(a
 + b)) - (32*a*b^2 + 30*a^2*b + 3*a^3)/(48*a^3*b*f*(a + b)) + ((2*a + 4*b)
*(8*a*b^2 + 8*a^2*b + a^3))/(48*a^4*b*f*(a + b))))/a - (((32*a*b^2 + 30*a^
2*b + 3*a^3)/(48*a^3*b*f*(a + b)) - ((2*a + 4*b)*(8*a*b^2 + 8*a^2*b + a^3)
)/(48*a^4*b*f*(a + b)))*(2*a + 4*b))/a - (8*a*b^2 + 8*a^2*b + a^3)/(48*a^3
*b*f*(a + b)) + (16*a*b + a^2 + 20*b^2)/(24*a^2*b*f*(a + b)) + (32*a*b^2 +
 40*a^2*b + 3*a^3)/(48*a^3*b*f*(a + b))) + exp(e*1i + f*x*1i)*(((2*a + 4*b
)*((((32*a*b^2 + 30*a^2*b + 3*a^3)/(48*a^3*b*f*(a + b)) - ((2*a + 4*b)*(8*
a*b^2 + 8*a^2*b + a^3))/(48*a^4*b*f*(a + b)))*(2*a + 4*b))/a + (8*a*b^2 +
8*a^2*b + a^3)/(48*a^3*b*f*(a + b)) - (16*a*b + a^2 + 20*b^2)/(24*a^2*b*f*
(a + b))))/a + (48*a*b^2 + 18*a^2*b + a^3 + 32*b^3)/(48*a^3*b*f*(a + b)) -
 (a + 6*b)/(24*a*b*f*(a + b)) - (32*a*b^2 + 30*a^2*b + 3*a^3)/(48*a^3*b*f*
(a + b)) + ((2*a + 4*b)*(8*a*b^2 + 8*a^2*b + a^3))/(48*a^4*b*f*(a + b))))*
(2*exp(e*2i + f*x*2i) + exp(e*4i + f*x*4i) + 1))/((exp(e*2i + f*x*2i) + 1)
*(a + exp(e*2i + f*x*2i)*(2*a + 4*b) + a*exp(e*4i + f*x*4i))) - ((a + b/(e
xp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2)^2)^(1/2)*(exp(e*1i + f*x*...
```

3.122 $\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

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3.122.1 Optimal result

Integrand size = 23, antiderivative size = 127

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{(a+b)^{5/2}f} - \frac{b\sec(e+fx)}{3a(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{b(5a+2b)\sec(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sec^2(e+fx)}}$$

output
```
-arctanh(sec(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))/(a+b)^(5/2)/f-1/
3*b*sec(f*x+e)/a/(a+b)/f/(a+b*sec(f*x+e)^2)^(3/2)-1/3*b*(5*a+2*b)*sec(f*x+
e)/a^2/(a+b)^2/f/(a+b*sec(f*x+e)^2)^(1/2)
```

3.122.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 3.86 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.85

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{(a+2b+a\cos(2(e+fx)))\sec^5(e+fx)\left(a^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\right.\right.}{6a^2(a+b)f\left(a+b\sec^2(e\right.}$$

input
```
Integrate[Csc[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^5*(a^2*Hypergeometric2F1[-3/2
, 1, -1/2, 1 - (a*Sin[e + f*x]^2)/(a + b)] + (a + b)*(-2*(2*a + b) + 3*a*S
in[e + f*x]^2)))/(6*a^2*(a + b)*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

3.122.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4622, 25, 316, 402, 27, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sin(e+fx)\,(a+b\sec(e+fx)^2)^{5/2}}dx$$

$$\downarrow 4622$$

$$\frac{\int -\frac{1}{(1-\sec^2(e+fx))(b\sec^2(e+fx)+a)^{5/2}}d\sec(e+fx)}{f}$$

$$\downarrow 25$$

$$-\frac{\int \frac{1}{(1-\sec^2(e+fx))(b\sec^2(e+fx)+a)^{5/2}}d\sec(e+fx)}{f}$$

$$\downarrow 316$$

$$\frac{\int \frac{2b\sec^2(e+fx)+b-3(a+b)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec(e+fx)}{3a(a+b)} - \frac{b\sec(e+fx)}{3a(a+b)(a+b\sec^2(e+fx))^{3/2}}}{f}$$

$$\downarrow 402$$

$$\frac{-\frac{\int \frac{3a^2}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{a(a+b)} - \frac{b(5a+2b)\sec(e+fx)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{3a(a+b)} - \frac{b\sec(e+fx)}{3a(a+b)(a+b\sec^2(e+fx))^{3/2}}}{f}$$

$$\downarrow 27$$

$$\frac{-\frac{3a\int \frac{1}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{a+b}-\frac{b(5a+2b)\sec(e+fx)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{3a(a+b)}-\frac{b\sec(e+fx)}{3a(a+b)(a+b\sec^2(e+fx))^{3/2}}}{f}$$

$$\downarrow 291$$

$$\frac{-\frac{3a\int \frac{1}{1-\frac{(a+b)\sec^2(e+fx)}{b\sec^2(e+fx)+a}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}}{a+b}-\frac{b(5a+2b)\sec(e+fx)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{3a(a+b)}-\frac{b\sec(e+fx)}{3a(a+b)(a+b\sec^2(e+fx))^{3/2}}}{f}$$

$$\downarrow 219$$

$$\frac{-\frac{3a\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{(a+b)^{3/2}}-\frac{b(5a+2b)\sec(e+fx)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{3a(a+b)}-\frac{b\sec(e+fx)}{3a(a+b)(a+b\sec^2(e+fx))^{3/2}}}{f}$$

input
```
Int[Csc[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(-1/3*(b*Sec[e + f*x])/(a*(a + b)*(a + b*Sec[e + f*x]^2)^(3/2)) + ((-3*a*A
rcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/(a + b)^(3/
2) - (b*(5*a + 2*b)*Sec[e + f*x])/(a*(a + b)*Sqrt[a + b*Sec[e + f*x]^2]))/
(3*a*(a + b)))/f
```

3.122.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

3.122.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3289 vs. $2(113) = 226$.

Time = 5.23 (sec) , antiderivative size = 3290, normalized size of antiderivative = 25.91

method	result	size
default	Expression too large to display	3290

input
```
int(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6/f/a^2/(2*(-a*b)^(1/2)+a-b)^2/(2*(-a*b)^(1/2)-a+b)^2/(a+b)^(1/2)*(b+a*
cos(f*x+e)^2)*(10*(a+b)^(5/2)*a*b^2+3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*cos(f*x+e)^3*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^5+3*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*cos(f*x+e)^3*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*a^5+12*cos(f*x+e)^2*(a+b
)^(5/2)*a^2*b+6*cos(f*x+e)^2*(a+b)^(5/2)*a*b^2+3*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*cos(f*x+e)^2*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^5+3*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)^2*ln(-4*(((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*a^5+3*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^4*b+6*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/...
```

3.122.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(113) = 226$.

Time $= 0.36$ (sec) , antiderivative size $= 592$, normalized size of antiderivative $= 4.66$

$$\int \frac{\csc(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \left[\frac{3\left(a^4\cos\left(fx+e\right)^4+2\,a^3b\cos\left(fx+e\right)^2+a^2b^2\right)\sqrt{a+b}\log\left(\frac{2\left(a\cos(fx+e)^2-\right.\right.}{6\left(\left(a^7+3\,a^6b+3\,a^5b^2+a^4b^3\right)f\cos\left(\right.\right.}\right.$$

input
```
integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(3*(a^4*cos(f*x + e)^4 + 2*a^3*b*cos(f*x + e)^2 + a^2*b^2)*sqrt(a + b
)*log(2*(a*cos(f*x + e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1)) - 2*(3*(2*a^3*b
+ 3*a^2*b^2 + a*b^3)*cos(f*x + e)^3 + (5*a^2*b^2 + 7*a*b^3 + 2*b^4)*cos(f*
x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^7 + 3*a^6*b + 3*a
^5*b^2 + a^4*b^3)*f*cos(f*x + e)^4 + 2*(a^6*b + 3*a^5*b^2 + 3*a^4*b^3 + a^
3*b^4)*f*cos(f*x + e)^2 + (a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^5)*f),
1/3*(3*(a^4*cos(f*x + e)^4 + 2*a^3*b*cos(f*x + e)^2 + a^2*b^2)*sqrt(-a - b
)*arctan(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x
+ e)/(a + b)) - (3*(2*a^3*b + 3*a^2*b^2 + a*b^3)*cos(f*x + e)^3 + (5*a^2*b
^2 + 7*a*b^3 + 2*b^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^4 + 2*(a^6*b
+ 3*a^5*b^2 + 3*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^2 + (a^5*b^2 + 3*a^4*b^3
 + 3*a^3*b^4 + a^2*b^5)*f)]
```

3.122.6 Sympy [F]

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral(csc(e + f*x)/(a + b*sec(e + f*x)**2)**(5/2), x)
```

3.122.7 Maxima [F]

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\csc(fx+e)}{(b\sec(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(csc(f*x + e)/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

3.122.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1097 vs. 2(113) = 226.

Time = 1.29 (sec) , antiderivative size = 1097, normalized size of antiderivative = 8.64

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
-1/6*(4*((((3*a^6*b^3 + 16*a^5*b^4 + 35*a^4*b^5 + 40*a^3*b^6 + 25*a^2*b^7
+ 8*a*b^8 + b^9)*tan(1/2*f*x + 1/2*e)^2/(a^8*b^2*sgn(cos(f*x + e)) + 6*a^7
*b^3*sgn(cos(f*x + e)) + 15*a^6*b^4*sgn(cos(f*x + e)) + 20*a^5*b^5*sgn(cos
(f*x + e)) + 15*a^4*b^6*sgn(cos(f*x + e)) + 6*a^3*b^7*sgn(cos(f*x + e)) +
a^2*b^8*sgn(cos(f*x + e))) - 3*(a^6*b^3 + 2*a^5*b^4 - 3*a^4*b^5 - 12*a^3*b
^6 - 13*a^2*b^7 - 6*a*b^8 - b^9)/(a^8*b^2*sgn(cos(f*x + e)) + 6*a^7*b^3*sg
n(cos(f*x + e)) + 15*a^6*b^4*sgn(cos(f*x + e)) + 20*a^5*b^5*sgn(cos(f*x +
e)) + 15*a^4*b^6*sgn(cos(f*x + e)) + 6*a^3*b^7*sgn(cos(f*x + e)) + a^2*b^8
*sgn(cos(f*x + e))))*tan(1/2*f*x + 1/2*e)^2 - 3*(a^6*b^3 + 2*a^5*b^4 - 3*a
^4*b^5 - 12*a^3*b^6 - 13*a^2*b^7 - 6*a*b^8 - b^9)/(a^8*b^2*sgn(cos(f*x + e
)) + 6*a^7*b^3*sgn(cos(f*x + e)) + 15*a^6*b^4*sgn(cos(f*x + e)) + 20*a^5*b
^5*sgn(cos(f*x + e)) + 15*a^4*b^6*sgn(cos(f*x + e)) + 6*a^3*b^7*sgn(cos(f*
x + e)) + a^2*b^8*sgn(cos(f*x + e))))*tan(1/2*f*x + 1/2*e)^2 + (3*a^6*b^3
+ 16*a^5*b^4 + 35*a^4*b^5 + 40*a^3*b^6 + 25*a^2*b^7 + 8*a*b^8 + b^9)/(a^8*
b^2*sgn(cos(f*x + e)) + 6*a^7*b^3*sgn(cos(f*x + e)) + 15*a^6*b^4*sgn(cos(f
*x + e)) + 20*a^5*b^5*sgn(cos(f*x + e)) + 15*a^4*b^6*sgn(cos(f*x + e)) + 6
*a^3*b^7*sgn(cos(f*x + e)) + a^2*b^8*sgn(cos(f*x + e))))/(a*tan(1/2*f*x +
1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan
(1/2*f*x + 1/2*e)^2 + a + b)^(3/2) - 3*log(abs(-sqrt(a + b)*tan(1/2*f*x +
1/2*e)^2 + sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2...
```

3.122.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{1}{\sin(e+fx)\left(a+\frac{b}{\cos(e+fx)^2}\right)^{5/2}}\,dx$$

input
```
int(1/(sin(e + f*x)*(a + b/cos(e + f*x)^2)^(5/2)),x)
```

output
```
int(1/(sin(e + f*x)*(a + b/cos(e + f*x)^2)^(5/2)), x)
```

3.123 $\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

3.123.1 Optimal result 1034
3.123.2 Mathematica [C] (verified) 1034
3.123.3 Rubi [A] (verified) 1035
3.123.4 Maple [B] (warning: unable to verify) 1038
3.123.5 Fricas [B] (verification not implemented) 1039
3.123.6 Sympy [F] 1040
3.123.7 Maxima [F(-1)] 1040
3.123.8 Giac [F] 1040
3.123.9 Mupad [F(-1)] 1041

3.123.1 Optimal result

Integrand size = 25, antiderivative size = 171

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{(a-4b)\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{2(a+b)^{7/2}f} - \frac{\cot(e+fx)\csc(e+fx)}{2(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{5b\sec(e+fx)}{6(a+b)^2 f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{(13a-2b)b\sec(e+fx)}{6a(a+b)^3 f\sqrt{a+b\sec^2(e+fx)}}$$

output
```
-1/2*(a-4*b)*arctanh(sec(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2))/(a+b
)^(7/2)/f-1/2*cot(f*x+e)*csc(f*x+e)/(a+b)/f/(a+b*sec(f*x+e)^2)^(3/2)-5/6*b
*sec(f*x+e)/(a+b)^2/f/(a+b*sec(f*x+e)^2)^(3/2)-1/6*(13*a-2*b)*b*sec(f*x+e)
/a/(a+b)^3/f/(a+b*sec(f*x+e)^2)^(1/2)
```

3.123.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.03 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.88

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))\left((a+b)\left(3a^2+6ab-2b^2+(3a^2+2b^2)\cos(2(e+fx))\right)\csc^2(e+fx)-3a(a+\right.}{24a(a+b)^3 f\left(a+b\sec^2\right.}$$

input
```
Integrate[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
-1/24*((a + 2*b + a*Cos[2*(e + f*x)])*((a + b)*(3*a^2 + 6*a*b - 2*b^2 + (3
*a^2 + 2*b^2)*Cos[2*(e + f*x)])*Csc[e + f*x]^2 - 3*a*(a - 4*b)*(a + 2*b +
a*Cos[2*(e + f*x)])*Hypergeometric2F1[-1/2, 1, 1/2, 1 - (a*Sin[e + f*x]^2)
/(a + b)])*Sec[e + f*x]^5)/(a*(a + b)^3*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

3.123.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4622, 373, 402, 25, 27, 402, 27, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sin(e+fx)^3\left(a+b\sec(e+fx)^2\right)^{5/2}}dx$$

$$\downarrow 4622$$

$$\frac{\int \frac{\sec^2(e+fx)}{(1-\sec^2(e+fx))^2(b\sec^2(e+fx)+a)^{5/2}}d\sec(e+fx)}{f}$$

$$\downarrow 373$$

$$\frac{\frac{\sec(e+fx)}{2(a+b)(1-\sec^2(e+fx))(a+b\sec^2(e+fx))^{3/2}} - \frac{\int \frac{a-4b\sec^2(e+fx)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{5/2}}d\sec(e+fx)}{2(a+b)}}{f}$$

$$\downarrow 402$$

$$\frac{\frac{\sec(e+fx)}{2(a+b)(1-\sec^2(e+fx))(a+b\sec^2(e+fx))^{3/2}} - \frac{\frac{5b\sec(e+fx)}{3(a+b)\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\int -\frac{a\left(-10b\sec^2(e+fx)+3a-2b\right)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec(e+fx)}{3a(a+b)}}{2(a+b)}}{f}$$

$$\downarrow 25$$

$$\frac{\sec(e+fx)}{2(a+b)(1-\sec^2(e+fx))(a+b\sec^2(e+fx))^{3/2}} - \frac{\frac{\int \frac{a\left(-10b\sec^2(e+fx)+3a-2b\right)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec(e+fx)}{3a(a+b)} + \frac{5b\sec(e+fx)}{3(a+b)\left(a+b\sec^2(e+fx)\right)^{3/2}}}{2(a+b)}$$

f

$\downarrow$ 27

$$\frac{\sec(e+fx)}{2(a+b)(1-\sec^2(e+fx))(a+b\sec^2(e+fx))^{3/2}} - \frac{\frac{\int \frac{-10b\sec^2(e+fx)+3a-2b}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec(e+fx)}{3(a+b)} + \frac{5b\sec(e+fx)}{3(a+b)\left(a+b\sec^2(e+fx)\right)^{3/2}}}{2(a+b)}$$

f

$\downarrow$ 402

$$\frac{\sec(e+fx)}{2(a+b)(1-\sec^2(e+fx))(a+b\sec^2(e+fx))^{3/2}} - \frac{\frac{b(13a-2b)\sec(e+fx)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}} - \frac{\int -\frac{3a(a-4b)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{a(a+b)}}{3(a+b)} + \frac{5b\sec(e+fx)}{3(a+b)\left(a+b\sec^2(e+fx)\right)}$$

f

$\downarrow$ 27

$$\frac{\sec(e+fx)}{2(a+b)(1-\sec^2(e+fx))(a+b\sec^2(e+fx))^{3/2}} - \frac{\frac{3(a-4b)\int \frac{1}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{a+b} + \frac{b(13a-2b)\sec(e+fx)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{3(a+b)} + \frac{5b\sec(e+f}{3(a+b)\left(a+b\sec^2(e}$$

f

$\downarrow$ 291

$$\frac{\sec(e+fx)}{2(a+b)(1-\sec^2(e+fx))(a+b\sec^2(e+fx))^{3/2}} - \frac{\frac{3(a-4b)\int \frac{1}{1-\frac{(a+b)\sec^2(e+fx)}{b\sec^2(e+fx)+a}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}}{a+b} + \frac{b(13a-2b)\sec(e+fx)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{3(a+b)} + \frac{5b\sec(e+fx)}{3(a+b)\left(a+b\sec^2(e+fx)\right)}$$

f

$\downarrow$ 219

$$\frac{\sec(e+fx)}{2(a+b)(1-\sec^2(e+fx))(a+b\sec^2(e+fx))^{3/2}} - \frac{\frac{3(a-4b)\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{(a+b)^{3/2}} + \frac{b(13a-2b)\sec(e+fx)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{3(a+b)} + \frac{5b\sec(e+fx)}{3(a+b)\left(a+b\sec^2(e+fx)\right)^{3/2}}}{2(a+b)}$$

f

3.123. $\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

input
```
Int[Csc[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(Sec[e + f*x]/(2*(a + b)*(1 - Sec[e + f*x]^2)*(a + b*Sec[e + f*x]^2)^(3/2)
) - ((5*b*Sec[e + f*x])/(3*(a + b)*(a + b*Sec[e + f*x]^2)^(3/2)) + ((3*(a
- 4*b)*ArcTanh[(Sqrt[a + b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/(a
+ b)^(3/2) + ((13*a - 2*b)*b*Sec[e + f*x])/(a*(a + b)*Sqrt[a + b*Sec[e + f
*x]^2]))/(3*(a + b)))/(2*(a + b)))/f
```

3.123.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

3.123.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 8688 vs. $2(151) = 302$.

Time = 5.54 (sec) , antiderivative size = 8689, normalized size of antiderivative = 50.81

method	result	size
default	Expression too large to display	8689

input
```
int(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.123.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(151) = 302$.

Time = 0.43 (sec) , antiderivative size = 941, normalized size of antiderivative = 5.50

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \left[-\frac{3\left((a^4-4\,a^3b)\cos(fx+e)^6-(a^4-6\,a^3b+8\,a^2b^2)\cos(fx+e)^4-a^2b^2\right.}{12\left((a^7+4\,a^6b+6\,a^5b^2\right.}\right.$$

input
```
integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[-1/12*(3*((a^4 - 4*a^3*b)*cos(f*x + e)^6 - (a^4 - 6*a^3*b + 8*a^2*b^2)*co
s(f*x + e)^4 - a^2*b^2 + 4*a*b^3 - (2*a^3*b - 9*a^2*b^2 + 4*a*b^3)*cos(f*x
 + e)^2)*sqrt(a + b)*log(2*(a*cos(f*x + e)^2 + 2*sqrt(a + b)*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1
)) - 2*(3*(a^4 - 3*a^3*b - 4*a^2*b^2)*cos(f*x + e)^5 + 2*(9*a^3*b + 4*a^2*
b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^3 + (13*a^2*b^2 + 11*a*b^3 - 2*b^4)*cos(
f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^7 + 4*a^6*b + 6
*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^6 - (a^7 + 2*a^6*b - 2*a^5*
b^2 - 8*a^4*b^3 - 7*a^3*b^4 - 2*a^2*b^5)*f*cos(f*x + e)^4 - (2*a^6*b + 7*a
^5*b^2 + 8*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - a*b^6)*f*cos(f*x + e)^2 - (a^
5*b^2 + 4*a^4*b^3 + 6*a^3*b^4 + 4*a^2*b^5 + a*b^6)*f), 1/6*(3*((a^4 - 4*a^
3*b)*cos(f*x + e)^6 - (a^4 - 6*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 - a^2*b^2
 + 4*a*b^3 - (2*a^3*b - 9*a^2*b^2 + 4*a*b^3)*cos(f*x + e)^2)*sqrt(-a - b)*
arctan(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x +
e)/(a + b)) + (3*(a^4 - 3*a^3*b - 4*a^2*b^2)*cos(f*x + e)^5 + 2*(9*a^3*b +
 4*a^2*b^2 - 4*a*b^3 + b^4)*cos(f*x + e)^3 + (13*a^2*b^2 + 11*a*b^3 - 2*b^
4)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^7 + 4*a^
6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^6 - (a^7 + 2*a^6*b -
 2*a^5*b^2 - 8*a^4*b^3 - 7*a^3*b^4 - 2*a^2*b^5)*f*cos(f*x + e)^4 - (2*a^6*
b + 7*a^5*b^2 + 8*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - a*b^6)*f*cos(f*x + ...
```

3.123.6 Sympy [F]

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)**3/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral(csc(e + f*x)**3/(a + b*sec(e + f*x)**2)**(5/2), x)
```

3.123.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.123.8 Giac [F]

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\csc\left(fx+e\right)^3}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.123.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{1}{\sin(e+fx)^3\left(a+\frac{b}{\cos(e+fx)^2}\right)^{5/2}}\,dx$$

input
```
int(1/(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^(5/2)),x)
```

output
```
int(1/(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^(5/2)), x)
```

3.124 $\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

3.124.1 Optimal result 1042
3.124.2 Mathematica [C] (verified) 1043
3.124.3 Rubi [A] (verified) 1043
3.124.4 Maple [B] (warning: unable to verify) 1047
3.124.5 Fricas [B] (verification not implemented) 1048
3.124.6 Sympy [F] 1048
3.124.7 Maxima [F(-1)] 1049
3.124.8 Giac [F] 1049
3.124.9 Mupad [F(-1)] 1049

3.124.1 Optimal result

Integrand size = 25, antiderivative size = 234

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{(3a^2-24ab+8b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{8(a+b)^{9/2}f} - \frac{(5a-2b)\cot(e+fx)\csc(e+fx)}{8(a+b)^2 f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)\csc(e+fx)}{4(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{(23a-12b)b\sec(e+fx)}{24(a+b)^3 f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{5(11a-10b)b\sec(e+fx)}{24(a+b)^4 f\sqrt{a+b\sec^2(e+fx)}}$$

output

```
-1/8*(3*a^2-24*a*b+8*b^2)*arctanh(sec(f*x+e)*(a+b)^(1/2)/(a+b*sec(f*x+e)^2
)^(1/2))/(a+b)^(9/2)/f-1/8*(5*a-2*b)*cot(f*x+e)*csc(f*x+e)/(a+b)^2/f/(a+b*
sec(f*x+e)^2)^(3/2)-1/4*cot(f*x+e)^3*csc(f*x+e)/(a+b)/f/(a+b*sec(f*x+e)^2)
^(3/2)-1/24*(23*a-12*b)*b*sec(f*x+e)/(a+b)^3/f/(a+b*sec(f*x+e)^2)^(3/2)-5/
24*(11*a-10*b)*b*sec(f*x+e)/(a+b)^4/f/(a+b*sec(f*x+e)^2)^(1/2)
```

3.124.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.31 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.55

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx =$$
$$-\frac{(a+2b+a\cos(2(e+fx)))\left(3(a+b)(3a-4b+(a+8b)\cos(2(e+fx)))\csc^4(e+fx)-2(3a^2-24ab+8\right.}{96(a+b)^3 f\,(a+b\sec^2(e+fx))^{5/2}}$$

input
```
Integrate[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
-1/96*((a + 2*b + a*Cos[2*(e + f*x)])*(3*(a + b)*(3*a - 4*b + (a + 8*b)*Co
s[2*(e + f*x)])*Csc[e + f*x]^4 - 2*(3*a^2 - 24*a*b + 8*b^2)*Hypergeometric
2F1[-3/2, 1, -1/2, 1 - (a*Sin[e + f*x]^2)/(a + b)])*Sec[e + f*x]^5)/((a +
b)^3*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

3.124.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.12, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 4622, 25, 372, 402, 25, 402, 25, 27, 402, 27, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{\sin(e+fx)^5\,(a+b\sec(e+fx)^2)^{5/2}}dx$$
$$\downarrow 4622$$
$$\frac{\int -\frac{\sec^4(e+fx)}{(1-\sec^2(e+fx))^3(b\sec^2(e+fx)+a)^{5/2}}d\sec(e+fx)}{f}$$
$$\downarrow 25$$

$$-\frac{\int \frac{\sec^4(e+fx)}{(1-\sec^2(e+fx))^3(b\sec^2(e+fx)+a)^{5/2}}\,d\sec(e+fx)}{f}$$

$$\downarrow 372$$

$$\frac{\int \frac{2(2a-b)\sec^2(e+fx)+a}{\left(1-\sec^2(e+fx)\right)^2\left(b\sec^2(e+fx)+a\right)^{5/2}}\,d\sec(e+fx)}{4(a+b)} - \frac{\sec(e+fx)}{4(a+b)(1-\sec^2(e+fx))^2(a+b\sec^2(e+fx))^{3/2}}}{f}$$

$$\downarrow 402$$

$$\frac{\frac{\int -\frac{a(3a-4b)-4(5a-2b)b\sec^2(e+fx)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{5/2}}\,d\sec(e+fx)}{2(a+b)} + \frac{(5a-2b)\sec(e+fx)}{2(a+b)\left(1-\sec^2(e+fx)\right)\left(a+b\sec^2(e+fx)\right)^{3/2}}}{4(a+b)} - \frac{\sec(e+fx)}{4(a+b)(1-\sec^2(e+fx))^2(a+b\sec^2(e+fx))^{3/2}}}{f}$$

$$\downarrow 25$$

$$\frac{\frac{(5a-2b)\sec(e+fx)}{2(a+b)\left(1-\sec^2(e+fx)\right)\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\int \frac{a(3a-4b)-4(5a-2b)b\sec^2(e+fx)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{5/2}}\,d\sec(e+fx)}{2(a+b)}}{4(a+b)} - \frac{\sec(e+fx)}{4(a+b)(1-\sec^2(e+fx))^2(a+b\sec^2(e+fx))^{3/2}}}{f}$$

$$\downarrow 402$$

$$\frac{\frac{(5a-2b)\sec(e+fx)}{2(a+b)\left(1-\sec^2(e+fx)\right)\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\frac{b(23a-12b)\sec(e+fx)}{3(a+b)\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\int -\frac{a\left(a(9a-26b)-2(23a-12b)b\sec^2(e+fx)\right)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{3/2}}\,d\sec(e+fx)}{3a(a+b)}}{2(a+b)}}{4(a+b)} - \frac{}{4(a+b)(1-\sec^2(e}}{f}$$

$$\downarrow 25$$

$$\frac{\frac{(5a-2b)\sec(e+fx)}{2(a+b)\left(1-\sec^2(e+fx)\right)\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\frac{\int \frac{a\left(a(9a-26b)-2(23a-12b)b\sec^2(e+fx)\right)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{3/2}}\,d\sec(e+fx)}{3a(a+b)} + \frac{b(23a-12b)\sec(e+fx)}{3(a+b)\left(a+b\sec^2(e+fx)\right)^{3/2}}}{2(a+b)}}{4(a+b)} - \frac{}{4(a+b)(1-\sec^2(e+}}{f}$$

$$\downarrow 27$$

3.124. $\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

$$\frac{(5a-2b)\sec(e+fx)}{2(a+b)\left(1-\sec^2(e+fx)\right)\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{\frac{\int\frac{a(9a-26b)-2(23a-12b)b\sec^2(e+fx)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec(e+fx)}{3(a+b)}+\frac{b(23a-12b)\sec(e+fx)}{3(a+b)\left(a+b\sec^2(e+fx)\right)^{3/2}}}{2(a+b)}}{4(a+b)}-\frac{\sec}{4(a+b)(1-\sec^2(e+f}$$

f

$\downarrow$ 402

$$\frac{(5a-2b)\sec(e+fx)}{2(a+b)\left(1-\sec^2(e+fx)\right)\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{\frac{\frac{5b(11a-10b)\sec(e+fx)}{(a+b)\sqrt{a+b\sec^2(e+fx)}}-\frac{\int-\frac{3a\left(3a^2-24ba+8b^2\right)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{a(a+b)}}{3(a+b)}+\frac{b(23a-12b)\sec(e+fx)}{3(a+b)\left(a+b\sec^2(e+fx)\right)^{3/2}}}{2(a+b)}}{4(a+b)}-$$

f

$\downarrow$ 27

$$\frac{(5a-2b)\sec(e+fx)}{2(a+b)\left(1-\sec^2(e+fx)\right)\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{\frac{\frac{3\left(3a^2-24ab+8b^2\right)\int\frac{1}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{a+b}+\frac{5b(11a-10b)\sec(e+fx)}{(a+b)\sqrt{a+b\sec^2(e+fx)}}}{3(a+b)}+\frac{b(23a-12b)}{3(a+b)\left(a+b\sec}}{2(a+b)}}{4(a+b)}$$

f

$\downarrow$ 291

$$\frac{(5a-2b)\sec(e+fx)}{2(a+b)\left(1-\sec^2(e+fx)\right)\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{\frac{\frac{3\left(3a^2-24ab+8b^2\right)\int\frac{1}{1-\frac{(a+b)\sec^2(e+fx)}{b\sec^2(e+fx)+a}}d\frac{\sec(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}}{a+b}+\frac{5b(11a-10b)\sec(e+fx)}{(a+b)\sqrt{a+b\sec^2(e+fx)}}}{3(a+b)}+\frac{b(23a-12b)\sec(e-}{3(a+b)\left(a+b\sec^2(e+}}{2(a+b)}}{4(a+b)}$$

f

$\downarrow$ 219

$$\frac{(5a-2b)\sec(e+fx)}{2(a+b)\left(1-\sec^2(e+fx)\right)\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{\frac{\frac{3\left(3a^2-24ab+8b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{a+b}\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\right)}{(a+b)^{3/2}}+\frac{5b(11a-10b)\sec(e+fx)}{(a+b)\sqrt{a+b\sec^2(e+fx)}}}{3(a+b)}+\frac{b(23a-12b)\sec(e+fx)}{3(a+b)\left(a+b\sec^2(e+fx)\right)^{3/2}}}{2(a+b)}}{4(a+b)}$$

f

3.124. $\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

input
```
Int[Csc[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(-1/4*Sec[e + f*x]/((a + b)*(1 - Sec[e + f*x]^2)^2*(a + b*Sec[e + f*x]^2)^
(3/2)) + (((5*a - 2*b)*Sec[e + f*x])/(2*(a + b)*(1 - Sec[e + f*x]^2)*(a + 
b*Sec[e + f*x]^2)^(3/2)) - (((23*a - 12*b)*b*Sec[e + f*x])/(3*(a + b)*(a + 
 b*Sec[e + f*x]^2)^(3/2)) + ((3*(3*a^2 - 24*a*b + 8*b^2)*ArcTanh[(Sqrt[a + 
 b]*Sec[e + f*x])/Sqrt[a + b*Sec[e + f*x]^2]])/(a + b)^(3/2) + (5*(11*a - 
10*b)*b*Sec[e + f*x])/((a + b)*Sqrt[a + b*Sec[e + f*x]^2]))/(3*(a + b)))/(
2*(a + b)))/(4*(a + b)))/f
```

3.124.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c, 
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) + 
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, 
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

3.124.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 8114 vs. 2(210) = 420.

Time = 6.06 (sec) , antiderivative size = 8115, normalized size of antiderivative = 34.68

method	result	size
default	Expression too large to display	8115

input
```
int(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.124.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 644 vs. 2(210) = 420.

Time = 0.56 (sec) , antiderivative size = 1307, normalized size of antiderivative = 5.59

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/48*(3*((3*a^4 - 24*a^3*b + 8*a^2*b^2)*cos(f*x + e)^8 - 2*(3*a^4 - 27*a^
3*b + 32*a^2*b^2 - 8*a*b^3)*cos(f*x + e)^6 + (3*a^4 - 36*a^3*b + 107*a^2*b
^2 - 56*a*b^3 + 8*b^4)*cos(f*x + e)^4 + 3*a^2*b^2 - 24*a*b^3 + 8*b^4 + 2*(
3*a^3*b - 27*a^2*b^2 + 32*a*b^3 - 8*b^4)*cos(f*x + e)^2)*sqrt(a + b)*log(2
*(a*cos(f*x + e)^2 - 2*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)*cos(f*x + e) + a + 2*b)/(cos(f*x + e)^2 - 1)) + 2*(3*(3*a^4 - 21*a^3*
b - 16*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^7 - (15*a^4 - 117*a^3*b + 4*a^2*b^2
 + 104*a*b^3 - 32*b^4)*cos(f*x + e)^5 - (78*a^3*b - 71*a^2*b^2 - 61*a*b^3
+ 88*b^4)*cos(f*x + e)^3 - 5*(11*a^2*b^2 + a*b^3 - 10*b^4)*cos(f*x + e))*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^7 + 5*a^6*b + 10*a^5*b^2 +
 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^8 - 2*(a^7 + 4*a^6*b + 5
*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*f*cos(f*x + e)^6 + (a^7 + a^6*b
- 9*a^5*b^2 - 25*a^4*b^3 - 25*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*f*cos(f*x
 + e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 - 5*a^2*b^5 - 4*a*b^6 - b^7)*f*
cos(f*x + e)^2 + (a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6
+ b^7)*f), 1/24*(3*((3*a^4 - 24*a^3*b + 8*a^2*b^2)*cos(f*x + e)^8 - 2*(3*a
^4 - 27*a^3*b + 32*a^2*b^2 - 8*a*b^3)*cos(f*x + e)^6 + (3*a^4 - 36*a^3*b +
 107*a^2*b^2 - 56*a*b^3 + 8*b^4)*cos(f*x + e)^4 + 3*a^2*b^2 - 24*a*b^3 + 8
*b^4 + 2*(3*a^3*b - 27*a^2*b^2 + 32*a*b^3 - 8*b^4)*cos(f*x + e)^2)*sqrt(-a
 - b)*arctan(sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*c...
```

3.124.6 Sympy [F]

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)**5/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral(csc(e + f*x)**5/(a + b*sec(e + f*x)**2)**(5/2), x)
```

3.124.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.124.8 Giac [F]

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\csc\left(fx+e\right)^5}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.124.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Hanged}$$

input
```
int(1/(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^(5/2)),x)
```

output
```
\text{Hanged}
```

3.125 $\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

3.125.1 Optimal result 1050
3.125.2 Mathematica [B] (warning: unable to verify) 1051
3.125.3 Rubi [A] (verified) 1051
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3.125.8 Giac [F] 1059
3.125.9 Mupad [F(-1)] 1059

3.125.1 Optimal result

Integrand size = 25, antiderivative size = 288

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{5(a+b)\left(a^2+14ab+21b^2\right)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{11/2}f}$$
$$-\frac{(a+b)(11a+21b)\cos(e+fx)\sin(e+fx)}{16a^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}}$$
$$+\frac{3(a+b)\cos^3(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}}+\frac{\cos^3(e+fx)\sin^3(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)^{3/2}}$$
$$-\frac{7b(a+b)(7a+15b)\tan(e+fx)}{48a^4f\left(a+b+b\tan^2(e+fx)\right)^{3/2}}-\frac{b\left(113a^2+420ab+315b^2\right)\tan(e+fx)}{48a^5f\sqrt{a+b+b\tan^2(e+fx)}}$$

output

```
5/16*(a+b)*(a^2+14*a*b+21*b^2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)
^2)^(1/2))/a^(11/2)/f-1/48*b*(113*a^2+420*a*b+315*b^2)*tan(f*x+e)/a^5/f/(a
+b+b*tan(f*x+e)^2)^(1/2)-1/16*(a+b)*(11*a+21*b)*cos(f*x+e)*sin(f*x+e)/a^3/
f/(a+b+b*tan(f*x+e)^2)^(3/2)+3/8*(a+b)*cos(f*x+e)^3*sin(f*x+e)/a^2/f/(a+b+
b*tan(f*x+e)^2)^(3/2)+1/6*cos(f*x+e)^3*sin(f*x+e)^3/a/f/(a+b+b*tan(f*x+e)^
2)^(3/2)-7/48*b*(a+b)*(7*a+15*b)*tan(f*x+e)/a^4/f/(a+b+b*tan(f*x+e)^2)^(3/
2)
```

3.125.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1705 vs. $2(288) = 576$.

Time = 16.24 (sec) , antiderivative size = 1705, normalized size of antiderivative = 5.92

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
-1/3072*(((a + 2*b + a*Cos[2*(e + f*x)])/(a + b))^(3/2)*(a + 2*b + a*Cos[2
*e + 2*f*x])^(5/2)*Sec[e + f*x]^5*(-60*Sqrt[a + b]*(3*a^3 + 17*a^2*b + 28*
a*b^2 + 14*b^3)*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*(a + 2*b + a*Co
s[2*(e + f*x)])^2 + Sqrt[a]*Sin[e + f*x]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(
a + b)]*(3*(239*a^5 + 1839*a^4*b + 5200*a^3*b^2 + 6960*a^2*b^3 + 4480*a*b^
4 + 1120*b^5) - 2*a*(459*a^4 + 3180*a^3*b + 7200*a^2*b^2 + 6720*a*b^3 + 22
40*b^4)*Sin[e + f*x]^2 + 672*a^2*b*(a + b)^2*Sin[e + f*x]^4 + 192*a^3*(a +
 b)^2*Sin[e + f*x]^6)))/(Sqrt[2]*a^(9/2)*f*(a + 2*b + a*Cos[2*(e + f*x)])^
(7/2)*(a + b*Sec[e + f*x]^2)^(5/2)) + (((a + 2*b + a*Cos[2*(e + f*x)])/(a
+ b))^(3/2)*(a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^5*(420*Sqrt[
a + b]*(a^4 + 9*a^3*b + 26*a^2*b^2 + 30*a*b^3 + 12*b^4)*ArcSin[(Sqrt[a]*Si
n[e + f*x])/Sqrt[a + b]]*(a + 2*b + a*Cos[2*(e + f*x)])^2 - Sqrt[a]*Sin[e
+ f*x]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)]*(3*(561*a^6 + 6161*a^5*b +
 25200*a^4*b^2 + 50960*a^3*b^3 + 54880*a^2*b^4 + 30240*a*b^5 + 6720*b^6) -
 2*a*(1151*a^5 + 11230*a^4*b + 39200*a^3*b^2 + 62720*a^2*b^3 + 47040*a*b^4
 + 13440*b^5)*Sin[e + f*x]^2 + 672*a^2*(a + b)^2*(a^2 + 3*a*b + 6*b^2)*Sin
[e + f*x]^4 - 576*a^3*(a - 2*b)*(a + b)^2*Sin[e + f*x]^6 + 512*a^4*(a + b)
^2*Sin[e + f*x]^8)))/(3072*Sqrt[2]*a^(11/2)*f*(a + 2*b + a*Cos[2*(e + f*x)
])^(7/2)*(a + b*Sec[e + f*x]^2)^(5/2)) - (5*(a + 2*b + a*Cos[2*e + 2*f*x])
^(5/2)*Csc[e + f*x]*Sec[e + f*x]^5*(Sin[e + f*x]^2/(a + b) + ((a + 2*b ...
```

3.125.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.09, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 4620, 372, 27, 440, 27, 402, 402, 27, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sin(e+fx)^6}{(a+b\sec(e+fx)^2)^{5/2}}\,dx$$

$$\downarrow 4620$$

$$\frac{\int \frac{\tan^6(e+fx)}{(\tan^2(e+fx)+1)^4(b\tan^2(e+fx)+a+b)^{5/2}}\,d\tan(e+fx)}{f}$$

$$\downarrow 372$$

$$\frac{\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)^{3/2}} - \frac{\int \frac{3\tan^2(e+fx)\left(-2(a+b)\tan^2(e+fx)+a+b\right)}{\left(\tan^2(e+fx)+1\right)^3\left(b\tan^2(e+fx)+a+b\right)^{5/2}}\,d\tan(e+fx)}{6a}}{f}$$

$$\downarrow 27$$

$$\frac{\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)^{3/2}} - \frac{\int \frac{\tan^2(e+fx)\left(-2(a+b)\tan^2(e+fx)+a+b\right)}{\left(\tan^2(e+fx)+1\right)^3\left(b\tan^2(e+fx)+a+b\right)^{5/2}}\,d\tan(e+fx)}{2a}}{f}$$

$$\downarrow 440$$

$$\frac{\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)^{3/2}} - \frac{\frac{(a+b)\int \frac{3(a+b)-2(4a+9b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^{5/2}}\,d\tan(e+fx)}{4a} - \frac{3(a+b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\left(a+b\tan^2(e+fx)+b\right)^{3/2}}}{2a}}{f}$$

$$\downarrow 27$$

$$\frac{\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)^{3/2}} - \frac{\frac{(a+b)\int \frac{3(a+b)-2(4a+9b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^{5/2}}\,d\tan(e+fx)}{4a} - \frac{3(a+b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\left(a+b\tan^2(e+fx)+\right.}}{2a}}{f}$$

$$\downarrow 402$$

$$\frac{\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)^{3/2}}-\frac{\frac{(a+b)\left(\frac{(11a+21b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-\frac{\int\frac{(a+b)(5a+21b)-4b(11a+21b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{5/2}}d\tan(e\cdots}{2a}\right)}{4a}}{2a}}{f}$$

$\downarrow$ 402

$$\frac{\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)^{3/2}}-\frac{\frac{(a+b)\left(\frac{(11a+21b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-\frac{\frac{\int\frac{(a+b)\left(15a^2+112ba+105b^2-14b(7a+15b)\tan^2(e+\cdots\right.}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}}{3a(a+b)}}{2a}\right)}{4a}}{2a}}{f}$$

$\downarrow$ 27

$$\frac{\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)^{3/2}}-\frac{\frac{(a+b)\left(\frac{(11a+21b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-\frac{\frac{\int\frac{15a^2+112ba+105b^2-14b(7a+15b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan\cdots}{3a}}{2a}\right)}{4a}}{2a}}{f}$$

$\downarrow$ 402

$$\frac{\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)^{3/2}}-\frac{\frac{(a+b)\left(\frac{(11a+21b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-\frac{\frac{\int\frac{15(a+b)\left(a^2+14ba+21b^2\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx\cdots}{a(a+b)}}{3a}\right)}{4a}}{f}$$

$\downarrow$ 27

3.125. $\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

$$\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)^{3/2}} - \frac{(a+b)\left(\frac{(11a+21b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{3/2}} - \frac{15\left(a^2+14ab+21b^2\right)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+}}}{a}\right)}{4a}}{f}$$

$$\downarrow 291$$

$$\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)^{3/2}} - \frac{(a+b)\left(\frac{(11a+21b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{3/2}} - \frac{15\left(a^2+14ab+21b^2\right)\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1} d\frac{}{\sqrt{b\,t}}}{a}\right)}{4a}}{f}$$

$$\downarrow 216$$

$$\frac{\tan^3(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)^{3/2}} - \frac{(a+b)\left(\frac{(11a+21b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{3/2}} - \frac{15\left(a^2+14ab+21b^2\right)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}}}{3a}\right)}{4a}}{f}$$

input
```
Int[Sin[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

3.125. $\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

output
```
(Tan[e + f*x]^3/(6*a*(1 + Tan[e + f*x]^2)^3*(a + b + b*Tan[e + f*x]^2)^(3/
2)) - ((-3*(a + b)*Tan[e + f*x])/(4*a*(1 + Tan[e + f*x]^2)^2*(a + b + b*Ta
n[e + f*x]^2)^(3/2)) + ((a + b)*(((11*a + 21*b)*Tan[e + f*x])/(2*a*(1 + Ta
n[e + f*x]^2)*(a + b + b*Tan[e + f*x]^2)^(3/2)) - ((-7*b*(7*a + 15*b)*Tan[
e + f*x])/(3*a*(a + b + b*Tan[e + f*x]^2)^(3/2)) + ((15*(a^2 + 14*a*b + 21
*b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/a^(3/
2) - (b*(113*a^2 + 420*a*b + 315*b^2)*Tan[e + f*x])/(a*(a + b)*Sqrt[a + b
+ b*Tan[e + f*x]^2]))/(3*a))/(2*a)))/(4*a))/(2*a))/f
```

3.125.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.125.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2210 vs. $2(260) = 520$.

Time = 10.46 (sec) , antiderivative size = 2211, normalized size of antiderivative = 7.68

method	result	size
default	Expression too large to display	2211

input
```
int(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/48/f/a^5/(-a)^(1/2)*(b+a*cos(f*x+e)^2)*(-315*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*
sin(f*x+e)*a)*a*b^3*cos(f*x+e)^2+162*(-a)^(1/2)*a^3*b*cos(f*x+e)^2*sin(f*x
+e)+574*(-a)^(1/2)*a^2*b^2*cos(f*x+e)^2*sin(f*x+e)+420*(-a)^(1/2)*a*b^3*co
s(f*x+e)^2*sin(f*x+e)-15*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*
(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(
1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^3*b*cos
(f*x+e)-225*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^2*b^2*cos(f*x+e)-525
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a*b^3*cos(f*x+e)-18*(-a)^(1/2)*a^
3*b*cos(f*x+e)^6*sin(f*x+e)+96*(-a)^(1/2)*a^3*b*cos(f*x+e)^4*sin(f*x+e)+63
*(-a)^(1/2)*a^2*b^2*cos(f*x+e)^4*sin(f*x+e)-225*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*
sin(f*x+e)*a)*a^3*b*cos(f*x+e)^3-525*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(...
```

3.125.5 Fricas [A] (verification not implemented)

Time = 24.11 (sec) , antiderivative size = 1003, normalized size of antiderivative = 3.48

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output

```
[-1/384*(15*(a^3*b^2 + 15*a^2*b^3 + 35*a*b^4 + 21*b^5 + (a^5 + 15*a^4*b +
35*a^3*b^2 + 21*a^2*b^3)*cos(f*x + e)^4 + 2*(a^4*b + 15*a^3*b^2 + 35*a^2*b
^3 + 21*a*b^4)*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(
a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x +
e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b +
7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a
^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^
3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(8*a^5*cos(f*x + e)^9 - 2*(13*a^5
+ 9*a^4*b)*cos(f*x + e)^7 + 3*(11*a^5 + 32*a^4*b + 21*a^3*b^2)*cos(f*x + e
)^5 + 2*(81*a^4*b + 287*a^3*b^2 + 210*a^2*b^3)*cos(f*x + e)^3 + (113*a^3*b
^2 + 420*a^2*b^3 + 315*a*b^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)*sin(f*x + e))/(a^8*f*cos(f*x + e)^4 + 2*a^7*b*f*cos(f*x + e)
^2 + a^6*b^2*f), -1/192*(15*(a^3*b^2 + 15*a^2*b^3 + 35*a*b^4 + 21*b^5 + (a
^5 + 15*a^4*b + 35*a^3*b^2 + 21*a^2*b^3)*cos(f*x + e)^4 + 2*(a^4*b + 15*a^
3*b^2 + 35*a^2*b^3 + 21*a*b^4)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*c
os(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x
 + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x
 + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) +
 4*(8*a^5*cos(f*x + e)^9 - 2*(13*a^5 + 9*a^4*b)*cos(f*x + e)^7 + 3*(11*...
```

3.125.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input

```
integrate(sin(f*x+e)**6/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output

```
Timed out
```

3.125.7 Maxima [F]

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sin(fx+e)^6}{(b\sec(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sin(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

3.125.8 Giac [F]

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sin(fx+e)^6}{(b\sec(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.125.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sin(e+fx)^6}{\left(a+\frac{b}{\cos(e+fx)^2}\right)^{5/2}}\,dx$$

input
```
int(sin(e + f*x)^6/(a + b/cos(e + f*x)^2)^(5/2),x)
```

output
```
int(sin(e + f*x)^6/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.126 $\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

3.126.1 Optimal result 1060
3.126.2 Mathematica [B] (warning: unable to verify) 1061
3.126.3 Rubi [A] (verified) 1062
3.126.4 Maple [B] (verified) 1066
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3.126.8 Giac [F] 1068
3.126.9 Mupad [F(-1)] 1069

3.126.1 Optimal result

Integrand size = 25, antiderivative size = 227

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{(3a^2+30ab+35b^2)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{9/2}f} - \frac{(5a+7b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(23a+35b)\tan(e+fx)}{24a^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{5b(11a+21b)\tan(e+fx)}{24a^4f\sqrt{a+b+b\tan^2(e+fx)}}$$

output

```
1/8*(3*a^2+30*a*b+35*b^2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(
1/2))/a^(9/2)/f-5/24*b*(11*a+21*b)*tan(f*x+e)/a^4/f/(a+b+b*tan(f*x+e)^2)^(
1/2)-1/8*(5*a+7*b)*cos(f*x+e)*sin(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)^(3/2)+
1/4*cos(f*x+e)^3*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)^(3/2)-1/24*b*(23*a+35
*b)*tan(f*x+e)/a^3/f/(a+b+b*tan(f*x+e)^2)^(3/2)
```

3.126.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1315 vs. $2(227) = 454$.

Time $= 12.11$ (sec) , antiderivative size $= 1315$, normalized size of antiderivative $= 5.79$

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx =$$

$$-\frac{\left(\frac{a+2b+a\cos(2(e+fx))}{a+b}\right)^{3/2}(a+2b+a\cos(2e+2fx))^{5/2}\sec^5(e+fx)\left(-60\sqrt{a+b}(3a^3+17a^2b+28ab^2+14\right.}{}$$

$$-\frac{(a+2b+a\cos(2e+2fx))^{5/2}\csc(e+fx)\sec^5(e+fx)\left(\frac{\sin^2(e+fx)}{a+b}+\frac{(a+2b+a\cos(2(e+fx)))\sin^2(e+fx)}{(a+b)^2}-\frac{12\sin^4(e\right.}{768\sqrt{2}f\,(a+b\sec^2(e+fx))}$$

$$+\frac{(a+2b+a\cos(2e+2fx))^{5/2}\csc(e+fx)\sec^5(e+fx)\left(\frac{\sin^2(e+fx)}{a+b}+\frac{(a+2b+a\cos(2(e+fx)))\sin^2(e+fx)}{(a+b)^2}-\frac{24\sin^4(e+\right.}{}$$

$$+\frac{(2a+3b+a\cos(2(e+fx)))(a+2b+a\cos(2e+2fx))^{5/2}\sec^4(e+fx)\tan(e+fx)}{256(a+b)^2f(a+2b+a\cos(2(e+fx)))^{3/2}\,(a+b\sec^2(e+fx))^{5/2}}$$

$$-\frac{(b+(3a+2b)\cos(2(e+fx)))(a+2b+a\cos(2e+2fx))^{5/2}\sec^4(e+fx)\tan(e+fx)}{384(a+b)^2f(a+2b+a\cos(2(e+fx)))^{3/2}\,(a+b\sec^2(e+fx))^{5/2}}$$

input
```
Integrate[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output

```
-1/768*(((a + 2*b + a*Cos[2*(e + f*x)])/(a + b))^(3/2)*(a + 2*b + a*Cos[2*
e + 2*f*x])^(5/2)*Sec[e + f*x]^5*(-60*Sqrt[a + b]*(3*a^3 + 17*a^2*b + 28*a
*b^2 + 14*b^3)*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*(a + 2*b + a*Cos
[2*(e + f*x)])^2 + Sqrt[a]*Sin[e + f*x]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(a
 + b)]*(3*(239*a^5 + 1839*a^4*b + 5200*a^3*b^2 + 6960*a^2*b^3 + 4480*a*b^4
 + 1120*b^5) - 2*a*(459*a^4 + 3180*a^3*b + 7200*a^2*b^2 + 6720*a*b^3 + 224
0*b^4)*Sin[e + f*x]^2 + 672*a^2*b*(a + b)^2*Sin[e + f*x]^4 + 192*a^3*(a +
b)^2*Sin[e + f*x]^6)))/(Sqrt[2]*a^(9/2)*f*(a + 2*b + a*Cos[2*(e + f*x)])^(
7/2)*(a + b*Sec[e + f*x]^2)^(5/2)) - ((a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)
*Csc[e + f*x]*Sec[e + f*x]^5*(Sin[e + f*x]^2/(a + b) + ((a + 2*b + a*Cos[2
*(e + f*x)])*Sin[e + f*x]^2)/(a + b)^2 - (12*Sin[e + f*x]^4)/(a + b) + (16
*(a + b - a*Sin[e + f*x]^2)*(1 - (a*Sin[e + f*x]^2)/(a + b))*((-6*a*(a + b
)*Sin[e + f*x]^2)/(a + 2*b + a*Cos[2*(e + f*x)]) + (a^2*(a + b)*Sin[e + f*
x]^4)/(a + b - a*Sin[e + f*x]^2)^2 + (3*Sqrt[a]*Sqrt[a + b]*ArcSin[(Sqrt[a
]*Sin[e + f*x])/Sqrt[a + b]]*Sin[e + f*x])/Sqrt[(a + b - a*Sin[e + f*x]^2)
/(a + b)]))/a^3))/(768*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^(5/2)*(a + b - a*S
in[e + f*x]^2)^(3/2)) + ((a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Csc[e + f*x]
*Sec[e + f*x]^5*(Sin[e + f*x]^2/(a + b) + ((a + 2*b + a*Cos[2*(e + f*x)])*
Sin[e + f*x]^2)/(a + b)^2 - (24*Sin[e + f*x]^4)/(a + b) + (96*Sin[e + f*x]
^6)/a + (80*(a + b - a*Sin[e + f*x]^2)*(1 - (a*Sin[e + f*x]^2)/(a + b))...
```

3.126.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4620, 372, 402, 402, 27, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sin(e+fx)^4}{\left(a+b\sec(e+fx)^2\right)^{5/2}}\,dx$$

$$\downarrow 4620$$

$$\frac{\int \frac{\tan^4(e+fx)}{\left(\tan^2(e+fx)+1\right)^3\left(b\tan^2(e+fx)+a+b\right)^{5/2}}\,d\tan(e+fx)}{f}$$

$$\downarrow 372$$

$$\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)^{3/2}} - \frac{\int \frac{-2(2a+3b)\tan^2(e+fx)+a+b}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^{5/2}} d\tan(e+fx)}{4a}$$
$$f$$

$$\downarrow 402$$

$$\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)^{3/2}} - \frac{\frac{(5a+7b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{3/2}} - \frac{\int \frac{(a+b)(3a+7b)-4b(5a+7b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{5/2}} d\tan(e+fx)}{2a}}{4a}$$
$$f$$

$$\downarrow 402$$

$$\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)^{3/2}} - \frac{\frac{(5a+7b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{3/2}} - \frac{\int \frac{(a+b)\left((a+b)(9a+35b)-2b(23a+35b)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}} d\tan(e+fx)}{3a(a+b)}}{4a}$$
$$f$$

$$\downarrow 27$$

$$\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)^{3/2}} - \frac{\frac{(5a+7b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{3/2}} - \frac{\int \frac{(a+b)(9a+35b)-2b(23a+35b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}} d\tan(e+fx)}{3a}}{4a}$$
$$f$$

$$\downarrow 402$$

$$\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)^{3/2}} - \frac{\frac{(5a+7b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{3/2}} - \frac{\int \frac{3(a+b)\left(3a^2+30ba+35b^2\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}} d\tan(e+fx)}{a(a+b)}}{4a}$$
$$f$$

$$\downarrow 27$$

$$\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)^{3/2}} - \frac{\frac{(5a+7b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{3/2}} - \frac{\frac{3\left(3a^2+30ab+35b^2\right)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+}}}{a}}{3a}}{4a}}{f}$$

$$\downarrow 291$$

$$\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)^{3/2}} - \frac{\frac{(5a+7b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{3/2}} - \frac{\frac{3\left(3a^2+30ab+35b^2\right)\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1} d\frac{\tan(e+}{\sqrt{b\tan^2(e+}}}{a}}{3a}}{4a}}{f}$$

$$\downarrow 216$$

$$\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)^{3/2}} - \frac{\frac{(5a+7b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{3/2}} - \frac{\frac{3\left(3a^2+30ab+35b^2\right)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}} - \frac{5b(1}{a\sqrt{}}}{3a}}{2a}}{4a}}{f}$$

input
```
Int[Sin[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(Tan[e + f*x]/(4*a*(1 + Tan[e + f*x]^2)^2*(a + b + b*Tan[e + f*x]^2)^(3/2)
) - (((5*a + 7*b)*Tan[e + f*x])/(2*a*(1 + Tan[e + f*x]^2)*(a + b + b*Tan[e
 + f*x]^2)^(3/2)) - (-1/3*(b*(23*a + 35*b)*Tan[e + f*x])/(a*(a + b + b*Tan
[e + f*x]^2)^(3/2)) + ((3*(3*a^2 + 30*a*b + 35*b^2)*ArcTan[(Sqrt[a]*Tan[e
+ f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/a^(3/2) - (5*b*(11*a + 21*b)*Tan[
e + f*x])/(a*Sqrt[a + b + b*Tan[e + f*x]^2]))/(3*a))/(2*a))/(4*a))/f
```

3.126. $\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

3.126.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.126.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1610 vs. 2(203) = 406.

Time = 8.52 (sec) , antiderivative size = 1611, normalized size of antiderivative = 7.10

method	result	size
default	Expression too large to display	1611

input
```
int(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/24/f/a^4/(-a)^(1/2)*(b+a*cos(f*x+e)^2)*(6*(-a)^(1/2)*a^3*cos(f*x+e)^6*si
n(f*x+e)-15*(-a)^(1/2)*a^3*cos(f*x+e)^4*sin(f*x+e)-21*(-a)^(1/2)*a^2*b*cos
(f*x+e)^4*sin(f*x+e)+9*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+
e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a
)*a^3+90*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^2*b*cos(f*x+e)^3+105*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a*b^2*cos(f*x+e)^3+9*cos(f*x+e)^2*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^3+90*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin
(f*x+e)*a)*a^2*b*cos(f*x+e)^2+105*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)
+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*
a*b^2*cos(f*x+e)^2-78*(-a)^(1/2)*a^2*b*cos(f*x+e)^2*sin(f*x+e)-140*(-a)...
```

3.126.5 Fricas [A] (verification not implemented)

Time = 7.53 (sec) , antiderivative size = 873, normalized size of antiderivative = 3.85

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \left[-\frac{3\left((3\,a^4+30\,a^3b+35\,a^2b^2)\cos(fx+e)^4+3\,a^2b^2+30\,ab^3+35\,b^4+2\,(3\right.}{}\right.$$

$$\left. -\frac{3\left((3\,a^4+30\,a^3b+35\,a^2b^2)\cos(fx+e)^4+3\,a^2b^2+30\,ab^3+35\,b^4+2\,(3\,a^3b+30\,a^2b^2+35\,ab^3)\cos(fx+\right.}{}\right]$$

input
```
integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[-1/192*(3*((3*a^4 + 30*a^3*b + 35*a^2*b^2)*cos(f*x + e)^4 + 3*a^2*b^2 + 3
0*a*b^3 + 35*b^4 + 2*(3*a^3*b + 30*a^2*b^2 + 35*a*b^3)*cos(f*x + e)^2)*sqr
t(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(
5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2
 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2
+ 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 -
14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f
*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)
) - 8*(6*a^4*cos(f*x + e)^7 - 3*(5*a^4 + 7*a^3*b)*cos(f*x + e)^5 - 2*(39*a
^3*b + 70*a^2*b^2)*cos(f*x + e)^3 - 5*(11*a^2*b^2 + 21*a*b^3)*cos(f*x + e)
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^7*f*cos(f*x
 + e)^4 + 2*a^6*b*f*cos(f*x + e)^2 + a^5*b^2*f), -1/96*(3*((3*a^4 + 30*a^3
*b + 35*a^2*b^2)*cos(f*x + e)^4 + 3*a^2*b^2 + 30*a*b^3 + 35*b^4 + 2*(3*a^3
*b + 30*a^2*b^2 + 35*a*b^3)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(
f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x +
e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x +
e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*
(6*a^4*cos(f*x + e)^7 - 3*(5*a^4 + 7*a^3*b)*cos(f*x + e)^5 - 2*(39*a^3*b +
 70*a^2*b^2)*cos(f*x + e)^3 - 5*(11*a^2*b^2 + 21*a*b^3)*cos(f*x + e))*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^7*f*cos(f*x + ...
```

3.126.6 Sympy [F]

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)**4/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral(sin(e + f*x)**4/(a + b*sec(e + f*x)**2)**(5/2), x)
```

3.126.7 Maxima [F]

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sin(fx+e)^4}{(b\sec(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sin(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

3.126.8 Giac [F]

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sin(fx+e)^4}{(b\sec(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.126.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sin(e+fx)^4}{\left(a+\frac{b}{\cos(e+fx)^2}\right)^{5/2}}\,dx$$

input
```
int(sin(e + f*x)^4/(a + b/cos(e + f*x)^2)^(5/2),x)
```

output
```
int(sin(e + f*x)^4/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.127 $\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

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3.127.1 Optimal result

Integrand size = 25, antiderivative size = 167

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{(a+5b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2a^{7/2}f} - \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{5b\tan(e+fx)}{6a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(13a+15b)\tan(e+fx)}{6a^3(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

output

```
1/2*(a+5*b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(7/2)/
f-1/6*b*(13*a+15*b)*tan(f*x+e)/a^3/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/2*
cos(f*x+e)*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)^(3/2)-5/6*b*tan(f*x+e)/a^2/
f/(a+b+b*tan(f*x+e)^2)^(3/2)
```

3.127.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 983 vs. 2(167) = 334.

Time = 8.40 (sec) , antiderivative size = 983, normalized size of antiderivative = 5.89

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx =$$

$$-\frac{(a+2b+a\cos(2e+2fx))^{5/2}\csc(e+fx)\sec^5(e+fx)\left(\frac{\sin^2(e+fx)}{a+b}+\frac{(a+2b+a\cos(2(e+fx)))\sin^2(e+fx)}{(a+b)^2}-\frac{12\sin^4(e+}{a+b}\right.}{256\sqrt{2}f\,(a+b\sec^2(e+fx))^{5}}$$

$$-\frac{(a+2b+a\cos(2e+2fx))^{5/2}\csc(e+fx)\sec^5(e+fx)\left(\frac{\sin^2(e+fx)}{a+b}+\frac{(a+2b+a\cos(2(e+fx)))\sin^2(e+fx)}{(a+b)^2}-\frac{24\sin^4(e+}{a+b}\right.}{}$$

$$+\frac{5(2a+3b+a\cos(2(e+fx)))(a+2b+a\cos(2e+2fx))^{5/2}\sec^4(e+fx)\tan(e+fx)}{384(a+b)^2f(a+2b+a\cos(2(e+fx)))^{3/2}\,(a+b\sec^2(e+fx))^{5/2}}$$

$$-\frac{(b+(3a+2b)\cos(2(e+fx)))(a+2b+a\cos(2e+2fx))^{5/2}\sec^4(e+fx)\tan(e+fx)}{384(a+b)^2f(a+2b+a\cos(2(e+fx)))^{3/2}\,(a+b\sec^2(e+fx))^{5/2}}$$

input
```
Integrate[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output

```
-1/256*((a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Csc[e + f*x]*Sec[e + f*x]^5*(
Sin[e + f*x]^2/(a + b) + ((a + 2*b + a*Cos[2*(e + f*x)])*Sin[e + f*x]^2)/(
a + b)^2 - (12*Sin[e + f*x]^4)/(a + b) + (16*(a + b - a*Sin[e + f*x]^2)*(1
 - (a*Sin[e + f*x]^2)/(a + b))*((-6*a*(a + b)*Sin[e + f*x]^2)/(a + 2*b + a
*Cos[2*(e + f*x)]) + (a^2*(a + b)*Sin[e + f*x]^4)/(a + b - a*Sin[e + f*x]^
2)^2 + (3*Sqrt[a]*Sqrt[a + b]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*S
in[e + f*x])/Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)]))/a^3))/(Sqrt[2]*f*(
a + b*Sec[e + f*x]^2)^(5/2)*(a + b - a*Sin[e + f*x]^2)^(3/2)) - ((a + 2*b
+ a*Cos[2*e + 2*f*x])^(5/2)*Csc[e + f*x]*Sec[e + f*x]^5*(Sin[e + f*x]^2/(a
 + b) + ((a + 2*b + a*Cos[2*(e + f*x)])*Sin[e + f*x]^2)/(a + b)^2 - (24*Si
n[e + f*x]^4)/(a + b) + (96*Sin[e + f*x]^6)/a + (80*(a + b - a*Sin[e + f*x
]^2)*(1 - (a*Sin[e + f*x]^2)/(a + b))*((-6*a*(a + b)*Sin[e + f*x]^2)/(a +
2*b + a*Cos[2*(e + f*x)]) + (a^2*(a + b)*Sin[e + f*x]^4)/(a + b - a*Sin[e
+ f*x]^2)^2 + (3*Sqrt[a]*Sqrt[a + b]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a
+ b]]*Sin[e + f*x])/Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)]))/a^3 - (160*
(a + b - a*Sin[e + f*x]^2)*(1 - (a*Sin[e + f*x]^2)/(a + b))*((-6*a*(a + b)
^2*Sin[e + f*x]^2)/(a + 2*b + a*Cos[2*(e + f*x)]) + (3*Sqrt[a]*(a + b)^(3/
2)*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*Sin[e + f*x])/Sqrt[(a + b -
a*Sin[e + f*x]^2)/(a + b)] + (a^2*Sin[e + f*x]^4)/(-1 + (a*Sin[e + f*x]^2)
/(a + b))^2))/a^4))/(768*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^(5/2)*(a + b ...
```

3.127.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4620, 373, 402, 27, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sin(e+fx)^2}{(a+b\sec(e+fx)^2)^{5/2}}\,dx$$

$$\downarrow 4620$$

$$\frac{\int \frac{\tan^2(e+fx)}{(\tan^2(e+fx)+1)^2(b\tan^2(e+fx)+a+b)^{5/2}}\,d\tan(e+fx)}{f}$$

$$\downarrow 373$$

$$\frac{\int \frac{-4b\tan^2(e+fx)+a+b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{5/2}}\,d\tan(e+fx)}{2a} - \frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\downarrow 402$$

$$\frac{\int \frac{(a+b)\left(-10b\tan^2(e+fx)+3a+5b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}\,d\tan(e+fx)}{3a(a+b)} - \frac{5b\tan(e+fx)}{3a\left(a+b\tan^2(e+fx)+b\right)^{3/2}}}{2a} - \frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\downarrow 27$$

$$\frac{\int \frac{-10b\tan^2(e+fx)+3a+5b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}\,d\tan(e+fx)}{3a} - \frac{5b\tan(e+fx)}{3a\left(a+b\tan^2(e+fx)+b\right)^{3/2}}}{2a} - \frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\downarrow 402$$

$$\frac{\int \frac{3(a+b)(a+5b)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}\,d\tan(e+fx)}{a(a+b)} - \frac{b(13a+15b)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a} - \frac{5b\tan(e+fx)}{3a\left(a+b\tan^2(e+fx)+b\right)^{3/2}}}{2a} - \frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+}}{f}$$

$$\downarrow 27$$

$$\frac{3(a+5b)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}\,d\tan(e+fx)}{a} - \frac{b(13a+15b)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a} - \frac{5b\tan(e+fx)}{3a\left(a+b\tan^2(e+fx)+b\right)^{3/2}}}{2a} - \frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+}}{f}$$

$$\downarrow 291$$

$$\frac{3(a+5b)\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}\,d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{a} - \frac{b(13a+15b)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a} - \frac{5b\tan(e+fx)}{3a\left(a+b\tan^2(e+fx)+b\right)^{3/2}}}{2a} - \frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+f}}{f}$$

3.127. $\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

$$\downarrow 216$$

$$\frac{\frac{3(a+5b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}}-\frac{b(13a+15b)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a}-\frac{5b\tan(e+fx)}{3a\left(a+b\tan^2(e+fx)+b\right)^{3/2}}}{2a}-\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)^{3/2}}$$
$$f$$

input
```
Int[Sin[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(-1/2*Tan[e + f*x]/(a*(1 + Tan[e + f*x]^2)*(a + b + b*Tan[e + f*x]^2)^(3/2
)) + ((-5*b*Tan[e + f*x])/(3*a*(a + b + b*Tan[e + f*x]^2)^(3/2)) + ((3*(a
+ 5*b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/a^(3
/2) - (b*(13*a + 15*b)*Tan[e + f*x])/(a*(a + b)*Sqrt[a + b + b*Tan[e + f*x
]^2]))/(3*a))/(2*a))/f
```

3.127.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.127.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1591 vs. $2(147) = 294$.

Time = 7.01 (sec) , antiderivative size = 1592, normalized size of antiderivative = 9.53

method	result	size
default	Expression too large to display	1592

input
```
int(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
-1/6/f/(a+b)/a^3/(-a)^(1/2)*(b+a*cos(f*x+e)^2)*(3*(-a)^(1/2)*a^3*cos(f*x+e
)^4*sin(f*x+e)+3*(-a)^(1/2)*a^2*b*cos(f*x+e)^4*sin(f*x+e)-3*cos(f*x+e)^3*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^3-18*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*si
n(f*x+e)*a)*a^2*b*cos(f*x+e)^3-15*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)
+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*
a*b^2*cos(f*x+e)^3+18*(-a)^(1/2)*a^2*b*cos(f*x+e)^2*sin(f*x+e)+20*(-a)^(1/
2)*a*b^2*cos(f*x+e)^2*sin(f*x+e)-3*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*
sin(f*x+e)*a)*a^3-18*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)
^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^2*b*cos(f*x
+e)^2-15*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a*b^2*cos(f*x+e)^2-3*l...
```

3.127.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(147) = 294$.

Time $= 2.09$ (sec) , antiderivative size $= 879$, normalized size of antiderivative $= 5.26$

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \left[-\frac{3\left(\left(a^4+6\,a^3b+5\,a^2b^2\right)\cos\left(fx+e\right)^4+a^2b^2+6\,ab^3+5\,b^4+2\left(a^3b+6\,a\right.\right.}{} \right.$$

$$\left. 3\left(\left(a^4+6\,a^3b+5\,a^2b^2\right)\cos\left(fx+e\right)^4+a^2b^2+6\,ab^3+5\,b^4+2\left(a^3b+6\,a^2b^2+5\,ab^3\right)\cos\left(fx+e\right)^2\right)\sqrt{a}\,\mathrm{arct}\right.$$

$$\left. -\frac{}{} \right]$$

input

```
integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[-1/48*(3*((a^4 + 6*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^2*b^2 + 6*a*b^3
+ 5*b^4 + 2*(a^3*b + 6*a^2*b^2 + 5*a*b^3)*cos(f*x + e)^2)*sqrt(-a)*log(128
*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^
3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 +
 b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*c
os(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*
a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt
(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(3*(a^4
 + a^3*b)*cos(f*x + e)^5 + 2*(9*a^3*b + 10*a^2*b^2)*cos(f*x + e)^3 + (13*a
^2*b^2 + 15*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^
2)*sin(f*x + e))/((a^7 + a^6*b)*f*cos(f*x + e)^4 + 2*(a^6*b + a^5*b^2)*f*c
os(f*x + e)^2 + (a^5*b^2 + a^4*b^3)*f), -1/24*(3*((a^4 + 6*a^3*b + 5*a^2*b
^2)*cos(f*x + e)^4 + a^2*b^2 + 6*a*b^3 + 5*b^4 + 2*(a^3*b + 6*a^2*b^2 + 5*
a*b^3)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 -
 a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2
- (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) + 4*(3*(a^4 + a^3*b)*cos(
f*x + e)^5 + 2*(9*a^3*b + 10*a^2*b^2)*cos(f*x + e)^3 + (13*a^2*b^2 + 15*a*
b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e
))/((a^7 + a^6*b)*f*cos(f*x + e)^4 + 2*(a^6*b + a^5*b^2)*f*cos(f*x + e)...
```

3.127.6 Sympy [F]

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)**2/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral(sin(e + f*x)**2/(a + b*sec(e + f*x)**2)**(5/2), x)
```

3.127.7 Maxima [F]

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sin\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sin(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

3.127.8 Giac [F]

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sin\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.127.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sin\left(e+f\,x\right)^2}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

input
```
int(sin(e + f*x)^2/(a + b/cos(e + f*x)^2)^(5/2),x)
```

output
```
int(sin(e + f*x)^2/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.128 $\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

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3.128.1 Optimal result

Integrand size = 16, antiderivative size = 125

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} - \frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(5a+3b)\tan(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

output
```
arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/f-1/3*b*(5*a
+3*b)*tan(f*x+e)/a^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/3*b*tan(f*x+e)
/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(3/2)
```

3.128.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 15.75 (sec) , antiderivative size = 1927, normalized size of antiderivative = 15.42

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(-5/2),x]
```

output

```
(3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/
(a + b)]*Cos[e + f*x]^4*Sin[e + f*x])/(4*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^
(5/2)*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2, 5/2, 3
/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -2, 7
/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[
3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*
x]^2)*((15*a*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e
+ f*x]^2)/(a + b)]*Cos[e + f*x]^5*Sin[e + f*x]^2)/(4*Sqrt[2]*(a + b - a*Si
n[e + f*x]^2)^(7/2)*(3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2,
 (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*
x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2,
Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)
*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*C
os[e + f*x]^5)/(4*Sqrt[2]*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*Appe
llF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a
*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] -
 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/
(a + b)])*Sin[e + f*x]^2)) - (3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e
+ f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^3*Sin[e + f*x]^2)/(Sqrt
[2]*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2, 5/2, ...
```

3.128.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {3042, 4616, 316, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{(a+b\sec(e+fx)^2)^{5/2}}dx$$

$$\downarrow 4616$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^{5/2}}d\tan(e+fx)}{f}$$

$$\downarrow 316$$

$$\frac{\int \frac{-2b\tan^2(e+fx)+3a+b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}\,d\tan(e+fx)}{3a(a+b)} - \frac{b\tan(e+fx)}{3a(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}$$

$$\Big\downarrow f$$

$$\Big\downarrow 402$$

$$\frac{\int \frac{3(a+b)^2}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}\,d\tan(e+fx)}{a(a+b)} - \frac{b(5a+3b)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}} - \frac{b\tan(e+fx)}{3a(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}$$

$$\Big\downarrow f$$

$$\Big\downarrow 27$$

$$\frac{3(a+b)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}\,d\tan(e+fx)}{a} - \frac{b(5a+3b)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}} - \frac{b\tan(e+fx)}{3a(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}$$

$$\Big\downarrow f$$

$$\Big\downarrow 291$$

$$\frac{3(a+b)\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}\,d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{a} - \frac{b(5a+3b)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}} - \frac{b\tan(e+fx)}{3a(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}$$

$$\Big\downarrow f$$

$$\Big\downarrow 216$$

$$\frac{3(a+b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}} - \frac{b(5a+3b)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}} - \frac{b\tan(e+fx)}{3a(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}$$

$$\Big\downarrow f$$

input

```
Int[(a + b*Sec[e + f*x]^2)^(-5/2),x]
```

output

```
(-1/3*(b*Tan[e + f*x])/(a*(a + b)*(a + b + b*Tan[e + f*x]^2)^(3/2)) + ((3*
(a + b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/a^(
3/2) - (b*(5*a + 3*b)*Tan[e + f*x])/(a*(a + b)*Sqrt[a + b + b*Tan[e + f*x]
^2]))/(3*a*(a + b)))/f
```

3.128.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

3.128.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1262 vs. 2(111) = 222.

Time = 5.54 (sec) , antiderivative size = 1263, normalized size of antiderivative = 10.10

method	result	size
default	Expression too large to display	1263

input
```
int(1/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/f/(a+b)^2/a^2/(-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*
x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^
2*csc(f*x+e)^2+a+b)*(-12*(-a)^(1/2)*a^2*b*(1-cos(f*x+e))^5*csc(f*x+e)^5-18
*(-a)^(1/2)*a*b^2*(1-cos(f*x+e))^5*csc(f*x+e)^5-6*(-a)^(1/2)*b^3*(1-cos(f*
x+e))^5*csc(f*x+e)^5+24*(1-cos(f*x+e))^3*a^2*(-a)^(1/2)*b*csc(f*x+e)^3-4*(
-a)^(1/2)*a*b^2*(1-cos(f*x+e))^3*csc(f*x+e)^3-12*(-a)^(1/2)*b^3*(1-cos(f*x
+e))^3*csc(f*x+e)^3+3*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*
csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*
x+e)^2+a+b)^(3/2)*ln(4*((-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-c
os(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x
+e))^2*csc(f*x+e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(f*x+e)))/((1-cos(f*x+e)
)^2*csc(f*x+e)^2+1))*a^2+6*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e
))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*c
sc(f*x+e)^2+a+b)^(3/2)*ln(4*((-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b
*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-co
s(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(f*x+e)))/((1-cos(f
*x+e))^2*csc(f*x+e)^2+1))*a*b+3*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(
f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e)
)^2*csc(f*x+e)^2+a+b)^(3/2)*ln(4*((-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e
)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2...
```

3.128.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(111) = 222$.

Time $= 0.71$ (sec) , antiderivative size $= 881$, normalized size of antiderivative $= 7.05$

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \left[-\frac{3\left(\left(a^4+2\,a^3b+a^2b^2\right)\cos\left(fx+e\right)^4+a^2b^2+2\,ab^3+b^4+2\left(a^3b+2\,a^2b^2\right.\right.}{}\right.$$

$$\left. -\frac{3\left(\left(a^4+2\,a^3b+a^2b^2\right)\cos\left(fx+e\right)^4+a^2b^2+2\,ab^3+b^4+2\left(a^3b+2\,a^2b^2+ab^3\right)\cos\left(fx+e\right)^2\right)\sqrt{a}\arctan\left(\right.}{12\left(\left(a^7+2\,a^6b+a^5b^2\right).\right.}\right.$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[-1/24*(3*((a^4 + 2*a^3*b + a^2*b^2)*cos(f*x + e)^4 + a^2*b^2 + 2*a*b^3 +
b^4 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*c
os(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b +
5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 -
 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x
 + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)
*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(2*(3*a^3*b +
 2*a^2*b^2)*cos(f*x + e)^3 + (5*a^2*b^2 + 3*a*b^3)*cos(f*x + e))*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2
)*f*cos(f*x + e)^4 + 2*(a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a
^5*b^2 + 2*a^4*b^3 + a^3*b^4)*f), -1/12*(3*((a^4 + 2*a^3*b + a^2*b^2)*cos(
f*x + e)^4 + a^2*b^2 + 2*a*b^3 + b^4 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*cos(f
*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*
x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a
^2*b)*cos(f*x + e)^2)*sin(f*x + e))) + 4*(2*(3*a^3*b + 2*a^2*b^2)*cos(f*x
+ e)^3 + (5*a^2*b^2 + 3*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*x + e)^4 +
 2*(a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*a^4*b^...
```

3.128.6 Sympy [F]

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{1}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(-5/2), x)
```

3.128.7 Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.128.8 Giac [F]

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{1}{(b\sec(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(-5/2), x)
```

3.128.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(e+fx)^2}\right)^{5/2}}\,dx$$

input
```
int(1/(a + b/cos(e + f*x)^2)^(5/2),x)
```

output
```
int(1/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.129 $\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

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3.129.1 Optimal result

Integrand size = 25, antiderivative size = 106

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{\cot(e+fx)}{(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{4b\tan(e+fx)}{3(a+b)^2 f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{8b\tan(e+fx)}{3(a+b)^3 f\sqrt{a+b+b\tan^2(e+fx)}}$$

output
```
-8/3*b*tan(f*x+e)/(a+b)^3/f/(a+b+b*tan(f*x+e)^2)^(1/2)-cot(f*x+e)/(a+b)/f/
(a+b+b*tan(f*x+e)^2)^(3/2)-4/3*b*tan(f*x+e)/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)
^(3/2)
```

3.129.2 Mathematica [A] (verified)

Time = 1.56 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.02

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))\left(3a^2-6ab-b^2-6(a^2-b^2)\csc^2(e+fx)+3(a+b)^2\csc^4(e+fx)\right)\sec^2(e+f}{6(a+b)^3 f\left(a+b\sec^2(e+fx)\right)^{5/2}}$$

input
```
Integrate[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
-1/6*((a + 2*b + a*Cos[2*(e + f*x)])*(3*a^2 - 6*a*b - b^2 - 6*(a^2 - b^2)*
Csc[e + f*x]^2 + 3*(a + b)^2*Csc[e + f*x]^4)*Sec[e + f*x]^2*Tan[e + f*x]^3
)/((a + b)^3*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

3.129.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4620, 245, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sin(e+fx)^2\left(a+b\sec(e+fx)^2\right)^{5/2}}dx$$

$$\downarrow 4620$$

$$\frac{\int \frac{\cot^2(e+fx)}{(b\tan^2(e+fx)+a+b)^{5/2}}d\tan(e+fx)}{f}$$

$$\downarrow 245$$

$$\frac{-\frac{4b\int \frac{1}{\left(b\tan^2(e+fx)+a+b\right)^{5/2}}d\tan(e+fx)}{a+b}-\frac{\cot(e+fx)}{(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\downarrow 209$$

$$\frac{-\frac{4b\left(\frac{2\int \frac{1}{\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{3(a+b)}+\frac{\tan(e+fx)}{3(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}\right)}{a+b}-\frac{\cot(e+fx)}{(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\downarrow 208$$

$$\frac{-\frac{4b\left(\frac{2\tan(e+fx)}{3(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}+\frac{\tan(e+fx)}{3(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}\right)}{a+b}-\frac{\cot(e+fx)}{(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

input
```
Int[Csc[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(-(Cot[e + f*x]/((a + b)*(a + b + b*Tan[e + f*x]^2)^(3/2))) - (4*b*(Tan[e
+ f*x]/(3*(a + b)*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (2*Tan[e + f*x])/(3*
(a + b)^2*Sqrt[a + b + b*Tan[e + f*x]^2])))/(a + b))/f
```

3.129.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.129.4 Maple [A] (verified)

Time = 5.67 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.28

method	result
default	$-\frac{\left(b+a\cos(fx+e)^2\right)\left(3\cos(fx+e)^4a^2-6\cos(fx+e)^4ab-\cos(fx+e)^4b^2+12\cos(fx+e)^2ab-4\cos(fx+e)^2b^2+8b^2\right)\sec(fx+e)^5\csc(f}{3f(a^3+3a^2b+3ab^2+b^3)\left(a+b\sec(fx+e)^2\right)^{\frac{5}{2}}}$

input
```
int(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/f/(a^3+3*a^2*b+3*a*b^2+b^3)*(b+a*cos(f*x+e)^2)*(3*cos(f*x+e)^4*a^2-6*
cos(f*x+e)^4*a*b-cos(f*x+e)^4*b^2+12*cos(f*x+e)^2*a*b-4*cos(f*x+e)^2*b^2+8
*b^2)/(a+b*sec(f*x+e)^2)^(5/2)*sec(f*x+e)^5*csc(f*x+e)
```

3.129.5 Fricas [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.81

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx =$$

$$-\frac{\left((3a^2-6ab-b^2)\cos(fx+e)^5+4(3ab-b^2)\cos(fx+e)^3+8b^2\cos(fx+e)\right)\sqrt{\frac{a}{}}}{3\left((a^5+3a^4b+3a^3b^2+a^2b^3)f\cos(fx+e)^4+2(a^4b+3a^3b^2+3a^2b^3+ab^4)f\cos(fx+e)^2+(a^3b^2+3}$$

input
```
integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*((3*a^2 - 6*a*b - b^2)*cos(f*x + e)^5 + 4*(3*a*b - b^2)*cos(f*x + e)^
3 + 8*b^2*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(((a^5
 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*f*cos(f*x + e)^4 + 2*(a^4*b + 3*a^3*b^2
+ 3*a^2*b^3 + a*b^4)*f*cos(f*x + e)^2 + (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b
^5)*f)*sin(f*x + e))
```

3.129.6 Sympy [F]

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)**2/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral(csc(e + f*x)**2/(a + b*sec(e + f*x)**2)**(5/2), x)
```

3.129.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.89

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{\frac{8\,b\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+b}(a+b)^3} + \frac{4\,b\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}(a+b)^2} + \frac{3}{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}(a+b)\tan(fx+e)}}{3\,f}$$

input
```
integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*(8*b*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^3) + 4*b*ta
n(f*x + e)/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)^2) + 3/((b*tan(f*x +
e)^2 + a + b)^(3/2)*(a + b)*tan(f*x + e)))/f
```

3.129.8 Giac [F]

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\csc\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.129.9 Mupad [B] (verification not implemented)

Time = 29.97 (sec) , antiderivative size = 336, normalized size of antiderivative = 3.17

$$\int \frac{\csc^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx =$$

$$-\frac{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\sqrt{a+\frac{b}{\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}\right)^2}}\left(-a\,b\,6\mathrm{i}+a^2\,3\mathrm{i}-b^2\,1\mathrm{i}+a^2\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,12\mathrm{i}+a^2\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,18\mathrm{i}+\right.}{3\,f\,(}$$

input
```
int(1/(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^(5/2)),x)
```

output
```
-((exp(e*2i + f*x*2i) + 1)*(a + b/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x
*1i)/2)^2)^(1/2)*(a^2*3i - a*b*6i - b^2*1i + a^2*exp(e*2i + f*x*2i)*12i +
a^2*exp(e*4i + f*x*4i)*18i + a^2*exp(e*6i + f*x*6i)*12i + a^2*exp(e*8i + f
*x*8i)*3i - b^2*exp(e*2i + f*x*2i)*20i + b^2*exp(e*4i + f*x*4i)*90i - b^2*
exp(e*6i + f*x*6i)*20i - b^2*exp(e*8i + f*x*8i)*1i + a*b*exp(e*2i + f*x*2i
)*24i + a*b*exp(e*4i + f*x*4i)*60i + a*b*exp(e*6i + f*x*6i)*24i - a*b*exp(
e*8i + f*x*8i)*6i))/(3*f*(a + b)^3*(exp(e*2i + f*x*2i) - 1)*(a + 2*a*exp(e
*2i + f*x*2i) + a*exp(e*4i + f*x*4i) + 4*b*exp(e*2i + f*x*2i))^2)
```

3.130 $\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

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3.130.1 Optimal result

Integrand size = 25, antiderivative size = 158

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{(a-b)\cot(e+fx)}{(a+b)^2 f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{\cot^3(e+fx)}{3(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{4(a-b)b\tan(e+fx)}{3(a+b)^3 f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{8(a-b)b\tan(e+fx)}{3(a+b)^4 f\sqrt{a+b+b\tan^2(e+fx)}}$$

output
```
-8/3*(a-b)*b*tan(f*x+e)/(a+b)^4/f/(a+b+b*tan(f*x+e)^2)^(1/2)-(a-b)*cot(f*x
+e)/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(3/2)-1/3*cot(f*x+e)^3/(a+b)/f/(a+b+b*t
an(f*x+e)^2)^(3/2)-4/3*(a-b)*b*tan(f*x+e)/(a+b)^3/f/(a+b+b*tan(f*x+e)^2)^(
3/2)
```

3.130.2 Mathematica [A] (verified)

Time = 4.51 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.87

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{(a+2b+a\cos(2(e+fx)))^3\left(\frac{4b^2(a+b)}{(a+2b+a\cos(2(e+fx)))^2} + \frac{4b(-3a+b)}{a+2b+a\cos(2(e+fx))} - 2(a\right.}{24(a+b)^4 f\,(a+b\sec$$

input
```
Integrate[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])^3*((4*b^2*(a + b))/(a + 2*b + a*Cos[2*(e +
 f*x)])^2 + (4*b*(-3*a + b))/(a + 2*b + a*Cos[2*(e + f*x)]) - 2*(a - 3*b)*
Csc[e + f*x]^2 - (a + b)*Csc[e + f*x]^4)*Sec[e + f*x]^4*Tan[e + f*x])/(24*
(a + b)^4*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

3.130.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 4620, 359, 245, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sin(e+fx)^4\,(a+b\sec(e+fx)^2)^{5/2}}\,dx$$

$$\downarrow 4620$$

$$\frac{\int \frac{\cot^4(e+fx)(\tan^2(e+fx)+1)}{(b\tan^2(e+fx)+a+b)^{5/2}}\,d\tan(e+fx)}{f}$$

$$\downarrow 359$$

$$\frac{\frac{(a-b)\int \frac{\cot^2(e+fx)}{\left(b\tan^2(e+fx)+a+b\right)^{5/2}}\,d\tan(e+fx)}{a+b} - \frac{\cot^3(e+fx)}{3(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\downarrow 245$$

$$\frac{(a-b)\left(-\frac{4b\int \frac{1}{\left(b\tan^2(e+fx)+a+b\right)^{5/2}}\,d\tan(e+fx)}{a+b} - \frac{\cot(e+fx)}{(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}\right)}{a+b} - \frac{\cot^3(e+fx)}{3(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\downarrow 209$$

$$\frac{(a-b)\left(-\frac{4b\left(\frac{2\int \frac{1}{\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{3(a+b)}+\frac{\tan(e+fx)}{3(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}\right)}{a+b}-\frac{\cot(e+fx)}{(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}\right)}{a+b}-\frac{\cot^3(e+fx)}{3(a+b)(a+b\tan^2(e+fx)}}{f}$$

$$\downarrow 208$$

$$\frac{(a-b)\left(-\frac{4b\left(\frac{2\tan(e+fx)}{3(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}+\frac{\tan(e+fx)}{3(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}\right)}{a+b}-\frac{\cot(e+fx)}{(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}\right)}{a+b}-\frac{\cot^3(e+fx)}{3(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

input
```
Int[Csc[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(-1/3*Cot[e + f*x]^3/((a + b)*(a + b + b*Tan[e + f*x]^2)^(3/2)) + ((a - b)
*(-(Cot[e + f*x]/((a + b)*(a + b + b*Tan[e + f*x]^2)^(3/2))) - (4*b*(Tan[e
 + f*x]/(3*(a + b)*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (2*Tan[e + f*x])/(3
*(a + b)^2*Sqrt[a + b + b*Tan[e + f*x]^2])))/(a + b)))/(a + b))/f
```

3.130.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.130.4 Maple [A] (verified)

Time = 7.22 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.42

method	result
default	$\frac{\left(b+a\cos(fx+e)^2\right)\left(2a^3\cos(fx+e)^6-12\cos(fx+e)^6a^2b+2\cos(fx+e)^6ab^2-3\cos(fx+e)^4a^3+21\cos(fx+e)^4a^2b-21\cos(fx+e)^4ab^2}{3f(a^4+4a^3b+6a^2b^2+4ab^3+b^4)\left(a+}$

input
```
int(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/f/(a^4+4*a^3*b+6*a^2*b^2+4*a*b^3+b^4)*(b+a*cos(f*x+e)^2)*(2*a^3*cos(f*
x+e)^6-12*cos(f*x+e)^6*a^2*b+2*cos(f*x+e)^6*a*b^2-3*cos(f*x+e)^4*a^3+21*co
s(f*x+e)^4*a^2*b-21*cos(f*x+e)^4*a*b^2+3*cos(f*x+e)^4*b^3-12*cos(f*x+e)^2*
a^2*b+24*cos(f*x+e)^2*a*b^2-12*cos(f*x+e)^2*b^3-8*a*b^2+8*b^3)/(a+b*sec(f*
x+e)^2)^(5/2)*sec(f*x+e)^5*csc(f*x+e)^3
```

3.130.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(144) = 288$.

Time = 2.35 (sec) , antiderivative size = 320, normalized size of antiderivative = 2.03

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx =$$

$$-\frac{\left(2\left(a^3-6\,a^2b+ab^2\right)\cos\left(fx+e\right)^7-3\left(a^3-7\,a^2b+7\,ab^2-b^3\right)\cos\right.}{3\left(\left(a^6+4\,a^5b+6\,a^4b^2+4\,a^3b^3+a^2b^4\right)f\cos\left(fx+e\right)^6-\left(a^6+2\,a^5b-2\,a^4b^2-8\,a^3b^3-7\,a^2b^4-2\,ab^5\right)f\,\mathrm{c}\right.}$$

input
```
integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*(2*(a^3 - 6*a^2*b + a*b^2)*cos(f*x + e)^7 - 3*(a^3 - 7*a^2*b + 7*a*b^
2 - b^3)*cos(f*x + e)^5 - 12*(a^2*b - 2*a*b^2 + b^3)*cos(f*x + e)^3 - 8*(a
*b^2 - b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(((a
^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*f*cos(f*x + e)^6 - (a^6 +
2*a^5*b - 2*a^4*b^2 - 8*a^3*b^3 - 7*a^2*b^4 - 2*a*b^5)*f*cos(f*x + e)^4 -
(2*a^5*b + 7*a^4*b^2 + 8*a^3*b^3 + 2*a^2*b^4 - 2*a*b^5 - b^6)*f*cos(f*x +
e)^2 - (a^4*b^2 + 4*a^3*b^3 + 6*a^2*b^4 + 4*a*b^5 + b^6)*f)*sin(f*x + e))
```

3.130.6 Sympy [F]

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\csc^4\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)**4/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral(csc(e + f*x)**4/(a + b*sec(e + f*x)**2)**(5/2), x)
```

3.130.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.37

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx =$$

$$-\frac{\frac{8b\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+b}(a+b)^3}+\frac{4b\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}(a+b)^2}-\frac{16b^2\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+b}(a+b)^4}-\frac{8b^2\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}(a+b)^3}+\frac{}{\left(b\tan\right.}}{3f}$$

input
```
integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*(8*b*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^3) + 4*b*ta
n(f*x + e)/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)^2) - 16*b^2*tan(f*x +
 e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^4) - 8*b^2*tan(f*x + e)/((b*ta
n(f*x + e)^2 + a + b)^(3/2)*(a + b)^3) + 3/((b*tan(f*x + e)^2 + a + b)^(3/
2)*(a + b)*tan(f*x + e)) - 6*b/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)^2
*tan(f*x + e)) + 1/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)*tan(f*x + e)^
3))/f
```

3.130.8 Giac [F]

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\csc\left(fx+e\right)^4}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.130.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Hanged}$$

input
```
int(1/(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^(5/2)),x)
```

output
```
\text{Hanged}
```

3.131 $\int \frac{\csc^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

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3.131.1 Optimal result

Integrand size = 25, antiderivative size = 226

$$\int \frac{\csc^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = -\frac{(5a^2-10ab+b^2)\cot(e+fx)}{5(a+b)^3 f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{2(5a+b)\cot^3(e+fx)}{15(a+b)^2 f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{\cot^5(e+fx)}{5(a+b) f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{4b(5a^2-10ab+b^2)\tan(e+fx)}{15(a+b)^4 f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{8b(5a^2-10ab+b^2)\tan(e+fx)}{15(a+b)^5 f\sqrt{a+b+b\tan^2(e+fx)}}$$

output

```
-8/15*b*(5*a^2-10*a*b+b^2)*tan(f*x+e)/(a+b)^5/f/(a+b+b*tan(f*x+e)^2)^(1/2)
-1/5*(5*a^2-10*a*b+b^2)*cot(f*x+e)/(a+b)^3/f/(a+b+b*tan(f*x+e)^2)^(3/2)-2/
15*(5*a+b)*cot(f*x+e)^3/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(3/2)-1/5*cot(f*x+e
)^5/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(3/2)-4/15*b*(5*a^2-10*a*b+b^2)*tan(f*x+e
)/(a+b)^4/f/(a+b+b*tan(f*x+e)^2)^(3/2)
```

3.131.2 Mathematica [A] (verified)

Time = 4.93 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.77

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{(a+2b+a\cos(2(e+fx)))^3\left(\frac{20ab^2(a+b)}{(a+2b+a\cos(2(e+fx)))^2}+\frac{10ab(-6a+5b)}{a+2b+a\cos(2(e+fx))}+(-8\right.}{}$$

input
```
Integrate[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])^3*((20*a*b^2*(a + b))/(a + 2*b + a*Cos[2*(
e + f*x)])^2 + (10*a*b*(-6*a + 5*b))/(a + 2*b + a*Cos[2*(e + f*x)]) + (-8*
a^2 + 50*a*b - 15*b^2)*Csc[e + f*x]^2 + 2*(a + b)*(-2*a + 5*b)*Csc[e + f*x
]^4 - 3*(a + b)^2*Csc[e + f*x]^6)*Sec[e + f*x]^4*Tan[e + f*x])/(120*(a + b
)^5*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

3.131.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.93, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4620, 365, 359, 245, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{\sin(e+fx)^6\left(a+b\sec(e+fx)^2\right)^{5/2}}dx$$
$$\downarrow 4620$$
$$\frac{\int \frac{\cot^6(e+fx)\left(\tan^2(e+fx)+1\right)^2}{(b\tan^2(e+fx)+a+b)^{5/2}}d\tan(e+fx)}{f}$$
$$\downarrow 365$$
$$\frac{\int \frac{\cot^4(e+fx)\left(5(a+b)\tan^2(e+fx)+2(5a+b)\right)}{\left(b\tan^2(e+fx)+a+b\right)^{5/2}}d\tan(e+fx)}{5(a+b)} - \frac{\cot^5(e+fx)}{5(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\downarrow 359$$

$$\frac{\left(5a^2-10ab+b^2\right)\int \frac{\cot^2(e+fx)}{\left(b\tan^2(e+fx)+a+b\right)^{5/2}}d\tan(e+fx)}{a+b}-\frac{2(5a+b)\cot^3(e+fx)}{3(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}}{5(a+b)}-\frac{\cot^5(e+fx)}{5(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\downarrow 245$$

$$\frac{\left(5a^2-10ab+b^2\right)\left(-\frac{4b\int \frac{1}{\left(b\tan^2(e+fx)+a+b\right)^{5/2}}d\tan(e+fx)}{a+b}-\frac{\cot(e+fx)}{(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}\right)}{a+b}-\frac{2(5a+b)\cot^3(e+fx)}{3(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}}{5(a+b)}-\frac{\cot^5(e+fx)}{5(a+b)(a+b\tan^2(e+f}}{f}$$

$$\downarrow 209$$

$$\frac{\left(5a^2-10ab+b^2\right)\left(-\frac{4b\left(\frac{2\int \frac{1}{\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{3(a+b)}+\frac{\tan(e+fx)}{3(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}\right)}{a+b}-\frac{\cot(e+fx)}{(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}\right)}{a+b}-\frac{2(5a+b)\cot^3(e}{3(a+b)\left(a+b\tan^2(e+}}{5(a+b)}}{f}$$

$$\downarrow 208$$

$$\frac{\left(5a^2-10ab+b^2\right)\left(-\frac{4b\left(\frac{2\tan(e+fx)}{3(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}}+\frac{\tan(e+fx)}{3(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}\right)}{a+b}-\frac{\cot(e+fx)}{(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}\right)}{a+b}-\frac{2(5a+b)\cot^3(e+fx)}{3(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}}{5(a+b)}}{f}$$

input
```
Int[Csc[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

3.131. $\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

output
```
(-1/5*Cot[e + f*x]^5/((a + b)*(a + b + b*Tan[e + f*x]^2)^(3/2)) + ((-2*(5*
a + b)*Cot[e + f*x]^3)/(3*(a + b)*(a + b + b*Tan[e + f*x]^2)^(3/2)) + ((5*
a^2 - 10*a*b + b^2)*(-(Cot[e + f*x]/((a + b)*(a + b + b*Tan[e + f*x]^2)^(3
/2))) - (4*b*(Tan[e + f*x]/(3*(a + b)*(a + b + b*Tan[e + f*x]^2)^(3/2)) +
(2*Tan[e + f*x])/(3*(a + b)^2*Sqrt[a + b + b*Tan[e + f*x]^2])))/(a + b)))/
(a + b))/(5*(a + b)))/f
```

3.131.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.131.4 Maple [A] (verified)

Time = 6.23 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.47

method	result
default	$-\frac{\left(b+a\cos(fx+e)^2\right)\left(8\cos(fx+e)^8a^4-80\cos(fx+e)^8a^3b+40\cos(fx+e)^8a^2b^2-20a^4\cos(fx+e)^6+212a^3b\cos(fx+e)^6-220\cos(fx$

input
```
int(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15/f/(a^5+5*a^4*b+10*a^3*b^2+10*a^2*b^3+5*a*b^4+b^5)*(b+a*cos(f*x+e)^2)
*(8*cos(f*x+e)^8*a^4-80*cos(f*x+e)^8*a^3*b+40*cos(f*x+e)^8*a^2*b^2-20*a^4*
cos(f*x+e)^6+212*a^3*b*cos(f*x+e)^6-220*cos(f*x+e)^6*a^2*b^2+60*cos(f*x+e)
^6*a*b^3+15*a^4*cos(f*x+e)^4-180*cos(f*x+e)^4*a^3*b+378*a^2*b^2*cos(f*x+e)
^4-180*cos(f*x+e)^4*a*b^3+15*cos(f*x+e)^4*b^4+60*cos(f*x+e)^2*a^3*b-220*a^
2*b^2*cos(f*x+e)^2+212*a*b^3*cos(f*x+e)^2-20*cos(f*x+e)^2*b^4+40*a^2*b^2-8
0*a*b^3+8*b^4)/(a+b*sec(f*x+e)^2)^(5/2)*sec(f*x+e)^5*csc(f*x+e)^5
```

3.131.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 460 vs. 2(206) = 412.

Time = 9.34 (sec) , antiderivative size = 460, normalized size of antiderivative = 2.04

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx =$$

$$-\frac{\left(8\left(a^4-10\,a^3b+5\,a^2b^2\right)\cos\left(fx+e\right)^9-4\left(5\,a^4-53\,a^3b+55\,a^2b^2\right.\right.}{15\left(\left(a^7+5\,a^6b+10\,a^5b^2+10\,a^4b^3+5\,a^3b^4+a^2b^5\right)f\cos\left(fx+e\right)^8-2\left(a^7+4\,a^6b+5\,a^5b^2-5\,a^3b^4-4\,a^2\right.\right.}$$

input
```
integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/15*(8*(a^4 - 10*a^3*b + 5*a^2*b^2)*cos(f*x + e)^9 - 4*(5*a^4 - 53*a^3*b
 + 55*a^2*b^2 - 15*a*b^3)*cos(f*x + e)^7 + 3*(5*a^4 - 60*a^3*b + 126*a^2*b
^2 - 60*a*b^3 + 5*b^4)*cos(f*x + e)^5 + 4*(15*a^3*b - 55*a^2*b^2 + 53*a*b^
3 - 5*b^4)*cos(f*x + e)^3 + 8*(5*a^2*b^2 - 10*a*b^3 + b^4)*cos(f*x + e))*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(((a^7 + 5*a^6*b + 10*a^5*b^2 +
 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^8 - 2*(a^7 + 4*a^6*b + 5
*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*f*cos(f*x + e)^6 + (a^7 + a^6*b
 - 9*a^5*b^2 - 25*a^4*b^3 - 25*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*f*cos(f*x
 + e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 - 5*a^2*b^5 - 4*a*b^6 - b^7)*f*
cos(f*x + e)^2 + (a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6
 + b^7)*f)*sin(f*x + e))
```

3.131.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**6/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Timed out
```

3.131.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.65

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx =$$
$$-\frac{\frac{40\,b\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+b}(a+b)^3}+\frac{20\,b\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}(a+b)^2}-\frac{160\,b^2\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+b}(a+b)^4}-\frac{80\,b^2\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}(a+b)^3}+\frac{1}{\sqrt{b\tan}}$$

input
```
integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/15*(40*b*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^3) + 20*b
*tan(f*x + e)/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)^2) - 160*b^2*tan(f
*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^4) - 80*b^2*tan(f*x + e)/(
(b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)^3) + 128*b^3*tan(f*x + e)/(sqrt(b
*tan(f*x + e)^2 + a + b)*(a + b)^5) + 64*b^3*tan(f*x + e)/((b*tan(f*x + e)
^2 + a + b)^(3/2)*(a + b)^4) + 15/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b
)*tan(f*x + e)) - 60*b/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)^2*tan(f*x
 + e)) + 48*b^2/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)^3*tan(f*x + e))
+ 10/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)*tan(f*x + e)^3) - 8*b/((b*t
an(f*x + e)^2 + a + b)^(3/2)*(a + b)^2*tan(f*x + e)^3) + 3/((b*tan(f*x + e
)^2 + a + b)^(3/2)*(a + b)*tan(f*x + e)^5))/f
```

3.131.8 Giac [F]

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\csc\left(fx+e\right)^6}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.131.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Hanged}$$

input
```
int(1/(sin(e + f*x)^6*(a + b/cos(e + f*x)^2)^(5/2)),x)
```

output
```
\text{Hanged}
```

3.132 $\int (a+b\sec^2(e+fx))^p (d\sin(e+fx))^m\,dx$

3.132.1 Optimal result 1107
3.132.2 Mathematica [B] (warning: unable to verify) 1107
3.132.3 Rubi [F] 1108
3.132.4 Maple [F] 1109
3.132.5 Fricas [F] 1109
3.132.6 Sympy [F(-1)] 1109
3.132.7 Maxima [F] 1110
3.132.8 Giac [F] 1110
3.132.9 Mupad [F(-1)] 1110

3.132.1 Optimal result

Integrand size = 25, antiderivative size = 123

$$\int (a+b\sec^2(e+fx))^p (d\sin(e+fx))^m\,dx$$
$$= \frac{\text{AppellF1}\left(\frac{1+m}{2}, \frac{1}{2}+p, -p, \frac{3+m}{2}, \sin^2(e+fx), \frac{a\sin^2(e+fx)}{a+b}\right)\cos^2(e+fx)^{\frac{1}{2}+p}(a+b\sec^2(e+fx))^p (d\sin(e+$$
$$f(1+m)$$

output
```
AppellF1(1/2+1/2*m,1/2+p,-p,3/2+1/2*m,sin(f*x+e)^2,a*sin(f*x+e)^2/(a+b))*(
cos(f*x+e)^2)^(1/2+p)*(a+b*sec(f*x+e)^2)^p*(d*sin(f*x+e))^m*tan(f*x+e)/f/(
1+m)/(((a+b-a*sin(f*x+e)^2)/(a+b))^p)
```

3.132.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 286 vs. 2(123) = 246.

Time = 4.20 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.33

$$\int (a+b\sec^2(e+fx))^p (d\sin(e+fx))^m\,dx$$
$$= \frac{\text{AppellF1}\left(\frac{1+m}{2}, \frac{2+m}{2}, -p, \frac{3+m}{2}, -\tan^2(e+fx), -\frac{b\tan^2(e+fx)}{a+b}\right)\cos(e+}{f(1+m)\left(\text{AppellF1}\left(\frac{1+m}{2}, \frac{2+m}{2}, -p, \frac{3+m}{2}, -\tan^2(e+fx), -\frac{b\tan^2(e+fx)}{a+b}\right) - \frac{\left(-2bp\,\text{AppellF1}\left(\frac{3+m}{2}, \frac{2+m}{2}, 1-p, \frac{5+m}{2},\right.\right.}{}\right.}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^p*(d*Sin[e + f*x])^m,x]
```

output
```
(AppellF1[(1 + m)/2, (2 + m)/2, -p, (3 + m)/2, -Tan[e + f*x]^2, -((b*Tan[e
 + f*x]^2)/(a + b))]*Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]*(d
*Sin[e + f*x])^m)/(f*(1 + m)*(AppellF1[(1 + m)/2, (2 + m)/2, -p, (3 + m)/2
, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - ((-2*b*p*AppellF1[(3 +
 m)/2, (2 + m)/2, 1 - p, (5 + m)/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/
(a + b))] + (a + b)*(2 + m)*AppellF1[(3 + m)/2, (4 + m)/2, -p, (5 + m)/2,
-Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2)/((a + b)*
(3 + m))))
```

3.132.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\sin(e+fx))^m \left(a+b\sec^2(e+fx)\right)^p dx$$

$$\downarrow 3042$$

$$\int (d\sin(e+fx))^m \left(a+b\sec(e+fx)^2\right)^p dx$$

$$\downarrow 4623$$

$$\int (d\sin(e+fx))^m \left(a+b\sec^2(e+fx)\right)^p dx$$

input
```
Int[(a + b*Sec[e + f*x]^2)^p*(d*Sin[e + f*x])^m,x]
```

output
```
$Aborted
```

3.132.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4623
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*((d_.)*sin[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Unintegrable[(a + b*(c*Sec[e + f*x])
^n)^p*(d*Sin[e + f*x])^m, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

3.132.4 Maple [F]

$$\int \left(a+b\sec\left(fx+e\right)^2\right)^p \left(d\sin\left(fx+e\right)\right)^m dx$$

input
```
int((a+b*sec(f*x+e)^2)^p*(d*sin(f*x+e))^m,x)
```

output
```
int((a+b*sec(f*x+e)^2)^p*(d*sin(f*x+e))^m,x)
```

3.132.5 Fricas [F]

$$\int \left(a+b\sec^2(e+fx)\right)^p \left(d\sin(e+fx)\right)^m dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p \left(d\sin\left(fx+e\right)\right)^m dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*(d*sin(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*(d*sin(f*x + e))^m, x)
```

3.132.6 Sympy [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^p \left(d\sin(e+fx)\right)^m dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e)**2)**p*(d*sin(f*x+e))**m,x)
```

output
```
Timed out
```

3.132.7 Maxima [F]

$$\int (a+b\sec^2(e+fx))^p (d\sin(e+fx))^m\,dx = \int (b\sec(fx+e)^2+a)^p (d\sin(fx+e))^m\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*(d*sin(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*(d*sin(f*x + e))^m, x)
```

3.132.8 Giac [F]

$$\int (a+b\sec^2(e+fx))^p (d\sin(e+fx))^m\,dx = \int (b\sec(fx+e)^2+a)^p (d\sin(fx+e))^m\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*(d*sin(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*(d*sin(f*x + e))^m, x)
```

3.132.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\sec^2(e+fx))^p (d\sin(e+fx))^m\,dx = \int (d\,\sin(e+f\,x))^m \left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p dx$$

input
```
int((d*sin(e + f*x))^m*(a + b/cos(e + f*x)^2)^p,x)
```

output
```
int((d*sin(e + f*x))^m*(a + b/cos(e + f*x)^2)^p, x)
```

3.133 $\int (a+b\sec^2(e+fx))^p \sin^5(e+fx)\,dx$

3.133.1 Optimal result 1111
3.133.2 Mathematica [A] (verified) 1112
3.133.3 Rubi [A] (verified) 1112
3.133.4 Maple [F] 1115
3.133.5 Fricas [F] 1115
3.133.6 Sympy [F(-1)] 1115
3.133.7 Maxima [F] 1116
3.133.8 Giac [F] 1116
3.133.9 Mupad [F(-1)] 1116

3.133.1 Optimal result

Integrand size = 23, antiderivative size = 182

$$\int (a+b\sec^2(e+fx))^p \sin^5(e+fx)\,dx$$
$$= \frac{(10a+b(3-2p))\cos^3(e+fx)\,(a+b\sec^2(e+fx))^{1+p}}{15a^2f}$$
$$- \frac{\cos^5(e+fx)\,(a+b\sec^2(e+fx))^{1+p}}{5af}$$
$$- \frac{(15a^2+10ab(1-2p)+b^2(3-8p+4p^2))\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\sec^2(e+fx)}{a}\right)(a}{15a^2f}$$

output

```
1/15*(10*a+b*(3-2*p))*cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(p+1)/a^2/f-1/5*cos(
f*x+e)^5*(a+b*sec(f*x+e)^2)^(p+1)/a/f-1/15*(15*a^2+10*a*b*(1-2*p)+b^2*(4*p
^2-8*p+3))*cos(f*x+e)*hypergeom([-1/2, -p],[1/2],-b*sec(f*x+e)^2/a)*(a+b*s
ec(f*x+e)^2)^p/a^2/f/((1+b*sec(f*x+e)^2/a)^p)
```

3.133.2 Mathematica [A] (verified)

Time = 3.66 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.39

$$\int (a+b\sec^2(e+fx))^p \sin^5(e+fx)\,dx$$

$$= \frac{2\cos(e+fx)\,(a+b\sec^2(e+fx))^p \sin^4(e+fx)\left(4(15a^2+10ab(1-2p)+b^2(3-8p+4p^2))\,\text{Hypergeome}\right.}{15a^2 f\left(4\cos(2(e+fx))\left(\frac{a+b+b\tan^2(e+fx)}{a}\right)^p - 2^{-p}\left(3\right.\right.}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^5,x]
```

output
```
(2*Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^4*(4*(15*a^2 + 10*a*
b*(1 - 2*p) + b^2*(3 - 8*p + 4*p^2))*Hypergeometric2F1[-1/2, -p, 1/2, -((b
*Sec[e + f*x]^2)/a)] + (a + 2*b + a*Cos[2*(e + f*x)])*(-17*a - 6*b + 4*b*p
 + 3*a*Cos[2*(e + f*x)])*((a + b + b*Tan[e + f*x]^2)/a)^p))/(15*a^2*f*(4*C
os[2*(e + f*x)]*((a + b + b*Tan[e + f*x]^2)/a)^p - (3*(((a + 2*b + a*Cos[2
*(e + f*x)])*Sec[e + f*x]^2)/a)^p + 2^p*Cos[4*(e + f*x)]*((a + b + b*Tan[e
 + f*x]^2)/a)^p)/2^p))
```

3.133.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4622, 365, 25, 359, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^5(e+fx)\,(a+b\sec^2(e+fx))^p\,dx$$

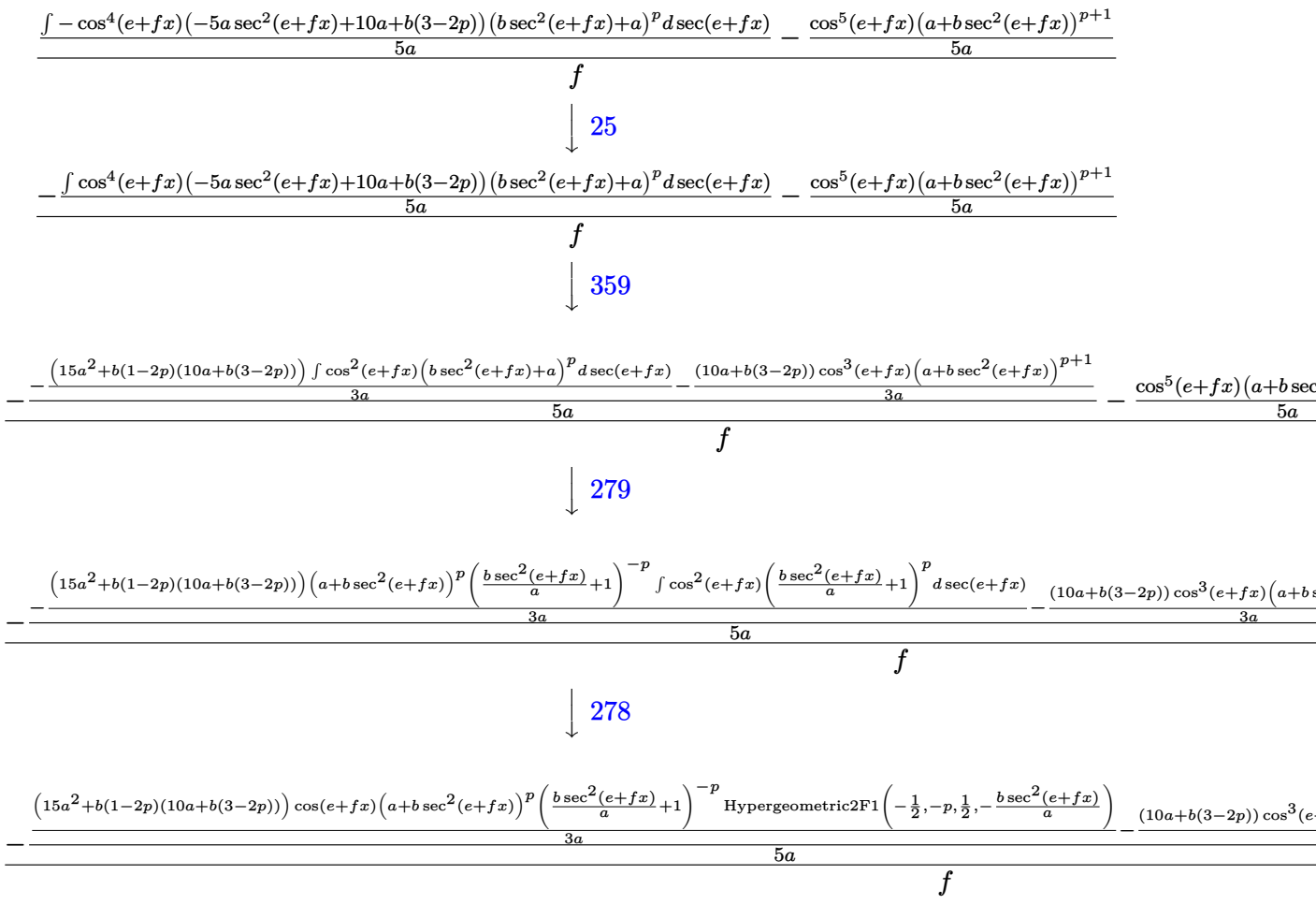
$$\downarrow 3042$$

$$\int \sin(e+fx)^5\,(a+b\sec(e+fx)^2)^p\,dx$$

$$\downarrow 4622$$

$$\frac{\int \cos^6(e+fx)\,(1-\sec^2(e+fx))^2\,(b\sec^2(e+fx)+a)^p\,d\sec(e+fx)}{f}$$

$$\downarrow 365$$



input
```
Int[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^5,x]
```

output
```
(-1/5*(Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(1 + p))/a - (-1/3*((10*a + b
*(3 - 2*p))*Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(1 + p))/a + ((15*a^2 +
b*(10*a + b*(3 - 2*p))*(1 - 2*p))*Cos[e + f*x]*Hypergeometric2F1[-1/2, -p,
 1/2, -((b*Sec[e + f*x]^2)/a)]*(a + b*Sec[e + f*x]^2)^p)/(3*a*(1 + (b*Sec[
e + f*x]^2)/a)^p))/(5*a))/f
```

3.133.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

3.133.4 Maple [F]

$$\int (a + b \sec (fx + e)^2)^p \sin (fx + e)^5 \, dx$$

input
```
int((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^5,x)
```

output
```
int((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^5,x)
```

3.133.5 Fricas [F]

$$\int (a + b \sec^2(e + fx))^p \sin^5(e + fx) \, dx = \int (b \sec (fx + e)^2 + a)^p \sin (fx + e)^5 \, dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^5,x, algorithm="fricas")
```

output
```
integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*(b*sec(f*x + e)^2 + a)^p*
sin(f*x + e), x)
```

3.133.6 Sympy [F(-1)]

Timed out.

$$\int (a + b \sec^2(e + fx))^p \sin^5(e + fx) \, dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e)**2)**p*sin(f*x+e)**5,x)
```

output
```
Timed out
```

3.133.7 Maxima [F]

$$\int (a+b\sec^2(e+fx))^p \sin^5(e+fx)\,dx = \int (b\sec(fx+e)^2+a)^p \sin(fx+e)^5\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^5,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e)^5, x)
```

3.133.8 Giac [F]

$$\int (a+b\sec^2(e+fx))^p \sin^5(e+fx)\,dx = \int (b\sec(fx+e)^2+a)^p \sin(fx+e)^5\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^5,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e)^5, x)
```

3.133.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\sec^2(e+fx))^p \sin^5(e+fx)\,dx = \int \sin(e+fx)^5 \left(a+\frac{b}{\cos(e+fx)^2}\right)^p dx$$

input
```
int(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^p,x)
```

output
```
int(sin(e + f*x)^5*(a + b/cos(e + f*x)^2)^p, x)
```

3.134 $\int (a+b\sec^2(e+fx))^p \sin^3(e+fx)\,dx$

3.134.1 Optimal result 1117
3.134.2 Mathematica [A] (verified) 1117
3.134.3 Rubi [A] (verified) 1118
3.134.4 Maple [F] 1120
3.134.5 Fricas [F] 1120
3.134.6 Sympy [F(-1)] 1120
3.134.7 Maxima [F] 1121
3.134.8 Giac [F] 1121
3.134.9 Mupad [F(-1)] 1121

3.134.1 Optimal result

Integrand size = 23, antiderivative size = 117

$$\int (a+b\sec^2(e+fx))^p \sin^3(e+fx)\,dx = \frac{\cos^3(e+fx)\,(a+b\sec^2(e+fx))^{1+p}}{3af} - \frac{(3a+b-2bp)\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\sec^2(e+fx)}{a}\right)(a+b\sec^2(e+fx))^p\left(1+\frac{b\sec}{}\right.}{3af}$$

output
```
1/3*cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(p+1)/a/f-1/3*(-2*b*p+3*a+b)*cos(f*x+e
)*hypergeom([-1/2, -p],[1/2],-b*sec(f*x+e)^2/a)*(a+b*sec(f*x+e)^2)^p/a/f/(
(1+b*sec(f*x+e)^2/a)^p)
```

3.134.2 Mathematica [A] (verified)

Time = 2.40 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.52

$$\int (a+b\sec^2(e+fx))^p \sin^3(e+fx)\,dx = -\frac{\cos(e+fx)\,(a+b\sec^2(e+fx))^p \sin^2(e+fx)\left(-2(3a+b-2bp)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\sec}{}\right.\right.}{3af\left(-2\left(1+\frac{b\sec^2(e+fx)}{a}\right)^p+\left(\frac{a+b+b\tan^2(e+fx)}{a}\right)^p+\cos(2(e+fx)\right.}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^3,x]
```

output
```
-1/3*(Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^2*(-2*(3*a + b -
2*b*p)*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Sec[e + f*x]^2)/a)] + (a + 2*
b + a*Cos[2*(e + f*x)])*((a + b + b*Tan[e + f*x]^2)/a)^p))/(a*f*(-2*(1 + (
b*Sec[e + f*x]^2)/a)^p + ((a + b + b*Tan[e + f*x]^2)/a)^p + Cos[2*(e + f*x
)]*((a + b + b*Tan[e + f*x]^2)/a)^p))
```

3.134.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4622, 25, 359, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$

$$\downarrow 3042$$

$$\int \sin(e+fx)^3\left(a+b\sec(e+fx)^2\right)^p\,dx$$

$$\downarrow 4622$$

$$\frac{\int -\cos^4(e+fx)\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^p\,d\sec(e+fx)}{f}$$

$$\downarrow 25$$

$$-\frac{\int \cos^4(e+fx)\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^p\,d\sec(e+fx)}{f}$$

$$\downarrow 359$$

$$\frac{\frac{(3a-2bp+b)\int \cos^2(e+fx)\left(b\sec^2(e+fx)+a\right)^p d\sec(e+fx)}{3a}+\frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{p+1}}{3a}}{f}$$

$$\downarrow 279$$

$$\frac{\frac{(3a-2bp+b)\left(a+b\sec^2(e+fx)\right)^p\left(\frac{b\sec^2(e+fx)}{a}+1\right)^{-p}\int \cos^2(e+fx)\left(\frac{b\sec^2(e+fx)}{a}+1\right)^p d\sec(e+fx)}{3a}+\frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{p+1}}{3a}}{f}$$

$$\downarrow 278$$

$$\frac{\cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{p+1}}{3a} - \frac{(3a-2bp+b)\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{p}\left(\frac{b\sec^2(e+fx)}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\sec^2(e+}{a}\right.}{3a}$$

$$f$$

input
```
Int[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^3,x]
```

output
```
((Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(1 + p))/(3*a) - ((3*a + b - 2*b*p
)*Cos[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Sec[e + f*x]^2)/a)]*(
a + b*Sec[e + f*x]^2)^p)/(3*a*(1 + (b*Sec[e + f*x]^2)/a)^p))/f
```

3.134.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

3.134.4 Maple [F]

$$\int (a + b \sec (fx + e)^2)^p \sin (fx + e)^3 \, dx$$

input
```
int((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^3,x)
```

output
```
int((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^3,x)
```

3.134.5 Fricas [F]

$$\int (a + b \sec^2(e + fx))^p \sin^3(e + fx) \, dx = \int (b \sec (fx + e)^2 + a)^p \sin (fx + e)^3 \, dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^3,x, algorithm="fricas")
```

output
```
integral(-(cos(f*x + e)^2 - 1)*(b*sec(f*x + e)^2 + a)^p*sin(f*x + e), x)
```

3.134.6 Sympy [F(-1)]

Timed out.

$$\int (a + b \sec^2(e + fx))^p \sin^3(e + fx) \, dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e)**2)**p*sin(f*x+e)**3,x)
```

output
```
Timed out
```

3.134.7 Maxima [F]

$$\int (a+b\sec^2(e+fx))^p \sin^3(e+fx)\,dx = \int (b\sec(fx+e)^2+a)^p \sin(fx+e)^3\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^3,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e)^3, x)
```

3.134.8 Giac [F]

$$\int (a+b\sec^2(e+fx))^p \sin^3(e+fx)\,dx = \int (b\sec(fx+e)^2+a)^p \sin(fx+e)^3\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^3,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e)^3, x)
```

3.134.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\sec^2(e+fx))^p \sin^3(e+fx)\,dx = \int \sin(e+fx)^3 \left(a+\frac{b}{\cos(e+fx)^2}\right)^p dx$$

input
```
int(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^p,x)
```

output
```
int(sin(e + f*x)^3*(a + b/cos(e + f*x)^2)^p, x)
```

3.135 $\int (a+b\sec^2(e+fx))^p \sin(e+fx)\,dx$

3.135.1 Optimal result . . . 1122
3.135.2 Mathematica [A] (verified) . . . 1122
3.135.3 Rubi [A] (verified) . . . 1123
3.135.4 Maple [F] . . . 1124
3.135.5 Fricas [F] . . . 1125
3.135.6 Sympy [F] . . . 1125
3.135.7 Maxima [F] . . . 1125
3.135.8 Giac [F] . . . 1126
3.135.9 Mupad [B] (verification not implemented) . . . 1126

3.135.1 Optimal result

Integrand size = 21, antiderivative size = 68

$$\int (a+b\sec^2(e+fx))^p \sin(e+fx)\,dx = -\frac{\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\sec^2(e+fx)}{a}\right)(a+b\sec^2(e+fx))^p\left(1+\frac{b\sec^2(e+fx)}{a}\right)^{-p}}{f}$$

output
```
-cos(f*x+e)*hypergeom([-1/2, -p],[1/2],-b*sec(f*x+e)^2/a)*(a+b*sec(f*x+e)^
2)^p/f/((1+b*sec(f*x+e)^2/a)^p)
```

3.135.2 Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00

$$\int (a+b\sec^2(e+fx))^p \sin(e+fx)\,dx = -\frac{\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\sec^2(e+fx)}{a}\right)(a+b\sec^2(e+fx))^p\left(1+\frac{b\sec^2(e+fx)}{a}\right)^{-p}}{f}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x],x]
```

output
```
-((Cos[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Sec[e + f*x]^2)/a)]*
(a + b*Sec[e + f*x]^2)^p)/(f*(1 + (b*Sec[e + f*x]^2)/a)^p))
```

3.135.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4622, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(e+fx)\left(a+b\sec^2(e+fx)\right)^p dx$$

$$\downarrow 3042$$

$$\int \sin(e+fx)\left(a+b\sec(e+fx)^2\right)^p dx$$

$$\downarrow 4622$$

$$\frac{\int \cos^2(e+fx)\left(b\sec^2(e+fx)+a\right)^p d\sec(e+fx)}{f}$$

$$\downarrow 279$$

$$\frac{\left(a+b\sec^2(e+fx)\right)^p\left(\frac{b\sec^2(e+fx)}{a}+1\right)^{-p}\int \cos^2(e+fx)\left(\frac{b\sec^2(e+fx)}{a}+1\right)^p d\sec(e+fx)}{f}$$

$$\downarrow 278$$

$$-\frac{\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\left(\frac{b\sec^2(e+fx)}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\sec^2(e+fx)}{a}\right)}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x],x]
```

output
```
-((Cos[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Sec[e + f*x]^2)/a)]*
(a + b*Sec[e + f*x]^2)^p)/(f*(1 + (b*Sec[e + f*x]^2)/a)^p))
```

3.135.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

3.135.4 Maple [F]

$$\int \left(a + b \sec\left(fx + e\right)^2\right)^p \sin\left(fx + e\right) dx$$

input
```
int((a+b*sec(f*x+e)^2)^p*sin(f*x+e),x)
```

output
```
int((a+b*sec(f*x+e)^2)^p*sin(f*x+e),x)
```

3.135.5 Fricas [F]

$$\int (a+b\sec^2(e+fx))^p \sin(e+fx)\,dx = \int (b\sec(fx+e)^2+a)^p \sin(fx+e)\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e),x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*sin(f*x + e), x)
```

3.135.6 Sympy [F]

$$\int (a+b\sec^2(e+fx))^p \sin(e+fx)\,dx = \int (a+b\sec^2(e+fx))^p \sin(e+fx)\,dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**p*sin(f*x+e),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**p*sin(e + f*x), x)
```

3.135.7 Maxima [F]

$$\int (a+b\sec^2(e+fx))^p \sin(e+fx)\,dx = \int (b\sec(fx+e)^2+a)^p \sin(fx+e)\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e), x)
```

3.135.8 Giac [F]

$$\int (a+b\sec^2(e+fx))^p \sin(e+fx)\,dx = \int \left(b\sec(fx+e)^2+a\right)^p \sin(fx+e)\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e), x)
```

3.135.9 Mupad [B] (verification not implemented)

Time = 19.17 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.16

$$\int (a+b\sec^2(e+fx))^p \sin(e+fx)\,dx = \frac{\cos(e+fx)\left(a+\frac{b}{\cos(e+fx)^2}\right)^p {}_2\mathrm{F}_1\left(\frac{1}{2}-p,-p;\ \frac{3}{2}-p;\ -\frac{a\cos(e+fx)^2}{b}\right)}{f\,(2p-1)\left(\frac{a\cos(e+fx)^2}{b}+1\right)^p}$$

input
```
int(sin(e + f*x)*(a + b/cos(e + f*x)^2)^p,x)
```

output
```
(cos(e + f*x)*(a + b/cos(e + f*x)^2)^p*hypergeom([1/2 - p, -p], 3/2 - p, -
(a*cos(e + f*x)^2)/b))/(f*(2*p - 1)*((a*cos(e + f*x)^2)/b + 1)^p)
```

3.136 $\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

3.136.1 Optimal result 1127
3.136.2 Mathematica [B] (warning: unable to verify) 1127
3.136.3 Rubi [A] (verified) 1128
3.136.4 Maple [F] 1130
3.136.5 Fricas [F] 1130
3.136.6 Sympy [F(-1)] 1130
3.136.7 Maxima [F] 1131
3.136.8 Giac [F] 1131
3.136.9 Mupad [F(-1)] 1131

3.136.1 Optimal result

Integrand size = 21, antiderivative size = 77

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = -\frac{\text{AppellF1}\left(\frac{1}{2},1,-p,\frac{3}{2},\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a}\right)\sec(e+fx)\left(a+b\sec^2(e+fx)\right)^p\left(1+\frac{b\sec^2(e+fx)}{a}\right)^{-p}}{f}$$

output
```
-AppellF1(1/2,1,-p,3/2,sec(f*x+e)^2,-b*sec(f*x+e)^2/a)*sec(f*x+e)*(a+b*sec
(f*x+e)^2)^p/f/((1+b*sec(f*x+e)^2/a)^p)
```

3.136.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1532 vs. 2(77) = 154.

Time = 16.05 (sec) , antiderivative size = 1532, normalized size of antiderivative = 19.90

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \text{Too large to display}$$

input
```
Integrate[Csc[e + f*x]*(a + b*Sec[e + f*x]^2)^p,x]
```

output

```
((a + 2*b + a*Cos[2*(e + f*x)])^p*Csc[e + f*x]*(Sec[e + f*x]^2)^p*(a + b*S
ec[e + f*x]^2)^p*((2*AppellF1[-1/2 - p, -1/2, -p, 1/2 - p, -Cot[e + f*x]^2
, -(((a + b)*Cot[e + f*x]^2)/b)]*Sqrt[Sec[e + f*x]^2])/((1 + 2*p)*(1 + ((a
 + b)*Cot[e + f*x]^2)/b)^p*Sqrt[Csc[e + f*x]^2]) - (AppellF1[1, 1/2, -p, 2
, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*Tan[e + f*x]^2)/((a + b
+ b*Tan[e + f*x]^2)/(a + b))^p))/(2*f*(-(a*p*(a + 2*b + a*Cos[2*(e + f*x)]
)^(-1 + p)*(Sec[e + f*x]^2)^p*Sin[2*(e + f*x)]*((2*AppellF1[-1/2 - p, -1/2
, -p, 1/2 - p, -Cot[e + f*x]^2, -(((a + b)*Cot[e + f*x]^2)/b)]*Sqrt[Sec[e
+ f*x]^2])/((1 + 2*p)*(1 + ((a + b)*Cot[e + f*x]^2)/b)^p*Sqrt[Csc[e + f*x]
^2]) - (AppellF1[1, 1/2, -p, 2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a +
 b))]*Tan[e + f*x]^2)/((a + b + b*Tan[e + f*x]^2)/(a + b))^p)) + p*(a + 2*
b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*Tan[e + f*x]*((2*AppellF1[-1/
2 - p, -1/2, -p, 1/2 - p, -Cot[e + f*x]^2, -(((a + b)*Cot[e + f*x]^2)/b)]*
Sqrt[Sec[e + f*x]^2])/((1 + 2*p)*(1 + ((a + b)*Cot[e + f*x]^2)/b)^p*Sqrt[C
sc[e + f*x]^2]) - (AppellF1[1, 1/2, -p, 2, -Tan[e + f*x]^2, -((b*Tan[e + f
*x]^2)/(a + b))]*Tan[e + f*x]^2)/((a + b + b*Tan[e + f*x]^2)/(a + b))^p) +
 ((a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*((2*AppellF1[-1/2 -
p, -1/2, -p, 1/2 - p, -Cot[e + f*x]^2, -(((a + b)*Cot[e + f*x]^2)/b)]*Cot[
e + f*x]*Sqrt[Sec[e + f*x]^2])/((1 + 2*p)*(1 + ((a + b)*Cot[e + f*x]^2)/b)
^p*Sqrt[Csc[e + f*x]^2]) + (4*(a + b)*p*AppellF1[-1/2 - p, -1/2, -p, 1/...
```

3.136.3 Rubi [A] (verified)

Time $= 0.25$ (sec) , antiderivative size $= 77$, normalized size of antiderivative $= 1.00$, number of steps used $= 6$, number of rules used $= 5$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used $= \{3042, 4622, 25, 334, 333\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^p}{\sin(e+fx)}dx$$

$$\downarrow 4622$$

$$\frac{\int -\frac{\left(b\sec^2(e+fx)+a\right)^p}{1-\sec^2(e+fx)}d\sec(e+fx)}{f}$$

$$\downarrow 25$$

$$-\frac{\int \frac{(b\sec^2(e+fx)+a)^p}{1-\sec^2(e+fx)} d\sec(e+fx)}{f}$$

$$\downarrow 334$$

$$-\frac{(a+b\sec^2(e+fx))^p \left(\frac{b\sec^2(e+fx)}{a}+1\right)^{-p} \int \frac{\left(\frac{b\sec^2(e+fx)}{a}+1\right)^p}{1-\sec^2(e+fx)} d\sec(e+fx)}{f}$$

$$\downarrow 333$$

$$-\frac{\sec(e+fx)\,(a+b\sec^2(e+fx))^p \left(\frac{b\sec^2(e+fx)}{a}+1\right)^{-p} \text{AppellF1}\left(\frac{1}{2},1,-p,\frac{3}{2},\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a}\right)}{f}$$

input
```
Int[Csc[e + f*x]*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
-((AppellF1[1/2, 1, -p, 3/2, Sec[e + f*x]^2, -((b*Sec[e + f*x]^2)/a)]*Sec[
e + f*x]*(a + b*Sec[e + f*x]^2)^p)/(f*(1 + (b*Sec[e + f*x]^2)/a)^p))
```

3.136.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

3.136.4 Maple [F]

$$\int \csc(fx+e)\left(a+b\sec(fx+e)^2\right)^p dx$$

input
```
int(csc(f*x+e)*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int(csc(f*x+e)*(a+b*sec(f*x+e)^2)^p,x)
```

3.136.5 Fricas [F]

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^p dx = \int \left(b\sec(fx+e)^2+a\right)^p \csc(fx+e)\, dx$$

input
```
integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*csc(f*x + e), x)
```

3.136.6 Sympy [F(-1)]

Timed out.

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^p dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)*(a+b*sec(f*x+e)**2)**p,x)
```

output
```
Timed out
```

3.136.7 Maxima [F]

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p \csc\left(fx+e\right)\,dx$$

input
```
integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e), x)
```

3.136.8 Giac [F]

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p \csc\left(fx+e\right)\,dx$$

input
```
integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e), x)
```

3.136.9 Mupad [F(-1)]

Timed out.

$$\int \csc(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p}{\sin\left(e+f\,x\right)}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^p/sin(e + f*x),x)
```

output
```
int((a + b/cos(e + f*x)^2)^p/sin(e + f*x), x)
```

3.137 $\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

3.137.1 Optimal result 1132
3.137.2 Mathematica [B] (warning: unable to verify) 1132
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3.137.6 Sympy [F(-1)] 1135
3.137.7 Maxima [F] 1135
3.137.8 Giac [F] 1136
3.137.9 Mupad [F(-1)] 1136

3.137.1 Optimal result

Integrand size = 23, antiderivative size = 81

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{AppellF1}\left(\frac{3}{2},2,-p,\frac{5}{2},\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a}\right)\sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\left(1+\frac{b\sec^2(e+fx)}{a}\right)^{-p}}{3f}$$

output
```
1/3*AppellF1(3/2,2,-p,5/2,sec(f*x+e)^2,-b*sec(f*x+e)^2/a)*sec(f*x+e)^3*(a+
b*sec(f*x+e)^2)^p/f/((1+b*sec(f*x+e)^2/a)^p)
```

3.137.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 266 vs. 2(81) = 162.

Time = 3.22 (sec) , antiderivative size = 266, normalized size of antiderivative = 3.28

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{AppellF1}\left(\frac{1}{2}-p,-\frac{1}{2},-p,\frac{3}{2}-p,-\cot^2(e+fx\right.}{f(-1+2p)\left(-\frac{\left(2(a+b)p\,\text{AppellF1}\left(\frac{3}{2}-p,-\frac{1}{2},1-p,\frac{5}{2}-p,-\cot^2(e+fx),-\frac{(a+b)\cot^2(e+fx)}{b}\right)+b\,\text{AppellF1}\left(\frac{3}{2}-p,\frac{1}{2},-p,\frac{5}{2}-p,-\cot^2(e+fx),-\right.\right.}{b(-3+2p)}}$$

input
```
Integrate[Csc[e + f*x]^3*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
(AppellF1[1/2 - p, -1/2, -p, 3/2 - p, -Cot[e + f*x]^2, -(((a + b)*Cot[e +
f*x]^2)/b)]*Csc[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p)/(f*(-1 + 2*p)*(-(((2*
(a + b)*p*AppellF1[3/2 - p, -1/2, 1 - p, 5/2 - p, -Cot[e + f*x]^2, -(((a +
 b)*Cot[e + f*x]^2)/b)] + b*AppellF1[3/2 - p, 1/2, -p, 5/2 - p, -Cot[e + f
*x]^2, -(((a + b)*Cot[e + f*x]^2)/b)])*Cot[e + f*x]*Csc[e + f*x])/(b*(-3 +
 2*p))) + AppellF1[1/2 - p, -1/2, -p, 3/2 - p, -Cot[e + f*x]^2, -(((a + b)
*Cot[e + f*x]^2)/b)]*Sec[e + f*x]))
```

3.137.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4622, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^p}{\sin(e+fx)^3}dx$$

$$\downarrow 4622$$

$$\frac{\int \frac{\sec^2(e+fx)\left(b\sec^2(e+fx)+a\right)^p}{\left(1-\sec^2(e+fx)\right)^2}d\sec(e+fx)}{f}$$

$$\downarrow 395$$

$$\frac{\left(a+b\sec^2(e+fx)\right)^p\left(\frac{b\sec^2(e+fx)}{a}+1\right)^{-p}\int \frac{\sec^2(e+fx)\left(\frac{b\sec^2(e+fx)}{a}+1\right)^p}{\left(1-\sec^2(e+fx)\right)^2}d\sec(e+fx)}{f}$$

$$\downarrow 394$$

$$\frac{\sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\left(\frac{b\sec^2(e+fx)}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{3}{2},2,-p,\frac{5}{2},\sec^2(e+fx),-\frac{b\sec^2(e+fx)}{a}\right)}{3f}$$

input
```
Int[Csc[e + f*x]^3*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
(AppellF1[3/2, 2, -p, 5/2, Sec[e + f*x]^2, -((b*Sec[e + f*x]^2)/a)]*Sec[e
+ f*x]^3*(a + b*Sec[e + f*x]^2)^p)/(3*f*(1 + (b*Sec[e + f*x]^2)/a)^p)
```

3.137.3.1 Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4622
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*sin[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Si
mp[1/(f*ff^m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p
/x^(m + 1)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4])
```

3.137.4 Maple [F]

$$\int \csc(fx+e)^3 \left(a+b\sec(fx+e)^2\right)^p dx$$

input
```
int(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x)
```

3.137.5 Fricas [F]

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p \csc\left(fx+e\right)^3\,dx$$

input
```
integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^3, x)
```

3.137.6 Sympy [F(-1)]

Timed out.

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**3*(a+b*sec(f*x+e)**2)**p,x)
```

output
```
Timed out
```

3.137.7 Maxima [F]

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p \csc\left(fx+e\right)^3\,dx$$

input
```
integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^3, x)
```

3.137.8 Giac [F]

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p \csc\left(fx+e\right)^3\,dx$$

input
```
integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^3, x)
```

3.137.9 Mupad [F(-1)]

Timed out.

$$\int \csc^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p}{\sin\left(e+f\,x\right)^3}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^p/sin(e + f*x)^3,x)
```

output
```
int((a + b/cos(e + f*x)^2)^p/sin(e + f*x)^3, x)
```

3.138 $\int (a+b\sec^2(e+fx))^p \sin^4(e+fx)\,dx$

3.138.1 Optimal result 1137
3.138.2 Mathematica [B] (warning: unable to verify) 1137
3.138.3 Rubi [A] (verified) 1138
3.138.4 Maple [F] 1139
3.138.5 Fricas [F] 1140
3.138.6 Sympy [F(-1)] 1140
3.138.7 Maxima [F] 1140
3.138.8 Giac [F] 1141
3.138.9 Mupad [F(-1)] 1141

3.138.1 Optimal result

Integrand size = 23, antiderivative size = 88

$$\int (a+b\sec^2(e+fx))^p \sin^4(e+fx)\,dx$$
$$= \frac{\text{AppellF1}\left(\frac{5}{2},3,-p,\frac{7}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan^5(e+fx)\,(a+b+b\tan^2(e+fx))^p\left(1+\frac{b\tan^2(e+}{a+b}\right.}{5f}$$

output
```
1/5*AppellF1(5/2,3,-p,7/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)^
5*(a+b+b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

3.138.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 5878 vs. 2(88) = 176.

Time = 25.84 (sec) , antiderivative size = 5878, normalized size of antiderivative = 66.80

$$\int (a+b\sec^2(e+fx))^p \sin^4(e+fx)\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^4,x]
```

output
```
Result too large to show
```

3.138.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4620, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$

$$\downarrow 3042$$

$$\int \sin(e+fx)^4\left(a+b\sec(e+fx)^2\right)^p\,dx$$

$$\downarrow 4620$$

$$\frac{\int \frac{\tan^4(e+fx)(b\tan^2(e+fx)+a+b)^p}{(\tan^2(e+fx)+1)^3}d\tan(e+fx)}{f}$$

$$\downarrow 395$$

$$\frac{(a+b\tan^2(e+fx)+b)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\int \frac{\tan^4(e+fx)\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^p}{(\tan^2(e+fx)+1)^3}d\tan(e+fx)}{f}$$

$$\downarrow 394$$

$$\frac{\tan^5(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\text{AppellF1}\left(\frac{5}{2},3,-p,\frac{7}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{5f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^4,x]
```

output
```
(AppellF1[5/2, 3, -p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
*Tan[e + f*x]^5*(a + b + b*Tan[e + f*x]^2)^p)/(5*f*(1 + (b*Tan[e + f*x]^2)
/(a + b))^p)
```

3.138.3.1 Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.138.4 Maple [F]

$$\int \left(a + b \sec\left(fx + e\right)^2\right)^p \sin\left(fx + e\right)^4 \, dx$$

input
```
int((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^4,x)
```

output
```
int((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^4,x)
```

3.138.5 Fricas [F]

$$\int (a+b\sec^2(e+fx))^p \sin^4(e+fx)\,dx = \int (b\sec(fx+e)^2+a)^p \sin(fx+e)^4\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^4,x, algorithm="fricas")
```

output
```
integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*(b*sec(f*x + e)^2 + a)^p,
 x)
```

3.138.6 Sympy [F(-1)]

Timed out.

$$\int (a+b\sec^2(e+fx))^p \sin^4(e+fx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e)**2)**p*sin(f*x+e)**4,x)
```

output
```
Timed out
```

3.138.7 Maxima [F]

$$\int (a+b\sec^2(e+fx))^p \sin^4(e+fx)\,dx = \int (b\sec(fx+e)^2+a)^p \sin(fx+e)^4\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^4,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e)^4, x)
```

3.138.8 Giac [F]

$$\int (a+b\sec^2(e+fx))^p \sin^4(e+fx)\,dx = \int (b\sec(fx+e)^2+a)^p \sin(fx+e)^4\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^4,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e)^4, x)
```

3.138.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\sec^2(e+fx))^p \sin^4(e+fx)\,dx = \int \sin(e+f\,x)^4 \left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p dx$$

input
```
int(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^p,x)
```

output
```
int(sin(e + f*x)^4*(a + b/cos(e + f*x)^2)^p, x)
```

3.139 $\int (a+b\sec^2(e+fx))^p \sin^2(e+fx)\,dx$

3.139.1 Optimal result 1142
3.139.2 Mathematica [B] (warning: unable to verify) 1142
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3.139.4 Maple [F] 1145
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3.139.7 Maxima [F] 1146
3.139.8 Giac [F] 1146
3.139.9 Mupad [F(-1)] 1146

3.139.1 Optimal result

Integrand size = 23, antiderivative size = 88

$$\int (a+b\sec^2(e+fx))^p \sin^2(e+fx)\,dx$$
$$= \frac{\text{AppellF1}\left(\frac{3}{2},2,-p,\frac{5}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan^3(e+fx)\,(a+b+b\tan^2(e+fx))^p\left(1+\frac{b\tan^2(e+}{a+b}\right.}{3f}$$

output
```
1/3*AppellF1(3/2,2,-p,5/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)^
3*(a+b+b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

3.139.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 3781 vs. 2(88) = 176.

Time = 20.19 (sec) , antiderivative size = 3781, normalized size of antiderivative = 42.97

$$\int (a+b\sec^2(e+fx))^p \sin^2(e+fx)\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^2,x]
```

output

```
(3*(a + b)*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-2 + p)*(a +
 b*Sec[e + f*x]^2)^p*Sin[e + f*x]^2*Tan[e + f*x]*(AppellF1[1/2, 2, -p, 3/2
, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]/(-3*(a + b)*AppellF1[1/2
, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(-(b*p*A
ppellF1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))
]) + 2*(a + b)*AppellF1[3/2, 3, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x
]^2)/(a + b))])*Tan[e + f*x]^2) + (AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e +
f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sec[e + f*x]^2)/(3*(a + b)*AppellF1[1/2
, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*App
ellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]
- (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e
+ f*x]^2])*Tan[e + f*x]^2)))/(f*(3*(a + b)*(a + 2*b + a*Cos[2*(e + f*x)])^
p*(Sec[e + f*x]^2)^(-1 + p)*(AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -(
(b*Tan[e + f*x]^2)/(a + b))]/(-3*(a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e
+ f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(-(b*p*AppellF1[3/2, 2, 1 - p
, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]) + 2*(a + b)*Appell
F1[3/2, 3, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e
 + f*x]^2) + (AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Ta
n[e + f*x]^2]*Sec[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Ta
n[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, ...
```

3.139.3 Rubi [A] (verified)

Time $= 0.26$ (sec) , antiderivative size $= 88$, normalized size of antiderivative $= 1.00$, number of steps used $= 5$, number of rules used $= 4$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used $= \{3042, 4620, 395, 394\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$

$$\downarrow 3042$$

$$\int \sin(e+fx)^2\left(a+b\sec(e+fx)^2\right)^p\,dx$$

$$\downarrow 4620$$

$$\frac{\int \frac{\tan^2(e+fx)\left(b\tan^2(e+fx)+a+b\right)^p}{\left(\tan^2(e+fx)+1\right)^2}d\tan(e+fx)}{f}$$

$$\downarrow 395$$

$$\frac{(a+b\tan^2(e+fx)+b)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\int\frac{\tan^2(e+fx)\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^p}{(\tan^2(e+fx)+1)^2}d\tan(e+fx)}{f}$$

$$\downarrow 394$$

$$\frac{\tan^3(e+fx)\,(a+b\tan^2(e+fx)+b)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\text{AppellF1}\left(\frac{3}{2},2,-p,\frac{5}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{3f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]^2,x]
```

output
```
(AppellF1[3/2, 2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
*Tan[e + f*x]^3*(a + b + b*Tan[e + f*x]^2)^p)/(3*f*(1 + (b*Tan[e + f*x]^2)
/(a + b))^p)
```

3.139.3.1 Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.139.4 Maple [F]

$$\int (a + b \sec (fx + e)^2)^p \sin (fx + e)^2 \, dx$$

input
```
int((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^2,x)
```

output
```
int((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^2,x)
```

3.139.5 Fricas [F]

$$\int (a + b \sec^2(e + fx))^p \sin^2(e + fx) \, dx = \int (b \sec (fx + e)^2 + a)^p \sin (fx + e)^2 \, dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^2,x, algorithm="fricas")
```

output
```
integral(-(cos(f*x + e)^2 - 1)*(b*sec(f*x + e)^2 + a)^p, x)
```

3.139.6 Sympy [F(-1)]

Timed out.

$$\int (a + b \sec^2(e + fx))^p \sin^2(e + fx) \, dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e)**2)**p*sin(f*x+e)**2,x)
```

output
```
Timed out
```

3.139.7 Maxima [F]

$$\int (a+b\sec^2(e+fx))^p \sin^2(e+fx)\,dx = \int (b\sec(fx+e)^2+a)^p \sin(fx+e)^2\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e)^2, x)
```

3.139.8 Giac [F]

$$\int (a+b\sec^2(e+fx))^p \sin^2(e+fx)\,dx = \int (b\sec(fx+e)^2+a)^p \sin(fx+e)^2\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*sin(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*sin(f*x + e)^2, x)
```

3.139.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\sec^2(e+fx))^p \sin^2(e+fx)\,dx = \int \sin(e+fx)^2 \left(a+\frac{b}{\cos(e+fx)^2}\right)^p dx$$

input
```
int(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^p,x)
```

output
```
int(sin(e + f*x)^2*(a + b/cos(e + f*x)^2)^p, x)
```

3.140 $\int (a+b\sec^2(e+fx))^p\,dx$

3.140.1 Optimal result 1147
3.140.2 Mathematica [B] (warning: unable to verify) 1147
3.140.3 Rubi [A] (verified) 1148
3.140.4 Maple [F] 1150
3.140.5 Fricas [F] 1150
3.140.6 Sympy [F] 1150
3.140.7 Maxima [F] 1151
3.140.8 Giac [F] 1151
3.140.9 Mupad [F(-1)] 1151

3.140.1 Optimal result

Integrand size = 14, antiderivative size = 83

$$\int (a+b\sec^2(e+fx))^p\,dx$$
$$= \frac{\text{AppellF1}\left(\frac{1}{2},1,-p,\frac{3}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\,(a+b+b\tan^2(e+fx))^p\left(1+\frac{b\tan^2(e+f}{a+b}\right.}{f}$$

output
```
AppellF1(1/2,1,-p,3/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)*(a+b
+b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

3.140.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2137 vs. 2(83) = 166.

Time = 14.98 (sec) , antiderivative size = 2137, normalized size of antiderivative = 25.75

$$\int (a+b\sec^2(e+fx))^p\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^p,x]
```

output

```
(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e
 + f*x]^2]*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^
p*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x])/(f*(3*(a + b)*AppellF1[1/2, -p, 1
, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3
/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a +
b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^
2])*Tan[e + f*x]^2)*((3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x
]^2)/(a + b)), -Tan[e + f*x]^2]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e +
f*x]^2)^(-1 + p))/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2
)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Ta
n[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/
2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) - (3*(
a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f
*x]^2]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]^2)
/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[
e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a
+ b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f
*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) + (6*(a + b)*p*AppellF1
[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*(a + 2*b
 + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]^2)/(3*(a + b)*...
```

3.140.3 Rubi [A] (verified)

Time $= 0.22$ (sec) , antiderivative size $= 83$, normalized size of antiderivative $= 1.00$, number of steps used $= 5$, number of rules used $= 4$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used $= \{3042, 4616, 334, 333\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + b\sec^2(e + fx))^p \, dx$$

$$\downarrow 3042$$

$$\int (a + b\sec(e + fx)^2)^p \, dx$$

$$\downarrow 4616$$

$$\frac{\int \frac{(b\tan^2(e+fx)+a+b)^p}{\tan^2(e+fx)+1} d\tan(e + fx)}{f}$$

$$\downarrow 334$$

$$\frac{(a+b\tan^2(e+fx)+b)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\int\frac{\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^p}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\downarrow 333$$

$$\frac{\tan(e+fx)\,(a+b\tan^2(e+fx)+b)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},1,-p,\frac{3}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
(AppellF1[1/2, 1, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
*Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a
+ b))^p)
```

3.140.3.1 Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

3.140.4 Maple [F]

$$\int (a + b \sec (fx + e)^2)^p \, dx$$

input
```
int((a+b*sec(f*x+e)^2)^p,x)
```

output
```
int((a+b*sec(f*x+e)^2)^p,x)
```

3.140.5 Fricas [F]

$$\int (a + b \sec^2(e + fx))^p \, dx = \int (b \sec (fx + e)^2 + a)^p \, dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p, x)
```

3.140.6 Sympy [F]

$$\int (a + b \sec^2(e + fx))^p \, dx = \int (a + b \sec^2 (e + fx))^p \, dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**p,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**p, x)
```

3.140.7 Maxima [F]

$$\int (a + b \sec^2(e + fx))^p \, dx = \int (b \sec(fx + e)^2 + a)^p \, dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p, x)
```

3.140.8 Giac [F]

$$\int (a + b \sec^2(e + fx))^p \, dx = \int (b \sec(fx + e)^2 + a)^p \, dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p, x)
```

3.140.9 Mupad [F(-1)]

Timed out.

$$\int (a + b \sec^2(e + fx))^p \, dx = \int \left(a + \frac{b}{\cos(e + fx)^2}\right)^p dx$$

input
```
int((a + b/cos(e + f*x)^2)^p,x)
```

output
```
int((a + b/cos(e + f*x)^2)^p, x)
```

3.141 $\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

3.141.1 Optimal result 1152
3.141.2 Mathematica [A] (verified) 1152
3.141.3 Rubi [A] (verified) 1153
3.141.4 Maple [F] 1154
3.141.5 Fricas [F] 1155
3.141.6 Sympy [F(-1)] 1155
3.141.7 Maxima [F] 1155
3.141.8 Giac [F] 1156
3.141.9 Mupad [F(-1)] 1156

3.141.1 Optimal result

Integrand size = 23, antiderivative size = 73

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = -\frac{\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-}}{f}$$

output
```
-cot(f*x+e)*hypergeom([-1/2, -p],[1/2],-b*tan(f*x+e)^2/(a+b))*(a+b+b*tan(f
*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

3.141.2 Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.99

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = -\frac{\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)\left(a+b\sec^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}}{f}$$

input
```
Integrate[Csc[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
-((Cot[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/(a +
 b))]*(a + b*Sec[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a + b))^p))
```

3.141.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4620, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^p}{\sin(e+fx)^2}dx$$

$$\downarrow 4620$$

$$\frac{\int \cot^2(e+fx)\left(b\tan^2(e+fx)+a+b\right)^p d\tan(e+fx)}{f}$$

$$\downarrow 279$$

$$\frac{\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\int \cot^2(e+fx)\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^p d\tan(e+fx)}{f}$$

$$\downarrow 278$$

$$-\frac{\cot(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

input
```
Int[Csc[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
-((Cot[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/(a +
 b))]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a + b))^p)
)
```

3.141.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.141.4 Maple [F]

$$\int \csc(fx+e)^2 \left(a+b\sec(fx+e)^2\right)^p dx$$

input
```
int(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x)
```

3.141.5 Fricas [F]

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p \csc\left(fx+e\right)^2\,dx$$

input
```
integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^2, x)
```

3.141.6 Sympy [F(-1)]

Timed out.

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**2*(a+b*sec(f*x+e)**2)**p,x)
```

output
```
Timed out
```

3.141.7 Maxima [F]

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p \csc\left(fx+e\right)^2\,dx$$

input
```
integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^2, x)
```

3.141.8 Giac [F]

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p \csc\left(fx+e\right)^2\,dx$$

input
```
integrate(csc(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^2, x)
```

3.141.9 Mupad [F(-1)]

Timed out.

$$\int \csc^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p}{\sin\left(e+f\,x\right)^2}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^p/sin(e + f*x)^2,x)
```

output
```
int((a + b/cos(e + f*x)^2)^p/sin(e + f*x)^2, x)
```

3.142 $\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

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3.142.1 Optimal result

Integrand size = 23, antiderivative size = 128

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = -\frac{\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{3(a+b)f}$$
$$-\frac{(3a+2b(1+p))\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)\left(a+b+b\tan^2(e+fx)\right)^p\left($$
$$3(a+b)f$$

output
```
-1/3*cot(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(p+1)/(a+b)/f-1/3*(3*a+2*b*(p+1))*c
ot(f*x+e)*hypergeom([-1/2, -p],[1/2],-b*tan(f*x+e)^2/(a+b))*(a+b+b*tan(f*x
+e)^2)^p/(a+b)/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

3.142.2 Mathematica [A] (verified)

Time = 1.58 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.03

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx =$$
$$-\frac{\cot(e+fx)\left(a+b\sec^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\left((3a+2b(1+p))\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\right.\right.$$
$$3(a+b)f$$

input
```
Integrate[Csc[e + f*x]^4*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
-1/3*(Cot[e + f*x]*(a + b*Sec[e + f*x]^2)^p*((3*a + 2*b*(1 + p))*Hypergeom
etric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/(a + b))] + Cot[e + f*x]^2*(a
 + b + b*Tan[e + f*x]^2)*(1 + (b*Tan[e + f*x]^2)/(a + b))^p))/((a + b)*f*(
1 + (b*Tan[e + f*x]^2)/(a + b))^p)
```

3.142.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4620, 359, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^p}{\sin(e+fx)^4}dx$$

$$\downarrow 4620$$

$$\frac{\int \cot^4(e+fx)\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^p d\tan(e+fx)}{f}$$

$$\downarrow 359$$

$$\frac{\frac{(3a+2b(p+1))\int \cot^2(e+fx)\left(b\tan^2(e+fx)+a+b\right)^p d\tan(e+fx)}{3(a+b)} - \frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{p+1}}{3(a+b)}}{f}$$

$$\downarrow 279$$

$$\frac{\frac{(3a+2b(p+1))\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\int \cot^2(e+fx)\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^p d\tan(e+fx)}{3(a+b)} - \frac{\cot^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{p+}}{3(a+b)}}{f}$$

$$\downarrow 278$$

$$\frac{-\frac{(3a+2b(p+1))\cot(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)}{3(a+b)} - \frac{\cot^3(e+fx)\left(a+b\tan\right.}{3(a+b}}{f}$$

input
```
Int[Csc[e + f*x]^4*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
(-1/3*(Cot[e + f*x]^3*(a + b + b*Tan[e + f*x]^2)^(1 + p))/(a + b) - ((3*a
+ 2*b*(1 + p))*Cot[e + f*x]*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e +
f*x]^2)/(a + b))]*(a + b + b*Tan[e + f*x]^2)^p)/(3*(a + b)*(1 + (b*Tan[e +
 f*x]^2)/(a + b))^p))/f
```

3.142.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.142.4 Maple [F]

$$\int \csc(fx+e)^4 \left(a+b\sec(fx+e)^2\right)^p dx$$

input
```
int(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x)
```

3.142.5 Fricas [F]

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p dx = \int \left(b\sec(fx+e)^2+a\right)^p \csc(fx+e)^4 dx$$

input
```
integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^4, x)
```

3.142.6 Sympy [F(-1)]

Timed out.

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**4*(a+b*sec(f*x+e)**2)**p,x)
```

output
```
Timed out
```

3.142.7 Maxima [F]

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p \csc\left(fx+e\right)^4\,dx$$

input
```
integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^4, x)
```

3.142.8 Giac [F]

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p \csc\left(fx+e\right)^4\,dx$$

input
```
integrate(csc(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^4, x)
```

3.142.9 Mupad [F(-1)]

Timed out.

$$\int \csc^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p}{\sin\left(e+f\,x\right)^4}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^p/sin(e + f*x)^4,x)
```

output
```
int((a + b/cos(e + f*x)^2)^p/sin(e + f*x)^4, x)
```

3.143 $\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

3.143.1 Optimal result 1162
3.143.2 Mathematica [A] (verified) 1163
3.143.3 Rubi [A] (verified) 1163
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3.143.7 Maxima [F] 1166
3.143.8 Giac [F] 1167
3.143.9 Mupad [F(-1)] 1167

3.143.1 Optimal result

Integrand size = 23, antiderivative size = 192

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$
$$= -\frac{(10a+b(7+2p))\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{15(a+b)^2 f}$$
$$-\frac{\cot^5(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{5(a+b)f}$$
$$-\frac{\left(15a^2+20ab(1+p)+4b^2(2+3p+p^2)\right)\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)(a}{15(a+b)^2 f}$$

output

```
-1/15*(10*a+b*(7+2*p))*cot(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(p+1)/(a+b)^2/f-1
/5*cot(f*x+e)^5*(a+b+b*tan(f*x+e)^2)^(p+1)/(a+b)/f-1/15*(15*a^2+20*a*b*(p+
1)+4*b^2*(p^2+3*p+2))*cot(f*x+e)*hypergeom([-1/2, -p],[1/2],-b*tan(f*x+e)^
2/(a+b))*(a+b+b*tan(f*x+e)^2)^p/(a+b)^2/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

3.143.2 Mathematica [A] (verified)

Time = 1.44 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.78

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx =$$

$$-\frac{\cot(e+fx)\left(3\cot^4(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},-p,-\frac{3}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)+10\cot^2(e+fx)\,\text{Hyperg}\right.}{}$$

input
```
Integrate[Csc[e + f*x]^6*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
-1/15*(Cot[e + f*x]*(3*Cot[e + f*x]^4*Hypergeometric2F1[-5/2, -p, -3/2, -(
(b*Tan[e + f*x]^2)/(a + b))] + 10*Cot[e + f*x]^2*Hypergeometric2F1[-3/2, -
p, -1/2, -((b*Tan[e + f*x]^2)/(a + b))] + 15*Hypergeometric2F1[-1/2, -p, 1
/2, -((b*Tan[e + f*x]^2)/(a + b))])*(a + b*Sec[e + f*x]^2)^p)/(f*(1 + (b*T
an[e + f*x]^2)/(a + b))^p)
```

3.143.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4620, 365, 359, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^p}{\sin(e+fx)^6}dx$$

$$\downarrow 4620$$

$$\frac{\int \cot^6(e+fx)\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^p d\tan(e+fx)}{f}$$

$$\downarrow 365$$

$$\frac{\int \cot^4(e+fx)\left(b\tan^2(e+fx)+a+b\right)^p\left(5(a+b)\tan^2(e+fx)+10a+b(2p+7)\right)d\tan(e+fx)}{5(a+b)} - \frac{\cot^5(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{p+1}}{5(a+b)}$$
$$f$$
$$\downarrow 359$$
$$\frac{\left(15a^2+20ab(p+1)+4b^2\left(p^2+3p+2\right)\right)\int \cot^2(e+fx)\left(b\tan^2(e+fx)+a+b\right)^p d\tan(e+fx)}{3(a+b)} - \frac{(10a+b(2p+7))\cot^3(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{p+1}}{3(a+b)} - \frac{\cot^5(e+fx)(a}{5(a+b)}$$
$$f$$
$$\downarrow 279$$
$$\frac{\left(15a^2+20ab(p+1)+4b^2\left(p^2+3p+2\right)\right)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\int \cot^2(e+fx)\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^p d\tan(e+fx)}{3(a+b)} - \frac{(10a+b(2p+7))\cot^3(e+fx)\left(a}{3(a+b)}$$
$$5(a+b)$$
$$f$$
$$\downarrow 278$$
$$-\frac{\left(15a^2+20ab(p+1)+4b^2\left(p^2+3p+2\right)\right)\cot(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)}{3(a+b)} - \frac{(10a+b(2p+7)}{}$$
$$5(a+b)$$
$$f$$

input
```
Int[Csc[e + f*x]^6*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
(-1/5*(Cot[e + f*x]^5*(a + b + b*Tan[e + f*x]^2)^(1 + p))/(a + b) + (-1/3*
((10*a + b*(7 + 2*p))*Cot[e + f*x]^3*(a + b + b*Tan[e + f*x]^2)^(1 + p))/(
a + b) - ((15*a^2 + 20*a*b*(1 + p) + 4*b^2*(2 + 3*p + p^2))*Cot[e + f*x]*H
ypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + b + b*
Tan[e + f*x]^2)^p)/(3*(a + b)*(1 + (b*Tan[e + f*x]^2)/(a + b))^p))/(5*(a +
 b)))/f
```

3.143.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4620
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff^(m
+ 1)/f   Subst[Int[x^m*(ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p/(1 + f
f^2*x^2)^(m/2 + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p},
 x] && IntegerQ[m/2] && IntegerQ[n/2]
```

3.143.4 Maple [F]

$$\int \csc(fx+e)^6 \left(a+b\sec(fx+e)^2\right)^p dx$$

input
```
int(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x)
```

3.143.5 Fricas [F]

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p \csc\left(fx+e\right)^6\,dx$$

input
```
integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^6, x)
```

3.143.6 Sympy [F(-1)]

Timed out.

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**6*(a+b*sec(f*x+e)**2)**p,x)
```

output
```
Timed out
```

3.143.7 Maxima [F]

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p \csc\left(fx+e\right)^6\,dx$$

input
```
integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^6, x)
```

3.143.8 Giac [F]

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p \csc\left(fx+e\right)^6\,dx$$

input
```
integrate(csc(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*csc(f*x + e)^6, x)
```

3.143.9 Mupad [F(-1)]

Timed out.

$$\int \csc^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p}{\sin\left(e+f\,x\right)^6}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^p/sin(e + f*x)^6,x)
```

output
```
int((a + b/cos(e + f*x)^2)^p/sin(e + f*x)^6, x)
```

3.144 $\int (a - a\sec^2(c+dx))^4\,dx$

3.144.1 Optimal result 1168
3.144.2 Mathematica [A] (verified) 1168
3.144.3 Rubi [A] (verified) 1169
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3.144.7 Maxima [A] (verification not implemented) 1172
3.144.8 Giac [A] (verification not implemented) 1173
3.144.9 Mupad [B] (verification not implemented) 1173

3.144.1 Optimal result

Integrand size = 15, antiderivative size = 74

$$\int (a - a\sec^2(c+dx))^4\,dx = a^4x - \frac{a^4\tan(c+dx)}{d} + \frac{a^4\tan^3(c+dx)}{3d} - \frac{a^4\tan^5(c+dx)}{5d} + \frac{a^4\tan^7(c+dx)}{7d}$$

output
```
a^4*x-a^4*tan(d*x+c)/d+1/3*a^4*tan(d*x+c)^3/d-1/5*a^4*tan(d*x+c)^5/d+1/7*a
^4*tan(d*x+c)^7/d
```

3.144.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.97

$$\int (a - a\sec^2(c+dx))^4\,dx = a^4\left(\frac{\arctan(\tan(c+dx))}{d} - \frac{\tan(c+dx)}{d} + \frac{\tan^3(c+dx)}{3d} - \frac{\tan^5(c+dx)}{5d} + \frac{\tan^7(c+dx)}{7d}\right)$$

input
```
Integrate[(a - a*Sec[c + d*x]^2)^4,x]
```

output
```
a^4*(ArcTan[Tan[c + d*x]]/d - Tan[c + d*x]/d + Tan[c + d*x]^3/(3*d) - Tan[
c + d*x]^5/(5*d) + Tan[c + d*x]^7/(7*d))
```

3.144.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.733$, Rules used = {3042, 4608, 3042, 3954, 3042, 3954, 3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a - a\sec^2(c+dx))^4\,dx$$

$$\downarrow 3042$$

$$\int (a - a\sec(c+dx)^2)^4\,dx$$

$$\downarrow 4608$$

$$a^4 \int \tan^8(c+dx)dx$$

$$\downarrow 3042$$

$$a^4 \int \tan(c+dx)^8 dx$$

$$\downarrow 3954$$

$$a^4\left(\frac{\tan^7(c+dx)}{7d} - \int \tan^6(c+dx)dx\right)$$

$$\downarrow 3042$$

$$a^4\left(\frac{\tan^7(c+dx)}{7d} - \int \tan(c+dx)^6 dx\right)$$

$$\downarrow 3954$$

$$a^4\left(\int \tan^4(c+dx)dx + \frac{\tan^7(c+dx)}{7d} - \frac{\tan^5(c+dx)}{5d}\right)$$

$$\downarrow 3042$$

$$a^4\left(\int \tan(c+dx)^4 dx + \frac{\tan^7(c+dx)}{7d} - \frac{\tan^5(c+dx)}{5d}\right)$$

$$\downarrow 3954$$

$$a^4\left(-\int \tan^2(c+dx)dx + \frac{\tan^7(c+dx)}{7d} - \frac{\tan^5(c+dx)}{5d} + \frac{\tan^3(c+dx)}{3d}\right)$$

$$\downarrow 3042$$

$$a^4\left(-\int \tan(c+dx)^2dx+\frac{\tan^7(c+dx)}{7d}-\frac{\tan^5(c+dx)}{5d}+\frac{\tan^3(c+dx)}{3d}\right)$$

$$\downarrow 3954$$

$$a^4\left(\int 1dx+\frac{\tan^7(c+dx)}{7d}-\frac{\tan^5(c+dx)}{5d}+\frac{\tan^3(c+dx)}{3d}-\frac{\tan(c+dx)}{d}\right)$$

$$\downarrow 24$$

$$a^4\left(\frac{\tan^7(c+dx)}{7d}-\frac{\tan^5(c+dx)}{5d}+\frac{\tan^3(c+dx)}{3d}-\frac{\tan(c+dx)}{d}+x\right)$$

input
```
Int[(a - a*Sec[c + d*x]^2)^4,x]
```

output
```
a^4*(x - Tan[c + d*x]/d + Tan[c + d*x]^3/(3*d) - Tan[c + d*x]^5/(5*d) + Ta
n[c + d*x]^7/(7*d))
```

3.144.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4608
```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
b^p   Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.144.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.86 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.31

method	result
risch	$a^4x-\frac{8ia^4\left(105\,e^{12i(dx+c)}+315\,e^{10i(dx+c)}+770\,e^{8i(dx+c)}+770\,e^{6i(dx+c)}+609\,e^{4i(dx+c)}+203\,e^{2i(dx+c)}+44\right)}{105d\left(e^{2i(dx+c)}+1\right)^7}$
derivativedivides	$\frac{a^4(dx+c)-4a^4\tan(dx+c)-6a^4\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+4a^4\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)-a^4\left($}{d}$
default	$\frac{a^4(dx+c)-4a^4\tan(dx+c)-6a^4\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+4a^4\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)-a^4\left($}{d}$
parts	$a^4x-\frac{a^4\left(-\frac{16}{35}-\frac{\sec(dx+c)^6}{7}-\frac{6\sec(dx+c)^4}{35}-\frac{8\sec(dx+c)^2}{35}\right)\tan(dx+c)}{d}-\frac{4a^4\tan(dx+c)}{d}-\frac{6a^4\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)t$
parallelrisch	$\frac{a^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}xd-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}xd+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{13}+21\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}xd-\frac{44\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{3}-35\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8\right.$}{}$
norman	$\frac{a^4x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}-a^4x+\frac{2a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{44a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}+\frac{706a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{15d}-\frac{3048a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{35d}+\frac{706a^4\tan\left(\frac{dx}{2}\right.}{15d}$

input
```
int((a-a*sec(d*x+c)^2)^4,x,method=_RETURNVERBOSE)
```

output
```
a^4*x-8/105*I*a^4*(105*exp(12*I*(d*x+c))+315*exp(10*I*(d*x+c))+770*exp(8*I
*(d*x+c))+770*exp(6*I*(d*x+c))+609*exp(4*I*(d*x+c))+203*exp(2*I*(d*x+c))+4
4)/d/(exp(2*I*(d*x+c))+1)^7
```

3.144.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.11

$$\int \left(a-a\sec^2(c+dx)\right)^4\,dx$$
$$=\frac{105\,a^4dx\cos\left(dx+c\right)^7-\left(176\,a^4\cos\left(dx+c\right)^6-122\,a^4\cos\left(dx+c\right)^4+66\,a^4\cos\left(dx+c\right)^2-15\,a^4\right)\sin\left(dx\right.}{105\,d\cos\left(dx+c\right)^7}$$

input
```
integrate((a-a*sec(d*x+c)^2)^4,x, algorithm="fricas")
```

output
```
1/105*(105*a^4*d*x*cos(d*x + c)^7 - (176*a^4*cos(d*x + c)^6 - 122*a^4*cos(
d*x + c)^4 + 66*a^4*cos(d*x + c)^2 - 15*a^4)*sin(d*x + c))/(d*cos(d*x + c)
^7)
```

3.144.6 Sympy [F]

$$\int \left(a - a\sec^2(c+dx)\right)^4 dx = a^4\left(\int 1\,dx + \int \left(-4\sec^2(c+dx)\right)\,dx + \int 6\sec^4(c+dx)\,dx + \int \left(-4\sec^6(c+dx)\right)\,dx + \int \sec^8(c+dx)\,dx\right)$$

input
```
integrate((a-a*sec(d*x+c)**2)**4,x)
```

output
```
a**4*(Integral(1, x) + Integral(-4*sec(c + d*x)**2, x) + Integral(6*sec(c
+ d*x)**4, x) + Integral(-4*sec(c + d*x)**6, x) + Integral(sec(c + d*x)**8
, x))
```

3.144.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.74

$$\begin{aligned}&\int \left(a - a\sec^2(c+dx)\right)^4 dx \\ &= a^4x + \frac{\left(5\tan(dx+c)^7 + 21\tan(dx+c)^5 + 35\tan(dx+c)^3 + 35\tan(dx+c)\right)a^4}{35\,d} \\ &\quad - \frac{4\left(3\tan(dx+c)^5 + 10\tan(dx+c)^3 + 15\tan(dx+c)\right)a^4}{15\,d} \\ &\quad + \frac{2\left(\tan(dx+c)^3 + 3\tan(dx+c)\right)a^4}{d} - \frac{4\,a^4\tan(dx+c)}{d}\end{aligned}$$

input
```
integrate((a-a*sec(d*x+c)^2)^4,x, algorithm="maxima")
```

output
```
a^4*x + 1/35*(5*tan(d*x + c)^7 + 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 3
5*tan(d*x + c))*a^4/d - 4/15*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*ta
n(d*x + c))*a^4/d + 2*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^4/d - 4*a^4*tan(
d*x + c)/d
```

3.144.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.89

$$\int \left(a - a\sec^2(c+dx)\right)^4 dx$$
$$= \frac{15\,a^4\tan(dx+c)^7 - 21\,a^4\tan(dx+c)^5 + 35\,a^4\tan(dx+c)^3 + 105\,(dx+c)a^4 - 105\,a^4\tan(dx+c)}{105\,d}$$

input
```
integrate((a-a*sec(d*x+c)^2)^4,x, algorithm="giac")
```

output
```
1/105*(15*a^4*tan(d*x + c)^7 - 21*a^4*tan(d*x + c)^5 + 35*a^4*tan(d*x + c)
^3 + 105*(d*x + c)*a^4 - 105*a^4*tan(d*x + c))/d
```

3.144.9 Mupad [B] (verification not implemented)

Time = 19.40 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.82

$$\int \left(a - a\sec^2(c+dx)\right)^4 dx$$
$$= \frac{\frac{a^4\tan(c+d\,x)^7}{7} - \frac{a^4\tan(c+d\,x)^5}{5} + \frac{a^4\tan(c+d\,x)^3}{3} - a^4\tan(c+d\,x) + d\,x\,a^4}{d}$$

input
```
int((a - a/cos(c + d*x)^2)^4,x)
```

output
```
((a^4*tan(c + d*x)^3)/3 - a^4*tan(c + d*x) - (a^4*tan(c + d*x)^5)/5 + (a^4
*tan(c + d*x)^7)/7 + a^4*d*x)/d
```

3.145 $\int (a - a\sec^2(c+dx))^3\, dx$

3.145.1 Optimal result 1174
3.145.2 Mathematica [A] (verified) 1174
3.145.3 Rubi [A] (verified) 1175
3.145.4 Maple [C] (verified) 1177
3.145.5 Fricas [A] (verification not implemented) 1177
3.145.6 Sympy [F] 1178
3.145.7 Maxima [A] (verification not implemented) 1178
3.145.8 Giac [A] (verification not implemented) 1178
3.145.9 Mupad [B] (verification not implemented) 1179

3.145.1 Optimal result

Integrand size = 15, antiderivative size = 56

$$\int (a - a\sec^2(c+dx))^3\, dx = a^3 x - \frac{a^3\tan(c+dx)}{d} + \frac{a^3\tan^3(c+dx)}{3d} - \frac{a^3\tan^5(c+dx)}{5d}$$

output
```
a^3*x-a^3*tan(d*x+c)/d+1/3*a^3*tan(d*x+c)^3/d-1/5*a^3*tan(d*x+c)^5/d
```

3.145.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.04

$$\int (a - a\sec^2(c+dx))^3\, dx = -a^3\left(-\frac{\arctan(\tan(c+dx))}{d} + \frac{\tan(c+dx)}{d} - \frac{\tan^3(c+dx)}{3d} + \frac{\tan^5(c+dx)}{5d}\right)$$

input
```
Integrate[(a - a*Sec[c + d*x]^2)^3,x]
```

output
```
-(a^3*(-(ArcTan[Tan[c + d*x]]/d) + Tan[c + d*x]/d - Tan[c + d*x]^3/(3*d) +
 Tan[c + d*x]^5/(5*d)))
```

3.145.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.88, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 4608, 3042, 3954, 3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a - a\sec^2(c+dx)\right)^3 dx$$

$$\downarrow 3042$$

$$\int \left(a - a\sec(c+dx)^2\right)^3 dx$$

$$\downarrow 4608$$

$$-a^3 \int \tan^6(c+dx)dx$$

$$\downarrow 3042$$

$$-a^3 \int \tan(c+dx)^6 dx$$

$$\downarrow 3954$$

$$-a^3\left(\frac{\tan^5(c+dx)}{5d} - \int \tan^4(c+dx)dx\right)$$

$$\downarrow 3042$$

$$-a^3\left(\frac{\tan^5(c+dx)}{5d} - \int \tan(c+dx)^4 dx\right)$$

$$\downarrow 3954$$

$$-a^3\left(\int \tan^2(c+dx)dx + \frac{\tan^5(c+dx)}{5d} - \frac{\tan^3(c+dx)}{3d}\right)$$

$$\downarrow 3042$$

$$-a^3\left(\int \tan(c+dx)^2 dx + \frac{\tan^5(c+dx)}{5d} - \frac{\tan^3(c+dx)}{3d}\right)$$

$$\downarrow 3954$$

$$-a^3\left(-\int 1dx + \frac{\tan^5(c+dx)}{5d} - \frac{\tan^3(c+dx)}{3d} + \frac{\tan(c+dx)}{d}\right)$$

$$\downarrow 24$$

$$-a^3\left(\frac{\tan^5(c+dx)}{5d}-\frac{\tan^3(c+dx)}{3d}+\frac{\tan(c+dx)}{d}-x\right)$$

input
```
Int[(a - a*Sec[c + d*x]^2)^3,x]
```

output
```
-(a^3*(-x + Tan[c + d*x]/d - Tan[c + d*x]^3/(3*d) + Tan[c + d*x]^5/(5*d)))
```

3.145.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4608
```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
b^p   Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.145.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.34

method	result
risch	$a^3x-\frac{2ia^3\left(45\,e^{8i(dx+c)}+90\,e^{6i(dx+c)}+140\,e^{4i(dx+c)}+70\,e^{2i(dx+c)}+23\right)}{15d\left(e^{2i(dx+c)}+1\right)^5}$
derivativedivides	$\frac{a^3(dx+c)-3a^3\tan(dx+c)-3a^3\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+a^3\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)}{d}$
default	$\frac{a^3(dx+c)-3a^3\tan(dx+c)-3a^3\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+a^3\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)}{d}$
parts	$a^3x-\frac{3a^3\tan(dx+c)}{d}-\frac{3a^3\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}+\frac{a^3\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)}{d}$
parallelrisch	$\frac{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}xd-5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8xd+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9+10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6xd-\frac{32\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{3}-10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4xd+\frac{35}{}\right.}{d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}$
norman	$\frac{a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}-a^3x+\frac{2a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{32a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}+\frac{356a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{15d}-\frac{32a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{3d}+\frac{2a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)^5}$

input
```
int((a-a*sec(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
a^3*x-2/15*I*a^3*(45*exp(8*I*(d*x+c))+90*exp(6*I*(d*x+c))+140*exp(4*I*(d*x
+c))+70*exp(2*I*(d*x+c))+23)/d/(exp(2*I*(d*x+c))+1)^5
```

3.145.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.23

$$\int \left(a-a\sec^2(c+dx)\right)^3\,dx = \frac{15\,a^3dx\cos\left(dx+c\right)^5-\left(23\,a^3\cos\left(dx+c\right)^4-11\,a^3\cos\left(dx+c\right)^2+3\,a^3\right)\sin\left(dx+c\right)}{15\,d\cos\left(dx+c\right)^5}$$

input
```
integrate((a-a*sec(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/15*(15*a^3*d*x*cos(d*x + c)^5 - (23*a^3*cos(d*x + c)^4 - 11*a^3*cos(d*x
+ c)^2 + 3*a^3)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

3.145.6 Sympy [F]

$$\int \left(a - a\sec^2(c+dx)\right)^3\,dx = -a^3\left(\int (-1)\,dx + \int 3\sec^2(c+dx)\,dx + \int \left(-3\sec^4(c+dx)\right)\,dx + \int \sec^6(c+dx)\,dx\right)$$

input
```
integrate((a-a*sec(d*x+c)**2)**3,x)
```

output
```
-a**3*(Integral(-1, x) + Integral(3*sec(c + d*x)**2, x) + Integral(-3*sec(
c + d*x)**4, x) + Integral(sec(c + d*x)**6, x))
```

3.145.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.45

$$\int \left(a - a\sec^2(c+dx)\right)^3\,dx = a^3x - \frac{\left(3\tan(dx+c)^5 + 10\tan(dx+c)^3 + 15\tan(dx+c)\right)a^3}{15\,d} + \frac{\left(\tan(dx+c)^3 + 3\tan(dx+c)\right)a^3}{d} - \frac{3\,a^3\tan(dx+c)}{d}$$

input
```
integrate((a-a*sec(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
a^3*x - 1/15*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^3/
d + (tan(d*x + c)^3 + 3*tan(d*x + c))*a^3/d - 3*a^3*tan(d*x + c)/d
```

3.145.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95

$$\int \left(a - a\sec^2(c+dx)\right)^3\,dx = -\frac{3\,a^3\tan(dx+c)^5 - 5\,a^3\tan(dx+c)^3 - 15\,(dx+c)a^3 + 15\,a^3\tan(dx+c)}{15\,d}$$

input
```
integrate((a-a*sec(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
-1/15*(3*a^3*tan(d*x + c)^5 - 5*a^3*tan(d*x + c)^3 - 15*(d*x + c)*a^3 + 15
*a^3*tan(d*x + c))/d
```

3.145.9 Mupad [B] (verification not implemented)

Time = 19.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.88

$$\int \left(a - a\sec^2(c+dx)\right)^3\,dx = -\frac{\frac{a^3\tan(c+dx)^5}{5} - \frac{a^3\tan(c+dx)^3}{3} + a^3\tan(c+dx) - dx\,a^3}{d}$$

input
```
int((a - a/cos(c + d*x)^2)^3,x)
```

output
```
-(a^3*tan(c + d*x) - (a^3*tan(c + d*x)^3)/3 + (a^3*tan(c + d*x)^5)/5 - a^3
*d*x)/d
```

3.146 $\int (a - a\sec^2(c+dx))^2\,dx$

3.146.1 Optimal result 1180
3.146.2 Mathematica [A] (verified) 1180
3.146.3 Rubi [A] (verified) 1181
3.146.4 Maple [A] (verified) 1182
3.146.5 Fricas [A] (verification not implemented) 1183
3.146.6 Sympy [F] 1183
3.146.7 Maxima [A] (verification not implemented) 1183
3.146.8 Giac [A] (verification not implemented) 1184
3.146.9 Mupad [B] (verification not implemented) 1184

3.146.1 Optimal result

Integrand size = 15, antiderivative size = 38

$$\int (a - a\sec^2(c+dx))^2\,dx = a^2x - \frac{a^2\tan(c+dx)}{d} + \frac{a^2\tan^3(c+dx)}{3d}$$

output
```
a^2*x-a^2*tan(d*x+c)/d+1/3*a^2*tan(d*x+c)^3/d
```

3.146.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

$$\int (a - a\sec^2(c+dx))^2\,dx = a^2\left(\frac{\arctan(\tan(c+dx))}{d} - \frac{\tan(c+dx)}{d} + \frac{\tan^3(c+dx)}{3d}\right)$$

input
```
Integrate[(a - a*Sec[c + d*x]^2)^2,x]
```

output
```
a^2*(ArcTan[Tan[c + d*x]]/d - Tan[c + d*x]/d + Tan[c + d*x]^3/(3*d))
```

3.146.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {3042, 4608, 3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a - a\sec^2(c+dx))^2\,dx$$

$$\downarrow 3042$$

$$\int (a - a\sec(c+dx)^2)^2\,dx$$

$$\downarrow 4608$$

$$a^2 \int \tan^4(c+dx)dx$$

$$\downarrow 3042$$

$$a^2 \int \tan(c+dx)^4 dx$$

$$\downarrow 3954$$

$$a^2\left(\frac{\tan^3(c+dx)}{3d} - \int \tan^2(c+dx)dx\right)$$

$$\downarrow 3042$$

$$a^2\left(\frac{\tan^3(c+dx)}{3d} - \int \tan(c+dx)^2 dx\right)$$

$$\downarrow 3954$$

$$a^2\left(\int 1dx + \frac{\tan^3(c+dx)}{3d} - \frac{\tan(c+dx)}{d}\right)$$

$$\downarrow 24$$

$$a^2\left(\frac{\tan^3(c+dx)}{3d} - \frac{\tan(c+dx)}{d} + x\right)$$

input
```
Int[(a - a*Sec[c + d*x]^2)^2,x]
```

output
```
a^2*(x - Tan[c + d*x]/d + Tan[c + d*x]^3/(3*d))
```

3.146.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4608
```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
b^p   Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.146.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.24

method	result
parts	$a^2x-\frac{a^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}-\frac{2a^2\tan(dx+c)}{d}$
derivativedivides	$\frac{a^2(dx+c)-2a^2\tan(dx+c)-a^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}$
default	$\frac{a^2(dx+c)-2a^2\tan(dx+c)-a^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}$
risch	$a^2x-\frac{4ia^2\left(3\mathrm{e}^{4i(dx+c)}+3\mathrm{e}^{2i(dx+c)}+2\right)}{3d\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}$
parallelrisch	$\frac{a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6xd-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4xd+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5+3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2xd-\frac{20\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-dx+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}$
norman	$\frac{a^2x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6-a^2x+\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{20a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}+\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d}+3a^2x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-3a^2x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)^3}$

input
```
int((a-a*sec(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x-a^2/d*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)-2*a^2*tan(d*x+c)/d
```

3.146.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.47

$$\int \left(a - a\sec^2(c+dx)\right)^2 \, dx = \frac{3\,a^2 dx \cos\left(dx+c\right)^3 - \left(4\,a^2\cos\left(dx+c\right)^2 - a^2\right)\sin\left(dx+c\right)}{3\,d\cos\left(dx+c\right)^3}$$

input
```
integrate((a-a*sec(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/3*(3*a^2*d*x*cos(d*x + c)^3 - (4*a^2*cos(d*x + c)^2 - a^2)*sin(d*x + c))
/(d*cos(d*x + c)^3)
```

3.146.6 Sympy [F]

$$\int \left(a - a\sec^2(c+dx)\right)^2 \, dx = a^2\left(\int 1\,dx + \int \left(-2\sec^2\left(c+dx\right)\right)\,dx + \int \sec^4\left(c+dx\right)dx\right)$$

input
```
integrate((a-a*sec(d*x+c)**2)**2,x)
```

output
```
a**2*(Integral(1, x) + Integral(-2*sec(c + d*x)**2, x) + Integral(sec(c +
d*x)**4, x))
```

3.146.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.18

$$\int \left(a - a\sec^2(c+dx)\right)^2 \, dx = a^2 x + \frac{\left(\tan\left(dx+c\right)^3 + 3\,\tan\left(dx+c\right)\right)a^2}{3\,d} - \frac{2\,a^2\tan\left(dx+c\right)}{d}$$

input
```
integrate((a-a*sec(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
a^2*x + 1/3*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2/d - 2*a^2*tan(d*x + c)/d
```

3.146.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

$$\int \left(a - a\sec^2(c+dx)\right)^2 \, dx = \frac{a^2 \tan(dx+c)^3 + 3\,(dx+c)a^2 - 3\,a^2 \tan(dx+c)}{3\,d}$$

input
```
integrate((a-a*sec(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/3*(a^2*tan(d*x + c)^3 + 3*(d*x + c)*a^2 - 3*a^2*tan(d*x + c))/d
```

3.146.9 Mupad [B] (verification not implemented)

Time = 18.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \left(a - a\sec^2(c+dx)\right)^2 \, dx = a^2\,x - \frac{a^2\left(3\tan(c+d\,x) - \tan(c+d\,x)^3\right)}{3\,d}$$

input
```
int((a - a/cos(c + d*x)^2)^2,x)
```

output
```
a^2*x - (a^2*(3*tan(c + d*x) - tan(c + d*x)^3))/(3*d)
```

3.147 $\int (a - a\sec^2(c+dx))\,dx$

3.147.1 Optimal result 1185
3.147.2 Mathematica [A] (verified) 1185
3.147.3 Rubi [A] (verified) 1186
3.147.4 Maple [A] (verified) 1186
3.147.5 Fricas [A] (verification not implemented) 1187
3.147.6 Sympy [F] 1187
3.147.7 Maxima [A] (verification not implemented) 1187
3.147.8 Giac [A] (verification not implemented) 1188
3.147.9 Mupad [B] (verification not implemented) 1188

3.147.1 Optimal result

Integrand size = 13, antiderivative size = 16

$$\int (a - a\sec^2(c+dx))\,dx = ax - \frac{a\tan(c+dx)}{d}$$

output
```
a*x-a*tan(d*x+c)/d
```

3.147.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int (a - a\sec^2(c+dx))\,dx = -a\left(-\frac{\arctan(\tan(c+dx))}{d} + \frac{\tan(c+dx)}{d}\right)$$

input
```
Integrate[a - a*Sec[c + d*x]^2,x]
```

output
```
-(a*(-(ArcTan[Tan[c + d*x]]/d) + Tan[c + d*x]/d))
```

3.147.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a - a\sec^2(c+dx))\,dx$$

$$\downarrow \text{2009}$$

$$ax - \frac{a\tan(c+dx)}{d}$$

input
```
Int[a - a*Sec[c + d*x]^2,x]
```

output
```
a*x - (a*Tan[c + d*x])/d
```

3.147.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.147.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result	size
default	$ax - \frac{a\tan(dx+c)}{d}$	17
parts	$ax - \frac{a\tan(dx+c)}{d}$	17
derivativedivides	$\frac{(dx+c)a - a\tan(dx+c)}{d}$	22
risch	$ax - \frac{2ia}{d\left(e^{2i(dx+c)}+1\right)}$	25
parallelrisch	$\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)} + ax$	35
norman	$\frac{ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 - ax + \frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}$	51

input
```
int(a-a*sec(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
a*x-a*tan(d*x+c)/d
```

3.147.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.00

$$\int \left(a - a\sec^2(c+dx)\right)\, dx = \frac{adx\cos\left(dx+c\right) - a\sin\left(dx+c\right)}{d\cos\left(dx+c\right)}$$

input
```
integrate(a-a*sec(d*x+c)^2,x, algorithm="fricas")
```

output
```
(a*d*x*cos(d*x + c) - a*sin(d*x + c))/(d*cos(d*x + c))
```

3.147.6 Sympy [F]

$$\int \left(a - a\sec^2(c+dx)\right)\, dx = -a\left(\int (-1)\, dx + \int \sec^2\left(c+dx\right) dx\right)$$

input
```
integrate(a-a*sec(d*x+c)**2,x)
```

output
```
-a*(Integral(-1, x) + Integral(sec(c + d*x)**2, x))
```

3.147.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \left(a - a\sec^2(c+dx)\right)\, dx = ax - \frac{a\tan\left(dx+c\right)}{d}$$

input
```
integrate(a-a*sec(d*x+c)^2,x, algorithm="maxima")
```

output
```
a*x - a*tan(d*x + c)/d
```

3.147.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \left(a - a\sec^2(c+dx)\right)\,dx = ax - \frac{a\tan\left(dx+c\right)}{d}$$

input
```
integrate(a-a*sec(d*x+c)^2,x, algorithm="giac")
```

output
```
a*x - a*tan(d*x + c)/d
```

3.147.9 Mupad [B] (verification not implemented)

Time = 18.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \left(a - a\sec^2(c+dx)\right)\,dx = a\,x - \frac{a\tan(c+d\,x)}{d}$$

input
```
int(a - a/cos(c + d*x)^2,x)
```

output
```
a*x - (a*tan(c + d*x))/d
```

3.148 $\int \frac{1}{a-a\sec^2(c+dx)}\,dx$

3.148.1 Optimal result 1189
3.148.2 Mathematica [C] (verified) 1189
3.148.3 Rubi [A] (verified) 1190
3.148.4 Maple [A] (verified) 1191
3.148.5 Fricas [A] (verification not implemented) 1192
3.148.6 Sympy [F] 1192
3.148.7 Maxima [A] (verification not implemented) 1192
3.148.8 Giac [B] (verification not implemented) 1193
3.148.9 Mupad [B] (verification not implemented) 1193

3.148.1 Optimal result

Integrand size = 15, antiderivative size = 19

$$\int \frac{1}{a-a\sec^2(c+dx)}\,dx = \frac{x}{a}+\frac{\cot(c+dx)}{ad}$$

output
```
x/a+cot(d*x+c)/a/d
```

3.148.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.63

$$\int \frac{1}{a-a\sec^2(c+dx)}\,dx = \frac{\cot(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(c+dx)\right)}{ad}$$

input
```
Integrate[(a - a*Sec[c + d*x]^2)^(-1),x]
```

output
```
(Cot[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c + d*x]^2])/(a*d)
```

3.148.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4608, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a - a\sec^2(c+dx)}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{a - a\sec(c+dx)^2}\,dx$$
$$\downarrow 4608$$
$$-\frac{\int \cot^2(c+dx)dx}{a}$$
$$\downarrow 3042$$
$$-\frac{\int \tan\left(c+dx+\frac{\pi}{2}\right)^2 dx}{a}$$
$$\downarrow 3954$$
$$-\frac{-\int 1dx - \frac{\cot(c+dx)}{d}}{a}$$
$$\downarrow 24$$
$$-\frac{-\frac{\cot(c+dx)}{d} - x}{a}$$

input
```
Int[(a - a*Sec[c + d*x]^2)^(-1),x]
```

output
```
-((-x - Cot[c + d*x]/d)/a)
```

3.148.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4608
```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
b^p   Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.148.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

method	result	size
derivativedivides	$\frac{\frac{1}{\tan(dx+c)}+\arctan(\tan(dx+c))}{da}$	24
default	$\frac{\frac{1}{\tan(dx+c)}+\arctan(\tan(dx+c))}{da}$	24
risch	$\frac{x}{a}+\frac{2i}{da\left(e^{2i(dx+c)}-1\right)}$	29
parallelrisch	$\frac{2dx-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\cot\left(\frac{dx}{2}+\frac{c}{2}\right)}{2da}$	34
norman	$\frac{\frac{x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}+\frac{1}{2ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2ad}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}$	55

input
```
int(1/(a-a*sec(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(1/tan(d*x+c)+arctan(tan(d*x+c)))
```

3.148.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.63

$$\int \frac{1}{a - a\sec^2(c+dx)}\,dx = \frac{dx\sin(dx+c) + \cos(dx+c)}{ad\sin(dx+c)}$$

input
```
integrate(1/(a-a*sec(d*x+c)^2),x, algorithm="fricas")
```

output
```
(d*x*sin(d*x + c) + cos(d*x + c))/(a*d*sin(d*x + c))
```

3.148.6 Sympy [F]

$$\int \frac{1}{a - a\sec^2(c+dx)}\,dx = -\frac{\int \frac{1}{\sec^2(c+dx)-1}\,dx}{a}$$

input
```
integrate(1/(a-a*sec(d*x+c)**2),x)
```

output
```
-Integral(1/(sec(c + d*x)**2 - 1), x)/a
```

3.148.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int \frac{1}{a - a\sec^2(c+dx)}\,dx = \frac{\frac{dx+c}{a} + \frac{1}{a\tan(dx+c)}}{d}$$

input
```
integrate(1/(a-a*sec(d*x+c)^2),x, algorithm="maxima")
```

output
```
((d*x + c)/a + 1/(a*tan(d*x + c)))/d
```

3.148.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(19) = 38$.

Time $= 0.27$ (sec) , antiderivative size $= 45$, normalized size of antiderivative $= 2.37$

$$\int \frac{1}{a - a\sec^2(c+dx)}\,dx = \frac{\frac{2\,(dx+c)}{a} - \frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a} + \frac{1}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

input
```
integrate(1/(a-a*sec(d*x+c)^2),x, algorithm="giac")
```

output
```
1/2*(2*(d*x + c)/a - tan(1/2*d*x + 1/2*c)/a + 1/(a*tan(1/2*d*x + 1/2*c)))/
d
```

3.148.9 Mupad [B] (verification not implemented)

Time $= 18.65$ (sec) , antiderivative size $= 19$, normalized size of antiderivative $= 1.00$

$$\int \frac{1}{a - a\sec^2(c+dx)}\,dx = \frac{x}{a} + \frac{\cot(c+d\,x)}{a\,d}$$

input
```
int(1/(a - a/cos(c + d*x)^2),x)
```

output
```
x/a + cot(c + d*x)/(a*d)
```

3.149 $\int \frac{1}{(a-a\sec^2(c+dx))^2}\,dx$

3.149.1 Optimal result 1194
3.149.2 Mathematica [C] (verified) 1194
3.149.3 Rubi [A] (verified) 1195
3.149.4 Maple [A] (verified) 1196
3.149.5 Fricas [B] (verification not implemented) 1197
3.149.6 Sympy [F] 1197
3.149.7 Maxima [A] (verification not implemented) 1197
3.149.8 Giac [B] (verification not implemented) 1198
3.149.9 Mupad [B] (verification not implemented) 1198

3.149.1 Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \frac{1}{(a-a\sec^2(c+dx))^2}\,dx = \frac{x}{a^2} + \frac{\cot(c+dx)}{a^2 d} - \frac{\cot^3(c+dx)}{3a^2 d}$$

output
```
x/a^2+cot(d*x+c)/a^2/d-1/3*cot(d*x+c)^3/a^2/d
```

3.149.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int \frac{1}{(a-a\sec^2(c+dx))^2}\,dx = -\frac{\cot^3(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2(c+dx)\right)}{3a^2 d}$$

input
```
Integrate[(a - a*Sec[c + d*x]^2)^(-2),x]
```

output
```
-1/3*(Cot[c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^2])/(a
^2*d)
```

3.149.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {3042, 4608, 3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a - a\sec^2(c+dx))^2}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{(a - a\sec(c+dx)^2)^2}dx$$
$$\downarrow 4608$$
$$\frac{\int \cot^4(c+dx)dx}{a^2}$$
$$\downarrow 3042$$
$$\frac{\int \tan\left(c+dx+\frac{\pi}{2}\right)^4 dx}{a^2}$$
$$\downarrow 3954$$
$$\frac{-\int \cot^2(c+dx)dx - \frac{\cot^3(c+dx)}{3d}}{a^2}$$
$$\downarrow 3042$$
$$\frac{-\int \tan\left(c+dx+\frac{\pi}{2}\right)^2 dx - \frac{\cot^3(c+dx)}{3d}}{a^2}$$
$$\downarrow 3954$$
$$\frac{\int 1dx - \frac{\cot^3(c+dx)}{3d} + \frac{\cot(c+dx)}{d}}{a^2}$$
$$\downarrow 24$$
$$\frac{-\frac{\cot^3(c+dx)}{3d} + \frac{\cot(c+dx)}{d} + x}{a^2}$$

input
```
Int[(a - a*Sec[c + d*x]^2)^(-2),x]
```

output
```
(x + Cot[c + d*x]/d - Cot[c + d*x]^3/(3*d))/a^2
```

3.149.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4608
```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
b^p   Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.149.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{\arctan(\tan(dx+c))-\frac{1}{3\tan(dx+c)^3}+\frac{1}{\tan(dx+c)}}{d\,a^2}$	34
default	$\frac{\arctan(\tan(dx+c))-\frac{1}{3\tan(dx+c)^3}+\frac{1}{\tan(dx+c)}}{d\,a^2}$	34
risch	$\frac{x}{a^2}+\frac{4i\left(3\,\mathrm{e}^{4i(dx+c)}-3\,\mathrm{e}^{2i(dx+c)}+2\right)}{3d\,a^2\left(\mathrm{e}^{2i(dx+c)}-1\right)^3}$	53
parallelrisch	$\frac{-\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+24dx+15\cot\left(\frac{dx}{2}+\frac{c}{2}\right)-15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{24d\,a^2}$	60
norman	$\frac{\frac{x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{a}-\frac{1}{24ad}+\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8ad}-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{8ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{24ad}}{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}$	98

input
```
int(1/(a-a*sec(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^2*(arctan(tan(d*x+c))-1/3/tan(d*x+c)^3+1/tan(d*x+c))
```

3.149.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(35) = 70$.

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.03

$$\int \frac{1}{\left(a - a\sec^2(c+dx)\right)^2}\,dx = \frac{4\cos(dx+c)^3 + 3\left(dx\cos(dx+c)^2 - dx\right)\sin(dx+c) - 3\cos(dx+c)}{3\left(a^2 d\cos(dx+c)^2 - a^2 d\right)\sin(dx+c)}$$

input
```
integrate(1/(a-a*sec(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/3*(4*cos(d*x + c)^3 + 3*(d*x*cos(d*x + c)^2 - d*x)*sin(d*x + c) - 3*cos(
d*x + c))/((a^2*d*cos(d*x + c)^2 - a^2*d)*sin(d*x + c))
```

3.149.6 Sympy [F]

$$\int \frac{1}{\left(a - a\sec^2(c+dx)\right)^2}\,dx = \frac{\int \frac{1}{\sec^4(c+dx) - 2\sec^2(c+dx) + 1}\,dx}{a^2}$$

input
```
integrate(1/(a-a*sec(d*x+c)**2)**2,x)
```

output
```
Integral(1/(sec(c + d*x)**4 - 2*sec(c + d*x)**2 + 1), x)/a**2
```

3.149.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(a - a\sec^2(c+dx)\right)^2}\,dx = \frac{\frac{3(dx+c)}{a^2} + \frac{3\tan(dx+c)^2 - 1}{a^2\tan(dx+c)^3}}{3\,d}$$

input
```
integrate(1/(a-a*sec(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/3*(3*(d*x + c)/a^2 + (3*tan(d*x + c)^2 - 1)/(a^2*tan(d*x + c)^3))/d
```

3.149.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(35) = 70.

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.16

$$\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^2}\,dx = \frac{\frac{24\,(dx+c)}{a^2}+\frac{15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}{a^2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}+\frac{a^4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-15\,a^4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{24\,d}$$

input
```
integrate(1/(a-a*sec(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/24*(24*(d*x + c)/a^2 + (15*tan(1/2*d*x + 1/2*c)^2 - 1)/(a^2*tan(1/2*d*x
+ 1/2*c)^3) + (a^4*tan(1/2*d*x + 1/2*c)^3 - 15*a^4*tan(1/2*d*x + 1/2*c))/a
^6)/d
```

3.149.9 Mupad [B] (verification not implemented)

Time = 18.37 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^2}\,dx = \frac{x}{a^2}+\frac{\tan(c+d\,x)^2-\frac{1}{3}}{a^2\,d\,\tan\left(c+d\,x\right)^3}$$

input
```
int(1/(a - a/cos(c + d*x)^2)^2,x)
```

output
```
x/a^2 + (tan(c + d*x)^2 - 1/3)/(a^2*d*tan(c + d*x)^3)
```

3.150 $\int \frac{1}{(a-a\sec^2(c+dx))^3}\,dx$

3.150.1 Optimal result . . . 1199
3.150.2 Mathematica [C] (verified) . . . 1199
3.150.3 Rubi [A] (verified) . . . 1200
3.150.4 Maple [A] (verified) . . . 1202
3.150.5 Fricas [B] (verification not implemented) . . . 1202
3.150.6 Sympy [F] . . . 1203
3.150.7 Maxima [A] (verification not implemented) . . . 1203
3.150.8 Giac [B] (verification not implemented) . . . 1203
3.150.9 Mupad [B] (verification not implemented) . . . 1204

3.150.1 Optimal result

Integrand size = 15, antiderivative size = 55

$$\int \frac{1}{(a-a\sec^2(c+dx))^3}\,dx = \frac{x}{a^3} + \frac{\cot(c+dx)}{a^3 d} - \frac{\cot^3(c+dx)}{3a^3 d} + \frac{\cot^5(c+dx)}{5a^3 d}$$

output
```
x/a^3+cot(d*x+c)/a^3/d-1/3*cot(d*x+c)^3/a^3/d+1/5*cot(d*x+c)^5/a^3/d
```

3.150.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.65

$$\int \frac{1}{(a-a\sec^2(c+dx))^3}\,dx = \frac{\cot^5(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-\tan^2(c+dx)\right)}{5a^3 d}$$

input
```
Integrate[(a - a*Sec[c + d*x]^2)^(-3),x]
```

output
```
(Cot[c + d*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[c + d*x]^2])/(5*a^3*
d)
```

3.150.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 4608, 3042, 3954, 3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a - a\sec^2(c+dx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{(a - a\sec(c+dx)^2)^3}dx$$

$$\downarrow 4608$$

$$-\frac{\int \cot^6(c+dx)dx}{a^3}$$

$$\downarrow 3042$$

$$-\frac{\int \tan\left(c+dx+\frac{\pi}{2}\right)^6 dx}{a^3}$$

$$\downarrow 3954$$

$$-\frac{-\int \cot^4(c+dx)dx - \frac{\cot^5(c+dx)}{5d}}{a^3}$$

$$\downarrow 3042$$

$$-\frac{-\int \tan\left(c+dx+\frac{\pi}{2}\right)^4 dx - \frac{\cot^5(c+dx)}{5d}}{a^3}$$

$$\downarrow 3954$$

$$-\frac{\int \cot^2(c+dx)dx - \frac{\cot^5(c+dx)}{5d} + \frac{\cot^3(c+dx)}{3d}}{a^3}$$

$$\downarrow 3042$$

$$-\frac{\int \tan\left(c+dx+\frac{\pi}{2}\right)^2 dx - \frac{\cot^5(c+dx)}{5d} + \frac{\cot^3(c+dx)}{3d}}{a^3}$$

$$\downarrow 3954$$

$$-\frac{-\int 1dx - \frac{\cot^5(c+dx)}{5d} + \frac{\cot^3(c+dx)}{3d} - \frac{\cot(c+dx)}{d}}{a^3}$$

$$\downarrow 24$$

$$-\frac{-\frac{\cot^5(c+dx)}{5d}+\frac{\cot^3(c+dx)}{3d}-\frac{\cot(c+dx)}{d}-x}{a^3}$$

input
```
Int[(a - a*Sec[c + d*x]^2)^(-3),x]
```

output
```
-((-x - Cot[c + d*x]/d + Cot[c + d*x]^3/(3*d) - Cot[c + d*x]^5/(5*d))/a^3)
```

3.150.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4608
```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
b^p   Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.150.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.80

method	result
derivativedivides	$\frac{\arctan(\tan(dx+c))-\frac{1}{3\tan(dx+c)^3}+\frac{1}{5\tan(dx+c)^5}+\frac{1}{\tan(dx+c)}}{d\,a^3}$
default	$\frac{\arctan(\tan(dx+c))-\frac{1}{3\tan(dx+c)^3}+\frac{1}{5\tan(dx+c)^5}+\frac{1}{\tan(dx+c)}}{d\,a^3}$
risch	$\frac{x}{a^3}+\frac{2i\left(45\,e^{8i(dx+c)}-90\,e^{6i(dx+c)}+140\,e^{4i(dx+c)}-70\,e^{2i(dx+c)}+23\right)}{15d\,a^3\left(e^{2i(dx+c)}-1\right)^5}$
parallelrisch	$\frac{-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5+3\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^5+35\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3-35\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^3+480dx-330\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+330\cot\left(\frac{dx}{2}+\frac{c}{2}\right)}{480d\,a^3}$
norman	$\frac{\frac{x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{a}+\frac{1}{160ad}-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{96ad}+\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{16ad}-\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{16ad}+\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{96ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{160ad}}{a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}$

input
```
int(1/(a-a*sec(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^3*(arctan(tan(d*x+c))-1/3/tan(d*x+c)^3+1/5/tan(d*x+c)^5+1/tan(d*x+c)
)
```

3.150.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(51) = 102.

Time = 0.24 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.98

$$\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^3}\,dx$$
$$= \frac{23\cos(dx+c)^5-35\cos(dx+c)^3+15\left(dx\cos(dx+c)^4-2\,dx\cos(dx+c)^2+dx\right)\sin(dx+c)+15\cos}{15\left(a^3d\cos(dx+c)^4-2\,a^3d\cos(dx+c)^2+a^3d\right)\sin(dx+c)}$$

input
```
integrate(1/(a-a*sec(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/15*(23*cos(d*x + c)^5 - 35*cos(d*x + c)^3 + 15*(d*x*cos(d*x + c)^4 - 2*d
*x*cos(d*x + c)^2 + d*x)*sin(d*x + c) + 15*cos(d*x + c))/((a^3*d*cos(d*x +
 c)^4 - 2*a^3*d*cos(d*x + c)^2 + a^3*d)*sin(d*x + c))
```

3.150.6 Sympy [F]

$$\int \frac{1}{(a - a\sec^2(c+dx))^3}\,dx = -\frac{\int \frac{1}{\sec^6(c+dx) - 3\sec^4(c+dx) + 3\sec^2(c+dx) - 1}\,dx}{a^3}$$

input
```
integrate(1/(a-a*sec(d*x+c)**2)**3,x)
```

output
```
-Integral(1/(sec(c + d*x)**6 - 3*sec(c + d*x)**4 + 3*sec(c + d*x)**2 - 1),
 x)/a**3
```

3.150.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int \frac{1}{(a - a\sec^2(c+dx))^3}\,dx = \frac{\frac{15\,(dx+c)}{a^3} + \frac{15\tan(dx+c)^4 - 5\tan(dx+c)^2 + 3}{a^3\tan(dx+c)^5}}{15\,d}$$

input
```
integrate(1/(a-a*sec(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/15*(15*(d*x + c)/a^3 + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/(a^3*t
an(d*x + c)^5))/d
```

3.150.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(51) = 102.

Time = 0.29 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.02

$$\int \frac{1}{(a - a\sec^2(c+dx))^3}\,dx = \frac{\frac{480\,(dx+c)}{a^3} + \frac{330\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 - 35\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 + 3}{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5} - \frac{3a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 - 35a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 330a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{15}}}{480\,d}$$

input
```
integrate(1/(a-a*sec(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/480*(480*(d*x + c)/a^3 + (330*tan(1/2*d*x + 1/2*c)^4 - 35*tan(1/2*d*x +
1/2*c)^2 + 3)/(a^3*tan(1/2*d*x + 1/2*c)^5) - (3*a^12*tan(1/2*d*x + 1/2*c)^
5 - 35*a^12*tan(1/2*d*x + 1/2*c)^3 + 330*a^12*tan(1/2*d*x + 1/2*c))/a^15)/
d
```

3.150.9 Mupad [B] (verification not implemented)

Time = 18.41 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.75

$$\int \frac{1}{(a - a\sec^2(c + dx))^3}\,dx = \frac{x}{a^3} + \frac{\tan(c + d\,x)^4 - \frac{\tan(c+d\,x)^2}{3} + \frac{1}{5}}{a^3\,d\tan(c + d\,x)^5}$$

input
```
int(1/(a - a/cos(c + d*x)^2)^3,x)
```

output
```
x/a^3 + (tan(c + d*x)^4 - tan(c + d*x)^2/3 + 1/5)/(a^3*d*tan(c + d*x)^5)
```

3.151 $\int \frac{1}{(a-a\sec^2(c+dx))^4}\,dx$

3.151.1 Optimal result . . . 1205
3.151.2 Mathematica [C] (verified) . . . 1205
3.151.3 Rubi [A] (verified) . . . 1206
3.151.4 Maple [A] (verified) . . . 1208
3.151.5 Fricas [B] (verification not implemented) . . . 1208
3.151.6 Sympy [F] . . . 1209
3.151.7 Maxima [A] (verification not implemented) . . . 1209
3.151.8 Giac [B] (verification not implemented) . . . 1209
3.151.9 Mupad [B] (verification not implemented) . . . 1210

3.151.1 Optimal result

Integrand size = 15, antiderivative size = 73

$$\int \frac{1}{(a-a\sec^2(c+dx))^4}\,dx = \frac{x}{a^4}+\frac{\cot(c+dx)}{a^4 d}-\frac{\cot^3(c+dx)}{3a^4 d}+\frac{\cot^5(c+dx)}{5a^4 d}-\frac{\cot^7(c+dx)}{7a^4 d}$$

output
```
x/a^4+cot(d*x+c)/a^4/d-1/3*cot(d*x+c)^3/a^4/d+1/5*cot(d*x+c)^5/a^4/d-1/7*c
ot(d*x+c)^7/a^4/d
```

3.151.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.49

$$\int \frac{1}{(a-a\sec^2(c+dx))^4}\,dx = -\frac{\cot^7(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{7}{2},1,-\frac{5}{2},-\tan^2(c+dx)\right)}{7a^4 d}$$

input
```
Integrate[(a - a*Sec[c + d*x]^2)^(-4),x]
```

output
```
-1/7*(Cot[c + d*x]^7*Hypergeometric2F1[-7/2, 1, -5/2, -Tan[c + d*x]^2])/(a
^4*d)
```

3.151.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.84, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.733$, Rules used = {3042, 4608, 3042, 3954, 3042, 3954, 3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a - a\sec^2(c+dx))^4}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{(a - a\sec(c+dx)^2)^4}dx$$
$$\downarrow 4608$$
$$\frac{\int \cot^8(c+dx)dx}{a^4}$$
$$\downarrow 3042$$
$$\frac{\int \tan\left(c+dx+\frac{\pi}{2}\right)^8 dx}{a^4}$$
$$\downarrow 3954$$
$$\frac{-\int \cot^6(c+dx)dx - \frac{\cot^7(c+dx)}{7d}}{a^4}$$
$$\downarrow 3042$$
$$\frac{-\int \tan\left(c+dx+\frac{\pi}{2}\right)^6 dx - \frac{\cot^7(c+dx)}{7d}}{a^4}$$
$$\downarrow 3954$$
$$\frac{\int \cot^4(c+dx)dx - \frac{\cot^7(c+dx)}{7d} + \frac{\cot^5(c+dx)}{5d}}{a^4}$$
$$\downarrow 3042$$
$$\frac{\int \tan\left(c+dx+\frac{\pi}{2}\right)^4 dx - \frac{\cot^7(c+dx)}{7d} + \frac{\cot^5(c+dx)}{5d}}{a^4}$$
$$\downarrow 3954$$
$$\frac{-\int \cot^2(c+dx)dx - \frac{\cot^7(c+dx)}{7d} + \frac{\cot^5(c+dx)}{5d} - \frac{\cot^3(c+dx)}{3d}}{a^4}$$

$$\downarrow 3042$$

$$\frac{-\int \tan\left(c+dx+\frac{\pi}{2}\right)^2 dx - \frac{\cot^7(c+dx)}{7d} + \frac{\cot^5(c+dx)}{5d} - \frac{\cot^3(c+dx)}{3d}}{a^4}$$

$$\downarrow 3954$$

$$\frac{\int 1dx - \frac{\cot^7(c+dx)}{7d} + \frac{\cot^5(c+dx)}{5d} - \frac{\cot^3(c+dx)}{3d} + \frac{\cot(c+dx)}{d}}{a^4}$$

$$\downarrow 24$$

$$\frac{-\frac{\cot^7(c+dx)}{7d} + \frac{\cot^5(c+dx)}{5d} - \frac{\cot^3(c+dx)}{3d} + \frac{\cot(c+dx)}{d} + x}{a^4}$$

input
```
Int[(a - a*Sec[c + d*x]^2)^(-4),x]
```

output
```
(x + Cot[c + d*x]/d - Cot[c + d*x]^3/(3*d) + Cot[c + d*x]^5/(5*d) - Cot[c
+ d*x]^7/(7*d))/a^4
```

3.151.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2  Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4608
```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
b^p  Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.151.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.74

method	result
derivativedivides	$\frac{-\frac{1}{7\tan(dx+c)^7}-\frac{1}{3\tan(dx+c)^3}+\frac{1}{5\tan(dx+c)^5}+\frac{1}{\tan(dx+c)}+\arctan(\tan(dx+c))}{d\,a^4}$
default	$\frac{-\frac{1}{7\tan(dx+c)^7}-\frac{1}{3\tan(dx+c)^3}+\frac{1}{5\tan(dx+c)^5}+\frac{1}{\tan(dx+c)}+\arctan(\tan(dx+c))}{d\,a^4}$
risch	$\frac{x}{a^4}+\frac{8i\left(105\,e^{12i(dx+c)}-315\,e^{10i(dx+c)}+770\,e^{8i(dx+c)}-770\,e^{6i(dx+c)}+609\,e^{4i(dx+c)}-203\,e^{2i(dx+c)}+44\right)}{105d\,a^4\left(e^{2i(dx+c)}-1\right)^7}$
parallelrisch	$\frac{-15\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^7+15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7+189\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^5-189\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5-1295\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^3+1295\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+1}{13440d\,a^4}$
norman	$\frac{\frac{x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{a}-\frac{1}{896ad}+\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{640ad}-\frac{37\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{384ad}+\frac{93\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{128ad}-\frac{93\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{128ad}+\frac{37\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{384ad}-\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{640}$
	$a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7$

input
```
int(1/(a-a*sec(d*x+c)^2)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^4*(-1/7/tan(d*x+c)^7-1/3/tan(d*x+c)^3+1/5/tan(d*x+c)^5+1/tan(d*x+c)+
arctan(tan(d*x+c)))
```

3.151.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. 2(67) = 134.

Time = 0.24 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.01

$$\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^4}\,dx$$
$$= \frac{176\cos(dx+c)^7-406\cos(dx+c)^5+350\cos(dx+c)^3+105\left(dx\cos(dx+c)^6-3\,dx\cos(dx+c)^4+3\right.}{105\left(a^4d\cos(dx+c)^6-3\,a^4d\cos(dx+c)^4+3\,a^4d\cos(dx+c)^2-\right.}$$

input
```
integrate(1/(a-a*sec(d*x+c)^2)^4,x, algorithm="fricas")
```

output
```
1/105*(176*cos(d*x + c)^7 - 406*cos(d*x + c)^5 + 350*cos(d*x + c)^3 + 105*
(d*x*cos(d*x + c)^6 - 3*d*x*cos(d*x + c)^4 + 3*d*x*cos(d*x + c)^2 - d*x)*s
in(d*x + c) - 105*cos(d*x + c))/((a^4*d*cos(d*x + c)^6 - 3*a^4*d*cos(d*x +
 c)^4 + 3*a^4*d*cos(d*x + c)^2 - a^4*d)*sin(d*x + c))
```

3.151.6 Sympy [F]

$$\int \frac{1}{(a - a\sec^2(c+dx))^4}\,dx = \frac{\int \frac{1}{\sec^8(c+dx) - 4\sec^6(c+dx) + 6\sec^4(c+dx) - 4\sec^2(c+dx) + 1}\,dx}{a^4}$$

input
```
integrate(1/(a-a*sec(d*x+c)**2)**4,x)
```

output
```
Integral(1/(sec(c + d*x)**8 - 4*sec(c + d*x)**6 + 6*sec(c + d*x)**4 - 4*se
c(c + d*x)**2 + 1), x)/a**4
```

3.151.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

$$\int \frac{1}{(a - a\sec^2(c+dx))^4}\,dx = \frac{\frac{105\,(dx+c)}{a^4} + \frac{105\tan(dx+c)^6 - 35\tan(dx+c)^4 + 21\tan(dx+c)^2 - 15}{a^4\tan(dx+c)^7}}{105\,d}$$

input
```
integrate(1/(a-a*sec(d*x+c)^2)^4,x, algorithm="maxima")
```

output
```
1/105*(105*(d*x + c)/a^4 + (105*tan(d*x + c)^6 - 35*tan(d*x + c)^4 + 21*ta
n(d*x + c)^2 - 15)/(a^4*tan(d*x + c)^7))/d
```

3.151.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(67) = 134.

Time = 0.31 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.90

$$\int \frac{1}{(a - a\sec^2(c+dx))^4}\,dx$$
$$= \frac{\frac{13440\,(dx+c)}{a^4} + \frac{9765\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6 - 1295\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 + 189\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - 15}{a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7} + \frac{15\,a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7 - 189\,a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 +}{}}{13440\,d}$$

input
```
integrate(1/(a-a*sec(d*x+c)^2)^4,x, algorithm="giac")
```

output
```
1/13440*(13440*(d*x + c)/a^4 + (9765*tan(1/2*d*x + 1/2*c)^6 - 1295*tan(1/2
*d*x + 1/2*c)^4 + 189*tan(1/2*d*x + 1/2*c)^2 - 15)/(a^4*tan(1/2*d*x + 1/2*
c)^7) + (15*a^24*tan(1/2*d*x + 1/2*c)^7 - 189*a^24*tan(1/2*d*x + 1/2*c)^5
+ 1295*a^24*tan(1/2*d*x + 1/2*c)^3 - 9765*a^24*tan(1/2*d*x + 1/2*c))/a^28)
/d
```

3.151.9 Mupad [B] (verification not implemented)

Time = 19.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.70

$$\int \frac{1}{(a-a\sec^2(c+dx))^4}\,dx = \frac{x}{a^4} + \frac{\tan(c+d\,x)^6 - \frac{\tan(c+d\,x)^4}{3} + \frac{\tan(c+d\,x)^2}{5} - \frac{1}{7}}{a^4\,d\tan(c+d\,x)^7}$$

input
```
int(1/(a - a/cos(c + d*x)^2)^4,x)
```

output
```
x/a^4 + (tan(c + d*x)^2/5 - tan(c + d*x)^4/3 + tan(c + d*x)^6 - 1/7)/(a^4*
d*tan(c + d*x)^7)
```

3.152 $\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

3.152.1 Optimal result 1211
3.152.2 Mathematica [A] (verified) 1212
3.152.3 Rubi [A] (verified) 1212
3.152.4 Maple [A] (verified) 1214
3.152.5 Fricas [A] (verification not implemented) 1215
3.152.6 Sympy [F] 1215
3.152.7 Maxima [A] (verification not implemented) 1216
3.152.8 Giac [A] (verification not implemented) 1216
3.152.9 Mupad [B] (verification not implemented) 1217

3.152.1 Optimal result

Integrand size = 21, antiderivative size = 98

$$\begin{aligned}\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx &= \frac{(6a+5b)\operatorname{arctanh}(\sin(e+fx))}{16f}\\ &+\frac{(6a+5b)\sec(e+fx)\tan(e+fx)}{16f}\\ &+\frac{(6a+5b)\sec^3(e+fx)\tan(e+fx)}{24f}\\ &+\frac{b\sec^5(e+fx)\tan(e+fx)}{6f}\end{aligned}$$

output
```
1/16*(6*a+5*b)*arctanh(sin(f*x+e))/f+1/16*(6*a+5*b)*sec(f*x+e)*tan(f*x+e)/
f+1/24*(6*a+5*b)*sec(f*x+e)^3*tan(f*x+e)/f+1/6*b*sec(f*x+e)^5*tan(f*x+e)/f
```

3.152.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.40

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{3a\operatorname{arctanh}(\sin(e+fx))}{8f} + \frac{5b\operatorname{arctanh}(\sin(e+fx))}{16f} + \frac{3a\sec(e+fx)\tan(e+fx)}{8f} + \frac{5b\sec(e+fx)\tan(e+fx)}{16f} + \frac{a\sec^3(e+fx)\tan(e+fx)}{4f} + \frac{5b\sec^3(e+fx)\tan(e+fx)}{24f} + \frac{b\sec^5(e+fx)\tan(e+fx)}{6f}$$

input
```
Integrate[Sec[e + f*x]^5*(a + b*Sec[e + f*x]^2),x]
```

output
```
(3*a*ArcTanh[Sin[e + f*x]])/(8*f) + (5*b*ArcTanh[Sin[e + f*x]])/(16*f) + (
3*a*Sec[e + f*x]*Tan[e + f*x])/(8*f) + (5*b*Sec[e + f*x]*Tan[e + f*x])/(16
*f) + (a*Sec[e + f*x]^3*Tan[e + f*x])/(4*f) + (5*b*Sec[e + f*x]^3*Tan[e +
f*x])/(24*f) + (b*Sec[e + f*x]^5*Tan[e + f*x])/(6*f)
```

3.152.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 4534, 3042, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)^5\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)^2\right)dx$$

$$\downarrow 4534$$

$$\frac{1}{6}(6a+5b)\int \sec^5(e+fx)dx + \frac{b\tan(e+fx)\sec^5(e+fx)}{6f}$$

$$\downarrow 3042$$

$$\frac{1}{6}(6a+5b)\int \csc\left(e+fx+\frac{\pi}{2}\right)^5 dx + \frac{b\tan(e+fx)\sec^5(e+fx)}{6f}$$

$$\downarrow 4255$$

$$\frac{1}{6}(6a+5b)\left(\frac{3}{4}\int \sec^3(e+fx)dx + \frac{\tan(e+fx)\sec^3(e+fx)}{4f}\right) + \frac{b\tan(e+fx)\sec^5(e+fx)}{6f}$$

$$\downarrow 3042$$

$$\frac{1}{6}(6a+5b)\left(\frac{3}{4}\int \csc\left(e+fx+\frac{\pi}{2}\right)^3 dx + \frac{\tan(e+fx)\sec^3(e+fx)}{4f}\right) + \frac{b\tan(e+fx)\sec^5(e+fx)}{6f}$$

$$\downarrow 4255$$

$$\frac{1}{6}(6a+5b)\left(\frac{3}{4}\left(\frac{1}{2}\int \sec(e+fx)dx + \frac{\tan(e+fx)\sec(e+fx)}{2f}\right) + \frac{\tan(e+fx)\sec^3(e+fx)}{4f}\right) + \frac{b\tan(e+fx)\sec^5(e+fx)}{6f}$$

$$\downarrow 3042$$

$$\frac{1}{6}(6a+5b)\left(\frac{3}{4}\left(\frac{1}{2}\int \csc\left(e+fx+\frac{\pi}{2}\right)dx + \frac{\tan(e+fx)\sec(e+fx)}{2f}\right) + \frac{\tan(e+fx)\sec^3(e+fx)}{4f}\right) + \frac{b\tan(e+fx)\sec^5(e+fx)}{6f}$$

$$\downarrow 4257$$

$$\frac{1}{6}(6a+5b)\left(\frac{3}{4}\left(\frac{\operatorname{arctanh}(\sin(e+fx))}{2f} + \frac{\tan(e+fx)\sec(e+fx)}{2f}\right) + \frac{\tan(e+fx)\sec^3(e+fx)}{4f}\right) + \frac{b\tan(e+fx)\sec^5(e+fx)}{6f}$$

input
```
Int[Sec[e + f*x]^5*(a + b*Sec[e + f*x]^2),x]
```

output
```
(b*Sec[e + f*x]^5*Tan[e + f*x])/(6*f) + ((6*a + 5*b)*((Sec[e + f*x]^3*Tan[
e + f*x])/(4*f) + (3*(ArcTanh[Sin[e + f*x]]/(2*f) + (Sec[e + f*x]*Tan[e +
f*x])/(2*f)))/4))/6
```

3.152.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

3.152.4 Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{a\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)+b\left(-\left(-\frac{\sec(fx+e)^5}{6}-\frac{5\sec(fx+e)^3}{24}-\frac{5\sec(fx}{16}\right.\right.}{f}$
default	$\frac{a\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)+b\left(-\left(-\frac{\sec(fx+e)^5}{6}-\frac{5\sec(fx+e)^3}{24}-\frac{5\sec(fx}{16}\right.\right.}{f}$
parts	$\frac{a\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)}{f}+\frac{b\left(-\left(-\frac{\sec(fx+e)^5}{6}-\frac{5\sec(fx+e)^3}{24}-\frac{5\sec(}{}\right.\right.}{}$
parallelrisch	$\frac{-270\left(a+\frac{5b}{6}\right)\left(\frac{2}{3}+\frac{\cos(6fx+6e)}{15}+\frac{2\cos(4fx+4e)}{5}+\cos(2fx+2e)\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+270\left(a+\frac{5b}{6}\right)\left(\frac{2}{3}+\frac{\cos(6fx+6e)}{15}+\frac{2\cos}{}\right.}{48f(10+\cos(6fx+6e)+6}$
norman	$\frac{\frac{(2a+15b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{4f}+\frac{(2a+15b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{4f}+\frac{(10a+11b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{8f}+\frac{(10a+11b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}{8f}-\frac{(42a-5b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{24f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^6}$
risch	$-\frac{ie^{i(fx+e)}\left(18a\,e^{10i(fx+e)}+15b\,e^{10i(fx+e)}+102a\,e^{8i(fx+e)}+85b\,e^{8i(fx+e)}+84a\,e^{6i(fx+e)}+198b\,e^{6i(fx+e)}-84a\,e^{4i(fx+}\right.}{24f\left(e^{2i(fx+e)}+1\right)^6}$

input
```
int(sec(f*x+e)^5*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(a*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)+t
an(f*x+e)))+b*(-(-1/6*sec(f*x+e)^5-5/24*sec(f*x+e)^3-5/16*sec(f*x+e))*tan(
f*x+e)+5/16*ln(sec(f*x+e)+tan(f*x+e))))
```

3.152.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.16

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$= \frac{3\left(6\,a+5\,b\right)\cos\left(fx+e\right)^6\log\left(\sin\left(fx+e\right)+1\right)-3\left(6\,a+5\,b\right)\cos\left(fx+e\right)^6\log\left(-\sin\left(fx+e\right)+1\right)+2\left(3\right.}{96\,f\cos\left(fx+e\right)^6}$$

input
```
integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/96*(3*(6*a + 5*b)*cos(f*x + e)^6*log(sin(f*x + e) + 1) - 3*(6*a + 5*b)*c
os(f*x + e)^6*log(-sin(f*x + e) + 1) + 2*(3*(6*a + 5*b)*cos(f*x + e)^4 + 2
*(6*a + 5*b)*cos(f*x + e)^2 + 8*b)*sin(f*x + e))/(f*cos(f*x + e)^6)
```

3.152.6 Sympy [F]

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)\sec^5\left(e+fx\right)dx$$

input
```
integrate(sec(f*x+e)**5*(a+b*sec(f*x+e)**2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)*sec(e + f*x)**5, x)
```

3.152.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.29

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$= \frac{3\,(6\,a+5\,b)\log\left(\sin\left(fx+e\right)+1\right)-3\,(6\,a+5\,b)\log\left(\sin\left(fx+e\right)-1\right)-\frac{2\left(3\,(6\,a+5\,b)\sin(fx+e)^5-8\,(6\,a+5\,b)\sin(fx\right.}{\sin(fx+e)^6-3\,\sin(fx+e)^4+}}{96\,f}$$

input
```
integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/96*(3*(6*a + 5*b)*log(sin(f*x + e) + 1) - 3*(6*a + 5*b)*log(sin(f*x + e)
 - 1) - 2*(3*(6*a + 5*b)*sin(f*x + e)^5 - 8*(6*a + 5*b)*sin(f*x + e)^3 + 3
*(10*a + 11*b)*sin(f*x + e))/(sin(f*x + e)^6 - 3*sin(f*x + e)^4 + 3*sin(f*
x + e)^2 - 1))/f
```

3.152.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.23

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$= \frac{3\,(6\,a+5\,b)\log\left(\left|\sin\left(fx+e\right)+1\right|\right)-3\,(6\,a+5\,b)\log\left(\left|\sin\left(fx+e\right)-1\right|\right)-\frac{2\left(18\,a\sin(fx+e)^5+15\,b\sin(fx+e)^5-48\right.}{96\,f}$$

input
```
integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/96*(3*(6*a + 5*b)*log(abs(sin(f*x + e) + 1)) - 3*(6*a + 5*b)*log(abs(sin
(f*x + e) - 1)) - 2*(18*a*sin(f*x + e)^5 + 15*b*sin(f*x + e)^5 - 48*a*sin(
f*x + e)^3 - 40*b*sin(f*x + e)^3 + 30*a*sin(f*x + e) + 33*b*sin(f*x + e))/
(sin(f*x + e)^2 - 1)^3)/f
```

3.152.9 Mupad [B] (verification not implemented)

Time = 18.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.04

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$= \frac{\operatorname{atanh}(\sin(e+fx))\,\left(\frac{3a}{8}+\frac{5b}{16}\right)}{f}$$
$$- \frac{\left(\frac{3a}{8}+\frac{5b}{16}\right)\sin(e+fx)^5+\left(-a-\frac{5b}{6}\right)\sin(e+fx)^3+\left(\frac{5a}{8}+\frac{11b}{16}\right)\sin(e+fx)}{f\left(\sin(e+fx)^6-3\sin(e+fx)^4+3\sin(e+fx)^2-1\right)}$$

input
```
int((a + b/cos(e + f*x)^2)/cos(e + f*x)^5,x)
```

output
```
(atanh(sin(e + f*x))*((3*a)/8 + (5*b)/16))/f - (sin(e + f*x)^5*((3*a)/8 +
(5*b)/16) + sin(e + f*x)*((5*a)/8 + (11*b)/16) - sin(e + f*x)^3*(a + (5*b)
/6))/(f*(3*sin(e + f*x)^2 - 3*sin(e + f*x)^4 + sin(e + f*x)^6 - 1))
```

3.153 $\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

3.153.1 Optimal result 1218
3.153.2 Mathematica [A] (verified) 1218
3.153.3 Rubi [A] (verified) 1219
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3.153.1 Optimal result

Integrand size = 21, antiderivative size = 70

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{(4a+3b)\operatorname{arctanh}(\sin(e+fx))}{8f} + \frac{(4a+3b)\sec(e+fx)\tan(e+fx)}{8f} + \frac{b\sec^3(e+fx)\tan(e+fx)}{4f}$$

output
```
1/8*(4*a+3*b)*arctanh(sin(f*x+e))/f+1/8*(4*a+3*b)*sec(f*x+e)*tan(f*x+e)/f+
1/4*b*sec(f*x+e)^3*tan(f*x+e)/f
```

3.153.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.33

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{a\operatorname{arctanh}(\sin(e+fx))}{2f} + \frac{3b\operatorname{arctanh}(\sin(e+fx))}{8f} + \frac{a\sec(e+fx)\tan(e+fx)}{2f} + \frac{3b\sec(e+fx)\tan(e+fx)}{8f} + \frac{b\sec^3(e+fx)\tan(e+fx)}{4f}$$

input
```
Integrate[Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2),x]
```

output
```
(a*ArcTanh[Sin[e + f*x]])/(2*f) + (3*b*ArcTanh[Sin[e + f*x]])/(8*f) + (a*S
ec[e + f*x]*Tan[e + f*x])/(2*f) + (3*b*Sec[e + f*x]*Tan[e + f*x])/(8*f) +
(b*Sec[e + f*x]^3*Tan[e + f*x])/(4*f)
```

3.153.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4534, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)^3\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)^2\right)dx$$

$$\downarrow 4534$$

$$\frac{1}{4}(4a+3b)\int \sec^3(e+fx)dx+\frac{b\tan(e+fx)\sec^3(e+fx)}{4f}$$

$$\downarrow 3042$$

$$\frac{1}{4}(4a+3b)\int \csc\left(e+fx+\frac{\pi}{2}\right)^3 dx+\frac{b\tan(e+fx)\sec^3(e+fx)}{4f}$$

$$\downarrow 4255$$

$$\frac{1}{4}(4a+3b)\left(\frac{1}{2}\int \sec(e+fx)dx+\frac{\tan(e+fx)\sec(e+fx)}{2f}\right)+\frac{b\tan(e+fx)\sec^3(e+fx)}{4f}$$

$$\downarrow 3042$$

$$\frac{1}{4}(4a+3b)\left(\frac{1}{2}\int \csc\left(e+fx+\frac{\pi}{2}\right)dx+\frac{\tan(e+fx)\sec(e+fx)}{2f}\right)+\frac{b\tan(e+fx)\sec^3(e+fx)}{4f}$$

$$\downarrow 4257$$

$$\frac{1}{4}(4a+3b)\left(\frac{\operatorname{arctanh}(\sin(e+fx))}{2f}+\frac{\tan(e+fx)\sec(e+fx)}{2f}\right)+\frac{b\tan(e+fx)\sec^3(e+fx)}{4f}$$

input
```
Int[Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2),x]
```

output
```
(b*Sec[e + f*x]^3*Tan[e + f*x])/(4*f) + ((4*a + 3*b)*(ArcTanh[Sin[e + f*x]
]/(2*f) + (Sec[e + f*x]*Tan[e + f*x])/(2*f)))/4
```

3.153.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

3.153.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.21

method	result
derivativedivides	$\frac{a\left(\frac{\tan(fx+e)\sec(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+b\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx$
default	$\frac{a\left(\frac{\tan(fx+e)\sec(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+b\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx$
parts	$\frac{a\left(\frac{\tan(fx+e)\sec(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}+\frac{b\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan$
parallelrisch	$\frac{-8\left(\frac{3}{4}+\frac{\cos(4fx+4e)}{4}+\cos(2fx+2e)\right)\left(a+\frac{3b}{4}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+8\left(\frac{3}{4}+\frac{\cos(4fx+4e)}{4}+\cos(2fx+2e)\right)\left(a+\frac{3b}{4}\right)\ln\left(\tan($
norman	$\frac{-\frac{(4a-3b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{4f}-\frac{(4a-3b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{4f}+\frac{(4a+5b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4f}+\frac{(4a+5b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{4f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^4}-\frac{(4a+3b)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}{8f}$
risch	$-\frac{ie^{i(fx+e)}\left(4a\,e^{6i(fx+e)}+3b\,e^{6i(fx+e)}+4a\,e^{4i(fx+e)}+11b\,e^{4i(fx+e)}-4a\,e^{2i(fx+e)}-11b\,e^{2i(fx+e)}-4a-3b\right)}{4f\left(e^{2i(fx+e)}+1\right)^4}+\frac{\ln\left(e^{i(fx}{2}$

input
```
int(sec(f*x+e)^3*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(a*(1/2*tan(f*x+e)*sec(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))+b*(-(-1/4
*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)+tan(f*x+e))))
```

3.153.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.36

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$=\frac{(4\,a+3\,b)\cos(fx+e)^4\log(\sin(fx+e)+1)-(4\,a+3\,b)\cos(fx+e)^4\log(-\sin(fx+e)+1)+2\left((4\,a$$
$$16\,f\cos(fx+e)^4$$

input
```
integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/16*((4*a + 3*b)*cos(f*x + e)^4*log(sin(f*x + e) + 1) - (4*a + 3*b)*cos(f
*x + e)^4*log(-sin(f*x + e) + 1) + 2*((4*a + 3*b)*cos(f*x + e)^2 + 2*b)*si
n(f*x + e))/(f*cos(f*x + e)^4)
```

3.153.6 Sympy [F]

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \int \left(a+b\sec^2(e+fx)\right)\sec^3(e+fx)\,dx$$

input
```
integrate(sec(f*x+e)**3*(a+b*sec(f*x+e)**2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)*sec(e + f*x)**3, x)
```

3.153.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.39

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$= \frac{(4a+3b)\log\left(\sin(fx+e)+1\right)-(4a+3b)\log\left(\sin(fx+e)-1\right)-\frac{2\left((4a+3b)\sin(fx+e)^3-(4a+5b)\sin(fx+e)\right)}{\sin(fx+e)^4-2\sin(fx+e)^2+1}}{16f}$$

input
```
integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/16*((4*a + 3*b)*log(sin(f*x + e) + 1) - (4*a + 3*b)*log(sin(f*x + e) - 1
) - 2*((4*a + 3*b)*sin(f*x + e)^3 - (4*a + 5*b)*sin(f*x + e))/(sin(f*x + e
)^4 - 2*sin(f*x + e)^2 + 1))/f
```

3.153.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.40

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$= \frac{(4a+3b)\log\left(|\sin(fx+e)+1|\right)-(4a+3b)\log\left(|\sin(fx+e)-1|\right)-\frac{2\left(4a\sin(fx+e)^3+3b\sin(fx+e)^3-4a\sin(fx+e)\right.}{\left(\sin(fx+e)^2-1\right)^2}}{16f}$$

input
```
integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/16*((4*a + 3*b)*log(abs(sin(f*x + e) + 1)) - (4*a + 3*b)*log(abs(sin(f*x
 + e) - 1)) - 2*(4*a*sin(f*x + e)^3 + 3*b*sin(f*x + e)^3 - 4*a*sin(f*x + e
) - 5*b*sin(f*x + e))/(sin(f*x + e)^2 - 1)^2)/f
```

3.153.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.11

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{\operatorname{atanh}(\sin(e+fx))\,\left(\frac{a}{2}+\frac{3b}{8}\right)}{f} - \frac{\sin(e+fx)^3\left(\frac{a}{2}+\frac{3b}{8}\right)-\sin(e+fx)\,\left(\frac{a}{2}+\frac{5b}{8}\right)}{f\,\left(\sin(e+fx)^4-2\sin(e+fx)^2+1\right)}$$

input
```
int((a + b/cos(e + f*x)^2)/cos(e + f*x)^3,x)
```

output
```
(atanh(sin(e + f*x))*(a/2 + (3*b)/8))/f - (sin(e + f*x)^3*(a/2 + (3*b)/8)
- sin(e + f*x)*(a/2 + (5*b)/8))/(f*(sin(e + f*x)^4 - 2*sin(e + f*x)^2 + 1)
)
```

3.154 $\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

3.154.1 Optimal result 1224
3.154.2 Mathematica [A] (verified) 1224
3.154.3 Rubi [A] (verified) 1225
3.154.4 Maple [A] (verified) 1226
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3.154.7 Maxima [A] (verification not implemented) 1227
3.154.8 Giac [A] (verification not implemented) 1227
3.154.9 Mupad [B] (verification not implemented) 1228

3.154.1 Optimal result

Integrand size = 19, antiderivative size = 40

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{(2a+b)\operatorname{arctanh}(\sin(e+fx))}{2f} + \frac{b\sec(e+fx)\tan(e+fx)}{2f}$$

output
```
1/2*(2*a+b)*arctanh(sin(f*x+e))/f+1/2*b*sec(f*x+e)*tan(f*x+e)/f
```

3.154.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{a\operatorname{arctanh}(\sin(e+fx))}{f} + \frac{b\operatorname{arctanh}(\sin(e+fx))}{2f} + \frac{b\sec(e+fx)\tan(e+fx)}{2f}$$

input
```
Integrate[Sec[e + f*x]*(a + b*Sec[e + f*x]^2),x]
```

output
```
(a*ArcTanh[Sin[e + f*x]])/f + (b*ArcTanh[Sin[e + f*x]])/(2*f) + (b*Sec[e +
 f*x]*Tan[e + f*x])/(2*f)
```

3.154.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 4534, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)^2\right)dx$$

$$\downarrow 4534$$

$$\frac{1}{2}(2a+b)\int \sec(e+fx)dx+\frac{b\tan(e+fx)\sec(e+fx)}{2f}$$

$$\downarrow 3042$$

$$\frac{1}{2}(2a+b)\int \csc\left(e+fx+\frac{\pi}{2}\right)dx+\frac{b\tan(e+fx)\sec(e+fx)}{2f}$$

$$\downarrow 4257$$

$$\frac{(2a+b)\text{arctanh}(\sin(e+fx))}{2f}+\frac{b\tan(e+fx)\sec(e+fx)}{2f}$$

input
```
Int[Sec[e + f*x]*(a + b*Sec[e + f*x]^2),x]
```

output
```
((2*a + b)*ArcTanh[Sin[e + f*x]])/(2*f) + (b*Sec[e + f*x]*Tan[e + f*x])/(2
*f)
```

3.154.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

3.154.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.38

method	result	size
derivativedivides	$\frac{a\ln(\sec(fx+e)+\tan(fx+e))+b\left(\frac{\tan(fx+e)\sec(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}$	55
default	$\frac{a\ln(\sec(fx+e)+\tan(fx+e))+b\left(\frac{\tan(fx+e)\sec(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}$	55
parts	$\frac{b\left(\frac{\tan(fx+e)\sec(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}+\frac{a\ln(\sec(fx+e)+\tan(fx+e))}{f}$	57
parallelrisch	$\frac{-\left(a+\frac{b}{2}\right)(1+\cos(2fx+2e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+\left(a+\frac{b}{2}\right)(1+\cos(2fx+2e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)+\sin(fx+e)b}{f(1+\cos(2fx+2e))}$	86
norman	$\frac{\frac{b\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{b\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^2}-\frac{(2a+b)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2f}+\frac{(2a+b)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2f}$	93
risch	$-\frac{ib\left(e^{3i(fx+e)}-e^{i(fx+e)}\right)}{f\left(e^{2i(fx+e)}+1\right)^2}+\frac{\ln\left(e^{i(fx+e)}+i\right)a}{f}+\frac{\ln\left(e^{i(fx+e)}+i\right)b}{2f}-\frac{\ln\left(e^{i(fx+e)}-i\right)a}{f}-\frac{\ln\left(e^{i(fx+e)}-i\right)b}{2f}$	118

input
```
int(sec(f*x+e)*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(a*ln(sec(f*x+e)+tan(f*x+e))+b*(1/2*tan(f*x+e)*sec(f*x+e)+1/2*ln(sec(f
*x+e)+tan(f*x+e))))
```

3.154.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.80

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$=\frac{(2\,a+b)\cos(fx+e)^2\log(\sin(fx+e)+1)-(2\,a+b)\cos(fx+e)^2\log(-\sin(fx+e)+1)+2\,b\sin(fx+e)}{4\,f\cos(fx+e)^2}$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/4*((2*a + b)*cos(f*x + e)^2*log(sin(f*x + e) + 1) - (2*a + b)*cos(f*x +
e)^2*log(-sin(f*x + e) + 1) + 2*b*sin(f*x + e))/(f*cos(f*x + e)^2)
```

3.154.6 Sympy [F]

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \int \left(a+b\sec^2(e+fx)\right)\sec(e+fx)\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e)**2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)*sec(e + f*x), x)
```

3.154.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.45

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{(2a+b)\log\left(\sin(fx+e)+1\right)-(2a+b)\log\left(\sin(fx+e)-1\right)-\frac{2b\sin(fx+e)}{\sin(fx+e)^2-1}}{4f}$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/4*((2*a + b)*log(sin(f*x + e) + 1) - (2*a + b)*log(sin(f*x + e) - 1) - 2
*b*sin(f*x + e)/(sin(f*x + e)^2 - 1))/f
```

3.154.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.50

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{(2a+b)\log\left(|\sin(fx+e)+1|\right)-(2a+b)\log\left(|\sin(fx+e)-1|\right)-\frac{2b\sin(fx+e)}{\sin(fx+e)^2-1}}{4f}$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/4*((2*a + b)*log(abs(sin(f*x + e) + 1)) - (2*a + b)*log(abs(sin(f*x + e)
 - 1)) - 2*b*sin(f*x + e)/(sin(f*x + e)^2 - 1))/f
```

3.154.9 Mupad [B] (verification not implemented)

Time = 18.46 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{\operatorname{atanh}(\sin(e+fx))\,\left(a+\frac{b}{2}\right)}{f} - \frac{b\sin(e+fx)}{2f\,\left(\sin(e+fx)^2-1\right)}$$

input
```
int((a + b/cos(e + f*x)^2)/cos(e + f*x),x)
```

output
```
(atanh(sin(e + f*x))*(a + b/2))/f - (b*sin(e + f*x))/(2*f*(sin(e + f*x)^2
- 1))
```

3.155 $\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

3.155.1 Optimal result 1229
3.155.2 Mathematica [A] (verified) 1229
3.155.3 Rubi [A] (verified) 1230
3.155.4 Maple [A] (verified) 1231
3.155.5 Fricas [A] (verification not implemented) 1231
3.155.6 Sympy [F] 1232
3.155.7 Maxima [A] (verification not implemented) 1232
3.155.8 Giac [A] (verification not implemented) 1232
3.155.9 Mupad [B] (verification not implemented) 1233

3.155.1 Optimal result

Integrand size = 19, antiderivative size = 24

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{b\operatorname{arctanh}(\sin(e+fx))}{f}+\frac{a\sin(e+fx)}{f}$$

output
```
b*arctanh(sin(f*x+e))/f+a*sin(f*x+e)/f
```

3.155.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{b\operatorname{arctanh}(\sin(e+fx))}{f}+\frac{a\cos(fx)\sin(e)}{f}+\frac{a\cos(e)\sin(fx)}{f}$$

input
```
Integrate[Cos[e + f*x]*(a + b*Sec[e + f*x]^2),x]
```

output
```
(b*ArcTanh[Sin[e + f*x]])/f + (a*Cos[f*x]*Sin[e])/f + (a*Cos[e]*Sin[f*x])/
f
```

3.155.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 4533, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \frac{a+b\csc\left(e+fx+\frac{\pi}{2}\right)^2}{\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\downarrow 4533$$

$$b\int \sec(e+fx)dx+\frac{a\sin(e+fx)}{f}$$

$$\downarrow 3042$$

$$b\int \csc\left(e+fx+\frac{\pi}{2}\right)dx+\frac{a\sin(e+fx)}{f}$$

$$\downarrow 4257$$

$$\frac{a\sin(e+fx)}{f}+\frac{b\text{arctanh}(\sin(e+fx))}{f}$$

input
```
Int[Cos[e + f*x]*(a + b*Sec[e + f*x]^2),x]
```

output
```
(b*ArcTanh[Sin[e + f*x]])/f + (a*Sin[e + f*x])/f
```

3.155.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

3.155.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25

method	result	size
derivativedivides	$\frac{\sin(fx+e)a+b\ln(\sec(fx+e)+\tan(fx+e))}{f}$	30
default	$\frac{\sin(fx+e)a+b\ln(\sec(fx+e)+\tan(fx+e))}{f}$	30
parallelrisch	$\frac{\sin(fx+e)a-\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)b+\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)b}{f}$	43
risch	$-\frac{ia\,e^{i(fx+e)}}{2f}+\frac{ia\,e^{-i(fx+e)}}{2f}+\frac{\ln\left(e^{i(fx+e)}+i\right)b}{f}-\frac{\ln\left(e^{i(fx+e)}-i\right)b}{f}$	71
norman	$\frac{-\frac{2a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{2a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{f}}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)}+\frac{b\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{f}-\frac{b\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{f}$	101

input
```
int(cos(f*x+e)*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(sin(f*x+e)*a+b*ln(sec(f*x+e)+tan(f*x+e)))
```

3.155.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.67

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{b\log\left(\sin\left(fx+e\right)+1\right)-b\log\left(-\sin\left(fx+e\right)+1\right)+2\,a\sin\left(fx+e\right)}{2\,f}$$

input
```
integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/2*(b*log(sin(f*x + e) + 1) - b*log(-sin(f*x + e) + 1) + 2*a*sin(f*x + e)
)/f
```

3.155.6 Sympy [F]

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \int \left(a+b\sec^2(e+fx)\right)\cos(e+fx)\,dx$$

input
```
integrate(cos(f*x+e)*(a+b*sec(f*x+e)**2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)*cos(e + f*x), x)
```

3.155.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.58

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{b(\log(\sin(fx+e)+1)-\log(\sin(fx+e)-1))+2\,a\sin(fx+e)}{2\,f}$$

input
```
integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/2*(b*(log(sin(f*x + e) + 1) - log(sin(f*x + e) - 1)) + 2*a*sin(f*x + e))
/f
```

3.155.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.67

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{b\log(|\sin(fx+e)+1|)-b\log(|\sin(fx+e)-1|)+2\,a\sin(fx+e)}{2\,f}$$

input
```
integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/2*(b*log(abs(sin(f*x + e) + 1)) - b*log(abs(sin(f*x + e) - 1)) + 2*a*sin
(f*x + e))/f
```

3.155.9 Mupad [B] (verification not implemented)

Time = 18.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{a\,\sin(e+f\,x)+b\,\mathrm{atanh}(\sin(e+f\,x))}{f}$$

input
```
int(cos(e + f*x)*(a + b/cos(e + f*x)^2),x)
```

output
```
(a*sin(e + f*x) + b*atanh(sin(e + f*x)))/f
```

3.156 $\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

3.156.1 Optimal result 1234
3.156.2 Mathematica [A] (verified) 1234
3.156.3 Rubi [A] (verified) 1235
3.156.4 Maple [A] (verified) 1236
3.156.5 Fricas [A] (verification not implemented) 1237
3.156.6 Sympy [F] 1237
3.156.7 Maxima [A] (verification not implemented) 1237
3.156.8 Giac [A] (verification not implemented) 1238
3.156.9 Mupad [B] (verification not implemented) 1238

3.156.1 Optimal result

Integrand size = 21, antiderivative size = 30

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{(a+b)\sin(e+fx)}{f} - \frac{a\sin^3(e+fx)}{3f}$$

output
```
(a+b)*sin(f*x+e)/f-1/3*a*sin(f*x+e)^3/f
```

3.156.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.67

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{b\cos(fx)\sin(e)}{f} + \frac{b\cos(e)\sin(fx)}{f} + \frac{a\sin(e+fx)}{f} - \frac{a\sin^3(e+fx)}{3f}$$

input
```
Integrate[Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2),x]
```

output
```
(b*Cos[f*x]*Sin[e])/f + (b*Cos[e]*Sin[f*x])/f + (a*Sin[e + f*x])/f - (a*Si
n[e + f*x]^3)/(3*f)
```

3.156.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4532, 3042, 3492, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \frac{a+b\csc\left(e+fx+\frac{\pi}{2}\right)^2}{\csc\left(e+fx+\frac{\pi}{2}\right)^3}dx$$

$$\downarrow 4532$$

$$\int \cos(e+fx)\left(a\cos^2(e+fx)+b\right)dx$$

$$\downarrow 3042$$

$$\int \sin\left(e+fx+\frac{\pi}{2}\right)\left(a\sin\left(e+fx+\frac{\pi}{2}\right)^2+b\right)dx$$

$$\downarrow 3492$$

$$-\frac{\int\left(-a\sin^2(e+fx)+a+b\right)d(-\sin(e+fx))}{f}$$

$$\downarrow 2009$$

$$-\frac{\frac{1}{3}a\sin^3(e+fx)-(a+b)\sin(e+fx)}{f}$$

input
```
Int[Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2),x]
```

output
```
-((-((a + b)*Sin[e + f*x]) + (a*Sin[e + f*x]^3)/3)/f)
```

3.156.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3492
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2
), x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

rule 4532
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

3.156.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

method	result	size
parallelrisch	$\frac{\sin(3fx+3e)a+9\left(a+\frac{4b}{3}\right)\sin(fx+e)}{12f}$	31
derivativedivides	$\frac{\frac{a\left(\cos(fx+e)^2+2\right)\sin(fx+e)}{3}+\sin(fx+e)b}{f}$	33
default	$\frac{\frac{a\left(\cos(fx+e)^2+2\right)\sin(fx+e)}{3}+\sin(fx+e)b}{f}$	33
risch	$\frac{3a\sin(fx+e)}{4f}+\frac{\sin(fx+e)b}{f}+\frac{a\sin(3fx+3e)}{12f}$	40
norman	$\frac{\frac{2(a-3b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3f}-\frac{2(a-3b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{3f}-\frac{2(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{2(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{f}}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)}$	111

input
```
int(cos(f*x+e)^3*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/12*(sin(3*f*x+3*e)*a+9*(a+4/3*b)*sin(f*x+e))/f
```

3.156.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{\left(a\cos\left(fx+e\right)^2+2\,a+3\,b\right)\sin\left(fx+e\right)}{3\,f}$$

input
```
integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/3*(a*cos(f*x + e)^2 + 2*a + 3*b)*sin(f*x + e)/f
```

3.156.6 Sympy [F]

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)\cos^3\left(e+fx\right)\,dx$$

input
```
integrate(cos(f*x+e)**3*(a+b*sec(f*x+e)**2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)*cos(e + f*x)**3, x)
```

3.156.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{a\sin\left(fx+e\right)^3-3\left(a+b\right)\sin\left(fx+e\right)}{3\,f}$$

input
```
integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
-1/3*(a*sin(f*x + e)^3 - 3*(a + b)*sin(f*x + e))/f
```

3.156.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{a\sin(fx+e)^3 - 3\,a\sin(fx+e) - 3\,b\sin(fx+e)}{3\,f}$$

input
```
integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
-1/3*(a*sin(f*x + e)^3 - 3*a*sin(f*x + e) - 3*b*sin(f*x + e))/f
```

3.156.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{\frac{a\sin(e+f\,x)^3}{3} - \sin(e+f\,x)\,(a+b)}{f}$$

input
```
int(cos(e + f*x)^3*(a + b/cos(e + f*x)^2),x)
```

output
```
-((a*sin(e + f*x)^3)/3 - sin(e + f*x)*(a + b))/f
```

3.157 $\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

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3.157.1 Optimal result

Integrand size = 21, antiderivative size = 50

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{(a+b)\sin(e+fx)}{f} - \frac{(2a+b)\sin^3(e+fx)}{3f} + \frac{a\sin^5(e+fx)}{5f}$$

output
```
(a+b)*sin(f*x+e)/f-1/3*(2*a+b)*sin(f*x+e)^3/f+1/5*a*sin(f*x+e)^5/f
```

3.157.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.42

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{a\sin(e+fx)}{f} + \frac{b\sin(e+fx)}{f} - \frac{2a\sin^3(e+fx)}{3f} - \frac{b\sin^3(e+fx)}{3f} + \frac{a\sin^5(e+fx)}{5f}$$

input
```
Integrate[Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2),x]
```

output
```
(a*Sin[e + f*x])/f + (b*Sin[e + f*x])/f - (2*a*Sin[e + f*x]^3)/(3*f) - (b*
Sin[e + f*x]^3)/(3*f) + (a*Sin[e + f*x]^5)/(5*f)
```

3.157.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4532, 3042, 3492, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \frac{a+b\csc\left(e+fx+\frac{\pi}{2}\right)^2}{\csc\left(e+fx+\frac{\pi}{2}\right)^5}dx$$

$$\downarrow 4532$$

$$\int \cos^3(e+fx)\left(a\cos^2(e+fx)+b\right)dx$$

$$\downarrow 3042$$

$$\int \sin\left(e+fx+\frac{\pi}{2}\right)^3\left(a\sin\left(e+fx+\frac{\pi}{2}\right)^2+b\right)dx$$

$$\downarrow 3492$$

$$-\frac{\int \left(1-\sin^2(e+fx)\right)\left(-a\sin^2(e+fx)+a+b\right)d(-\sin(e+fx))}{f}$$

$$\downarrow 290$$

$$-\frac{\int \left(a\sin^4(e+fx)-(2a+b)\sin^2(e+fx)+a\left(\frac{b}{a}+1\right)\right)d(-\sin(e+fx))}{f}$$

$$\downarrow 2009$$

$$-\frac{\frac{1}{3}(2a+b)\sin^3(e+fx)-(a+b)\sin(e+fx)-\frac{1}{5}a\sin^5(e+fx)}{f}$$

input
```
Int[Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2),x]
```

output
```
-((-((a + b)*Sin[e + f*x]) + ((2*a + b)*Sin[e + f*x]^3)/3 - (a*Sin[e + f*x
]^5)/5)/f)
```

3.157.3.1 Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3492
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2
), x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

rule 4532
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

3.157.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

method	result
parallelrisch	$\frac{(25a+20b)\sin(3fx+3e)+3\sin(5fx+5e)a+150\sin(fx+e)\left(a+\frac{6b}{5}\right)}{240f}$
derivativedivides	$\frac{\frac{a\left(\frac{8}{3}+\cos(fx+e)^4+\frac{4\cos(fx+e)^2}{3}\right)\sin(fx+e)}{5}+\frac{b\left(\cos(fx+e)^2+2\right)\sin(fx+e)}{3}}{f}$
default	$\frac{\frac{a\left(\frac{8}{3}+\cos(fx+e)^4+\frac{4\cos(fx+e)^2}{3}\right)\sin(fx+e)}{5}+\frac{b\left(\cos(fx+e)^2+2\right)\sin(fx+e)}{3}}{f}$
risch	$\frac{5a\sin(fx+e)}{8f}+\frac{3\sin(fx+e)b}{4f}+\frac{a\sin(5fx+5e)}{80f}+\frac{5a\sin(3fx+3e)}{48f}+\frac{\sin(3fx+3e)b}{12f}$
norman	$-\frac{\frac{2(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{2(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}{f}-\frac{2(a+5b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3f}+\frac{2(a+5b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{3f}-\frac{4(19a+5b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{15f}+\frac{4(19}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)^5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)}$

input
```
int(cos(f*x+e)^5*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/240*((25*a+20*b)*sin(3*f*x+3*e)+3*sin(5*f*x+5*e)*a+150*sin(f*x+e)*(a+6/5
*b))/f
```

3.157.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{\left(3\,a\cos\left(fx+e\right)^4+\left(4\,a+5\,b\right)\cos\left(fx+e\right)^2+8\,a+10\,b\right)\sin\left(fx+e\right)}{15\,f}$$

input
```
integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/15*(3*a*cos(f*x + e)^4 + (4*a + 5*b)*cos(f*x + e)^2 + 8*a + 10*b)*sin(f*
x + e)/f
```

3.157.6 Sympy [F]

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)\cos^5\left(e+fx\right)\,dx$$

input
```
integrate(cos(f*x+e)**5*(a+b*sec(f*x+e)**2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)*cos(e + f*x)**5, x)
```

3.157.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{3\,a\sin(fx+e)^5 - 5\,(2\,a+b)\sin(fx+e)^3 + 15\,(a+b)\sin(fx+e)}{15\,f}$$

input
```
integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/15*(3*a*sin(f*x + e)^5 - 5*(2*a + b)*sin(f*x + e)^3 + 15*(a + b)*sin(f*x
 + e))/f
```

3.157.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.14

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{3\,a\sin(fx+e)^5 - 10\,a\sin(fx+e)^3 - 5\,b\sin(fx+e)^3 + 15\,a\sin(fx+e) + 15\,b\sin(fx+e)}{15\,f}$$

input
```
integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/15*(3*a*sin(f*x + e)^5 - 10*a*sin(f*x + e)^3 - 5*b*sin(f*x + e)^3 + 15*a
*sin(f*x + e) + 15*b*sin(f*x + e))/f
```

3.157.9 Mupad [B] (verification not implemented)

Time = 18.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{\frac{a\sin(e+f\,x)^5}{5} + \left(-\frac{2\,a}{3} - \frac{b}{3}\right)\sin(e+f\,x)^3 + (a+b)\sin(e+f\,x)}{f}$$

input
```
int(cos(e + f*x)^5*(a + b/cos(e + f*x)^2),x)
```

output
```
((a*sin(e + f*x)^5)/5 - sin(e + f*x)^3*((2*a)/3 + b/3) + sin(e + f*x)*(a +
 b))/f
```

3.158 $\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

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3.158.1 Optimal result

Integrand size = 21, antiderivative size = 87

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{(7a+6b)\tan(e+fx)}{7f} + \frac{b\sec^6(e+fx)\tan(e+fx)}{7f} + \frac{2(7a+6b)\tan^3(e+fx)}{21f} + \frac{(7a+6b)\tan^5(e+fx)}{35f}$$

output
```
1/7*(7*a+6*b)*tan(f*x+e)/f+1/7*b*sec(f*x+e)^6*tan(f*x+e)/f+2/21*(7*a+6*b)*
tan(f*x+e)^3/f+1/35*(7*a+6*b)*tan(f*x+e)^5/f
```

3.158.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.93

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{a\left(\tan(e+fx)+\frac{2}{3}\tan^3(e+fx)+\frac{1}{5}\tan^5(e+fx)\right)}{f} + \frac{b\left(\tan(e+fx)+\tan^3(e+fx)+\frac{3}{5}\tan^5(e+fx)+\frac{1}{7}\tan^7(e+fx)\right)}{f}$$

input
```
Integrate[Sec[e + f*x]^6*(a + b*Sec[e + f*x]^2),x]
```

output
```
(a*(Tan[e + f*x] + (2*Tan[e + f*x]^3)/3 + Tan[e + f*x]^5/5))/f + (b*(Tan[e
 + f*x] + Tan[e + f*x]^3 + (3*Tan[e + f*x]^5)/5 + Tan[e + f*x]^7/7))/f
```

3.158.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.80, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4534, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)^6\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)^2\right)dx$$

$$\downarrow 4534$$

$$\frac{1}{7}(7a+6b)\int \sec^6(e+fx)dx+\frac{b\tan(e+fx)\sec^6(e+fx)}{7f}$$

$$\downarrow 3042$$

$$\frac{1}{7}(7a+6b)\int \csc\left(e+fx+\frac{\pi}{2}\right)^6 dx+\frac{b\tan(e+fx)\sec^6(e+fx)}{7f}$$

$$\downarrow 4254$$

$$\frac{b\tan(e+fx)\sec^6(e+fx)}{7f}-\frac{(7a+6b)\int\left(\tan^4(e+fx)+2\tan^2(e+fx)+1\right)d(-\tan(e+fx))}{7f}$$

$$\downarrow 2009$$

$$\frac{b\tan(e+fx)\sec^6(e+fx)}{7f}-\frac{(7a+6b)\left(-\frac{1}{5}\tan^5(e+fx)-\frac{2}{3}\tan^3(e+fx)-\tan(e+fx)\right)}{7f}$$

input
```
Int[Sec[e + f*x]^6*(a + b*Sec[e + f*x]^2),x]
```

output
```
(b*Sec[e + f*x]^6*Tan[e + f*x])/(7*f) - ((7*a + 6*b)*(-Tan[e + f*x] - (2*T
an[e + f*x]^3)/3 - Tan[e + f*x]^5/5))/(7*f)
```

3.158.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

3.158.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{-a\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)-b\left(-\frac{16}{35}-\frac{\sec(fx+e)^6}{7}-\frac{6\sec(fx+e)^4}{35}-\frac{8\sec(fx+e)^2}{35}\right)\tan(fx+e)}{f}$
default	$\frac{-a\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)-b\left(-\frac{16}{35}-\frac{\sec(fx+e)^6}{7}-\frac{6\sec(fx+e)^4}{35}-\frac{8\sec(fx+e)^2}{35}\right)\tan(fx+e)}{f}$
parts	$-\frac{a\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)}{f}-\frac{b\left(-\frac{16}{35}-\frac{\sec(fx+e)^6}{7}-\frac{6\sec(fx+e)^4}{35}-\frac{8\sec(fx+e)^2}{35}\right)\tan(fx+e)}{f}$
risch	$\frac{16i\left(70a\,e^{8i(fx+e)}+175a\,e^{6i(fx+e)}+210b\,e^{6i(fx+e)}+147a\,e^{4i(fx+e)}+126b\,e^{4i(fx+e)}+49a\,e^{2i(fx+e)}+42b\,e^{2i(fx+e)}+7a+\right.}{105f\left(e^{2i(fx+e)}+1\right)^7}$
parallelrisch	$\frac{(1176a+1008b)\sin(3fx+3e)+(392a+336b)\sin(5fx+5e)+(56a+48b)\sin(7fx+7e)+840\sin(fx+e)(a+2b)}{105f(\cos(7fx+7e)+7\cos(5fx+5e)+21\cos(3fx+3e)+35\cos(fx+e))}$
norman	$-\frac{2(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{2(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{13}}{f}+\frac{4(5a+3b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3f}+\frac{4(5a+3b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}{3f}+\frac{8(91a+53b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{35f}-$ $\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^7$

input
```
int(sec(f*x+e)^6*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-a*(-8/15-1/5*sec(f*x+e)^4-4/15*sec(f*x+e)^2)*tan(f*x+e)-b*(-16/35-1/
7*sec(f*x+e)^6-6/35*sec(f*x+e)^4-8/35*sec(f*x+e)^2)*tan(f*x+e))
```

3.158.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.85

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$=\frac{\left(8\left(7\,a+6\,b\right)\cos\left(fx+e\right)^6+4\left(7\,a+6\,b\right)\cos\left(fx+e\right)^4+3\left(7\,a+6\,b\right)\cos\left(fx+e\right)^2+15\,b\right)\sin\left(fx+e\right)}{105\,f\cos\left(fx+e\right)^7}$$

input
```
integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/105*(8*(7*a + 6*b)*cos(f*x + e)^6 + 4*(7*a + 6*b)*cos(f*x + e)^4 + 3*(7*
a + 6*b)*cos(f*x + e)^2 + 15*b)*sin(f*x + e)/(f*cos(f*x + e)^7)
```

3.158.6 Sympy [F]

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \int \left(a+b\sec^2(e+fx)\right)\sec^6(e+fx)\,dx$$

input
```
integrate(sec(f*x+e)**6*(a+b*sec(f*x+e)**2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)*sec(e + f*x)**6, x)
```

3.158.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.69

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$= \frac{15\,b\tan(fx+e)^7 + 21\,(a+3\,b)\tan(fx+e)^5 + 35\,(2\,a+3\,b)\tan(fx+e)^3 + 105\,(a+b)\tan(fx+e)}{105\,f}$$

input
```
integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/105*(15*b*tan(f*x + e)^7 + 21*(a + 3*b)*tan(f*x + e)^5 + 35*(2*a + 3*b)*
tan(f*x + e)^3 + 105*(a + b)*tan(f*x + e))/f
```

3.158.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.91

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$= \frac{15\,b\tan(fx+e)^7 + 21\,a\tan(fx+e)^5 + 63\,b\tan(fx+e)^5 + 70\,a\tan(fx+e)^3 + 105\,b\tan(fx+e)^3 + 1}{105\,f}$$

input
```
integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/105*(15*b*tan(f*x + e)^7 + 21*a*tan(f*x + e)^5 + 63*b*tan(f*x + e)^5 + 7
0*a*tan(f*x + e)^3 + 105*b*tan(f*x + e)^3 + 105*a*tan(f*x + e) + 105*b*tan
(f*x + e))/f
```

3.158.9 Mupad [B] (verification not implemented)

Time = 18.16 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.64

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{\frac{b\tan(e+fx)^7}{7}+\left(\frac{a}{5}+\frac{3b}{5}\right)\tan(e+fx)^5+\left(\frac{2a}{3}+b\right)\tan(e+fx)^3+(a+b)\tan(e+fx)}{f}$$

input
```
int((a + b/cos(e + f*x)^2)/cos(e + f*x)^6,x)
```

output
```
(tan(e + f*x)^5*(a/5 + (3*b)/5) + (b*tan(e + f*x)^7)/7 + tan(e + f*x)^3*((
2*a)/3 + b) + tan(e + f*x)*(a + b))/f
```

3.159 $\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

3.159.1 Optimal result 1251
3.159.2 Mathematica [A] (verified) 1251
3.159.3 Rubi [A] (verified) 1252
3.159.4 Maple [A] (verified) 1253
3.159.5 Fricas [A] (verification not implemented) 1254
3.159.6 Sympy [F] 1254
3.159.7 Maxima [A] (verification not implemented) 1254
3.159.8 Giac [A] (verification not implemented) 1255
3.159.9 Mupad [B] (verification not implemented) 1255

3.159.1 Optimal result

Integrand size = 21, antiderivative size = 65

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{(5a+4b)\tan(e+fx)}{5f} + \frac{b\sec^4(e+fx)\tan(e+fx)}{5f} + \frac{(5a+4b)\tan^3(e+fx)}{15f}$$

output
```
1/5*(5*a+4*b)*tan(f*x+e)/f+1/5*b*sec(f*x+e)^4*tan(f*x+e)/f+1/15*(5*a+4*b)*
tan(f*x+e)^3/f
```

3.159.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{a\left(\tan(e+fx)+\frac{1}{3}\tan^3(e+fx)\right)}{f} + \frac{b\left(\tan(e+fx)+\frac{2}{3}\tan^3(e+fx)+\frac{1}{5}\tan^5(e+fx)\right)}{f}$$

input
```
Integrate[Sec[e + f*x]^4*(a + b*Sec[e + f*x]^2),x]
```

output
```
(a*(Tan[e + f*x] + Tan[e + f*x]^3/3))/f + (b*(Tan[e + f*x] + (2*Tan[e + f*
x]^3)/3 + Tan[e + f*x]^5/5))/f
```

3.159.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4534, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)^4\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)^2\right)dx$$

$$\downarrow 4534$$

$$\frac{1}{5}(5a+4b)\int \sec^4(e+fx)dx+\frac{b\tan(e+fx)\sec^4(e+fx)}{5f}$$

$$\downarrow 3042$$

$$\frac{1}{5}(5a+4b)\int \csc\left(e+fx+\frac{\pi}{2}\right)^4 dx+\frac{b\tan(e+fx)\sec^4(e+fx)}{5f}$$

$$\downarrow 4254$$

$$\frac{b\tan(e+fx)\sec^4(e+fx)}{5f}-\frac{(5a+4b)\int\left(\tan^2(e+fx)+1\right)d(-\tan(e+fx))}{5f}$$

$$\downarrow 2009$$

$$\frac{b\tan(e+fx)\sec^4(e+fx)}{5f}-\frac{(5a+4b)\left(-\frac{1}{3}\tan^3(e+fx)-\tan(e+fx)\right)}{5f}$$

input
```
Int[Sec[e + f*x]^4*(a + b*Sec[e + f*x]^2),x]
```

output
```
(b*Sec[e + f*x]^4*Tan[e + f*x])/(5*f) - ((5*a + 4*b)*(-Tan[e + f*x] - Tan[
e + f*x]^3/3))/(5*f)
```

3.159.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

3.159.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

method	result	si
derivativedivides	$\frac{-a\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)-b\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)}{f}$	5
default	$\frac{-a\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)-b\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)}{f}$	5
parts	$-\frac{a\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)}{f}-\frac{b\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)}{f}$	6
parallelrisch	$\frac{(50a+40b)\sin(3fx+3e)+(10a+8b)\sin(5fx+5e)+40\sin(fx+e)(a+2b)}{15f(\cos(5fx+5e)+5\cos(3fx+3e)+10\cos(fx+e))}$	8
risch	$\frac{4i\left(15a\,e^{6i(fx+e)}+35a\,e^{4i(fx+e)}+40b\,e^{4i(fx+e)}+25a\,e^{2i(fx+e)}+20b\,e^{2i(fx+e)}+5a+4b\right)}{15f\left(e^{2i(fx+e)}+1\right)^5}$	8
norman	$\frac{-\frac{2(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{2(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{f}+\frac{8(2a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3f}+\frac{8(2a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{3f}-\frac{4(25a+29b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{15f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^5}$	1

input
```
int(sec(f*x+e)^4*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-a*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)-b*(-8/15-1/5*sec(f*x+e)^4-4/15*
sec(f*x+e)^2)*tan(f*x+e))
```

3.159.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{\left(2\left(5\,a+4\,b\right)\cos\left(fx+e\right)^4+\left(5\,a+4\,b\right)\cos\left(fx+e\right)^2+3\,b\right)\sin\left(fx+e\right)}{15\,f\cos\left(fx+e\right)^5}$$

input
```
integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/15*(2*(5*a + 4*b)*cos(f*x + e)^4 + (5*a + 4*b)*cos(f*x + e)^2 + 3*b)*sin
(f*x + e)/(f*cos(f*x + e)^5)
```

3.159.6 Sympy [F]

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)\sec^4\left(e+fx\right)dx$$

input
```
integrate(sec(f*x+e)**4*(a+b*sec(f*x+e)**2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)*sec(e + f*x)**4, x)
```

3.159.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.66

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{3\,b\tan\left(fx+e\right)^5+5\left(a+2\,b\right)\tan\left(fx+e\right)^3+15\left(a+b\right)\tan\left(fx+e\right)}{15\,f}$$

input
```
integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/15*(3*b*tan(f*x + e)^5 + 5*(a + 2*b)*tan(f*x + e)^3 + 15*(a + b)*tan(f*x
 + e))/f
```

3.159.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$=\frac{3\,b\tan\left(fx+e\right)^5+5\,a\tan\left(fx+e\right)^3+10\,b\tan\left(fx+e\right)^3+15\,a\tan\left(fx+e\right)+15\,b\tan\left(fx+e\right)}{15\,f}$$

input
```
integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/15*(3*b*tan(f*x + e)^5 + 5*a*tan(f*x + e)^3 + 10*b*tan(f*x + e)^3 + 15*a
*tan(f*x + e) + 15*b*tan(f*x + e))/f
```

3.159.9 Mupad [B] (verification not implemented)

Time = 18.42 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.65

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$=\frac{\frac{b\tan(e+f\,x)^5}{5}+\left(\frac{a}{3}+\frac{2\,b}{3}\right)\,\tan(e+f\,x)^3+(a+b)\,\tan(e+f\,x)}{f}$$

input
```
int((a + b/cos(e + f*x)^2)/cos(e + f*x)^4,x)
```

output
```
(tan(e + f*x)^3*(a/3 + (2*b)/3) + (b*tan(e + f*x)^5)/5 + tan(e + f*x)*(a +
 b))/f
```

3.160 $\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

3.160.1 Optimal result 1256
3.160.2 Mathematica [A] (verified) 1256
3.160.3 Rubi [A] (verified) 1257
3.160.4 Maple [A] (verified) 1258
3.160.5 Fricas [A] (verification not implemented) 1259
3.160.6 Sympy [F] 1259
3.160.7 Maxima [A] (verification not implemented) 1259
3.160.8 Giac [A] (verification not implemented) 1260
3.160.9 Mupad [B] (verification not implemented) 1260

3.160.1 Optimal result

Integrand size = 21, antiderivative size = 43

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{(3a+2b)\tan(e+fx)}{3f} + \frac{b\sec^2(e+fx)\tan(e+fx)}{3f}$$

output
```
1/3*(3*a+2*b)*tan(f*x+e)/f+1/3*b*sec(f*x+e)^2*tan(f*x+e)/f
```

3.160.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{a\tan(e+fx)}{f} + \frac{b\left(\tan(e+fx)+\frac{1}{3}\tan^3(e+fx)\right)}{f}$$

input
```
Integrate[Sec[e + f*x]^2*(a + b*Sec[e + f*x]^2),x]
```

output
```
(a*Tan[e + f*x])/f + (b*(Tan[e + f*x] + Tan[e + f*x]^3/3))/f
```

3.160.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4534, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)^2\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)^2\right)dx$$

$$\downarrow 4534$$

$$\frac{1}{3}(3a+2b)\int \sec^2(e+fx)dx+\frac{b\tan(e+fx)\sec^2(e+fx)}{3f}$$

$$\downarrow 3042$$

$$\frac{1}{3}(3a+2b)\int \csc\left(e+fx+\frac{\pi}{2}\right)^2 dx+\frac{b\tan(e+fx)\sec^2(e+fx)}{3f}$$

$$\downarrow 4254$$

$$\frac{b\tan(e+fx)\sec^2(e+fx)}{3f}-\frac{(3a+2b)\int 1d(-\tan(e+fx))}{3f}$$

$$\downarrow 24$$

$$\frac{(3a+2b)\tan(e+fx)}{3f}+\frac{b\tan(e+fx)\sec^2(e+fx)}{3f}$$

input
```
Int[Sec[e + f*x]^2*(a + b*Sec[e + f*x]^2),x]
```

output
```
((3*a + 2*b)*Tan[e + f*x])/(3*f) + (b*Sec[e + f*x]^2*Tan[e + f*x])/(3*f)
```

3.160.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

3.160.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{a\tan(fx+e)-b\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)}{f}$	35
default	$\frac{a\tan(fx+e)-b\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)}{f}$	35
parts	$\frac{a\tan(fx+e)}{f}-\frac{b\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)}{f}$	37
parallelrisch	$\frac{(3a+2b)\sin(3fx+3e)+3\sin(fx+e)(a+2b)}{3f(\cos(3fx+3e)+3\cos(fx+e))}$	57
risch	$\frac{2i\left(3a\,\mathrm{e}^{4i(fx+e)}+6a\,\mathrm{e}^{2i(fx+e)}+6b\,\mathrm{e}^{2i(fx+e)}+3a+2b\right)}{3f\left(\mathrm{e}^{2i(fx+e)}+1\right)^3}$	63
norman	$\frac{-\frac{2(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{2(a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{f}+\frac{4(3a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3}$	75

input
```
int(sec(f*x+e)^2*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(a*tan(f*x+e)-b*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e))
```

3.160.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{\left((3\,a+2\,b)\cos\left(fx+e\right)^2+b\right)\sin\left(fx+e\right)}{3\,f\cos\left(fx+e\right)^3}$$

input
```
integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/3*((3*a + 2*b)*cos(f*x + e)^2 + b)*sin(f*x + e)/(f*cos(f*x + e)^3)
```

3.160.6 Sympy [F]

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)\sec^2\left(e+fx\right)dx$$

input
```
integrate(sec(f*x+e)**2*(a+b*sec(f*x+e)**2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)*sec(e + f*x)**2, x)
```

3.160.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)b+3\,a\tan\left(fx+e\right)}{3\,f}$$

input
```
integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/3*((tan(f*x + e)^3 + 3*tan(f*x + e))*b + 3*a*tan(f*x + e))/f
```

3.160.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{b\tan(fx+e)^3+3\,a\tan(fx+e)+3\,b\tan(fx+e)}{3\,f}$$

input
```
integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/3*(b*tan(f*x + e)^3 + 3*a*tan(f*x + e) + 3*b*tan(f*x + e))/f
```

3.160.9 Mupad [B] (verification not implemented)

Time = 17.96 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.65

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{b\tan(e+f\,x)^3}{3\,f}+\frac{\tan(e+f\,x)\,(a+b)}{f}$$

input
```
int((a + b/cos(e + f*x)^2)/cos(e + f*x)^2,x)
```

output
```
(b*tan(e + f*x)^3)/(3*f) + (tan(e + f*x)*(a + b))/f
```

3.161 $\int (a + b \sec^2(e + fx))\, dx$

3.161.1 Optimal result 1261
3.161.2 Mathematica [A] (verified) 1261
3.161.3 Rubi [A] (verified) 1262
3.161.4 Maple [A] (verified) 1262
3.161.5 Fricas [B] (verification not implemented) 1263
3.161.6 Sympy [F] 1263
3.161.7 Maxima [A] (verification not implemented) 1263
3.161.8 Giac [A] (verification not implemented) 1264
3.161.9 Mupad [B] (verification not implemented) 1264

3.161.1 Optimal result

Integrand size = 12, antiderivative size = 15

$$\int (a + b \sec^2(e + fx))\, dx = ax + \frac{b \tan(e + fx)}{f}$$

output
```
a*x+b*tan(f*x+e)/f
```

3.161.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int (a + b \sec^2(e + fx))\, dx = ax + \frac{b \tan(e + fx)}{f}$$

input
```
Integrate[a + b*Sec[e + f*x]^2,x]
```

output
```
a*x + (b*Tan[e + f*x])/f
```

3.161.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + b\sec^2(e+fx))\,dx$$

$$\downarrow \text{2009}$$

$$ax + \frac{b\tan(e+fx)}{f}$$

input
```
Int[a + b*Sec[e + f*x]^2,x]
```

output
```
a*x + (b*Tan[e + f*x])/f
```

3.161.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.161.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result	size
default	$ax + \frac{b\tan(fx+e)}{f}$	16
parts	$ax + \frac{b\tan(fx+e)}{f}$	16
derivativedivides	$\frac{(fx+e)a+b\tan(fx+e)}{f}$	21
risch	$ax + \frac{2ib}{f(e^{2i(fx+e)}+1)}$	25
parallelrisch	$-\frac{2b\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)} + ax$	35
norman	$\frac{ax\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2 - ax - \frac{2b\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1}$	51

input
```
int(a+b*sec(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
a*x+b*tan(f*x+e)/f
```

3.161.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(15) = 30$.

Time $= 0.25$ (sec) , antiderivative size $= 31$, normalized size of antiderivative $= 2.07$

$$\int \left(a + b\sec^2(e + fx)\right)\, dx = \frac{afx\cos\left(fx + e\right) + b\sin\left(fx + e\right)}{f\cos\left(fx + e\right)}$$

input
```
integrate(a+b*sec(f*x+e)^2,x, algorithm="fricas")
```

output
```
(a*f*x*cos(f*x + e) + b*sin(f*x + e))/(f*cos(f*x + e))
```

3.161.6 Sympy [F]

$$\int \left(a + b\sec^2(e + fx)\right)\, dx = \int \left(a + b\sec^2\left(e + fx\right)\right)\, dx$$

input
```
integrate(a+b*sec(f*x+e)**2,x)
```

output
```
Integral(a + b*sec(e + f*x)**2, x)
```

3.161.7 Maxima [A] (verification not implemented)

Time $= 0.18$ (sec) , antiderivative size $= 15$, normalized size of antiderivative $= 1.00$

$$\int \left(a + b\sec^2(e + fx)\right)\, dx = ax + \frac{b\tan\left(fx + e\right)}{f}$$

input
```
integrate(a+b*sec(f*x+e)^2,x, algorithm="maxima")
```

output
```
a*x + b*tan(f*x + e)/f
```

3.161.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \left(a + b\sec^2(e + fx)\right)\, dx = ax + \frac{b\tan\left(fx + e\right)}{f}$$

input
```
integrate(a+b*sec(f*x+e)^2,x, algorithm="giac")
```

output
```
a*x + b*tan(f*x + e)/f
```

3.161.9 Mupad [B] (verification not implemented)

Time = 18.42 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \left(a + b\sec^2(e + fx)\right)\, dx = \frac{b\tan(e + f\,x) + a\,f\,x}{f}$$

input
```
int(a + b/cos(e + f*x)^2,x)
```

output
```
(b*tan(e + f*x) + a*f*x)/f
```

3.162 $\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

3.162.1 Optimal result 1265
3.162.2 Mathematica [A] (verified) 1265
3.162.3 Rubi [A] (verified) 1266
3.162.4 Maple [A] (verified) 1267
3.162.5 Fricas [A] (verification not implemented) 1267
3.162.6 Sympy [A] (verification not implemented) 1268
3.162.7 Maxima [A] (verification not implemented) 1268
3.162.8 Giac [A] (verification not implemented) 1268
3.162.9 Mupad [B] (verification not implemented) 1269

3.162.1 Optimal result

Integrand size = 21, antiderivative size = 31

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{1}{2}(a+2b)x + \frac{a\cos(e+fx)\sin(e+fx)}{2f}$$

output
```
1/2*(a+2*b)*x+1/2*a*cos(f*x+e)*sin(f*x+e)/f
```

3.162.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = bx + \frac{a(e+fx)}{2f} + \frac{a\sin(2(e+fx))}{4f}$$

input
```
Integrate[Cos[e + f*x]^2*(a + b*Sec[e + f*x]^2),x]
```

output
```
b*x + (a*(e + f*x))/(2*f) + (a*Sin[2*(e + f*x)])/(4*f)
```

3.162.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4533, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \frac{a+b\csc\left(e+fx+\frac{\pi}{2}\right)^2}{\csc\left(e+fx+\frac{\pi}{2}\right)^2}dx$$

$$\downarrow 4533$$

$$\frac{1}{2}(a+2b)\int 1dx+\frac{a\sin(e+fx)\cos(e+fx)}{2f}$$

$$\downarrow 24$$

$$\frac{1}{2}x(a+2b)+\frac{a\sin(e+fx)\cos(e+fx)}{2f}$$

input
```
Int[Cos[e + f*x]^2*(a + b*Sec[e + f*x]^2),x]
```

output
```
((a + 2*b)*x)/2 + (a*Cos[e + f*x]*Sin[e + f*x])/(2*f)
```

3.162.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

3.162.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

method	result
risch	$\frac{ax}{2}+xb+\frac{a\sin(2fx+2e)}{4f}$
parallelrisch	$\frac{a\sin(2fx+2e)+2(a+2b)xf}{4f}$
derivativedivides	$\frac{a\left(\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+b(fx+e)}{f}$
default	$\frac{a\left(\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+b(fx+e)}{f}$
norman	$\frac{(-\frac{a}{2}-b)x+(-\frac{a}{2}-b)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+(\frac{a}{2}+b)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+(\frac{a}{2}+b)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6-\frac{a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{2a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{f}-$ $\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)$

input
```
int(cos(f*x+e)^2*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x+x*b+1/4*a/f*sin(2*f*x+2*e)
```

3.162.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx=\frac{(a+2\,b)fx+a\cos(fx+e)\sin(fx+e)}{2\,f}$$

input
```
integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/2*((a + 2*b)*f*x + a*cos(f*x + e)*sin(f*x + e))/f
```

3.162.6 Sympy [A] (verification not implemented)

Time = 2.47 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.65

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$= a\left(\begin{cases}\frac{x\sin^2(e+fx)}{2}+\frac{x\cos^2(e+fx)}{2}+\frac{\sin(e+fx)\cos(e+fx)}{2f} & \text{for } f\neq 0\\ x\cos^2(e) & \text{otherwise}\end{cases}\right)+bx$$

input
```
integrate(cos(f*x+e)**2*(a+b*sec(f*x+e)**2),x)
```

output
```
a*Piecewise((x*sin(e + f*x)**2/2 + x*cos(e + f*x)**2/2 + sin(e + f*x)*cos(
e + f*x)/(2*f), Ne(f, 0)), (x*cos(e)**2, True)) + b*x
```

3.162.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{(fx+e)(a+2\,b)+\frac{a\tan(fx+e)}{\tan(fx+e)^2+1}}{2\,f}$$

input
```
integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/2*((f*x + e)*(a + 2*b) + a*tan(f*x + e)/(tan(f*x + e)^2 + 1))/f
```

3.162.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{(fx+e)(a+2\,b)+\frac{a\tan(fx+e)}{\tan(fx+e)^2+1}}{2\,f}$$

input
```
integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/2*((f*x + e)*(a + 2*b) + a*tan(f*x + e)/(tan(f*x + e)^2 + 1))/f
```

3.162.9 Mupad [B] (verification not implemented)

Time = 19.58 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{\frac{a\sin(2e+2fx)}{4}+fx\left(\frac{a}{2}+b\right)}{f}$$

input
```
int(cos(e + f*x)^2*(a + b/cos(e + f*x)^2),x)
```

output
```
((a*sin(2*e + 2*f*x))/4 + f*x*(a/2 + b))/f
```

3.163 $\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

3.163.1 Optimal result 1270
3.163.2 Mathematica [A] (verified) 1270
3.163.3 Rubi [A] (verified) 1271
3.163.4 Maple [A] (verified) 1272
3.163.5 Fricas [A] (verification not implemented) 1273
3.163.6 Sympy [F] 1273
3.163.7 Maxima [A] (verification not implemented) 1273
3.163.8 Giac [A] (verification not implemented) 1274
3.163.9 Mupad [B] (verification not implemented) 1274

3.163.1 Optimal result

Integrand size = 21, antiderivative size = 61

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{1}{8}(3a+4b)x + \frac{(3a+4b)\cos(e+fx)\sin(e+fx)}{8f} + \frac{a\cos^3(e+fx)\sin(e+fx)}{4f}$$

output
```
1/8*(3*a+4*b)*x+1/8*(3*a+4*b)*cos(f*x+e)*sin(f*x+e)/f+1/4*a*cos(f*x+e)^3*s
in(f*x+e)/f
```

3.163.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{4(3a+4b)(e+fx)+8(a+b)\sin(2(e+fx))+a\sin(4(e+fx))}{32f}$$

input
```
Integrate[Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2),x]
```

output
```
(4*(3*a + 4*b)*(e + f*x) + 8*(a + b)*Sin[2*(e + f*x)] + a*Sin[4*(e + f*x)]
)/(32*f)
```

3.163.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4533, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \frac{a+b\csc\left(e+fx+\frac{\pi}{2}\right)^2}{\csc\left(e+fx+\frac{\pi}{2}\right)^4}dx$$

$$\downarrow 4533$$

$$\frac{1}{4}(3a+4b)\int \cos^2(e+fx)dx+\frac{a\sin(e+fx)\cos^3(e+fx)}{4f}$$

$$\downarrow 3042$$

$$\frac{1}{4}(3a+4b)\int \sin\left(e+fx+\frac{\pi}{2}\right)^2 dx+\frac{a\sin(e+fx)\cos^3(e+fx)}{4f}$$

$$\downarrow 3115$$

$$\frac{1}{4}(3a+4b)\left(\frac{\int 1dx}{2}+\frac{\sin(e+fx)\cos(e+fx)}{2f}\right)+\frac{a\sin(e+fx)\cos^3(e+fx)}{4f}$$

$$\downarrow 24$$

$$\frac{1}{4}(3a+4b)\left(\frac{\sin(e+fx)\cos(e+fx)}{2f}+\frac{x}{2}\right)+\frac{a\sin(e+fx)\cos^3(e+fx)}{4f}$$

input
```
Int[Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2),x]
```

output
```
(a*Cos[e + f*x]^3*Sin[e + f*x])/(4*f) + ((3*a + 4*b)*(x/2 + (Cos[e + f*x]*
Sin[e + f*x])/(2*f)))/4
```

3.163.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

3.163.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.72

method	result
parallelrisch	$\frac{(8a+8b)\sin(2fx+2e)+\sin(4fx+4e)a+12\left(a+\frac{4b}{3}\right)xf}{32f}$
risch	$\frac{3ax}{8}+\frac{xb}{2}+\frac{\sin(4fx+4e)a}{32f}+\frac{a\sin(2fx+2e)}{4f}+\frac{\sin(2fx+2e)b}{4f}$
derivativedivides	$\frac{a\left(\frac{\left(\cos(fx+e)^3+\frac{3\cos(fx+e)}{2}\right)\sin(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+b\left(\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}$
default	$\frac{a\left(\frac{\left(\cos(fx+e)^3+\frac{3\cos(fx+e)}{2}\right)\sin(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+b\left(\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}$
norman	$\frac{\left(-\frac{3a}{8}-\frac{b}{2}\right)x+\left(-\frac{9a}{8}-\frac{3b}{2}\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+\left(-\frac{3a}{4}-b\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+\left(\frac{3a}{4}+b\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6+\left(\frac{3a}{8}+\frac{b}{2}\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)$ $\left(1+\tan\left(\frac{fx}{2}+\right.\right.$

input
```
int(cos(f*x+e)^4*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/32*((8*a+8*b)*sin(2*f*x+2*e)+sin(4*f*x+4*e)*a+12*(a+4/3*b)*x*f)/f
```

3.163.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.80

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{(3\,a+4\,b)fx+\left(2\,a\cos\left(fx+e\right)^3+(3\,a+4\,b)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{8\,f}$$

input
```
integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/8*((3*a + 4*b)*f*x + (2*a*cos(f*x + e)^3 + (3*a + 4*b)*cos(f*x + e))*sin
(f*x + e))/f
```

3.163.6 Sympy [F]

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)\cos^4\left(e+fx\right)\,dx$$

input
```
integrate(cos(f*x+e)**4*(a+b*sec(f*x+e)**2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)*cos(e + f*x)**4, x)
```

3.163.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.20

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{(fx+e)(3\,a+4\,b)+\frac{(3\,a+4\,b)\tan(fx+e)^3+(5\,a+4\,b)\tan(fx+e)}{\tan(fx+e)^4+2\,\tan(fx+e)^2+1}}{8\,f}$$

input
```
integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/8*((f*x + e)*(3*a + 4*b) + ((3*a + 4*b)*tan(f*x + e)^3 + (5*a + 4*b)*tan
(f*x + e))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1))/f
```

3.163.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.20

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{(fx+e)(3\,a+4\,b)+\frac{3\,a\tan(fx+e)^3+4\,b\tan(fx+e)^3+5\,a\tan(fx+e)+4\,b\tan(fx+e)}{\left(\tan(fx+e)^2+1\right)^2}}{8\,f}$$

input
```
integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/8*((f*x + e)*(3*a + 4*b) + (3*a*tan(f*x + e)^3 + 4*b*tan(f*x + e)^3 + 5*
a*tan(f*x + e) + 4*b*tan(f*x + e))/(tan(f*x + e)^2 + 1)^2)/f
```

3.163.9 Mupad [B] (verification not implemented)

Time = 18.81 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.10

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = x\left(\frac{3\,a}{8}+\frac{b}{2}\right) + \frac{\left(\frac{3\,a}{8}+\frac{b}{2}\right)\tan(e+f\,x)^3+\left(\frac{5\,a}{8}+\frac{b}{2}\right)\tan(e+f\,x)}{f\left(\tan(e+f\,x)^4+2\tan(e+f\,x)^2+1\right)}$$

input
```
int(cos(e + f*x)^4*(a + b/cos(e + f*x)^2),x)
```

output
```
x*((3*a)/8 + b/2) + (tan(e + f*x)^3*((3*a)/8 + b/2) + tan(e + f*x)*((5*a)/
8 + b/2))/(f*(2*tan(e + f*x)^2 + tan(e + f*x)^4 + 1))
```

3.164 $\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

3.164.1 Optimal result 1275
3.164.2 Mathematica [A] (verified) 1275
3.164.3 Rubi [A] (verified) 1276
3.164.4 Maple [A] (verified) 1277
3.164.5 Fricas [A] (verification not implemented) 1278
3.164.6 Sympy [F(-1)] 1279
3.164.7 Maxima [A] (verification not implemented) 1279
3.164.8 Giac [A] (verification not implemented) 1279
3.164.9 Mupad [B] (verification not implemented) 1280

3.164.1 Optimal result

Integrand size = 21, antiderivative size = 89

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{1}{16}(5a+6b)x + \frac{(5a+6b)\cos(e+fx)\sin(e+fx)}{16f} + \frac{(5a+6b)\cos^3(e+fx)\sin(e+fx)}{24f} + \frac{a\cos^5(e+fx)\sin(e+fx)}{6f}$$

output
```
1/16*(5*a+6*b)*x+1/16*(5*a+6*b)*cos(f*x+e)*sin(f*x+e)/f+1/24*(5*a+6*b)*cos
(f*x+e)^3*sin(f*x+e)/f+1/6*a*cos(f*x+e)^5*sin(f*x+e)/f
```

3.164.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{60ae+72be+60afx+72bfx+(45a+48b)\sin(2(e+fx))+(9a+6b)\sin(4(e+fx))+a\sin(6(e+fx))}{192f}$$

input
```
Integrate[Cos[e + f*x]^6*(a + b*Sec[e + f*x]^2),x]
```

output
```
(60*a*e + 72*b*e + 60*a*f*x + 72*b*f*x + (45*a + 48*b)*Sin[2*(e + f*x)] +
(9*a + 6*b)*Sin[4*(e + f*x)] + a*Sin[6*(e + f*x)])/(192*f)
```

3.164.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4533, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \frac{a+b\csc\left(e+fx+\frac{\pi}{2}\right)^2}{\csc\left(e+fx+\frac{\pi}{2}\right)^6}dx$$

$$\downarrow 4533$$

$$\frac{1}{6}(5a+6b)\int \cos^4(e+fx)dx+\frac{a\sin(e+fx)\cos^5(e+fx)}{6f}$$

$$\downarrow 3042$$

$$\frac{1}{6}(5a+6b)\int \sin\left(e+fx+\frac{\pi}{2}\right)^4 dx+\frac{a\sin(e+fx)\cos^5(e+fx)}{6f}$$

$$\downarrow 3115$$

$$\frac{1}{6}(5a+6b)\left(\frac{3}{4}\int \cos^2(e+fx)dx+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)+\frac{a\sin(e+fx)\cos^5(e+fx)}{6f}$$

$$\downarrow 3042$$

$$\frac{1}{6}(5a+6b)\left(\frac{3}{4}\int \sin\left(e+fx+\frac{\pi}{2}\right)^2 dx+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)+\frac{a\sin(e+fx)\cos^5(e+fx)}{6f}$$

$$\downarrow 3115$$

$$\frac{1}{6}(5a+6b)\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(e+fx)\cos(e+fx)}{2f}\right)+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)+\frac{a\sin(e+fx)\cos^5(e+fx)}{6f}$$

$$\downarrow 24$$

$$\frac{1}{6}(5a+6b)\left(\frac{\sin(e+fx)\cos^3(e+fx)}{4f}+\frac{3}{4}\left(\frac{\sin(e+fx)\cos(e+fx)}{2f}+\frac{x}{2}\right)\right)+\frac{a\sin(e+fx)\cos^5(e+fx)}{6f}$$

input
```
Int[Cos[e + f*x]^6*(a + b*Sec[e + f*x]^2),x]
```

output
```
(a*Cos[e + f*x]^5*Sin[e + f*x])/(6*f) + ((5*a + 6*b)*((Cos[e + f*x]^3*Sin[
e + f*x])/(4*f) + (3*(x/2 + (Cos[e + f*x]*Sin[e + f*x])/(2*f)))/4))/6
```

3.164.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

3.164.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.69

method	result
parallelrisch	$\frac{(45a+48b)\sin(2fx+2e)+(9a+6b)\sin(4fx+4e)+a\sin(6fx+6e)+60xf\left(a+\frac{6b}{5}\right)}{192f}$
risch	$\frac{5ax}{16}+\frac{3xb}{8}+\frac{a\sin(6fx+6e)}{192f}+\frac{3\sin(4fx+4e)a}{64f}+\frac{\sin(4fx+4e)b}{32f}+\frac{15a\sin(2fx+2e)}{64f}+\frac{\sin(2fx+2e)b}{4f}$
derivativedivides	$\frac{a\left(\frac{\left(\cos(fx+e)^5+\frac{5\cos(fx+e)^3}{4}+\frac{15\cos(fx+e)}{8}\right)\sin(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)+b\left(\frac{\left(\cos(fx+e)^3+\frac{3\cos(fx+e)}{2}\right)\sin(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right.}{f}$
default	$\frac{a\left(\frac{\left(\cos(fx+e)^5+\frac{5\cos(fx+e)^3}{4}+\frac{15\cos(fx+e)}{8}\right)\sin(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)+b\left(\frac{\left(\cos(fx+e)^3+\frac{3\cos(fx+e)}{2}\right)\sin(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right.}{f}$
norman	$\left(-\frac{5a}{16}-\frac{3b}{8}\right)x+\left(-\frac{45a}{16}-\frac{27b}{8}\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+\left(-\frac{25a}{16}-\frac{15b}{8}\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+\left(-\frac{25a}{16}-\frac{15b}{8}\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6+\left(\frac{5a}{16}+\frac{3b}{8}\right)$

input
```
int(cos(f*x+e)^6*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/192*((45*a+48*b)*sin(2*f*x+2*e)+(9*a+6*b)*sin(4*f*x+4*e)+a*sin(6*f*x+6*e
)+60*x*f*(a+6/5*b))/f
```

3.164.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$=\frac{3\left(5\,a+6\,b\right)fx+\left(8\,a\cos\left(fx+e\right)^5+2\left(5\,a+6\,b\right)\cos\left(fx+e\right)^3+3\left(5\,a+6\,b\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{48\,f}$$

input
```
integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/48*(3*(5*a + 6*b)*f*x + (8*a*cos(f*x + e)^5 + 2*(5*a + 6*b)*cos(f*x + e)
^3 + 3*(5*a + 6*b)*cos(f*x + e))*sin(f*x + e))/f
```

3.164.6 Sympy [F(-1)]

Timed out.

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**6*(a+b*sec(f*x+e)**2),x)
```

output
```
Timed out
```

3.164.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.16

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$= \frac{3\,(fx+e)(5\,a+6\,b)+\frac{3\,(5\,a+6\,b)\tan(fx+e)^5+8\,(5\,a+6\,b)\tan(fx+e)^3+3\,(11\,a+10\,b)\tan(fx+e)}{\tan(fx+e)^6+3\tan(fx+e)^4+3\tan(fx+e)^2+1}}{48\,f}$$

input
```
integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/48*(3*(f*x + e)*(5*a + 6*b) + (3*(5*a + 6*b)*tan(f*x + e)^5 + 8*(5*a + 6
*b)*tan(f*x + e)^3 + 3*(11*a + 10*b)*tan(f*x + e))/(tan(f*x + e)^6 + 3*tan
(f*x + e)^4 + 3*tan(f*x + e)^2 + 1))/f
```

3.164.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.08

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$= \frac{3\,(fx+e)(5\,a+6\,b)+\frac{15\,a\tan(fx+e)^5+18\,b\tan(fx+e)^5+40\,a\tan(fx+e)^3+48\,b\tan(fx+e)^3+33\,a\tan(fx+e)+30\,b\tan(fx+e)}{\left(\tan(fx+e)^2+1\right)^3}}{48\,f}$$

input
```
integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/48*(3*(f*x + e)*(5*a + 6*b) + (15*a*tan(f*x + e)^5 + 18*b*tan(f*x + e)^5
 + 40*a*tan(f*x + e)^3 + 48*b*tan(f*x + e)^3 + 33*a*tan(f*x + e) + 30*b*ta
n(f*x + e))/(tan(f*x + e)^2 + 1)^3)/f
```

3.164.9 Mupad [B] (verification not implemented)

Time = 19.16 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.02

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$= x\left(\frac{5\,a}{16}+\frac{3\,b}{8}\right)$$
$$+\frac{\left(\frac{5\,a}{16}+\frac{3\,b}{8}\right)\tan(e+f\,x)^5+\left(\frac{5\,a}{6}+b\right)\tan(e+f\,x)^3+\left(\frac{11\,a}{16}+\frac{5\,b}{8}\right)\tan(e+f\,x)}{f\,\left(\tan\left(e+f\,x\right)^6+3\tan\left(e+f\,x\right)^4+3\tan\left(e+f\,x\right)^2+1\right)}$$

input
```
int(cos(e + f*x)^6*(a + b/cos(e + f*x)^2),x)
```

output
```
x*((5*a)/16 + (3*b)/8) + (tan(e + f*x)^5*((5*a)/16 + (3*b)/8) + tan(e + f*
x)*((11*a)/16 + (5*b)/8) + tan(e + f*x)^3*((5*a)/6 + b))/(f*(3*tan(e + f*x
)^2 + 3*tan(e + f*x)^4 + tan(e + f*x)^6 + 1))
```

3.165 $\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

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3.165.1 Optimal result

Integrand size = 23, antiderivative size = 165

$$\begin{aligned}
&\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx \\
&= \frac{(48a^2+80ab+35b^2)\operatorname{arctanh}(\sin(e+fx))}{128f} \\
&\quad + \frac{(48a^2+80ab+35b^2)\sec(e+fx)\tan(e+fx)}{128f} \\
&\quad + \frac{(48a^2+80ab+35b^2)\sec^3(e+fx)\tan(e+fx)}{192f} + \frac{b(10a+7b)\sec^5(e+fx)\tan(e+fx)}{48f} \\
&\quad + \frac{b\sec^7(e+fx)\left(a+b-a\sin^2(e+fx)\right)\tan(e+fx)}{8f}
\end{aligned}$$

output

```
1/128*(48*a^2+80*a*b+35*b^2)*arctanh(sin(f*x+e))/f+1/128*(48*a^2+80*a*b+35
*b^2)*sec(f*x+e)*tan(f*x+e)/f+1/192*(48*a^2+80*a*b+35*b^2)*sec(f*x+e)^3*ta
n(f*x+e)/f+1/48*b*(10*a+7*b)*sec(f*x+e)^5*tan(f*x+e)/f+1/8*b*sec(f*x+e)^7*
(a+b-a*sin(f*x+e)^2)*tan(f*x+e)/f
```

3.165.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.56

$$\begin{aligned}
&\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx \\
&= \frac{3a^2\operatorname{arctanh}(\sin(e+fx))}{8f} + \frac{5ab\operatorname{arctanh}(\sin(e+fx))}{8f} \\
&\quad + \frac{35b^2\operatorname{arctanh}(\sin(e+fx))}{128f} + \frac{3a^2\sec(e+fx)\tan(e+fx)}{8f} \\
&\quad + \frac{5ab\sec(e+fx)\tan(e+fx)}{8f} + \frac{35b^2\sec(e+fx)\tan(e+fx)}{128f} \\
&\quad + \frac{a^2\sec^3(e+fx)\tan(e+fx)}{4f} + \frac{5ab\sec^3(e+fx)\tan(e+fx)}{12f} \\
&\quad + \frac{35b^2\sec^3(e+fx)\tan(e+fx)}{192f} + \frac{ab\sec^5(e+fx)\tan(e+fx)}{3f} \\
&\quad + \frac{7b^2\sec^5(e+fx)\tan(e+fx)}{48f} + \frac{b^2\sec^7(e+fx)\tan(e+fx)}{8f}
\end{aligned}$$

input
```
Integrate[Sec[e + f*x]^5*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(3*a^2*ArcTanh[Sin[e + f*x]])/(8*f) + (5*a*b*ArcTanh[Sin[e + f*x]])/(8*f)
+ (35*b^2*ArcTanh[Sin[e + f*x]])/(128*f) + (3*a^2*Sec[e + f*x]*Tan[e + f*x
])/(8*f) + (5*a*b*Sec[e + f*x]*Tan[e + f*x])/(8*f) + (35*b^2*Sec[e + f*x]*
Tan[e + f*x])/(128*f) + (a^2*Sec[e + f*x]^3*Tan[e + f*x])/(4*f) + (5*a*b*S
ec[e + f*x]^3*Tan[e + f*x])/(12*f) + (35*b^2*Sec[e + f*x]^3*Tan[e + f*x])/
(192*f) + (a*b*Sec[e + f*x]^5*Tan[e + f*x])/(3*f) + (7*b^2*Sec[e + f*x]^5*
Tan[e + f*x])/(48*f) + (b^2*Sec[e + f*x]^7*Tan[e + f*x])/(8*f)
```

3.165.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4635, 315, 25, 298, 215, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$

$$\downarrow 3042$$

$$\int \sec(e+fx)^5 \left(a+b\sec(e+fx)^2\right)^2 dx$$

$$\downarrow 4635$$

$$\frac{\int \frac{\left(-a\sin^2(e+fx)+a+b\right)^2}{\left(1-\sin^2(e+fx)\right)^5} d\sin(e+fx)}{f}$$

$$\downarrow 315$$

$$\frac{\frac{b\sin(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}{8\left(1-\sin^2(e+fx)\right)^4} - \frac{1}{8}\int -\frac{(a+b)(8a+7b)-a(8a+5b)\sin^2(e+fx)}{\left(1-\sin^2(e+fx)\right)^4} d\sin(e+fx)}{f}$$

$$\downarrow 25$$

$$\frac{\frac{1}{8}\int \frac{(a+b)(8a+7b)-a(8a+5b)\sin^2(e+fx)}{\left(1-\sin^2(e+fx)\right)^4} d\sin(e+fx) + \frac{b\sin(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}{8\left(1-\sin^2(e+fx)\right)^4}}{f}$$

$$\downarrow 298$$

$$\frac{\frac{1}{8}\left(\frac{1}{6}\left(48a^2+80ab+35b^2\right)\int \frac{1}{\left(1-\sin^2(e+fx)\right)^3} d\sin(e+fx) + \frac{b(10a+7b)\sin(e+fx)}{6\left(1-\sin^2(e+fx)\right)^3}\right) + \frac{b\sin(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}{8\left(1-\sin^2(e+fx)\right)^4}}{f}$$

$$\downarrow 215$$

$$\frac{\frac{1}{8}\left(\frac{1}{6}\left(48a^2+80ab+35b^2\right)\left(\frac{3}{4}\int \frac{1}{\left(1-\sin^2(e+fx)\right)^2} d\sin(e+fx) + \frac{\sin(e+fx)}{4\left(1-\sin^2(e+fx)\right)^2}\right) + \frac{b(10a+7b)\sin(e+fx)}{6\left(1-\sin^2(e+fx)\right)^3}\right) + \frac{b\sin(e+fx)}{8(1-}}{f}$$

$$\downarrow 215$$

$$\frac{\frac{1}{8}\left(\frac{1}{6}\left(48a^2+80ab+35b^2\right)\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{1-\sin^2(e+fx)} d\sin(e+fx) + \frac{\sin(e+fx)}{2\left(1-\sin^2(e+fx)\right)}\right) + \frac{\sin(e+fx)}{4\left(1-\sin^2(e+fx)\right)^2}\right) + \frac{b(10a+7b)\sin(e}{6\left(1-\sin^2(e+f}}{f}$$

$$\downarrow 219$$

$$\frac{\frac{1}{8}\left(\frac{1}{6}\left(48a^2+80ab+35b^2\right)\left(\frac{3}{4}\left(\frac{1}{2}\operatorname{arctanh}(\sin(e+fx)) + \frac{\sin(e+fx)}{2\left(1-\sin^2(e+fx)\right)}\right) + \frac{\sin(e+fx)}{4\left(1-\sin^2(e+fx)\right)^2}\right) + \frac{b(10a+7b)\sin(e+fx)}{6\left(1-\sin^2(e+fx)\right)^3}\right)}{f}$$

input
```
Int[Sec[e + f*x]^5*(a + b*Sec[e + f*x]^2)^2,x]
```

3.165. $\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx$

output
```
((b*Sin[e + f*x]*(a + b - a*Sin[e + f*x]^2))/(8*(1 - Sin[e + f*x]^2)^4) +
((b*(10*a + 7*b)*Sin[e + f*x])/(6*(1 - Sin[e + f*x]^2)^3) + ((48*a^2 + 80*
a*b + 35*b^2)*(Sin[e + f*x]/(4*(1 - Sin[e + f*x]^2)^2) + (3*(ArcTanh[Sin[e
 + f*x]]/2 + Sin[e + f*x]/(2*(1 - Sin[e + f*x]^2))))/4))/6)/8)/f
```

3.165.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.165.4 Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{a^2\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)+2ab\left(-\left(-\frac{\sec(fx+e)^5}{6}-\frac{5\sec(fx+e)^3}{24}-\frac{5\sec}{}\right.\right.}{}$
default	$\frac{a^2\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)+2ab\left(-\left(-\frac{\sec(fx+e)^5}{6}-\frac{5\sec(fx+e)^3}{24}-\frac{5\sec}{}\right.\right.}{}$
parts	$\frac{a^2\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)}{f}+\frac{b^2\left(-\left(-\frac{\sec(fx+e)^7}{8}-\frac{7\sec(fx+e)^5}{48}-\frac{35}{}\right.\right.}{}$
parallelrisch	$\frac{-8064\left(\frac{\cos(8fx+8e)}{56}+\frac{\cos(6fx+6e)}{7}+\frac{\cos(4fx+4e)}{2}+\cos(2fx+2e)+\frac{5}{8}\right)\left(a^2+\frac{5}{3}ab+\frac{35}{48}b^2\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+8064\left(\frac{\cos(8f}{5}\right.}{}$
norman	$\frac{\left(80a^2+176ab+93b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{64f}+\frac{\left(80a^2+176ab+93b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{15}}{64f}-\frac{\left(432a^2+1360ab-1085b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{192f}-\frac{\left(432a^2+1360\right.}{}$
risch	$-\frac{ie^{i(fx+e)}\left(144a^2e^{14i(fx+e)}+240ab\,e^{14i(fx+e)}+105b^2e^{14i(fx+e)}+1104a^2e^{12i(fx+e)}+1840ab\,e^{12i(fx+e)}+805b^2e^{12i(fx}\right.}{}$

input

```
int(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output

```
1/f*(a^2*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)
+tan(f*x+e)))+2*a*b*(-(-1/6*sec(f*x+e)^5-5/24*sec(f*x+e)^3-5/16*sec(f*x+e)
)*tan(f*x+e)+5/16*ln(sec(f*x+e)+tan(f*x+e)))+b^2*(-(-1/8*sec(f*x+e)^7-7/48
*sec(f*x+e)^5-35/192*sec(f*x+e)^3-35/128*sec(f*x+e))*tan(f*x+e)+35/128*ln(
sec(f*x+e)+tan(f*x+e))))
```

3.165.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.02

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$= \frac{3\left(48a^2+80ab+35b^2\right)\cos(fx+e)^8\log(\sin(fx+e)+1)-3\left(48a^2+80ab+35b^2\right)\cos(fx+e)^8\log(-$$

input
```
integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/768*(3*(48*a^2 + 80*a*b + 35*b^2)*cos(f*x + e)^8*log(sin(f*x + e) + 1) -
 3*(48*a^2 + 80*a*b + 35*b^2)*cos(f*x + e)^8*log(-sin(f*x + e) + 1) + 2*(3
*(48*a^2 + 80*a*b + 35*b^2)*cos(f*x + e)^6 + 2*(48*a^2 + 80*a*b + 35*b^2)*
cos(f*x + e)^4 + 8*(16*a*b + 7*b^2)*cos(f*x + e)^2 + 48*b^2)*sin(f*x + e))
/(f*cos(f*x + e)^8)
```

3.165.6 Sympy [F]

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \int \left(a+b\sec^2(e+fx)\right)^2\sec^5(e+fx)\,dx$$

input
```
integrate(sec(f*x+e)**5*(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**2*sec(e + f*x)**5, x)
```

3.165.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.21

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$= \frac{3\left(48a^2+80ab+35b^2\right)\log(\sin(fx+e)+1)-3\left(48a^2+80ab+35b^2\right)\log(\sin(fx+e)-1)-\frac{2\left(3\left(48a^2\right.\right.}{768}$$

input
```
integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/768*(3*(48*a^2 + 80*a*b + 35*b^2)*log(sin(f*x + e) + 1) - 3*(48*a^2 + 80
*a*b + 35*b^2)*log(sin(f*x + e) - 1) - 2*(3*(48*a^2 + 80*a*b + 35*b^2)*sin
(f*x + e)^7 - 11*(48*a^2 + 80*a*b + 35*b^2)*sin(f*x + e)^5 + (624*a^2 + 11
68*a*b + 511*b^2)*sin(f*x + e)^3 - 3*(80*a^2 + 176*a*b + 93*b^2)*sin(f*x +
 e))/(sin(f*x + e)^8 - 4*sin(f*x + e)^6 + 6*sin(f*x + e)^4 - 4*sin(f*x + e
)^2 + 1))/f
```

3.165.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.34

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$

$$= \frac{3\left(48a^2+80ab+35b^2\right)\log\left(\left|\sin\left(fx+e\right)+1\right|\right)-3\left(48a^2+80ab+35b^2\right)\log\left(\left|\sin\left(fx+e\right)-1\right|\right)-\frac{2\left(144\right.}{}}{}$$

input
```
integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/768*(3*(48*a^2 + 80*a*b + 35*b^2)*log(abs(sin(f*x + e) + 1)) - 3*(48*a^2
 + 80*a*b + 35*b^2)*log(abs(sin(f*x + e) - 1)) - 2*(144*a^2*sin(f*x + e)^7
 + 240*a*b*sin(f*x + e)^7 + 105*b^2*sin(f*x + e)^7 - 528*a^2*sin(f*x + e)^
5 - 880*a*b*sin(f*x + e)^5 - 385*b^2*sin(f*x + e)^5 + 624*a^2*sin(f*x + e)
^3 + 1168*a*b*sin(f*x + e)^3 + 511*b^2*sin(f*x + e)^3 - 240*a^2*sin(f*x +
e) - 528*a*b*sin(f*x + e) - 279*b^2*sin(f*x + e))/(sin(f*x + e)^2 - 1)^4)/
f
```

3.165.9 Mupad [B] (verification not implemented)

Time = 18.83 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.03

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$

$$= \frac{\left(-\frac{3a^2}{8}-\frac{5ab}{8}-\frac{35b^2}{128}\right)\sin\left(e+fx\right)^7+\left(\frac{11a^2}{8}+\frac{55ab}{24}+\frac{385b^2}{384}\right)\sin\left(e+fx\right)^5+\left(-\frac{13a^2}{8}-\frac{73ab}{24}-\frac{511b^2}{384}\right)\sin}{f\left(\sin\left(e+fx\right)^8-4\sin\left(e+fx\right)^6+6\sin\left(e+fx\right)^4-4\sin\left(e+f\right.\right.}$$

$$+\frac{\operatorname{atanh}\left(\sin\left(e+fx\right)\right)\left(\frac{3a^2}{8}+\frac{5ab}{8}+\frac{35b^2}{128}\right)}{f}$$

input
```
int((a + b/cos(e + f*x)^2)^2/cos(e + f*x)^5,x)
```

output
```
(sin(e + f*x)*((11*a*b)/8 + (5*a^2)/8 + (93*b^2)/128) - sin(e + f*x)^7*((5
*a*b)/8 + (3*a^2)/8 + (35*b^2)/128) + sin(e + f*x)^5*((55*a*b)/24 + (11*a^
2)/8 + (385*b^2)/384) - sin(e + f*x)^3*((73*a*b)/24 + (13*a^2)/8 + (511*b^
2)/384))/(f*(6*sin(e + f*x)^4 - 4*sin(e + f*x)^2 - 4*sin(e + f*x)^6 + sin(
e + f*x)^8 + 1)) + (atanh(sin(e + f*x))*((5*a*b)/8 + (3*a^2)/8 + (35*b^2)/
128))/f
```

3.166 $\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

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3.166.1 Optimal result

Integrand size = 23, antiderivative size = 129

$$\begin{aligned}
&\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx \\
&= \frac{(8a^2+12ab+5b^2)\operatorname{arctanh}(\sin(e+fx))}{16f} \\
&\quad + \frac{(8a^2+12ab+5b^2)\sec(e+fx)\tan(e+fx)}{16f} + \frac{b(8a+5b)\sec^3(e+fx)\tan(e+fx)}{24f} \\
&\quad + \frac{b\sec^5(e+fx)\left(a+b-a\sin^2(e+fx)\right)\tan(e+fx)}{6f}
\end{aligned}$$

output

```
1/16*(8*a^2+12*a*b+5*b^2)*arctanh(sin(f*x+e))/f+1/16*(8*a^2+12*a*b+5*b^2)*
sec(f*x+e)*tan(f*x+e)/f+1/24*b*(8*a+5*b)*sec(f*x+e)^3*tan(f*x+e)/f+1/6*b*s
ec(f*x+e)^5*(a+b-a*sin(f*x+e)^2)*tan(f*x+e)/f
```

3.166.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.45

$$\begin{aligned}
&\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx \\
&= \frac{a^2\operatorname{arctanh}(\sin(e+fx))}{2f} + \frac{3ab\operatorname{arctanh}(\sin(e+fx))}{4f} + \frac{5b^2\operatorname{arctanh}(\sin(e+fx))}{16f} \\
&\quad + \frac{a^2\sec(e+fx)\tan(e+fx)}{2f} + \frac{3ab\sec(e+fx)\tan(e+fx)}{4f} \\
&\quad + \frac{5b^2\sec(e+fx)\tan(e+fx)}{16f} + \frac{ab\sec^3(e+fx)\tan(e+fx)}{2f} \\
&\quad + \frac{5b^2\sec^3(e+fx)\tan(e+fx)}{24f} + \frac{b^2\sec^5(e+fx)\tan(e+fx)}{6f}
\end{aligned}$$

input
```
Integrate[Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(a^2*ArcTanh[Sin[e + f*x]])/(2*f) + (3*a*b*ArcTanh[Sin[e + f*x]])/(4*f) +
(5*b^2*ArcTanh[Sin[e + f*x]])/(16*f) + (a^2*Sec[e + f*x]*Tan[e + f*x])/(2*
f) + (3*a*b*Sec[e + f*x]*Tan[e + f*x])/(4*f) + (5*b^2*Sec[e + f*x]*Tan[e +
 f*x])/(16*f) + (a*b*Sec[e + f*x]^3*Tan[e + f*x])/(2*f) + (5*b^2*Sec[e + f
*x]^3*Tan[e + f*x])/(24*f) + (b^2*Sec[e + f*x]^5*Tan[e + f*x])/(6*f)
```

3.166.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4635, 315, 25, 298, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$

$$\downarrow \text{3042}$$

$$\int \sec(e+fx)^3\left(a+b\sec(e+fx)^2\right)^2\,dx$$

$$\downarrow \text{4635}$$

$$\frac{\int \frac{\left(-a\sin^2(e+fx)+a+b\right)^2}{\left(1-\sin^2(e+fx)\right)^4} d\sin(e+fx)}{f}$$

$$\downarrow 315$$

$$\frac{\frac{b\sin(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}{6\left(1-\sin^2(e+fx)\right)^3} - \frac{1}{6}\int -\frac{(a+b)(6a+5b)-3a(2a+b)\sin^2(e+fx)}{\left(1-\sin^2(e+fx)\right)^3} d\sin(e+fx)}{f}$$

$$\downarrow 25$$

$$\frac{\frac{1}{6}\int \frac{(a+b)(6a+5b)-3a(2a+b)\sin^2(e+fx)}{\left(1-\sin^2(e+fx)\right)^3} d\sin(e+fx) + \frac{b\sin(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}{6\left(1-\sin^2(e+fx)\right)^3}}{f}$$

$$\downarrow 298$$

$$\frac{\frac{1}{6}\left(\frac{3}{4}\left(8a^2+12ab+5b^2\right)\int \frac{1}{\left(1-\sin^2(e+fx)\right)^2} d\sin(e+fx) + \frac{b(8a+5b)\sin(e+fx)}{4\left(1-\sin^2(e+fx)\right)^2}\right) + \frac{b\sin(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}{6\left(1-\sin^2(e+fx)\right)^3}}{f}$$

$$\downarrow 215$$

$$\frac{\frac{1}{6}\left(\frac{3}{4}\left(8a^2+12ab+5b^2\right)\left(\frac{1}{2}\int \frac{1}{1-\sin^2(e+fx)} d\sin(e+fx) + \frac{\sin(e+fx)}{2\left(1-\sin^2(e+fx)\right)}\right) + \frac{b(8a+5b)\sin(e+fx)}{4\left(1-\sin^2(e+fx)\right)^2}\right) + \frac{b\sin(e+fx)\left(-a\sin^2(e}{6\left(1-\sin^2(e+}}{f}$$

$$\downarrow 219$$

$$\frac{\frac{1}{6}\left(\frac{3}{4}\left(8a^2+12ab+5b^2\right)\left(\frac{1}{2}\operatorname{arctanh}(\sin(e+fx)) + \frac{\sin(e+fx)}{2\left(1-\sin^2(e+fx)\right)}\right) + \frac{b(8a+5b)\sin(e+fx)}{4\left(1-\sin^2(e+fx)\right)^2}\right) + \frac{b\sin(e+fx)\left(-a\sin^2(e+fx)+\right.}{6\left(1-\sin^2(e+fx)\right)^3}}{f}$$

input
```
Int[Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((b*Sin[e + f*x]*(a + b - a*Sin[e + f*x]^2))/(6*(1 - Sin[e + f*x]^2)^3) +
((b*(8*a + 5*b)*Sin[e + f*x])/(4*(1 - Sin[e + f*x]^2)^2) + (3*(8*a^2 + 12*
a*b + 5*b^2)*(ArcTanh[Sin[e + f*x]]/2 + Sin[e + f*x]/(2*(1 - Sin[e + f*x]^
2))))/4)/6)/f
```

3.166. $\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx$

3.166.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.166.4 Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.14

method	result
derivativedivides	$\frac{a^2\left(\frac{\tan(fx+e)\sec(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+2ab\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan}{8}}{f}$
default	$\frac{a^2\left(\frac{\tan(fx+e)\sec(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+2ab\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan}{8}}{f}$
parts	$\frac{a^2\left(\frac{\tan(fx+e)\sec(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}+\frac{b^2\left(-\left(-\frac{\sec(fx+e)^5}{6}-\frac{5\sec(fx+e)^3}{24}-\frac{5\sec(fx+e)}{16}\right)\tan(fx+e)+\frac{5}{f}\right.}{f}$
parallelrisch	$\frac{-360\left(a^2+\frac{3}{2}ab+\frac{5}{8}b^2\right)\left(\frac{2}{3}+\frac{\cos(6fx+6e)}{15}+\frac{2\cos(4fx+4e)}{5}+\cos(2fx+2e)\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+360\left(a^2+\frac{3}{2}ab+\frac{5}{8}b^2\right)\left(\frac{2}{3}+\frac{\cos}{48f(}\right.}{48f(}$
norman	$\frac{\left(8a^2+4ab+15b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{4f}+\frac{\left(8a^2+4ab+15b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{4f}+\frac{\left(8a^2+20ab+11b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{8f}+\frac{\left(8a^2+20ab+11b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right.}{8f}$ $\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^6$
risch	$-\frac{ie^{i(fx+e)}\left(24a^2e^{10i(fx+e)}+36ab\,e^{10i(fx+e)}+15b^2e^{10i(fx+e)}+72a^2e^{8i(fx+e)}+204ab\,e^{8i(fx+e)}+85b^2e^{8i(fx+e)}+48a^2e\right.}{}$

input
```
int(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(a^2*(1/2*tan(f*x+e)*sec(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))+2*a*b*(
-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)+tan(f*x+e
)))+b^2*(-(-1/6*sec(f*x+e)^5-5/24*sec(f*x+e)^3-5/16*sec(f*x+e))*tan(f*x+e)
+5/16*ln(sec(f*x+e)+tan(f*x+e))))
```

3.166.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.11

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$

$$=\frac{3\left(8a^2+12ab+5b^2\right)\cos(fx+e)^6\log(\sin(fx+e)+1)-3\left(8a^2+12ab+5b^2\right)\cos(fx+e)^6\log(-\sin}{96f\cos(}$$

input
```
integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/96*(3*(8*a^2 + 12*a*b + 5*b^2)*cos(f*x + e)^6*log(sin(f*x + e) + 1) - 3*
(8*a^2 + 12*a*b + 5*b^2)*cos(f*x + e)^6*log(-sin(f*x + e) + 1) + 2*(3*(8*a
^2 + 12*a*b + 5*b^2)*cos(f*x + e)^4 + 2*(12*a*b + 5*b^2)*cos(f*x + e)^2 +
8*b^2)*sin(f*x + e))/(f*cos(f*x + e)^6)
```

3.166.6 Sympy [F]

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \int \left(a+b\sec^2(e+fx)\right)^2\sec^3(e+fx)\,dx$$

input
```
integrate(sec(f*x+e)**3*(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**2*sec(e + f*x)**3, x)
```

3.166.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.29

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$

$$= \frac{3\left(8a^2+12ab+5b^2\right)\log\left(\sin(fx+e)+1\right) - 3\left(8a^2+12ab+5b^2\right)\log\left(\sin(fx+e)-1\right) - \frac{2\left(3\left(8a^2+12ab-\right.\right.}{}}{96f}$$

input
```
integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/96*(3*(8*a^2 + 12*a*b + 5*b^2)*log(sin(f*x + e) + 1) - 3*(8*a^2 + 12*a*b
 + 5*b^2)*log(sin(f*x + e) - 1) - 2*(3*(8*a^2 + 12*a*b + 5*b^2)*sin(f*x +
e)^5 - 8*(6*a^2 + 12*a*b + 5*b^2)*sin(f*x + e)^3 + 3*(8*a^2 + 20*a*b + 11*
b^2)*sin(f*x + e))/(sin(f*x + e)^6 - 3*sin(f*x + e)^4 + 3*sin(f*x + e)^2 -
 1))/f
```

3.166.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.42

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$

$$= \frac{3\left(8a^2+12ab+5b^2\right)\log\left(\left|\sin(fx+e)+1\right|\right)-3\left(8a^2+12ab+5b^2\right)\log\left(\left|\sin(fx+e)-1\right|\right)-\frac{2\left(24a^2\sin(f\right.}{}}{}$$

input
```
integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/96*(3*(8*a^2 + 12*a*b + 5*b^2)*log(abs(sin(f*x + e) + 1)) - 3*(8*a^2 + 1
2*a*b + 5*b^2)*log(abs(sin(f*x + e) - 1)) - 2*(24*a^2*sin(f*x + e)^5 + 36*
a*b*sin(f*x + e)^5 + 15*b^2*sin(f*x + e)^5 - 48*a^2*sin(f*x + e)^3 - 96*a*
b*sin(f*x + e)^3 - 40*b^2*sin(f*x + e)^3 + 24*a^2*sin(f*x + e) + 60*a*b*si
n(f*x + e) + 33*b^2*sin(f*x + e))/(sin(f*x + e)^2 - 1)^3)/f
```

3.166.9 Mupad [B] (verification not implemented)

Time = 18.61 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.04

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{\operatorname{atanh}(\sin(e+fx))\left(\frac{a^2}{2}+\frac{3ab}{4}+\frac{5b^2}{16}\right)}{f}$$
$$-\frac{\left(\frac{a^2}{2}+\frac{3ab}{4}+\frac{5b^2}{16}\right)\sin(e+fx)^5+\left(-a^2-2ab-\frac{5b^2}{6}\right)\sin(e+fx)^3+\left(\frac{a^2}{2}+\frac{5ab}{4}+\frac{11b^2}{16}\right)\sin(e+fx}{f\left(\sin(e+fx)^6-3\sin(e+fx)^4+3\sin(e+fx)^2-1\right)}$$

input
```
int((a + b/cos(e + f*x)^2)^2/cos(e + f*x)^3,x)
```

output
```
(atanh(sin(e + f*x))*((3*a*b)/4 + a^2/2 + (5*b^2)/16))/f - (sin(e + f*x)*(
(5*a*b)/4 + a^2/2 + (11*b^2)/16) - sin(e + f*x)^3*(2*a*b + a^2 + (5*b^2)/6
) + sin(e + f*x)^5*((3*a*b)/4 + a^2/2 + (5*b^2)/16))/(f*(3*sin(e + f*x)^2
- 3*sin(e + f*x)^4 + sin(e + f*x)^6 - 1))
```

3.167 $\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

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3.167.2 Mathematica [A] (verified) 1296
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3.167.8 Giac [A] (verification not implemented) 1301
3.167.9 Mupad [B] (verification not implemented) 1301

3.167.1 Optimal result

Integrand size = 21, antiderivative size = 91

$$\begin{aligned}\int &\sec(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx \\ &= \frac{(8a^2+8ab+3b^2)\operatorname{arctanh}(\sin(e+fx))}{8f}+\frac{3b(2a+b)\sec(e+fx)\tan(e+fx)}{8f} \\ &\quad+\frac{b\sec^3(e+fx)\left(a+b-a\sin^2(e+fx)\right)\tan(e+fx)}{4f}\end{aligned}$$

output
```
1/8*(8*a^2+8*a*b+3*b^2)*arctanh(sin(f*x+e))/f+3/8*b*(2*a+b)*sec(f*x+e)*tan
(f*x+e)/f+1/4*b*sec(f*x+e)^3*(a+b-a*sin(f*x+e)^2)*tan(f*x+e)/f
```

3.167.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.20

$$\begin{aligned}\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = &\frac{a^2\operatorname{arctanh}(\sin(e+fx))}{f}+\frac{ab\operatorname{arctanh}(\sin(e+fx))}{f} \\ &+\frac{3b^2\operatorname{arctanh}(\sin(e+fx))}{8f} \\ &+\frac{ab\sec(e+fx)\tan(e+fx)}{f} \\ &+\frac{3b^2\sec(e+fx)\tan(e+fx)}{8f} \\ &+\frac{b^2\sec^3(e+fx)\tan(e+fx)}{4f}\end{aligned}$$

input
```
Integrate[Sec[e + f*x]*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(a^2*ArcTanh[Sin[e + f*x]])/f + (a*b*ArcTanh[Sin[e + f*x]])/f + (3*b^2*Arc
Tanh[Sin[e + f*x]])/(8*f) + (a*b*Sec[e + f*x]*Tan[e + f*x])/f + (3*b^2*Sec
[e + f*x]*Tan[e + f*x])/(8*f) + (b^2*Sec[e + f*x]^3*Tan[e + f*x])/(4*f)
```

3.167.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4635, 315, 25, 298, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int \sec(e+fx)\left(a+b\sec(e+fx)^2\right)^2 dx$$

$$\downarrow 4635$$

$$\frac{\int \frac{\left(-a\sin^2(e+fx)+a+b\right)^2}{\left(1-\sin^2(e+fx)\right)^3} d\sin(e+fx)}{f}$$

$$\downarrow 315$$

$$\frac{\frac{b\sin(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}{4\left(1-\sin^2(e+fx)\right)^2} - \frac{1}{4}\int -\frac{(a+b)(4a+3b)-a(4a+b)\sin^2(e+fx)}{\left(1-\sin^2(e+fx)\right)^2} d\sin(e+fx)}{f}$$

$$\downarrow 25$$

$$\frac{\frac{1}{4}\int \frac{(a+b)(4a+3b)-a(4a+b)\sin^2(e+fx)}{\left(1-\sin^2(e+fx)\right)^2} d\sin(e+fx) + \frac{b\sin(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}{4\left(1-\sin^2(e+fx)\right)^2}}{f}$$

$$\downarrow 298$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(8a^2+8ab+3b^2\right)\int \frac{1}{1-\sin^2(e+fx)} d\sin(e+fx) + \frac{3b(2a+b)\sin(e+fx)}{2\left(1-\sin^2(e+fx)\right)}\right) + \frac{b\sin(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}{4\left(1-\sin^2(e+fx)\right)^2}}{f}$$

$$\downarrow 219$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(8a^2+8ab+3b^2\right)\operatorname{arctanh}(\sin(e+fx))+\frac{3b(2a+b)\sin(e+fx)}{2\left(1-\sin^2(e+fx)\right)}\right)+\frac{b\sin(e+fx)\left(-a\sin^2(e+fx)+a+b\right)}{4\left(1-\sin^2(e+fx)\right)^2}}{f}$$

input
```
Int[Sec[e + f*x]*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((b*Sin[e + f*x]*(a + b - a*Sin[e + f*x]^2))/(4*(1 - Sin[e + f*x]^2)^2) +
(((8*a^2 + 8*a*b + 3*b^2)*ArcTanh[Sin[e + f*x]])/2 + (3*b*(2*a + b)*Sin[e
+ f*x])/(2*(1 - Sin[e + f*x]^2)))/4)/f
```

3.167.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.167.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.18

method	result
derivativedivides	$\frac{a^2\ln(\sec(fx+e)+\tan(fx+e))+2ab\left(\frac{\tan(fx+e)\sec(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+b^2\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right.\right.}{f}$
default	$\frac{a^2\ln(\sec(fx+e)+\tan(fx+e))+2ab\left(\frac{\tan(fx+e)\sec(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+b^2\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right.\right.}{f}$
parts	$\frac{a^2\ln(\sec(fx+e)+\tan(fx+e))}{f}+\frac{b^2\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)}{f}+\frac{ab\,\mathrm{ta}}{}$
parallelrisch	$\frac{-4\left(a^2+ab+\frac{3}{8}b^2\right)\left(\frac{3}{4}+\frac{\cos(4fx+4e)}{4}+\cos(2fx+2e)\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+4\left(a^2+ab+\frac{3}{8}b^2\right)\left(\frac{3}{4}+\frac{\cos(4fx+4e)}{4}+\cos(2fx+2e\right.}{f(\cos(4fx+4e)+4\cos(2fx+2e)+3)}$
norman	$\frac{-\frac{b(8a-3b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{4f}-\frac{b(8a-3b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{4f}+\frac{b(8a+5b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4f}+\frac{b(8a+5b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{4f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^4}-\frac{\left(8a^2+8ab+3b^2\right)\ln\left(\mathrm{t}\right.}{8f}$
risch	$-\frac{ib\,\mathrm{e}^{i(fx+e)}\left(8a\,\mathrm{e}^{6i(fx+e)}+3b\,\mathrm{e}^{6i(fx+e)}+8a\,\mathrm{e}^{4i(fx+e)}+11b\,\mathrm{e}^{4i(fx+e)}-8a\,\mathrm{e}^{2i(fx+e)}-11b\,\mathrm{e}^{2i(fx+e)}-8a-3b\right)}{4f\left(\mathrm{e}^{2i(fx+e)}+1\right)^4}-\frac{\ln\left(\mathrm{e}^{i(f}\right.}{}$

input

```
int(sec(f*x+e)*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output

```
1/f*(a^2*ln(sec(f*x+e)+tan(f*x+e))+2*a*b*(1/2*tan(f*x+e)*sec(f*x+e)+1/2*ln
(sec(f*x+e)+tan(f*x+e)))+b^2*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+
e)+3/8*ln(sec(f*x+e)+tan(f*x+e))))
```

3.167.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.27

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$

$$=\frac{(8\,a^2+8\,ab+3\,b^2)\cos(fx+e)^4\log(\sin(fx+e)+1)-(8\,a^2+8\,ab+3\,b^2)\cos(fx+e)^4\log(-\sin(fx+$$

$$16\,f\cos(fx+e)^4$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/16*((8*a^2 + 8*a*b + 3*b^2)*cos(f*x + e)^4*log(sin(f*x + e) + 1) - (8*a^
2 + 8*a*b + 3*b^2)*cos(f*x + e)^4*log(-sin(f*x + e) + 1) + 2*((8*a*b + 3*b
^2)*cos(f*x + e)^2 + 2*b^2)*sin(f*x + e))/(f*cos(f*x + e)^4)
```

3.167.6 Sympy [F]

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\int \left(a+b\sec^2(e+fx)\right)^2\sec(e+fx)\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**2*sec(e + f*x), x)
```

3.167.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.31

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$

$$=\frac{(8\,a^2+8\,ab+3\,b^2)\log(\sin(fx+e)+1)-(8\,a^2+8\,ab+3\,b^2)\log(\sin(fx+e)-1)-\frac{2\left((8\,ab+3\,b^2)\sin(fx+e)\right.}{\sin(fx+e)^4-}}{16\,f}$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/16*((8*a^2 + 8*a*b + 3*b^2)*log(sin(f*x + e) + 1) - (8*a^2 + 8*a*b + 3*b
^2)*log(sin(f*x + e) - 1) - 2*((8*a*b + 3*b^2)*sin(f*x + e)^3 - (8*a*b + 5
*b^2)*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1))/f
```

3.167.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.32

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$

$$= \frac{(8a^2+8ab+3b^2)\log\left(|\sin(fx+e)+1|\right)-(8a^2+8ab+3b^2)\log\left(|\sin(fx+e)-1|\right)-\frac{2\left(8ab\sin(fx+e)^3+3\right.}{16f}$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/16*((8*a^2 + 8*a*b + 3*b^2)*log(abs(sin(f*x + e) + 1)) - (8*a^2 + 8*a*b
+ 3*b^2)*log(abs(sin(f*x + e) - 1)) - 2*(8*a*b*sin(f*x + e)^3 + 3*b^2*sin(
f*x + e)^3 - 8*a*b*sin(f*x + e) - 5*b^2*sin(f*x + e))/(sin(f*x + e)^2 - 1)
^2)/f
```

3.167.9 Mupad [B] (verification not implemented)

Time = 18.69 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.95

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$

$$= \frac{\operatorname{atanh}(\sin(e+fx))\left(a^2+ab+\frac{3b^2}{8}\right)}{f} + \frac{\sin(e+fx)\left(\frac{5b^2}{8}+ab\right)-\sin(e+fx)^3\left(\frac{3b^2}{8}+ab\right)}{f\left(\sin(e+fx)^4-2\sin(e+fx)^2+1\right)}$$

input
```
int((a + b/cos(e + f*x)^2)^2/cos(e + f*x),x)
```

output
```
(atanh(sin(e + f*x))*(a*b + a^2 + (3*b^2)/8))/f + (sin(e + f*x)*(a*b + (5*
b^2)/8) - sin(e + f*x)^3*(a*b + (3*b^2)/8))/(f*(sin(e + f*x)^4 - 2*sin(e +
 f*x)^2 + 1))
```

3.168 $\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

3.168.1 Optimal result 1302
3.168.2 Mathematica [A] (verified) 1302
3.168.3 Rubi [A] (verified) 1303
3.168.4 Maple [A] (verified) 1304
3.168.5 Fricas [A] (verification not implemented) 1305
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3.168.7 Maxima [A] (verification not implemented) 1305
3.168.8 Giac [A] (verification not implemented) 1306
3.168.9 Mupad [B] (verification not implemented) 1306

3.168.1 Optimal result

Integrand size = 21, antiderivative size = 56

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{b(4a+b)\operatorname{arctanh}(\sin(e+fx))}{2f} + \frac{a^2\sin(e+fx)}{f} + \frac{b^2\sec(e+fx)\tan(e+fx)}{2f}$$

output
```
1/2*b*(4*a+b)*arctanh(sin(f*x+e))/f+a^2*sin(f*x+e)/f+1/2*b^2*sec(f*x+e)*ta
n(f*x+e)/f
```

3.168.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.43

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{2ab\operatorname{arctanh}(\sin(e+fx))}{f} + \frac{b^2\operatorname{arctanh}(\sin(e+fx))}{2f} + \frac{a^2\cos(fx)\sin(e)}{f} + \frac{a^2\cos(e)\sin(fx)}{f} + \frac{b^2\sec(e+fx)\tan(e+fx)}{2f}$$

input
```
Integrate[Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(2*a*b*ArcTanh[Sin[e + f*x]])/f + (b^2*ArcTanh[Sin[e + f*x]])/(2*f) + (a^2
*Cos[f*x]*Sin[e])/f + (a^2*Cos[e]*Sin[f*x])/f + (b^2*Sec[e + f*x]*Tan[e +
f*x])/(2*f)
```

3.168.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4635, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^2}{\sec(e+fx)} dx$$

$$\downarrow 4635$$

$$\frac{\int \frac{\left(-a\sin^2(e+fx)+a+b\right)^2}{\left(1-\sin^2(e+fx)\right)^2} d\sin(e+fx)}{f}$$

$$\downarrow 300$$

$$\frac{\int \left(a^2+\frac{b(2a+b)-2ab\sin^2(e+fx)}{\left(1-\sin^2(e+fx)\right)^2}\right) d\sin(e+fx)}{f}$$

$$\downarrow 2009$$

$$\frac{a^2\sin(e+fx)+\frac{1}{2}b(4a+b)\operatorname{arctanh}(\sin(e+fx))+\frac{b^2\sin(e+fx)}{2\left(1-\sin^2(e+fx)\right)}}{f}$$

input
```
Int[Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((b*(4*a + b)*ArcTanh[Sin[e + f*x]])/2 + a^2*Sin[e + f*x] + (b^2*Sin[e + f
*x])/(2*(1 - Sin[e + f*x]^2)))/f
```

3.168.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.168.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.23

method	result
derivativedivides	$\frac{\sin(fx+e)a^2+2ab\ln(\sec(fx+e)+\tan(fx+e))+b^2\left(\frac{\tan(fx+e)\sec(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}$
default	$\frac{\sin(fx+e)a^2+2ab\ln(\sec(fx+e)+\tan(fx+e))+b^2\left(\frac{\tan(fx+e)\sec(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}$
parallelrisch	$\frac{-4b(1+\cos(2fx+2e))\left(a+\frac{b}{4}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+4b(1+\cos(2fx+2e))\left(a+\frac{b}{4}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)+\sin(3fx+3e)a^2+}{2f(1+\cos(2fx+2e))}$
risch	$-\frac{ia^2e^{i(fx+e)}}{2f}+\frac{ia^2e^{-i(fx+e)}}{2f}-\frac{ib^2\left(e^{3i(fx+e)}-e^{i(fx+e)}\right)}{f\left(e^{2i(fx+e)}+1\right)^2}-\frac{2\ln\left(e^{i(fx+e)}-i\right)ab}{f}-\frac{\ln\left(e^{i(fx+e)}-i\right)b^2}{2f}+\frac{2\ln(e^{i}$
norman	$\frac{\left(2a^2+b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{f}+\frac{\left(6a^2-b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{f}-\frac{\left(2a^2+b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{\left(6a^2-b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{f}}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3}-\frac{b(4a+b)\ln\left(\tan\left(\frac{f}{2}\right.\right.}{2f}$

input
```
int(cos(f*x+e)*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(sin(f*x+e)*a^2+2*a*b*ln(sec(f*x+e)+tan(f*x+e))+b^2*(1/2*tan(f*x+e)*se
c(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e))))
```

3.168.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.68

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$= \frac{(4\,ab+b^2)\cos(fx+e)^2\log(\sin(fx+e)+1)-(4\,ab+b^2)\cos(fx+e)^2\log(-\sin(fx+e)+1)+2\,(2\,a}{4\,f\cos(fx+e)^2}$$

input
```
integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/4*((4*a*b + b^2)*cos(f*x + e)^2*log(sin(f*x + e) + 1) - (4*a*b + b^2)*co
s(f*x + e)^2*log(-sin(f*x + e) + 1) + 2*(2*a^2*cos(f*x + e)^2 + b^2)*sin(f
*x + e))/(f*cos(f*x + e)^2)
```

3.168.6 Sympy [F]

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \int \left(a+b\sec^2(e+fx)\right)^2\cos(e+fx)\,dx$$

input
```
integrate(cos(f*x+e)*(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**2*cos(e + f*x), x)
```

3.168.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.55

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx =$$
$$-\frac{b^2\left(\frac{2\sin(fx+e)}{\sin(fx+e)^2-1}-\log(\sin(fx+e)+1)+\log(\sin(fx+e)-1)\right)-4\,ab(\log(\sin(fx+e)+1)-\log(\sin}{4\,f}$$

input
```
integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/4*(b^2*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + l
og(sin(f*x + e) - 1)) - 4*a*b*(log(sin(f*x + e) + 1) - log(sin(f*x + e) -
1)) - 4*a^2*sin(f*x + e))/f
```

3.168.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.41

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{4\,a^2\sin(fx+e)+(4\,ab+b^2)\log\left(\left|\sin(fx+e)+1\right|\right)-(4\,ab+b^2)\log\left(\left|\sin(fx+e)-1\right|\right)-\frac{2\,b^2\sin(fx+e)}{\sin(fx+e)^2-1}}{4\,f}$$

input
```
integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/4*(4*a^2*sin(f*x + e) + (4*a*b + b^2)*log(abs(sin(f*x + e) + 1)) - (4*a*
b + b^2)*log(abs(sin(f*x + e) - 1)) - 2*b^2*sin(f*x + e)/(sin(f*x + e)^2 -
 1))/f
```

3.168.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.98

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{a^2\,\sin(e+f\,x)+\frac{b\,\mathrm{atanh}(\sin(e+f\,x))\,(4\,a+b)}{2}-\frac{b^2\,\sin(e+f\,x)}{2\left(\sin(e+f\,x)^2-1\right)}}{f}$$

input
```
int(cos(e + f*x)*(a + b/cos(e + f*x)^2)^2,x)
```

output
```
(a^2*sin(e + f*x) + (b*atanh(sin(e + f*x))*(4*a + b))/2 - (b^2*sin(e + f*x
))/(2*(sin(e + f*x)^2 - 1)))/f
```

3.169 $\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

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3.169.1 Optimal result

Integrand size = 23, antiderivative size = 49

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{b^2\operatorname{arctanh}(\sin(e+fx))}{f} + \frac{a(a+2b)\sin(e+fx)}{f} - \frac{a^2\sin^3(e+fx)}{3f}$$

output
```
b^2*arctanh(sin(f*x+e))/f+a*(a+2*b)*sin(f*x+e)/f-1/3*a^2*sin(f*x+e)^3/f
```

3.169.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.47

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{b^2\operatorname{arctanh}(\sin(e+fx))}{f} + \frac{2ab\cos(fx)\sin(e)}{f} + \frac{2ab\cos(e)\sin(fx)}{f} + \frac{a^2\sin(e+fx)}{f} - \frac{a^2\sin^3(e+fx)}{3f}$$

input
```
Integrate[Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(b^2*ArcTanh[Sin[e + f*x]])/f + (2*a*b*Cos[f*x]*Sin[e])/f + (2*a*b*Cos[e]*
Sin[f*x])/f + (a^2*Sin[e + f*x])/f - (a^2*Sin[e + f*x]^3)/(3*f)
```

3.169.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4635, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^2}{\sec(e+fx)^3} dx$$

$$\downarrow 4635$$

$$\frac{\int \frac{\left(-a\sin^2(e+fx)+a+b\right)^2}{1-\sin^2(e+fx)} d\sin(e+fx)}{f}$$

$$\downarrow 300$$

$$\frac{\int \left(\frac{b^2}{1-\sin^2(e+fx)} - a^2\sin^2(e+fx) + a(a+2b)\right) d\sin(e+fx)}{f}$$

$$\downarrow 2009$$

$$\frac{-\frac{1}{3}a^2\sin^3(e+fx) + a(a+2b)\sin(e+fx) + b^2\text{arctanh}(\sin(e+fx))}{f}$$

input
```
Int[Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(b^2*ArcTanh[Sin[e + f*x]] + a*(a + 2*b)*Sin[e + f*x] - (a^2*Sin[e + f*x]^
3)/3)/f
```

3.169.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.169.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{\frac{a^2\left(\cos(fx+e)^2+2\right)\sin(fx+e)}{3}+2\sin(fx+e)ab+b^2\ln(\sec(fx+e)+\tan(fx+e))}{f}$
default	$\frac{\frac{a^2\left(\cos(fx+e)^2+2\right)\sin(fx+e)}{3}+2\sin(fx+e)ab+b^2\ln(\sec(fx+e)+\tan(fx+e))}{f}$
parallelrisch	$\frac{-12\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)b^2+12\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)b^2+\sin(3fx+3e)a^2+9\left(a+\frac{8b}{3}\right)a\sin(fx+e)}{12f}$
risch	$-\frac{3ia^2e^{i(fx+e)}}{8f}-\frac{ie^{i(fx+e)}ab}{f}+\frac{3ia^2e^{-i(fx+e)}}{8f}+\frac{ie^{-i(fx+e)}ab}{f}+\frac{\ln\left(e^{i(fx+e)}+i\right)b^2}{f}-\frac{\ln\left(e^{i(fx+e)}-i\right)b^2}{f}+\frac{a^2}{}$
norman	$\frac{-\frac{4a(a-2b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{f}+\frac{4a(a-2b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{f}-\frac{2a(a+2b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{2a(a+2b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}{f}+\frac{2a(7a+6b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{}}{3f}}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3}$

input
```
int(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/3*a^2*(cos(f*x+e)^2+2)*sin(f*x+e)+2*sin(f*x+e)*a*b+b^2*ln(sec(f*x+e
)+tan(f*x+e)))
```

3.169.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.35

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$= \frac{3\,b^2\log\left(\sin\left(fx+e\right)+1\right)-3\,b^2\log\left(-\sin\left(fx+e\right)+1\right)+2\left(a^2\cos\left(fx+e\right)^2+2\,a^2+6\,ab\right)\sin\left(fx+e\right)}{6\,f}$$

input
```
integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/6*(3*b^2*log(sin(f*x + e) + 1) - 3*b^2*log(-sin(f*x + e) + 1) + 2*(a^2*c
os(f*x + e)^2 + 2*a^2 + 6*a*b)*sin(f*x + e))/f
```

3.169.6 Sympy [F]

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)^2\cos^3\left(e+fx\right)dx$$

input
```
integrate(cos(f*x+e)**3*(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**2*cos(e + f*x)**3, x)
```

3.169.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.29

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx =$$
$$-\frac{2\,a^2\sin\left(fx+e\right)^3-3\,b^2\log\left(\sin\left(fx+e\right)+1\right)+3\,b^2\log\left(\sin\left(fx+e\right)-1\right)-6\left(a^2+2\,ab\right)\sin\left(fx+e\right)}{6\,f}$$

input
```
integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/6*(2*a^2*sin(f*x + e)^3 - 3*b^2*log(sin(f*x + e) + 1) + 3*b^2*log(sin(f
*x + e) - 1) - 6*(a^2 + 2*a*b)*sin(f*x + e))/f
```

3.169.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.43

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx =$$
$$-\frac{2\,a^2\sin(fx+e)^3-3\,b^2\log\left(\left|\sin(fx+e)+1\right|\right)+3\,b^2\log\left(\left|\sin(fx+e)-1\right|\right)-6\,a^2\sin(fx+e)-12\,ab}{6\,f}$$

input
```
integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-1/6*(2*a^2*sin(f*x + e)^3 - 3*b^2*log(abs(sin(f*x + e) + 1)) + 3*b^2*log(
abs(sin(f*x + e) - 1)) - 6*a^2*sin(f*x + e) - 12*a*b*sin(f*x + e))/f
```

3.169.9 Mupad [B] (verification not implemented)

Time = 17.97 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=-\frac{\sin(e+f\,x)\,\left(a^2-2\,a\,(a+b)\right)+\frac{a^2\sin(e+f\,x)^3}{3}-b^2\,\mathrm{atanh}(\sin(e+f\,x))}{f}$$

input
```
int(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^2,x)
```

output
```
-(sin(e + f*x)*(a^2 - 2*a*(a + b)) + (a^2*sin(e + f*x)^3)/3 - b^2*atanh(si
n(e + f*x)))/f
```

3.170 $\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

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3.170.7 Maxima [A] (verification not implemented) 1315
3.170.8 Giac [A] (verification not implemented) 1316
3.170.9 Mupad [B] (verification not implemented) 1316

3.170.1 Optimal result

Integrand size = 23, antiderivative size = 53

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{(a+b)^2\sin(e+fx)}{f} - \frac{2a(a+b)\sin^3(e+fx)}{3f} + \frac{a^2\sin^5(e+fx)}{5f}$$

output
```
(a+b)^2*sin(f*x+e)/f-2/3*a*(a+b)*sin(f*x+e)^3/f+1/5*a^2*sin(f*x+e)^5/f
```

3.170.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.00

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{b^2\cos(fx)\sin(e)}{f} + \frac{b^2\cos(e)\sin(fx)}{f} + \frac{a^2\sin(e+fx)}{f} + \frac{2ab\sin(e+fx)}{f} - \frac{2a^2\sin^3(e+fx)}{3f} - \frac{2ab\sin^3(e+fx)}{3f} + \frac{a^2\sin^5(e+fx)}{5f}$$

input
```
Integrate[Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(b^2*Cos[f*x]*Sin[e])/f + (b^2*Cos[e]*Sin[f*x])/f + (a^2*Sin[e + f*x])/f +
 (2*a*b*Sin[e + f*x])/f - (2*a^2*Sin[e + f*x]^3)/(3*f) - (2*a*b*Sin[e + f*
x]^3)/(3*f) + (a^2*Sin[e + f*x]^5)/(5*f)
```

3.170.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4635, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^2}{\sec(e+fx)^5}dx$$

$$\downarrow 4635$$

$$\frac{\int \left(-a\sin^2(e+fx)+a+b\right)^2 d\sin(e+fx)}{f}$$

$$\downarrow 210$$

$$\frac{\int \left(a^2\sin^4(e+fx)-2a^2\left(\frac{b}{a}+1\right)\sin^2(e+fx)+a^2\left(\frac{b(2a+b)}{a^2}+1\right)\right)d\sin(e+fx)}{f}$$

$$\downarrow 2009$$

$$\frac{\frac{1}{5}a^2\sin^5(e+fx)-\frac{2}{3}a(a+b)\sin^3(e+fx)+(a+b)^2\sin(e+fx)}{f}$$

input
```
Int[Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((a + b)^2*Sin[e + f*x] - (2*a*(a + b)*Sin[e + f*x]^3)/3 + (a^2*Sin[e + f*
x]^5)/5)/f
```

3.170.3.1 Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.170.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.26

method	result
derivativedivides	$\frac{\frac{a^2\left(\frac{8}{3}+\cos(fx+e)^4+\frac{4\cos(fx+e)^2}{3}\right)\sin(fx+e)}{5}+\frac{2ab\left(\cos(fx+e)^2+2\right)\sin(fx+e)}{3}+\sin(fx+e)b^2}{f}$
default	$\frac{\frac{a^2\left(\frac{8}{3}+\cos(fx+e)^4+\frac{4\cos(fx+e)^2}{3}\right)\sin(fx+e)}{5}+\frac{2ab\left(\cos(fx+e)^2+2\right)\sin(fx+e)}{3}+\sin(fx+e)b^2}{f}$
parallelrisch	$\frac{150\sin(fx+e)a^2+360\sin(fx+e)ab+240\sin(fx+e)b^2+3\sin(5fx+5e)a^2+25\sin(3fx+3e)a^2+40\sin(3fx+3e)ab}{240f}$
risch	$\frac{5a^2\sin(fx+e)}{8f}+\frac{3\sin(fx+e)ab}{2f}+\frac{\sin(fx+e)b^2}{f}+\frac{a^2\sin(5fx+5e)}{80f}+\frac{5a^2\sin(3fx+3e)}{48f}+\frac{\sin(3fx+3e)ab}{6f}$
norman	$-\frac{2\left(a^2+2ab+b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{2\left(a^2+2ab+b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{15}}{f}+\frac{2\left(5a^2+2ab-3b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3f}-\frac{2\left(5a^2+2ab-3b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right.}{3f}$ $\left(1+\tan\right.$

input
```
int(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/5*a^2*(8/3+cos(f*x+e)^4+4/3*cos(f*x+e)^2)*sin(f*x+e)+2/3*a*b*(cos(f
*x+e)^2+2)*sin(f*x+e)+sin(f*x+e)*b^2)
```

3.170.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.11

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{\left(3\,a^2\cos(fx+e)^4+2\left(2\,a^2+5\,ab\right)\cos(fx+e)^2+8\,a^2+20\,ab+15\,b^2\right)\sin(fx+e)}{15\,f}$$

input
```
integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/15*(3*a^2*cos(f*x + e)^4 + 2*(2*a^2 + 5*a*b)*cos(f*x + e)^2 + 8*a^2 + 20
*a*b + 15*b^2)*sin(f*x + e)/f
```

3.170.6 Sympy [F(-1)]

Timed out.

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**5*(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Timed out
```

3.170.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{3\,a^2\sin(fx+e)^5-10\left(a^2+ab\right)\sin(fx+e)^3+15\left(a^2+2\,ab+b^2\right)\sin(fx+e)}{15\,f}$$

input
```
integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/15*(3*a^2*sin(f*x + e)^5 - 10*(a^2 + a*b)*sin(f*x + e)^3 + 15*(a^2 + 2*a
*b + b^2)*sin(f*x + e))/f
```

3.170.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.43

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{3\,a^2\sin(fx+e)^5-10\,a^2\sin(fx+e)^3-10\,ab\sin(fx+e)^3+15\,a^2\sin(fx+e)+30\,ab\sin(fx+e)+15}{15\,f}$$

input
```
integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/15*(3*a^2*sin(f*x + e)^5 - 10*a^2*sin(f*x + e)^3 - 10*a*b*sin(f*x + e)^3
 + 15*a^2*sin(f*x + e) + 30*a*b*sin(f*x + e) + 15*b^2*sin(f*x + e))/f
```

3.170.9 Mupad [B] (verification not implemented)

Time = 18.54 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{\sin(e+fx)\,(a+b)^2+\frac{a^2\sin(e+fx)^5}{5}-\frac{2\,a\sin(e+fx)^3\,(a+b)}{3}}{f}$$

input
```
int(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^2,x)
```

output
```
(sin(e + f*x)*(a + b)^2 + (a^2*sin(e + f*x)^5)/5 - (2*a*sin(e + f*x)^3*(a
+ b))/3)/f
```

3.171 $\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

3.171.1 Optimal result 1317
3.171.2 Mathematica [A] (verified) 1317
3.171.3 Rubi [A] (verified) 1318
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3.171.8 Giac [A] (verification not implemented) 1321
3.171.9 Mupad [B] (verification not implemented) 1322

3.171.1 Optimal result

Integrand size = 23, antiderivative size = 106

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{(a+b)^2\tan(e+fx)}{f} + \frac{2(a+b)(a+2b)\tan^3(e+fx)}{3f} + \frac{(a^2+6ab+6b^2)\tan^5(e+fx)}{5f} + \frac{2b(a+2b)\tan^7(e+fx)}{7f} + \frac{b^2\tan^9(e+fx)}{9f}$$

output
```
(a+b)^2*tan(f*x+e)/f+2/3*(a+b)*(a+2*b)*tan(f*x+e)^3/f+1/5*(a^2+6*a*b+6*b^2
)*tan(f*x+e)^5/f+2/7*b*(a+2*b)*tan(f*x+e)^7/f+1/9*b^2*tan(f*x+e)^9/f
```

3.171.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.91

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{315(a+b)^2\tan(e+fx)+210(a^2+3ab+2b^2)\tan^3(e+fx)+63(a^2+6ab+6b^2)\tan^5(e+fx)+90b(a+$$

input
```
Integrate[Sec[e + f*x]^6*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(315*(a + b)^2*Tan[e + f*x] + 210*(a^2 + 3*a*b + 2*b^2)*Tan[e + f*x]^3 + 6
3*(a^2 + 6*a*b + 6*b^2)*Tan[e + f*x]^5 + 90*b*(a + 2*b)*Tan[e + f*x]^7 + 3
5*b^2*Tan[e + f*x]^9)/(315*f)
```

3.171.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4634, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int \sec(e+fx)^6\left(a+b\sec(e+fx)^2\right)^2 dx$$

$$\downarrow 4634$$

$$\frac{\int \left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^2 d\tan(e+fx)}{f}$$

$$\downarrow 290$$

$$\frac{\int \left(b^2\tan^8(e+fx)+2b(a+2b)\tan^6(e+fx)+\left(a^2+6ba+6b^2\right)\tan^4(e+fx)+2(a+b)(a+2b)\tan^2(e+fx)+\right.}{f}$$

$$\downarrow 2009$$

$$\frac{\frac{1}{5}\left(a^2+6ab+6b^2\right)\tan^5(e+fx)+\frac{2}{7}b(a+2b)\tan^7(e+fx)+\frac{2}{3}(a+b)(a+2b)\tan^3(e+fx)+(a+b)^2\tan(e+fx)}{f}$$

input
```
Int[Sec[e + f*x]^6*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((a + b)^2*Tan[e + f*x] + (2*(a + b)*(a + 2*b)*Tan[e + f*x]^3)/3 + ((a^2 +
 6*a*b + 6*b^2)*Tan[e + f*x]^5)/5 + (2*b*(a + 2*b)*Tan[e + f*x]^7)/7 + (b^
2*Tan[e + f*x]^9)/9)/f
```

3.171.3.1 Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.171.4 Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.26

method	result
derivativedivides	$\frac{-a^2\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)-2ab\left(-\frac{16}{35}-\frac{\sec(fx+e)^6}{7}-\frac{6\sec(fx+e)^4}{35}-\frac{8\sec(fx+e)^2}{35}\right)\tan(fx+e)-b}{f}$
default	$\frac{-a^2\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)-2ab\left(-\frac{16}{35}-\frac{\sec(fx+e)^6}{7}-\frac{6\sec(fx+e)^4}{35}-\frac{8\sec(fx+e)^2}{35}\right)\tan(fx+e)-b}{f}$
parts	$-\frac{a^2\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)}{f}-\frac{b^2\left(-\frac{128}{315}-\frac{\sec(fx+e)^8}{9}-\frac{8\sec(fx+e)^6}{63}-\frac{16\sec(fx+e)^4}{105}-\frac{64\sec(fx+}{315}\right.}{f}$
parallelrisch	$\frac{\left(10752a^2+24192ab+10752b^2\right)\sin(3fx+3e)+\left(6048a^2+10368ab+4608b^2\right)\sin(5fx+5e)+\left(1512a^2+2592ab+1152b^2\right)\sin(}{315f(\cos(9fx+9e)+9\cos(7fx+7e)+36\cos(5fx+5e)+84\cos(3}$
risch	$\frac{16i\left(210a^2e^{12i(fx+e)}+945a^2e^{10i(fx+e)}+1260ab\,e^{10i(fx+e)}+1701a^2e^{8i(fx+e)}+3276ab\,e^{8i(fx+e)}+2016b^2e^{8i(fx+e)}+155}{}$

input
```
int(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-a^2*(-8/15-1/5*sec(f*x+e)^4-4/15*sec(f*x+e)^2)*tan(f*x+e)-2*a*b*(-16
/35-1/7*sec(f*x+e)^6-6/35*sec(f*x+e)^4-8/35*sec(f*x+e)^2)*tan(f*x+e)-b^2*(
-128/315-1/9*sec(f*x+e)^8-8/63*sec(f*x+e)^6-16/105*sec(f*x+e)^4-64/315*sec
(f*x+e)^2)*tan(f*x+e))
```

3.171.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.13

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$= \frac{\left(8\left(21\,a^2+36\,ab+16\,b^2\right)\cos\left(fx+e\right)^8+4\left(21\,a^2+36\,ab+16\,b^2\right)\cos\left(fx+e\right)^6+3\left(21\,a^2+36\,ab+16\,b^2\right.\right.}{315\,f\cos\left(fx+e\right)^9}$$

input
```
integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/315*(8*(21*a^2 + 36*a*b + 16*b^2)*cos(f*x + e)^8 + 4*(21*a^2 + 36*a*b +
16*b^2)*cos(f*x + e)^6 + 3*(21*a^2 + 36*a*b + 16*b^2)*cos(f*x + e)^4 + 10*
(9*a*b + 4*b^2)*cos(f*x + e)^2 + 35*b^2)*sin(f*x + e)/(f*cos(f*x + e)^9)
```

3.171.6 Sympy [F]

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)^2\sec^6\left(e+fx\right)dx$$

input
```
integrate(sec(f*x+e)**6*(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**2*sec(e + f*x)**6, x)
```

3.171.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.97

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{35\,b^2\tan(fx+e)^9+90\,(ab+2\,b^2)\tan(fx+e)^7+63\,(a^2+6\,ab+6\,b^2)\tan(fx+e)^5+210\,(a^2+3\,ab+2}{315\,f}$$

input
```
integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/315*(35*b^2*tan(f*x + e)^9 + 90*(a*b + 2*b^2)*tan(f*x + e)^7 + 63*(a^2 +
 6*a*b + 6*b^2)*tan(f*x + e)^5 + 210*(a^2 + 3*a*b + 2*b^2)*tan(f*x + e)^3
+ 315*(a^2 + 2*a*b + b^2)*tan(f*x + e))/f
```

3.171.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.43

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{35\,b^2\tan(fx+e)^9+90\,ab\tan(fx+e)^7+180\,b^2\tan(fx+e)^7+63\,a^2\tan(fx+e)^5+378\,ab\tan(fx+e}$$

input
```
integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/315*(35*b^2*tan(f*x + e)^9 + 90*a*b*tan(f*x + e)^7 + 180*b^2*tan(f*x + e
)^7 + 63*a^2*tan(f*x + e)^5 + 378*a*b*tan(f*x + e)^5 + 378*b^2*tan(f*x + e
)^5 + 210*a^2*tan(f*x + e)^3 + 630*a*b*tan(f*x + e)^3 + 420*b^2*tan(f*x +
e)^3 + 315*a^2*tan(f*x + e) + 630*a*b*tan(f*x + e) + 315*b^2*tan(f*x + e))
/f
```

3.171.9 Mupad [B] (verification not implemented)

Time = 18.23 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.89

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{\tan(e+f\,x)\,(a+b)^2+\frac{b^2\tan(e+f\,x)^9}{9}+\tan(e+f\,x)^3\left(\frac{2\,a^2}{3}+2\,a\,b+\frac{4\,b^2}{3}\right)+\tan(e+f\,x)^5\left(\frac{a^2}{5}+\frac{6\,a\,b}{5}+\frac{6\,b^2}{5}\right.}{f}$$

input
```
int((a + b/cos(e + f*x)^2)^2/cos(e + f*x)^6,x)
```

output
```
(tan(e + f*x)*(a + b)^2 + (b^2*tan(e + f*x)^9)/9 + tan(e + f*x)^3*(2*a*b +
 (2*a^2)/3 + (4*b^2)/3) + tan(e + f*x)^5*((6*a*b)/5 + a^2/5 + (6*b^2)/5) +
 (2*b*tan(e + f*x)^7*(a + 2*b))/7)/f
```

3.172 $\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

3.172.1 Optimal result 1323
3.172.2 Mathematica [A] (verified) 1323
3.172.3 Rubi [A] (verified) 1324
3.172.4 Maple [A] (verified) 1325
3.172.5 Fricas [A] (verification not implemented) 1326
3.172.6 Sympy [F] 1326
3.172.7 Maxima [A] (verification not implemented) 1327
3.172.8 Giac [A] (verification not implemented) 1327
3.172.9 Mupad [B] (verification not implemented) 1327

3.172.1 Optimal result

Integrand size = 23, antiderivative size = 80

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{(a+b)^2\tan(e+fx)}{f}+\frac{(a+b)(a+3b)\tan^3(e+fx)}{3f} + \frac{b(2a+3b)\tan^5(e+fx)}{5f}+\frac{b^2\tan^7(e+fx)}{7f}$$

output
```
(a+b)^2*tan(f*x+e)/f+1/3*(a+b)*(a+3*b)*tan(f*x+e)^3/f+1/5*b*(2*a+3*b)*tan(
f*x+e)^5/f+1/7*b^2*tan(f*x+e)^7/f
```

3.172.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{105(a+b)^2\tan(e+fx)+35(a^2+4ab+3b^2)\tan^3(e+fx)+21b(2a+3b)\tan^5(e+fx)+15b^2\tan^7(e+}{105f}$$

input
```
Integrate[Sec[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(105*(a + b)^2*Tan[e + f*x] + 35*(a^2 + 4*a*b + 3*b^2)*Tan[e + f*x]^3 + 21
*b*(2*a + 3*b)*Tan[e + f*x]^5 + 15*b^2*Tan[e + f*x]^7)/(105*f)
```

3.172.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4634, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int \sec(e+fx)^4\left(a+b\sec(e+fx)^2\right)^2 dx$$

$$\downarrow 4634$$

$$\frac{\int \left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2 d\tan(e+fx)}{f}$$

$$\downarrow 290$$

$$\frac{\int \left(b^2\tan^6(e+fx)+b(2a+3b)\tan^4(e+fx)+(a+b)(a+3b)\tan^2(e+fx)+(a+b)^2\right) d\tan(e+fx)}{f}$$

$$\downarrow 2009$$

$$\frac{\frac{1}{5}b(2a+3b)\tan^5(e+fx)+\frac{1}{3}(a+b)(a+3b)\tan^3(e+fx)+(a+b)^2\tan(e+fx)+\frac{1}{7}b^2\tan^7(e+fx)}{f}$$

input
```
Int[Sec[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((a + b)^2*Tan[e + f*x] + ((a + b)*(a + 3*b)*Tan[e + f*x]^3)/3 + (b*(2*a +
 3*b)*Tan[e + f*x]^5)/5 + (b^2*Tan[e + f*x]^7)/7)/f
```

3.172.3.1 Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.172.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.30

method	result
derivativedivides	$\frac{-a^2\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)-2ab\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)-b^2\left(-\frac{16}{35}-\frac{\sec(fx+e)^6}{7}-\frac{6\sec(fx}{35}\right.}{f}$
default	$\frac{-a^2\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)-2ab\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)-b^2\left(-\frac{16}{35}-\frac{\sec(fx+e)^6}{7}-\frac{6\sec(fx}{35}\right.}{f}$
parts	$-\frac{a^2\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)}{f}-\frac{b^2\left(-\frac{16}{35}-\frac{\sec(fx+e)^6}{7}-\frac{6\sec(fx+e)^4}{35}-\frac{8\sec(fx+e)^2}{35}\right)\tan(fx+e)}{f}-\frac{2ab\left(-\frac{8}{15}-}$
parallelrisch	$\frac{(1050a^2+2352ab+1008b^2)\sin(3fx+3e)+(490a^2+784ab+336b^2)\sin(5fx+5e)+(70a^2+112ab+48b^2)\sin(7fx+7e)+630}{105f(\cos(7fx+7e)+7\cos(5fx+5e)+21\cos(3fx+3e)+35\cos(fx+e))}$
risch	$\frac{4i\left(105a^2e^{10i(fx+e)}+455a^2e^{8i(fx+e)}+560abe^{8i(fx+e)}+770a^2e^{6i(fx+e)}+1400abe^{6i(fx+e)}+840b^2e^{6i(fx+e)}+630a^2e^{4i(f}{105f\left(e^{2i(fx+}\right.}$
norman	$-\frac{2\left(a^2+2ab+b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{2\left(a^2+2ab+b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{13}}{f}+\frac{4\left(7a^2+10ab+3b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3f}+\frac{4\left(7a^2+10ab+3b^2\right)\tan\left(\frac{fx}{2}+}{3f}$ $\left(\tan\left(\frac{fx}{2}+\right.\right.$

input
```
int(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-a^2*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)-2*a*b*(-8/15-1/5*sec(f*x+e)^4
-4/15*sec(f*x+e)^2)*tan(f*x+e)-b^2*(-16/35-1/7*sec(f*x+e)^6-6/35*sec(f*x+e
)^4-8/35*sec(f*x+e)^2)*tan(f*x+e))
```

3.172.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.18

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$= \frac{\left(2\left(35\,a^2+56\,ab+24\,b^2\right)\cos\left(fx+e\right)^6+\left(35\,a^2+56\,ab+24\,b^2\right)\cos\left(fx+e\right)^4+6\left(7\,ab+3\,b^2\right)\cos\left(fx+e\right)^2+15\,b^2\right)\sin\left(fx+e\right)}{105\,f\cos\left(fx+e\right)^7}$$

input
```
integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/105*(2*(35*a^2 + 56*a*b + 24*b^2)*cos(f*x + e)^6 + (35*a^2 + 56*a*b + 24
*b^2)*cos(f*x + e)^4 + 6*(7*a*b + 3*b^2)*cos(f*x + e)^2 + 15*b^2)*sin(f*x
+ e)/(f*cos(f*x + e)^7)
```

3.172.6 Sympy [F]

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)^2\sec^4\left(e+fx\right)dx$$

input
```
integrate(sec(f*x+e)**4*(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**2*sec(e + f*x)**4, x)
```

3.172.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.01

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{15\,b^2\tan(fx+e)^7+21\,(2\,ab+3\,b^2)\tan(fx+e)^5+35\,(a^2+4\,ab+3\,b^2)\tan(fx+e)^3+105\,(a^2+2\,ab+$$
$$105\,f$$

input
```
integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/105*(15*b^2*tan(f*x + e)^7 + 21*(2*a*b + 3*b^2)*tan(f*x + e)^5 + 35*(a^2
 + 4*a*b + 3*b^2)*tan(f*x + e)^3 + 105*(a^2 + 2*a*b + b^2)*tan(f*x + e))/f
```

3.172.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.42

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{15\,b^2\tan(fx+e)^7+42\,ab\tan(fx+e)^5+63\,b^2\tan(fx+e)^5+35\,a^2\tan(fx+e)^3+140\,ab\tan(fx+e)}{105\,f}$$

input
```
integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/105*(15*b^2*tan(f*x + e)^7 + 42*a*b*tan(f*x + e)^5 + 63*b^2*tan(f*x + e)
^5 + 35*a^2*tan(f*x + e)^3 + 140*a*b*tan(f*x + e)^3 + 105*b^2*tan(f*x + e)
^3 + 105*a^2*tan(f*x + e) + 210*a*b*tan(f*x + e) + 105*b^2*tan(f*x + e))/f
```

3.172.9 Mupad [B] (verification not implemented)

Time = 18.59 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{\tan(e+f\,x)\,(a+b)^2+\tan(e+f\,x)^3\left(\frac{a^2}{3}+\frac{4\,a\,b}{3}+b^2\right)+\frac{b^2\tan(e+f\,x)^7}{7}+\frac{b\tan(e+f\,x)^5\,(2\,a+3\,b)}{5}}{f}$$

input
```
int((a + b/cos(e + f*x)^2)^2/cos(e + f*x)^4,x)
```

output
```
(tan(e + f*x)*(a + b)^2 + tan(e + f*x)^3*((4*a*b)/3 + a^2/3 + b^2) + (b^2*
tan(e + f*x)^7)/7 + (b*tan(e + f*x)^5*(2*a + 3*b))/5)/f
```

3.173 $\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

3.173.1 Optimal result 1329
3.173.2 Mathematica [A] (verified) 1329
3.173.3 Rubi [A] (verified) 1330
3.173.4 Maple [A] (verified) 1331
3.173.5 Fricas [A] (verification not implemented) 1332
3.173.6 Sympy [F] 1332
3.173.7 Maxima [A] (verification not implemented) 1332
3.173.8 Giac [A] (verification not implemented) 1333
3.173.9 Mupad [B] (verification not implemented) 1333

3.173.1 Optimal result

Integrand size = 23, antiderivative size = 53

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{(a+b)^2\tan(e+fx)}{f} + \frac{2b(a+b)\tan^3(e+fx)}{3f} + \frac{b^2\tan^5(e+fx)}{5f}$$

output
```
(a+b)^2*tan(f*x+e)/f+2/3*b*(a+b)*tan(f*x+e)^3/f+1/5*b^2*tan(f*x+e)^5/f
```

3.173.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{15(a+b)^2\tan(e+fx)+10b(a+b)\tan^3(e+fx)+3b^2\tan^5(e+fx)}{15f}$$

input
```
Integrate[Sec[e + f*x]^2*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(15*(a + b)^2*Tan[e + f*x] + 10*b*(a + b)*Tan[e + f*x]^3 + 3*b^2*Tan[e + f
*x]^5)/(15*f)
```

3.173.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4634, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int \sec(e+fx)^2\left(a+b\sec(e+fx)^2\right)^2 dx$$

$$\downarrow 4634$$

$$\frac{\int \left(b\tan^2(e+fx)+a+b\right)^2 d\tan(e+fx)}{f}$$

$$\downarrow 210$$

$$\frac{\int \left(b^2\tan^4(e+fx)+2ab\left(\frac{b}{a}+1\right)\tan^2(e+fx)+a^2\left(\frac{b(2a+b)}{a^2}+1\right)\right) d\tan(e+fx)}{f}$$

$$\downarrow 2009$$

$$\frac{\frac{2}{3}b(a+b)\tan^3(e+fx)+(a+b)^2\tan(e+fx)+\frac{1}{5}b^2\tan^5(e+fx)}{f}$$

input
```
Int[Sec[e + f*x]^2*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((a + b)^2*Tan[e + f*x] + (2*b*(a + b)*Tan[e + f*x]^3)/3 + (b^2*Tan[e + f*
x]^5)/5)/f
```

3.173.3.1 Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.173.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.34

method	result
derivativedivides	$\frac{a^2\tan(fx+e)-2ab\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)-b^2\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)}{f}$
default	$\frac{a^2\tan(fx+e)-2ab\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)-b^2\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)}{f}$
parts	$\frac{a^2\tan(fx+e)}{f}-\frac{b^2\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)}{f}-\frac{2ab\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)}{f}$
parallelrisch	$\frac{(45a^2+100ab+40b^2)\sin(3fx+3e)+(15a^2+20ab+8b^2)\sin(5fx+5e)+30\sin(fx+e)\left(a^2+\frac{8}{3}ab+\frac{8}{3}b^2\right)}{15f(\cos(5fx+5e)+5\cos(3fx+3e)+10\cos(fx+e))}$
risch	$\frac{2i\left(15a^2e^{8i(fx+e)}+60a^2e^{6i(fx+e)}+60ab\,e^{6i(fx+e)}+90a^2e^{4i(fx+e)}+140ab\,e^{4i(fx+e)}+80b^2e^{4i(fx+e)}+60a^2e^{2i(fx+e)}+10\right.}{15f\left(e^{2i(fx+e)}+1\right)^5}$
norman	$-\frac{2\left(a^2+2ab+b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{2\left(a^2+2ab+b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{f}+\frac{8\left(3a^2+4ab+b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3f}+\frac{8\left(3a^2+4ab+b^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{3f}-$ $\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^5$

input
```
int(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(a^2*tan(f*x+e)-2*a*b*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)-b^2*(-8/15-1/
5*sec(f*x+e)^4-4/15*sec(f*x+e)^2)*tan(f*x+e))
```

3.173.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.30

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$= \frac{\left(\left(15\,a^2+20\,ab+8\,b^2\right)\cos\left(fx+e\right)^4+2\left(5\,ab+2\,b^2\right)\cos\left(fx+e\right)^2+3\,b^2\right)\sin\left(fx+e\right)}{15\,f\cos\left(fx+e\right)^5}$$

input
```
integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/15*((15*a^2 + 20*a*b + 8*b^2)*cos(f*x + e)^4 + 2*(5*a*b + 2*b^2)*cos(f*x
 + e)^2 + 3*b^2)*sin(f*x + e)/(f*cos(f*x + e)^5)
```

3.173.6 Sympy [F]

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)^2\sec^2\left(e+fx\right)dx$$

input
```
integrate(sec(f*x+e)**2*(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**2*sec(e + f*x)**2, x)
```

3.173.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.34

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$= \frac{10\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)ab+\left(3\,\tan\left(fx+e\right)^5+10\,\tan\left(fx+e\right)^3+15\,\tan\left(fx+e\right)\right)b^2+15\,a}{15\,f}$$

input
```
integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/15*(10*(tan(f*x + e)^3 + 3*tan(f*x + e))*a*b + (3*tan(f*x + e)^5 + 10*ta
n(f*x + e)^3 + 15*tan(f*x + e))*b^2 + 15*a^2*tan(f*x + e))/f
```

3.173.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.43

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$= \frac{3\,b^2\tan(fx+e)^5 + 10\,ab\tan(fx+e)^3 + 10\,b^2\tan(fx+e)^3 + 15\,a^2\tan(fx+e) + 30\,ab\tan(fx+e) + }{15\,f}$$

input
```
integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/15*(3*b^2*tan(f*x + e)^5 + 10*a*b*tan(f*x + e)^3 + 10*b^2*tan(f*x + e)^3
 + 15*a^2*tan(f*x + e) + 30*a*b*tan(f*x + e) + 15*b^2*tan(f*x + e))/f
```

3.173.9 Mupad [B] (verification not implemented)

Time = 18.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$= \frac{\tan(e+f\,x)\,(a+b)^2 + \frac{b^2\tan(e+f\,x)^5}{5} + \frac{2\,b\tan(e+f\,x)^3\,(a+b)}{3}}{f}$$

input
```
int((a + b/cos(e + f*x)^2)^2/cos(e + f*x)^2,x)
```

output
```
(tan(e + f*x)*(a + b)^2 + (b^2*tan(e + f*x)^5)/5 + (2*b*tan(e + f*x)^3*(a
+ b))/3)/f
```

3.174 $\int (a + b\sec^2(e + fx))^2 \, dx$

3.174.1 Optimal result 1334
3.174.2 Mathematica [A] (verified) 1334
3.174.3 Rubi [A] (verified) 1335
3.174.4 Maple [A] (verified) 1336
3.174.5 Fricas [A] (verification not implemented) 1337
3.174.6 Sympy [F] 1337
3.174.7 Maxima [A] (verification not implemented) 1337
3.174.8 Giac [A] (verification not implemented) 1338
3.174.9 Mupad [B] (verification not implemented) 1338

3.174.1 Optimal result

Integrand size = 14, antiderivative size = 40

$$\int (a + b\sec^2(e + fx))^2 \, dx = a^2 x + \frac{b(2a + b)\tan(e + fx)}{f} + \frac{b^2 \tan^3(e + fx)}{3f}$$

output
```
a^2*x+b*(2*a+b)*tan(f*x+e)/f+1/3*b^2*tan(f*x+e)^3/f
```

3.174.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int (a + b\sec^2(e + fx))^2 \, dx = \frac{3a^2 fx + 3b(2a + b)\tan(e + fx) + b^2 \tan^3(e + fx)}{3f}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(3*a^2*f*x + 3*b*(2*a + b)*Tan[e + f*x] + b^2*Tan[e + f*x]^3)/(3*f)
```

3.174.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4616, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + b\sec^2(e+fx))^2\,dx$$

$$\downarrow 3042$$

$$\int (a + b\sec(e+fx)^2)^2\,dx$$

$$\downarrow 4616$$

$$\frac{\int \frac{(b\tan^2(e+fx)+a+b)^2}{\tan^2(e+fx)+1} d\tan(e+fx)}{f}$$

$$\downarrow 300$$

$$\frac{\int \left(\frac{a^2}{\tan^2(e+fx)+1} + b^2\tan^2(e+fx) + b(2a+b)\right) d\tan(e+fx)}{f}$$

$$\downarrow 2009$$

$$\frac{a^2\arctan(\tan(e+fx)) + b(2a+b)\tan(e+fx) + \frac{1}{3}b^2\tan^3(e+fx)}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(a^2*ArcTan[Tan[e + f*x]] + b*(2*a + b)*Tan[e + f*x] + (b^2*Tan[e + f*x]^3
)/3)/f
```

3.174.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f  Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

3.174.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

method	result
parts	$a^2x-\frac{b^2\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)}{f}+\frac{2ab\tan(fx+e)}{f}$
derivativedivides	$\frac{a^2(fx+e)+2ab\tan(fx+e)-b^2\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)}{f}$
default	$\frac{a^2(fx+e)+2ab\tan(fx+e)-b^2\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)}{f}$
risch	$a^2x+\frac{4ib\left(3a\,\mathrm{e}^{4i(fx+e)}+6a\,\mathrm{e}^{2i(fx+e)}+3b\,\mathrm{e}^{2i(fx+e)}+3a+b\right)}{3f\left(\mathrm{e}^{2i(fx+e)}+1\right)^3}$
norman	$\frac{a^2x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6-a^2x+3a^2x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-3a^2x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4-\frac{2b(2a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{2b(2a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{f}+\frac{4b(6a}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3}$
parallelrisch	$\frac{3x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6a^2f+(-12ab-6b^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5-9x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4a^2f+(24ab+4b^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3+9x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2$

input
```
int((a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x-b^2/f*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+2*a*b/f*tan(f*x+e)
```

3.174.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.45

$$\int \left(a + b\sec^2(e+fx)\right)^2 dx$$
$$= \frac{3\,a^2 f x \cos\left(fx+e\right)^3 + \left(2\left(3\,ab+b^2\right)\cos\left(fx+e\right)^2 + b^2\right)\sin\left(fx+e\right)}{3\,f\cos\left(fx+e\right)^3}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/3*(3*a^2*f*x*cos(f*x + e)^3 + (2*(3*a*b + b^2)*cos(f*x + e)^2 + b^2)*sin
(f*x + e))/(f*cos(f*x + e)^3)
```

3.174.6 Sympy [F]

$$\int \left(a + b\sec^2(e+fx)\right)^2 dx = \int \left(a + b\sec^2\left(e+fx\right)\right)^2 dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**2, x)
```

3.174.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10

$$\int \left(a + b\sec^2(e+fx)\right)^2 dx = a^2 x + \frac{\left(\tan\left(fx+e\right)^3 + 3\,\tan\left(fx+e\right)\right)b^2}{3\,f} + \frac{2\,ab\tan\left(fx+e\right)}{f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
a^2*x + 1/3*(tan(f*x + e)^3 + 3*tan(f*x + e))*b^2/f + 2*a*b*tan(f*x + e)/f
```

3.174.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.22

$$\int \left(a + b\sec^2(e+fx)\right)^2 dx = \frac{b^2\tan(fx+e)^3 + 3(fx+e)a^2 + 6ab\tan(fx+e) + 3b^2\tan(fx+e)}{3f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/3*(b^2*tan(f*x + e)^3 + 3*(f*x + e)*a^2 + 6*a*b*tan(f*x + e) + 3*b^2*tan
(f*x + e))/f
```

3.174.9 Mupad [B] (verification not implemented)

Time = 18.49 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \left(a + b\sec^2(e+fx)\right)^2 dx = \frac{\frac{b^2\tan(e+fx)^3}{3} - \tan(e+fx)\left(b^2 - 2b(a+b)\right) + a^2 fx}{f}$$

input
```
int((a + b/cos(e + f*x)^2)^2,x)
```

output
```
((b^2*tan(e + f*x)^3)/3 - tan(e + f*x)*(b^2 - 2*b*(a + b)) + a^2*f*x)/f
```

3.175 $\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

3.175.1 Optimal result 1339
3.175.2 Mathematica [A] (verified) 1339
3.175.3 Rubi [A] (verified) 1340
3.175.4 Maple [A] (verified) 1341
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3.175.6 Sympy [F] 1342
3.175.7 Maxima [A] (verification not implemented) 1342
3.175.8 Giac [A] (verification not implemented) 1343
3.175.9 Mupad [B] (verification not implemented) 1343

3.175.1 Optimal result

Integrand size = 23, antiderivative size = 47

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{1}{2}a(a+4b)x + \frac{a^2\cos(e+fx)\sin(e+fx)}{2f} + \frac{b^2\tan(e+fx)}{f}$$

output
```
1/2*a*(a+4*b)*x+1/2*a^2*cos(f*x+e)*sin(f*x+e)/f+b^2*tan(f*x+e)/f
```

3.175.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.11

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = 2abx + \frac{a^2(e+fx)}{2f} + \frac{a^2\sin(2(e+fx))}{4f} + \frac{b^2\tan(e+fx)}{f}$$

input
```
Integrate[Cos[e + f*x]^2*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
2*a*b*x + (a^2*(e + f*x))/(2*f) + (a^2*Sin[2*(e + f*x)])/(4*f) + (b^2*Tan[
e + f*x])/f
```

3.175.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.21, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4634, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^2}{\sec(e+fx)^2} dx$$

$$\downarrow 4634$$

$$\frac{\int \frac{\left(b\tan^2(e+fx)+a+b\right)^2}{\left(\tan^2(e+fx)+1\right)^2} d\tan(e+fx)}{f}$$

$$\downarrow 300$$

$$\frac{\int \left(b^2+\frac{2ab\tan^2(e+fx)+a(a+2b)}{\left(\tan^2(e+fx)+1\right)^2}\right) d\tan(e+fx)}{f}$$

$$\downarrow 2009$$

$$\frac{\frac{a^2\tan(e+fx)}{2\left(\tan^2(e+fx)+1\right)}+\frac{1}{2}a(a+4b)\arctan(\tan(e+fx))+b^2\tan(e+fx)}{f}$$

input
```
Int[Cos[e + f*x]^2*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((a*(a + 4*b)*ArcTan[Tan[e + f*x]])/2 + b^2*Tan[e + f*x] + (a^2*Tan[e + f*
x])/(2*(1 + Tan[e + f*x]^2)))/f
```

3.175.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.175.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{a^2\left(\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+2ab(fx+e)+b^2\tan(fx+e)}{f}$
default	$\frac{a^2\left(\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+2ab(fx+e)+b^2\tan(fx+e)}{f}$
parallelrisch	$\frac{\sin(3fx+3e)a^2+4afx(a+4b)\cos(fx+e)+\sin(fx+e)(a^2+8b^2)}{8f\cos(fx+e)}$
risch	$\frac{a^2x}{2}+2xab-\frac{ie^{2i(fx+e)}a^2}{8f}+\frac{ie^{-2i(fx+e)}a^2}{8f}+\frac{2ib^2}{f(e^{2i(fx+e)}+1)}$
norman	$\frac{(-\frac{1}{2}a^2-2ab)x+(-\frac{1}{2}a^2-2ab)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8+(\frac{1}{2}a^2+2ab)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+(\frac{1}{2}a^2+2ab)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}+(-a^2-4ab)x$ $\left(1-$

input
```
int(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(a^2*(1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+2*a*b*(f*x+e)+b^2*tan(f
*x+e))
```

3.175.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.19

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{(a^2+4\,ab)fx\cos(fx+e)+\left(a^2\cos(fx+e)^2+2\,b^2\right)\sin(fx+e)}{2\,f\cos(fx+e)}$$

input
```
integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/2*((a^2 + 4*a*b)*f*x*cos(f*x + e) + (a^2*cos(f*x + e)^2 + 2*b^2)*sin(f*x
 + e))/(f*cos(f*x + e))
```

3.175.6 Sympy [F]

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \int \left(a+b\sec^2(e+fx)\right)^2\cos^2(e+fx)\,dx$$

input
```
integrate(cos(f*x+e)**2*(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**2*cos(e + f*x)**2, x)
```

3.175.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.13

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{2\,b^2\tan(fx+e)+(a^2+4\,ab)(fx+e)+\frac{a^2\tan(fx+e)}{\tan(fx+e)^2+1}}{2\,f}$$

input
```
integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/2*(2*b^2*tan(f*x + e) + (a^2 + 4*a*b)*(f*x + e) + a^2*tan(f*x + e)/(tan(
f*x + e)^2 + 1))/f
```

3.175.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.13

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{2\,b^2\tan(fx+e)+(a^2+4\,ab)(fx+e)+\frac{a^2\tan(fx+e)}{\tan(fx+e)^2+1}}{2\,f}$$

input
```
integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/2*(2*b^2*tan(f*x + e) + (a^2 + 4*a*b)*(f*x + e) + a^2*tan(f*x + e)/(tan(
f*x + e)^2 + 1))/f
```

3.175.9 Mupad [B] (verification not implemented)

Time = 18.78 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.40

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{b^2\tan(e+f\,x)}{f}+\frac{a^2\,\sin(2\,e+2\,f\,x)}{4\,f}+\frac{a\,\mathrm{atan}\left(\frac{a\tan(e+f\,x)\,(a+4\,b)}{2\left(\frac{a^2}{2}+2\,b\,a\right)}\right)\,(a+4\,b)}{2\,f}$$

input
```
int(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^2,x)
```

output
```
(b^2*tan(e + f*x))/f + (a^2*sin(2*e + 2*f*x))/(4*f) + (a*atan((a*tan(e + f
*x)*(a + 4*b))/(2*(2*a*b + a^2/2)))*(a + 4*b))/(2*f)
```

3.176 $\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

3.176.1 Optimal result 1344
3.176.2 Mathematica [A] (verified) 1344
3.176.3 Rubi [A] (verified) 1345
3.176.4 Maple [A] (verified) 1347
3.176.5 Fricas [A] (verification not implemented) 1347
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3.176.7 Maxima [A] (verification not implemented) 1348
3.176.8 Giac [A] (verification not implemented) 1348
3.176.9 Mupad [B] (verification not implemented) 1349

3.176.1 Optimal result

Integrand size = 23, antiderivative size = 81

$$\begin{aligned}&\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx\\&=\frac{1}{8}\left(3a^2+8ab+8b^2\right)x+\frac{3a(a+2b)\cos(e+fx)\sin(e+fx)}{8f}\\&\quad+\frac{a\cos^3(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)}{4f}\end{aligned}$$

output
```
1/8*(3*a^2+8*a*b+8*b^2)*x+3/8*a*(a+2*b)*cos(f*x+e)*sin(f*x+e)/f+1/4*a*cos(
f*x+e)^3*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)/f
```

3.176.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.72

$$\begin{aligned}&\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx\\&=\frac{4\left(3a^2+8ab+8b^2\right)(e+fx)+8a(a+2b)\sin(2(e+fx))+a^2\sin(4(e+fx))}{32f}\end{aligned}$$

input
```
Integrate[Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(4*(3*a^2 + 8*a*b + 8*b^2)*(e + f*x) + 8*a*(a + 2*b)*Sin[2*(e + f*x)] + a^
2*Sin[4*(e + f*x)])/(32*f)
```

3.176.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.23, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4634, 315, 298, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^2}{\sec(e+fx)^4} dx$$

$$\downarrow 4634$$

$$\frac{\int \frac{\left(b\tan^2(e+fx)+a+b\right)^2}{\left(\tan^2(e+fx)+1\right)^3} d\tan(e+fx)}{f}$$

$$\downarrow 315$$

$$\frac{\frac{1}{4}\int \frac{b(a+4b)\tan^2(e+fx)+(a+b)(3a+4b)}{\left(\tan^2(e+fx)+1\right)^2} d\tan(e+fx) + \frac{a\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)}{4\left(\tan^2(e+fx)+1\right)^2}}{f}$$

$$\downarrow 298$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(3a^2+8ab+8b^2\right)\int \frac{1}{\tan^2(e+fx)+1} d\tan(e+fx) + \frac{3a(a+2b)\tan(e+fx)}{2\left(\tan^2(e+fx)+1\right)}\right) + \frac{a\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)}{4\left(\tan^2(e+fx)+1\right)^2}}{f}$$

$$\downarrow 216$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(3a^2+8ab+8b^2\right)\arctan(\tan(e+fx)) + \frac{3a(a+2b)\tan(e+fx)}{2\left(\tan^2(e+fx)+1\right)}\right) + \frac{a\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)}{4\left(\tan^2(e+fx)+1\right)^2}}{f}$$

input
```
Int[Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((a*Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2))/(4*(1 + Tan[e + f*x]^2)^2) +
(((3*a^2 + 8*a*b + 8*b^2)*ArcTan[Tan[e + f*x]])/2 + (3*a*(a + 2*b)*Tan[e +
 f*x])/(2*(1 + Tan[e + f*x]^2)))/4)/f
```

3.176.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.176.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.67

method	result
parallelrisch	$\frac{8a(a+2b)\sin(2fx+2e)+a^2\sin(4fx+4e)+12xf\left(a^2+\frac{8}{3}ab+\frac{8}{3}b^2\right)}{32f}$
risch	$\frac{3a^2x}{8}+xab+xb^2+\frac{a^2\sin(4fx+4e)}{32f}+\frac{\sin(2fx+2e)a^2}{4f}+\frac{\sin(2fx+2e)ab}{2f}$
derivativedivides	$\frac{a^2\left(\frac{\left(\cos(fx+e)^3+\frac{3\cos(fx+e)}{2}\right)\sin(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+2ab\left(\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+b^2(fx+e)}{f}$
default	$\frac{a^2\left(\frac{\left(\cos(fx+e)^3+\frac{3\cos(fx+e)}{2}\right)\sin(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+2ab\left(\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+b^2(fx+e)}{f}$
norman	$\frac{(-\frac{3}{8}a^2-ab-b^2)x+(-\frac{9}{8}a^2-3ab-3b^2)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8+(-\frac{9}{8}a^2-3ab-3b^2)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}+(-\frac{3}{8}a^2-ab-b^2)x\tan\left(\frac{fx}{2}+$

input
```
int(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/32*(8*a*(a+2*b)*sin(2*f*x+2*e)+a^2*sin(4*f*x+4*e)+12*x*f*(a^2+8/3*a*b+8/
3*b^2))/f
```

3.176.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.77

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{(3\,a^2+8\,ab+8\,b^2)fx+\left(2\,a^2\cos(fx+e)^3+(3\,a^2+8\,ab)\cos(fx+e)\right)\sin(fx+e)}{8\,f}$$

input
```
integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/8*((3*a^2 + 8*a*b + 8*b^2)*f*x + (2*a^2*cos(f*x + e)^3 + (3*a^2 + 8*a*b)
*cos(f*x + e))*sin(f*x + e))/f
```

3.176.6 Sympy [F]

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \int \left(a+b\sec^2(e+fx)\right)^2\cos^4(e+fx)\,dx$$

input
```
integrate(cos(f*x+e)**4*(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**2*cos(e + f*x)**4, x)
```

3.176.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.07

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{(3a^2+8ab+8b^2)(fx+e)+\frac{(3a^2+8ab)\tan(fx+e)^3+(5a^2+8ab)\tan(fx+e)}{\tan(fx+e)^4+2\tan(fx+e)^2+1}}{8f}$$

input
```
integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/8*((3*a^2 + 8*a*b + 8*b^2)*(f*x + e) + ((3*a^2 + 8*a*b)*tan(f*x + e)^3 +
 (5*a^2 + 8*a*b)*tan(f*x + e))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1))/f
```

3.176.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.07

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{(3a^2+8ab+8b^2)(fx+e)+\frac{3a^2\tan(fx+e)^3+8ab\tan(fx+e)^3+5a^2\tan(fx+e)+8ab\tan(fx+e)}{\left(\tan(fx+e)^2+1\right)^2}}{8f}$$

input
```
integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/8*((3*a^2 + 8*a*b + 8*b^2)*(f*x + e) + (3*a^2*tan(f*x + e)^3 + 8*a*b*tan
(f*x + e)^3 + 5*a^2*tan(f*x + e) + 8*a*b*tan(f*x + e))/(tan(f*x + e)^2 + 1
)^2)/f
```

3.176.9 Mupad [B] (verification not implemented)

Time = 18.62 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.94

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$= x\left(\frac{3a^2}{8}+ab+b^2\right)+\frac{\left(\frac{3a^2}{8}+ba\right)\tan(e+fx)^3+\left(\frac{5a^2}{8}+ba\right)\tan(e+fx)}{f\left(\tan(e+fx)^4+2\tan(e+fx)^2+1\right)}$$

input
```
int(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^2,x)
```

output
```
x*(a*b + (3*a^2)/8 + b^2) + (tan(e + f*x)*(a*b + (5*a^2)/8) + tan(e + f*x)
^3*(a*b + (3*a^2)/8))/(f*(2*tan(e + f*x)^2 + tan(e + f*x)^4 + 1))
```

3.177 $\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

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3.177.2 Mathematica [A] (verified) 1350
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3.177.9 Mupad [B] (verification not implemented) 1355

3.177.1 Optimal result

Integrand size = 23, antiderivative size = 119

$$\begin{aligned}
&\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx \\
&= \frac{1}{16}\left(5a^2+12ab+8b^2\right)x + \frac{\left(5a^2+12ab+8b^2\right)\cos(e+fx)\sin(e+fx)}{16f} \\
&\quad + \frac{a(5a+8b)\cos^3(e+fx)\sin(e+fx)}{24f} \\
&\quad + \frac{a\cos^5(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)}{6f}
\end{aligned}$$

output
```
1/16*(5*a^2+12*a*b+8*b^2)*x+1/16*(5*a^2+12*a*b+8*b^2)*cos(f*x+e)*sin(f*x+e
)/f+1/24*a*(5*a+8*b)*cos(f*x+e)^3*sin(f*x+e)/f+1/6*a*cos(f*x+e)^5*sin(f*x+
e)*(a+b+b*tan(f*x+e)^2)/f
```

3.177.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.83

$$\begin{aligned}
&\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx \\
&= \frac{60a^2e+144abe+96b^2e+60a^2fx+144abfx+96b^2fx+\left(45a^2+96ab+48b^2\right)\sin(2(e+fx))+3a(3a+}{192f}
\end{aligned}$$

input
```
Integrate[Cos[e + f*x]^6*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(60*a^2*e + 144*a*b*e + 96*b^2*e + 60*a^2*f*x + 144*a*b*f*x + 96*b^2*f*x +
 (45*a^2 + 96*a*b + 48*b^2)*Sin[2*(e + f*x)] + 3*a*(3*a + 4*b)*Sin[4*(e +
f*x)] + a^2*Sin[6*(e + f*x)])/(192*f)
```

3.177.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4634, 315, 298, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^2}{\sec(e+fx)^6}dx$$

$$\downarrow 4634$$

$$\frac{\int \frac{\left(b\tan^2(e+fx)+a+b\right)^2}{\left(\tan^2(e+fx)+1\right)^4}d\tan(e+fx)}{f}$$

$$\downarrow 315$$

$$\frac{\frac{1}{6}\int \frac{3b(a+2b)\tan^2(e+fx)+(a+b)(5a+6b)}{\left(\tan^2(e+fx)+1\right)^3}d\tan(e+fx)+\frac{a\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)}{6\left(\tan^2(e+fx)+1\right)^3}}{f}$$

$$\downarrow 298$$

$$\frac{\frac{1}{6}\left(\frac{3}{4}\left(5a^2+12ab+8b^2\right)\int \frac{1}{\left(\tan^2(e+fx)+1\right)^2}d\tan(e+fx)+\frac{a(5a+8b)\tan(e+fx)}{4\left(\tan^2(e+fx)+1\right)^2}\right)+\frac{a\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)}{6\left(\tan^2(e+fx)+1\right)^3}}{f}$$

$$\downarrow 215$$

$$\frac{\frac{1}{6}\left(\frac{3}{4}\left(5a^2+12ab+8b^2\right)\left(\frac{1}{2}\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)+\frac{\tan(e+fx)}{2\left(\tan^2(e+fx)+1\right)}\right)+\frac{a(5a+8b)\tan(e+fx)}{4\left(\tan^2(e+fx)+1\right)^2}\right)+\frac{a\tan(e+fx)(a+b\tan}{6\left(\tan^2(e+fx)\right.}}{f}$$

$$\downarrow 216$$

$$\frac{\frac{1}{6}\left(\frac{3}{4}\left(5a^2+12ab+8b^2\right)\left(\frac{1}{2}\arctan(\tan(e+fx))+\frac{\tan(e+fx)}{2(\tan^2(e+fx)+1)}\right)+\frac{a(5a+8b)\tan(e+fx)}{4(\tan^2(e+fx)+1)^2}\right)+\frac{a\tan(e+fx)\left(a+b\tan^2(e+fx)\right)}{6(\tan^2(e+fx)+1)^3}}{f}$$

input
```
Int[Cos[e + f*x]^6*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((a*Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2))/(6*(1 + Tan[e + f*x]^2)^3) +
((a*(5*a + 8*b)*Tan[e + f*x])/(4*(1 + Tan[e + f*x]^2)^2) + (3*(5*a^2 + 12*
a*b + 8*b^2)*(ArcTan[Tan[e + f*x]]/2 + Tan[e + f*x]/(2*(1 + Tan[e + f*x]^2
))))/4)/6)/f
```

3.177.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.177.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.69

method	result
parallelrisch	$\frac{\left(45a^2+96ab+48b^2\right)\sin(2fx+2e)+\left(9a^2+12ab\right)\sin(4fx+4e)+a^2\sin(6fx+6e)+60x\left(a^2+\frac{12}{5}ab+\frac{8}{5}b^2\right)f}{192f}$
derivativedivides	$\frac{a^2\left(\frac{\left(\cos(fx+e)^5+\frac{5\cos(fx+e)^3}{4}+\frac{15\cos(fx+e)}{8}\right)\sin(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)+2ab\left(\frac{\left(\cos(fx+e)^3+\frac{3\cos(fx+e)}{2}\right)\sin(fx+e)}{4}+\frac{3fx}{8}+\right.}{f}$
default	$\frac{a^2\left(\frac{\left(\cos(fx+e)^5+\frac{5\cos(fx+e)^3}{4}+\frac{15\cos(fx+e)}{8}\right)\sin(fx+e)}{6}+\frac{5fx}{16}+\frac{5e}{16}\right)+2ab\left(\frac{\left(\cos(fx+e)^3+\frac{3\cos(fx+e)}{2}\right)\sin(fx+e)}{4}+\frac{3fx}{8}+\right.}{f}$
risch	$\frac{5a^2x}{16}+\frac{3xab}{4}+\frac{xb^2}{2}+\frac{a^2\sin(6fx+6e)}{192f}+\frac{3a^2\sin(4fx+4e)}{64f}+\frac{\sin(4fx+4e)ab}{16f}+\frac{15\sin(2fx+2e)a^2}{64f}+\frac{\sin(2fx}{2}$

input
```
int(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/192*((45*a^2+96*a*b+48*b^2)*sin(2*f*x+2*e)+(9*a^2+12*a*b)*sin(4*f*x+4*e)
+a^2*sin(6*f*x+6*e)+60*x*(a^2+12/5*a*b+8/5*b^2)*f)/f
```

3.177.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.75

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{3\left(5a^2+12ab+8b^2\right)fx+\left(8a^2\cos(fx+e)^5+2\left(5a^2+12ab\right)\cos(fx+e)^3+3\left(5a^2+12ab+8b^2\right)\cos\right.}{48f}$$

input
```
integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/48*(3*(5*a^2 + 12*a*b + 8*b^2)*f*x + (8*a^2*cos(f*x + e)^5 + 2*(5*a^2 +
12*a*b)*cos(f*x + e)^3 + 3*(5*a^2 + 12*a*b + 8*b^2)*cos(f*x + e))*sin(f*x
+ e))/f
```

3.177.6 Sympy [F(-1)]

Timed out.

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**6*(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Timed out
```

3.177.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.13

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{3\left(5a^2+12ab+8b^2\right)(fx+e)+\frac{3\left(5a^2+12ab+8b^2\right)\tan(fx+e)^5+8\left(5a^2+12ab+6b^2\right)\tan(fx+e)^3+3\left(11a^2+20ab+8b^2\right)\tan(fx+\right.}{\tan(fx+e)^6+3\tan(fx+e)^4+3\tan(fx+e)^2+1}}{48f}$$

input
```
integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/48*(3*(5*a^2 + 12*a*b + 8*b^2)*(f*x + e) + (3*(5*a^2 + 12*a*b + 8*b^2)*t
an(f*x + e)^5 + 8*(5*a^2 + 12*a*b + 6*b^2)*tan(f*x + e)^3 + 3*(11*a^2 + 20
*a*b + 8*b^2)*tan(f*x + e))/(tan(f*x + e)^6 + 3*tan(f*x + e)^4 + 3*tan(f*x
 + e)^2 + 1))/f
```

3.177.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.26

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$= \frac{3\left(5\,a^2+12\,ab+8\,b^2\right)(fx+e)+\frac{15\,a^2\tan(fx+e)^5+36\,ab\tan(fx+e)^5+24\,b^2\tan(fx+e)^5+40\,a^2\tan(fx+e)^3+96\,ab\tan(fx+e)^3+48}{\left(\tan(fx+e)^2+1\right)^3}}{48\,f}$$

input
```
integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/48*(3*(5*a^2 + 12*a*b + 8*b^2)*(f*x + e) + (15*a^2*tan(f*x + e)^5 + 36*a
*b*tan(f*x + e)^5 + 24*b^2*tan(f*x + e)^5 + 40*a^2*tan(f*x + e)^3 + 96*a*b
*tan(f*x + e)^3 + 48*b^2*tan(f*x + e)^3 + 33*a^2*tan(f*x + e) + 60*a*b*tan
(f*x + e) + 24*b^2*tan(f*x + e))/(tan(f*x + e)^2 + 1)^3)/f
```

3.177.9 Mupad [B] (verification not implemented)

Time = 19.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.03

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = x\left(\frac{5\,a^2}{16}+\frac{3\,a\,b}{4}+\frac{b^2}{2}\right)$$
$$+\frac{\left(\frac{5\,a^2}{16}+\frac{3\,a\,b}{4}+\frac{b^2}{2}\right)\tan(e+f\,x)^5+\left(\frac{5\,a^2}{6}+2\,a\,b+b^2\right)\tan(e+f\,x)^3+\left(\frac{11\,a^2}{16}+\frac{5\,a\,b}{4}+\frac{b^2}{2}\right)\tan(e+f\,x)}{f\left(\tan(e+f\,x)^6+3\tan(e+f\,x)^4+3\tan(e+f\,x)^2+1\right)}$$

input
```
int(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^2,x)
```

output
```
x*((3*a*b)/4 + (5*a^2)/16 + b^2/2) + (tan(e + f*x)*((5*a*b)/4 + (11*a^2)/1
6 + b^2/2) + tan(e + f*x)^3*(2*a*b + (5*a^2)/6 + b^2) + tan(e + f*x)^5*((3
*a*b)/4 + (5*a^2)/16 + b^2/2))/(f*(3*tan(e + f*x)^2 + 3*tan(e + f*x)^4 + t
an(e + f*x)^6 + 1))
```

3.178 $\int (a + b\sec^2(c+dx))^3\, dx$

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3.178.2 Mathematica [A] (verified) . . . 1356
3.178.3 Rubi [A] (verified) . . . 1357
3.178.4 Maple [A] (verified) . . . 1358
3.178.5 Fricas [A] (verification not implemented) . . . 1359
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3.178.7 Maxima [A] (verification not implemented) . . . 1359
3.178.8 Giac [A] (verification not implemented) . . . 1360
3.178.9 Mupad [B] (verification not implemented) . . . 1360

3.178.1 Optimal result

Integrand size = 14, antiderivative size = 73

$$\int (a + b\sec^2(c+dx))^3\, dx = a^3x + \frac{b(3a^2+3ab+b^2)\tan(c+dx)}{d} + \frac{b^2(3a+2b)\tan^3(c+dx)}{3d} + \frac{b^3\tan^5(c+dx)}{5d}$$

output
```
a^3*x+b*(3*a^2+3*a*b+b^2)*tan(d*x+c)/d+1/3*b^2*(3*a+2*b)*tan(d*x+c)^3/d+1/
5*b^3*tan(d*x+c)^5/d
```

3.178.2 Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

$$\int (a + b\sec^2(c+dx))^3\, dx = \frac{15a^3dx + 15b(3a^2+3ab+b^2)\tan(c+dx) + 5b^2(3a+2b)\tan^3(c+dx) + 3b^3\tan^5(c+dx)}{15d}$$

input
```
Integrate[(a + b*Sec[c + d*x]^2)^3,x]
```

output
```
(15*a^3*d*x + 15*b*(3*a^2 + 3*a*b + b^2)*Tan[c + d*x] + 5*b^2*(3*a + 2*b)*
Tan[c + d*x]^3 + 3*b^3*Tan[c + d*x]^5)/(15*d)
```

3.178.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4616, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + b\sec^2(c+dx))^3\,dx$$

$$\downarrow 3042$$

$$\int (a + b\sec(c+dx)^2)^3\,dx$$

$$\downarrow 4616$$

$$\frac{\int \frac{(b\tan^2(c+dx)+a+b)^3}{\tan^2(c+dx)+1} d\tan(c+dx)}{d}$$

$$\downarrow 300$$

$$\frac{\int \left(b^3\tan^4(c+dx) + b^2(3a+2b)\tan^2(c+dx) + b\left(3a^2+3ba+b^2\right) + \frac{a^3}{\tan^2(c+dx)+1}\right) d\tan(c+dx)}{d}$$

$$\downarrow 2009$$

$$\frac{a^3\arctan(\tan(c+dx)) + b\left(3a^2+3ab+b^2\right)\tan(c+dx) + \frac{1}{3}b^2(3a+2b)\tan^3(c+dx) + \frac{1}{5}b^3\tan^5(c+dx)}{d}$$

input
```
Int[(a + b*Sec[c + d*x]^2)^3,x]
```

output
```
(a^3*ArcTan[Tan[c + d*x]] + b*(3*a^2 + 3*a*b + b^2)*Tan[c + d*x] + (b^2*(3
*a + 2*b)*Tan[c + d*x]^3)/3 + (b^3*Tan[c + d*x]^5)/5)/d
```

3.178.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

3.178.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.15

method	result
derivativedivides	$\frac{a^3(dx+c)+3a^2b\tan(dx+c)-3ab^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)-b^3\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)}{d}$
default	$\frac{a^3(dx+c)+3a^2b\tan(dx+c)-3ab^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)-b^3\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)}{d}$
parts	$a^3x-\frac{b^3\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)}{d}+\frac{3a^2b\tan(dx+c)}{d}-\frac{3ab^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}$
risch	$a^3x+\frac{2ib(45a^2e^{8i(dx+c)}+180a^2e^{6i(dx+c)}+90abe^{6i(dx+c)}+270a^2e^{4i(dx+c)}+210abe^{4i(dx+c)}+80b^2e^{4i(dx+c)}+180a^2e$ $15d(e^{2i(dx+c)}+1)^5$
norman	$\frac{a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}-a^3x+5a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-10a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+10a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6-5a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8-\frac{2b\left(3a^2$
parallelrisch	$\frac{15a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}xd+(-90a^2b-90ab^2-30b^3)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9-75a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8xd+360b\left(a+\frac{b}{3}\right)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7+15$

input
```
int((a+b*sec(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(d*x+c)+3*a^2*b*tan(d*x+c)-3*a*b^2*(-2/3-1/3*sec(d*x+c)^2)*tan(d*
x+c)-b^3*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c))
```

3.178.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.23

$$\int \left(a+b\sec^2(c+dx)\right)^3\,dx$$
$$= \frac{15\,a^3 dx\cos\left(dx+c\right)^5 + \left(\left(45\,a^2b+30\,ab^2+8\,b^3\right)\cos\left(dx+c\right)^4 + 3\,b^3 + \left(15\,ab^2+4\,b^3\right)\cos\left(dx+c\right)^2\right)\sin\left(d\right.}{15\,d\cos\left(dx+c\right)^5}$$

input
```
integrate((a+b*sec(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/15*(15*a^3*d*x*cos(d*x + c)^5 + ((45*a^2*b + 30*a*b^2 + 8*b^3)*cos(d*x +
 c)^4 + 3*b^3 + (15*a*b^2 + 4*b^3)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*
x + c)^5)
```

3.178.6 Sympy [F]

$$\int \left(a+b\sec^2(c+dx)\right)^3\,dx = \int \left(a+b\sec^2\left(c+dx\right)\right)^3\,dx$$

input
```
integrate((a+b*sec(d*x+c)**2)**3,x)
```

output
```
Integral((a + b*sec(c + d*x)**2)**3, x)
```

3.178.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.14

$$\int \left(a+b\sec^2(c+dx)\right)^3\,dx = a^3x + \frac{\left(\tan\left(dx+c\right)^3 + 3\,\tan\left(dx+c\right)\right)ab^2}{d} + \frac{\left(3\,\tan\left(dx+c\right)^5 + 10\,\tan\left(dx+c\right)^3 + 15\,\tan\left(dx+c\right)\right)b^3}{15\,d} + \frac{3\,a^2b\tan\left(dx+c\right)}{d}$$

input
```
integrate((a+b*sec(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
a^3*x + (tan(d*x + c)^3 + 3*tan(d*x + c))*a*b^2/d + 1/15*(3*tan(d*x + c)^5
 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*b^3/d + 3*a^2*b*tan(d*x + c)/d
```

3.178.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.25

$$\int \left(a + b\sec^2(c+dx)\right)^3 \, dx$$
$$= \frac{3\,b^3\tan(dx+c)^5 + 15\,ab^2\tan(dx+c)^3 + 10\,b^3\tan(dx+c)^3 + 15\,(dx+c)a^3 + 45\,a^2b\tan(dx+c) + 45\,a}{15\,d}$$

input
```
integrate((a+b*sec(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/15*(3*b^3*tan(d*x + c)^5 + 15*a*b^2*tan(d*x + c)^3 + 10*b^3*tan(d*x + c)
^3 + 15*(d*x + c)*a^3 + 45*a^2*b*tan(d*x + c) + 45*a*b^2*tan(d*x + c) + 15
*b^3*tan(d*x + c))/d
```

3.178.9 Mupad [B] (verification not implemented)

Time = 18.69 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int \left(a + b\sec^2(c+dx)\right)^3 \, dx$$
$$= \frac{\tan(c+d\,x)\,\left(3\,b\,(a+b)^2 - 3\,b^2\,(a+b) + b^3\right) + \frac{b^3\tan(c+d\,x)^5}{5} + \tan(c+d\,x)^3\left(b^2\,(a+b) - \frac{b^3}{3}\right) + a^3\,d\,x}{d}$$

input
```
int((a + b/cos(c + d*x)^2)^3,x)
```

output
```
(tan(c + d*x)*(3*b*(a + b)^2 - 3*b^2*(a + b) + b^3) + (b^3*tan(c + d*x)^5)
/5 + tan(c + d*x)^3*(b^2*(a + b) - b^3/3) + a^3*d*x)/d
```

3.179 $\int (a + b\sec^2(c + dx))^4 \, dx$

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3.179.2 Mathematica [A] (verified) 1361
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3.179.1 Optimal result

Integrand size = 14, antiderivative size = 111

$$\int (a + b\sec^2(c + dx))^4 \, dx = a^4 x + \frac{b(2a + b)\left(2a^2 + 2ab + b^2\right)\tan(c + dx)}{d} + \frac{b^2(6a^2 + 8ab + 3b^2)\tan^3(c + dx)}{3d} + \frac{b^3(4a + 3b)\tan^5(c + dx)}{5d} + \frac{b^4\tan^7(c + dx)}{7d}$$

output
```
a^4*x+b*(2*a+b)*(2*a^2+2*a*b+b^2)*tan(d*x+c)/d+1/3*b^2*(6*a^2+8*a*b+3*b^2)
*tan(d*x+c)^3/d+1/5*b^3*(4*a+3*b)*tan(d*x+c)^5/d+1/7*b^4*tan(d*x+c)^7/d
```

3.179.2 Mathematica [A] (verified)

Time = 3.60 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.95

$$\int (a + b\sec^2(c + dx))^4 \, dx = \frac{105a^4 dx + 105b(4a^3 + 6a^2 b + 4ab^2 + b^3)\tan(c + dx) + 35b^2(6a^2 + 8ab + 3b^2)\tan^3(c + dx) + 21b^3(4a + 3}{105d}$$

input
```
Integrate[(a + b*Sec[c + d*x]^2)^4,x]
```

output
```
(105*a^4*d*x + 105*b*(4*a^3 + 6*a^2*b + 4*a*b^2 + b^3)*Tan[c + d*x] + 35*b
^2*(6*a^2 + 8*a*b + 3*b^2)*Tan[c + d*x]^3 + 21*b^3*(4*a + 3*b)*Tan[c + d*x
]^5 + 15*b^4*Tan[c + d*x]^7)/(105*d)
```

3.179.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4616, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a + b\sec^2(c+dx)\right)^4 dx$$

$$\downarrow 3042$$

$$\int \left(a + b\sec(c+dx)^2\right)^4 dx$$

$$\downarrow 4616$$

$$\frac{\int \frac{\left(b\tan^2(c+dx)+a+b\right)^4}{\tan^2(c+dx)+1} d\tan(c+dx)}{d}$$

$$\downarrow 300$$

$$\frac{\int \left(b^4\tan^6(c+dx) + b^3(4a+3b)\tan^4(c+dx) + b^2\left(6a^2+8ba+3b^2\right)\tan^2(c+dx) + b(2a+b)\left(2a^2+2ba+b^2\right) +}{d}$$

$$\downarrow 2009$$

$$\frac{a^4\arctan(\tan(c+dx)) + \frac{1}{3}b^2\left(6a^2+8ab+3b^2\right)\tan^3(c+dx) + b(2a+b)\left(2a^2+2ab+b^2\right)\tan(c+dx) + \frac{1}{5}b^3(4a+}{d}$$

input
```
Int[(a + b*Sec[c + d*x]^2)^4,x]
```

output
```
(a^4*ArcTan[Tan[c + d*x]] + b*(2*a + b)*(2*a^2 + 2*a*b + b^2)*Tan[c + d*x]
 + (b^2*(6*a^2 + 8*a*b + 3*b^2)*Tan[c + d*x]^3)/3 + (b^3*(4*a + 3*b)*Tan[c
 + d*x]^5)/5 + (b^4*Tan[c + d*x]^7)/7)/d
```

3.179.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

3.179.4 Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.17

method	result
derivativedivides	$\frac{a^4(dx+c)+4a^3b\tan(dx+c)-6a^2b^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)-4ab^3\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)-}{d}$
default	$\frac{a^4(dx+c)+4a^3b\tan(dx+c)-6a^2b^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)-4ab^3\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)-}{d}$
parts	$a^4x-\frac{b^4\left(-\frac{16}{35}-\frac{\sec(dx+c)^6}{7}-\frac{6\sec(dx+c)^4}{35}-\frac{8\sec(dx+c)^2}{35}\right)\tan(dx+c)}{d}+\frac{4a^3b\tan(dx+c)}{d}-\frac{6a^2b^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right.}{d}$
risch	$a^4x+\frac{8ib(105a^3e^{12i(dx+c)}+630a^3e^{10i(dx+c)}+315a^2be^{10i(dx+c)}+1575a^3e^{8i(dx+c)}+1365a^2be^{8i(dx+c)}+560ab^2e^{8i(dx}{}$
norman	$\frac{a^4x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}-a^4x+7a^4x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-21a^4x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+35a^4x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6-35a^4x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8+21a^4x}{}$
parallelrisch	$\frac{105a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}xd+\left(-840a^3b-1260a^2b^2-840ab^3-210b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{13}-735a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}xd+(5040a^3b+5}{}$

input
```
int((a+b*sec(d*x+c)^2)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^4*(d*x+c)+4*a^3*b*tan(d*x+c)-6*a^2*b^2*(-2/3-1/3*sec(d*x+c)^2)*tan(
d*x+c)-4*a*b^3*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)-b^4*(
-16/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*tan(d*x+c))
```

3.179.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.17

$$\int \left(a + b\sec^2(c+dx)\right)^4 dx$$
$$= \frac{105\,a^4 dx\cos\left(dx+c\right)^7 + \left(4\left(105\,a^3b + 105\,a^2b^2 + 56\,ab^3 + 12\,b^4\right)\cos\left(dx+c\right)^6 + 2\left(105\,a^2b^2 + 56\,ab^3 + 12\right.\right.}{105\,d\cos\left(dx+c\right)^7}$$

input
```
integrate((a+b*sec(d*x+c)^2)^4,x, algorithm="fricas")
```

output
```
1/105*(105*a^4*d*x*cos(d*x + c)^7 + (4*(105*a^3*b + 105*a^2*b^2 + 56*a*b^3
 + 12*b^4)*cos(d*x + c)^6 + 2*(105*a^2*b^2 + 56*a*b^3 + 12*b^4)*cos(d*x +
c)^4 + 15*b^4 + 6*(14*a*b^3 + 3*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(
d*x + c)^7)
```

3.179.6 Sympy [F]

$$\int \left(a + b\sec^2(c+dx)\right)^4 dx = \int \left(a + b\sec^2\left(c+dx\right)\right)^4 dx$$

input
```
integrate((a+b*sec(d*x+c)**2)**4,x)
```

output
```
Integral((a + b*sec(c + d*x)**2)**4, x)
```

3.179.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.21

$$\begin{aligned}
&\int \left(a + b\sec^2(c+dx)\right)^4 \, dx \\
&= a^4 x + \frac{2\left(\tan(dx+c)^3 + 3\tan(dx+c)\right)a^2b^2}{d} \\
&\quad + \frac{4\left(3\tan(dx+c)^5 + 10\tan(dx+c)^3 + 15\tan(dx+c)\right)ab^3}{15\,d} \\
&\quad + \frac{\left(5\tan(dx+c)^7 + 21\tan(dx+c)^5 + 35\tan(dx+c)^3 + 35\tan(dx+c)\right)b^4}{35\,d} \\
&\quad + \frac{4\,a^3 b\tan(dx+c)}{d}
\end{aligned}$$

input
```
integrate((a+b*sec(d*x+c)^2)^4,x, algorithm="maxima")
```

output
```
a^4*x + 2*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2*b^2/d + 4/15*(3*tan(d*x +
c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a*b^3/d + 1/35*(5*tan(d*x + c)
^7 + 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 35*tan(d*x + c))*b^4/d + 4*a^
3*b*tan(d*x + c)/d
```

3.179.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.33

$$\begin{aligned}
&\int \left(a + b\sec^2(c+dx)\right)^4 \, dx \\
&= \frac{15\,b^4\tan(dx+c)^7 + 84\,ab^3\tan(dx+c)^5 + 63\,b^4\tan(dx+c)^5 + 210\,a^2b^2\tan(dx+c)^3 + 280\,ab^3\tan(dx+}{}
\end{aligned}$$

input
```
integrate((a+b*sec(d*x+c)^2)^4,x, algorithm="giac")
```

output
```
1/105*(15*b^4*tan(d*x + c)^7 + 84*a*b^3*tan(d*x + c)^5 + 63*b^4*tan(d*x +
c)^5 + 210*a^2*b^2*tan(d*x + c)^3 + 280*a*b^3*tan(d*x + c)^3 + 105*b^4*tan
(d*x + c)^3 + 105*(d*x + c)*a^4 + 420*a^3*b*tan(d*x + c) + 630*a^2*b^2*tan
(d*x + c) + 420*a*b^3*tan(d*x + c) + 105*b^4*tan(d*x + c))/d
```

3.179.9 Mupad [B] (verification not implemented)

Time = 18.71 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.07

$$\int \left(a+b\sec^2(c+dx)\right)^4\,dx$$

$$=\frac{\tan(c+dx)\,\left(4\,b\,(a+b)^3+4\,b^3\,(a+b)-6\,b^2\,(a+b)^2-b^4\right)+\tan(c+dx)^3\left(2\,b^2\,(a+b)^2-\frac{4\,b^3\,(a+b)}{3}+\frac{b^4}{3}\right.}{d}$$

input
```
int((a + b/cos(c + d*x)^2)^4,x)
```

output
```
(tan(c + d*x)*(4*b*(a + b)^3 + 4*b^3*(a + b) - 6*b^2*(a + b)^2 - b^4) + ta
n(c + d*x)^3*(2*b^2*(a + b)^2 - (4*b^3*(a + b))/3 + b^4/3) + (b^4*tan(c +
d*x)^7)/7 + tan(c + d*x)^5*((4*b^3*(a + b))/5 - b^4/5) + a^4*d*x)/d
```

3.180 $\int \frac{\sec^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$

3.180.1 Optimal result 1367
3.180.2 Mathematica [C] (warning: unable to verify) 1367
3.180.3 Rubi [A] (verified) 1368
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3.180.8 Giac [A] (verification not implemented) 1373
3.180.9 Mupad [B] (verification not implemented) 1373

3.180.1 Optimal result

Integrand size = 23, antiderivative size = 86

$$\int \frac{\sec^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{(2a-b)\operatorname{arctanh}(\sin(e+fx))}{2b^2 f} + \frac{a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{b^2\sqrt{a+b}f} + \frac{\sec(e+fx)\tan(e+fx)}{2bf}$$

output
```
-1/2*(2*a-b)*arctanh(sin(f*x+e))/b^2/f+a^(3/2)*arctanh(sin(f*x+e)*a^(1/2)/
(a+b)^(1/2))/b^2/f/(a+b)^(1/2)+1/2*sec(f*x+e)*tan(f*x+e)/b/f
```

3.180.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.77 (sec) , antiderivative size = 1195, normalized size of antiderivative = 13.90

$$\int \frac{\sec^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(4a\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-2b\log\left(\cos\left(\frac{1}{2}(e-\right.\right.\right.}{}$$

input
```
Integrate[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(4*a*Log[Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2]] - 2*b*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - 4*a*Lo
g[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + 2*b*Log[Cos[(e + f*x)/2] + Sin[(e
 + f*x)/2]] + (a^(3/2)*Cos[e]*Log[a + 2*(a + b)*Cos[2*e] - a*Cos[2*(e + f*
x)] - (2*I)*a*Sin[2*e] - (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Co
s[e] - I*Sin[e])^2]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[
e])^2]*Sin[2*e + f*x]])/(Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]) - (a^(3/
2)*Cos[e]*Log[-a - 2*(a + b)*Cos[2*e] + a*Cos[2*(e + f*x)] + (2*I)*a*Sin[2
*e] + (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]
*Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[2*e + f*
x]])/(Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]) + ((2*I)*a^(3/2)*ArcTan[(2*
Sin[e]*(I*a + I*b + I*(a + b)*Cos[2*e] + Sqrt[a]*Sqrt[a + b]*Cos[f*x]*Sqrt
[(Cos[e] - I*Sin[e])^2] - Sqrt[a]*Sqrt[a + b]*Cos[2*e + f*x]*Sqrt[(Cos[e]
 - I*Sin[e])^2] + a*Sin[2*e] + b*Sin[2*e] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos
[e] - I*Sin[e])^2]*Sin[f*x] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e
])^2]*Sin[2*e + f*x]))/(I*(a + 3*b)*Cos[e] + I*(a + b)*Cos[3*e] + I*a*Cos[
e + 2*f*x] + I*a*Cos[3*e + 2*f*x] + 3*a*Sin[e] + b*Sin[e] + a*Sin[3*e] + b
*Sin[3*e] + a*Sin[e + 2*f*x] - a*Sin[3*e + 2*f*x])]*Sqrt[(Cos[e] - I*Sin[e
])^2]*(Cos[e] + I*Sin[e]))/Sqrt[a + b] - (I*a^(3/2)*Log[a + 2*(a + b)*Cos[
2*e] - a*Cos[2*(e + f*x)] - (2*I)*a*Sin[2*e] - (2*I)*b*Sin[2*e] + 2*Sqr...
```

3.180.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4635, 316, 25, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\sec(e+fx)^5}{a+b\sec(e+fx)^2}\,dx$$
$$\downarrow 4635$$

$$\frac{\int \frac{1}{\left(1-\sin^2(e+fx)\right)^2\left(-a\sin^2(e+fx)+a+b\right)}d\sin(e+fx)}{f}$$

$$\downarrow 316$$

$$\frac{\int -\frac{a\sin^2(e+fx)+a-b}{\left(1-\sin^2(e+fx)\right)\left(-a\sin^2(e+fx)+a+b\right)}d\sin(e+fx)}{2b}+\frac{\sin(e+fx)}{2b\left(1-\sin^2(e+fx)\right)}}{f}$$

$$\downarrow 25$$

$$\frac{\frac{\sin(e+fx)}{2b\left(1-\sin^2(e+fx)\right)}-\frac{\int \frac{a\sin^2(e+fx)+a-b}{\left(1-\sin^2(e+fx)\right)\left(-a\sin^2(e+fx)+a+b\right)}d\sin(e+fx)}{2b}}{f}$$

$$\downarrow 397$$

$$\frac{\frac{\sin(e+fx)}{2b\left(1-\sin^2(e+fx)\right)}-\frac{(2a-b)\int \frac{1}{1-\sin^2(e+fx)}d\sin(e+fx)}{b}-\frac{2a^2\int \frac{1}{-a\sin^2(e+fx)+a+b}d\sin(e+fx)}{b}}{2b}}{f}$$

$$\downarrow 219$$

$$\frac{\frac{\sin(e+fx)}{2b\left(1-\sin^2(e+fx)\right)}-\frac{(2a-b)\operatorname{arctanh}(\sin(e+fx))}{b}-\frac{2a^2\int \frac{1}{-a\sin^2(e+fx)+a+b}d\sin(e+fx)}{b}}{2b}}{f}$$

$$\downarrow 221$$

$$\frac{\frac{\sin(e+fx)}{2b\left(1-\sin^2(e+fx)\right)}-\frac{(2a-b)\operatorname{arctanh}(\sin(e+fx))}{b}-\frac{2a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{b\sqrt{a+b}}}{2b}}{f}$$

input
```
Int[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

output
```
(-1/2*(((2*a - b)*ArcTanh[Sin[e + f*x]])/b - (2*a^(3/2)*ArcTanh[(Sqrt[a]*S
in[e + f*x])/Sqrt[a + b]])/(b*Sqrt[a + b]))/b + Sin[e + f*x]/(2*b*(1 - Sin
[e + f*x]^2)))/f
```

3.180.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.180.4 Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.23

method	result
derivativedivides	$\frac{\frac{a^2 \operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{b^2\sqrt{a(a+b)}} - \frac{1}{4b(\sin(fx+e)+1)} + \frac{(-2a+b)\ln(\sin(fx+e)+1)}{4b^2} - \frac{1}{4b(\sin(fx+e)-1)} + \frac{(2a-b)\ln(\sin(fx+e)-1)}{4b^2}}{f}$
default	$\frac{\frac{a^2 \operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{b^2\sqrt{a(a+b)}} - \frac{1}{4b(\sin(fx+e)+1)} + \frac{(-2a+b)\ln(\sin(fx+e)+1)}{4b^2} - \frac{1}{4b(\sin(fx+e)-1)} + \frac{(2a-b)\ln(\sin(fx+e)-1)}{4b^2}}{f}$
risch	$-\frac{i\left(e^{3i(fx+e)}-e^{i(fx+e)}\right)}{fb\left(e^{2i(fx+e)}+1\right)^2} + \frac{\ln\left(e^{i(fx+e)}-i\right)a}{b^2 f} - \frac{\ln\left(e^{i(fx+e)}-i\right)}{2bf} - \frac{\ln\left(e^{i(fx+e)}+i\right)a}{b^2 f} + \frac{\ln\left(e^{i(fx+e)}+i\right)}{2bf} + \frac{\sqrt{a(a+}$

input
```
int(sec(f*x+e)^5/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(a^2/b^2/(a*(a+b))^(1/2)*arctanh(a*sin(f*x+e)/(a*(a+b))^(1/2))-1/4/b/(
sin(f*x+e)+1)+1/4/b^2*(-2*a+b)*ln(sin(f*x+e)+1)-1/4/b/(sin(f*x+e)-1)+1/4*(
2*a-b)/b^2*ln(sin(f*x+e)-1))
```

3.180.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 272, normalized size of antiderivative = 3.16

$$\int \frac{\sec^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$= \left[\frac{2\,a\sqrt{\frac{a}{a+b}}\cos(fx+e)^2\log\left(-\frac{a\cos(fx+e)^2-2\,(a+b)\sqrt{\frac{a}{a+b}}\sin(fx+e)-2\,a-b}{a\cos(fx+e)^2+b}\right)-(2\,a-b)\cos(fx+e)^2\log(\sin(fx\right.$$

$$\left.\frac{}{4\,b^2 f\cos(fx+e)^2}\right.$$

$$-\frac{4\,a\sqrt{-\frac{a}{a+b}}\arctan\left(\sqrt{-\frac{a}{a+b}}\sin(fx+e)\right)\cos(fx+e)^2+(2\,a-b)\cos(fx+e)^2\log(\sin(fx+e)+1)-}{4\,b^2 f\cos(fx+e)^2}$$

input
```
integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/4*(2*a*sqrt(a/(a + b))*cos(f*x + e)^2*log(-(a*cos(f*x + e)^2 - 2*(a + b
)*sqrt(a/(a + b))*sin(f*x + e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) - (2*a -
 b)*cos(f*x + e)^2*log(sin(f*x + e) + 1) + (2*a - b)*cos(f*x + e)^2*log(-s
in(f*x + e) + 1) + 2*b*sin(f*x + e))/(b^2*f*cos(f*x + e)^2), -1/4*(4*a*sqr
t(-a/(a + b))*arctan(sqrt(-a/(a + b))*sin(f*x + e))*cos(f*x + e)^2 + (2*a
- b)*cos(f*x + e)^2*log(sin(f*x + e) + 1) - (2*a - b)*cos(f*x + e)^2*log(-
sin(f*x + e) + 1) - 2*b*sin(f*x + e))/(b^2*f*cos(f*x + e)^2)]
```

3.180.6 Sympy [F]

$$\int \frac{\sec^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\sec^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

input
```
integrate(sec(f*x+e)**5/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(sec(e + f*x)**5/(a + b*sec(e + f*x)**2), x)
```

3.180.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.44

$$\int \frac{\sec^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{2a^2\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)ab^2}} + \frac{(2a-b)\log(\sin(fx+e)+1)}{b^2} - \frac{(2a-b)\log(\sin(fx+e)-1)}{b^2} + \frac{2\sin(fx+e)}{b\sin(fx+e)^2-b}}{4f}$$

input
```
integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
-1/4*(2*a^2*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x + e) + sqrt(
(a + b)*a)))/(sqrt((a + b)*a)*b^2) + (2*a - b)*log(sin(f*x + e) + 1)/b^2 -
 (2*a - b)*log(sin(f*x + e) - 1)/b^2 + 2*sin(f*x + e)/(b*sin(f*x + e)^2 -
b))/f
```

3.180.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.31

$$\int \frac{\sec^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$= -\frac{\frac{4a^2\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{\sqrt{-a^2-ab}b^2} + \frac{(2a-b)\log(|\sin(fx+e)+1|)}{b^2} - \frac{(2a-b)\log(|\sin(fx+e)-1|)}{b^2} + \frac{2\sin(fx+e)}{\left(\sin(fx+e)^2-1\right)b}}{4f}$$

input
```
integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
-1/4*(4*a^2*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/(sqrt(-a^2 - a*b)*b^2)
 + (2*a - b)*log(abs(sin(f*x + e) + 1))/b^2 - (2*a - b)*log(abs(sin(f*x +
e) - 1))/b^2 + 2*sin(f*x + e)/((sin(f*x + e)^2 - 1)*b))/f
```

3.180.9 Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 591, normalized size of antiderivative = 6.87

$$\int \frac{\sec^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$= \frac{b\left(a\sin\left(e+f\,x\right) - a\,\mathrm{atanh}(\sin\left(e+f\,x\right)) + a\sin\left(e+f\,x\right)^2\,\mathrm{atanh}(\sin\left(e+f\,x\right))\right) + b^2\left(\sin\left(e+f\,x\right) + \mathrm{ata}\right.}{}$$

input
```
int(1/(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)),x)
```

output

```
(b*(a*sin(e + f*x) - a*atanh(sin(e + f*x)) + a*sin(e + f*x)^2*atanh(sin(e
+ f*x))) + atan((a^5*sin(e + f*x)*(a^3*b + a^4)^(1/2)*8i - b*sin(e + f*x)*
(a^3*b + a^4)^(3/2)*4i - a*sin(e + f*x)*(a^3*b + a^4)^(3/2)*8i - a^2*b^3*s
in(e + f*x)*(a^3*b + a^4)^(1/2)*2i + a^3*b^2*sin(e + f*x)*(a^3*b + a^4)^(1
/2)*1i + a*b^4*sin(e + f*x)*(a^3*b + a^4)^(1/2)*1i + a^4*b*sin(e + f*x)*(a
^3*b + a^4)^(1/2)*12i)/(a^3*b^4 - a^2*b^5 + 5*a^4*b^3 + 3*a^5*b^2))*(a^3*b
 + a^4)^(1/2)*2i + b^2*(sin(e + f*x) + atanh(sin(e + f*x)) - sin(e + f*x)^
2*atanh(sin(e + f*x))) - 2*a^2*atanh(sin(e + f*x)) - atan((a^5*sin(e + f*x
)*(a^3*b + a^4)^(1/2)*8i - b*sin(e + f*x)*(a^3*b + a^4)^(3/2)*4i - a*sin(e
 + f*x)*(a^3*b + a^4)^(3/2)*8i - a^2*b^3*sin(e + f*x)*(a^3*b + a^4)^(1/2)*
2i + a^3*b^2*sin(e + f*x)*(a^3*b + a^4)^(1/2)*1i + a*b^4*sin(e + f*x)*(a^3
*b + a^4)^(1/2)*1i + a^4*b*sin(e + f*x)*(a^3*b + a^4)^(1/2)*12i)/(a^3*b^4
- a^2*b^5 + 5*a^4*b^3 + 3*a^5*b^2))*sin(e + f*x)^2*(a^3*b + a^4)^(1/2)*2i
+ 2*a^2*sin(e + f*x)^2*atanh(sin(e + f*x)))/(f*(2*a*b^2 + 2*b^3 - 2*b^3*si
n(e + f*x)^2 - 2*a*b^2*sin(e + f*x)^2))
```

3.181 $\int \frac{\sec^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$

3.181.1 Optimal result 1375
3.181.2 Mathematica [A] (verified) 1375
3.181.3 Rubi [A] (verified) 1376
3.181.4 Maple [A] (verified) 1377
3.181.5 Fricas [A] (verification not implemented) 1378
3.181.6 Sympy [F] 1378
3.181.7 Maxima [A] (verification not implemented) 1379
3.181.8 Giac [A] (verification not implemented) 1379
3.181.9 Mupad [B] (verification not implemented) 1379

3.181.1 Optimal result

Integrand size = 23, antiderivative size = 55

$$\int \frac{\sec^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\operatorname{arctanh}(\sin(e+fx))}{bf} - \frac{\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{b\sqrt{a+b}f}$$

output
```
arctanh(sin(f*x+e))/b/f-arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))*a^(1/2)/b/
f/(a+b)^(1/2)
```

3.181.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

$$\int \frac{\sec^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\operatorname{arctanh}(\sin(e+fx)) - \frac{\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a+b}}}{bf}$$

input
```
Integrate[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

output
```
(ArcTanh[Sin[e + f*x]] - (Sqrt[a]*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a +
b]])/Sqrt[a + b])/(b*f)
```

3.181.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4635, 303, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sec(e+fx)^3}{a+b\sec(e+fx)^2}\,dx$$

$$\downarrow 4635$$

$$\frac{\int \frac{1}{(1-\sin^2(e+fx))(-a\sin^2(e+fx)+a+b)}\,d\sin(e+fx)}{f}$$

$$\downarrow 303$$

$$\frac{\frac{\int \frac{1}{1-\sin^2(e+fx)}\,d\sin(e+fx)}{b} - \frac{a\int \frac{1}{-a\sin^2(e+fx)+a+b}\,d\sin(e+fx)}{b}}{f}$$

$$\downarrow 219$$

$$\frac{\frac{\operatorname{arctanh}(\sin(e+fx))}{b} - \frac{a\int \frac{1}{-a\sin^2(e+fx)+a+b}\,d\sin(e+fx)}{b}}{f}$$

$$\downarrow 221$$

$$\frac{\frac{\operatorname{arctanh}(\sin(e+fx))}{b} - \frac{\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{b\sqrt{a+b}}}{f}$$

input
```
Int[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

output
```
(ArcTanh[Sin[e + f*x]]/b - (Sqrt[a]*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a
+ b]])/(b*Sqrt[a + b]))/f
```

3.181.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 303
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Simp[b/(b
*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x
^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.181.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.15

method	result
derivativedivides	$\frac{-\frac{a \operatorname{arctanh}\left(\frac{a \sin(fx+e)}{\sqrt{a(a+b)}}\right)}{b\sqrt{a(a+b)}} - \frac{\ln(\sin(fx+e)-1)}{2b} + \frac{\ln(\sin(fx+e)+1)}{2b}}{f}$
default	$\frac{-\frac{a \operatorname{arctanh}\left(\frac{a \sin(fx+e)}{\sqrt{a(a+b)}}\right)}{b\sqrt{a(a+b)}} - \frac{\ln(\sin(fx+e)-1)}{2b} + \frac{\ln(\sin(fx+e)+1)}{2b}}{f}$
risch	$\frac{\ln\left(e^{i(fx+e)}+i\right)}{bf} - \frac{\ln\left(e^{i(fx+e)}-i\right)}{bf} + \frac{\sqrt{a(a+b)}\,\ln\left(e^{2i(fx+e)} - \frac{2i\sqrt{a(a+b)}\,e^{i(fx+e)}}{a} - 1\right)}{2(a+b)fb} - \frac{\sqrt{a(a+b)}\,\ln\left(e^{2i(fx+e)}+\right.}{2(a+$

input
```
int(sec(f*x+e)^3/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-a/b/(a*(a+b))^(1/2)*arctanh(a*sin(f*x+e)/(a*(a+b))^(1/2))-1/2/b*ln(s
in(f*x+e)-1)+1/2/b*ln(sin(f*x+e)+1))
```

3.181.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.85

$$\int \frac{\sec^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \left[\frac{\sqrt{\frac{a}{a+b}}\log\left(-\frac{a\cos(fx+e)^2+2\,(a+b)\sqrt{\frac{a}{a+b}}\sin(fx+e)-2\,a-b}{a\cos(fx+e)^2+b}\right)+\log\left(\sin\left(fx+e\right)+1\right)-\log\left(-\sin\left(fx+e\right)+1\right)}{2\,bf}, -\right.$$

input
```
integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a/(a + b))*log(-(a*cos(f*x + e)^2 + 2*(a + b)*sqrt(a/(a + b))*s
in(f*x + e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) + log(sin(f*x + e) + 1) - l
og(-sin(f*x + e) + 1))/(b*f), 1/2*(2*sqrt(-a/(a + b))*arctan(sqrt(-a/(a +
b))*sin(f*x + e)) + log(sin(f*x + e) + 1) - log(-sin(f*x + e) + 1))/(b*f)]
```

3.181.6 Sympy [F]

$$\int \frac{\sec^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\sec^3\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

input
```
integrate(sec(f*x+e)**3/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(sec(e + f*x)**3/(a + b*sec(e + f*x)**2), x)
```

3.181.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.51

$$\int \frac{\sec^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{a\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)ab}} + \frac{\log(\sin(fx+e)+1)}{b} - \frac{\log(\sin(fx+e)-1)}{b}}{2f}$$

input
```
integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/2*(a*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x + e) + sqrt((a +
b)*a)))/(sqrt((a + b)*a)*b) + log(sin(f*x + e) + 1)/b - log(sin(f*x + e) -
 1)/b)/f
```

3.181.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.35

$$\int \frac{\sec^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{2a\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{\sqrt{-a^2-ab}b} + \frac{\log(|\sin(fx+e)+1|)}{b} - \frac{\log(|\sin(fx+e)-1|)}{b}}{2f}$$

input
```
integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/2*(2*a*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/(sqrt(-a^2 - a*b)*b) + lo
g(abs(sin(f*x + e) + 1))/b - log(abs(sin(f*x + e) - 1))/b)/f
```

3.181.9 Mupad [B] (verification not implemented)

Time = 18.74 (sec) , antiderivative size = 456, normalized size of antiderivative = 8.29

$$\int \frac{\sec^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\operatorname{atanh}(\sin(e+fx))}{bf}$$

$$+\frac{\operatorname{atan}\left(\frac{\left(2a^3\sin(e+fx)+\frac{\left(2a^2b^2-\frac{\sin(e+fx)\left(16a^3b^2+8a^2b^3\right)\sqrt{a(a+b)}}{4\left(b^2+ab\right)}\right)\sqrt{a(a+b)}}{2\left(b^2+ab\right)}\right)\sqrt{a(a+b)}\,1i}{b^2+ab} + \frac{\left(2a^3\sin(e+fx)-\frac{\left(2a^2b^2+\frac{\sin(e+fx)\left(16a^3\right.\right.}{4\left(\right.}\right.}{2\left(\right.}}{b^2+ab}}{\frac{\left(2a^3\sin(e+fx)+\frac{\left(2a^2b^2-\frac{\sin(e+fx)\left(16a^3b^2+8a^2b^3\right)\sqrt{a(a+b)}}{4\left(b^2+ab\right)}\right)\sqrt{a(a+b)}}{2\left(b^2+ab\right)}\right)\sqrt{a(a+b)}}{b^2+ab} - \frac{\left(2a^3\sin(e+fx)-\frac{\left(2a^2b^2+\frac{\sin(e+fx)\left(16a^3\right.\right.}{4\left(\right.}\right.}{2\left(\right.}}{b^2+}}$$

$$f\left(b^2+ab\right)$$

input
```
int(1/(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)),x)
```

output
```
atanh(sin(e + f*x))/(b*f) + (atan((((2*a^3*sin(e + f*x) + ((2*a^2*b^2 - (s
in(e + f*x)*(8*a^2*b^3 + 16*a^3*b^2)*(a*(a + b))^(1/2))/(4*(a*b + b^2)))*(
a*(a + b))^(1/2))/(2*(a*b + b^2)))*(a*(a + b))^(1/2)*1i)/(a*b + b^2) + ((2
*a^3*sin(e + f*x) - ((2*a^2*b^2 + (sin(e + f*x)*(8*a^2*b^3 + 16*a^3*b^2)*(
a*(a + b))^(1/2))/(4*(a*b + b^2)))*(a*(a + b))^(1/2))/(2*(a*b + b^2)))*(a*
(a + b))^(1/2)*1i)/(a*b + b^2))/(((2*a^3*sin(e + f*x) + ((2*a^2*b^2 - (sin
(e + f*x)*(8*a^2*b^3 + 16*a^3*b^2)*(a*(a + b))^(1/2))/(4*(a*b + b^2)))*(a*
(a + b))^(1/2))/(2*(a*b + b^2)))*(a*(a + b))^(1/2))/(a*b + b^2) - ((2*a^3*
sin(e + f*x) - ((2*a^2*b^2 + (sin(e + f*x)*(8*a^2*b^3 + 16*a^3*b^2)*(a*(a
+ b))^(1/2))/(4*(a*b + b^2)))*(a*(a + b))^(1/2))/(2*(a*b + b^2)))*(a*(a +
b))^(1/2))/(a*b + b^2)))*(a*(a + b))^(1/2)*1i)/(f*(a*b + b^2))
```

3.182 $\int \frac{\sec(e+fx)}{a+b\sec^2(e+fx)}\,dx$

3.182.1 Optimal result 1381
3.182.2 Mathematica [A] (verified) 1381
3.182.3 Rubi [A] (verified) 1382
3.182.4 Maple [A] (verified) 1383
3.182.5 Fricas [A] (verification not implemented) 1383
3.182.6 Sympy [F] 1384
3.182.7 Maxima [A] (verification not implemented) 1384
3.182.8 Giac [A] (verification not implemented) 1384
3.182.9 Mupad [B] (verification not implemented) 1385

3.182.1 Optimal result

Integrand size = 21, antiderivative size = 36

$$\int \frac{\sec(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a}\sqrt{a+b}f}$$

output
```
arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))/f/a^(1/2)/(a+b)^(1/2)
```

3.182.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{\sec(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a}\sqrt{a+b}f}$$

input
```
Integrate[Sec[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

output
```
ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]/(Sqrt[a]*Sqrt[a + b]*f)
```

3.182.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4635, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\sec(e+fx)}{a+b\sec(e+fx)^2}\,dx$$
$$\downarrow 4635$$
$$\frac{\int \frac{1}{-a\sin^2(e+fx)+a+b}\,d\sin(e+fx)}{f}$$
$$\downarrow 221$$
$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a}f\sqrt{a+b}}$$

input
```
Int[Sec[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

output
```
ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]/(Sqrt[a]*Sqrt[a + b]*f)
```

3.182.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.182.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{f\sqrt{a(a+b)}}$	28
default	$\frac{\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{f\sqrt{a(a+b)}}$	28
risch	$\frac{\ln\left(e^{2i(fx+e)}+\frac{2i(a+b)e^{i(fx+e)}}{\sqrt{a^2+ab}}-1\right)}{2\sqrt{a^2+ab}\,f}-\frac{\ln\left(e^{2i(fx+e)}-\frac{2i(a+b)e^{i(fx+e)}}{\sqrt{a^2+ab}}-1\right)}{2\sqrt{a^2+ab}\,f}$	102

input
```
int(sec(f*x+e)/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f/(a*(a+b))^(1/2)*arctanh(a*sin(f*x+e)/(a*(a+b))^(1/2))
```

3.182.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 117, normalized size of antiderivative = 3.25

$$\int \frac{\sec(e+fx)}{a+b\sec^2(e+fx)}\,dx = \left[\frac{\log\left(-\frac{a\cos(fx+e)^2-2\sqrt{a^2+ab}\sin(fx+e)-2a-b}{a\cos(fx+e)^2+b}\right)}{2\sqrt{a^2+ab}f},\right.$$
$$\left.-\frac{\sqrt{-a^2-ab}\arctan\left(\frac{\sqrt{-a^2-ab}\sin(fx+e)}{a+b}\right)}{(a^2+ab)f}\right]$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/2*log(-(a*cos(f*x + e)^2 - 2*sqrt(a^2 + a*b)*sin(f*x + e) - 2*a - b)/(a
*cos(f*x + e)^2 + b))/(sqrt(a^2 + a*b)*f), -sqrt(-a^2 - a*b)*arctan(sqrt(-
a^2 - a*b)*sin(f*x + e)/(a + b))/((a^2 + a*b)*f)]
```

3.182.6 Sympy [F]

$$\int \frac{\sec(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\sec(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(sec(e + f*x)/(a + b*sec(e + f*x)**2), x)
```

3.182.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.39

$$\int \frac{\sec(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{2\sqrt{(a+b)a}f}$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
-1/2*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x + e) + sqrt((a + b)
*a)))/(sqrt((a + b)*a)*f)
```

3.182.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\sec(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{\sqrt{-a^2-ab}f}$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
-arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/(sqrt(-a^2 - a*b)*f)
```

3.182.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \frac{\sec(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\operatorname{atanh}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a}\,f\,\sqrt{a+b}}$$

input
```
int(1/(cos(e + f*x)*(a + b/cos(e + f*x)^2)),x)
```

output
```
atanh((a^(1/2)*sin(e + f*x))/(a + b)^(1/2))/(a^(1/2)*f*(a + b)^(1/2))
```

3.183 $\int \frac{\cos(e+fx)}{a+b\sec^2(e+fx)}\,dx$

3.183.1 Optimal result 1386
3.183.2 Mathematica [A] (verified) 1386
3.183.3 Rubi [A] (verified) 1387
3.183.4 Maple [A] (verified) 1388
3.183.5 Fricas [A] (verification not implemented) 1389
3.183.6 Sympy [F] 1389
3.183.7 Maxima [A] (verification not implemented) 1389
3.183.8 Giac [A] (verification not implemented) 1390
3.183.9 Mupad [B] (verification not implemented) 1390

3.183.1 Optimal result

Integrand size = 21, antiderivative size = 52

$$\int \frac{\cos(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{a^{3/2}\sqrt{a+b}f} + \frac{\sin(e+fx)}{af}$$

output
```
sin(f*x+e)/a/f-b*arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))/a^(3/2)/f/(a+b)^(
1/2)
```

3.183.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \frac{\cos(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{-\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a+b}} + \sqrt{a}\sin(e+fx)}{a^{3/2}f}$$

input
```
Integrate[Cos[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

output
```
(-((b*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/Sqrt[a + b]) + Sqrt[a]*
Sin[e + f*x])/(a^(3/2)*f)
```

3.183.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4635, 299, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sec(e+fx)\left(a+b\sec(e+fx)^2\right)}dx$$

$$\downarrow 4635$$

$$\frac{\int \frac{1-\sin^2(e+fx)}{-a\sin^2(e+fx)+a+b}d\sin(e+fx)}{f}$$

$$\downarrow 299$$

$$\frac{\frac{\sin(e+fx)}{a} - \frac{b\int \frac{1}{-a\sin^2(e+fx)+a+b}d\sin(e+fx)}{a}}{f}$$

$$\downarrow 221$$

$$\frac{\frac{\sin(e+fx)}{a} - \frac{b\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{a^{3/2}\sqrt{a+b}}}{f}$$

input
```
Int[Cos[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

output
```
(-((b*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(a^(3/2)*Sqrt[a + b]))
+ Sin[e + f*x]/a)/f
```

3.183.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.183.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.87

method	result	size
derivativedivides	$\frac{\frac{\sin(fx+e)}{a}-\frac{b\,\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{a\sqrt{a(a+b)}}}{f}$	45
default	$\frac{\frac{\sin(fx+e)}{a}-\frac{b\,\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{a\sqrt{a(a+b)}}}{f}$	45
risch	$-\frac{i\mathrm{e}^{i(fx+e)}}{2af}+\frac{i\mathrm{e}^{-i(fx+e)}}{2af}+\frac{b\ln\left(\mathrm{e}^{2i(fx+e)}-\frac{2i(a+b)\mathrm{e}^{i(fx+e)}}{\sqrt{a^2+ab}}-1\right)}{2\sqrt{a^2+ab}\,fa}-\frac{b\ln\left(\mathrm{e}^{2i(fx+e)}+\frac{2i(a+b)\mathrm{e}^{i(fx+e)}}{\sqrt{a^2+ab}}-1\right)}{2\sqrt{a^2+ab}\,fa}$	146

input
```
int(cos(f*x+e)/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(sin(f*x+e)/a-b/a/(a*(a+b))^(1/2)*arctanh(a*sin(f*x+e)/(a*(a+b))^(1/2)
))
```

3.183.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 164, normalized size of antiderivative = 3.15

$$\int \frac{\cos(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \left[\frac{\sqrt{a^2+ab}\,b\log\left(-\frac{a\cos(fx+e)^2+2\sqrt{a^2+ab}\sin(fx+e)-2a-b}{a\cos(fx+e)^2+b}\right)+2\left(a^2+ab\right)\sin\left(fx+e\right)}{2\left(a^3+a^2b\right)f}, \frac{\sqrt{-a^2-ab}\,b\arctan\left(\frac{\sqrt{-a}\right.}{}\right.$$

input
```
integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a^2 + a*b)*b*log(-(a*cos(f*x + e)^2 + 2*sqrt(a^2 + a*b)*sin(f*x
 + e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) + 2*(a^2 + a*b)*sin(f*x + e))/((a
^3 + a^2*b)*f), (sqrt(-a^2 - a*b)*b*arctan(sqrt(-a^2 - a*b)*sin(f*x + e)/(
a + b)) + (a^2 + a*b)*sin(f*x + e))/((a^3 + a^2*b)*f)]
```

3.183.6 Sympy [F]

$$\int \frac{\cos(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\cos\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

input
```
integrate(cos(f*x+e)/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(cos(e + f*x)/(a + b*sec(e + f*x)**2), x)
```

3.183.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.29

$$\int \frac{\cos(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{b\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}a}+\frac{2\sin(fx+e)}{a}}{2f}$$

input
```
integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/2*(b*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x + e) + sqrt((a +
b)*a)))/(sqrt((a + b)*a)*a) + 2*sin(f*x + e)/a)/f
```

3.183.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

$$\int \frac{\cos(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{b\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{\sqrt{-a^2-ab}a} + \frac{\sin(fx+e)}{a}}{f}$$

input
```
integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
(b*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/(sqrt(-a^2 - a*b)*a) + sin(f*x
+ e)/a)/f
```

3.183.9 Mupad [B] (verification not implemented)

Time = 18.39 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int \frac{\cos(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\sin(e+fx)}{af} - \frac{b\,\mathrm{atanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{a^{3/2}f\sqrt{a+b}}$$

input
```
int(cos(e + f*x)/(a + b/cos(e + f*x)^2),x)
```

output
```
sin(e + f*x)/(a*f) - (b*atanh((a^(1/2)*sin(e + f*x))/(a + b)^(1/2)))/(a^(3
/2)*f*(a + b)^(1/2))
```

3.184 $\int \frac{\cos^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$

3.184.1 Optimal result 1391
3.184.2 Mathematica [A] (verified) 1391
3.184.3 Rubi [A] (verified) 1392
3.184.4 Maple [A] (verified) 1393
3.184.5 Fricas [A] (verification not implemented) 1394
3.184.6 Sympy [F(-1)] 1394
3.184.7 Maxima [A] (verification not implemented) 1395
3.184.8 Giac [A] (verification not implemented) 1395
3.184.9 Mupad [B] (verification not implemented) 1395

3.184.1 Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \frac{\cos^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{b^2\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{a^{5/2}\sqrt{a+b}f} + \frac{(a-b)\sin(e+fx)}{a^2 f} - \frac{\sin^3(e+fx)}{3af}$$

output
```
(a-b)*sin(f*x+e)/a^2/f-1/3*sin(f*x+e)^3/a/f+b^2*arctanh(sin(f*x+e)*a^(1/2)
/(a+b)^(1/2))/a^(5/2)/f/(a+b)^(1/2)
```

3.184.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.38

$$\int \frac{\cos^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{6b^2\left(-\log\left(\sqrt{a+b}-\sqrt{a}\sin(e+fx)\right)+\log\left(\sqrt{a+b}+\sqrt{a}\sin(e+fx)\right)\right)}{\sqrt{a+b}} + 3\sqrt{a}(3a-4b)\sin(e+fx) + a^{3/2}\sin(3(e+fx))}{12a^{5/2}f}$$

input
```
Integrate[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

output
```
((6*b^2*(-Log[Sqrt[a + b] - Sqrt[a]*Sin[e + f*x]] + Log[Sqrt[a + b] + Sqrt
[a]*Sin[e + f*x]]))/Sqrt[a + b] + 3*Sqrt[a]*(3*a - 4*b)*Sin[e + f*x] + a^(
3/2)*Sin[3*(e + f*x)])/(12*a^(5/2)*f)
```

3.184.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4635, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sec(e+fx)^3\left(a+b\sec(e+fx)^2\right)}dx$$

$$\downarrow 4635$$

$$\frac{\int \frac{\left(1-\sin^2(e+fx)\right)^2}{-a\sin^2(e+fx)+a+b}d\sin(e+fx)}{f}$$

$$\downarrow 300$$

$$\frac{\int \left(\frac{b^2}{a^2\left(-a\sin^2(e+fx)+a+b\right)}-\frac{\sin^2(e+fx)}{a}+\frac{a-b}{a^2}\right)d\sin(e+fx)}{f}$$

$$\downarrow 2009$$

$$\frac{\frac{b^2\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{a^{5/2}\sqrt{a+b}}+\frac{(a-b)\sin(e+fx)}{a^2}-\frac{\sin^3(e+fx)}{3a}}{f}$$

input
```
Int[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

output
```
((b^2*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(a^(5/2)*Sqrt[a + b]) +
 ((a - b)*Sin[e + f*x])/a^2 - Sin[e + f*x]^3/(3*a))/f
```

3.184.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.184.4 Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{-\frac{\frac{a\sin(fx+e)^3}{3}-\sin(fx+e)a+\sin(fx+e)b}{a^2}+\frac{b^2\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{a^2\sqrt{a(a+b)}}}{f}$
default	$\frac{-\frac{\frac{a\sin(fx+e)^3}{3}-\sin(fx+e)a+\sin(fx+e)b}{a^2}+\frac{b^2\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{a^2\sqrt{a(a+b)}}}{f}$
risch	$-\frac{3ie^{i(fx+e)}}{8af}+\frac{ie^{i(fx+e)}b}{2a^2f}+\frac{3ie^{-i(fx+e)}}{8af}-\frac{ie^{-i(fx+e)}b}{2a^2f}+\frac{b^2\ln\left(e^{2i(fx+e)}+\frac{2i(a+b)e^{i(fx+e)}}{\sqrt{a^2+ab}}-1\right)}{2\sqrt{a^2+ab}\,f\,a^2}-\frac{b^2\ln\left(e^{2i(}\right.}$

input
```
int(cos(f*x+e)^3/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/a^2*(1/3*a*sin(f*x+e)^3-sin(f*x+e)*a+sin(f*x+e)*b)+b^2/a^2/(a*(a+b
))^(1/2)*arctanh(a*sin(f*x+e)/(a*(a+b))^(1/2)))
```

3.184.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 230, normalized size of antiderivative = 3.03

$$\int \frac{\cos^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \left[\frac{3\sqrt{a^2+ab}b^2\log\left(-\frac{a\cos(fx+e)^2-2\sqrt{a^2+ab}\sin(fx+e)-2a-b}{a\cos(fx+e)^2+b}\right)+2\left(2a^3-a^2b-3ab^2+\left(a^3+a^2b\right)\cos\left(fx+e\right)^2\right.}{6\left(a^4+a^3b\right)f}\right.$$
$$\left.-\frac{3\sqrt{-a^2-ab}b^2\arctan\left(\frac{\sqrt{-a^2-ab}\sin(fx+e)}{a+b}\right)-\left(2a^3-a^2b-3ab^2+\left(a^3+a^2b\right)\cos\left(fx+e\right)^2\right)\sin\left(fx+e\right)}{3\left(a^4+a^3b\right)f}\right]$$

input
```
integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(a^2 + a*b)*b^2*log(-(a*cos(f*x + e)^2 - 2*sqrt(a^2 + a*b)*sin
(f*x + e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) + 2*(2*a^3 - a^2*b - 3*a*b^2
+ (a^3 + a^2*b)*cos(f*x + e)^2)*sin(f*x + e))/((a^4 + a^3*b)*f), -1/3*(3*s
qrt(-a^2 - a*b)*b^2*arctan(sqrt(-a^2 - a*b)*sin(f*x + e)/(a + b)) - (2*a^3
 - a^2*b - 3*a*b^2 + (a^3 + a^2*b)*cos(f*x + e)^2)*sin(f*x + e))/((a^4 + a
^3*b)*f)]
```

3.184.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**3/(a+b*sec(f*x+e)**2),x)
```

output
```
Timed out
```

3.184.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.16

$$\int \frac{\cos^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{3b^2\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}a^2}+\frac{2\left(a\sin(fx+e)^3-3(a-b)\sin(fx+e)\right)}{a^2}}{6f}$$

input
```
integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
-1/6*(3*b^2*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x + e) + sqrt(
(a + b)*a)))/(sqrt((a + b)*a)*a^2) + 2*(a*sin(f*x + e)^3 - 3*(a - b)*sin(f
*x + e))/a^2)/f
```

3.184.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.12

$$\int \frac{\cos^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{3b^2\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{\sqrt{-a^2-ab}a^2}+\frac{a^2\sin(fx+e)^3-3a^2\sin(fx+e)+3ab\sin(fx+e)}{a^3}}{3f}$$

input
```
integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
-1/3*(3*b^2*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/(sqrt(-a^2 - a*b)*a^2)
 + (a^2*sin(f*x + e)^3 - 3*a^2*sin(f*x + e) + 3*a*b*sin(f*x + e))/a^3)/f
```

3.184.9 Mupad [B] (verification not implemented)

Time = 18.60 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.95

$$\int \frac{\cos^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{b^2\,\mathrm{atanh}\left(\frac{\sqrt{a}\,\sin(e+f\,x)}{\sqrt{a+b}}\right)}{a^{5/2}\,f\,\sqrt{a+b}}-\frac{\sin(e+f\,x)^3}{3\,a\,f}-\frac{\sin(e+f\,x)\,\left(\frac{a+b}{a^2}-\frac{2}{a}\right)}{f}$$

input
```
int(cos(e + f*x)^3/(a + b/cos(e + f*x)^2),x)
```

output
```
(b^2*atanh((a^(1/2)*sin(e + f*x))/(a + b)^(1/2)))/(a^(5/2)*f*(a + b)^(1/2)
) - sin(e + f*x)^3/(3*a*f) - (sin(e + f*x)*((a + b)/a^2 - 2/a))/f
```

3.185 $\int \frac{\cos^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$

3.185.1 Optimal result 1396
3.185.2 Mathematica [A] (verified) 1396
3.185.3 Rubi [A] (verified) 1397
3.185.4 Maple [A] (verified) 1398
3.185.5 Fricas [A] (verification not implemented) 1399
3.185.6 Sympy [F(-1)] 1399
3.185.7 Maxima [A] (verification not implemented) 1400
3.185.8 Giac [A] (verification not implemented) 1400
3.185.9 Mupad [B] (verification not implemented) 1401

3.185.1 Optimal result

Integrand size = 23, antiderivative size = 108

$$\int \frac{\cos^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{b^3\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{a^{7/2}\sqrt{a+b}f} + \frac{(a^2-ab+b^2)\sin(e+fx)}{a^3 f} - \frac{(2a-b)\sin^3(e+fx)}{3a^2 f} + \frac{\sin^5(e+fx)}{5af}$$

output
```
(a^2-a*b+b^2)*sin(f*x+e)/a^3/f-1/3*(2*a-b)*sin(f*x+e)^3/a^2/f+1/5*sin(f*x+
e)^5/a/f-b^3*arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))/a^(7/2)/f/(a+b)^(1/2)
```

3.185.2 Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.26

$$\int \frac{\cos^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{120b^3\left(\log\left(\sqrt{a+b}-\sqrt{a}\sin(e+fx)\right)-\log\left(\sqrt{a+b}+\sqrt{a}\sin(e+fx)\right)\right)}{\sqrt{a+b}} + 30\sqrt{a}(5a^2-6ab+8b^2)\sin(e+fx) + 5a^{3/2}(5a-4b)\sin}{240a^{7/2}f}$$

input
```
Integrate[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

output
```
((120*b^3*(Log[Sqrt[a + b] - Sqrt[a]*Sin[e + f*x]] - Log[Sqrt[a + b] + Sqr
t[a]*Sin[e + f*x]]))/Sqrt[a + b] + 30*Sqrt[a]*(5*a^2 - 6*a*b + 8*b^2)*Sin[
e + f*x] + 5*a^(3/2)*(5*a - 4*b)*Sin[3*(e + f*x)] + 3*a^(5/2)*Sin[5*(e + f
*x)])/(240*a^(7/2)*f)
```

3.185.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4635, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sec(e+fx)^5\left(a+b\sec(e+fx)^2\right)}dx$$

$$\downarrow 4635$$

$$\frac{\int \frac{\left(1-\sin^2(e+fx)\right)^3}{-a\sin^2(e+fx)+a+b}d\sin(e+fx)}{f}$$

$$\downarrow 300$$

$$\frac{\int \left(\frac{\sin^4(e+fx)}{a}-\frac{(2a-b)\sin^2(e+fx)}{a^2}+\frac{a^2-ba+b^2}{a^3}-\frac{b^3}{a^3\left(-a\sin^2(e+fx)+a+b\right)}\right)d\sin(e+fx)}{f}$$

$$\downarrow 2009$$

$$\frac{-\frac{b^3\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{a^{7/2}\sqrt{a+b}}-\frac{(2a-b)\sin^3(e+fx)}{3a^2}+\frac{\left(a^2-ab+b^2\right)\sin(e+fx)}{a^3}+\frac{\sin^5(e+fx)}{5a}}{f}$$

input
```
Int[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

output
```
(-((b^3*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(a^(7/2)*Sqrt[a + b])
) + ((a^2 - a*b + b^2)*Sin[e + f*x])/a^3 - ((2*a - b)*Sin[e + f*x]^3)/(3*a
^2) + Sin[e + f*x]^5/(5*a))/f
```

3.185.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.185.4 Maple [A] (verified)

Time = 1.94 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{\frac{a^2\sin(fx+e)^5}{5}-\frac{2a^2\sin(fx+e)^3}{3}+\frac{a\sin(fx+e)^3b}{3}+\sin(fx+e)a^2-\sin(fx+e)ab+\sin(fx+e)b^2}{a^3}-\frac{b^3\arctanh\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{a^3\sqrt{a(a+b)}}}{f}$
default	$\frac{\frac{a^2\sin(fx+e)^5}{5}-\frac{2a^2\sin(fx+e)^3}{3}+\frac{a\sin(fx+e)^3b}{3}+\sin(fx+e)a^2-\sin(fx+e)ab+\sin(fx+e)b^2}{a^3}-\frac{b^3\arctanh\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{a^3\sqrt{a(a+b)}}}{f}$
risch	$-\frac{5ie^{i(fx+e)}}{16af}+\frac{3ie^{i(fx+e)}b}{8a^2f}-\frac{ie^{i(fx+e)}b^2}{2a^3f}+\frac{5ie^{-i(fx+e)}}{16af}-\frac{3ie^{-i(fx+e)}b}{8a^2f}+\frac{ie^{-i(fx+e)}b^2}{2a^3f}+\frac{b^3\ln\left(e^{2i(fx+e)}-\right.}{2\sqrt{a^2}}$

input
```
int(cos(f*x+e)^5/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/a^3*(1/5*a^2*sin(f*x+e)^5-2/3*a^2*sin(f*x+e)^3+1/3*a*sin(f*x+e)^3*b
+sin(f*x+e)*a^2-sin(f*x+e)*a*b+sin(f*x+e)*b^2)-b^3/a^3/(a*(a+b))^(1/2)*arc
tanh(a*sin(f*x+e)/(a*(a+b))^(1/2)))
```

3.185.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 305, normalized size of antiderivative = 2.82

$$\int \frac{\cos^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \left[\frac{15\sqrt{a^2+ab}b^3\log\left(-\frac{a\cos(fx+e)^2+2\sqrt{a^2+ab}\sin(fx+e)-2a-b}{a\cos(fx+e)^2+b}\right)+2\left(3\left(a^4+a^3b\right)\cos\left(fx+e\right)^4+8a^4-2a^3b+5\right.}{30\left(a^5+a^4b\right)f}\right.$$

input
```
integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/30*(15*sqrt(a^2 + a*b)*b^3*log(-(a*cos(f*x + e)^2 + 2*sqrt(a^2 + a*b)*s
in(f*x + e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) + 2*(3*(a^4 + a^3*b)*cos(f*
x + e)^4 + 8*a^4 - 2*a^3*b + 5*a^2*b^2 + 15*a*b^3 + (4*a^4 - a^3*b - 5*a^2
*b^2)*cos(f*x + e)^2)*sin(f*x + e))/((a^5 + a^4*b)*f), 1/15*(15*sqrt(-a^2
- a*b)*b^3*arctan(sqrt(-a^2 - a*b)*sin(f*x + e)/(a + b)) + (3*(a^4 + a^3*b
)*cos(f*x + e)^4 + 8*a^4 - 2*a^3*b + 5*a^2*b^2 + 15*a*b^3 + (4*a^4 - a^3*b
 - 5*a^2*b^2)*cos(f*x + e)^2)*sin(f*x + e))/((a^5 + a^4*b)*f)]
```

3.185.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**5/(a+b*sec(f*x+e)**2),x)
```

output
```
Timed out
```

3.185.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.08

$$\int \frac{\cos^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{\frac{15\,b^3\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}a^3} + \frac{2\left(3\,a^2\sin(fx+e)^5-5\,(2\,a^2-ab)\sin(fx+e)^3+15\,(a^2-ab+b^2)\sin(fx+e)\right)}{a^3}}{30\,f}$$

input
```
integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/30*(15*b^3*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x + e) + sqrt
((a + b)*a)))/(sqrt((a + b)*a)*a^3) + 2*(3*a^2*sin(f*x + e)^5 - 5*(2*a^2 -
 a*b)*sin(f*x + e)^3 + 15*(a^2 - a*b + b^2)*sin(f*x + e))/a^3)/f
```

3.185.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.19

$$\int \frac{\cos^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{\frac{15\,b^3\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{\sqrt{-a^2-ab}a^3} + \frac{3\,a^4\sin(fx+e)^5-10\,a^4\sin(fx+e)^3+5\,a^3b\sin(fx+e)^3+15\,a^4\sin(fx+e)-15\,a^3b\sin(fx+e)+15\,a^2b^2\sin(fx+e)}{a^5}}{15\,f}$$

input
```
integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/15*(15*b^3*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/(sqrt(-a^2 - a*b)*a^3
) + (3*a^4*sin(f*x + e)^5 - 10*a^4*sin(f*x + e)^3 + 5*a^3*b*sin(f*x + e)^3
 + 15*a^4*sin(f*x + e) - 15*a^3*b*sin(f*x + e) + 15*a^2*b^2*sin(f*x + e))/
a^5)/f
```

3.185.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.03

$$\int \frac{\cos^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\sin(e+fx)\left(\frac{3}{a}+\frac{(a+b)\left(\frac{a+b}{a^2}-\frac{3}{a}\right)}{a}\right)}{f} + \frac{\sin(e+fx)^5}{5af} + \frac{\sin(e+fx)^3\left(\frac{a+b}{3a^2}-\frac{1}{a}\right)}{f} - \frac{b^3\,\mathrm{atanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{a^{7/2}f\sqrt{a+b}}$$

input
```
int(cos(e + f*x)^5/(a + b/cos(e + f*x)^2),x)
```

output
```
(sin(e + f*x)*(3/a + ((a + b)*((a + b)/a^2 - 3/a))/a))/f + sin(e + f*x)^5/
(5*a*f) + (sin(e + f*x)^3*((a + b)/(3*a^2) - 1/a))/f - (b^3*atanh((a^(1/2)
*sin(e + f*x))/(a + b)^(1/2)))/(a^(7/2)*f*(a + b)^(1/2))
```

3.186 $\int \frac{\sec^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$

3.186.1 Optimal result 1402
3.186.2 Mathematica [C] (verified) 1402
3.186.3 Rubi [A] (verified) 1403
3.186.4 Maple [A] (verified) 1404
3.186.5 Fricas [B] (verification not implemented) 1405
3.186.6 Sympy [F] 1405
3.186.7 Maxima [A] (verification not implemented) 1406
3.186.8 Giac [A] (verification not implemented) 1406
3.186.9 Mupad [B] (verification not implemented) 1406

3.186.1 Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \frac{\sec^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{a^2\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{b^{5/2}\sqrt{a+b}f} - \frac{(a-b)\tan(e+fx)}{b^2 f} + \frac{\tan^3(e+fx)}{3bf}$$

output
```
a^2*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/b^(5/2)/f/(a+b)^(1/2)-(a-b)*tan
(f*x+e)/b^2/f+1/3*tan(f*x+e)^3/b/f
```

3.186.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.79 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.91

$$\int \frac{\sec^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(-3a^2\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(c}{6b^2\sqrt{a+b}f\,(a}$$

input
```
Integrate[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(-3*a^2*ArcTan[(Sec[f*x]*(C
os[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[
a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]) + Sqrt[a +
b]*Sec[e + f*x]*Sqrt[b*(I*Cos[e] + Sin[e])^4]*(Sec[e]*(-3*a + 2*b + b*Sec[
e + f*x]^2)*Sin[f*x] + b*Sec[e + f*x]*Tan[e])))/(6*b^2*Sqrt[a + b]*f*(a +
b*Sec[e + f*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

3.186.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4634, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sec(e+fx)^6}{a+b\sec(e+fx)^2}\,dx$$

$$\downarrow 4634$$

$$\frac{\int \frac{\left(\tan^2(e+fx)+1\right)^2}{b\tan^2(e+fx)+a+b}\,d\tan(e+fx)}{f}$$

$$\downarrow 300$$

$$\frac{\int \left(\frac{a^2}{b^2(b\tan^2(e+fx)+a+b)}+\frac{\tan^2(e+fx)}{b}-\frac{a-b}{b^2}\right)d\tan(e+fx)}{f}$$

$$\downarrow 2009$$

$$\frac{\frac{a^2\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{b^{5/2}\sqrt{a+b}}-\frac{(a-b)\tan(e+fx)}{b^2}+\frac{\tan^3(e+fx)}{3b}}{f}$$

input
```
Int[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

output
```
((a^2*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(b^(5/2)*Sqrt[a + b]) -
((a - b)*Tan[e + f*x])/b^2 + Tan[e + f*x]^3/(3*b))/f
```

3.186.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.186.4 Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{-\frac{-\frac{b\tan(fx+e)^3}{3}+a\tan(fx+e)-b\tan(fx+e)}{b^2}+\frac{a^2\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^2\sqrt{(a+b)b}}}{f}$
default	$\frac{-\frac{-\frac{b\tan(fx+e)^3}{3}+a\tan(fx+e)-b\tan(fx+e)}{b^2}+\frac{a^2\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^2\sqrt{(a+b)b}}}{f}$
risch	$-\frac{2i\left(3a\,e^{4i(fx+e)}+6a\,e^{2i(fx+e)}-6b\,e^{2i(fx+e)}+3a-2b\right)}{3f\,b^2\left(e^{2i(fx+e)}+1\right)^3}-\frac{a^2\ln\left(e^{2i(fx+e)}+\frac{2iba+2ib^2+a\sqrt{-ab-b^2}+2b\sqrt{-ab-b^2}}{a\sqrt{-ab-b^2}}\right)}{2\sqrt{-ab-b^2}\,f\,b^2}+$

input
```
int(sec(f*x+e)^6/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/b^2*(-1/3*b*tan(f*x+e)^3+a*tan(f*x+e)-b*tan(f*x+e))+a^2/b^2/((a+b)
*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))
```

3.186.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(67) = 134$.

Time = 0.28 (sec) , antiderivative size = 354, normalized size of antiderivative = 4.60

$$\int \frac{\sec^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$= \left[-\frac{3\sqrt{-ab-b^2}a^2\cos(fx+e)^3\log\left(\frac{(a^2+8ab+8b^2)\cos(fx+e)^4-2(3ab+4b^2)\cos(fx+e)^2+4\left((a+2b)\cos(fx+e)^3-b\cos(fx+e)\right.}{a^2\cos(fx+e)^4+2ab\cos(fx+e)^2+b^2}\right.}{12(ab^3+b^4)f\cos(fx+e)^3} \right.$$

$$\left. -\frac{3\sqrt{ab+b^2}a^2\arctan\left(\frac{(a+2b)\cos(fx+e)^2-b}{2\sqrt{ab+b^2}\cos(fx+e)\sin(fx+e)}\right)\cos(fx+e)^3-2\left(ab^2+b^3-(3a^2b+ab^2-2b^3)\cos(f\right.}{6(ab^3+b^4)f\cos(fx+e)^3}$$

input
```
integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[-1/12*(3*sqrt(-a*b - b^2)*a^2*cos(f*x + e)^3*log(((a^2 + 8*a*b + 8*b^2)*c
os(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a + 2*b)*cos(f*x +
e)^3 - b*cos(f*x + e))*sqrt(-a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x +
 e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) - 4*(a*b^2 + b^3 - (3*a^2*b + a*b^2 -
 2*b^3)*cos(f*x + e)^2)*sin(f*x + e))/((a*b^3 + b^4)*f*cos(f*x + e)^3), -1
/6*(3*sqrt(a*b + b^2)*a^2*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(
a*b + b^2)*cos(f*x + e)*sin(f*x + e)))*cos(f*x + e)^3 - 2*(a*b^2 + b^3 - (
3*a^2*b + a*b^2 - 2*b^3)*cos(f*x + e)^2)*sin(f*x + e))/((a*b^3 + b^4)*f*co
s(f*x + e)^3)]
```

3.186.6 Sympy [F]

$$\int \frac{\sec^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\sec^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

input
```
integrate(sec(f*x+e)**6/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(sec(e + f*x)**6/(a + b*sec(e + f*x)**2), x)
```

3.186.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.84

$$\int \frac{\sec^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{3\,a^2\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}b^2} + \frac{b\tan(fx+e)^3-3\,(a-b)\tan(fx+e)}{b^2}}{3\,f}$$

input
```
integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/3*(3*a^2*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*b^2) +
(b*tan(f*x + e)^3 - 3*(a - b)*tan(f*x + e))/b^2)/f
```

3.186.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.25

$$\int \frac{\sec^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{\frac{3\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)a^2}{\sqrt{ab+b^2}b^2} + \frac{b^2\tan(fx+e)^3-3\,ab\tan(fx+e)+3\,b^2\tan(fx+e)}{b^3}}{3\,f}$$

input
```
integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/3*(3*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a
*b + b^2)))*a^2/(sqrt(a*b + b^2)*b^2) + (b^2*tan(f*x + e)^3 - 3*a*b*tan(f*
x + e) + 3*b^2*tan(f*x + e))/b^3)/f
```

3.186.9 Mupad [B] (verification not implemented)

Time = 18.55 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.94

$$\int \frac{\sec^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\tan(e+f\,x)^3}{3\,b\,f} - \frac{\tan(e+f\,x)\,\left(\frac{a+b}{b^2}-\frac{2}{b}\right)}{f} + \frac{a^2\,\operatorname{atan}\left(\frac{\sqrt{b}\tan(e+f\,x)}{\sqrt{a+b}}\right)}{b^{5/2}\,f\,\sqrt{a+b}}$$

input
```
int(1/(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)),x)
```

output
```
tan(e + f*x)^3/(3*b*f) - (tan(e + f*x)*((a + b)/b^2 - 2/b))/f + (a^2*atan(
(b^(1/2)*tan(e + f*x))/(a + b)^(1/2)))/(b^(5/2)*f*(a + b)^(1/2))
```

3.187 $\int \frac{\sec^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$

3.187.1 Optimal result 1408
3.187.2 Mathematica [C] (verified) 1408
3.187.3 Rubi [A] (verified) 1409
3.187.4 Maple [A] (verified) 1410
3.187.5 Fricas [B] (verification not implemented) 1411
3.187.6 Sympy [F] 1411
3.187.7 Maxima [A] (verification not implemented) 1412
3.187.8 Giac [A] (verification not implemented) 1412
3.187.9 Mupad [B] (verification not implemented) 1412

3.187.1 Optimal result

Integrand size = 23, antiderivative size = 52

$$\int \frac{\sec^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{a\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{b^{3/2}\sqrt{a+b}f} + \frac{\tan(e+fx)}{bf}$$

output
```
-a*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/b^(3/2)/f/(a+b)^(1/2)+tan(f*x+e)
/b/f
```

3.187.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.65 (sec) , antiderivative size = 192, normalized size of antiderivative = 3.69

$$\int \frac{\sec^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(a\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(\cos(2$$
$$2b\sqrt{a+b}f\,(a+b\sec^2(e+fx))\,\sqrt{b(\cos(e)$$

input
```
Integrate[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(a*ArcTan[(Sec[f*x]*(Cos[2*
e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b
]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]) + Sqrt[a + b]*Se
c[e]*Sec[e + f*x]*Sqrt[b*(I*Cos[e] + Sin[e])^4]*Sin[f*x]))/(2*b*Sqrt[a + b
]*f*(a + b*Sec[e + f*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

3.187.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4634, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\sec(e+fx)^4}{a+b\sec(e+fx)^2}\,dx$$
$$\downarrow 4634$$
$$\frac{\int \frac{\tan^2(e+fx)+1}{b\tan^2(e+fx)+a+b}\,d\tan(e+fx)}{f}$$
$$\downarrow 299$$
$$\frac{\frac{\tan(e+fx)}{b} - \frac{a\int \frac{1}{b\tan^2(e+fx)+a+b}\,d\tan(e+fx)}{b}}{f}$$
$$\downarrow 218$$
$$\frac{\frac{\tan(e+fx)}{b} - \frac{a\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{b^{3/2}\sqrt{a+b}}}{f}$$

input
```
Int[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

output
```
(-((a*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(b^(3/2)*Sqrt[a + b])) +
 Tan[e + f*x]/b)/f
```

3.187.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.187.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.87

method	result
derivativedivides	$\frac{\frac{\tan(fx+e)}{b}-\frac{a\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b\sqrt{(a+b)b}}}{f}$
default	$\frac{\frac{\tan(fx+e)}{b}-\frac{a\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b\sqrt{(a+b)b}}}{f}$
risch	$\frac{2i}{fb\left(e^{2i(fx+e)}+1\right)}-\frac{a\ln\left(e^{2i(fx+e)}+\frac{-2iba-2ib^2+a\sqrt{-ab-b^2}+2b\sqrt{-ab-b^2}}{a\sqrt{-ab-b^2}}\right)}{2\sqrt{-ab-b^2}\,fb}+\frac{a\ln\left(e^{2i(fx+e)}+\frac{2iba+2ib^2+a\sqrt{-ab-b^2}}{a\sqrt{-ab-b^2}}\right.}{2\sqrt{-ab-b^2}\,fb}$

input
```
int(sec(f*x+e)^4/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(tan(f*x+e)/b-a/b/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))
)
```

3.187.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(44) = 88$.

Time = 0.28 (sec) , antiderivative size = 286, normalized size of antiderivative = 5.50

$$\int \frac{\sec^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$= \left[-\frac{\sqrt{-ab-b^2}a\cos(fx+e)\log\left(\frac{(a^2+8ab+8b^2)\cos(fx+e)^4-2(3ab+4b^2)\cos(fx+e)^2-4\left((a+2b)\cos(fx+e)^3-b\cos(fx+e)\right)\sqrt{-}}{a^2\cos(fx+e)^4+2ab\cos(fx+e)^2+b^2}\right.}{4(ab^2+b^3)f\cos(fx+e)}\right.$$

input
```
integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[-1/4*(sqrt(-a*b - b^2)*a*cos(f*x + e)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x
+ e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a + 2*b)*cos(f*x + e)^3 -
b*cos(f*x + e))*sqrt(-a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 +
 2*a*b*cos(f*x + e)^2 + b^2)) - 4*(a*b + b^2)*sin(f*x + e))/((a*b^2 + b^3)
*f*cos(f*x + e)), 1/2*(sqrt(a*b + b^2)*a*arctan(1/2*((a + 2*b)*cos(f*x + e
)^2 - b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e)))*cos(f*x + e) + 2*(a*
b + b^2)*sin(f*x + e))/((a*b^2 + b^3)*f*cos(f*x + e))]
```

3.187.6 Sympy [F]

$$\int \frac{\sec^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\sec^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

input
```
integrate(sec(f*x+e)**4/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(sec(e + f*x)**4/(a + b*sec(e + f*x)**2), x)
```

3.187.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.87

$$\int \frac{\sec^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{a\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}b} - \frac{\tan(fx+e)}{b}}{f}$$

input
```
integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
-(a*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*b) - tan(f*x +
 e)/b)/f
```

3.187.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.27

$$\int \frac{\sec^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)a}{\sqrt{ab+b^2}b} - \frac{\tan(fx+e)}{b}}{f}$$

input
```
integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
-((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b +
b^2)))*a/(sqrt(a*b + b^2)*b) - tan(f*x + e)/b)/f
```

3.187.9 Mupad [B] (verification not implemented)

Time = 18.46 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int \frac{\sec^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\tan(e+f\,x)}{b\,f} - \frac{a\,\operatorname{atan}\left(\frac{\sqrt{b}\tan(e+f\,x)}{\sqrt{a+b}}\right)}{b^{3/2}\,f\,\sqrt{a+b}}$$

input
```
int(1/(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)),x)
```

output
```
tan(e + f*x)/(b*f) - (a*atan((b^(1/2)*tan(e + f*x))/(a + b)^(1/2)))/(b^(3/
2)*f*(a + b)^(1/2))
```

3.188 $\int \frac{\sec^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$

3.188.1 Optimal result 1413
3.188.2 Mathematica [A] (verified) 1413
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3.188.8 Giac [A] (verification not implemented) 1417
3.188.9 Mupad [B] (verification not implemented) 1417

3.188.1 Optimal result

Integrand size = 23, antiderivative size = 36

$$\int \frac{\sec^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{b}\sqrt{a+b}f}$$

output
```
arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/f/b^(1/2)/(a+b)^(1/2)
```

3.188.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{b}\sqrt{a+b}f}$$

input
```
Integrate[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

output
```
ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]]/(Sqrt[b]*Sqrt[a + b]*f)
```

3.188.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4634, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\sec(e+fx)^2}{a+b\sec(e+fx)^2}\,dx$$
$$\downarrow 4634$$
$$\frac{\int \frac{1}{b\tan^2(e+fx)+a+b}\,d\tan(e+fx)}{f}$$
$$\downarrow 218$$
$$\frac{\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{b}f\sqrt{a+b}}$$

input
```
Int[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

output
```
ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]]/(Sqrt[b]*Sqrt[a + b]*f)
```

3.188.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.188.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{f\sqrt{(a+b)b}}$	28
default	$\frac{\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{f\sqrt{(a+b)b}}$	28
risch	$-\frac{\ln\left(e^{2i(fx+e)}+\frac{2iba+2ib^2+a\sqrt{-ab-b^2}+2b\sqrt{-ab-b^2}}{a\sqrt{-ab-b^2}}\right)}{2\sqrt{-ab-b^2}\,f}+\frac{\ln\left(e^{2i(fx+e)}+\frac{-2iba-2ib^2+a\sqrt{-ab-b^2}+2b\sqrt{-ab-b^2}}{a\sqrt{-ab-b^2}}\right)}{2\sqrt{-ab-b^2}\,f}$	172

input
```
int(sec(f*x+e)^2/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))
```

3.188.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(28) = 56.

Time = 0.26 (sec) , antiderivative size = 209, normalized size of antiderivative = 5.81

$$\int \frac{\sec^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \left[-\frac{\sqrt{-ab-b^2}\log\left(\frac{(a^2+8ab+8b^2)\cos(fx+e)^4-2(3ab+4b^2)\cos(fx+e)^2+4\left((a+2b)\cos(fx+e)^3-b\cos(fx+e)\right)\sqrt{-ab-b^2}\sin(fx+e)+}{a^2\cos(fx+e)^4+2ab\cos(fx+e)^2+b^2}\right.}{4(ab+b^2)f}\right.$$
$$\left.-\frac{\arctan\left(\frac{(a+2b)\cos(fx+e)^2-b}{2\sqrt{ab+b^2}\cos(fx+e)\sin(fx+e)}\right)}{2\sqrt{ab+b^2}f}\right]$$

input
```
integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[-1/4*sqrt(-a*b - b^2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*
b + 4*b^2)*cos(f*x + e)^2 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*
sqrt(-a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x +
 e)^2 + b^2))/((a*b + b^2)*f), -1/2*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 -
 b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e)))/(sqrt(a*b + b^2)*f)]
```

3.188.6 Sympy [F]

$$\int \frac{\sec^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\sec^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

input
```
integrate(sec(f*x+e)**2/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(sec(e + f*x)**2/(a + b*sec(e + f*x)**2), x)
```

3.188.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.75

$$\int \frac{\sec^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}f}$$

input
```
integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*f)
```

3.188.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.33

$$\int \frac{\sec^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\pi\left\lfloor \frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor \operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)}{\sqrt{ab+b^2}f}$$

input
```
integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^
2)))/(sqrt(a*b + b^2)*f)
```

3.188.9 Mupad [B] (verification not implemented)

Time = 18.86 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \frac{\sec^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\operatorname{atan}\left(\frac{b\tan(e+fx)}{\sqrt{b^2+ab}}\right)}{f\sqrt{b^2+ab}}$$

input
```
int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)),x)
```

output
```
atan((b*tan(e + f*x))/(a*b + b^2)^(1/2))/(f*(a*b + b^2)^(1/2))
```

3.189 $\int \frac{1}{a+b\sec^2(e+fx)}\,dx$

3.189.1 Optimal result 1418
3.189.2 Mathematica [C] (verified) 1418
3.189.3 Rubi [A] (verified) 1419
3.189.4 Maple [A] (verified) 1420
3.189.5 Fricas [A] (verification not implemented) 1421
3.189.6 Sympy [F] 1421
3.189.7 Maxima [A] (verification not implemented) 1422
3.189.8 Giac [A] (verification not implemented) 1422
3.189.9 Mupad [B] (verification not implemented) 1422

3.189.1 Optimal result

Integrand size = 14, antiderivative size = 45

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx = \frac{x}{a} + \frac{\sqrt{b}\arctan\left(\frac{\sqrt{a+b}\cot(e+fx)}{\sqrt{b}}\right)}{a\sqrt{a+b}f}$$

output
```
x/a+arctan(cot(f*x+e)*(a+b)^(1/2)/b^(1/2))*b^(1/2)/a/f/(a+b)^(1/2)
```

3.189.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 182, normalized size of antiderivative = 4.04

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(\sqrt{a+b}fx\sqrt{b(\cos(e)-i\sin(e))^4}+b\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e}{2\sqrt{a+b}}\right.\right.}{2a\sqrt{a+b}f\,(a+b\sec^2(e+fx))\sqrt{b(\cos(e)-i\sin(e))^4}}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(-1),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(Sqrt[a + b]*f*x*Sqrt[b*(Co
s[e] - I*Sin[e])^4] + b*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2
*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e
])^4])]*(Cos[2*e] - I*Sin[2*e])))/(2*a*Sqrt[a + b]*f*(a + b*Sec[e + f*x]^2
)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

3.189.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 4615, 3042, 3660, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{a+b\sec(e+fx)^2}\,dx$$
$$\downarrow 4615$$
$$\frac{x}{a} - \frac{b\int \frac{1}{a\cos^2(e+fx)+b}\,dx}{a}$$
$$\downarrow 3042$$
$$\frac{x}{a} - \frac{b\int \frac{1}{a\sin\left(e+fx+\frac{\pi}{2}\right)^2+b}\,dx}{a}$$
$$\downarrow 3660$$
$$\frac{b\int \frac{1}{(a+b)\cot^2(e+fx)+b}\,d\cot(e+fx)}{af} + \frac{x}{a}$$
$$\downarrow 218$$
$$\frac{\sqrt{b}\arctan\left(\frac{\sqrt{a+b}\cot(e+fx)}{\sqrt{b}}\right)}{af\sqrt{a+b}} + \frac{x}{a}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^(-1),x]
```

output
```
x/a + (Sqrt[b]*ArcTan[(Sqrt[a + b]*Cot[e + f*x])/Sqrt[b]])/(a*Sqrt[a + b]*
f)
```

3.189.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f  Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 4615
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> Simp[x/a, x
] - Simp[b/a  Int[1/(b + a*Cos[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f},
x] && NeQ[a + b, 0]
```

3.189.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

method	result	size
derivativedivides	$\frac{-\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a\sqrt{(a+b)b}}+\frac{\arctan(\tan(fx+e))}{a}}{f}$	46
default	$\frac{-\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a\sqrt{(a+b)b}}+\frac{\arctan(\tan(fx+e))}{a}}{f}$	46
risch	$\frac{x}{a}+\frac{\sqrt{-(a+b)b}\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-(a+b)b}+a+2b}{a}\right)}{2(a+b)fa}-\frac{\sqrt{-(a+b)b}\ln\left(e^{2i(fx+e)}-\frac{2i\sqrt{-(a+b)b}-a-2b}{a}\right)}{2(a+b)fa}$	114

input
```
int(1/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-b/a/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))+1/a*arctan(
tan(f*x+e)))
```

3.189.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 231, normalized size of antiderivative = 5.13

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx$$
$$= \left[\frac{4\,fx+\sqrt{-\frac{b}{a+b}}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos(fx+e)^2+4\left((a^2+3\,ab+2\,b^2)\cos(fx+e)^3-(ab+b^2)\cos(fx+e)\right)\sqrt{-}}{a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2}\right.}{4\,af}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/4*(4*f*x + sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 -
 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^
3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*co
s(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)))/(a*f), 1/2*(2*f*x + sqrt(b/(a
 + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*
x + e)*sin(f*x + e))))/(a*f)]
```

3.189.6 Sympy [F]

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx = \int \frac{1}{a+b\sec^2(e+fx)}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(1/(a + b*sec(e + f*x)**2), x)
```

3.189.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)ba}} - \frac{fx+e}{a}}{f}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
-(b*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a) - (f*x + e)
/a)/f
```

3.189.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.44

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b}{\sqrt{ab+b^2}a} - \frac{fx+e}{a}}{f}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
-((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b +
b^2)))*b/(sqrt(a*b + b^2)*a) - (f*x + e)/a)/f
```

3.189.9 Mupad [B] (verification not implemented)

Time = 20.04 (sec) , antiderivative size = 460, normalized size of antiderivative = 10.22

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx = \frac{x}{a}$$

$$-\frac{\operatorname{atan}\left(\frac{\left(\frac{2b^3\tan(e+fx)-\left(2a^2b^2-\frac{\tan(e+fx)\left(8a^3b^2+16a^2b^3\right)\sqrt{-b(a+b)}}{4\left(a^2+ba\right)}\right)\sqrt{-b(a+b)}}{2\left(a^2+ba\right)}\right)\sqrt{-b(a+b)}\,1i}{a^2+ba}+\frac{\left(2b^3\tan(e+fx)+\frac{\left(2a^2b^2+\frac{\tan(e+fx)\left(8\right.\right.}{}\right.}{}}{\frac{\left(\frac{2b^3\tan(e+fx)-\left(2a^2b^2-\frac{\tan(e+fx)\left(8a^3b^2+16a^2b^3\right)\sqrt{-b(a+b)}}{4\left(a^2+ba\right)}\right)\sqrt{-b(a+b)}}{2\left(a^2+ba\right)}\right)\sqrt{-b(a+b)}}{a^2+ba}-\frac{\left(2b^3\tan(e+fx)+\frac{\left(2a^2b^2+\frac{\tan(e+fx)\left(8\right.\right.}{}\right.}{}}\right)}{f\,(a^2+ba)}$$

input
```
int(1/(a + b/cos(e + f*x)^2),x)
```

output
```
x/a - (atan((((2*b^3*tan(e + f*x) - ((2*a^2*b^2 - (tan(e + f*x)*(16*a^2*b^
3 + 8*a^3*b^2)*(-b*(a + b))^(1/2))/(4*(a*b + a^2)))*(-b*(a + b))^(1/2))/(2
*(a*b + a^2)))*(-b*(a + b))^(1/2)*1i)/(a*b + a^2) + ((2*b^3*tan(e + f*x) +
 ((2*a^2*b^2 + (tan(e + f*x)*(16*a^2*b^3 + 8*a^3*b^2)*(-b*(a + b))^(1/2))/
(4*(a*b + a^2)))*(-b*(a + b))^(1/2))/(2*(a*b + a^2)))*(-b*(a + b))^(1/2)*1
i)/(a*b + a^2))/(((2*b^3*tan(e + f*x) - ((2*a^2*b^2 - (tan(e + f*x)*(16*a^
2*b^3 + 8*a^3*b^2)*(-b*(a + b))^(1/2))/(4*(a*b + a^2)))*(-b*(a + b))^(1/2)
)/(2*(a*b + a^2)))*(-b*(a + b))^(1/2))/(a*b + a^2) - ((2*b^3*tan(e + f*x)
+ ((2*a^2*b^2 + (tan(e + f*x)*(16*a^2*b^3 + 8*a^3*b^2)*(-b*(a + b))^(1/2))
/(4*(a*b + a^2)))*(-b*(a + b))^(1/2))/(2*(a*b + a^2)))*(-b*(a + b))^(1/2))
/(a*b + a^2)))*(-b*(a + b))^(1/2)*1i)/(f*(a*b + a^2))
```

3.190 $\int \frac{\cos^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$

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3.190.2 Mathematica [A] (verified) 1424
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3.190.1 Optimal result

Integrand size = 23, antiderivative size = 75

$$\int \frac{\cos^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{(a-2b)x}{2a^2} + \frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^2\sqrt{a+b}f} + \frac{\cos(e+fx)\sin(e+fx)}{2af}$$

output
```
1/2*(a-2*b)*x/a^2+1/2*cos(f*x+e)*sin(f*x+e)/a/f+b^(3/2)*arctan(b^(1/2)*tan
(f*x+e)/(a+b)^(1/2))/a^2/f/(a+b)^(1/2)
```

3.190.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.89

$$\int \frac{\cos^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{2(a-2b)(e+fx) + \frac{4b^{3/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a+b}} + a\sin(2(e+fx))}{4a^2f}$$

input
```
Integrate[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

output
```
(2*(a - 2*b)*(e + f*x) + (4*b^(3/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a +
 b]])/Sqrt[a + b] + a*Sin[2*(e + f*x)])/(4*a^2*f)
```

3.190.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.21, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4634, 316, 25, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sec(e+fx)^2\left(a+b\sec(e+fx)^2\right)}dx$$

$$\downarrow 4634$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)^2(b\tan^2(e+fx)+a+b)}d\tan(e+fx)}{f}$$

$$\downarrow 316$$

$$\frac{\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)} - \frac{\int -\frac{b\tan^2(e+fx)+a-b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2a}}{f}$$

$$\downarrow 25$$

$$\frac{\frac{\int \frac{b\tan^2(e+fx)+a-b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2a} + \frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)}}{f}$$

$$\downarrow 397$$

$$\frac{\frac{2b^2\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a} + \frac{(a-2b)\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}}{2a} + \frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)}}{f}$$

$$\downarrow 216$$

$$\frac{\frac{\frac{2b^2\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a} + \frac{(a-2b)\arctan(\tan(e+fx))}{a}}{2a} + \frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)}}{f}$$

$$\downarrow 218$$

$$\frac{\frac{2b^{3/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}+\frac{(a-2b)\arctan(\tan(e+fx))}{a}}{2a}+\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)}}{f}$$

input
```
Int[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

output
```
((((a - 2*b)*ArcTan[Tan[e + f*x]])/a + (2*b^(3/2)*ArcTan[(Sqrt[b]*Tan[e +
f*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(2*a) + Tan[e + f*x]/(2*a*(1 + Tan[e
+ f*x]^2)))/f
```

3.190.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.190.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{\frac{b^2 \arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a^2\sqrt{(a+b)b}}+\frac{\frac{a\tan(fx+e)}{2+2\tan(fx+e)^2}+\frac{(a-2b)\arctan(\tan(fx+e))}{2}}{a^2}}{f}$
default	$\frac{\frac{b^2 \arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a^2\sqrt{(a+b)b}}+\frac{\frac{a\tan(fx+e)}{2+2\tan(fx+e)^2}+\frac{(a-2b)\arctan(\tan(fx+e))}{2}}{a^2}}{f}$
risch	$\frac{x}{2a}-\frac{xb}{a^2}-\frac{ie^{2i(fx+e)}}{8af}+\frac{ie^{-2i(fx+e)}}{8af}+\frac{\sqrt{-(a+b)b}\,b\ln\left(e^{2i(fx+e)}-\frac{2i\sqrt{-(a+b)b}-a-2b}{a}\right)}{2(a+b)f\,a^2}-\frac{\sqrt{-(a+b)b}\,b\ln\left(e^{2i(}\right.$

input
```
int(cos(f*x+e)^2/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(b^2/a^2/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))+1/a^2*(1
/2*a*tan(f*x+e)/(1+tan(f*x+e)^2)+1/2*(a-2*b)*arctan(tan(f*x+e))))
```

3.190.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 272, normalized size of antiderivative = 3.63

$$\int \frac{\cos^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$= \left[\frac{2\,(a-2\,b)fx+2\,a\cos\left(fx+e\right)\sin\left(fx+e\right)+b\sqrt{-\frac{b}{a+b}}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos(fx+e)^2-4}{a^2\cos(fx}\right.\right.$$

$$\left.\left.\frac{}{4\,a^2f}\right.\right.$$

input
```
integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/4*(2*(a - 2*b)*f*x + 2*a*cos(f*x + e)*sin(f*x + e) + b*sqrt(-b/(a + b))
*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e
)^2 - 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*
sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x +
 e)^2 + b^2)))/(a^2*f), 1/2*((a - 2*b)*f*x + a*cos(f*x + e)*sin(f*x + e) -
 b*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b
))/(b*cos(f*x + e)*sin(f*x + e))))/(a^2*f)]
```

3.190.6 Sympy [F]

$$\int \frac{\cos^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\cos^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

input
```
integrate(cos(f*x+e)**2/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(cos(e + f*x)**2/(a + b*sec(e + f*x)**2), x)
```

3.190.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.96

$$\int \frac{\cos^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{2\,b^2\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}a^2} + \frac{(fx+e)(a-2\,b)}{a^2} + \frac{\tan(fx+e)}{a\tan(fx+e)^2+a}}{2\,f}$$

input
```
integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/2*(2*b^2*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^2) +
(f*x + e)*(a - 2*b)/a^2 + tan(f*x + e)/(a*tan(f*x + e)^2 + a))/f
```

3.190.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.25

$$\int \frac{\cos^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{\frac{2\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b^2}{\sqrt{ab+b^2}a^2}+\frac{(fx+e)(a-2b)}{a^2}+\frac{\tan(fx+e)}{\left(\tan(fx+e)^2+1\right)a}}{2f}$$

input
```
integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/2*(2*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a
*b + b^2)))*b^2/(sqrt(a*b + b^2)*a^2) + (f*x + e)*(a - 2*b)/a^2 + tan(f*x
+ e)/((tan(f*x + e)^2 + 1)*a))/f
```

3.190.9 Mupad [B] (verification not implemented)

Time = 20.23 (sec) , antiderivative size = 373, normalized size of antiderivative = 4.97

$$\int \frac{\cos^2(e+fx)}{a+b\sec^2(e+fx)}\,dx =$$
$$-\frac{2b^2\operatorname{atan}\left(\frac{\sin(e+fx)}{\cos(e+fx)}\right)-a\left(\frac{b\sin(2e+2fx)}{2}-b\operatorname{atan}\left(\frac{\sin(e+fx)}{\cos(e+fx)}\right)\right)-a^2\left(\frac{\sin(2e+2fx)}{2}+\operatorname{atan}\left(\frac{\sin(e+fx)}{\cos(e+fx)}\right)\right)+\text{at}}{}$$

input
```
int(cos(e + f*x)^2/(a + b/cos(e + f*x)^2),x)
```

output
```
-(atan((a*sin(e + f*x)*(- a*b^3 - b^4)^(3/2)*4i + b*sin(e + f*x)*(- a*b^3
- b^4)^(3/2)*8i + b^5*sin(e + f*x)*(- a*b^3 - b^4)^(1/2)*8i + a*b^4*sin(e
+ f*x)*(- a*b^3 - b^4)^(1/2)*12i + a^4*b*sin(e + f*x)*(- a*b^3 - b^4)^(1/2
)*1i + a^2*b^3*sin(e + f*x)*(- a*b^3 - b^4)^(1/2)*1i - a^3*b^2*sin(e + f*x
)*(- a*b^3 - b^4)^(1/2)*2i)/(3*a^2*b^5*cos(e + f*x) + 5*a^3*b^4*cos(e + f*
x) + a^4*b^3*cos(e + f*x) - a^5*b^2*cos(e + f*x)))*(- a*b^3 - b^4)^(1/2)*2
i + 2*b^2*atan(sin(e + f*x)/cos(e + f*x)) - a*((b*sin(2*e + 2*f*x))/2 - b*
atan(sin(e + f*x)/cos(e + f*x))) - a^2*(sin(2*e + 2*f*x)/2 + atan(sin(e +
f*x)/cos(e + f*x))))/(f*(2*a^2*b + 2*a^3))
```

3.191 $\int \frac{\cos^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$

3.191.1 Optimal result 1430
3.191.2 Mathematica [A] (verified) 1430
3.191.3 Rubi [A] (verified) 1431
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3.191.5 Fricas [A] (verification not implemented) 1434
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3.191.8 Giac [A] (verification not implemented) 1436
3.191.9 Mupad [B] (verification not implemented) 1436

3.191.1 Optimal result

Integrand size = 23, antiderivative size = 117

$$\int \frac{\cos^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{(3a^2-4ab+8b^2)\,x}{8a^3} - \frac{b^{5/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^3\sqrt{a+b}f} + \frac{(3a-4b)\cos(e+fx)\sin(e+fx)}{8a^2f} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af}$$

output
```
1/8*(3*a^2-4*a*b+8*b^2)*x/a^3+1/8*(3*a-4*b)*cos(f*x+e)*sin(f*x+e)/a^2/f+1/
4*cos(f*x+e)^3*sin(f*x+e)/a/f-b^(5/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2
))/a^3/f/(a+b)^(1/2)
```

3.191.2 Mathematica [A] (verified)

Time = 3.20 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.81

$$\int \frac{\cos^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{4(3a^2-4ab+8b^2)(e+fx) - \frac{32b^{5/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a+b}} + 8a(a-b)\sin(2(e+fx)) + a^2\sin(4(e+fx))}{32a^3f}$$

input
```
Integrate[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

output
```
(4*(3*a^2 - 4*a*b + 8*b^2)*(e + f*x) - (32*b^(5/2)*ArcTan[(Sqrt[b]*Tan[e +
 f*x])/Sqrt[a + b]])/Sqrt[a + b] + 8*a*(a - b)*Sin[2*(e + f*x)] + a^2*Sin[
4*(e + f*x)])/(32*a^3*f)
```

3.191.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.21, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4634, 316, 25, 402, 25, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sec(e+fx)^4\left(a+b\sec(e+fx)^2\right)}dx$$

$$\downarrow 4634$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)^3(b\tan^2(e+fx)+a+b)}d\tan(e+fx)}{f}$$

$$\downarrow 316$$

$$\frac{\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2}-\frac{\int -\frac{3b\tan^2(e+fx)+3a-b}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{4a}}{f}$$

$$\downarrow 25$$

$$\frac{\frac{\int \frac{3b\tan^2(e+fx)+3a-b}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{4a}+\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2}}{f}$$

$$\downarrow 402$$

$$\frac{\frac{\frac{(3a-4b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)}-\frac{\int -\frac{3a^2-ba+4b^2+(3a-4b)b\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2a}}{4a}+\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2}}{f}$$

$$\downarrow 25$$

$$\frac{\frac{\int \frac{3a^2-ba+4b^2+(3a-4b)b\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2a}+\frac{(3a-4b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)}}{4a}+\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2}}{f}$$

$$\downarrow 397$$

$$\frac{\frac{\left(3a^2-4ab+8b^2\right)\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}-\frac{8b^3\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{2a}+\frac{(3a-4b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)}}{4a}+\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2}}{f}$$

$$\downarrow 216$$

$$\frac{\frac{\left(3a^2-4ab+8b^2\right)\arctan(\tan(e+fx))}{a}-\frac{8b^3\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{2a}+\frac{(3a-4b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)}}{4a}+\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2}}{f}$$

$$\downarrow 218$$

$$\frac{\frac{\left(3a^2-4ab+8b^2\right)\arctan(\tan(e+fx))}{a}-\frac{8b^{5/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{2a}+\frac{(3a-4b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)}}{4a}+\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2}}{f}$$

input
```
Int[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

output
```
(Tan[e + f*x]/(4*a*(1 + Tan[e + f*x]^2)^2) + ((((3*a^2 - 4*a*b + 8*b^2)*Ar
cTan[Tan[e + f*x]])/a - (8*b^(5/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a +
b]])/(a*Sqrt[a + b]))/(2*a) + ((3*a - 4*b)*Tan[e + f*x])/(2*a*(1 + Tan[e +
 f*x]^2)))/(4*a))/f
```

3.191.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.191.4 Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.99

method	result
derivativedivides	$\frac{-\frac{b^3 \arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a^3\sqrt{(a+b)b}}+\frac{\left(\frac{3}{8}a^2-\frac{1}{2}ab\right)\tan(fx+e)^3+\left(-\frac{1}{2}ab+\frac{5}{8}a^2\right)\tan(fx+e)}{\left(1+\tan(fx+e)^2\right)^2}+\frac{\left(3a^2-4ab+8b^2\right)\arctan(\tan(fx+e))}{8}}{a^3}}{f}$
default	$\frac{-\frac{b^3 \arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a^3\sqrt{(a+b)b}}+\frac{\left(\frac{3}{8}a^2-\frac{1}{2}ab\right)\tan(fx+e)^3+\left(-\frac{1}{2}ab+\frac{5}{8}a^2\right)\tan(fx+e)}{\left(1+\tan(fx+e)^2\right)^2}+\frac{\left(3a^2-4ab+8b^2\right)\arctan(\tan(fx+e))}{8}}{a^3}}{f}$
risch	$\frac{3x}{8a}-\frac{xb}{2a^2}+\frac{x\,b^2}{a^3}-\frac{ie^{2i(fx+e)}}{8af}+\frac{ie^{2i(fx+e)}b}{8a^2f}+\frac{ie^{-2i(fx+e)}}{8af}-\frac{ie^{-2i(fx+e)}b}{8a^2f}+\frac{\sqrt{-(a+b)b}\,b^2\ln\left(e^{2i(fx+e)}+2\right.}{2(a+b)fa^3}$

input
```
int(cos(f*x+e)^4/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-b^3/a^3/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))+1/a^3*(
((3/8*a^2-1/2*a*b)*tan(f*x+e)^3+(-1/2*a*b+5/8*a^2)*tan(f*x+e))/(1+tan(f*x+
e)^2)^2+1/8*(3*a^2-4*a*b+8*b^2)*arctan(tan(f*x+e))))
```

3.191.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.93

$$\int \frac{\cos^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \left[\frac{2\,b^2\sqrt{-\frac{b}{a+b}}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos(fx+e)^2+4\left((a^2+3\,ab+2\,b^2)\cos(fx+e)^3-(ab+b^2)\cos(fx+e)\right)\sqrt{-\frac{b}{a+b}}}{a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2}\right.}{8\,a^3f}\right.$$

input
```
integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/8*(2*b^2*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2
*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3
- (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(
f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) + (3*a^2 - 4*a*b + 8*b^2)*f*x +
(2*a^2*cos(f*x + e)^3 + (3*a^2 - 4*a*b)*cos(f*x + e))*sin(f*x + e))/(a^3*f
), 1/8*(4*b^2*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sq
rt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e))) + (3*a^2 - 4*a*b + 8*b^2)*f*x
 + (2*a^2*cos(f*x + e)^3 + (3*a^2 - 4*a*b)*cos(f*x + e))*sin(f*x + e))/(a^
3*f)]
```

3.191.6 Sympy [F]

$$\int \frac{\cos^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\cos^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

input
```
integrate(cos(f*x+e)**4/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(cos(e + f*x)**4/(a + b*sec(e + f*x)**2), x)
```

3.191.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.08

$$\int \frac{\cos^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{8b^3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}a^3} - \frac{(3a-4b)\tan(fx+e)^3+(5a-4b)\tan(fx+e)}{a^2\tan(fx+e)^4+2a^2\tan(fx+e)^2+a^2} - \frac{(3a^2-4ab+8b^2)(fx+e)}{a^3}}{8f}$$

input
```
integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
-1/8*(8*b^3*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^3) -
 ((3*a - 4*b)*tan(f*x + e)^3 + (5*a - 4*b)*tan(f*x + e))/(a^2*tan(f*x + e)
^4 + 2*a^2*tan(f*x + e)^2 + a^2) - (3*a^2 - 4*a*b + 8*b^2)*(f*x + e)/a^3)/
f
```

3.191.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.21

$$\int \frac{\cos^4(e+fx)}{a+b\sec^2(e+fx)}\,dx =$$

$$-\frac{\frac{8\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b^3}{\sqrt{ab+b^2}a^3} - \frac{(3a^2-4ab+8b^2)(fx+e)}{a^3} - \frac{3a\tan(fx+e)^3-4b\tan(fx+e)^3+5a\tan(fx+e)-4b\tan(}{\left(\tan(fx+e)^2+1\right)^2a^2}}{8f}$$

input
```
integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
-1/8*(8*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(
a*b + b^2)))*b^3/(sqrt(a*b + b^2)*a^3) - (3*a^2 - 4*a*b + 8*b^2)*(f*x + e)
/a^3 - (3*a*tan(f*x + e)^3 - 4*b*tan(f*x + e)^3 + 5*a*tan(f*x + e) - 4*b*t
an(f*x + e))/((tan(f*x + e)^2 + 1)^2*a^2))/f
```

3.191.9 Mupad [B] (verification not implemented)

Time = 20.26 (sec) , antiderivative size = 1114, normalized size of antiderivative = 9.52

$$\int \frac{\cos^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
int(cos(e + f*x)^4/(a + b/cos(e + f*x)^2),x)
```

output

```
((tan(e + f*x)*(5*a - 4*b))/(8*a^2) + (tan(e + f*x)^3*(3*a - 4*b))/(8*a^2)
)/(f*(2*tan(e + f*x)^2 + tan(e + f*x)^4 + 1)) - (atan((((-b^5*(a + b))^(1/
2)*((tan(e + f*x)*(128*b^7 - 64*a*b^6 + 64*a^2*b^5 - 24*a^3*b^4 + 9*a^4*b^
3))/(64*a^4) - ((-b^5*(a + b))^(1/2)*((2*a^6*b^4 - (a^7*b^3)/2 + (3*a^8*b^
2)/2)/(2*a^6) - (tan(e + f*x)*(512*a^6*b^3 + 256*a^7*b^2)*(-b^5*(a + b))^(
1/2))/(128*a^4*(a^3*b + a^4))))/(2*(a^3*b + a^4)))*1i)/(a^3*b + a^4) + ((-
b^5*(a + b))^(1/2)*((tan(e + f*x)*(128*b^7 - 64*a*b^6 + 64*a^2*b^5 - 24*a^
3*b^4 + 9*a^4*b^3))/(64*a^4) + ((-b^5*(a + b))^(1/2)*((2*a^6*b^4 - (a^7*b^
3)/2 + (3*a^8*b^2)/2)/(2*a^6) + (tan(e + f*x)*(512*a^6*b^3 + 256*a^7*b^2)*
(-b^5*(a + b))^(1/2))/(128*a^4*(a^3*b + a^4))))/(2*(a^3*b + a^4)))*1i)/(a^
3*b + a^4))/(((5*a*b^7)/4 - b^8 - (3*a^2*b^6)/4 + (9*a^3*b^5)/32)/a^6 + ((
-b^5*(a + b))^(1/2)*((tan(e + f*x)*(128*b^7 - 64*a*b^6 + 64*a^2*b^5 - 24*a
^3*b^4 + 9*a^4*b^3))/(64*a^4) - ((-b^5*(a + b))^(1/2)*((2*a^6*b^4 - (a^7*b
^3)/2 + (3*a^8*b^2)/2)/(2*a^6) - (tan(e + f*x)*(512*a^6*b^3 + 256*a^7*b^2)
*(-b^5*(a + b))^(1/2))/(128*a^4*(a^3*b + a^4))))/(2*(a^3*b + a^4))))/(a^3*
b + a^4) - ((-b^5*(a + b))^(1/2)*((tan(e + f*x)*(128*b^7 - 64*a*b^6 + 64*a
^2*b^5 - 24*a^3*b^4 + 9*a^4*b^3))/(64*a^4) + ((-b^5*(a + b))^(1/2)*((2*a^6
*b^4 - (a^7*b^3)/2 + (3*a^8*b^2)/2)/(2*a^6) + (tan(e + f*x)*(512*a^6*b^3 +
 256*a^7*b^2)*(-b^5*(a + b))^(1/2))/(128*a^4*(a^3*b + a^4))))/(2*(a^3*b +
a^4)))/(a^3*b + a^4)))*(-b^5*(a + b))^(1/2)*1i)/(f*(a^3*b + a^4)) - (a...
```

3.192 $\int \frac{\cos^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$

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3.192.1 Optimal result

Integrand size = 23, antiderivative size = 163

$$\int \frac{\cos^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{(5a^3-6a^2b+8ab^2-16b^3)\,x}{16a^4} + \frac{b^{7/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a^4\sqrt{a+b}f} + \frac{(5a^2-6ab+8b^2)\cos(e+fx)\sin(e+fx)}{16a^3f} + \frac{(5a-6b)\cos^3(e+fx)\sin(e+fx)}{24a^2f} + \frac{\cos^5(e+fx)\sin(e+fx)}{6af}$$

output
```
1/16*(5*a^3-6*a^2*b+8*a*b^2-16*b^3)*x/a^4+1/16*(5*a^2-6*a*b+8*b^2)*cos(f*x
+e)*sin(f*x+e)/a^3/f+1/24*(5*a-6*b)*cos(f*x+e)^3*sin(f*x+e)/a^2/f+1/6*cos(
f*x+e)^5*sin(f*x+e)/a/f+b^(7/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^4
/f/(a+b)^(1/2)
```

3.192.2 Mathematica [A] (verified)

Time = 1.86 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.82

$$\int \frac{\cos^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{12(5a^3-6a^2b+8ab^2-16b^3)\,(e+fx)+\frac{192b^{7/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a+b}}+3a(15a^2-16ab+16b^2)\sin(2(e+fx))}{192a^4f}$$

input
```
Integrate[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

output
```
(12*(5*a^3 - 6*a^2*b + 8*a*b^2 - 16*b^3)*(e + f*x) + (192*b^(7/2)*ArcTan[(
Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/Sqrt[a + b] + 3*a*(15*a^2 - 16*a*b + 1
6*b^2)*Sin[2*(e + f*x)] + 3*a^2*(3*a - 2*b)*Sin[4*(e + f*x)] + a^3*Sin[6*(
e + f*x)])/(192*a^4*f)
```

3.192.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.21, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 4634, 316, 25, 402, 27, 402, 25, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sec(e+fx)^6\left(a+b\sec(e+fx)^2\right)}dx$$

$$\downarrow 4634$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)^4(b\tan^2(e+fx)+a+b)}d\tan(e+fx)}{f}$$

$$\downarrow 316$$

$$\frac{\frac{\tan(e+fx)}{6a(\tan^2(e+fx)+1)^3} - \frac{\int -\frac{5b\tan^2(e+fx)+5a-b}{\left(\tan^2(e+fx)+1\right)^3\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{6a}}{f}$$

$$\downarrow 25$$

$$\frac{\frac{\int \frac{5b\tan^2(e+fx)+5a-b}{\left(\tan^2(e+fx)+1\right)^3\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{6a} + \frac{\tan(e+fx)}{6a(\tan^2(e+fx)+1)^3}}{f}$$

$$\downarrow 402$$

$$\frac{\frac{(5a-6b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2}-\frac{\int -\frac{3\left(5a^2-ba+2b^2+(5a-6b)b\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{4a}}{6a}+\frac{\tan(e+fx)}{6a(\tan^2(e+fx)+1)^3}}{f}$$

$$\downarrow 27$$

$$\frac{\frac{3\int \frac{5a^2-ba+2b^2+(5a-6b)b\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{4a}+\frac{(5a-6b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2}}{6a}+\frac{\tan(e+fx)}{6a(\tan^2(e+fx)+1)^3}}{f}$$

$$\downarrow 402$$

$$\frac{3\left(\frac{\left(5a^2-6ab+8b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)}-\frac{\int -\frac{5a^3-ba^2+2b^2a-8b^3+b\left(5a^2-6ba+8b^2\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2a}\right)}{4a}+\frac{(5a-6b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2}}{6a}+\frac{\tan(e+fx)}{6a(\tan^2(e+fx)+1)^3}}{f}$$

$$\downarrow 25$$

$$\frac{3\left(\frac{\int \frac{5a^3-ba^2+2b^2a-8b^3+b\left(5a^2-6ba+8b^2\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2a}+\frac{\left(5a^2-6ab+8b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)}\right)}{4a}+\frac{(5a-6b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2}}{6a}+\frac{\tan(e+fx)}{6a(\tan^2(e+fx)+1)^3}}{f}$$

$$\downarrow 397$$

$$\frac{3\left(\frac{\left(5a^3-6a^2b+8ab^2-16b^3\right)\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}+\frac{16b^4\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}+\frac{\left(5a^2-6ab+8b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)}\right)}{4a}+\frac{(5a-6b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2}}{6a}+\frac{\tan(e+fx)}{6a(\tan^2(e+fx)+1)^3}}{f}$$

$$\downarrow 216$$

$$\frac{3\left(\frac{16b^4\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}+\frac{\left(5a^3-6a^2b+8ab^2-16b^3\right)\arctan(\tan(e+fx))}{a}+\frac{\left(5a^2-6ab+8b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)}\right)}{4a}+\frac{(5a-6b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2}}{6a}+\frac{\tan(e}{6a(\tan^2(e}}{f}$$

3.192. $\int \frac{\cos^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$

$$\downarrow 218$$

$$\frac{3\left(\frac{\left(5a^2-6ab+8b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)}+\frac{\frac{\left(5a^3-6a^2b+8ab^2-16b^3\right)\arctan(\tan(e+fx))}{a}+\frac{16b^{7/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{2a}\right)}{4a}+\frac{(5a-6b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2}+\frac{\tan(e+fx)}{6a(\tan^2(e+fx)+}$$
$$6a$$
$$f$$

input
```
Int[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

output
```
(Tan[e + f*x]/(6*a*(1 + Tan[e + f*x]^2)^3) + (((5*a - 6*b)*Tan[e + f*x])/(
4*a*(1 + Tan[e + f*x]^2)^2) + (3*((((5*a^3 - 6*a^2*b + 8*a*b^2 - 16*b^3)*A
rcTan[Tan[e + f*x]])/a + (16*b^(7/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a
+ b]])/(a*Sqrt[a + b]))/(2*a) + ((5*a^2 - 6*a*b + 8*b^2)*Tan[e + f*x])/(2*
a*(1 + Tan[e + f*x]^2))))/(4*a))/(6*a))/f
```

3.192.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.192.4 Maple [A] (verified)

Time = 2.62 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{\frac{\left(\frac{5}{16}a^3-\frac{3}{8}a^2b+\frac{1}{2}ab^2\right)\tan(fx+e)^5+\left(ab^2+\frac{5}{6}a^3-a^2b\right)\tan(fx+e)^3+\left(-\frac{5}{8}a^2b+\frac{1}{2}ab^2+\frac{11}{16}a^3\right)\tan(fx+e)}{\left(1+\tan(fx+e)^2\right)^3}+\frac{\left(5a^3-6a^2b+8ab^2-16b^3\right.}{16}}{a^4}}{f}$
default	$\frac{\frac{\left(\frac{5}{16}a^3-\frac{3}{8}a^2b+\frac{1}{2}ab^2\right)\tan(fx+e)^5+\left(ab^2+\frac{5}{6}a^3-a^2b\right)\tan(fx+e)^3+\left(-\frac{5}{8}a^2b+\frac{1}{2}ab^2+\frac{11}{16}a^3\right)\tan(fx+e)}{\left(1+\tan(fx+e)^2\right)^3}+\frac{\left(5a^3-6a^2b+8ab^2-16b^3\right.}{16}}{a^4}}{f}$
risch	$\frac{5x}{16a}-\frac{3xb}{8a^2}+\frac{xb^2}{2a^3}-\frac{xb^3}{a^4}-\frac{15ie^{2i(fx+e)}}{128af}+\frac{ie^{2i(fx+e)}b}{8a^2f}-\frac{ie^{2i(fx+e)}b^2}{8a^3f}+\frac{15ie^{-2i(fx+e)}}{128af}-\frac{ie^{-2i(fx+e)}b}{8a^2f}+$

input
```
int(cos(f*x+e)^6/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/a^4*(((5/16*a^3-3/8*a^2*b+1/2*a*b^2)*tan(f*x+e)^5+(a*b^2+5/6*a^3-a^
2*b)*tan(f*x+e)^3+(-5/8*a^2*b+1/2*a*b^2+11/16*a^3)*tan(f*x+e))/(1+tan(f*x+
e)^2)^3+1/16*(5*a^3-6*a^2*b+8*a*b^2-16*b^3)*arctan(tan(f*x+e)))+b^4/a^4/((
a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))
```

3.192.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 424, normalized size of antiderivative = 2.60

$$\int \frac{\cos^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$= \left[\frac{12\,b^3\sqrt{-\frac{b}{a+b}}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos(fx+e)^2-4\left((a^2+3\,ab+2\,b^2)\cos(fx+e)^3-(ab+b^2)\cos(fx+e)\right)\sqrt{-\frac{b}{a+}}\right.}{a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2}\right.$$

$$\left.-\frac{24\,b^3\sqrt{\frac{b}{a+b}}\arctan\left(\frac{\left((a+2\,b)\cos(fx+e)^2-b\right)\sqrt{\frac{b}{a+b}}}{2\,b\cos(fx+e)\sin(fx+e)}\right)-3\,(5\,a^3-6\,a^2b+8\,ab^2-16\,b^3)fx-\left(8\,a^3\cos(fx+e)^5\right.}{48\,a^4f}\right.$$

input
```
integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/48*(12*b^3*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 -
 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^
3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*co
s(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) + 3*(5*a^3 - 6*a^2*b + 8*a*b^2
 - 16*b^3)*f*x + (8*a^3*cos(f*x + e)^5 + 2*(5*a^3 - 6*a^2*b)*cos(f*x + e)^
3 + 3*(5*a^3 - 6*a^2*b + 8*a*b^2)*cos(f*x + e))*sin(f*x + e))/(a^4*f), -1/
48*(24*b^3*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(
b/(a + b))/(b*cos(f*x + e)*sin(f*x + e))) - 3*(5*a^3 - 6*a^2*b + 8*a*b^2 -
 16*b^3)*f*x - (8*a^3*cos(f*x + e)^5 + 2*(5*a^3 - 6*a^2*b)*cos(f*x + e)^3
+ 3*(5*a^3 - 6*a^2*b + 8*a*b^2)*cos(f*x + e))*sin(f*x + e))/(a^4*f)]
```

3.192.6 Sympy [F]

$$\int \frac{\cos^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\cos^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

input
```
integrate(cos(f*x+e)**6/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(cos(e + f*x)**6/(a + b*sec(e + f*x)**2), x)
```

3.192.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.16

$$\int \frac{\cos^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{\frac{48\,b^4\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}a^4} + \frac{3\left(5\,a^2-6\,ab+8\,b^2\right)\tan(fx+e)^5+8\left(5\,a^2-6\,ab+6\,b^2\right)\tan(fx+e)^3+3\left(11\,a^2-10\,ab+8\,b^2\right)\tan(fx+e)}{a^3\tan(fx+e)^6+3\,a^3\tan(fx+e)^4+3\,a^3\tan(fx+e)^2+a^3} + \frac{3\left(5\,a^3-6}{48\,f}$$

input
```
integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/48*(48*b^4*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^4)
+ (3*(5*a^2 - 6*a*b + 8*b^2)*tan(f*x + e)^5 + 8*(5*a^2 - 6*a*b + 6*b^2)*ta
n(f*x + e)^3 + 3*(11*a^2 - 10*a*b + 8*b^2)*tan(f*x + e))/(a^3*tan(f*x + e)
^6 + 3*a^3*tan(f*x + e)^4 + 3*a^3*tan(f*x + e)^2 + a^3) + 3*(5*a^3 - 6*a^2
*b + 8*a*b^2 - 16*b^3)*(f*x + e)/a^4)/f
```

3.192.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.33

$$\int \frac{\cos^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$= \frac{\frac{48\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b^4}{\sqrt{ab+b^2}a^4}+\frac{3\left(5a^3-6a^2b+8ab^2-16b^3\right)(fx+e)}{a^4}+\frac{15a^2\tan(fx+e)^5-18ab\tan(fx+e)^5+24b^2\tan(}{}}{48f}$$

input
```
integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/48*(48*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt
(a*b + b^2)))*b^4/(sqrt(a*b + b^2)*a^4) + 3*(5*a^3 - 6*a^2*b + 8*a*b^2 - 1
6*b^3)*(f*x + e)/a^4 + (15*a^2*tan(f*x + e)^5 - 18*a*b*tan(f*x + e)^5 + 24
*b^2*tan(f*x + e)^5 + 40*a^2*tan(f*x + e)^3 - 48*a*b*tan(f*x + e)^3 + 48*b
^2*tan(f*x + e)^3 + 33*a^2*tan(f*x + e) - 30*a*b*tan(f*x + e) + 24*b^2*tan
(f*x + e))/((tan(f*x + e)^2 + 1)^3*a^3))/f
```

3.192.9 Mupad [B] (verification not implemented)

Time = 21.55 (sec) , antiderivative size = 1979, normalized size of antiderivative = 12.14

$$\int \frac{\cos^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
int(cos(e + f*x)^6/(a + b/cos(e + f*x)^2),x)
```

output

```
((tan(e + f*x)*(11*a^2 - 10*a*b + 8*b^2))/(16*a^3) + (tan(e + f*x)^3*(5*a^
2 - 6*a*b + 6*b^2))/(6*a^3) + (tan(e + f*x)^5*(5*a^2 - 6*a*b + 8*b^2))/(16
*a^3))/(f*(3*tan(e + f*x)^2 + 3*tan(e + f*x)^4 + tan(e + f*x)^6 + 1)) + (a
tan(((((((2*a^8*b^5 - (a^9*b^4)/2 + (a^10*b^3)/4 - (5*a^11*b^2)/4)/a^9 - (
tan(e + f*x)*(2048*a^8*b^3 + 1024*a^9*b^2)*(a*b^2*8i - a^2*b*6i + a^3*5i -
 b^3*16i))/(4096*a^10))*(a*b^2*8i - a^2*b*6i + a^3*5i - b^3*16i))/(32*a^4)
 - (tan(e + f*x)*(512*b^9 - 256*a*b^8 + 256*a^2*b^7 - 256*a^3*b^6 + 116*a^
4*b^5 - 60*a^5*b^4 + 25*a^6*b^3))/(128*a^6))*(a*b^2*8i - a^2*b*6i + a^3*5i
 - b^3*16i)*1i)/(32*a^4) - (((((2*a^8*b^5 - (a^9*b^4)/2 + (a^10*b^3)/4 - (
5*a^11*b^2)/4)/a^9 + (tan(e + f*x)*(2048*a^8*b^3 + 1024*a^9*b^2)*(a*b^2*8i
 - a^2*b*6i + a^3*5i - b^3*16i))/(4096*a^10))*(a*b^2*8i - a^2*b*6i + a^3*5
i - b^3*16i))/(32*a^4) + (tan(e + f*x)*(512*b^9 - 256*a*b^8 + 256*a^2*b^7
 - 256*a^3*b^6 + 116*a^4*b^5 - 60*a^5*b^4 + 25*a^6*b^3))/(128*a^6))*(a*b^2*
8i - a^2*b*6i + a^3*5i - b^3*16i)*1i)/(32*a^4))/((((((2*a^8*b^5 - (a^9*b^4
)/2 + (a^10*b^3)/4 - (5*a^11*b^2)/4)/a^9 - (tan(e + f*x)*(2048*a^8*b^3 + 1
024*a^9*b^2)*(a*b^2*8i - a^2*b*6i + a^3*5i - b^3*16i))/(4096*a^10))*(a*b^2
*8i - a^2*b*6i + a^3*5i - b^3*16i))/(32*a^4) - (tan(e + f*x)*(512*b^9 - 25
6*a*b^8 + 256*a^2*b^7 - 256*a^3*b^6 + 116*a^4*b^5 - 60*a^5*b^4 + 25*a^6*b^
3))/(128*a^6))*(a*b^2*8i - a^2*b*6i + a^3*5i - b^3*16i))/(32*a^4) - ((5*a*
b^10)/4 - b^11 - (11*a^2*b^9)/8 + (29*a^3*b^8)/32 - (15*a^4*b^7)/32 + (...
```

3.193 $\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

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3.193.1 Optimal result

Integrand size = 23, antiderivative size = 102

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\operatorname{arctanh}(\sin(e+fx))}{b^2 f} - \frac{\sqrt{a}(2a+3b)\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2b^2(a+b)^{3/2}f} - \frac{a\sin(e+fx)}{2b(a+b)f\left(a+b-a\sin^2(e+fx)\right)}$$

output
```
arctanh(sin(f*x+e))/b^2/f-1/2*a*sin(f*x+e)/b/(a+b)/f/(a+b-a*sin(f*x+e)^2)-
1/2*(2*a+3*b)*arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))*a^(1/2)/b^2/(a+b)^(3
/2)/f
```

3.193.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.73 (sec) , antiderivative size = 980, normalized size of antiderivative = 9.61

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$
$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^3(e+fx)\left(-2ia(2a+3b)\arctan\left(\frac{2\sin(e)\left(ia+ib+i(a+b)\cos(2e)+\sqrt{a}\sqrt{a+b}\cos(fx)\sqrt{(}\right.\right.}{i(a+3b)\cos}$$

input
```
Integrate[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*((-2*I)*a*(2*a + 3*b)*ArcTa
n[(2*Sin[e]*(I*a + I*b + I*(a + b)*Cos[2*e] + Sqrt[a]*Sqrt[a + b]*Cos[f*x]
*Sqrt[(Cos[e] - I*Sin[e])^2] - Sqrt[a]*Sqrt[a + b]*Cos[2*e + f*x]*Sqrt[(Co
s[e] - I*Sin[e])^2] + a*Sin[2*e] + b*Sin[2*e] - I*Sqrt[a]*Sqrt[a + b]*Sqrt
[(Cos[e] - I*Sin[e])^2]*Sin[f*x] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*
Sin[e])^2]*Sin[2*e + f*x]))/(I*(a + 3*b)*Cos[e] + I*(a + b)*Cos[3*e] + I*a
*Cos[e + 2*f*x] + I*a*Cos[3*e + 2*f*x] + 3*a*Sin[e] + b*Sin[e] + a*Sin[3*e
] + b*Sin[3*e] + a*Sin[e + 2*f*x] - a*Sin[3*e + 2*f*x])]*(a + 2*b + a*Cos[
2*(e + f*x)])*Sec[e + f*x]*(Cos[e] - I*Sin[e]) - a*(2*a + 3*b)*(a + 2*b +
a*Cos[2*(e + f*x)])*Log[a + 2*(a + b)*Cos[2*e] - a*Cos[2*(e + f*x)] - (2*I
)*a*Sin[2*e] - (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*S
in[e])^2]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin
[2*e + f*x]]*Sec[e + f*x]*(Cos[e] - I*Sin[e]) + a*(2*a + 3*b)*(a + 2*b + a
*Cos[2*(e + f*x)])*Log[-a - 2*(a + b)*Cos[2*e] + a*Cos[2*(e + f*x)] + (2*I
)*a*Sin[2*e] + (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*S
in[e])^2]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin
[2*e + f*x]]*Sec[e + f*x]*(Cos[e] - I*Sin[e]) - 8*Sqrt[a]*(a + b)^(3/2)*(a
 + 2*b + a*Cos[2*(e + f*x)])*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*Sec[
e + f*x]*Sqrt[(Cos[e] - I*Sin[e])^2] + 8*Sqrt[a]*(a + b)^(3/2)*(a + 2*b +
a*Cos[2*(e + f*x)])*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sec[e + f*...
```

3.193.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4635, 316, 25, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\sec(e+fx)^5}{\left(a+b\sec(e+fx)^2\right)^2}dx$$
$$\downarrow 4635$$

$$\frac{\int \frac{1}{\left(1-\sin^2(e+fx)\right)\left(-a\sin^2(e+fx)+a+b\right)^2}d\sin(e+fx)}{f}$$

$$\downarrow 316$$

$$\frac{-\frac{\int -\frac{a\sin^2(e+fx)+a+2b}{\left(1-\sin^2(e+fx)\right)\left(-a\sin^2(e+fx)+a+b\right)}d\sin(e+fx)}{2b(a+b)}-\frac{a\sin(e+fx)}{2b(a+b)\left(-a\sin^2(e+fx)+a+b\right)}}{f}$$

$$\downarrow 25$$

$$\frac{\frac{\int \frac{a\sin^2(e+fx)+a+2b}{\left(1-\sin^2(e+fx)\right)\left(-a\sin^2(e+fx)+a+b\right)}d\sin(e+fx)}{2b(a+b)}-\frac{a\sin(e+fx)}{2b(a+b)\left(-a\sin^2(e+fx)+a+b\right)}}{f}$$

$$\downarrow 397$$

$$\frac{\frac{2(a+b)\int \frac{1}{1-\sin^2(e+fx)}d\sin(e+fx)}{b}-\frac{a(2a+3b)\int \frac{1}{-a\sin^2(e+fx)+a+b}d\sin(e+fx)}{b}}{2b(a+b)}-\frac{a\sin(e+fx)}{2b(a+b)\left(-a\sin^2(e+fx)+a+b\right)}}{f}$$

$$\downarrow 219$$

$$\frac{\frac{2(a+b)\operatorname{arctanh}(\sin(e+fx))}{b}-\frac{a(2a+3b)\int \frac{1}{-a\sin^2(e+fx)+a+b}d\sin(e+fx)}{b}}{2b(a+b)}-\frac{a\sin(e+fx)}{2b(a+b)\left(-a\sin^2(e+fx)+a+b\right)}}{f}$$

$$\downarrow 221$$

$$\frac{\frac{2(a+b)\operatorname{arctanh}(\sin(e+fx))}{b}-\frac{\sqrt{a}(2a+3b)\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{b\sqrt{a+b}}}{2b(a+b)}-\frac{a\sin(e+fx)}{2b(a+b)\left(-a\sin^2(e+fx)+a+b\right)}}{f}$$

input
```
Int[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(((2*(a + b)*ArcTanh[Sin[e + f*x]])/b - (Sqrt[a]*(2*a + 3*b)*ArcTanh[(Sqrt
[a]*Sin[e + f*x])/Sqrt[a + b]])/(b*Sqrt[a + b]))/(2*b*(a + b)) - (a*Sin[e
+ f*x])/(2*b*(a + b)*(a + b - a*Sin[e + f*x]^2)))/f
```

3.193.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.193.4 Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{\frac{\ln(\sin(fx+e)+1)}{2b^2}+\frac{a\left(\frac{b\sin(fx+e)}{2(a+b)\left(a\sin(fx+e)^2-a-b\right)}-\frac{(2a+3b)\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{2(a+b)\sqrt{a(a+b)}}\right)}{b^2}-\frac{\ln(\sin(fx+e)-1)}{2b^2}}{f}$
default	$\frac{\frac{\ln(\sin(fx+e)+1)}{2b^2}+\frac{a\left(\frac{b\sin(fx+e)}{2(a+b)\left(a\sin(fx+e)^2-a-b\right)}-\frac{(2a+3b)\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{2(a+b)\sqrt{a(a+b)}}\right)}{b^2}-\frac{\ln(\sin(fx+e)-1)}{2b^2}}{f}$
risch	$\frac{ia\left(e^{3i(fx+e)}-e^{i(fx+e)}\right)}{b(a+b)f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)}-\frac{\ln\left(e^{i(fx+e)}-i\right)}{b^2f}+\frac{\ln\left(e^{i(fx+e)}+i\right)}{b^2f}+\frac{\sqrt{a(a+b)}\ln\left(e^{2i(fx+e}$

input
```
int(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/2/b^2*ln(sin(f*x+e)+1)+a/b^2*(1/2/(a+b)*b*sin(f*x+e)/(a*sin(f*x+e)^
2-a-b)-1/2*(2*a+3*b)/(a+b)/(a*(a+b))^(1/2)*arctanh(a*sin(f*x+e)/(a*(a+b))^
(1/2)))-1/2/b^2*ln(sin(f*x+e)-1))
```

3.193.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 392, normalized size of antiderivative = 3.84

$$\int \frac{\sec^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$= \left[-\frac{2\,ab\sin(fx+e) - \left((2\,a^2+3\,ab)\cos(fx+e)^2+2\,ab+3\,b^2\right)\sqrt{\frac{a}{a+b}}\log\left(-\frac{a\cos(fx+e)^2+2\,(a+b)\sqrt{\frac{a}{a+b}}\sin(fx+e)}{a\cos(fx+e)^2+b}\right.}{4\left((a^2b\right.}\right.$$

$$\left. -\frac{ab\sin(fx+e) - \left((2\,a^2+3\,ab)\cos(fx+e)^2+2\,ab+3\,b^2\right)\sqrt{-\frac{a}{a+b}}\arctan\left(\sqrt{-\frac{a}{a+b}}\sin(fx+e)\right) - ((a}{2\left((a^2b^2+ab^3)f\right.} \right]$$

input
```
integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[-1/4*(2*a*b*sin(f*x + e) - ((2*a^2 + 3*a*b)*cos(f*x + e)^2 + 2*a*b + 3*b^
2)*sqrt(a/(a + b))*log(-(a*cos(f*x + e)^2 + 2*(a + b)*sqrt(a/(a + b))*sin(
f*x + e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) - 2*((a^2 + a*b)*cos(f*x + e)^
2 + a*b + b^2)*log(sin(f*x + e) + 1) + 2*((a^2 + a*b)*cos(f*x + e)^2 + a*b
 + b^2)*log(-sin(f*x + e) + 1))/((a^2*b^2 + a*b^3)*f*cos(f*x + e)^2 + (a*b
^3 + b^4)*f), -1/2*(a*b*sin(f*x + e) - ((2*a^2 + 3*a*b)*cos(f*x + e)^2 + 2
*a*b + 3*b^2)*sqrt(-a/(a + b))*arctan(sqrt(-a/(a + b))*sin(f*x + e)) - ((a
^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)*log(sin(f*x + e) + 1) + ((a^2 + a*b)
*cos(f*x + e)^2 + a*b + b^2)*log(-sin(f*x + e) + 1))/((a^2*b^2 + a*b^3)*f*
cos(f*x + e)^2 + (a*b^3 + b^4)*f)]
```

3.193.6 Sympy [F]

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

input
```
integrate(sec(f*x+e)**5/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral(sec(e + f*x)**5/(a + b*sec(e + f*x)**2)**2, x)
```

3.193.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.43

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$
$$= \frac{\frac{(2a+3b)a\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{(ab^2+b^3)\sqrt{(a+b)a}} - \frac{2a\sin(fx+e)}{a^2b+2ab^2+b^3-(a^2b+ab^2)\sin(fx+e)^2} + \frac{2\log(\sin(fx+e)+1)}{b^2} - \frac{2\log(\sin(fx+e)-1)}{b^2}}{4f}$$

input
```
integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/4*((2*a + 3*b)*a*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x + e)
+ sqrt((a + b)*a)))/((a*b^2 + b^3)*sqrt((a + b)*a)) - 2*a*sin(f*x + e)/(a^
2*b + 2*a*b^2 + b^3 - (a^2*b + a*b^2)*sin(f*x + e)^2) + 2*log(sin(f*x + e)
 + 1)/b^2 - 2*log(sin(f*x + e) - 1)/b^2)/f
```

3.193.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.24

$$\int \frac{\sec^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\frac{(2a^2+3ab)\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{(ab^2+b^3)\sqrt{-a^2-ab}} + \frac{a\sin(fx+e)}{\left(a\sin(fx+e)^2-a-b\right)(ab+b^2)} + \frac{\log(|\sin(fx+e)+1|)}{b^2} - \frac{\log(|\sin(fx+e)-1|)}{b^2}}{2f}$$

input
```
integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/2*((2*a^2 + 3*a*b)*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/((a*b^2 + b^3
)*sqrt(-a^2 - a*b)) + a*sin(f*x + e)/((a*sin(f*x + e)^2 - a - b)*(a*b + b^
2)) + log(abs(sin(f*x + e) + 1))/b^2 - log(abs(sin(f*x + e) - 1))/b^2)/f
```

3.193.9 Mupad [B] (verification not implemented)

Time = 21.33 (sec) , antiderivative size = 2039, normalized size of antiderivative = 19.99

$$\int \frac{\sec^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^2),x)
```

output

```
(atan(((((4*a^2*b^6 + 6*a^3*b^5 + 2*a^4*b^4)/(2*(2*a*b^4 + b^5 + a^2*b^3)
) - (sin(e + f*x)*(16*a^2*b^7 + 64*a^3*b^6 + 80*a^4*b^5 + 32*a^5*b^4))/(8*
b^2*(2*a*b^3 + b^4 + a^2*b^2)))*1i)/(2*b^2) + (sin(e + f*x)*(20*a^4*b + 8*
a^5 + 13*a^3*b^2)*1i)/(4*(2*a*b^3 + b^4 + a^2*b^2)))/b^2 - ((((4*a^2*b^6 +
 6*a^3*b^5 + 2*a^4*b^4)/(2*(2*a*b^4 + b^5 + a^2*b^3)) + (sin(e + f*x)*(16*
a^2*b^7 + 64*a^3*b^6 + 80*a^4*b^5 + 32*a^5*b^4))/(8*b^2*(2*a*b^3 + b^4 + a
^2*b^2)))*1i)/(2*b^2) - (sin(e + f*x)*(20*a^4*b + 8*a^5 + 13*a^3*b^2)*1i)/
(4*(2*a*b^3 + b^4 + a^2*b^2)))/b^2)/((((4*a^2*b^6 + 6*a^3*b^5 + 2*a^4*b^4)
/(2*(2*a*b^4 + b^5 + a^2*b^3)) - (sin(e + f*x)*(16*a^2*b^7 + 64*a^3*b^6 +
80*a^4*b^5 + 32*a^5*b^4))/(8*b^2*(2*a*b^3 + b^4 + a^2*b^2)))/(2*b^2) + (si
n(e + f*x)*(20*a^4*b + 8*a^5 + 13*a^3*b^2))/(4*(2*a*b^3 + b^4 + a^2*b^2)))
/b^2 + (((4*a^2*b^6 + 6*a^3*b^5 + 2*a^4*b^4)/(2*(2*a*b^4 + b^5 + a^2*b^3))
 + (sin(e + f*x)*(16*a^2*b^7 + 64*a^3*b^6 + 80*a^4*b^5 + 32*a^5*b^4))/(8*b
^2*(2*a*b^3 + b^4 + a^2*b^2)))/(2*b^2) - (sin(e + f*x)*(20*a^4*b + 8*a^5 +
 13*a^3*b^2))/(4*(2*a*b^3 + b^4 + a^2*b^2)))/b^2 - ((3*a^3*b)/2 + a^4)/(2*
a*b^4 + b^5 + a^2*b^3)))*1i)/(b^2*f) - (atan(((((sin(e + f*x)*(20*a^4*b +
8*a^5 + 13*a^3*b^2))/(2*(2*a*b^3 + b^4 + a^2*b^2)) + ((a*(a + b)^3)^(1/2)*
((4*a^2*b^6 + 6*a^3*b^5 + 2*a^4*b^4)/(2*a*b^4 + b^5 + a^2*b^3) - (sin(e +
f*x)*(a*(a + b)^3)^(1/2)*(2*a + 3*b)*(16*a^2*b^7 + 64*a^3*b^6 + 80*a^4*b^5
 + 32*a^5*b^4))/(8*(2*a*b^3 + b^4 + a^2*b^2)*(3*a*b^4 + b^5 + 3*a^2*b^3...
```

3.194 $\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

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3.194.1 Optimal result

Integrand size = 23, antiderivative size = 74

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2\sqrt{a}(a+b)^{3/2}f} + \frac{\sin(e+fx)}{2(a+b)f\left(a+b-a\sin^2(e+fx)\right)}$$

output
```
1/2*sin(f*x+e)/(a+b)/f/(a+b-a*sin(f*x+e)^2)+1/2*arctanh(sin(f*x+e)*a^(1/2)
/(a+b)^(1/2))/(a+b)^(3/2)/f/a^(1/2)
```

3.194.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.96

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a}(a+b)^{3/2}} + \frac{2\sin(e+fx)}{(a+b)(a+2b+a\cos(2(e+fx)))}}{2f}$$

input
```
Integrate[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]/(Sqrt[a]*(a + b)^(3/2)) + (2*
Sin[e + f*x])/((a + b)*(a + 2*b + a*Cos[2*(e + f*x)])))/(2*f)
```

3.194.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4635, 215, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sec(e+fx)^3}{(a+b\sec(e+fx)^2)^2}\,dx$$

$$\downarrow 4635$$

$$\frac{\int \frac{1}{(-a\sin^2(e+fx)+a+b)^2}\,d\sin(e+fx)}{f}$$

$$\downarrow 215$$

$$\frac{\int \frac{1}{-a\sin^2(e+fx)+a+b}\,d\sin(e+fx)}{2(a+b)} + \frac{\sin(e+fx)}{2(a+b)(-a\sin^2(e+fx)+a+b)}}{f}$$

$$\downarrow 221$$

$$\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2\sqrt{a}(a+b)^{3/2}} + \frac{\sin(e+fx)}{2(a+b)(-a\sin^2(e+fx)+a+b)}}{f}$$

input
```
Int[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]/(2*Sqrt[a]*(a + b)^(3/2)) + S
in[e + f*x]/(2*(a + b)*(a + b - a*Sin[e + f*x]^2)))/f
```

3.194.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.194.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{-\frac{\sin(fx+e)}{2(a+b)\left(a\sin(fx+e)^2-a-b\right)}+\frac{\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{2(a+b)\sqrt{a(a+b)}}}{f}$
default	$\frac{-\frac{\sin(fx+e)}{2(a+b)\left(a\sin(fx+e)^2-a-b\right)}+\frac{\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{2(a+b)\sqrt{a(a+b)}}}{f}$
risch	$-\frac{i\left(e^{3i(fx+e)}-e^{i(fx+e)}\right)}{f(a+b)\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)}+\frac{\ln\left(e^{2i(fx+e)}+\frac{2i(a+b)e^{i(fx+e)}}{\sqrt{a^2+ab}}-1\right)}{4\sqrt{a^2+ab}\,(a+b)f}-\frac{\ln\left(e^{2i(fx+e)}-\frac{2i(a+$

input
```
int(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/2*sin(f*x+e)/(a+b)/(a*sin(f*x+e)^2-a-b)+1/2/(a+b)/(a*(a+b))^(1/2)*
arctanh(a*sin(f*x+e)/(a*(a+b))^(1/2)))
```

3.194.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 262, normalized size of antiderivative = 3.54

$$\int \frac{\sec^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$
$$= \left[\frac{\left(a\cos(fx+e)^2+b\right)\sqrt{a^2+ab}\log\left(-\frac{a\cos(fx+e)^2-2\sqrt{a^2+ab}\sin(fx+e)-2a-b}{a\cos(fx+e)^2+b}\right)+2\left(a^2+ab\right)\sin(fx+e)}{4\left(\left(a^4+2a^3b+a^2b^2\right)f\cos(fx+e)^2+\left(a^3b+2a^2b^2+ab^3\right)f\right)},\right.$$
$$\left.-\frac{\left(a\cos(fx+e)^2+b\right)\sqrt{-a^2-ab}\arctan\left(\frac{\sqrt{-a^2-ab}\sin(fx+e)}{a+b}\right)-\left(a^2+ab\right)\sin(fx+e)}{2\left(\left(a^4+2a^3b+a^2b^2\right)f\cos(fx+e)^2+\left(a^3b+2a^2b^2+ab^3\right)f\right)}\right]$$

input
```
integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[1/4*((a*cos(f*x + e)^2 + b)*sqrt(a^2 + a*b)*log(-(a*cos(f*x + e)^2 - 2*sq
rt(a^2 + a*b)*sin(f*x + e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) + 2*(a^2 + a
*b)*sin(f*x + e))/((a^4 + 2*a^3*b + a^2*b^2)*f*cos(f*x + e)^2 + (a^3*b + 2
*a^2*b^2 + a*b^3)*f), -1/2*((a*cos(f*x + e)^2 + b)*sqrt(-a^2 - a*b)*arctan
(sqrt(-a^2 - a*b)*sin(f*x + e)/(a + b)) - (a^2 + a*b)*sin(f*x + e))/((a^4
+ 2*a^3*b + a^2*b^2)*f*cos(f*x + e)^2 + (a^3*b + 2*a^2*b^2 + a*b^3)*f)]
```

3.194.6 Sympy [F]

$$\int \frac{\sec^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \int \frac{\sec^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

input
```
integrate(sec(f*x+e)**3/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral(sec(e + f*x)**3/(a + b*sec(e + f*x)**2)**2, x)
```

3.194.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.32

$$\int \frac{\sec^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{\frac{2\sin(fx+e)}{(a^2+ab)\sin(fx+e)^2-a^2-2ab-b^2}+\frac{\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}(a+b)}}{4f}$$

input
```
integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/4*(2*sin(f*x + e)/((a^2 + a*b)*sin(f*x + e)^2 - a^2 - 2*a*b - b^2) + lo
g((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x + e) + sqrt((a + b)*a)))/(
sqrt((a + b)*a)*(a + b)))/f
```

3.194.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.03

$$\int \frac{\sec^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{\frac{\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{\sqrt{-a^2-ab}(a+b)}+\frac{\sin(fx+e)}{\left(a\sin(fx+e)^2-a-b\right)(a+b)}}{2f}$$

input
```
integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-1/2*(arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/(sqrt(-a^2 - a*b)*(a + b)) +
 sin(f*x + e)/((a*sin(f*x + e)^2 - a - b)*(a + b)))/f
```

3.194.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

$$\int \frac{\sec^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\sin(e+fx)}{2f\,(a+b)\,\left(-a\sin(e+fx)^2+a+b\right)}+\frac{\operatorname{atanh}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)}{2\sqrt{a}\,f\,(a+b)^{3/2}}$$

input
```
int(1/(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^2),x)
```

output
```
sin(e + f*x)/(2*f*(a + b)*(a + b - a*sin(e + f*x)^2)) + atanh((a^(1/2)*sin
(e + f*x))/(a + b)^(1/2))/(2*a^(1/2)*f*(a + b)^(3/2))
```

3.195 $\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

3.195.1 Optimal result 1460
3.195.2 Mathematica [A] (verified) 1460
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3.195.8 Giac [A] (verification not implemented) 1464
3.195.9 Mupad [B] (verification not implemented) 1464

3.195.1 Optimal result

Integrand size = 21, antiderivative size = 83

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{(2a+b)\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{3/2}(a+b)^{3/2}f} - \frac{b\sin(e+fx)}{2a(a+b)f\left(a+b-a\sin^2(e+fx)\right)}$$

output
```
1/2*(2*a+b)*arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))/a^(3/2)/(a+b)^(3/2)/f-
1/2*b*sin(f*x+e)/a/(a+b)/f/(a+b-a*sin(f*x+e)^2)
```

3.195.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.99

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{(2a+b)\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} - \frac{2\sqrt{a}b\sin(e+fx)}{(a+b)(a+2b+a\cos(2(e+fx)))}}{2a^{3/2}f}$$

input
```
Integrate[Sec[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(((2*a + b)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(a + b)^(3/2) - (
2*Sqrt[a]*b*Sin[e + f*x])/((a + b)*(a + 2*b + a*Cos[2*(e + f*x)])))/(2*a^(
3/2)*f)
```

3.195.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4635, 298, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx)^2)^2}dx$$

$$\downarrow 4635$$

$$\frac{\int \frac{1-\sin^2(e+fx)}{(-a\sin^2(e+fx)+a+b)^2}d\sin(e+fx)}{f}$$

$$\downarrow 298$$

$$\frac{\frac{(2a+b)\int \frac{1}{-a\sin^2(e+fx)+a+b}d\sin(e+fx)}{2a(a+b)} - \frac{b\sin(e+fx)}{2a(a+b)(-a\sin^2(e+fx)+a+b)}}{f}$$

$$\downarrow 221$$

$$\frac{\frac{(2a+b)\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{3/2}(a+b)^{3/2}} - \frac{b\sin(e+fx)}{2a(a+b)(-a\sin^2(e+fx)+a+b)}}{f}$$

input
```
Int[Sec[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(((2*a + b)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(2*a^(3/2)*(a + b
)^(3/2)) - (b*Sin[e + f*x])/(2*a*(a + b)*(a + b - a*Sin[e + f*x]^2)))/f
```

3.195.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.195.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{\frac{b\sin(fx+e)}{2a(a+b)\left(a\sin(fx+e)^2-a-b\right)}+\frac{(2a+b)\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{2a(a+b)\sqrt{a(a+b)}}}{f}$
default	$\frac{\frac{b\sin(fx+e)}{2a(a+b)\left(a\sin(fx+e)^2-a-b\right)}+\frac{(2a+b)\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{2a(a+b)\sqrt{a(a+b)}}}{f}$
risch	$\frac{ib\left(e^{3i(fx+e)}-e^{i(fx+e)}\right)}{af(a+b)\left(ae^{4i(fx+e)}+2ae^{2i(fx+e)}+4be^{2i(fx+e)}+a\right)}+\frac{\ln\left(e^{2i(fx+e)}+\frac{2i(a+b)e^{i(fx+e)}}{\sqrt{a^2+ab}}-1\right)}{2\sqrt{a^2+ab}\,(a+b)f}+\frac{\ln\left(e^{2i(fx+e)}+\frac{2i(a+b}{\sqrt{}}\right.}{4\sqrt{a^2+ab}\,(a}$

input
```
int(sec(f*x+e)/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/2*b/a/(a+b)*sin(f*x+e)/(a*sin(f*x+e)^2-a-b)+1/2*(2*a+b)/a/(a+b)/(a*
(a+b))^(1/2)*arctanh(a*sin(f*x+e)/(a*(a+b))^(1/2)))
```

3.195.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 301, normalized size of antiderivative = 3.63

$$\int \frac{\sec(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$
$$= \left[\frac{\left((2a^2+ab)\cos(fx+e)^2+2ab+b^2\right)\sqrt{a^2+ab}\log\left(-\frac{a\cos(fx+e)^2-2\sqrt{a^2+ab}\sin(fx+e)-2a-b}{a\cos(fx+e)^2+b}\right)-2\left(a^2b+ab^2\right)}{4\left(\left(a^5+2a^4b+a^3b^2\right)f\cos(fx+e)^2+\left(a^4b+2a^3b^2+a^2b^3\right)f\right)}\right.$$
$$\left.-\frac{\left((2a^2+ab)\cos(fx+e)^2+2ab+b^2\right)\sqrt{-a^2-ab}\arctan\left(\frac{\sqrt{-a^2-ab}\sin(fx+e)}{a+b}\right)+\left(a^2b+ab^2\right)\sin(fx+e)}{2\left(\left(a^5+2a^4b+a^3b^2\right)f\cos(fx+e)^2+\left(a^4b+2a^3b^2+a^2b^3\right)f\right)}\right]$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[1/4*(((2*a^2 + a*b)*cos(f*x + e)^2 + 2*a*b + b^2)*sqrt(a^2 + a*b)*log(-(a
*cos(f*x + e)^2 - 2*sqrt(a^2 + a*b)*sin(f*x + e) - 2*a - b)/(a*cos(f*x + e
)^2 + b)) - 2*(a^2*b + a*b^2)*sin(f*x + e))/((a^5 + 2*a^4*b + a^3*b^2)*f*c
os(f*x + e)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*f), -1/2*(((2*a^2 + a*b)*cos
(f*x + e)^2 + 2*a*b + b^2)*sqrt(-a^2 - a*b)*arctan(sqrt(-a^2 - a*b)*sin(f*
x + e)/(a + b)) + (a^2*b + a*b^2)*sin(f*x + e))/((a^5 + 2*a^4*b + a^3*b^2)
*f*cos(f*x + e)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*f)]
```

3.195.6 Sympy [F]

$$\int \frac{\sec(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \int \frac{\sec(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral(sec(e + f*x)/(a + b*sec(e + f*x)**2)**2, x)
```

3.195.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.34

$$\int \frac{\sec(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{\frac{2\,b\sin(fx+e)}{a^3+2\,a^2b+ab^2-(a^3+a^2b)\sin(fx+e)^2}+\frac{(2\,a+b)\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}(a^2+ab)}}{4\,f}$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/4*(2*b*sin(f*x + e)/(a^3 + 2*a^2*b + a*b^2 - (a^3 + a^2*b)*sin(f*x + e)
^2) + (2*a + b)*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x + e) + s
qrt((a + b)*a)))/(sqrt((a + b)*a)*(a^2 + a*b)))/f
```

3.195.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.10

$$\int \frac{\sec(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{\frac{(2\,a+b)\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{(a^2+ab)\sqrt{-a^2-ab}}-\frac{b\sin(fx+e)}{\left(a\sin(fx+e)^2-a-b\right)(a^2+ab)}}{2\,f}$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-1/2*((2*a + b)*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/((a^2 + a*b)*sqrt(
-a^2 - a*b)) - b*sin(f*x + e)/((a*sin(f*x + e)^2 - a - b)*(a^2 + a*b)))/f
```

3.195.9 Mupad [B] (verification not implemented)

Time = 19.74 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.86

$$\int \frac{\sec(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\operatorname{atanh}\left(\frac{\sqrt{a}\,\sin(e+f\,x)}{\sqrt{a+b}}\right)\,(2\,a+b)}{2\,a^{3/2}\,f\,(a+b)^{3/2}} - \frac{b\,\sin(e+f\,x)}{2\,a\,f\,\left(a+b\right)\,\left(-a\,\sin(e+f\,x)^2+a+b\right)}$$

input
```
int(1/(cos(e + f*x)*(a + b/cos(e + f*x)^2)^2),x)
```

output
```
(atanh((a^(1/2)*sin(e + f*x))/(a + b)^(1/2))*(2*a + b))/(2*a^(3/2)*f*(a +
b)^(3/2)) - (b*sin(e + f*x))/(2*a*f*(a + b)*(a + b - a*sin(e + f*x)^2))
```

3.196 $\int \frac{\cos(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

3.196.1 Optimal result 1466
3.196.2 Mathematica [A] (verified) 1466
3.196.3 Rubi [A] (verified) 1467
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3.196.5 Fricas [A] (verification not implemented) 1469
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3.196.1 Optimal result

Integrand size = 21, antiderivative size = 101

$$\int \frac{\cos(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{b(4a+3b)\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{5/2}(a+b)^{3/2}f} + \frac{\sin(e+fx)}{a^2 f} + \frac{b^2\sin(e+fx)}{2a^2(a+b)f\left(a+b-a\sin^2(e+fx)\right)}$$

output
```
-1/2*b*(4*a+3*b)*arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))/a^(5/2)/(a+b)^(3/
2)/f+sin(f*x+e)/a^2/f+1/2*b^2*sin(f*x+e)/a^2/(a+b)/f/(a+b-a*sin(f*x+e)^2)
```

3.196.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.88

$$\int \frac{\cos(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{-\frac{b(4a+3b)\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} + \sqrt{a}\sin(e+fx)\left(2+\frac{b^2}{(a+b)(a+b-a\sin^2(e+fx))}\right)}{2a^{5/2}f}$$

input
```
Integrate[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(-((b*(4*a + 3*b)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(a + b)^(3/
2)) + Sqrt[a]*Sin[e + f*x]*(2 + b^2/((a + b)*(a + b - a*Sin[e + f*x]^2))))
/(2*a^(5/2)*f)
```

3.196.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4635, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sec(e+fx)\left(a+b\sec(e+fx)^2\right)^2}dx$$

$$\downarrow 4635$$

$$\frac{\int \frac{\left(1-\sin^2(e+fx)\right)^2}{\left(-a\sin^2(e+fx)+a+b\right)^2}d\sin(e+fx)}{f}$$

$$\downarrow 300$$

$$\frac{\int \left(\frac{1}{a^2}-\frac{b(2a+b)-2ab\sin^2(e+fx)}{a^2\left(-a\sin^2(e+fx)+a+b\right)^2}\right)d\sin(e+fx)}{f}$$

$$\downarrow 2009$$

$$\frac{-\frac{b(4a+3b)\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{5/2}(a+b)^{3/2}}+\frac{b^2\sin(e+fx)}{2a^2(a+b)\left(-a\sin^2(e+fx)+a+b\right)}+\frac{\sin(e+fx)}{a^2}}{f}$$

input
```
Int[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(-1/2*(b*(4*a + 3*b)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(a^(5/2)
*(a + b)^(3/2)) + Sin[e + f*x]/a^2 + (b^2*Sin[e + f*x])/(2*a^2*(a + b)*(a
+ b - a*Sin[e + f*x]^2)))/f
```

3.196.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.196.4 Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{\frac{\sin(fx+e)}{a^2}+\frac{b\left(-\frac{b\sin(fx+e)}{2(a+b)\left(a\sin(fx+e)^2-a-b\right)}-\frac{(4a+3b)\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{2(a+b)\sqrt{a(a+b)}}\right)}{a^2}}{f}$
default	$\frac{\frac{\sin(fx+e)}{a^2}+\frac{b\left(-\frac{b\sin(fx+e)}{2(a+b)\left(a\sin(fx+e)^2-a-b\right)}-\frac{(4a+3b)\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{2(a+b)\sqrt{a(a+b)}}\right)}{a^2}}{f}$
risch	$-\frac{ie^{i(fx+e)}}{2a^2f}+\frac{ie^{-i(fx+e)}}{2a^2f}-\frac{ib^2\left(e^{3i(fx+e)}-e^{i(fx+e)}\right)}{a^2(a+b)f\left(ae^{4i(fx+e)}+2ae^{2i(fx+e)}+4be^{2i(fx+e)}+a\right)}+\frac{\ln\left(e^{2i(fx+e)}-\frac{2i(a+b)e^{i(fx+}}{\sqrt{a^2+ab}}\right.}{\sqrt{a^2+ab}\,(a+b)fa}$

input
```
int(cos(f*x+e)/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(sin(f*x+e)/a^2+1/a^2*b*(-1/2/(a+b)*b*sin(f*x+e)/(a*sin(f*x+e)^2-a-b)-
1/2*(4*a+3*b)/(a+b)/(a*(a+b))^(1/2)*arctanh(a*sin(f*x+e)/(a*(a+b))^(1/2)))
)
```

3.196.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 391, normalized size of antiderivative = 3.87

$$\int \frac{\cos(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$
$$= \left[\frac{\left(4\,ab^2+3\,b^3+\left(4\,a^2b+3\,ab^2\right)\cos\left(fx+e\right)^2\right)\sqrt{a^2+ab}\log\left(-\frac{a\cos(fx+e)^2+2\sqrt{a^2+ab}\sin(fx+e)-2\,a-b}{a\cos(fx+e)^2+b}\right)+2\left(2\,a^3\right.}{4\left(\left(a^6+2\,a^5b+a^4b^2\right)f\cos\left(fx+e\right)^2+\left(a^5b+2\,a\right.\right.}\right.$$

input
```
integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[1/4*((4*a*b^2 + 3*b^3 + (4*a^2*b + 3*a*b^2)*cos(f*x + e)^2)*sqrt(a^2 + a*
b)*log(-(a*cos(f*x + e)^2 + 2*sqrt(a^2 + a*b)*sin(f*x + e) - 2*a - b)/(a*c
os(f*x + e)^2 + b)) + 2*(2*a^3*b + 5*a^2*b^2 + 3*a*b^3 + 2*(a^4 + 2*a^3*b
+ a^2*b^2)*cos(f*x + e)^2)*sin(f*x + e))/((a^6 + 2*a^5*b + a^4*b^2)*f*cos(
f*x + e)^2 + (a^5*b + 2*a^4*b^2 + a^3*b^3)*f), 1/2*((4*a*b^2 + 3*b^3 + (4*
a^2*b + 3*a*b^2)*cos(f*x + e)^2)*sqrt(-a^2 - a*b)*arctan(sqrt(-a^2 - a*b)*
sin(f*x + e)/(a + b)) + (2*a^3*b + 5*a^2*b^2 + 3*a*b^3 + 2*(a^4 + 2*a^3*b
+ a^2*b^2)*cos(f*x + e)^2)*sin(f*x + e))/((a^6 + 2*a^5*b + a^4*b^2)*f*cos(
f*x + e)^2 + (a^5*b + 2*a^4*b^2 + a^3*b^3)*f)]
```

3.196.6 Sympy [F]

$$\int \frac{\cos(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \int \frac{\cos\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^2}\,dx$$

input
```
integrate(cos(f*x+e)/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral(cos(e + f*x)/(a + b*sec(e + f*x)**2)**2, x)
```

3.196.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.32

$$\int \frac{\cos(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\frac{2b^2\sin(fx+e)}{a^4+2a^3b+a^2b^2-(a^4+a^3b)\sin(fx+e)^2}+\frac{(4ab+3b^2)\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{(a^3+a^2b)\sqrt{(a+b)a}}+\frac{4\sin(fx+e)}{a^2}}{4f}$$

input
```
integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/4*(2*b^2*sin(f*x + e)/(a^4 + 2*a^3*b + a^2*b^2 - (a^4 + a^3*b)*sin(f*x +
 e)^2) + (4*a*b + 3*b^2)*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x
 + e) + sqrt((a + b)*a)))/((a^3 + a^2*b)*sqrt((a + b)*a)) + 4*sin(f*x + e)
/a^2)/f
```

3.196.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.12

$$\int \frac{\cos(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{\frac{b^2\sin(fx+e)}{(a^3+a^2b)\left(a\sin(fx+e)^2-a-b\right)}-\frac{(4ab+3b^2)\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{(a^3+a^2b)\sqrt{-a^2-ab}}-\frac{2\sin(fx+e)}{a^2}}{2f}$$

input
```
integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-1/2*(b^2*sin(f*x + e)/((a^3 + a^2*b)*(a*sin(f*x + e)^2 - a - b)) - (4*a*b
 + 3*b^2)*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/((a^3 + a^2*b)*sqrt(-a^2
 - a*b)) - 2*sin(f*x + e)/a^2)/f
```

3.196.9 Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.93

$$\int \frac{\cos(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\sin(e+fx)}{a^2 f} + \frac{b^2\sin(e+fx)}{2f\,(a+b)\left(-a^3\sin(e+fx)^2+a^3+ba^2\right)} - \frac{b\,\operatorname{atanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)(4a+3b)}{2a^{5/2}f\,(a+b)^{3/2}}$$

input
```
int(cos(e + f*x)/(a + b/cos(e + f*x)^2)^2,x)
```

output
```
sin(e + f*x)/(a^2*f) + (b^2*sin(e + f*x))/(2*f*(a + b)*(a^2*b + a^3 - a^3*
sin(e + f*x)^2)) - (b*atanh((a^(1/2)*sin(e + f*x))/(a + b)^(1/2))*(4*a + 3
*b))/(2*a^(5/2)*f*(a + b)^(3/2))
```

3.197 $\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

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3.197.1 Optimal result

Integrand size = 23, antiderivative size = 126

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{b^2(6a+5b)\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{7/2}(a+b)^{3/2}f} + \frac{(a-2b)\sin(e+fx)}{a^3 f} - \frac{\sin^3(e+fx)}{3a^2 f} - \frac{b^3\sin(e+fx)}{2a^3(a+b)f\,(a+b-a\sin^2(e+fx))}$$

output
```
1/2*b^2*(6*a+5*b)*arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))/a^(7/2)/(a+b)^(3
/2)/f+(a-2*b)*sin(f*x+e)/a^3/f-1/3*sin(f*x+e)^3/a^2/f-1/2*b^3*sin(f*x+e)/a
^3/(a+b)/f/(a+b-a*sin(f*x+e)^2)
```

3.197.2 Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.10

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{-\frac{3b^2(6a+5b)\left(\log\left(\sqrt{a+b}-\sqrt{a}\sin(e+fx)\right)-\log\left(\sqrt{a+b}+\sqrt{a}\sin(e+fx)\right)\right)}{(a+b)^{3/2}} + 3\sqrt{a}\left(3a-8b-\frac{4b^3}{(a+b)(a+2b+a\cos(2(e+fx)))}\right)\sin(e+f}{12a^{7/2}f}$$

input
```
Integrate[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((-3*b^2*(6*a + 5*b)*(Log[Sqrt[a + b] - Sqrt[a]*Sin[e + f*x]] - Log[Sqrt[a
 + b] + Sqrt[a]*Sin[e + f*x]]))/(a + b)^(3/2) + 3*Sqrt[a]*(3*a - 8*b - (4*
b^3)/((a + b)*(a + 2*b + a*Cos[2*(e + f*x)])))*Sin[e + f*x] + a^(3/2)*Sin[
3*(e + f*x)])/(12*a^(7/2)*f)
```

3.197.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4635, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sec(e+fx)^3\,(a+b\sec(e+fx)^2)^2}\,dx$$

$$\downarrow 4635$$

$$\frac{\int \frac{(1-\sin^2(e+fx))^3}{(-a\sin^2(e+fx)+a+b)^2}\,d\sin(e+fx)}{f}$$

$$\downarrow 300$$

$$\frac{\int \left(-\frac{\sin^2(e+fx)}{a^2}+\frac{a-2b}{a^3}+\frac{b^2(3a+2b)-3ab^2\sin^2(e+fx)}{a^3(-a\sin^2(e+fx)+a+b)^2}\right)d\sin(e+fx)}{f}$$

$$\downarrow 2009$$

$$\frac{\frac{b^2(6a+5b)\text{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{7/2}(a+b)^{3/2}}-\frac{b^3\sin(e+fx)}{2a^3(a+b)(-a\sin^2(e+fx)+a+b)}+\frac{(a-2b)\sin(e+fx)}{a^3}-\frac{\sin^3(e+fx)}{3a^2}}{f}$$

input
```
Int[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((b^2*(6*a + 5*b)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(2*a^(7/2)*
(a + b)^(3/2)) + ((a - 2*b)*Sin[e + f*x])/a^3 - Sin[e + f*x]^3/(3*a^2) - (
b^3*Sin[e + f*x])/(2*a^3*(a + b)*(a + b - a*Sin[e + f*x]^2)))/f
```

3.197.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.197.4 Maple [A] (verified)

Time = 3.10 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{-\frac{\frac{a\sin(fx+e)^3}{3}-\sin(fx+e)a+2\sin(fx+e)b}{a^3}-\frac{b^2\left(-\frac{b\sin(fx+e)}{2(a+b)\left(a\sin(fx+e)^2-a-b\right)}-\frac{(6a+5b)\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{2(a+b)\sqrt{a(a+b)}}\right)}{a^3}}{f}$
default	$\frac{-\frac{\frac{a\sin(fx+e)^3}{3}-\sin(fx+e)a+2\sin(fx+e)b}{a^3}-\frac{b^2\left(-\frac{b\sin(fx+e)}{2(a+b)\left(a\sin(fx+e)^2-a-b\right)}-\frac{(6a+5b)\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{2(a+b)\sqrt{a(a+b)}}\right)}{a^3}}{f}$
risch	$-\frac{ie^{3i(fx+e)}}{24a^2f}-\frac{3ie^{i(fx+e)}}{8a^2f}+\frac{ie^{i(fx+e)}b}{a^3f}+\frac{3ie^{-i(fx+e)}}{8a^2f}-\frac{ie^{-i(fx+e)}b}{a^3f}+\frac{ie^{-3i(fx+e)}}{24a^2f}+\frac{ib^3\left(e^{3i}\right.}{a^3(a+b)f\left(ae^{4i(fx+e)}\right.}$

input
```
int(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/a^3*(1/3*a*sin(f*x+e)^3-sin(f*x+e)*a+2*sin(f*x+e)*b)-b^2/a^3*(-1/2
/(a+b)*b*sin(f*x+e)/(a*sin(f*x+e)^2-a-b)-1/2*(6*a+5*b)/(a+b)/(a*(a+b))^(1/
2)*arctanh(a*sin(f*x+e)/(a*(a+b))^(1/2))))
```

3.197.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(115) = 230$.

Time = 0.31 (sec) , antiderivative size = 490, normalized size of antiderivative = 3.89

$$\int \frac{\cos^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$= \left[\frac{3\left(6\,ab^3+5\,b^4+\left(6\,a^2b^2+5\,ab^3\right)\cos\left(fx+e\right)^2\right)\sqrt{a^2+ab}\log\left(-\frac{a\cos(fx+e)^2-2\sqrt{a^2+ab}\sin(fx+e)-2\,a-b}{a\cos(fx+e)^2+b}\right)+2\,(4}{12\left((a^7+2\,a^6b+a^5\right.}\right.$$

$$\left.-\frac{3\left(6\,ab^3+5\,b^4+\left(6\,a^2b^2+5\,ab^3\right)\cos\left(fx+e\right)^2\right)\sqrt{-a^2-ab}\arctan\left(\frac{\sqrt{-a^2-ab}\sin(fx+e)}{a+b}\right)-\left(4\,a^4b-4\,a^3b^2\right.}{6\left((a^7+2\,a^6b+a^5b^2\right)f\,c}\right]$$

input
```
integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[1/12*(3*(6*a*b^3 + 5*b^4 + (6*a^2*b^2 + 5*a*b^3)*cos(f*x + e)^2)*sqrt(a^2
 + a*b)*log(-(a*cos(f*x + e)^2 - 2*sqrt(a^2 + a*b)*sin(f*x + e) - 2*a - b)
/(a*cos(f*x + e)^2 + b)) + 2*(4*a^4*b - 4*a^3*b^2 - 23*a^2*b^3 - 15*a*b^4
+ 2*(a^5 + 2*a^4*b + a^3*b^2)*cos(f*x + e)^4 + 2*(2*a^5 - a^4*b - 8*a^3*b^
2 - 5*a^2*b^3)*cos(f*x + e)^2)*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*
cos(f*x + e)^2 + (a^6*b + 2*a^5*b^2 + a^4*b^3)*f), -1/6*(3*(6*a*b^3 + 5*b^
4 + (6*a^2*b^2 + 5*a*b^3)*cos(f*x + e)^2)*sqrt(-a^2 - a*b)*arctan(sqrt(-a^
2 - a*b)*sin(f*x + e)/(a + b)) - (4*a^4*b - 4*a^3*b^2 - 23*a^2*b^3 - 15*a*
b^4 + 2*(a^5 + 2*a^4*b + a^3*b^2)*cos(f*x + e)^4 + 2*(2*a^5 - a^4*b - 8*a^
3*b^2 - 5*a^2*b^3)*cos(f*x + e)^2)*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2
)*f*cos(f*x + e)^2 + (a^6*b + 2*a^5*b^2 + a^4*b^3)*f)]
```

3.197.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**3/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Timed out
```

3.197.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.22

$$\int \frac{\cos^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{\frac{6b^3\sin(fx+e)}{a^5+2a^4b+a^3b^2-(a^5+a^4b)\sin(fx+e)^2}+\frac{3\left(6ab^2+5b^3\right)\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{(a^4+a^3b)\sqrt{(a+b)a}}+\frac{4\left(a\sin(fx+e)^3-3(a-2b)\sin(fx+e)\right)}{a^3}}{12f}$$

input
```
integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/12*(6*b^3*sin(f*x + e)/(a^5 + 2*a^4*b + a^3*b^2 - (a^5 + a^4*b)*sin(f*x
 + e)^2) + 3*(6*a*b^2 + 5*b^3)*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*s
in(f*x + e) + sqrt((a + b)*a)))/((a^4 + a^3*b)*sqrt((a + b)*a)) + 4*(a*sin
(f*x + e)^3 - 3*(a - 2*b)*sin(f*x + e))/a^3)/f
```

3.197.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.16

$$\int \frac{\cos^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{\frac{3b^3\sin(fx+e)}{(a^4+a^3b)\left(a\sin(fx+e)^2-a-b\right)}-\frac{3\left(6ab^2+5b^3\right)\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{(a^4+a^3b)\sqrt{-a^2-ab}}-\frac{2\left(a^4\sin(fx+e)^3-3a^4\sin(fx+e)+6a^3b\sin(fx+e)\right)}{a^6}}{6f}$$

input
```
integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/6*(3*b^3*sin(f*x + e)/((a^4 + a^3*b)*(a*sin(f*x + e)^2 - a - b)) - 3*(6*
a*b^2 + 5*b^3)*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/((a^4 + a^3*b)*sqrt
(-a^2 - a*b)) - 2*(a^4*sin(f*x + e)^3 - 3*a^4*sin(f*x + e) + 6*a^3*b*sin(f
*x + e))/a^6)/f
```

3.197.9 Mupad [B] (verification not implemented)

Time = 19.79 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.98

$$\int \frac{\cos^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{b^2\,\operatorname{atanh}\left(\frac{\sqrt{a}\,\sin(e+f\,x)}{\sqrt{a+b}}\right)\,(6\,a+5\,b)}{2\,a^{7/2}\,f\,(a+b)^{3/2}} - \frac{\sin\left(e+f\,x\right)^3}{3\,a^2\,f} - \frac{b^3\,\sin\left(e+f\,x\right)}{2\,f\,\left(a+b\right)\,\left(-a^4\sin\left(e+f\,x\right)^2+a^4+b\,a^3\right)} - \frac{\sin\left(e+f\,x\right)\,\left(\frac{2\,(a+b)}{a^3}-\frac{3}{a^2}\right)}{f}$$

input
```
int(cos(e + f*x)^3/(a + b/cos(e + f*x)^2)^2,x)
```

output
```
(b^2*atanh((a^(1/2)*sin(e + f*x))/(a + b)^(1/2))*(6*a + 5*b))/(2*a^(7/2)*f
*(a + b)^(3/2)) - sin(e + f*x)^3/(3*a^2*f) - (b^3*sin(e + f*x))/(2*f*(a +
b)*(a^3*b + a^4 - a^4*sin(e + f*x)^2)) - (sin(e + f*x)*((2*(a + b))/a^3 -
3/a^2))/f
```

3.198 $\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

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3.198.2 Mathematica [A] (verified) 1478
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3.198.9 Mupad [B] (verification not implemented) 1483

3.198.1 Optimal result

Integrand size = 23, antiderivative size = 157

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{b^3(8a+7b)\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{9/2}(a+b)^{3/2}f} + \frac{(a^2-2ab+3b^2)\sin(e+fx)}{a^4 f} - \frac{2(a-b)\sin^3(e+fx)}{3a^3 f} + \frac{\sin^5(e+fx)}{5a^2 f} + \frac{b^4\sin(e+fx)}{2a^4(a+b)f\,(a+b-a\sin^2(e+fx))}$$

output
```
-1/2*b^3*(8*a+7*b)*arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))/a^(9/2)/(a+b)^(
3/2)/f+(a^2-2*a*b+3*b^2)*sin(f*x+e)/a^4/f-2/3*(a-b)*sin(f*x+e)^3/a^3/f+1/5
*sin(f*x+e)^5/a^2/f+1/2*b^4*sin(f*x+e)/a^4/(a+b)/f/(a+b-a*sin(f*x+e)^2)
```

3.198.2 Mathematica [A] (verified)

Time = 1.65 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.09

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{60b^3(8a+7b)\left(\log\left(\sqrt{a+b}-\sqrt{a}\sin(e+fx)\right)-\log\left(\sqrt{a+b}+\sqrt{a}\sin(e+fx)\right)\right)}{(a+b)^{3/2}} + 30\sqrt{a}\left(5a^2-12ab+8b^2\left(3+\frac{b^2}{(a+b)(a+2b+a\cos(2(e+fx)}\right.\right.}{240a^{9/2}f}$$

input
```
Integrate[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((60*b^3*(8*a + 7*b)*(Log[Sqrt[a + b] - Sqrt[a]*Sin[e + f*x]] - Log[Sqrt[a
 + b] + Sqrt[a]*Sin[e + f*x]]))/(a + b)^(3/2) + 30*Sqrt[a]*(5*a^2 - 12*a*b
 + 8*b^2*(3 + b^2/((a + b)*(a + 2*b + a*Cos[2*(e + f*x)])))*Sin[e + f*x]
+ 5*a^(3/2)*(5*a - 8*b)*Sin[3*(e + f*x)] + 3*a^(5/2)*Sin[5*(e + f*x)])/(24
0*a^(9/2)*f)
```

3.198.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4635, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sec(e+fx)^5\left(a+b\sec(e+fx)^2\right)^2}dx$$

$$\downarrow 4635$$

$$\frac{\int \frac{\left(1-\sin^2(e+fx)\right)^4}{\left(-a\sin^2(e+fx)+a+b\right)^2}d\sin(e+fx)}{f}$$

$$\downarrow 300$$

$$\frac{\int \left(\frac{\sin^4(e+fx)}{a^2}-\frac{2(a-b)\sin^2(e+fx)}{a^3}+\frac{a^2-2ba+3b^2}{a^4}-\frac{b^3(4a+3b)-4ab^3\sin^2(e+fx)}{a^4\left(-a\sin^2(e+fx)+a+b\right)^2}\right)d\sin(e+fx)}{f}$$

$$\downarrow 2009$$

$$\frac{-\frac{b^3(8a+7b)\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{9/2}(a+b)^{3/2}}+\frac{b^4\sin(e+fx)}{2a^4(a+b)\left(-a\sin^2(e+fx)+a+b\right)}-\frac{2(a-b)\sin^3(e+fx)}{3a^3}+\frac{\sin^5(e+fx)}{5a^2}+\frac{\left(a^2-2ab+3b^2\right)\sin(e+fx)}{a^4}}{f}$$

input
```
Int[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(-1/2*(b^3*(8*a + 7*b)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(a^(9/
2)*(a + b)^(3/2)) + ((a^2 - 2*a*b + 3*b^2)*Sin[e + f*x])/a^4 - (2*(a - b)*
Sin[e + f*x]^3)/(3*a^3) + Sin[e + f*x]^5/(5*a^2) + (b^4*Sin[e + f*x])/(2*a
^4*(a + b)*(a + b - a*Sin[e + f*x]^2)))/f
```

3.198.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.198.4 Maple [A] (verified)

Time = 4.72 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{\frac{a^2\sin(fx+e)^5}{5}-\frac{2a^2\sin(fx+e)^3}{3}+\frac{2a\sin(fx+e)^3b}{3}+\sin(fx+e)a^2-2\sin(fx+e)ab+3\sin(fx+e)b^2}{a^4}+\frac{b^3\left(-\frac{b\sin(fx+e)}{2(a+b)\left(a\sin(fx+e)^2-a-b\right.}\right.}{f}$
default	$\frac{\frac{a^2\sin(fx+e)^5}{5}-\frac{2a^2\sin(fx+e)^3}{3}+\frac{2a\sin(fx+e)^3b}{3}+\sin(fx+e)a^2-2\sin(fx+e)ab+3\sin(fx+e)b^2}{a^4}+\frac{b^3\left(-\frac{b\sin(fx+e)}{2(a+b)\left(a\sin(fx+e)^2-a-b\right.}\right.}{f}$
risch	$-\frac{5ie^{3i(fx+e)}}{96a^2f}+\frac{ie^{3i(fx+e)}b}{12a^3f}-\frac{3ie^{-i(fx+e)}b}{4a^3f}+\frac{3ie^{-i(fx+e)}b^2}{2fa^4}+\frac{5ie^{-3i(fx+e)}}{96a^2f}-\frac{3ie^{i(fx+e)}b^2}{2fa^4}-\frac{5ie^{i(fx+e)}}{16a^2f}+$

input
```
int(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/a^4*(1/5*a^2*sin(f*x+e)^5-2/3*a^2*sin(f*x+e)^3+2/3*a*sin(f*x+e)^3*b
+sin(f*x+e)*a^2-2*sin(f*x+e)*a*b+3*sin(f*x+e)*b^2)+b^3/a^4*(-1/2/(a+b)*b*s
in(f*x+e)/(a*sin(f*x+e)^2-a-b)-1/2*(8*a+7*b)/(a+b)/(a*(a+b))^(1/2)*arctanh
(a*sin(f*x+e)/(a*(a+b))^(1/2))))
```

3.198.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 583, normalized size of antiderivative = 3.71

$$\int \frac{\cos^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$
$$= \left[\frac{15\left(8\,ab^4+7\,b^5+\left(8\,a^2b^3+7\,ab^4\right)\cos\left(fx+e\right)^2\right)\sqrt{a^2+ab}\log\left(-\frac{a\cos(fx+e)^2+2\sqrt{a^2+ab}\sin(fx+e)-2\,a-b}{a\cos(fx+e)^2+b}\right)+2\,(\right.$$

input
```
integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[1/60*(15*(8*a*b^4 + 7*b^5 + (8*a^2*b^3 + 7*a*b^4)*cos(f*x + e)^2)*sqrt(a^
2 + a*b)*log(-(a*cos(f*x + e)^2 + 2*sqrt(a^2 + a*b)*sin(f*x + e) - 2*a - b
)/(a*cos(f*x + e)^2 + b)) + 2*(6*(a^6 + 2*a^5*b + a^4*b^2)*cos(f*x + e)^6
+ 16*a^5*b - 8*a^4*b^2 + 26*a^3*b^3 + 155*a^2*b^4 + 105*a*b^5 + 2*(4*a^6 +
 a^5*b - 10*a^4*b^2 - 7*a^3*b^3)*cos(f*x + e)^4 + 2*(8*a^6 + 11*a^4*b^2 +
54*a^3*b^3 + 35*a^2*b^4)*cos(f*x + e)^2)*sin(f*x + e))/((a^8 + 2*a^7*b + a
^6*b^2)*f*cos(f*x + e)^2 + (a^7*b + 2*a^6*b^2 + a^5*b^3)*f), 1/30*(15*(8*a
*b^4 + 7*b^5 + (8*a^2*b^3 + 7*a*b^4)*cos(f*x + e)^2)*sqrt(-a^2 - a*b)*arct
an(sqrt(-a^2 - a*b)*sin(f*x + e)/(a + b)) + (6*(a^6 + 2*a^5*b + a^4*b^2)*c
os(f*x + e)^6 + 16*a^5*b - 8*a^4*b^2 + 26*a^3*b^3 + 155*a^2*b^4 + 105*a*b^
5 + 2*(4*a^6 + a^5*b - 10*a^4*b^2 - 7*a^3*b^3)*cos(f*x + e)^4 + 2*(8*a^6 +
 11*a^4*b^2 + 54*a^3*b^3 + 35*a^2*b^4)*cos(f*x + e)^2)*sin(f*x + e))/((a^8
 + 2*a^7*b + a^6*b^2)*f*cos(f*x + e)^2 + (a^7*b + 2*a^6*b^2 + a^5*b^3)*f)]
```

3.198.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**5/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Timed out
```

3.198.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.17

$$\int \frac{\cos^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$
$$= \frac{\frac{30\,b^4\sin(fx+e)}{a^6+2\,a^5b+a^4b^2-(a^6+a^5b)\sin(fx+e)^2} + \frac{15\,(8\,ab^3+7\,b^4)\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{(a^5+a^4b)\sqrt{(a+b)a}} + \frac{4\left(3\,a^2\sin(fx+e)^5-10\,(a^2-ab)\sin(fx+e)^3+15\,(a^2-}{a^4}}{60\,f}$$

input
```
integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/60*(30*b^4*sin(f*x + e)/(a^6 + 2*a^5*b + a^4*b^2 - (a^6 + a^5*b)*sin(f*x
 + e)^2) + 15*(8*a*b^3 + 7*b^4)*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*
sin(f*x + e) + sqrt((a + b)*a)))/((a^5 + a^4*b)*sqrt((a + b)*a)) + 4*(3*a^
2*sin(f*x + e)^5 - 10*(a^2 - a*b)*sin(f*x + e)^3 + 15*(a^2 - 2*a*b + 3*b^2
)*sin(f*x + e))/a^4)/f
```

3.198.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.20

$$\int \frac{\cos^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx =$$
$$-\frac{\frac{15\,b^4\sin(fx+e)}{(a^5+a^4b)\left(a\sin(fx+e)^2-a-b\right)} - \frac{15\,(8\,ab^3+7\,b^4)\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{(a^5+a^4b)\sqrt{-a^2-ab}} - \frac{2\left(3\,a^8\sin(fx+e)^5-10\,a^8\sin(fx+e)^3+10\,a^7b\sin(fx+e)^3+15\,a}{a^{10}}}{30\,f}$$

input
```
integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-1/30*(15*b^4*sin(f*x + e)/((a^5 + a^4*b)*(a*sin(f*x + e)^2 - a - b)) - 15
*(8*a*b^3 + 7*b^4)*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/((a^5 + a^4*b)*
sqrt(-a^2 - a*b)) - 2*(3*a^8*sin(f*x + e)^5 - 10*a^8*sin(f*x + e)^3 + 10*a
^7*b*sin(f*x + e)^3 + 15*a^8*sin(f*x + e) - 30*a^7*b*sin(f*x + e) + 45*a^6
*b^2*sin(f*x + e))/a^10)/f
```

3.198.9 Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.10

$$\int \frac{\cos^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\sin(e+fx)^5}{5a^2f} + \frac{\sin(e+fx)^3\left(\frac{2(a+b)}{3a^3}-\frac{4}{3a^2}\right)}{f} + \frac{\sin(e+fx)\left(\frac{6}{a^2}-\frac{(a+b)^2}{a^4}+\frac{2(a+b)\left(\frac{2(a+b)}{a^3}-\frac{4}{a^2}\right)}{a}\right)}{f} + \frac{b^4\sin(e+fx)}{2f\,(a+b)\left(-a^5\sin(e+fx)^2+a^5+ba^4\right)} - \frac{b^3\,\mathrm{atanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)(8a+7b)}{2a^{9/2}f\,(a+b)^{3/2}}$$

input
```
int(cos(e + f*x)^5/(a + b/cos(e + f*x)^2)^2,x)
```

output
```
sin(e + f*x)^5/(5*a^2*f) + (sin(e + f*x)^3*((2*(a + b))/(3*a^3) - 4/(3*a^2
)))/f + (sin(e + f*x)*(6/a^2 - (a + b)^2/a^4 + (2*(a + b)*((2*(a + b))/a^3
 - 4/a^2))/a))/f + (b^4*sin(e + f*x))/(2*f*(a + b)*(a^4*b + a^5 - a^5*sin(
e + f*x)^2)) - (b^3*atanh((a^(1/2)*sin(e + f*x))/(a + b)^(1/2))*(8*a + 7*b
))/(2*a^(9/2)*f*(a + b)^(3/2))
```

3.199 $\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

3.199.1 Optimal result 1484
3.199.2 Mathematica [C] (warning: unable to verify) 1484
3.199.3 Rubi [A] (verified) 1485
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3.199.1 Optimal result

Integrand size = 23, antiderivative size = 100

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{a(3a+4b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2b^{5/2}(a+b)^{3/2}f} + \frac{\tan(e+fx)}{b^2 f} + \frac{a^2\tan(e+fx)}{2b^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)}$$

output
```
-1/2*a*(3*a+4*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/b^(5/2)/(a+b)^(3/2
)/f+tan(f*x+e)/b^2/f+1/2*a^2*tan(f*x+e)/b^2/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

3.199.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.16 (sec) , antiderivative size = 248, normalized size of antiderivative = 2.48

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^4(e+fx)\left(\frac{a(3a+4b)\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(a+2b+a}{(a+b)^{3/2}\sqrt{b(\cos(e)-i\sin(e))^4}}\right.}{8b^2 f\,(a}$$

input
```
Integrate[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*((a*(3*a + 4*b)*ArcTan[(Sec
[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/
(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x
)])*(Cos[2*e] - I*Sin[2*e]))/((a + b)^(3/2)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
 + 2*(a + 2*b + a*Cos[2*(e + f*x)])*Sec[e]*Sec[e + f*x]*Sin[f*x] + (a*(-((
a + 2*b)*Sin[2*e]) + a*Sin[2*f*x]))/((a + b)*(Cos[e] - Sin[e])*(Cos[e] + S
in[e])))/(8*b^2*f*(a + b*Sec[e + f*x]^2)^2)
```

3.199.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4634, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sec(e+fx)^6}{(a+b\sec(e+fx)^2)^2}\,dx$$

$$\downarrow 4634$$

$$\frac{\int \frac{(\tan^2(e+fx)+1)^2}{(b\tan^2(e+fx)+a+b)^2}\,d\tan(e+fx)}{f}$$

$$\downarrow 300$$

$$\frac{\int \left(\frac{1}{b^2}-\frac{2ab\tan^2(e+fx)+a(a+2b)}{b^2(b\tan^2(e+fx)+a+b)^2}\right)\,d\tan(e+fx)}{f}$$

$$\downarrow 2009$$

$$\frac{\frac{a^2\tan(e+fx)}{2b^2(a+b)(a+b\tan^2(e+fx)+b)}-\frac{a(3a+4b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2b^{5/2}(a+b)^{3/2}}+\frac{\tan(e+fx)}{b^2}}{f}$$

input
```
Int[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(-1/2*(a*(3*a + 4*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(b^(5/2)*
(a + b)^(3/2)) + Tan[e + f*x]/b^2 + (a^2*Tan[e + f*x])/(2*b^2*(a + b)*(a +
 b + b*Tan[e + f*x]^2)))/f
```

3.199.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.199.4 Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{\frac{\tan(fx+e)}{b^2}-\frac{a\left(-\frac{a\tan(fx+e)}{2(a+b)\left(a+b+b\tan(fx+e)^2\right)}+\frac{(3a+4b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}\right)}{b^2}}{f}$
default	$\frac{\frac{\tan(fx+e)}{b^2}-\frac{a\left(-\frac{a\tan(fx+e)}{2(a+b)\left(a+b+b\tan(fx+e)^2\right)}+\frac{(3a+4b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}\right)}{b^2}}{f}$
risch	$\frac{i\left(3a^2e^{4i(fx+e)}+4ab\,e^{4i(fx+e)}+6a^2e^{2i(fx+e)}+14ab\,e^{2i(fx+e)}+8b^2e^{2i(fx+e)}+3a^2+2ab\right)}{(a+b)b^2f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)\left(e^{2i(fx+e)}+1\right)}-\frac{3a^2\ln\left(e^{2i(fx+e)}-\frac{2iba+2i}{4\sqrt{-ab-b}}\right.}{}$

input
```
int(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(tan(f*x+e)/b^2-a/b^2*(-1/2*a/(a+b)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/
2*(3*a+4*b)/(a+b)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))))
```

3.199.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(88) = 176$.

Time $= 0.31$ (sec) , antiderivative size $= 516$, normalized size of antiderivative $= 5.16$

$$\int \frac{\sec^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$= \left[-\frac{\left((3\,a^3+4\,a^2b)\cos(fx+e)^3+(3\,a^2b+4\,ab^2)\cos(fx+e)\right)\sqrt{-ab-b^2}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,a\right.}{8\left((a^3b^3+2\,a^2b^4-\right.}\right.$$

input
```
integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[-1/8*(((3*a^3 + 4*a^2*b)*cos(f*x + e)^3 + (3*a^2*b + 4*a*b^2)*cos(f*x + e
))*sqrt(-a*b - b^2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b +
 4*b^2)*cos(f*x + e)^2 - 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqr
t(-a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)
^2 + b^2)) - 4*(2*a^2*b^2 + 4*a*b^3 + 2*b^4 + (3*a^3*b + 5*a^2*b^2 + 2*a*b
^3)*cos(f*x + e)^2)*sin(f*x + e))/((a^3*b^3 + 2*a^2*b^4 + a*b^5)*f*cos(f*x
 + e)^3 + (a^2*b^4 + 2*a*b^5 + b^6)*f*cos(f*x + e)), 1/4*(((3*a^3 + 4*a^2*
b)*cos(f*x + e)^3 + (3*a^2*b + 4*a*b^2)*cos(f*x + e))*sqrt(a*b + b^2)*arct
an(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*
x + e))) + 2*(2*a^2*b^2 + 4*a*b^3 + 2*b^4 + (3*a^3*b + 5*a^2*b^2 + 2*a*b^3
)*cos(f*x + e)^2)*sin(f*x + e))/((a^3*b^3 + 2*a^2*b^4 + a*b^5)*f*cos(f*x +
 e)^3 + (a^2*b^4 + 2*a*b^5 + b^6)*f*cos(f*x + e))]
```

3.199.6 Sympy [F]

$$\int \frac{\sec^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \int \frac{\sec^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

input
```
integrate(sec(f*x+e)**6/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral(sec(e + f*x)**6/(a + b*sec(e + f*x)**2)**2, x)
```

3.199.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.10

$$\int \frac{\sec^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\frac{a^2\tan(fx+e)}{a^2b^2+2\,ab^3+b^4+(ab^3+b^4)\tan(fx+e)^2} - \frac{(3\,a^2+4\,ab)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(ab^2+b^3)\sqrt{(a+b)b}} + \frac{2\tan(fx+e)}{b^2}}{2\,f}$$

input
```
integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/2*(a^2*tan(f*x + e)/(a^2*b^2 + 2*a*b^3 + b^4 + (a*b^3 + b^4)*tan(f*x + e
)^2) - (3*a^2 + 4*a*b)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a*b^2 + b^
3)*sqrt((a + b)*b)) + 2*tan(f*x + e)/b^2)/f
```

3.199.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.20

$$\int \frac{\sec^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$
$$= \frac{\frac{a^2\tan(fx+e)}{(ab^2+b^3)\left(b\tan(fx+e)^2+a+b\right)} - \frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(3a^2+4ab)}{(ab^2+b^3)\sqrt{ab+b^2}} + \frac{2\tan(fx+e)}{b^2}}{2f}$$

input
```
integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/2*(a^2*tan(f*x + e)/((a*b^2 + b^3)*(b*tan(f*x + e)^2 + a + b)) - (pi*flo
or((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))*(3
*a^2 + 4*a*b)/((a*b^2 + b^3)*sqrt(a*b + b^2)) + 2*tan(f*x + e)/b^2)/f
```

3.199.9 Mupad [B] (verification not implemented)

Time = 19.57 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.13

$$\int \frac{\sec^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\tan(e+fx)}{b^2 f} + \frac{a^2\tan(e+fx)}{2f\,(a+b)\,\left(b^3\tan(e+fx)^2+b^3+ab^2\right)} - \frac{a\,\operatorname{atan}\left(\frac{a\sqrt{b}\tan(e+fx)\,(3a+4b)}{\sqrt{a+b}\,(3a^2+4ba)}\right)\,(3a+4b)}{2b^{5/2}f\,(a+b)^{3/2}}$$

input
```
int(1/(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^2),x)
```

output
```
tan(e + f*x)/(b^2*f) + (a^2*tan(e + f*x))/(2*f*(a + b)*(a*b^2 + b^3 + b^3*
tan(e + f*x)^2)) - (a*atan((a*b^(1/2)*tan(e + f*x)*(3*a + 4*b))/((a + b)^(
1/2)*(4*a*b + 3*a^2)))*(3*a + 4*b))/(2*b^(5/2)*f*(a + b)^(3/2))
```

3.200 $\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

3.200.1 Optimal result 1490
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3.200.1 Optimal result

Integrand size = 23, antiderivative size = 82

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{(a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2b^{3/2}(a+b)^{3/2}f} - \frac{a\tan(e+fx)}{2b(a+b)f\left(a+b+b\tan^2(e+fx)\right)}$$

output
```
1/2*(a+2*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/b^(3/2)/(a+b)^(3/2)/f-1
/2*a*tan(f*x+e)/b/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

3.200.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.02

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{(a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} - \frac{a\sqrt{b}\sin(2(e+fx))}{(a+b)(a+2b+a\cos(2(e+fx)))}}{2b^{3/2}f}$$

input
```
Integrate[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(((a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(3/2) - (a
*Sqrt[b]*Sin[2*(e + f*x)])/((a + b)*(a + 2*b + a*Cos[2*(e + f*x)])))/(2*b^
(3/2)*f)
```

3.200.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4634, 298, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sec(e+fx)^4}{(a+b\sec(e+fx)^2)^2}\,dx$$

$$\downarrow 4634$$

$$\frac{\int \frac{\tan^2(e+fx)+1}{(b\tan^2(e+fx)+a+b)^2}\,d\tan(e+fx)}{f}$$

$$\downarrow 298$$

$$\frac{\frac{(a+2b)\int \frac{1}{b\tan^2(e+fx)+a+b}\,d\tan(e+fx)}{2b(a+b)} - \frac{a\tan(e+fx)}{2b(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

$$\downarrow 218$$

$$\frac{\frac{(a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2b^{3/2}(a+b)^{3/2}} - \frac{a\tan(e+fx)}{2b(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

input
```
Int[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(((a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(2*b^(3/2)*(a + b)
^(3/2)) - (a*Tan[e + f*x])/(2*b*(a + b)*(a + b + b*Tan[e + f*x]^2)))/f
```

3.200.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.200.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\dfrac{-\frac{a\tan(fx+e)}{2(a+b)b\left(a+b+b\tan(fx+e)^2\right)}+\frac{(a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)b\sqrt{(a+b)b}}}{f}$
default	$\dfrac{-\frac{a\tan(fx+e)}{2(a+b)b\left(a+b+b\tan(fx+e)^2\right)}+\frac{(a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)b\sqrt{(a+b)b}}}{f}$
risch	$-\frac{i\left(a\,\mathrm{e}^{2i(fx+e)}+2b\,\mathrm{e}^{2i(fx+e)}+a\right)}{bf(a+b)\left(a\,\mathrm{e}^{4i(fx+e)}+2a\,\mathrm{e}^{2i(fx+e)}+4b\,\mathrm{e}^{2i(fx+e)}+a\right)}-\frac{a\ln\left(\mathrm{e}^{2i(fx+e)}+\frac{2iba+2ib^2+a\sqrt{-ab-b^2}+2b\sqrt{-ab-b^2}}{a\sqrt{-ab-b^2}}\right)}{4\sqrt{-ab-b^2}\,(a+b)fb}-\ln$

input
```
int(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/2*a/(a+b)/b*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(a+2*b)/(a+b)/b/((
a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))
```

3.200.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(70) = 140.

Time = 0.28 (sec) , antiderivative size = 406, normalized size of antiderivative = 4.95

$$\int \frac{\sec^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$= \left[-\frac{4\left(a^2b+ab^2\right)\cos(fx+e)\sin(fx+e)+\left(\left(a^2+2ab\right)\cos(fx+e)^2+ab+2b^2\right)\sqrt{-ab-b^2}\log\left(\frac{(a^2+8a}{8\left(\left(a^3b^2+2a^2b^3+ab^4\right)f\cos(fx+e)^2+\right.}\right.$$

$$\left. -\frac{2\left(a^2b+ab^2\right)\cos(fx+e)\sin(fx+e)+\left(\left(a^2+2ab\right)\cos(fx+e)^2+ab+2b^2\right)\sqrt{ab+b^2}\arctan\left(\frac{(a+}{2\sqrt{ab+}}\right.}{4\left(\left(a^3b^2+2a^2b^3+ab^4\right)f\cos(fx+e)^2+\left(a^2b^3+2ab^4+b^5\right)f\right)}$$

input
```
integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[-1/8*(4*(a^2*b + a*b^2)*cos(f*x + e)*sin(f*x + e) + ((a^2 + 2*a*b)*cos(f*
x + e)^2 + a*b + 2*b^2)*sqrt(-a*b - b^2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*
x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a + 2*b)*cos(f*x + e)^3
 - b*cos(f*x + e))*sqrt(-a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4
 + 2*a*b*cos(f*x + e)^2 + b^2)))/((a^3*b^2 + 2*a^2*b^3 + a*b^4)*f*cos(f*x
 + e)^2 + (a^2*b^3 + 2*a*b^4 + b^5)*f), -1/4*(2*(a^2*b + a*b^2)*cos(f*x + e
)*sin(f*x + e) + ((a^2 + 2*a*b)*cos(f*x + e)^2 + a*b + 2*b^2)*sqrt(a*b + b
^2)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a*b + b^2)*cos(f*x + e
)*sin(f*x + e))))/((a^3*b^2 + 2*a^2*b^3 + a*b^4)*f*cos(f*x + e)^2 + (a^2*b
^3 + 2*a*b^4 + b^5)*f)]
```

3.200.6 Sympy [F]

$$\int \frac{\sec^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \int \frac{\sec^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

input
```
integrate(sec(f*x+e)**4/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral(sec(e + f*x)**4/(a + b*sec(e + f*x)**2)**2, x)
```

3.200.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.07

$$\int \frac{\sec^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{\frac{a\tan(fx+e)}{a^2b+2ab^2+b^3+(ab^2+b^3)\tan(fx+e)^2} - \frac{(a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}(ab+b^2)}}{2f}$$

input
```
integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/2*(a*tan(f*x + e)/(a^2*b + 2*a*b^2 + b^3 + (a*b^2 + b^3)*tan(f*x + e)^2
) - (a + 2*b)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*(a*b
 + b^2)))/f
```

3.200.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.09

$$\int \frac{\sec^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(a+2b)}{(ab+b^2)^{\frac{3}{2}}} - \frac{a\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)(ab+b^2)}}{2f}$$

input
```
integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/2*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
 + b^2)))*(a + 2*b)/(a*b + b^2)^(3/2) - a*tan(f*x + e)/((b*tan(f*x + e)^2
+ a + b)*(a*b + b^2)))/f
```

3.200.9 Mupad [B] (verification not implemented)

Time = 19.61 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.85

$$\int \frac{\sec^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)\,(a+2b)}{2\,b^{3/2}\,f\,(a+b)^{3/2}} - \frac{a\tan(e+fx)}{2\,b\,f\,\left(a+b\right)\,\left(b\tan\left(e+fx\right)^2+a+b\right)}$$

input
```
int(1/(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^2),x)
```

output
```
(atan((b^(1/2)*tan(e + f*x))/(a + b)^(1/2))*(a + 2*b))/(2*b^(3/2)*f*(a + b
)^(3/2)) - (a*tan(e + f*x))/(2*b*f*(a + b)*(a + b + b*tan(e + f*x)^2))
```

3.201 $\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

3.201.1 Optimal result 1496
3.201.2 Mathematica [C] (verified) 1496
3.201.3 Rubi [A] (verified) 1497
3.201.4 Maple [A] (verified) 1498
3.201.5 Fricas [B] (verification not implemented) 1499
3.201.6 Sympy [F] 1499
3.201.7 Maxima [A] (verification not implemented) 1500
3.201.8 Giac [A] (verification not implemented) 1500
3.201.9 Mupad [B] (verification not implemented) 1500

3.201.1 Optimal result

Integrand size = 23, antiderivative size = 73

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2\sqrt{b}(a+b)^{3/2}f} + \frac{\tan(e+fx)}{2(a+b)f\left(a+b+b\tan^2(e+fx)\right)}$$

output
```
1/2*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/(a+b)^(3/2)/f/b^(1/2)+1/2*tan(f
*x+e)/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

3.201.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.52 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.89

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^4(e+fx)\left(-\frac{\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(a+2b+a\cos(2(e-$$

$$8(a+b)f\left(a+b\sec^2(e+fx)\right)^2$$

input
```
Integrate[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*(-((ArcTan[(Sec[f*x]*(Cos[2
*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a +
b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[2*e
] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])) + (-((a + 2*
b)*Sin[2*e]) + a*Sin[2*f*x])/(a*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))))/(8*
(a + b)*f*(a + b*Sec[e + f*x]^2)^2)
```

3.201.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4634, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sec(e+fx)^2}{(a+b\sec(e+fx)^2)^2}\,dx$$

$$\downarrow 4634$$

$$\frac{\int \frac{1}{(b\tan^2(e+fx)+a+b)^2}\,d\tan(e+fx)}{f}$$

$$\downarrow 215$$

$$\frac{\frac{\int \frac{1}{b\tan^2(e+fx)+a+b}\,d\tan(e+fx)}{2(a+b)} + \frac{\tan(e+fx)}{2(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

$$\downarrow 218$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2\sqrt{b}(a+b)^{3/2}} + \frac{\tan(e+fx)}{2(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

input
```
Int[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]]/(2*Sqrt[b]*(a + b)^(3/2)) + Ta
n[e + f*x]/(2*(a + b)*(a + b + b*Tan[e + f*x]^2)))/f
```

3.201.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.201.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{\frac{\tan(fx+e)}{2(a+b)\left(a+b+b\tan(fx+e)^2\right)}+\frac{\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}}{f}$
default	$\frac{\frac{\tan(fx+e)}{2(a+b)\left(a+b+b\tan(fx+e)^2\right)}+\frac{\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}}{f}$
risch	$\frac{i\left(a\,e^{2i(fx+e)}+2b\,e^{2i(fx+e)}+a\right)}{af(a+b)\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)}-\frac{\ln\left(e^{2i(fx+e)}+\frac{2iba+2ib^2+a\sqrt{-ab-b^2}+2b\sqrt{-ab-b^2}}{a\sqrt{-ab-b^2}}\right)}{4\sqrt{-ab-b^2}\,(a+b)f}+\frac{\ln\left(e^{2}\right.}{}$

input
```
int(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/2*tan(f*x+e)/(a+b)/(a+b+b*tan(f*x+e)^2)+1/2/(a+b)/((a+b)*b)^(1/2)*a
rctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))
```

3.201.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(61) = 122$.

Time $= 0.28$ (sec) , antiderivative size $= 368$, normalized size of antiderivative $= 5.04$

$$\int \frac{\sec^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$= \left[\frac{4\left(ab+b^2\right)\cos\left(fx+e\right)\sin\left(fx+e\right) - \left(a\cos\left(fx+e\right)^2+b\right)\sqrt{-ab-b^2}\log\left(\frac{\left(a^2+8\,ab+8\,b^2\right)\cos(fx+e)^4-2\left(3\,ab+\right.\right.}{8\left(\left(a^3b+2\,a^2b^2+ab^3\right)f\cos\left(fx+e\right)^2+\left(a^2b^2+2\,ab\right.\right.}\right.$$

input
```
integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[1/8*(4*(a*b + b^2)*cos(f*x + e)*sin(f*x + e) - (a*cos(f*x + e)^2 + b)*sqr
t(-a*b - b^2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2
)*cos(f*x + e)^2 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b
 - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b
^2)))/((a^3*b + 2*a^2*b^2 + a*b^3)*f*cos(f*x + e)^2 + (a^2*b^2 + 2*a*b^3 +
 b^4)*f), 1/4*(2*(a*b + b^2)*cos(f*x + e)*sin(f*x + e) - (a*cos(f*x + e)^2
 + b)*sqrt(a*b + b^2)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a*b
+ b^2)*cos(f*x + e)*sin(f*x + e))))/((a^3*b + 2*a^2*b^2 + a*b^3)*f*cos(f*x
 + e)^2 + (a^2*b^2 + 2*a*b^3 + b^4)*f)]
```

3.201.6 Sympy [F]

$$\int \frac{\sec^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \int \frac{\sec^2\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^2}\,dx$$

input
```
integrate(sec(f*x+e)**2/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral(sec(e + f*x)**2/(a + b*sec(e + f*x)**2)**2, x)
```

3.201.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.97

$$\int \frac{\sec^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\frac{\tan(fx+e)}{(ab+b^2)\tan(fx+e)^2+a^2+2ab+b^2} + \frac{\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}(a+b)}}{2f}$$

input
```
integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/2*(tan(f*x + e)/((a*b + b^2)*tan(f*x + e)^2 + a^2 + 2*a*b + b^2) + arcta
n(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*(a + b)))/f
```

3.201.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.14

$$\int \frac{\sec^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\frac{\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)}{\sqrt{ab+b^2}(a+b)} + \frac{\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)(a+b)}}{2f}$$

input
```
integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/2*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
 + b^2)))/(sqrt(a*b + b^2)*(a + b)) + tan(f*x + e)/((b*tan(f*x + e)^2 + a
+ b)*(a + b)))/f
```

3.201.9 Mupad [B] (verification not implemented)

Time = 19.35 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.95

$$\int \frac{\sec^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$
$$= \frac{\tan(e+fx)}{2f\,(a+b)\left(b\tan(e+fx)^2+a+b\right)} + \frac{\operatorname{atan}\left(\frac{\sqrt{b}\tan(e+fx)\,(2a+2b)}{2\,(a+b)^{3/2}}\right)}{2\sqrt{b}f\,(a+b)^{3/2}}$$

input
```
int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^2),x)
```

output
```
tan(e + f*x)/(2*f*(a + b)*(a + b + b*tan(e + f*x)^2)) + atan((b^(1/2)*tan(
e + f*x)*(2*a + 2*b))/(2*(a + b)^(3/2)))/(2*b^(1/2)*f*(a + b)^(3/2))
```

3.202 $\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

3.202.1 Optimal result 1502
3.202.2 Mathematica [C] (warning: unable to verify) 1502
3.202.3 Rubi [A] (verified) 1503
3.202.4 Maple [A] (verified) 1505
3.202.5 Fricas [B] (verification not implemented) 1506
3.202.6 Sympy [F] 1506
3.202.7 Maxima [A] (verification not implemented) 1507
3.202.8 Giac [A] (verification not implemented) 1507
3.202.9 Mupad [B] (verification not implemented) 1508

3.202.1 Optimal result

Integrand size = 14, antiderivative size = 92

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{x}{a^2} - \frac{\sqrt{b}(3a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{3/2}f} - \frac{b\tan(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)}$$

output
```
x/a^2-1/2*(3*a+2*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(1/2)/a^2/(a+
b)^(3/2)/f-1/2*b*tan(f*x+e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

3.202.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.41 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.61

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^4(e+fx)\left(2x(a+2b+a\cos(2(e+fx))) + \frac{b(3a+2b)\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e}{2\sqrt{a+b}}\right.\right.}{8a^2\left(a+b\sec^2(e+\right.}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(-2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*(2*x*(a + 2*b + a*Cos[2*(e
+ f*x)]) + (b*(3*a + 2*b)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a +
 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin
[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[2*e] - I*Sin[2*e]))/((a + b)
^(3/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (b*((a + 2*b)*Sin[2*e] - a*Sin[2
*f*x]))/((a + b)*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e])))/(8*a^2*(a + b*Se
c[e + f*x]^2)^2)
```

3.202.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.21, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4616, 316, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sec^2(e+fx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{(a+b\sec(e+fx)^2)^2}\,dx$$

$$\downarrow 4616$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^2}\,d\tan(e+fx)}{f}$$

$$\downarrow 316$$

$$\frac{\int \frac{-b\tan^2(e+fx)+2a+b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}\,d\tan(e+fx)}{2a(a+b)} - \frac{b\tan(e+fx)}{2a(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

$$\downarrow 397$$

$$\frac{\frac{2(a+b)\int \frac{1}{\tan^2(e+fx)+1}\,d\tan(e+fx)}{a} - \frac{b(3a+2b)\int \frac{1}{b\tan^2(e+fx)+a+b}\,d\tan(e+fx)}{a}}{2a(a+b)} - \frac{b\tan(e+fx)}{2a(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

$$\downarrow 216$$

$$\frac{\frac{2(a+b)\arctan(\tan(e+fx))}{a}-\frac{b(3a+2b)\int\frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{2a(a+b)}-\frac{b\tan(e+fx)}{2a(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

$$\downarrow 218$$

$$\frac{\frac{2(a+b)\arctan(\tan(e+fx))}{a}-\frac{\sqrt{b}(3a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{2a(a+b)}-\frac{b\tan(e+fx)}{2a(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^(-2),x]
```

output
```
(((2*(a + b)*ArcTan[Tan[e + f*x]])/a - (Sqrt[b]*(3*a + 2*b)*ArcTan[(Sqrt[b
]*Tan[e + f*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(2*a*(a + b)) - (b*Tan[e +
f*x])/(2*a*(a + b)*(a + b + b*Tan[e + f*x]^2)))/f
```

3.202.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f  Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

3.202.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{a\tan(fx+e)}{2(a+b)\left(a+b+b\tan(fx+e)^2\right)}+\frac{(3a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}\right)}{a^2}+\frac{\arctan(\tan(fx+e))}{a^2}}{f}$
default	$\frac{-\frac{b\left(\frac{a\tan(fx+e)}{2(a+b)\left(a+b+b\tan(fx+e)^2\right)}+\frac{(3a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}\right)}{a^2}+\frac{\arctan(\tan(fx+e))}{a^2}}{f}$
risch	$\frac{x}{a^2}-\frac{ib\left(a\,e^{2i(fx+e)}+2b\,e^{2i(fx+e)}+a\right)}{a^2(a+b)f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)}+\frac{3\sqrt{-(a+b)b}\,\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-(a+b)b}+a+2b}{a}\right)}{4(a+b)^2fa}+$

input
```
int(1/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-b/a^2*(1/2*a/(a+b)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(3*a+2*b)/(a+
b)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))+1/a^2*arctan(tan(
f*x+e)))
```

3.202.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(80) = 160$.

Time $= 0.29$ (sec) , antiderivative size $= 435$, normalized size of antiderivative $= 4.73$

$$\int \frac{1}{\left(a + b\sec^2(e+fx)\right)^2}\,dx$$

$$= \left[\frac{8\left(a^2+ab\right)fx\cos\left(fx+e\right)^2 - 4\,ab\cos\left(fx+e\right)\sin\left(fx+e\right) + 8\left(ab+b^2\right)fx + \left(\left(3\,a^2+2\,ab\right)\cos\left(fx+e\right)\right.}{8\left(\left(a^4+a^{3}\right.\right.}\right.$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[1/8*(8*(a^2 + a*b)*f*x*cos(f*x + e)^2 - 4*a*b*cos(f*x + e)*sin(f*x + e) +
 8*(a*b + b^2)*f*x + ((3*a^2 + 2*a*b)*cos(f*x + e)^2 + 3*a*b + 2*b^2)*sqrt
(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)
*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*co
s(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a
*b*cos(f*x + e)^2 + b^2)))/((a^4 + a^3*b)*f*cos(f*x + e)^2 + (a^3*b + a^2*
b^2)*f), 1/4*(4*(a^2 + a*b)*f*x*cos(f*x + e)^2 - 2*a*b*cos(f*x + e)*sin(f*
x + e) + 4*(a*b + b^2)*f*x + ((3*a^2 + 2*a*b)*cos(f*x + e)^2 + 3*a*b + 2*b
^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a +
b))/(b*cos(f*x + e)*sin(f*x + e))))/((a^4 + a^3*b)*f*cos(f*x + e)^2 + (a^3
*b + a^2*b^2)*f)]
```

3.202.6 Sympy [F]

$$\int \frac{1}{\left(a + b\sec^2(e+fx)\right)^2}\,dx = \int \frac{1}{\left(a + b\sec^2\left(e+fx\right)\right)^2}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(-2), x)
```

3.202.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.15

$$\int \frac{1}{(a+b\sec^2(e+fx))^2}\,dx$$
$$= -\frac{\frac{b\tan(fx+e)}{a^3+2a^2b+ab^2+(a^2b+ab^2)\tan(fx+e)^2} + \frac{(3ab+2b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^3+a^2b)\sqrt{(a+b)b}} - \frac{2(fx+e)}{a^2}}{2f}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/2*(b*tan(f*x + e)/(a^3 + 2*a^2*b + a*b^2 + (a^2*b + a*b^2)*tan(f*x + e)
^2) + (3*a*b + 2*b^2)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^3 + a^2*b
)*sqrt((a + b)*b)) - 2*(f*x + e)/a^2)/f
```

3.202.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.24

$$\int \frac{1}{(a+b\sec^2(e+fx))^2}\,dx$$
$$= -\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(3ab+2b^2)}{(a^3+a^2b)\sqrt{ab+b^2}} + \frac{b\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)(a^2+ab)} - \frac{2(fx+e)}{a^2}}{2f}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-1/2*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*
b + b^2)))*(3*a*b + 2*b^2)/((a^3 + a^2*b)*sqrt(a*b + b^2)) + b*tan(f*x + e
)/((b*tan(f*x + e)^2 + a + b)*(a^2 + a*b)) - 2*(f*x + e)/a^2)/f
```

3.202.9 Mupad [B] (verification not implemented)

Time = 21.72 (sec) , antiderivative size = 2056, normalized size of antiderivative = 22.35

$$\int \frac{1}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(1/(a + b/cos(e + f*x)^2)^2,x)
```

output
```
atan((((((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)*1i)/(2*(2*a^4*b + a^5 + a^3*b
^2)) - (tan(e + f*x)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/
(8*a^2*(2*a^3*b + a^4 + a^2*b^2)))/(2*a^2) + (tan(e + f*x)*(20*a*b^4 + 8*b
^5 + 13*a^2*b^3))/(4*(2*a^3*b + a^4 + a^2*b^2)))/a^2 - ((((2*a^4*b^4 + 6*a
^5*b^3 + 4*a^6*b^2)*1i)/(2*(2*a^4*b + a^5 + a^3*b^2)) + (tan(e + f*x)*(32*
a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*a^2*(2*a^3*b + a^4 + a
^2*b^2)))/(2*a^2) - (tan(e + f*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3))/(4*(2*a
^3*b + a^4 + a^2*b^2)))/a^2)/((((((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)*1i)/
(2*(2*a^4*b + a^5 + a^3*b^2)) - (tan(e + f*x)*(32*a^4*b^5 + 80*a^5*b^4 + 6
4*a^6*b^3 + 16*a^7*b^2))/(8*a^2*(2*a^3*b + a^4 + a^2*b^2)))*1i)/(2*a^2) +
(tan(e + f*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3)*1i)/(4*(2*a^3*b + a^4 + a^2*
b^2)))/a^2 + (((((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)*1i)/(2*(2*a^4*b + a^5
 + a^3*b^2)) + (tan(e + f*x)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^
7*b^2))/(8*a^2*(2*a^3*b + a^4 + a^2*b^2)))*1i)/(2*a^2) - (tan(e + f*x)*(20
*a*b^4 + 8*b^5 + 13*a^2*b^3)*1i)/(4*(2*a^3*b + a^4 + a^2*b^2)))/a^2 + ((3*
a*b^3)/2 + b^4)/(2*a^4*b + a^5 + a^3*b^2)))/(a^2*f) + (atan((((tan(e + f*
x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3))/(2*(2*a^3*b + a^4 + a^2*b^2)) - ((-b*(
a + b)^3)^(1/2)*((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)/(2*a^4*b + a^5 + a^3*
b^2) - (tan(e + f*x)*(-b*(a + b)^3)^(1/2)*(3*a + 2*b)*(32*a^4*b^5 + 80*a^5
*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*(2*a^3*b + a^4 + a^2*b^2)*(3*a^4*b ...
```

3.203 $\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

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3.203.1 Optimal result

Integrand size = 23, antiderivative size = 142

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{(a-4b)x}{2a^3} + \frac{b^{3/2}(5a+4b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^3(a+b)^{3/2}f} + \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)} + \frac{b(a+2b)\tan(e+fx)}{2a^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)}$$

output
```
1/2*(a-4*b)*x/a^3+1/2*b^(3/2)*(5*a+4*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1
/2))/a^3/(a+b)^(3/2)/f+1/2*cos(f*x+e)*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)+
1/2*b*(a+2*b)*tan(f*x+e)/a^2/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

3.203.2 Mathematica [A] (verified)

Time = 1.23 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.73

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{2(a-4b)(e+fx) + \frac{2b^{3/2}(5a+4b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} + \left(a + \frac{2ab^2}{(a+b)(a+2b+a\cos(2(e+fx)))}\right)\sin(2(e+fx))}{4a^3 f}$$

input
```
Integrate[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(2*(a - 4*b)*(e + f*x) + (2*b^(3/2)*(5*a + 4*b)*ArcTan[(Sqrt[b]*Tan[e + f*
x])/Sqrt[a + b]])/(a + b)^(3/2) + (a + (2*a*b^2)/((a + b)*(a + 2*b + a*Cos
[2*(e + f*x)])))*Sin[2*(e + f*x)])/(4*a^3*f)
```

3.203.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.13, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4634, 316, 25, 402, 27, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sec(e+fx)^2\,(a+b\sec(e+fx)^2)^2}\,dx$$

$$\downarrow 4634$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)^2(b\tan^2(e+fx)+a+b)^2}\,d\tan(e+fx)}{f}$$

$$\downarrow 316$$

$$\frac{\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)} - \frac{\int -\frac{3b\tan^2(e+fx)+a-b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}\,d\tan(e+fx)}{2a}}{f}$$

$$\downarrow 25$$

$$\frac{\frac{\int \frac{3b\tan^2(e+fx)+a-b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}\,d\tan(e+fx)}{2a} + \frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)}}{f}$$

$$\downarrow 402$$

$$\frac{\frac{\int \frac{2\left(a^2-2ba-2b^2+b(a+2b)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}\,d\tan(e+fx)}{2a(a+b)} + \frac{b(a+2b)\tan(e+fx)}{a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{2a} + \frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)}}{f}$$

$$\downarrow 27$$

$$\frac{\frac{\int \frac{a^2-2ba-2b^2+b(a+2b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{a(a+b)}+\frac{b(a+2b)\tan(e+fx)}{a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{2a}+\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)}}{f}$$

$$\downarrow 397$$

$$\frac{\frac{b^2(5a+4b)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}+\frac{(a-4b)(a+b)\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}}{a(a+b)}+\frac{b(a+2b)\tan(e+fx)}{a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{2a}+\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e-}}{f}$$

$$\downarrow 216$$

$$\frac{\frac{\frac{b^2(5a+4b)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}+\frac{(a-4b)(a+b)\arctan(\tan(e+fx))}{a}}{a(a+b)}+\frac{b(a+2b)\tan(e+fx)}{a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{2a}+\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)}}{f}$$

$$\downarrow 218$$

$$\frac{\frac{\frac{b^{3/2}(5a+4b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}+\frac{(a-4b)(a+b)\arctan(\tan(e+fx))}{a}}{a(a+b)}+\frac{b(a+2b)\tan(e+fx)}{a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{2a}+\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)}}{f}$$

input
```
Int[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(Tan[e + f*x]/(2*a*(1 + Tan[e + f*x]^2)*(a + b + b*Tan[e + f*x]^2)) + ((((
a - 4*b)*(a + b)*ArcTan[Tan[e + f*x]])/a + (b^(3/2)*(5*a + 4*b)*ArcTan[(Sq
rt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(a*(a + b)) + (b*(a + 2
*b)*Tan[e + f*x])/(a*(a + b)*(a + b + b*Tan[e + f*x]^2)))/(2*a))/f
```

3.203.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.203.4 Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{b^2\left(\frac{a\tan(fx+e)}{2(a+b)\left(a+b+b\tan(fx+e)^2\right)}+\frac{(5a+4b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}\right)}{a^3}+\frac{\frac{a\tan(fx+e)}{2+2\tan(fx+e)^2}+\frac{(a-4b)\arctan(\tan(fx+e))}{2}}{a^3}}{f}$
default	$\frac{b^2\left(\frac{a\tan(fx+e)}{2(a+b)\left(a+b+b\tan(fx+e)^2\right)}+\frac{(5a+4b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}\right)}{a^3}+\frac{\frac{a\tan(fx+e)}{2+2\tan(fx+e)^2}+\frac{(a-4b)\arctan(\tan(fx+e))}{2}}{a^3}}{f}$
risch	$\frac{x}{2a^2}-\frac{2xb}{a^3}-\frac{ie^{2i(fx+e)}}{8a^2f}+\frac{ie^{-2i(fx+e)}}{8a^2f}+\frac{ib^2\left(ae^{2i(fx+e)}+2be^{2i(fx+e)}+a\right)}{a^3(a+b)f\left(ae^{4i(fx+e)}+2ae^{2i(fx+e)}+4be^{2i(fx+e)}+a\right)}+\frac{5\sqrt{-(a+b)b}$

input
```
int(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(b^2/a^3*(1/2*a/(a+b)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(5*a+4*b)/(a
+b)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))+1/a^3*(1/2*a*tan
(f*x+e)/(1+tan(f*x+e)^2)+1/2*(a-4*b)*arctan(tan(f*x+e))))
```

3.203.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 544, normalized size of antiderivative = 3.83

$$\int \frac{\cos^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$=\left[\frac{4\left(a^3-3a^2b-4ab^2\right)fx\cos\left(fx+e\right)^2+4\left(a^2b-3ab^2-4b^3\right)fx+\left(5ab^2+4b^3+\left(5a^2b+4ab^2\right)\cos\left(fx\right.\right.\right.}{}\right.$$

input
```
integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[1/8*(4*(a^3 - 3*a^2*b - 4*a*b^2)*f*x*cos(f*x + e)^2 + 4*(a^2*b - 3*a*b^2
- 4*b^3)*f*x + (5*a*b^2 + 4*b^3 + (5*a^2*b + 4*a*b^2)*cos(f*x + e)^2)*sqrt
(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)
*cos(f*x + e)^2 - 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*co
s(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a
*b*cos(f*x + e)^2 + b^2)) + 4*((a^3 + a^2*b)*cos(f*x + e)^3 + (a^2*b + 2*a
*b^2)*cos(f*x + e))*sin(f*x + e))/((a^5 + a^4*b)*f*cos(f*x + e)^2 + (a^4*b
 + a^3*b^2)*f), 1/4*(2*(a^3 - 3*a^2*b - 4*a*b^2)*f*x*cos(f*x + e)^2 + 2*(a
^2*b - 3*a*b^2 - 4*b^3)*f*x - (5*a*b^2 + 4*b^3 + (5*a^2*b + 4*a*b^2)*cos(f
*x + e)^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(
b/(a + b))/(b*cos(f*x + e)*sin(f*x + e))) + 2*((a^3 + a^2*b)*cos(f*x + e)^
3 + (a^2*b + 2*a*b^2)*cos(f*x + e))*sin(f*x + e))/((a^5 + a^4*b)*f*cos(f*x
 + e)^2 + (a^4*b + a^3*b^2)*f)]
```

3.203.6 Sympy [F]

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

input
```
integrate(cos(f*x+e)**2/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral(cos(e + f*x)**2/(a + b*sec(e + f*x)**2)**2, x)
```

3.203.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.23

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{(5ab^2+4b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^4+a^3b)\sqrt{(a+b)b}} + \frac{(ab+2b^2)\tan(fx+e)^3+(a^2+2ab+2b^2)\tan(fx+e)}{(a^3b+a^2b^2)\tan(fx+e)^4+a^4+2a^3b+a^2b^2+(a^4+3a^3b+2a^2b^2)\tan(fx+e)^2} + \frac{(fx+e)(a-4b)}{a^3}}{2f}$$

input
```
integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/2*((5*a*b^2 + 4*b^3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^4 + a^3*
b)*sqrt((a + b)*b)) + ((a*b + 2*b^2)*tan(f*x + e)^3 + (a^2 + 2*a*b + 2*b^2
)*tan(f*x + e))/((a^3*b + a^2*b^2)*tan(f*x + e)^4 + a^4 + 2*a^3*b + a^2*b^
2 + (a^4 + 3*a^3*b + 2*a^2*b^2)*tan(f*x + e)^2) + (f*x + e)*(a - 4*b)/a^3)
/f
```

3.203.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.35

$$\int \frac{\cos^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$
$$= \frac{\frac{(5ab^2+4b^3)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^4+a^3b)\sqrt{ab+b^2}} + \frac{ab\tan(fx+e)^3+2b^2\tan(fx+e)^3+a^2\tan(fx+e)+2ab\tan(fx+e)+2b^2\tan(fx+e}{\left(b\tan(fx+e)^4+a\tan(fx+e)^2+2b\tan(fx+e)^2+a+b\right)(a^3+a^2b)}}{2f}$$

input
```
integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/2*((5*a*b^2 + 4*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan
(f*x + e)/sqrt(a*b + b^2)))/((a^4 + a^3*b)*sqrt(a*b + b^2)) + (a*b*tan(f*x
 + e)^3 + 2*b^2*tan(f*x + e)^3 + a^2*tan(f*x + e) + 2*a*b*tan(f*x + e) + 2
*b^2*tan(f*x + e))/((b*tan(f*x + e)^4 + a*tan(f*x + e)^2 + 2*b*tan(f*x + e
)^2 + a + b)*(a^3 + a^2*b)) + (f*x + e)*(a - 4*b)/a^3)/f
```

3.203.9 Mupad [B] (verification not implemented)

Time = 22.47 (sec) , antiderivative size = 2401, normalized size of antiderivative = 16.91

$$\int \frac{\cos^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(cos(e + f*x)^2/(a + b/cos(e + f*x)^2)^2,x)
```

output

```
((tan(e + f*x)*(2*a*b + a^2 + 2*b^2))/(2*a^2*(a + b)) + (b*tan(e + f*x)^3*
(a + 2*b))/(2*a^2*(a + b)))/(f*(a + b + b*tan(e + f*x)^4 + tan(e + f*x)^2*
(a + 2*b))) - (atan(((((tan(e + f*x)*(64*a*b^6 + 32*b^7 + 26*a^2*b^5 - 6*a
^3*b^4 + a^4*b^3))/(2*(2*a^5*b + a^6 + a^4*b^2)) - (((4*a^6*b^5 + 8*a^7*b^
4 + 2*a^8*b^3 - 2*a^9*b^2)/(2*a^7*b + a^8 + a^6*b^2) - (tan(e + f*x)*(a*1i
 - b*4i)*(32*a^6*b^5 + 80*a^7*b^4 + 64*a^8*b^3 + 16*a^9*b^2))/(8*a^3*(2*a^
5*b + a^6 + a^4*b^2)))*(a*1i - b*4i))/(4*a^3))*(a*1i - b*4i)*1i)/(4*a^3) +
 (((tan(e + f*x)*(64*a*b^6 + 32*b^7 + 26*a^2*b^5 - 6*a^3*b^4 + a^4*b^3))/(
2*(2*a^5*b + a^6 + a^4*b^2)) + (((4*a^6*b^5 + 8*a^7*b^4 + 2*a^8*b^3 - 2*a^
9*b^2)/(2*a^7*b + a^8 + a^6*b^2) + (tan(e + f*x)*(a*1i - b*4i)*(32*a^6*b^5
 + 80*a^7*b^4 + 64*a^8*b^3 + 16*a^9*b^2))/(8*a^3*(2*a^5*b + a^6 + a^4*b^2)
))*(a*1i - b*4i))/(4*a^3))*(a*1i - b*4i)*1i)/(4*a^3))/((12*a*b^6 + 8*b^7 +
 (3*a^2*b^5)/2 - (5*a^3*b^4)/4)/(2*a^7*b + a^8 + a^6*b^2) - (((tan(e + f*x
)*(64*a*b^6 + 32*b^7 + 26*a^2*b^5 - 6*a^3*b^4 + a^4*b^3))/(2*(2*a^5*b + a^
6 + a^4*b^2)) - (((4*a^6*b^5 + 8*a^7*b^4 + 2*a^8*b^3 - 2*a^9*b^2)/(2*a^7*b
 + a^8 + a^6*b^2) - (tan(e + f*x)*(a*1i - b*4i)*(32*a^6*b^5 + 80*a^7*b^4 +
 64*a^8*b^3 + 16*a^9*b^2))/(8*a^3*(2*a^5*b + a^6 + a^4*b^2)))*(a*1i - b*4i
))/(4*a^3))*(a*1i - b*4i))/(4*a^3) + (((tan(e + f*x)*(64*a*b^6 + 32*b^7 +
26*a^2*b^5 - 6*a^3*b^4 + a^4*b^3))/(2*(2*a^5*b + a^6 + a^4*b^2)) + (((4*a^
6*b^5 + 8*a^7*b^4 + 2*a^8*b^3 - 2*a^9*b^2)/(2*a^7*b + a^8 + a^6*b^2) + ...
```

3.204 $\int \frac{\cos^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

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3.204.1 Optimal result

Integrand size = 23, antiderivative size = 203

$$\begin{aligned}\int \frac{\cos^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx &= \frac{\left(3a^2-8ab+24b^2\right)x}{8a^4} - \frac{b^{5/2}(7a+6b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^4(a+b)^{3/2}f} \\ &+ \frac{3(a-2b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)} \\ &+ \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)} \\ &+ \frac{(a-3b)b(3a+4b)\tan(e+fx)}{8a^3(a+b)f\left(a+b+b\tan^2(e+fx)\right)}\end{aligned}$$

output

```
1/8*(3*a^2-8*a*b+24*b^2)*x/a^4-1/2*b^(5/2)*(7*a+6*b)*arctan(b^(1/2)*tan(f*
x+e)/(a+b)^(1/2))/a^4/(a+b)^(3/2)/f+3/8*(a-2*b)*cos(f*x+e)*sin(f*x+e)/a^2/
f/(a+b+b*tan(f*x+e)^2)+1/4*cos(f*x+e)^3*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2
)+1/8*(a-3*b)*b*(3*a+4*b)*tan(f*x+e)/a^3/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

3.204.2 Mathematica [A] (verified)

Time = 1.57 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.68

$$\int \frac{\cos^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$
$$= \frac{4(3a^2-8ab+24b^2)(e+fx) - \frac{16b^{5/2}(7a+6b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} + 8a(a-2b)\sin(2(e+fx)) - \frac{16ab^3\sin(2(e}{(a+b)(a+2b+a\cos}}{32a^4f}$$

input
```
Integrate[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(4*(3*a^2 - 8*a*b + 24*b^2)*(e + f*x) - (16*b^(5/2)*(7*a + 6*b)*ArcTan[(Sq
rt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(3/2) + 8*a*(a - 2*b)*Sin[2*(e +
 f*x)] - (16*a*b^3*Sin[2*(e + f*x)])/((a + b)*(a + 2*b + a*Cos[2*(e + f*x)
])) + a^2*Sin[4*(e + f*x)])/(32*a^4*f)
```

3.204.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.14, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 4634, 316, 25, 402, 25, 402, 27, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{\sec(e+fx)^4\left(a+b\sec(e+fx)^2\right)^2}\,dx$$
$$\downarrow 4634$$
$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)^3\left(b\tan^2(e+fx)+a+b\right)^2}\,d\tan(e+fx)}{f}$$
$$\downarrow 316$$
$$\frac{\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)} - \frac{\int -\frac{5b\tan^2(e+fx)+3a-b}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^2}\,d\tan(e+fx)}{4a}}{f}$$

$$\downarrow 25$$

$$\frac{\int \frac{5b\tan^2(e+fx)+3a-b}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^2}\,d\tan(e+fx)}{4a}+\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)}}{f}$$

$$\downarrow 402$$

$$\frac{\frac{3(a-2b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)}-\frac{\int -\frac{3a^2+ba+6b^2+9(a-2b)b\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}\,d\tan(e+fx)}{2a}}{4a}+\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)}}{f}$$

$$\downarrow 25$$

$$\frac{\frac{\int \frac{3a^2+ba+6b^2+9(a-2b)b\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}\,d\tan(e+fx)}{2a}+\frac{3(a-2b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)}}{4a}+\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)}}{f}$$

$$\downarrow 402$$

$$\frac{\frac{\int \frac{2\left(3a^3-2ba^2+11b^2a+12b^3+(a-3b)b(3a+4b)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}\,d\tan(e+fx)}{2a(a+b)}+\frac{b(a-3b)(3a+4b)\tan(e+fx)}{a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{2a}+\frac{3(a-2b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)}}{4a}+\frac{}{4a(\tan^2}}{f}$$

$$\downarrow 27$$

$$\frac{\frac{\int \frac{3a^3-2ba^2+11b^2a+12b^3+(a-3b)b(3a+4b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}\,d\tan(e+fx)}{a(a+b)}+\frac{b(a-3b)(3a+4b)\tan(e+fx)}{a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{2a}+\frac{3(a-2b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)}}{4a}+\frac{}{4a(\tan^2(e+}}{f}$$

$$\downarrow 397$$

$$\frac{\frac{(a+b)\left(3a^2-8ab+24b^2\right)\int \frac{1}{\tan^2(e+fx)+1}\,d\tan(e+fx)}{a}-\frac{4b^3(7a+6b)\int \frac{1}{b\tan^2(e+fx)+a+b}\,d\tan(e+fx)}{a}}{a(a+b)}+\frac{b(a-3b)(3a+4b)\tan(e+fx)}{a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{2a}+\frac{3(a-2b)\tan(e+}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\right.}}{4a}}{f}$$

$$\downarrow 216$$

3.204. $\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

$$\frac{\frac{(a+b)\left(3a^2-8ab+24b^2\right)\arctan(\tan(e+fx))}{a}-\frac{4b^3(7a+6b)\int\frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}+\frac{b(a-3b)(3a+4b)\tan(e+fx)}{a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{2a}+\frac{3(a-2b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)\right.}$$
$$\frac{4a}{f}$$

$$\downarrow 218$$

$$\frac{\frac{(a+b)\left(3a^2-8ab+24b^2\right)\arctan(\tan(e+fx))}{a}-\frac{4b^{5/2}(7a+6b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}+\frac{b(a-3b)(3a+4b)\tan(e+fx)}{a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{2a}+\frac{3(a-2b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)}$$
$$\frac{4a}{f}$$

input
```
Int[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(Tan[e + f*x]/(4*a*(1 + Tan[e + f*x]^2)^2*(a + b + b*Tan[e + f*x]^2)) + ((
3*(a - 2*b)*Tan[e + f*x])/(2*a*(1 + Tan[e + f*x]^2)*(a + b + b*Tan[e + f*x
]^2)) + ((((a + b)*(3*a^2 - 8*a*b + 24*b^2)*ArcTan[Tan[e + f*x]])/a - (4*b
^(5/2)*(7*a + 6*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*Sqrt[a +
 b]))/(a*(a + b)) + ((a - 3*b)*b*(3*a + 4*b)*Tan[e + f*x])/(a*(a + b)*(a +
 b + b*Tan[e + f*x]^2)))/(2*a))/(4*a))/f
```

3.204.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.204.4 Maple [A] (verified)

Time = 3.49 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.79

method	result
derivativedivides	$\frac{\frac{\left(\frac{3}{8}a^2-ab\right)\tan(fx+e)^3+\left(-ab+\frac{5}{8}a^2\right)\tan(fx+e)}{\left(1+\tan(fx+e)^2\right)^2}+\frac{\left(3a^2-8ab+24b^2\right)\arctan(\tan(fx+e))}{8}}{a^4}-\frac{b^3\left(\frac{a\tan(fx+e)}{2(a+b)\left(a+b+b\tan(fx+e)^2\right)}+\frac{(7a+6$
default	$\frac{\frac{\left(\frac{3}{8}a^2-ab\right)\tan(fx+e)^3+\left(-ab+\frac{5}{8}a^2\right)\tan(fx+e)}{\left(1+\tan(fx+e)^2\right)^2}+\frac{\left(3a^2-8ab+24b^2\right)\arctan(\tan(fx+e))}{8}}{a^4}-\frac{b^3\left(\frac{a\tan(fx+e)}{2(a+b)\left(a+b+b\tan(fx+e)^2\right)}+\frac{(7a+6$
risch	$\frac{3x}{8a^2}-\frac{xb}{a^3}+\frac{3xb^2}{a^4}-\frac{ie^{4i(fx+e)}}{64a^2f}-\frac{ie^{2i(fx+e)}}{8a^2f}+\frac{ie^{2i(fx+e)}b}{4a^3f}+\frac{ie^{-2i(fx+e)}}{8a^2f}-\frac{ie^{-2i(fx+e)}b}{4a^3f}+\frac{ie^{-4i(fx+e)}}{64a^2f}-$

input
```
int(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/a^4*(((3/8*a^2-a*b)*tan(f*x+e)^3+(-a*b+5/8*a^2)*tan(f*x+e))/(1+tan(
f*x+e)^2)^2+1/8*(3*a^2-8*a*b+24*b^2)*arctan(tan(f*x+e)))-b^3/a^4*(1/2*a/(a
+b)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(7*a+6*b)/(a+b)/((a+b)*b)^(1/2)*ar
ctan(b*tan(f*x+e)/((a+b)*b)^(1/2))))
```

3.204.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 656, normalized size of antiderivative = 3.23

$$\int \frac{\cos^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$= \left[\frac{(3\,a^4-5\,a^3b+16\,a^2b^2+24\,ab^3)fx\cos(fx+e)^2+(3\,a^3b-5\,a^2b^2+16\,ab^3+24\,b^4)fx+\left(7\,ab^3+6\,b^4+\right.}{}\right.$$

input
```
integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[1/8*((3*a^4 - 5*a^3*b + 16*a^2*b^2 + 24*a*b^3)*f*x*cos(f*x + e)^2 + (3*a^
3*b - 5*a^2*b^2 + 16*a*b^3 + 24*b^4)*f*x + (7*a*b^3 + 6*b^4 + (7*a^2*b^2 +
 6*a*b^3)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(
f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*c
os(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) +
b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) + (2*(a^4 + a^3*b)
*cos(f*x + e)^5 + 3*(a^4 - a^3*b - 2*a^2*b^2)*cos(f*x + e)^3 + (3*a^3*b -
5*a^2*b^2 - 12*a*b^3)*cos(f*x + e))*sin(f*x + e))/((a^6 + a^5*b)*f*cos(f*x
 + e)^2 + (a^5*b + a^4*b^2)*f), 1/8*((3*a^4 - 5*a^3*b + 16*a^2*b^2 + 24*a*
b^3)*f*x*cos(f*x + e)^2 + (3*a^3*b - 5*a^2*b^2 + 16*a*b^3 + 24*b^4)*f*x +
2*(7*a*b^3 + 6*b^4 + (7*a^2*b^2 + 6*a*b^3)*cos(f*x + e)^2)*sqrt(b/(a + b))
*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)
*sin(f*x + e))) + (2*(a^4 + a^3*b)*cos(f*x + e)^5 + 3*(a^4 - a^3*b - 2*a^2
*b^2)*cos(f*x + e)^3 + (3*a^3*b - 5*a^2*b^2 - 12*a*b^3)*cos(f*x + e))*sin(
f*x + e))/((a^6 + a^5*b)*f*cos(f*x + e)^2 + (a^5*b + a^4*b^2)*f)]
```

3.204.6 Sympy [F]

$$\int \frac{\cos^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \int \frac{\cos^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

input
```
integrate(cos(f*x+e)**4/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral(cos(e + f*x)**4/(a + b*sec(e + f*x)**2)**2, x)
```

3.204.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.32

$$\int \frac{\cos^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{\frac{4\left(7ab^3+6b^4\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\left(a^5+a^4b\right)\sqrt{(a+b)b}} - \frac{\left(3a^2b-5ab^2-12b^3\right)\tan(fx+e)^5+\left(3a^3+3a^2b-16ab^2-24b^3\right)\tan(fx+e)^3+\left(5a^3+2a^2b-11ab^2-12\right.}{\left(a^4b+a^3b^2\right)\tan(fx+e)^6+a^5+2a^4b+a^3b^2+\left(a^5+4a^4b+3a^3b^2\right)\tan(fx+e)^4+\left(2a^5+5a^4b+3a^3b^2\right)}}{8f}$$

input
```
integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/8*(4*(7*a*b^3 + 6*b^4)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^5 + a
^4*b)*sqrt((a + b)*b)) - ((3*a^2*b - 5*a*b^2 - 12*b^3)*tan(f*x + e)^5 + (3
*a^3 + 3*a^2*b - 16*a*b^2 - 24*b^3)*tan(f*x + e)^3 + (5*a^3 + 2*a^2*b - 11
*a*b^2 - 12*b^3)*tan(f*x + e))/((a^4*b + a^3*b^2)*tan(f*x + e)^6 + a^5 + 2
*a^4*b + a^3*b^2 + (a^5 + 4*a^4*b + 3*a^3*b^2)*tan(f*x + e)^4 + (2*a^5 + 5
*a^4*b + 3*a^3*b^2)*tan(f*x + e)^2) - (3*a^2 - 8*a*b + 24*b^2)*(f*x + e)/a
^4)/f
```

3.204.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.96

$$\int \frac{\cos^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx =$$
$$-\frac{\frac{4\,b^3\tan(fx+e)}{(a^4+a^3b)\left(b\tan(fx+e)^2+a+b\right)}+\frac{4\,(7\,ab^3+6\,b^4)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^5+a^4b)\sqrt{ab+b^2}}-\frac{(3\,a^2-8\,ab+24\,b^2)(fx+e)}{a^4}-\frac{3\,a\tan(f}{}}{8\,f}$$

input
```
integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-1/8*(4*b^3*tan(f*x + e)/((a^4 + a^3*b)*(b*tan(f*x + e)^2 + a + b)) + 4*(7
*a*b^3 + 6*b^4)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x +
e)/sqrt(a*b + b^2)))/((a^5 + a^4*b)*sqrt(a*b + b^2)) - (3*a^2 - 8*a*b + 24
*b^2)*(f*x + e)/a^4 - (3*a*tan(f*x + e)^3 - 8*b*tan(f*x + e)^3 + 5*a*tan(f
*x + e) - 8*b*tan(f*x + e))/((tan(f*x + e)^2 + 1)^2*a^3))/f
```

3.204.9 Mupad [B] (verification not implemented)

Time = 23.00 (sec) , antiderivative size = 2880, normalized size of antiderivative = 14.19

$$\int \frac{\cos^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(cos(e + f*x)^4/(a + b/cos(e + f*x)^2)^2,x)
```

output

```
(atan(((((tan(e + f*x)*(2112*a*b^8 + 1152*b^9 + 800*a^2*b^7 - 16*a^3*b^6 +
 121*a^4*b^5 - 30*a^5*b^4 + 9*a^6*b^3))/(32*(2*a^7*b + a^8 + a^6*b^2)) - (
((6*a^8*b^6 + (23*a^9*b^5)/2 + (9*a^10*b^4)/2 + (a^11*b^3)/2 + (3*a^12*b^2
)/2)/(2*a^10*b + a^11 + a^9*b^2) - (tan(e + f*x)*(a^2*3i - a*b*8i + b^2*24
i)*(512*a^8*b^5 + 1280*a^9*b^4 + 1024*a^10*b^3 + 256*a^11*b^2))/(512*a^4*(
2*a^7*b + a^8 + a^6*b^2)))*(a^2*3i - a*b*8i + b^2*24i))/(16*a^4))*(a^2*3i
- a*b*8i + b^2*24i)*1i)/(16*a^4) + (((tan(e + f*x)*(2112*a*b^8 + 1152*b^9
+ 800*a^2*b^7 - 16*a^3*b^6 + 121*a^4*b^5 - 30*a^5*b^4 + 9*a^6*b^3))/(32*(2
*a^7*b + a^8 + a^6*b^2)) + (((6*a^8*b^6 + (23*a^9*b^5)/2 + (9*a^10*b^4)/2
+ (a^11*b^3)/2 + (3*a^12*b^2)/2)/(2*a^10*b + a^11 + a^9*b^2) + (tan(e + f*
x)*(a^2*3i - a*b*8i + b^2*24i)*(512*a^8*b^5 + 1280*a^9*b^4 + 1024*a^10*b^3
 + 256*a^11*b^2))/(512*a^4*(2*a^7*b + a^8 + a^6*b^2)))*(a^2*3i - a*b*8i +
b^2*24i))/(16*a^4))*(a^2*3i - a*b*8i + b^2*24i)*1i)/(16*a^4))/(((135*a*b^9
)/4 + 27*b^10 - (9*a^2*b^8)/2 - (149*a^3*b^7)/32 + (219*a^4*b^6)/64 - (63*
a^5*b^5)/64)/(2*a^10*b + a^11 + a^9*b^2) - (((tan(e + f*x)*(2112*a*b^8 + 1
152*b^9 + 800*a^2*b^7 - 16*a^3*b^6 + 121*a^4*b^5 - 30*a^5*b^4 + 9*a^6*b^3)
)/(32*(2*a^7*b + a^8 + a^6*b^2)) - (((6*a^8*b^6 + (23*a^9*b^5)/2 + (9*a^10
*b^4)/2 + (a^11*b^3)/2 + (3*a^12*b^2)/2)/(2*a^10*b + a^11 + a^9*b^2) - (ta
n(e + f*x)*(a^2*3i - a*b*8i + b^2*24i)*(512*a^8*b^5 + 1280*a^9*b^4 + 1024*
a^10*b^3 + 256*a^11*b^2))/(512*a^4*(2*a^7*b + a^8 + a^6*b^2)))*(a^2*3i ...
```

3.205 $\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

3.205.1 Optimal result 1526
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3.205.8 Giac [A] (verification not implemented) 1534
3.205.9 Mupad [B] (verification not implemented) 1535

3.205.1 Optimal result

Integrand size = 23, antiderivative size = 278

$$\begin{aligned}\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx &= \frac{\left(5a^3-12a^2b+24ab^2-64b^3\right)x}{16a^5}\\ &+\frac{b^{7/2}(9a+8b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^5(a+b)^{3/2}f}\\ &+\frac{\left(15a^2-26ab+48b^2\right)\cos(e+fx)\sin(e+fx)}{48a^3f\left(a+b+b\tan^2(e+fx)\right)}\\ &+\frac{(5a-8b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)}\\ &+\frac{\cos^5(e+fx)\sin(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)}\\ &+\frac{b(5a^3-7a^2b+12ab^2+32b^3)\tan(e+fx)}{16a^4(a+b)f\left(a+b+b\tan^2(e+fx)\right)}\end{aligned}$$

output

```
1/16*(5*a^3-12*a^2*b+24*a*b^2-64*b^3)*x/a^5+1/2*b^(7/2)*(9*a+8*b)*arctan(b
^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^5/(a+b)^(3/2)/f+1/48*(15*a^2-26*a*b+48*b^
2)*cos(f*x+e)*sin(f*x+e)/a^3/f/(a+b+b*tan(f*x+e)^2)+1/24*(5*a-8*b)*cos(f*x
+e)^3*sin(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)+1/6*cos(f*x+e)^5*sin(f*x+e)/a/
f/(a+b+b*tan(f*x+e)^2)+1/16*b*(5*a^3-7*a^2*b+12*a*b^2+32*b^3)*tan(f*x+e)/a
^4/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

3.205.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.75 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.79

$$\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^4(e+fx)\left(12(5a^3-12a^2b+24ab^2-64b^3)\,x(a+2b+a\cos(2(e+fx)))\right.}{}$$

input
```
Integrate[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*(12*(5*a^3 - 12*a^2*b + 24*
a*b^2 - 64*b^3)*x*(a + 2*b + a*Cos[2*(e + f*x)]) - (96*b^4*(9*a + 8*b)*Arc
Tan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e +
 f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*
(e + f*x)])*(Cos[2*e] - I*Sin[2*e]))/((a + b)^(3/2)*f*Sqrt[b*(Cos[e] - I*S
in[e])^4]) + (3*a*(15*a^2 - 32*a*b + 48*b^2)*Cos[2*f*x]*(a + 2*b + a*Cos[2
*(e + f*x)])*Sin[2*e])/f + (3*a^2*(3*a - 4*b)*Cos[4*f*x]*(a + 2*b + a*Cos[
2*(e + f*x)])*Sin[4*e])/f + (a^3*Cos[6*f*x]*(a + 2*b + a*Cos[2*(e + f*x)])
*Sin[6*e])/f + (3*a*(15*a^2 - 32*a*b + 48*b^2)*Cos[2*e]*(a + 2*b + a*Cos[2
*(e + f*x)])*Sin[2*f*x])/f - (96*b^4*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/
((a + b)*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e])) + (3*a^2*(3*a - 4*b)*Cos[4
*e]*(a + 2*b + a*Cos[2*(e + f*x)])*Sin[4*f*x])/f + (a^3*Cos[6*e]*(a + 2*b
+ a*Cos[2*(e + f*x)])*Sin[6*f*x])/f))/(768*a^5*(a + b*Sec[e + f*x]^2)^2)
```

3.205.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.14, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 4634, 316, 25, 402, 25, 402, 27, 402, 27, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sec(e+fx)^6\left(a+b\sec(e+fx)^2\right)^2}dx$$

$$\downarrow 4634$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)^4(b\tan^2(e+fx)+a+b)^2}d\tan(e+fx)}{f}$$

$$\downarrow 316$$

$$\frac{\frac{\tan(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)}-\frac{\int -\frac{7b\tan^2(e+fx)+5a-b}{\left(\tan^2(e+fx)+1\right)^3\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{6a}}{f}$$

$$\downarrow 25$$

$$\frac{\frac{\int \frac{7b\tan^2(e+fx)+5a-b}{\left(\tan^2(e+fx)+1\right)^3\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{6a}+\frac{\tan(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)}}{f}$$

$$\downarrow 402$$

$$\frac{\frac{(5a-8b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\left(a+b\tan^2(e+fx)+b\right)}-\frac{\int -\frac{15a^2-ba+8b^2+5(5a-8b)b\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{4a}}{6a}+\frac{\tan(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)}}{f}$$

$$\downarrow 25$$

$$\frac{\frac{\int \frac{15a^2-ba+8b^2+5(5a-8b)b\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{4a}+\frac{(5a-8b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\left(a+b\tan^2(e+fx)+b\right)}}{6a}+\frac{\tan(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)}}{f}$$

$$\downarrow 402$$

$$\frac{\frac{\left(15a^2-26ab+48b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)}-\frac{\int -\frac{3\left(5a^3+3ba^2-2b^2a-16b^3+b\left(15a^2-26ba+48b^2\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{2a}}{4a}+\frac{\frac{(5a-8b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\left(a+b\tan^2(e+fx)+\right.}}{6a}}{f}$$

$$\downarrow 27$$

3.205. $\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

$$\frac{3\int \frac{5a^3+3ba^2-2b^2a-16b^3+b\left(15a^2-26ba+48b^2\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}\,d\tan(e+fx)}{2a}+\frac{\left(15a^2-26ab+48b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)}+\frac{(5a-8b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\left(a+b\tan^2(e+fx)+b\right.}$$
$$\frac{}{4a}$$
$$\frac{}{6a}$$
$$f$$

$$\downarrow 402$$

$$\frac{3\left(\int \frac{2\left(5a^4-2ba^3+5b^2a^2-28b^3a-32b^4+b\left(5a^3-7ba^2+12b^2a+32b^3\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}\,d\tan(e+fx)}{2a(a+b)}+\frac{b\left(5a^3-7a^2b+12ab^2+32b^3\right)\tan(e+fx)}{a(a+b)\left(a+b\tan^2(e+fx)+b\right)}\right)}{2a}+\frac{\left(15a^2-26ab+48\right.}{2a\left(\tan^2(e+fx)+1\right)\left(\right.}$$
$$\frac{}{4a}$$
$$\frac{}{6a}$$
$$f$$

$$\downarrow 27$$

$$\frac{3\left(\int \frac{5a^4-2ba^3+5b^2a^2-28b^3a-32b^4+b\left(5a^3-7ba^2+12b^2a+32b^3\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}\,d\tan(e+fx)}{a(a+b)}+\frac{b\left(5a^3-7a^2b+12ab^2+32b^3\right)\tan(e+fx)}{a(a+b)\left(a+b\tan^2(e+fx)+b\right)}\right)}{2a}+\frac{\left(15a^2-26ab+48b^2\right)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\right.}$$
$$\frac{}{4a}$$
$$\frac{}{6a}$$
$$f$$

$$\downarrow 397$$

$$\frac{3\left(\frac{(a+b)\left(5a^3-12a^2b+24ab^2-64b^3\right)\int \frac{1}{\tan^2(e+fx)+1}\,d\tan(e+fx)}{a}+\frac{8b^4(9a+8b)\int \frac{1}{b\tan^2(e+fx)+a+b}\,d\tan(e+fx)}{a}+\frac{b\left(5a^3-7a^2b+12ab^2+32b^3\right)\tan(e+fx)}{a(a+b)\left(a+b\tan^2(e+fx)+b\right)}\right)}{2a(a+b)}+$$
$$\frac{}{4a}$$
$$\frac{}{6a}$$
$$f$$

$$\downarrow 216$$

$$\frac{3\left(\frac{8b^4(9a+8b)\int \frac{1}{b\tan^2(e+fx)+a+b}\,d\tan(e+fx)}{a}+\frac{(a+b)\left(5a^3-12a^2b+24ab^2-64b^3\right)\arctan(\tan(e+fx))}{a}+\frac{b\left(5a^3-7a^2b+12ab^2+32b^3\right)\tan(e+fx)}{a(a+b)\left(a+b\tan^2(e+fx)+b\right)}\right)}{2a}+\frac{\left(15a^2-\right.}{2a\left(\tan^2(e+\right.}$$
$$\frac{}{4a}$$
$$\frac{}{6a}$$
$$f$$

$$\downarrow 218$$

3.205. $\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

$$\frac{\frac{\frac{\frac{\left(15a^2-26ab+48b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)}+\frac{3\left(\frac{\frac{(a+b)\left(5a^3-12a^2b+24ab^2-64b^3\right)\arctan(\tan(e+fx))}{a}+\frac{8b^{7/2}(9a+8b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{a(a+b)}+\frac{b\left(5a^3-7a^2b+12ab\right.}{a(a+b)\left(a+b\,\mathrm{t}\right.}\right.}{2a}}{4a}}{6a}}{f}$$

input
```
Int[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(Tan[e + f*x]/(6*a*(1 + Tan[e + f*x]^2)^3*(a + b + b*Tan[e + f*x]^2)) + ((
(5*a - 8*b)*Tan[e + f*x])/(4*a*(1 + Tan[e + f*x]^2)^2*(a + b + b*Tan[e + f
*x]^2)) + (((15*a^2 - 26*a*b + 48*b^2)*Tan[e + f*x])/(2*a*(1 + Tan[e + f*x
]^2)*(a + b + b*Tan[e + f*x]^2)) + (3*((((a + b)*(5*a^3 - 12*a^2*b + 24*a*
b^2 - 64*b^3)*ArcTan[Tan[e + f*x]])/a + (8*b^(7/2)*(9*a + 8*b)*ArcTan[(Sqr
t[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(a*(a + b)) + (b*(5*a^3
- 7*a^2*b + 12*a*b^2 + 32*b^3)*Tan[e + f*x])/(a*(a + b)*(a + b + b*Tan[e +
 f*x]^2))))/(2*a))/(4*a))/(6*a))/f
```

3.205.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.205.4 Maple [A] (verified)

Time = 6.04 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.76

method	result
derivativedivides	$\frac{b^4\left(\frac{a\tan(fx+e)}{2(a+b)\left(a+b+b\tan(fx+e)^2\right)}+\frac{(9a+8b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}\right)}{a^5}+\frac{\left(\frac{5}{16}a^3-\frac{3}{4}a^2b+\frac{3}{2}ab^2\right)\tan(fx+e)^5+\left(3ab^2+\frac{5}{6}a^3-2a^2b\right)\tan}{\left(1+\tan(fx+e)^2\right)}$
default	$\frac{b^4\left(\frac{a\tan(fx+e)}{2(a+b)\left(a+b+b\tan(fx+e)^2\right)}+\frac{(9a+8b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}\right)}{a^5}+\frac{\left(\frac{5}{16}a^3-\frac{3}{4}a^2b+\frac{3}{2}ab^2\right)\tan(fx+e)^5+\left(3ab^2+\frac{5}{6}a^3-2a^2b\right)\tan}{\left(1+\tan(fx+e)^2\right)}$
risch	$\frac{5x}{16a^2}-\frac{3xb}{4a^3}+\frac{3xb^2}{2a^4}-\frac{4xb^3}{a^5}+\frac{ie^{4i(fx+e)}b}{32a^3f}+\frac{ib^4\left(ae^{2i(fx+e)}+2be^{2i(fx+e)}+a\right)}{a^5(a+b)f\left(ae^{4i(fx+e)}+2ae^{2i(fx+e)}+4be^{2i(fx+e)}+a\right)}-\frac{15ie^{2i(}}{128a}$

input
```
int(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(b^4/a^5*(1/2*a/(a+b)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(9*a+8*b)/(a
+b)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))+1/a^5*(((5/16*a^
3-3/4*a^2*b+3/2*a*b^2)*tan(f*x+e)^5+(3*a*b^2+5/6*a^3-2*a^2*b)*tan(f*x+e)^3
+(-5/4*a^2*b+3/2*a*b^2+11/16*a^3)*tan(f*x+e))/(1+tan(f*x+e)^2)^3+1/16*(5*a
^3-12*a^2*b+24*a*b^2-64*b^3)*arctan(tan(f*x+e))))
```

3.205.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 789, normalized size of antiderivative = 2.84

$$\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$
$$= \left[\frac{3\left(5a^5-7a^4b+12a^3b^2-40a^2b^3-64ab^4\right)fx\cos(fx+e)^2+3\left(5a^4b-7a^3b^2+12a^2b^3-40ab^4-64b\right.}{\right.}\right.$$

input
```
integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[1/48*(3*(5*a^5 - 7*a^4*b + 12*a^3*b^2 - 40*a^2*b^3 - 64*a*b^4)*f*x*cos(f*
x + e)^2 + 3*(5*a^4*b - 7*a^3*b^2 + 12*a^2*b^3 - 40*a*b^4 - 64*b^5)*f*x +
6*(9*a*b^4 + 8*b^5 + (9*a^2*b^3 + 8*a*b^4)*cos(f*x + e)^2)*sqrt(-b/(a + b)
)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x +
e)^2 - 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))
*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x
+ e)^2 + b^2)) + (8*(a^5 + a^4*b)*cos(f*x + e)^7 + 2*(5*a^5 - 3*a^4*b - 8*
a^3*b^2)*cos(f*x + e)^5 + (15*a^5 - 11*a^4*b + 22*a^3*b^2 + 48*a^2*b^3)*co
s(f*x + e)^3 + 3*(5*a^4*b - 7*a^3*b^2 + 12*a^2*b^3 + 32*a*b^4)*cos(f*x + e
))*sin(f*x + e))/((a^7 + a^6*b)*f*cos(f*x + e)^2 + (a^6*b + a^5*b^2)*f), 1
/48*(3*(5*a^5 - 7*a^4*b + 12*a^3*b^2 - 40*a^2*b^3 - 64*a*b^4)*f*x*cos(f*x
+ e)^2 + 3*(5*a^4*b - 7*a^3*b^2 + 12*a^2*b^3 - 40*a*b^4 - 64*b^5)*f*x - 12
*(9*a*b^4 + 8*b^5 + (9*a^2*b^3 + 8*a*b^4)*cos(f*x + e)^2)*sqrt(b/(a + b))*
arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*
sin(f*x + e))) + (8*(a^5 + a^4*b)*cos(f*x + e)^7 + 2*(5*a^5 - 3*a^4*b - 8*
a^3*b^2)*cos(f*x + e)^5 + (15*a^5 - 11*a^4*b + 22*a^3*b^2 + 48*a^2*b^3)*co
s(f*x + e)^3 + 3*(5*a^4*b - 7*a^3*b^2 + 12*a^2*b^3 + 32*a*b^4)*cos(f*x + e
))*sin(f*x + e))/((a^7 + a^6*b)*f*cos(f*x + e)^2 + (a^6*b + a^5*b^2)*f)]
```

3.205.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**6/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Timed out
```

3.205.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.33

$$\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$
$$= \frac{24\left(9ab^4+8b^5\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^6+a^5b)\sqrt{(a+b)b}} + \frac{3\left(5a^3b-7a^2b^2+12ab^3+32b^4\right)\tan(fx+e)^7+\left(15a^4+34a^3b-41a^2b^2+156ab^3+288b^4\right)\tan(fx+e)^5+(}{\left(a^5b+a^4b^2\right)\tan(fx+e)^8+\left(a^6+5a^5b+4a^4b^2\right)\tan(fx+e)^6+a^6+2a^5b+a}$$

input
```
integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/48*(24*(9*a*b^4 + 8*b^5)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^6 +
a^5*b)*sqrt((a + b)*b)) + (3*(5*a^3*b - 7*a^2*b^2 + 12*a*b^3 + 32*b^4)*tan
(f*x + e)^7 + (15*a^4 + 34*a^3*b - 41*a^2*b^2 + 156*a*b^3 + 288*b^4)*tan(f
*x + e)^5 + (40*a^4 + 17*a^3*b - 35*a^2*b^2 + 204*a*b^3 + 288*b^4)*tan(f*x
 + e)^3 + 3*(11*a^4 + 2*a^3*b - 5*a^2*b^2 + 28*a*b^3 + 32*b^4)*tan(f*x + e
))/((a^5*b + a^4*b^2)*tan(f*x + e)^8 + (a^6 + 5*a^5*b + 4*a^4*b^2)*tan(f*x
 + e)^6 + a^6 + 2*a^5*b + a^4*b^2 + 3*(a^6 + 3*a^5*b + 2*a^4*b^2)*tan(f*x
+ e)^4 + (3*a^6 + 7*a^5*b + 4*a^4*b^2)*tan(f*x + e)^2) + 3*(5*a^3 - 12*a^2
*b + 24*a*b^2 - 64*b^3)*(f*x + e)/a^5)/f
```

3.205.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.97

$$\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$= \frac{\frac{24\,b^4\tan(fx+e)}{(a^5+a^4b)\left(b\tan(fx+e)^2+a+b\right)} + \frac{24\,(9\,ab^4+8\,b^5)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^6+a^5b)\sqrt{ab+b^2}} + \frac{3\,(5\,a^3-12\,a^2b+24\,ab^2-64\,b^3)(fx+e)}{a^5} +}{}$$

input
```
integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/48*(24*b^4*tan(f*x + e)/((a^5 + a^4*b)*(b*tan(f*x + e)^2 + a + b)) + 24*
(9*a*b^4 + 8*b^5)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x
+ e)/sqrt(a*b + b^2)))/((a^6 + a^5*b)*sqrt(a*b + b^2)) + 3*(5*a^3 - 12*a^2
*b + 24*a*b^2 - 64*b^3)*(f*x + e)/a^5 + (15*a^2*tan(f*x + e)^5 - 36*a*b*ta
n(f*x + e)^5 + 72*b^2*tan(f*x + e)^5 + 40*a^2*tan(f*x + e)^3 - 96*a*b*tan(
f*x + e)^3 + 144*b^2*tan(f*x + e)^3 + 33*a^2*tan(f*x + e) - 60*a*b*tan(f*x
 + e) + 72*b^2*tan(f*x + e))/((tan(f*x + e)^2 + 1)^3*a^4))/f
```

3.205.9 Mupad [B] (verification not implemented)

Time = 23.68 (sec) , antiderivative size = 3310, normalized size of antiderivative = 11.91

$$\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(cos(e + f*x)^6/(a + b/cos(e + f*x)^2)^2,x)
```

output
```
((tan(e + f*x)*(28*a*b^3 + 2*a^3*b + 11*a^4 + 32*b^4 - 5*a^2*b^2))/(16*a^4
*(a + b)) + (tan(e + f*x)^5*(156*a*b^3 + 34*a^3*b + 15*a^4 + 288*b^4 - 41*
a^2*b^2))/(48*a^4*(a + b)) + (tan(e + f*x)^3*(204*a*b^3 + 17*a^3*b + 40*a^
4 + 288*b^4 - 35*a^2*b^2))/(48*a^4*(a + b)) + (b*tan(e + f*x)^7*(12*a*b^2
 - 7*a^2*b + 5*a^3 + 32*b^3))/(16*a^4*(a + b)))/(f*(a + b + tan(e + f*x)^2*
(3*a + 4*b) + tan(e + f*x)^4*(3*a + 6*b) + b*tan(e + f*x)^8 + tan(e + f*x)
^6*(a + 4*b))) - (atan(-((((((8*a^10*b^7 + 15*a^11*b^6 + (23*a^12*b^5)/4 -
 (3*a^13*b^4)/4 - (3*a^14*b^3)/4 - (5*a^15*b^2)/4)/(2*a^13*b + a^14 + a^12
*b^2) - (tan(e + f*x)*(a*b^2*24i - a^2*b*12i + a^3*5i - b^3*64i)*(2048*a^1
0*b^5 + 5120*a^11*b^4 + 4096*a^12*b^3 + 1024*a^13*b^2))/(4096*a^5*(2*a^9*b
 + a^10 + a^8*b^2)))*(a*b^2*24i - a^2*b*12i + a^3*5i - b^3*64i))/(32*a^5)
 - (tan(e + f*x)*(14336*a*b^10 + 8192*b^11 + 5248*a^2*b^9 - 64*a^3*b^8 + 64
*a^4*b^7 - 568*a^5*b^6 + 169*a^6*b^5 - 70*a^7*b^4 + 25*a^8*b^3))/(128*(2*a
^9*b + a^10 + a^8*b^2)))*(a*b^2*24i - a^2*b*12i + a^3*5i - b^3*64i)*1i)/(3
2*a^5) - (((((8*a^10*b^7 + 15*a^11*b^6 + (23*a^12*b^5)/4 - (3*a^13*b^4)/4
 - (3*a^14*b^3)/4 - (5*a^15*b^2)/4)/(2*a^13*b + a^14 + a^12*b^2) + (tan(e +
 f*x)*(a*b^2*24i - a^2*b*12i + a^3*5i - b^3*64i)*(2048*a^10*b^5 + 5120*a^1
1*b^4 + 4096*a^12*b^3 + 1024*a^13*b^2))/(4096*a^5*(2*a^9*b + a^10 + a^8*b^
2)))*(a*b^2*24i - a^2*b*12i + a^3*5i - b^3*64i))/(32*a^5) + (tan(e + f*x)*
(14336*a*b^10 + 8192*b^11 + 5248*a^2*b^9 - 64*a^3*b^8 + 64*a^4*b^7 - 56...
```

3.206 $\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

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3.206.1 Optimal result

Integrand size = 23, antiderivative size = 108

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{3\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8\sqrt{a}(a+b)^{5/2}f} + \frac{\sin(e+fx)}{4(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2} + \frac{3\sin(e+fx)}{8(a+b)^2 f\left(a+b-a\sin^2(e+fx)\right)}$$

output
```
1/4*sin(f*x+e)/(a+b)/f/(a+b-a*sin(f*x+e)^2)^2+3/8*sin(f*x+e)/(a+b)^2/f/(a+
b-a*sin(f*x+e)^2)+3/8*arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))/(a+b)^(5/2)/
f/a^(1/2)
```

3.206.2 Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.83

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{3\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a}(a+b)^{5/2}} + \frac{2(7a+10b+3a\cos(2(e+fx)))\sin(e+fx)}{(a+b)^2(a+2b+a\cos(2(e+fx)))^2}}{8f}$$

input
```
Integrate[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
((3*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/(Sqrt[a]*(a + b)^(5/2)) +
 (2*(7*a + 10*b + 3*a*Cos[2*(e + f*x)])*Sin[e + f*x])/((a + b)^2*(a + 2*b
+ a*Cos[2*(e + f*x)])^2))/(8*f)
```

3.206.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4635, 215, 215, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sec(e+fx)^5}{(a+b\sec(e+fx)^2)^3}\,dx$$

$$\downarrow 4635$$

$$\frac{\int \frac{1}{(-a\sin^2(e+fx)+a+b)^3}\,d\sin(e+fx)}{f}$$

$$\downarrow 215$$

$$\frac{3\int \frac{1}{\left(-a\sin^2(e+fx)+a+b\right)^2}\,d\sin(e+fx)}{4(a+b)} + \frac{\sin(e+fx)}{4(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2}}{f}$$

$$\downarrow 215$$

$$\frac{3\left(\frac{\int \frac{1}{-a\sin^2(e+fx)+a+b}\,d\sin(e+fx)}{2(a+b)} + \frac{\sin(e+fx)}{2(a+b)\left(-a\sin^2(e+fx)+a+b\right)}\right)}{4(a+b)} + \frac{\sin(e+fx)}{4(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2}}{f}$$

$$\downarrow 221$$

$$\frac{3\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2\sqrt{a}(a+b)^{3/2}} + \frac{\sin(e+fx)}{2(a+b)\left(-a\sin^2(e+fx)+a+b\right)}\right)}{4(a+b)} + \frac{\sin(e+fx)}{4(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2}}{f}$$

input
```
Int[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
(Sin[e + f*x]/(4*(a + b)*(a + b - a*Sin[e + f*x]^2)^2) + (3*(ArcTanh[(Sqrt
[a]*Sin[e + f*x])/Sqrt[a + b]]/(2*Sqrt[a]*(a + b)^(3/2)) + Sin[e + f*x]/(2
*(a + b)*(a + b - a*Sin[e + f*x]^2))))/(4*(a + b)))/f
```

3.206.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.206.4 Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{\frac{\sin(fx+e)}{4(a+b)\left(a\sin(fx+e)^2-a-b\right)^2}+\frac{-\frac{3\sin(fx+e)}{8(a+b)\left(a\sin(fx+e)^2-a-b\right)}+\frac{3\arctanh\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{8(a+b)\sqrt{a(a+b)}}}{a+b}}{f}$
default	$\frac{\frac{\sin(fx+e)}{4(a+b)\left(a\sin(fx+e)^2-a-b\right)^2}+\frac{-\frac{3\sin(fx+e)}{8(a+b)\left(a\sin(fx+e)^2-a-b\right)}+\frac{3\arctanh\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{8(a+b)\sqrt{a(a+b)}}}{a+b}}{f}$
risch	$-\frac{i\left(3a\,e^{7i(fx+e)}+11a\,e^{5i(fx+e)}+20b\,e^{5i(fx+e)}-11a\,e^{3i(fx+e)}-20b\,e^{3i(fx+e)}-3a\,e^{i(fx+e)}\right)}{4(a+b)^2f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)^2}+\frac{3\ln\left(e^{2i(fx+e)}+\frac{2i(a+b}{\sqrt{a}}\right.}{16\sqrt{a^2+ab}\,(a-}$

input
```
int(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/4*sin(f*x+e)/(a+b)/(a*sin(f*x+e)^2-a-b)^2+3/4/(a+b)*(-1/2*sin(f*x+e
)/(a+b)/(a*sin(f*x+e)^2-a-b)+1/2/(a+b)/(a*(a+b))^(1/2)*arctanh(a*sin(f*x+e
)/(a*(a+b))^(1/2))))
```

3.206.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(100) = 200$.

Time $= 0.30$ (sec) , antiderivative size $= 472$, normalized size of antiderivative $= 4.37$

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \left[\frac{3\left(a^2\cos(fx+e)^4+2ab\cos(fx+e)^2+b^2\right)\sqrt{a^2+ab}\log\left(-\frac{a\cos(fx+e)^2-2\sqrt{a^2+ab}\sin(fx+e)-2a-b}{a\cos(fx+e)^2+b}\right)+2\left(2a^{3}\right.}{16\left(\left(a^6+3a^5b+3a^4b^2+a^3b^3\right)f\cos(fx+e)^4+2\left(a^5b+3a^4b^2+3a^3b^3+a^2b^4\right)f\cos(f\right.}\right.$$
$$\left.-\frac{3\left(a^2\cos(fx+e)^4+2ab\cos(fx+e)^2+b^2\right)\sqrt{-a^2-ab}\arctan\left(\frac{\sqrt{-a^2-ab}\sin(fx+e)}{a+b}\right)-\left(2a^3+7a^2b+5a\right.}{8\left(\left(a^6+3a^5b+3a^4b^2+a^3b^3\right)f\cos(fx+e)^4+2\left(a^5b+3a^4b^2+3a^3b^3+a^2b^4\right)f\cos(fx+e)^{2}\right.}\right]$$

input
```
integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[1/16*(3*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*sqrt(a^2 + a*b)
*log(-(a*cos(f*x + e)^2 - 2*sqrt(a^2 + a*b)*sin(f*x + e) - 2*a - b)/(a*cos
(f*x + e)^2 + b)) + 2*(2*a^3 + 7*a^2*b + 5*a*b^2 + 3*(a^3 + a^2*b)*cos(f*x
 + e)^2)*sin(f*x + e))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*f*cos(f*x +
e)^4 + 2*(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*f*cos(f*x + e)^2 + (a^4
*b^2 + 3*a^3*b^3 + 3*a^2*b^4 + a*b^5)*f), -1/8*(3*(a^2*cos(f*x + e)^4 + 2*
a*b*cos(f*x + e)^2 + b^2)*sqrt(-a^2 - a*b)*arctan(sqrt(-a^2 - a*b)*sin(f*x
 + e)/(a + b)) - (2*a^3 + 7*a^2*b + 5*a*b^2 + 3*(a^3 + a^2*b)*cos(f*x + e)
^2)*sin(f*x + e))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*f*cos(f*x + e)^4
+ 2*(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*f*cos(f*x + e)^2 + (a^4*b^2
+ 3*a^3*b^3 + 3*a^2*b^4 + a*b^5)*f)]
```

3.206.6 Sympy [F]

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

input
```
integrate(sec(f*x+e)**5/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Integral(sec(e + f*x)**5/(a + b*sec(e + f*x)**2)**3, x)
```

3.206.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.66

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{2\left(3a\sin(fx+e)^3-5(a+b)\sin(fx+e)\right)}{(a^4+2a^3b+a^2b^2)\sin(fx+e)^4+a^4+4a^3b+6a^2b^2+4ab^3+b^4-2(a^4+3a^3b+3a^2b^2+ab^3)\sin(fx+e)^2}+\frac{3\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}(a^2+2ab+b^2)}}{16f}$$

input
```
integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
-1/16*(2*(3*a*sin(f*x + e)^3 - 5*(a + b)*sin(f*x + e))/((a^4 + 2*a^3*b + a
^2*b^2)*sin(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4 - 2*(a^
4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sin(f*x + e)^2) + 3*log((a*sin(f*x + e) -
 sqrt((a + b)*a))/(a*sin(f*x + e) + sqrt((a + b)*a)))/(sqrt((a + b)*a)*(a^
2 + 2*a*b + b^2)))/f
```

3.206.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.08

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{3\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{(a^2+2ab+b^2)\sqrt{-a^2-ab}}+\frac{3a\sin(fx+e)^3-5a\sin(fx+e)-5b\sin(fx+e)}{\left(a\sin(fx+e)^2-a-b\right)^2(a^2+2ab+b^2)}}{8f}$$

input
```
integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
-1/8*(3*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/((a^2 + 2*a*b + b^2)*sqrt(
-a^2 - a*b)) + (3*a*sin(f*x + e)^3 - 5*a*sin(f*x + e) - 5*b*sin(f*x + e))/
((a*sin(f*x + e)^2 - a - b)^2*(a^2 + 2*a*b + b^2)))/f
```

3.206.9 Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.05

$$\int \frac{\sec^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$
$$= \frac{\frac{5\sin(e+fx)}{8(a+b)} - \frac{3a\sin(e+fx)^3}{8(a+b)^2}}{f\left(2ab+a^2+b^2-\sin(e+fx)^2\left(2a^2+2ba\right)+a^2\sin(e+fx)^4\right)} + \frac{3\,\mathrm{atanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8\sqrt{a}f(a+b)^{5/2}}$$

input
```
int(1/(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^3),x)
```

output
```
((5*sin(e + f*x))/(8*(a + b)) - (3*a*sin(e + f*x)^3)/(8*(a + b)^2))/(f*(2*
a*b + a^2 + b^2 - sin(e + f*x)^2*(2*a*b + 2*a^2) + a^2*sin(e + f*x)^4)) +
(3*atanh((a^(1/2)*sin(e + f*x))/(a + b)^(1/2)))/(8*a^(1/2)*f*(a + b)^(5/2)
)
```

3.207 $\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

3.207.1 Optimal result . . . 1542
3.207.2 Mathematica [A] (verified) . . . 1542
3.207.3 Rubi [A] (verified) . . . 1543
3.207.4 Maple [A] (verified) . . . 1545
3.207.5 Fricas [B] (verification not implemented) . . . 1545
3.207.6 Sympy [F] . . . 1546
3.207.7 Maxima [A] (verification not implemented) . . . 1546
3.207.8 Giac [A] (verification not implemented) . . . 1547
3.207.9 Mupad [B] (verification not implemented) . . . 1547

3.207.1 Optimal result

Integrand size = 23, antiderivative size = 125

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{(4a+b)\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{3/2}(a+b)^{5/2}f} - \frac{b\sin(e+fx)}{4a(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2} + \frac{(4a+b)\sin(e+fx)}{8a(a+b)^2f\left(a+b-a\sin^2(e+fx)\right)}$$

output
```
1/8*(4*a+b)*arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))/a^(3/2)/(a+b)^(5/2)/f-
1/4*b*sin(f*x+e)/a/(a+b)/f/(a+b-a*sin(f*x+e)^2)^2+1/8*(4*a+b)*sin(f*x+e)/a
/(a+b)^2/f/(a+b-a*sin(f*x+e)^2)
```

3.207.2 Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.30

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))^3\sec^6(e+fx)\left(\frac{8\sin(e+fx)}{(a+b-a\sin^2(e+fx))^2} - (4a+b)\left(\frac{3\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{a}(a+b)^{5/2}} + \frac{4\sin(e-}{(a+b}\right.\right.}{192af\left(a+b\sec^2(e+fx)\right)^3}$$

input
```
Integrate[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
-1/192*((a + 2*b + a*Cos[2*(e + f*x)])^3*Sec[e + f*x]^6*((8*Sin[e + f*x])/
(a + b - a*Sin[e + f*x]^2)^2 - (4*a + b)*((3*ArcTanh[(Sqrt[a]*Sin[e + f*x]
)/Sqrt[a + b]])/(Sqrt[a]*(a + b)^(5/2)) + (4*Sin[e + f*x]*(5*(a + b) - 3*a
*Sin[e + f*x]^2))/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2))))/(a*f*(a
+ b*Sec[e + f*x]^2)^3)
```

3.207.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4635, 298, 215, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sec(e+fx)^3}{(a+b\sec(e+fx)^2)^3}\,dx$$

$$\downarrow 4635$$

$$\frac{\int \frac{1-\sin^2(e+fx)}{(-a\sin^2(e+fx)+a+b)^3}\,d\sin(e+fx)}{f}$$

$$\downarrow 298$$

$$\frac{\frac{(4a+b)\int \frac{1}{\left(-a\sin^2(e+fx)+a+b\right)^2}\,d\sin(e+fx)}{4a(a+b)} - \frac{b\sin(e+fx)}{4a(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2}}{f}$$

$$\downarrow 215$$

$$\frac{\frac{(4a+b)\left(\frac{\int \frac{1}{-a\sin^2(e+fx)+a+b}\,d\sin(e+fx)}{2(a+b)} + \frac{\sin(e+fx)}{2(a+b)\left(-a\sin^2(e+fx)+a+b\right)}\right)}{4a(a+b)} - \frac{b\sin(e+fx)}{4a(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2}}{f}$$

$$\downarrow 221$$

$$\frac{(4a+b)\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2\sqrt{a}(a+b)^{3/2}}+\frac{\sin(e+fx)}{2(a+b)\left(-a\sin^2(e+fx)+a+b\right)}\right)}{4a(a+b)}-\frac{b\sin(e+fx)}{4a(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2}}{f}$$

input
```
Int[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
(-1/4*(b*Sin[e + f*x])/(a*(a + b)*(a + b - a*Sin[e + f*x]^2)^2) + ((4*a +
b)*(ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]/(2*Sqrt[a]*(a + b)^(3/2))
+ Sin[e + f*x]/(2*(a + b)*(a + b - a*Sin[e + f*x]^2))))/(4*a*(a + b)))/f
```

3.207.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.207.4 Maple [A] (verified)

Time = 1.71 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.99

method	result
derivativedivides	$\frac{-\frac{(4a+b)\sin(fx+e)^3}{8\left(a^2+2ab+b^2\right)}+\frac{(4a-b)\sin(fx+e)}{8a(a+b)}}{\left(a\sin(fx+e)^2-a-b\right)^2}+\frac{(4a+b)\arctanh\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{8\left(a^2+2ab+b^2\right)a\sqrt{a(a+b)}}}{f}$
default	$\frac{-\frac{(4a+b)\sin(fx+e)^3}{8\left(a^2+2ab+b^2\right)}+\frac{(4a-b)\sin(fx+e)}{8a(a+b)}}{\left(a\sin(fx+e)^2-a-b\right)^2}+\frac{(4a+b)\arctanh\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{8\left(a^2+2ab+b^2\right)a\sqrt{a(a+b)}}}{f}$
risch	$-\frac{i\left(4a^2e^{7i(fx+e)}+ab\,e^{7i(fx+e)}+4a^2e^{5i(fx+e)}+9ab\,e^{5i(fx+e)}-4b^2e^{5i(fx+e)}-4a^2e^{3i(fx+e)}-9ab\,e^{3i(fx+e)}+4b^2e^{3i(fx+e)}\right.}{4a(a+b)^2f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)^2}$

input
```
int(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*((-1/8*(4*a+b)/(a^2+2*a*b+b^2)*sin(f*x+e)^3+1/8*(4*a-b)/a/(a+b)*sin(f*
x+e))/(a*sin(f*x+e)^2-a-b)^2+1/8*(4*a+b)/(a^2+2*a*b+b^2)/a/(a*(a+b))^(1/2)
*arctanh(a*sin(f*x+e)/(a*(a+b))^(1/2)))
```

3.207.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 260 vs. 2(117) = 234.

Time = 0.30 (sec) , antiderivative size = 544, normalized size of antiderivative = 4.35

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \left[\frac{\left((4a^3+a^2b)\cos(fx+e)^4+4ab^2+b^3+2(4a^2b+ab^2)\cos(fx+e)^2\right)\sqrt{a^2+ab}\log\left(-\frac{a\cos(fx+e)^2-2\sqrt{a^2-}}{a\cos(fx\right.}\right.$$
$$\left.16\left((a^7+3a^6b+3a^5b^2+a^4b^3)f\cos(fx+e)^4+2(a^6b+3a^5b^2+3a^4b^3\right.\right.$$
$$-\frac{\left((4a^3+a^2b)\cos(fx+e)^4+4ab^2+b^3+2(4a^2b+ab^2)\cos(fx+e)^2\right)\sqrt{-a^2-ab}\arctan\left(\frac{\sqrt{-a^2-ab}\sin(f}{a+b}\right.}{8\left((a^7+3a^6b+3a^5b^2+a^4b^3)f\cos(fx+e)^4+2(a^6b+3a^5b^2+3a^4b^3+a^3b\right.}$$

input
```
integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[1/16*(((4*a^3 + a^2*b)*cos(f*x + e)^4 + 4*a*b^2 + b^3 + 2*(4*a^2*b + a*b^
2)*cos(f*x + e)^2)*sqrt(a^2 + a*b)*log(-(a*cos(f*x + e)^2 - 2*sqrt(a^2 + a
*b)*sin(f*x + e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) + 2*(2*a^3*b + a^2*b^2
 - a*b^3 + (4*a^4 + 5*a^3*b + a^2*b^2)*cos(f*x + e)^2)*sin(f*x + e))/((a^7
 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^4 + 2*(a^6*b + 3*a^5*b^2
+ 3*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^2 + (a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4
 + a^2*b^5)*f), -1/8*(((4*a^3 + a^2*b)*cos(f*x + e)^4 + 4*a*b^2 + b^3 + 2*
(4*a^2*b + a*b^2)*cos(f*x + e)^2)*sqrt(-a^2 - a*b)*arctan(sqrt(-a^2 - a*b)
*sin(f*x + e)/(a + b)) - (2*a^3*b + a^2*b^2 - a*b^3 + (4*a^4 + 5*a^3*b + a
^2*b^2)*cos(f*x + e)^2)*sin(f*x + e))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^
3)*f*cos(f*x + e)^4 + 2*(a^6*b + 3*a^5*b^2 + 3*a^4*b^3 + a^3*b^4)*f*cos(f*
x + e)^2 + (a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^5)*f)]
```

3.207.6 Sympy [F]

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

input
```
integrate(sec(f*x+e)**3/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Integral(sec(e + f*x)**3/(a + b*sec(e + f*x)**2)**3, x)
```

3.207.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.70

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{(4a+b)\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{(a^3+2a^2b+ab^2)\sqrt{(a+b)a}} + \frac{2\left((4a^2+ab)\sin(fx+e)^3-(4a^2+3ab-b^2)\sin(fx+e)\right)}{a^5+4a^4b+6a^3b^2+4a^2b^3+ab^4+(a^5+2a^4b+a^3b^2)\sin(fx+e)^4-2(a^5+3a^4b+3a^3b^2+a^2b^3)\sin(fx+e)}}{16f}$$

input
```
integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
-1/16*((4*a + b)*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*sin(f*x + e) +
sqrt((a + b)*a)))/((a^3 + 2*a^2*b + a*b^2)*sqrt((a + b)*a)) + 2*((4*a^2 +
a*b)*sin(f*x + e)^3 - (4*a^2 + 3*a*b - b^2)*sin(f*x + e))/(a^5 + 4*a^4*b +
 6*a^3*b^2 + 4*a^2*b^3 + a*b^4 + (a^5 + 2*a^4*b + a^3*b^2)*sin(f*x + e)^4
- 2*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sin(f*x + e)^2))/f
```

3.207.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.24

$$\int \frac{\sec^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{\frac{(4a+b)\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{(a^3+2a^2b+ab^2)\sqrt{-a^2-ab}} + \frac{4a^2\sin(fx+e)^3+ab\sin(fx+e)^3-4a^2\sin(fx+e)-3ab\sin(fx+e)+b^2\sin(fx+e)}{(a^3+2a^2b+ab^2)\left(a\sin(fx+e)^2-a-b\right)^2}}{8f}$$

input
```
integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
-1/8*((4*a + b)*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/((a^3 + 2*a^2*b +
a*b^2)*sqrt(-a^2 - a*b)) + (4*a^2*sin(f*x + e)^3 + a*b*sin(f*x + e)^3 - 4*
a^2*sin(f*x + e) - 3*a*b*sin(f*x + e) + b^2*sin(f*x + e))/((a^3 + 2*a^2*b
+ a*b^2)*(a*sin(f*x + e)^2 - a - b)^2))/f
```

3.207.9 Mupad [B] (verification not implemented)

Time = 19.60 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.03

$$\int \frac{\sec^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\operatorname{atanh}\left(\frac{\sqrt{a}\,\sin(e+fx)}{\sqrt{a+b}}\right)(4a+b)}{8a^{3/2}f(a+b)^{5/2}} - \frac{\frac{\sin(e+fx)^3(4a+b)}{8(a+b)^2} - \frac{\sin(e+fx)(4a-b)}{8a(a+b)}}{f\left(2ab+a^2+b^2-\sin(e+fx)^2(2a^2+2ba)+a^2\sin(e+fx)^4\right)}$$

input
```
int(1/(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^3),x)
```

output
```
(atanh((a^(1/2)*sin(e + f*x))/(a + b)^(1/2))*(4*a + b))/(8*a^(3/2)*f*(a +
b)^(5/2)) - ((sin(e + f*x)^3*(4*a + b))/(8*(a + b)^2) - (sin(e + f*x)*(4*a
 - b))/(8*a*(a + b)))/(f*(2*a*b + a^2 + b^2 - sin(e + f*x)^2*(2*a*b + 2*a^
2) + a^2*sin(e + f*x)^4))
```

3.208 $\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

3.208.1 Optimal result . . . 1549
3.208.2 Mathematica [C] (warning: unable to verify) . . . 1549
3.208.3 Rubi [A] (verified) . . . 1550
3.208.4 Maple [A] (verified) . . . 1553
3.208.5 Fricas [B] (verification not implemented) . . . 1553
3.208.6 Sympy [F] . . . 1554
3.208.7 Maxima [A] (verification not implemented) . . . 1554
3.208.8 Giac [A] (verification not implemented) . . . 1555
3.208.9 Mupad [B] (verification not implemented) . . . 1555

3.208.1 Optimal result

Integrand size = 21, antiderivative size = 144

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{(8a^2+8ab+3b^2)\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{5/2}(a+b)^{5/2}f} - \frac{b\cos^2(e+fx)\sin(e+fx)}{4a(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2} - \frac{3b(2a+b)\sin(e+fx)}{8a^2(a+b)^2f\left(a+b-a\sin^2(e+fx)\right)}$$

output
```
1/8*(8*a^2+8*a*b+3*b^2)*arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))/a^(5/2)/(a
+b)^(5/2)/f-1/4*b*cos(f*x+e)^2*sin(f*x+e)/a/(a+b)/f/(a+b-a*sin(f*x+e)^2)^2
-3/8*b*(2*a+b)*sin(f*x+e)/a^2/(a+b)^2/f/(a+b-a*sin(f*x+e)^2)
```

3.208.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.45 (sec) , antiderivative size = 927, normalized size of antiderivative = 6.44

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{(a+2b+a\cos(2(e+fx)))\sec^5(e+fx)\left(-2i(8a^2+8ab+3b^2)\arctan\left(\frac{(a+b)\sin(e)}{(a+b)\cos(e)-\sqrt{a}\sqrt{a+b}\sqrt{(\cos(e)-i\sin(e))^2}}\right.\right.}{}$$

input
```
Integrate[Sec[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^5*((-2*I)*(8*a^2 + 8*a*b + 3*
b^2)*ArcTan[((a + b)*Sin[e])/((a + b)*Cos[e] - Sqrt[a]*Sqrt[a + b]*Sqrt[(C
os[e] - I*Sin[e])^2]*(Cos[2*e] + I*Sin[2*e])*Sin[e + f*x])]*(a + 2*b + a*C
os[2*(e + f*x)])^2*Sec[e + f*x]*(Cos[e] - I*Sin[e]) + (8*a^2 + 8*a*b + 3*b
^2)*(a + 2*b + a*Cos[2*(e + f*x)])^2*Log[a + 2*(a + b)*Cos[2*e] - a*Cos[2*
(e + f*x)] - (2*I)*a*Sin[2*e] - (2*I)*b*Sin[2*e] + 2*Sqrt[a]*Sqrt[a + b]*S
qrt[(Cos[e] - I*Sin[e])^2]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] -
 I*Sin[e])^2]*Sin[2*e + f*x]]*Sec[e + f*x]*(Cos[e] - I*Sin[e]) - (8*a^2 +
8*a*b + 3*b^2)*(a + 2*b + a*Cos[2*(e + f*x)])^2*Log[-a - 2*(a + b)*Cos[2*e
] + a*Cos[2*(e + f*x)] + (2*I)*a*Sin[2*e] + (2*I)*b*Sin[2*e] + 2*Sqrt[a]*S
qrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[f*x] + 2*Sqrt[a]*Sqrt[a + b]*Sq
rt[(Cos[e] - I*Sin[e])^2]*Sin[2*e + f*x]]*Sec[e + f*x]*(Cos[e] - I*Sin[e])
 + 2*(8*a^2 + 8*a*b + 3*b^2)*ArcTan[(2*Sin[e]*(I*a + I*b + I*(a + b)*Cos[2
*e] + Sqrt[a]*Sqrt[a + b]*Cos[f*x]*Sqrt[(Cos[e] - I*Sin[e])^2] - Sqrt[a]*S
qrt[a + b]*Cos[2*e + f*x]*Sqrt[(Cos[e] - I*Sin[e])^2] + a*Sin[2*e] + b*Sin
[2*e] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[f*x] - I*Sqr
t[a]*Sqrt[a + b]*Sqrt[(Cos[e] - I*Sin[e])^2]*Sin[2*e + f*x]))/(I*(a + 3*b)
*Cos[e] + I*(a + b)*Cos[3*e] + I*a*Cos[e + 2*f*x] + I*a*Cos[3*e + 2*f*x] +
 3*a*Sin[e] + b*Sin[e] + a*Sin[3*e] + b*Sin[3*e] + a*Sin[e + 2*f*x] - a*Si
n[3*e + 2*f*x])]*(a + 2*b + a*Cos[2*(e + f*x)])^2*Sec[e + f*x]*(I*Cos[e...
```

3.208.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4635, 315, 25, 298, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\sec(e+fx)}{\left(a+b\sec(e+fx)^2\right)^3}\,dx$$
$$\downarrow 4635$$

$$\frac{\int \frac{\left(1-\sin^2(e+fx)\right)^2}{\left(-a\sin^2(e+fx)+a+b\right)^3} d\sin(e+fx)}{f}$$

$$\downarrow 315$$

$$\frac{-\frac{\int -\frac{-\left((4a+3b)\sin^2(e+fx)\right)+4a+b}{\left(-a\sin^2(e+fx)+a+b\right)^2} d\sin(e+fx)}{4a(a+b)} - \frac{b\sin(e+fx)\left(1-\sin^2(e+fx)\right)}{4a(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2}}{f}$$

$$\downarrow 25$$

$$\frac{\frac{\int \frac{-\left((4a+3b)\sin^2(e+fx)\right)+4a+b}{\left(-a\sin^2(e+fx)+a+b\right)^2} d\sin(e+fx)}{4a(a+b)} - \frac{b\sin(e+fx)\left(1-\sin^2(e+fx)\right)}{4a(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2}}{f}$$

$$\downarrow 298$$

$$\frac{\frac{\left(8a^2+8ab+3b^2\right)\int \frac{1}{-a\sin^2(e+fx)+a+b} d\sin(e+fx)}{2a(a+b)} - \frac{3b(2a+b)\sin(e+fx)}{2a(a+b)\left(-a\sin^2(e+fx)+a+b\right)}}{4a(a+b)} - \frac{b\sin(e+fx)\left(1-\sin^2(e+fx)\right)}{4a(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2}}{f}$$

$$\downarrow 221$$

$$\frac{\frac{\left(8a^2+8ab+3b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{2a^{3/2}(a+b)^{3/2}} - \frac{3b(2a+b)\sin(e+fx)}{2a(a+b)\left(-a\sin^2(e+fx)+a+b\right)}}{4a(a+b)} - \frac{b\sin(e+fx)\left(1-\sin^2(e+fx)\right)}{4a(a+b)\left(-a\sin^2(e+fx)+a+b\right)^2}}{f}$$

input
```
Int[Sec[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
(-1/4*(b*Sin[e + f*x]*(1 - Sin[e + f*x]^2))/(a*(a + b)*(a + b - a*Sin[e +
f*x]^2)^2) + (((8*a^2 + 8*a*b + 3*b^2)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt
[a + b]])/(2*a^(3/2)*(a + b)^(3/2)) - (3*b*(2*a + b)*Sin[e + f*x])/(2*a*(a
 + b)*(a + b - a*Sin[e + f*x]^2)))/(4*a*(a + b)))/f
```

3.208.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.208.4 Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.99

method	result
derivativedivides	$\dfrac{-\dfrac{-\frac{b(8a+5b)\sin(fx+e)^3}{8a\left(a^2+2ab+b^2\right)}+\frac{(8a+3b)b\sin(fx+e)}{8a^2(a+b)}}{\left(a\sin(fx+e)^2-a-b\right)^2}+\dfrac{\left(8a^2+8ab+3b^2\right)\arctanh\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{8\left(a^2+2ab+b^2\right)a^2\sqrt{a(a+b)}}}{f}$
default	$\dfrac{-\dfrac{-\frac{b(8a+5b)\sin(fx+e)^3}{8a\left(a^2+2ab+b^2\right)}+\frac{(8a+3b)b\sin(fx+e)}{8a^2(a+b)}}{\left(a\sin(fx+e)^2-a-b\right)^2}+\dfrac{\left(8a^2+8ab+3b^2\right)\arctanh\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{8\left(a^2+2ab+b^2\right)a^2\sqrt{a(a+b)}}}{f}$
risch	$\dfrac{ib\left(8a^2e^{7i(fx+e)}+5abe^{7i(fx+e)}+8a^2e^{5i(fx+e)}+29abe^{5i(fx+e)}+12b^2e^{5i(fx+e)}-8a^2e^{3i(fx+e)}-29abe^{3i(fx+e)}-12b^2e^{3i(}\right.}{4a^2(a+b)^2f\left(ae^{4i(fx+e)}+2ae^{2i(fx+e)}+4be^{2i(fx+e)}+a\right)^2}$

input
```
int(sec(f*x+e)/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-(-1/8*b*(8*a+5*b)/a/(a^2+2*a*b+b^2)*sin(f*x+e)^3+1/8*(8*a+3*b)/a^2*b
/(a+b)*sin(f*x+e))/(a*sin(f*x+e)^2-a-b)^2+1/8*(8*a^2+8*a*b+3*b^2)/(a^2+2*a
*b+b^2)/a^2/(a*(a+b))^(1/2)*arctanh(a*sin(f*x+e)/(a*(a+b))^(1/2)))
```

3.208.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(136) = 272.

Time = 0.29 (sec) , antiderivative size = 613, normalized size of antiderivative = 4.26

$$\int \frac{\sec(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$
$$= \left[\frac{\left(\left(8a^4+8a^3b+3a^2b^2\right)\cos(fx+e)^4+8a^2b^2+8ab^3+3b^4+2\left(8a^3b+8a^2b^2+3ab^3\right)\cos(fx+e)^2\right)\sqrt{}}{16\left(\left(a^8+3a^7b+3a^6b^2+a^5b^3\right)f\cos(fx+e)^4+2(\right.}\right.$$
$$-\frac{\left(\left(8a^4+8a^3b+3a^2b^2\right)\cos(fx+e)^4+8a^2b^2+8ab^3+3b^4+2\left(8a^3b+8a^2b^2+3ab^3\right)\cos(fx+e)^2\right)\sqrt{}}{8\left(\left(a^8+3a^7b+3a^6b^2+a^5b^3\right)f\cos(fx+e)^4+2\left(a^7b+3\right.\right.}$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[1/16*(((8*a^4 + 8*a^3*b + 3*a^2*b^2)*cos(f*x + e)^4 + 8*a^2*b^2 + 8*a*b^3
 + 3*b^4 + 2*(8*a^3*b + 8*a^2*b^2 + 3*a*b^3)*cos(f*x + e)^2)*sqrt(a^2 + a*
b)*log(-(a*cos(f*x + e)^2 - 2*sqrt(a^2 + a*b)*sin(f*x + e) - 2*a - b)/(a*c
os(f*x + e)^2 + b)) - 2*(6*a^3*b^2 + 9*a^2*b^3 + 3*a*b^4 + (8*a^4*b + 13*a
^3*b^2 + 5*a^2*b^3)*cos(f*x + e)^2)*sin(f*x + e))/((a^8 + 3*a^7*b + 3*a^6*
b^2 + a^5*b^3)*f*cos(f*x + e)^4 + 2*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b
^4)*f*cos(f*x + e)^2 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*f), -1/
8*(((8*a^4 + 8*a^3*b + 3*a^2*b^2)*cos(f*x + e)^4 + 8*a^2*b^2 + 8*a*b^3 + 3
*b^4 + 2*(8*a^3*b + 8*a^2*b^2 + 3*a*b^3)*cos(f*x + e)^2)*sqrt(-a^2 - a*b)*
arctan(sqrt(-a^2 - a*b)*sin(f*x + e)/(a + b)) + (6*a^3*b^2 + 9*a^2*b^3 + 3
*a*b^4 + (8*a^4*b + 13*a^3*b^2 + 5*a^2*b^3)*cos(f*x + e)^2)*sin(f*x + e))/
((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*f*cos(f*x + e)^4 + 2*(a^7*b + 3*a^6
*b^2 + 3*a^5*b^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a^6*b^2 + 3*a^5*b^3 + 3*a^
4*b^4 + a^3*b^5)*f)]
```

3.208.6 Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Integral(sec(e + f*x)/(a + b*sec(e + f*x)**2)**3, x)
```

3.208.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.62

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx =$$
$$-\frac{\frac{(8a^2+8ab+3b^2)\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{(a^4+2a^3b+a^2b^2)\sqrt{(a+b)a}} - \frac{2\left((8a^2b+5ab^2)\sin(fx+e)^3-(8a^2b+11ab^2+3b^3)\sin(fx+e)\right)}{a^6+4a^5b+6a^4b^2+4a^3b^3+a^2b^4+(a^6+2a^5b+a^4b^2)\sin(fx+e)^4-2(a^6+3a^5b+3a^4b^2+a^3b^3}}{16f}$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
-1/16*((8*a^2 + 8*a*b + 3*b^2)*log((a*sin(f*x + e) - sqrt((a + b)*a))/(a*s
in(f*x + e) + sqrt((a + b)*a)))/((a^4 + 2*a^3*b + a^2*b^2)*sqrt((a + b)*a)
) - 2*((8*a^2*b + 5*a*b^2)*sin(f*x + e)^3 - (8*a^2*b + 11*a*b^2 + 3*b^3)*s
in(f*x + e))/(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4 + (a^6 + 2*a
^5*b + a^4*b^2)*sin(f*x + e)^4 - 2*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*s
in(f*x + e)^2))/f
```

3.208.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.24

$$\int \frac{\sec(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{\frac{\left(8a^2+8ab+3b^2\right)\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{\left(a^4+2a^3b+a^2b^2\right)\sqrt{-a^2-ab}} - \frac{8a^2b\sin(fx+e)^3+5ab^2\sin(fx+e)^3-8a^2b\sin(fx+e)-11ab^2\sin(fx+e)-3b^3\sin(fx+e)}{\left(a^4+2a^3b+a^2b^2\right)\left(a\sin(fx+e)^2-a-b\right)^2}}{8f}$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
-1/8*((8*a^2 + 8*a*b + 3*b^2)*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b))/((a^
4 + 2*a^3*b + a^2*b^2)*sqrt(-a^2 - a*b)) - (8*a^2*b*sin(f*x + e)^3 + 5*a*b
^2*sin(f*x + e)^3 - 8*a^2*b*sin(f*x + e) - 11*a*b^2*sin(f*x + e) - 3*b^3*s
in(f*x + e))/((a^4 + 2*a^3*b + a^2*b^2)*(a*sin(f*x + e)^2 - a - b)^2))/f
```

3.208.9 Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.03

$$\int \frac{\sec(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\frac{\sin(e+fx)^3\left(5b^2+8ab\right)}{8a(a+b)^2} - \frac{\sin(e+fx)\left(3b^2+8ab\right)}{8a^2(a+b)}}{f\left(2ab+a^2+b^2-\sin(e+fx)^2\left(2a^2+2ba\right)+a^2\sin(e+fx)^4\right)} + \frac{\operatorname{atanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)\left(8a^2+8ab+3b^2\right)}{8a^{5/2}f(a+b)^{5/2}}$$

input
```
int(1/(cos(e + f*x)*(a + b/cos(e + f*x)^2)^3),x)
```

output
```
((sin(e + f*x)^3*(8*a*b + 5*b^2))/(8*a*(a + b)^2) - (sin(e + f*x)*(8*a*b +
 3*b^2))/(8*a^2*(a + b)))/(f*(2*a*b + a^2 + b^2 - sin(e + f*x)^2*(2*a*b +
2*a^2) + a^2*sin(e + f*x)^4)) + (atanh((a^(1/2)*sin(e + f*x))/(a + b)^(1/2
))*(8*a*b + 8*a^2 + 3*b^2))/(8*a^(5/2)*f*(a + b)^(5/2))
```

3.209 $\int \frac{\cos(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$

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3.209.1 Optimal result

Integrand size = 21, antiderivative size = 156

$$\int \frac{\cos(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{3b(4(a+b)^2+(2a+b)^2)\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{7/2}(a+b)^{5/2}f} + \frac{\sin(e+fx)}{a^3 f} - \frac{b^3\sin(e+fx)}{4a^3(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2} + \frac{3b^2(4a+3b)\sin(e+fx)}{8a^3(a+b)^2 f\left(a+b-a\sin^2(e+fx)\right)}$$

output

```
-3/8*b*(4*(a+b)^2+(2*a+b)^2)*arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))/a^(7/
2)/(a+b)^(5/2)/f+sin(f*x+e)/a^3/f-1/4*b^3*sin(f*x+e)/a^3/(a+b)/f/(a+b-a*si
n(f*x+e)^2)^2+3/8*b^2*(4*a+3*b)*sin(f*x+e)/a^3/(a+b)^2/f/(a+b-a*sin(f*x+e)
^2)
```

3.209.2 Mathematica [A] (verified)

Time = 1.36 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.13

$$\int \frac{\cos(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{-\frac{3b(8a^2+12ab+5b^2)\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} + \frac{4\sqrt{a}\sin(e+fx)\left(8a^4+32a^3b+60a^2b^2+51ab^3+15b^4-a\left(16a^3+48a^2b+60ab^2+25b^3\right)\sin^2(e+f\right.}{(a+b)^2(a+2b+a\cos(2(e+fx)))^2}}{8a^{7/2}f}$$

input
```
Integrate[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
((-3*b*(8*a^2 + 12*a*b + 5*b^2)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]
])/(a + b)^(5/2) + (4*Sqrt[a]*Sin[e + f*x]*(8*a^4 + 32*a^3*b + 60*a^2*b^2
+ 51*a*b^3 + 15*b^4 - a*(16*a^3 + 48*a^2*b + 60*a*b^2 + 25*b^3)*Sin[e + f*
x]^2 + 8*a^2*(a + b)^2*Sin[e + f*x]^4))/((a + b)^2*(a + 2*b + a*Cos[2*(e +
 f*x)])^2))/(8*a^(7/2)*f)
```

3.209.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4635, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sec(e+fx)\,(a+b\sec(e+fx)^2)^3}\,dx$$

$$\downarrow 4635$$

$$\frac{\int \frac{(1-\sin^2(e+fx))^3}{(-a\sin^2(e+fx)+a+b)^3}\,d\sin(e+fx)}{f}$$

$$\downarrow 300$$

$$\frac{\int \left(\frac{1}{a^3} - \frac{3a^2b\sin^4(e+fx) - 3ab(2a+b)\sin^2(e+fx) + b(3a^2+3ba+b^2)}{a^3(-a\sin^2(e+fx)+a+b)^3}\right)\,d\sin(e+fx)}{f}$$

$$\downarrow 2009$$

$$\frac{-\frac{3b(4(a+b)^2+(2a+b)^2)\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{7/2}(a+b)^{5/2}} - \frac{b^3\sin(e+fx)}{4a^3(a+b)(-a\sin^2(e+fx)+a+b)^2} + \frac{3b^2(4a+3b)\sin(e+fx)}{8a^3(a+b)^2(-a\sin^2(e+fx)+a+b)} + \frac{\sin(e+fx)}{a^3}}{f}$$

input
```
Int[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
((-3*b*(4*(a + b)^2 + (2*a + b)^2)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a +
 b]])/(8*a^(7/2)*(a + b)^(5/2)) + Sin[e + f*x]/a^3 - (b^3*Sin[e + f*x])/(4
*a^3*(a + b)*(a + b - a*Sin[e + f*x]^2)^2) + (3*b^2*(4*a + 3*b)*Sin[e + f*
x])/(8*a^3*(a + b)^2*(a + b - a*Sin[e + f*x]^2)))/f
```

3.209.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.209.4 Maple [A] (verified)

Time = 3.14 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{\frac{\sin(fx+e)}{a^3}+\frac{b\left(\frac{-\frac{3ab(4a+3b)\sin(fx+e)^3}{8\left(a^2+2ab+b^2\right)}+\frac{(12a+7b)b\sin(fx+e)}{8a+8b}}{\left(a\sin(fx+e)^2-a-b\right)^2}-\frac{3\left(8a^2+12ab+5b^2\right)\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{8\left(a^2+2ab+b^2\right)\sqrt{a(a+b)}}\right)}{a^3}}{f}$
default	$\frac{\frac{\sin(fx+e)}{a^3}+\frac{b\left(\frac{-\frac{3ab(4a+3b)\sin(fx+e)^3}{8\left(a^2+2ab+b^2\right)}+\frac{(12a+7b)b\sin(fx+e)}{8a+8b}}{\left(a\sin(fx+e)^2-a-b\right)^2}-\frac{3\left(8a^2+12ab+5b^2\right)\operatorname{arctanh}\left(\frac{a\sin(fx+e)}{\sqrt{a(a+b)}}\right)}{8\left(a^2+2ab+b^2\right)\sqrt{a(a+b)}}\right)}{a^3}}{f}$
risch	$-\frac{ie^{i(fx+e)}}{2a^3f}+\frac{ie^{-i(fx+e)}}{2a^3f}-\frac{ib^2\left(12a^2e^{7i(fx+e)}+9abe^{7i(fx+e)}+12a^2e^{5i(fx+e)}+49abe^{5i(fx+e)}+28b^2e^{5i(fx+e)}-12a\right.}{4a^3(a+b)^2f\left(ae^{4i(fx+e)}+2ae^{2i(fx+\right.}$

input
```
int(cos(f*x+e)/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(sin(f*x+e)/a^3+1/a^3*b*((-3/8*a*b*(4*a+3*b)/(a^2+2*a*b+b^2)*sin(f*x+e
)^3+1/8*(12*a+7*b)*b/(a+b)*sin(f*x+e))/(a*sin(f*x+e)^2-a-b)^2-3/8*(8*a^2+1
2*a*b+5*b^2)/(a^2+2*a*b+b^2)/(a*(a+b))^(1/2)*arctanh(a*sin(f*x+e)/(a*(a+b)
)^(1/2))))
```

3.209.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(148) = 296$.

Time $= 0.33$ (sec) , antiderivative size $= 727$, normalized size of antiderivative $= 4.66$

$$\int \frac{\cos(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$

$$= \left[\frac{3\left(8a^2b^3+12ab^4+5b^5+\left(8a^4b+12a^3b^2+5a^2b^3\right)\cos(fx+e)^4+2\left(8a^3b^2+12a^2b^3+5ab^4\right)\cos(fx+\right.}{16\left((a^9+3a^\right.}\right.$$

input
```
integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[1/16*(3*(8*a^2*b^3 + 12*a*b^4 + 5*b^5 + (8*a^4*b + 12*a^3*b^2 + 5*a^2*b^3
)*cos(f*x + e)^4 + 2*(8*a^3*b^2 + 12*a^2*b^3 + 5*a*b^4)*cos(f*x + e)^2)*sq
rt(a^2 + a*b)*log(-(a*cos(f*x + e)^2 + 2*sqrt(a^2 + a*b)*sin(f*x + e) - 2*
a - b)/(a*cos(f*x + e)^2 + b)) + 2*(8*a^4*b^2 + 34*a^3*b^3 + 41*a^2*b^4 +
15*a*b^5 + 8*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cos(f*x + e)^4 + (16*a^
5*b + 60*a^4*b^2 + 69*a^3*b^3 + 25*a^2*b^4)*cos(f*x + e)^2)*sin(f*x + e))/
((a^9 + 3*a^8*b + 3*a^7*b^2 + a^6*b^3)*f*cos(f*x + e)^4 + 2*(a^8*b + 3*a^7
*b^2 + 3*a^6*b^3 + a^5*b^4)*f*cos(f*x + e)^2 + (a^7*b^2 + 3*a^6*b^3 + 3*a^
5*b^4 + a^4*b^5)*f), 1/8*(3*(8*a^2*b^3 + 12*a*b^4 + 5*b^5 + (8*a^4*b + 12*
a^3*b^2 + 5*a^2*b^3)*cos(f*x + e)^4 + 2*(8*a^3*b^2 + 12*a^2*b^3 + 5*a*b^4)
*cos(f*x + e)^2)*sqrt(-a^2 - a*b)*arctan(sqrt(-a^2 - a*b)*sin(f*x + e)/(a
+ b)) + (8*a^4*b^2 + 34*a^3*b^3 + 41*a^2*b^4 + 15*a*b^5 + 8*(a^6 + 3*a^5*b
 + 3*a^4*b^2 + a^3*b^3)*cos(f*x + e)^4 + (16*a^5*b + 60*a^4*b^2 + 69*a^3*b
^3 + 25*a^2*b^4)*cos(f*x + e)^2)*sin(f*x + e))/((a^9 + 3*a^8*b + 3*a^7*b^2
 + a^6*b^3)*f*cos(f*x + e)^4 + 2*(a^8*b + 3*a^7*b^2 + 3*a^6*b^3 + a^5*b^4)
*f*cos(f*x + e)^2 + (a^7*b^2 + 3*a^6*b^3 + 3*a^5*b^4 + a^4*b^5)*f)]
```

3.209.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Timed out
```

3.209.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.62

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \frac{3\left(8a^2b+12ab^2+5b^3\right)\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{\left(a^5+2a^4b+a^3b^2\right)\sqrt{(a+b)a}} - \frac{2\left(3\left(4a^2b^2+3ab^3\right)\sin(fx+e)^3-\left(12a^2b^2+19ab^3+7b^4\right)\sin(fx+e)\right)}{a^7+4a^6b+6a^5b^2+4a^4b^3+a^3b^4+\left(a^7+2a^6b+a^5b^2\right)\sin(fx+e)^4-2\left(a^7+3a^6b+3a^5b^2+a^4b\right.}$$
$$16f$$

input
```
integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
1/16*(3*(8*a^2*b + 12*a*b^2 + 5*b^3)*log((a*sin(f*x + e) - sqrt((a + b)*a)
)/(a*sin(f*x + e) + sqrt((a + b)*a)))/((a^5 + 2*a^4*b + a^3*b^2)*sqrt((a +
 b)*a)) - 2*(3*(4*a^2*b^2 + 3*a*b^3)*sin(f*x + e)^3 - (12*a^2*b^2 + 19*a*b
^3 + 7*b^4)*sin(f*x + e))/(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4
 + (a^7 + 2*a^6*b + a^5*b^2)*sin(f*x + e)^4 - 2*(a^7 + 3*a^6*b + 3*a^5*b^2
 + a^4*b^3)*sin(f*x + e)^2) + 16*sin(f*x + e)/a^3)/f
```

3.209.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.26

$$\int \frac{\cos(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$
$$= \frac{\frac{3\left(8a^2b+12ab^2+5b^3\right)\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{\left(a^5+2a^4b+a^3b^2\right)\sqrt{-a^2-ab}} - \frac{12a^2b^2\sin(fx+e)^3+9ab^3\sin(fx+e)^3-12a^2b^2\sin(fx+e)-19ab^3\sin(fx+e)-7b^4\sin(fx+e)}{\left(a^5+2a^4b+a^3b^2\right)\left(a\sin(fx+e)^2-a-b\right)^2} +$$
$$\frac{}{8f}$$

input
```
integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
1/8*(3*(8*a^2*b + 12*a*b^2 + 5*b^3)*arctan(a*sin(f*x + e)/sqrt(-a^2 - a*b)
)/((a^5 + 2*a^4*b + a^3*b^2)*sqrt(-a^2 - a*b)) - (12*a^2*b^2*sin(f*x + e)^
3 + 9*a*b^3*sin(f*x + e)^3 - 12*a^2*b^2*sin(f*x + e) - 19*a*b^3*sin(f*x +
e) - 7*b^4*sin(f*x + e))/((a^5 + 2*a^4*b + a^3*b^2)*(a*sin(f*x + e)^2 - a
- b)^2) + 8*sin(f*x + e)/a^3)/f
```

3.209.9 Mupad [B] (verification not implemented)

Time = 19.72 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.12

$$\int \frac{\cos(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$
$$= \frac{\sin(e+fx)}{a^3 f} + \frac{\frac{\sin(e+fx)\left(7b^3+12ab^2\right)}{8(a+b)} - \frac{3\sin(e+fx)^3\left(4a^2b^2+3ab^3\right)}{8(a+b)^2}}{f\left(2a^4b-\sin(e+fx)^2\left(2a^5+2ba^4\right)+a^5+a^3b^2+a^5\sin(e+fx)^4\right)}$$
$$- \frac{3b\,\mathrm{atanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)\left(8a^2+12ab+5b^2\right)}{8a^{7/2}f(a+b)^{5/2}}$$

input
```
int(cos(e + f*x)/(a + b/cos(e + f*x)^2)^3,x)
```

output
```
sin(e + f*x)/(a^3*f) + ((sin(e + f*x)*(12*a*b^2 + 7*b^3))/(8*(a + b)) - (3
*sin(e + f*x)^3*(3*a*b^3 + 4*a^2*b^2))/(8*(a + b)^2))/(f*(2*a^4*b - sin(e
+ f*x)^2*(2*a^4*b + 2*a^5) + a^5 + a^3*b^2 + a^5*sin(e + f*x)^4)) - (3*b*a
tanh((a^(1/2)*sin(e + f*x))/(a + b)^(1/2))*(12*a*b + 8*a^2 + 5*b^2))/(8*a^
(7/2)*f*(a + b)^(5/2))
```

3.210 $\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

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3.210.1 Optimal result

Integrand size = 23, antiderivative size = 181

$$\begin{aligned}\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx &= \frac{b^2(48a^2+80ab+35b^2)\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{9/2}(a+b)^{5/2}f}\\ &+\frac{(a-3b)\sin(e+fx)}{a^4f}-\frac{\sin^3(e+fx)}{3a^3f}\\ &+\frac{b^4\sin(e+fx)}{4a^4(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2}\\ &-\frac{b^3(16a+13b)\sin(e+fx)}{8a^4(a+b)^2f\left(a+b-a\sin^2(e+fx)\right)}\end{aligned}$$

output

```
1/8*b^2*(48*a^2+80*a*b+35*b^2)*arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))/a^(
9/2)/(a+b)^(5/2)/f+(a-3*b)*sin(f*x+e)/a^4/f-1/3*sin(f*x+e)^3/a^3/f+1/4*b^4
*sin(f*x+e)/a^4/(a+b)/f/(a+b-a*sin(f*x+e)^2)^2-1/8*b^3*(16*a+13*b)*sin(f*x
+e)/a^4/(a+b)^2/f/(a+b-a*sin(f*x+e)^2)
```

3.210.2 Mathematica [A] (verified)

Time = 3.25 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.07

$$\int \frac{\cos^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$
$$= \frac{-\frac{3b^2(48a^2+80ab+35b^2)\left(\log\left(\sqrt{a+b}-\sqrt{a}\sin(e+fx)\right)-\log\left(\sqrt{a+b}+\sqrt{a}\sin(e+fx)\right)\right)}{(a+b)^{5/2}}+12\sqrt{a}\left(-12b-\frac{b^4(9a+22b+13a\cos(2(e+fx)))}{(a+b)^2(a+2b+a\cos(2(e+fx)))^2}+}{48a^{9/2}f}$$

input
```
Integrate[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
((-3*b^2*(48*a^2 + 80*a*b + 35*b^2)*(Log[Sqrt[a + b] - Sqrt[a]*Sin[e + f*x
]] - Log[Sqrt[a + b] + Sqrt[a]*Sin[e + f*x]]))/(a + b)^(5/2) + 12*Sqrt[a]*
(-12*b - (b^4*(9*a + 22*b + 13*a*Cos[2*(e + f*x)]))/((a + b)^2*(a + 2*b +
a*Cos[2*(e + f*x)])^2) + a*(3 - (16*b^3)/((a + b)^2*(a + 2*b + a*Cos[2*(e
+ f*x)])))))*Sin[e + f*x] + 4*a^(3/2)*Sin[3*(e + f*x)])/(48*a^(9/2)*f)
```

3.210.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4635, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{\sec(e+fx)^3\left(a+b\sec(e+fx)^2\right)^3}\,dx$$
$$\downarrow 4635$$
$$\frac{\int \frac{\left(1-\sin^2(e+fx)\right)^4}{\left(-a\sin^2(e+fx)+a+b\right)^3}\,d\sin(e+fx)}{f}$$
$$\downarrow 300$$

$$\frac{\int \left(-\frac{\sin^2(e+fx)}{a^3}+\frac{a-3b}{a^4}+\frac{6a^2b^2\sin^4(e+fx)-4ab^2(3a+2b)\sin^2(e+fx)+b^2(6a^2+8ba+3b^2)}{a^4(-a\sin^2(e+fx)+a+b)^3}\right)d\sin(e+fx)}{f}$$

$$\downarrow \text{2009}$$

$$\frac{\frac{b^4\sin(e+fx)}{4a^4(a+b)(-a\sin^2(e+fx)+a+b)^2}-\frac{b^3(16a+13b)\sin(e+fx)}{8a^4(a+b)^2(-a\sin^2(e+fx)+a+b)}+\frac{(a-3b)\sin(e+fx)}{a^4}-\frac{\sin^3(e+fx)}{3a^3}+\frac{b^2(48a^2+80ab+35b^2)\text{arctanh}}{8a^{9/2}(a+b)^{5/2}}}{f}$$

input
```
Int[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
((b^2*(48*a^2 + 80*a*b + 35*b^2)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b
]])/(8*a^(9/2)*(a + b)^(5/2)) + ((a - 3*b)*Sin[e + f*x])/a^4 - Sin[e + f*x
]^3/(3*a^3) + (b^4*Sin[e + f*x])/(4*a^4*(a + b)*(a + b - a*Sin[e + f*x]^2)
^2) - (b^3*(16*a + 13*b)*Sin[e + f*x])/(8*a^4*(a + b)^2*(a + b - a*Sin[e +
 f*x]^2)))/f
```

3.210.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.210.4 Maple [A] (verified)

Time = 5.84 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{-\frac{\frac{a\sin(fx+e)^3}{3}-\sin(fx+e)a+3\sin(fx+e)b}{a^4}-b^2\left(\frac{-\frac{ab(16a+13b)\sin(fx+e)^3}{8\left(a^2+2ab+b^2\right)}+\frac{(16a+11b)b\sin(fx+e)}{8a+8b}}{\left(a\sin(fx+e)^2-a-b\right)^2}-\frac{\left(48a^2+80ab+35b^2\right)\arctanh$ $\frac{}{8\left(a^2+2ab+b^2\right)\sqrt{a}}$ $\frac{}{a^4}$ $\frac{}{f}$
default	$\frac{-\frac{\frac{a\sin(fx+e)^3}{3}-\sin(fx+e)a+3\sin(fx+e)b}{a^4}-b^2\left(\frac{-\frac{ab(16a+13b)\sin(fx+e)^3}{8\left(a^2+2ab+b^2\right)}+\frac{(16a+11b)b\sin(fx+e)}{8a+8b}}{\left(a\sin(fx+e)^2-a-b\right)^2}-\frac{\left(48a^2+80ab+35b^2\right)\arctanh$ $\frac{}{8\left(a^2+2ab+b^2\right)\sqrt{a}}$ $\frac{}{a^4}$ $\frac{}{f}$
risch	$-\frac{ie^{3i(fx+e)}}{24a^3f}-\frac{3ie^{i(fx+e)}}{8a^3f}+\frac{3ie^{i(fx+e)}b}{2a^4f}+\frac{3ie^{-i(fx+e)}}{8a^3f}-\frac{3ie^{-i(fx+e)}b}{2a^4f}+\frac{ie^{-3i(fx+e)}}{24a^3f}+\frac{ib^3\left(16a^2e^{7i(fx+e)}+$

input
```
int(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/a^4*(1/3*a*sin(f*x+e)^3-sin(f*x+e)*a+3*sin(f*x+e)*b)-b^2/a^4*((-1/
8*a*b*(16*a+13*b)/(a^2+2*a*b+b^2)*sin(f*x+e)^3+1/8*(16*a+11*b)*b/(a+b)*sin
(f*x+e))/(a*sin(f*x+e)^2-a-b)^2-1/8*(48*a^2+80*a*b+35*b^2)/(a^2+2*a*b+b^2)
/(a*(a+b))^(1/2)*arctanh(a*sin(f*x+e)/(a*(a+b))^(1/2))))
```

3.210.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 416 vs. 2(171) = 342.

Time = 0.32 (sec) , antiderivative size = 856, normalized size of antiderivative = 4.73

$$\int \frac{\cos^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$
$$= \left[\frac{3\left(48\,a^2b^4+80\,ab^5+35\,b^6+\left(48\,a^4b^2+80\,a^3b^3+35\,a^2b^4\right)\cos\left(fx+e\right)^4+2\left(48\,a^3b^3+80\,a^2b^4+35\,ab^5\right)\right.}{}\right.$$
$$\left.-\frac{3\left(48\,a^2b^4+80\,ab^5+35\,b^6+\left(48\,a^4b^2+80\,a^3b^3+35\,a^2b^4\right)\cos\left(fx+e\right)^4+2\left(48\,a^3b^3+80\,a^2b^4+35\,ab^5\right)\right.}{}\right.$$

input
```
integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[1/48*(3*(48*a^2*b^4 + 80*a*b^5 + 35*b^6 + (48*a^4*b^2 + 80*a^3*b^3 + 35*a
^2*b^4)*cos(f*x + e)^4 + 2*(48*a^3*b^3 + 80*a^2*b^4 + 35*a*b^5)*cos(f*x +
e)^2)*sqrt(a^2 + a*b)*log(-(a*cos(f*x + e)^2 - 2*sqrt(a^2 + a*b)*sin(f*x +
 e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) + 2*(16*a^5*b^2 - 24*a^4*b^3 - 210*
a^3*b^4 - 275*a^2*b^5 - 105*a*b^6 + 8*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3
)*cos(f*x + e)^6 + 8*(2*a^7 - a^6*b - 15*a^5*b^2 - 19*a^4*b^3 - 7*a^3*b^4)
*cos(f*x + e)^4 + (32*a^6*b - 40*a^5*b^2 - 360*a^4*b^3 - 463*a^3*b^4 - 175
*a^2*b^5)*cos(f*x + e)^2)*sin(f*x + e))/((a^10 + 3*a^9*b + 3*a^8*b^2 + a^7
*b^3)*f*cos(f*x + e)^4 + 2*(a^9*b + 3*a^8*b^2 + 3*a^7*b^3 + a^6*b^4)*f*cos
(f*x + e)^2 + (a^8*b^2 + 3*a^7*b^3 + 3*a^6*b^4 + a^5*b^5)*f), -1/24*(3*(48
*a^2*b^4 + 80*a*b^5 + 35*b^6 + (48*a^4*b^2 + 80*a^3*b^3 + 35*a^2*b^4)*cos(
f*x + e)^4 + 2*(48*a^3*b^3 + 80*a^2*b^4 + 35*a*b^5)*cos(f*x + e)^2)*sqrt(-
a^2 - a*b)*arctan(sqrt(-a^2 - a*b)*sin(f*x + e)/(a + b)) - (16*a^5*b^2 - 2
4*a^4*b^3 - 210*a^3*b^4 - 275*a^2*b^5 - 105*a*b^6 + 8*(a^7 + 3*a^6*b + 3*a
^5*b^2 + a^4*b^3)*cos(f*x + e)^6 + 8*(2*a^7 - a^6*b - 15*a^5*b^2 - 19*a^4*
b^3 - 7*a^3*b^4)*cos(f*x + e)^4 + (32*a^6*b - 40*a^5*b^2 - 360*a^4*b^3 - 4
63*a^3*b^4 - 175*a^2*b^5)*cos(f*x + e)^2)*sin(f*x + e))/((a^10 + 3*a^9*b +
 3*a^8*b^2 + a^7*b^3)*f*cos(f*x + e)^4 + 2*(a^9*b + 3*a^8*b^2 + 3*a^7*b^3
+ a^6*b^4)*f*cos(f*x + e)^2 + (a^8*b^2 + 3*a^7*b^3 + 3*a^6*b^4 + a^5*b^5)*
f)]
```

3.210.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**3/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Timed out
```

3.210.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.50

$$\int \frac{\cos^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx =$$

$$-\frac{\frac{3\left(48a^2b^2+80ab^3+35b^4\right)\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{\left(a^6+2a^5b+a^4b^2\right)\sqrt{(a+b)a}} - \frac{6\left(\left(16a^2b^3+13ab^4\right)\sin(fx+e)^3-\left(16a^2b^3+27ab^4+11b^5\right)\sin(fx+\right.}{a^8+4a^7b+6a^6b^2+4a^5b^3+a^4b^4+\left(a^8+2a^7b+a^6b^2\right)\sin(fx+e)^4-2\left(a^8+3a^7b+3a^6b\right.}}{48f}$$

input
```
integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
-1/48*(3*(48*a^2*b^2 + 80*a*b^3 + 35*b^4)*log((a*sin(f*x + e) - sqrt((a +
b)*a))/(a*sin(f*x + e) + sqrt((a + b)*a)))/((a^6 + 2*a^5*b + a^4*b^2)*sqrt
((a + b)*a)) - 6*((16*a^2*b^3 + 13*a*b^4)*sin(f*x + e)^3 - (16*a^2*b^3 + 2
7*a*b^4 + 11*b^5)*sin(f*x + e))/(a^8 + 4*a^7*b + 6*a^6*b^2 + 4*a^5*b^3 + a
^4*b^4 + (a^8 + 2*a^7*b + a^6*b^2)*sin(f*x + e)^4 - 2*(a^8 + 3*a^7*b + 3*a
^6*b^2 + a^5*b^3)*sin(f*x + e)^2) + 16*(a*sin(f*x + e)^3 - 3*(a - 3*b)*sin
(f*x + e))/a^4)/f
```

3.210.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.27

$$\int \frac{\cos^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx =$$

$$-\frac{\frac{3\left(48a^2b^2+80ab^3+35b^4\right)\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{\left(a^6+2a^5b+a^4b^2\right)\sqrt{-a^2-ab}} - \frac{3\left(16a^2b^3\sin(fx+e)^3+13ab^4\sin(fx+e)^3-16a^2b^3\sin(fx+e)-27ab^4\sin(fx+e)-11b^5\right.}{\left(a^6+2a^5b+a^4b^2\right)\left(a\sin(fx+e)^2-a-b\right)^2}}{24f}$$

input
```
integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
-1/24*(3*(48*a^2*b^2 + 80*a*b^3 + 35*b^4)*arctan(a*sin(f*x + e)/sqrt(-a^2
- a*b))/((a^6 + 2*a^5*b + a^4*b^2)*sqrt(-a^2 - a*b)) - 3*(16*a^2*b^3*sin(f
*x + e)^3 + 13*a*b^4*sin(f*x + e)^3 - 16*a^2*b^3*sin(f*x + e) - 27*a*b^4*s
in(f*x + e) - 11*b^5*sin(f*x + e))/((a^6 + 2*a^5*b + a^4*b^2)*(a*sin(f*x +
 e)^2 - a - b)^2) + 8*(a^6*sin(f*x + e)^3 - 3*a^6*sin(f*x + e) + 9*a^5*b*s
in(f*x + e))/a^9)/f
```

3.210.9 Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.41

$$\int \frac{\cos^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$
$$= \frac{b^2\,\ln\left(\sqrt{a+b}+\sqrt{a}\,\sin(e+f\,x)\right)\,\left(3\,a^2+5\,a\,b+\frac{35\,b^2}{16}\right)}{a^{9/2}\,f\,(a+b)^{5/2}}$$
$$- \frac{\frac{\sin(e+f\,x)\,\left(11\,b^4+16\,a\,b^3\right)}{8\,(a+b)} - \frac{\sin(e+f\,x)^3\,\left(16\,a^2\,b^3+13\,a\,b^4\right)}{8\,(a+b)^2}}{f\,\left(2\,a^5\,b-\sin(e+f\,x)^2\,\left(2\,a^6+2\,b\,a^5\right)+a^6+a^4\,b^2+a^6\,\sin(e+f\,x)^4\right)}$$
$$- \frac{\sin(e+f\,x)^3}{3\,a^3\,f} - \frac{b^2\,\ln\left(\sqrt{a}\,\sin(e+f\,x)-\sqrt{a+b}\right)\,\left(48\,a^2+80\,a\,b+35\,b^2\right)}{16\,a^{9/2}\,f\,(a+b)^{5/2}}$$
$$- \frac{\sin(e+f\,x)\,\left(\frac{3\,(a+b)}{a^4}-\frac{4}{a^3}\right)}{f}$$

input
```
int(cos(e + f*x)^3/(a + b/cos(e + f*x)^2)^3,x)
```

output
```
(b^2*log((a + b)^(1/2) + a^(1/2)*sin(e + f*x))*(5*a*b + 3*a^2 + (35*b^2)/1
6))/(a^(9/2)*f*(a + b)^(5/2)) - ((sin(e + f*x)*(16*a*b^3 + 11*b^4))/(8*(a
+ b)) - (sin(e + f*x)^3*(13*a*b^4 + 16*a^2*b^3))/(8*(a + b)^2))/(f*(2*a^5*
b - sin(e + f*x)^2*(2*a^5*b + 2*a^6) + a^6 + a^4*b^2 + a^6*sin(e + f*x)^4)
) - sin(e + f*x)^3/(3*a^3*f) - (b^2*log(a^(1/2)*sin(e + f*x) - (a + b)^(1/
2))*(80*a*b + 48*a^2 + 35*b^2))/(16*a^(9/2)*f*(a + b)^(5/2)) - (sin(e + f*
x)*((3*(a + b))/a^4 - 4/a^3))/f
```

3.211 $\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

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3.211.1 Optimal result

Integrand size = 23, antiderivative size = 214

$$\begin{aligned}\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx &= -\frac{b^3(80a^2+140ab+63b^2)\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{8a^{11/2}(a+b)^{5/2}f}\\ &+\frac{(a^2-3ab+6b^2)\sin(e+fx)}{a^5f}-\frac{(2a-3b)\sin^3(e+fx)}{3a^4f}\\ &+\frac{\sin^5(e+fx)}{5a^3f}-\frac{b^5\sin(e+fx)}{4a^5(a+b)f\left(a+b-a\sin^2(e+fx)\right)^2}\\ &+\frac{b^4(20a+17b)\sin(e+fx)}{8a^5(a+b)^2f\left(a+b-a\sin^2(e+fx)\right)}\end{aligned}$$

output

```
-1/8*b^3*(80*a^2+140*a*b+63*b^2)*arctanh(sin(f*x+e)*a^(1/2)/(a+b)^(1/2))/a
^(11/2)/(a+b)^(5/2)/f+(a^2-3*a*b+6*b^2)*sin(f*x+e)/a^5/f-1/3*(2*a-3*b)*sin
(f*x+e)^3/a^4/f+1/5*sin(f*x+e)^5/a^3/f-1/4*b^5*sin(f*x+e)/a^5/(a+b)/f/(a+b
-a*sin(f*x+e)^2)^2+1/8*b^4*(20*a+17*b)*sin(f*x+e)/a^5/(a+b)^2/f/(a+b-a*sin
(f*x+e)^2)
```

3.211.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.35 (sec) , antiderivative size = 2670, normalized size of antiderivative = 12.48

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
((5*a^2 - 18*a*b + 48*b^2)*Cos[f*x]*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e
 + f*x]^6*Sin[e])/(64*a^5*f*(a + b*Sec[e + f*x]^2)^3) + ((-80*a^2*b^3 - 14
0*a*b^4 - 63*b^5)*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(((I/128
)*ArcTan[((-I)*a*Cos[e] - I*b*Cos[e] + I*a*Cos[3*e] + I*b*Cos[3*e] + a*Sin
[e] + b*Sin[e] - Sqrt[a]*Sqrt[a + b]*Cos[e - f*x]*Sqrt[Cos[2*e] - I*Sin[2*
e]] + Sqrt[a]*Sqrt[a + b]*Cos[3*e + f*x]*Sqrt[Cos[2*e] - I*Sin[2*e]] + a*S
in[3*e] + b*Sin[3*e] - I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*S
in[e - f*x] - (2*I)*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[e
 + f*x] + I*Sqrt[a]*Sqrt[a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[3*e + f*x])
/(a*Cos[e] + 3*b*Cos[e] + a*Cos[3*e] + b*Cos[3*e] + a*Cos[e + 2*f*x] + a*C
os[3*e + 2*f*x] - (3*I)*a*Sin[e] - I*b*Sin[e] - I*a*Sin[3*e] - I*b*Sin[3*e
] - I*a*Sin[e + 2*f*x] + I*a*Sin[3*e + 2*f*x])]*Cos[e])/(a^(11/2)*Sqrt[a +
 b]*f*Sqrt[Cos[2*e] - I*Sin[2*e]]) + (ArcTan[((-I)*a*Cos[e] - I*b*Cos[e] +
 I*a*Cos[3*e] + I*b*Cos[3*e] + a*Sin[e] + b*Sin[e] - Sqrt[a]*Sqrt[a + b]*C
os[e - f*x]*Sqrt[Cos[2*e] - I*Sin[2*e]] + Sqrt[a]*Sqrt[a + b]*Cos[3*e + f*
x]*Sqrt[Cos[2*e] - I*Sin[2*e]] + a*Sin[3*e] + b*Sin[3*e] - I*Sqrt[a]*Sqrt[
a + b]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[e - f*x] - (2*I)*Sqrt[a]*Sqrt[a + b
]*Sqrt[Cos[2*e] - I*Sin[2*e]]*Sin[e + f*x] + I*Sqrt[a]*Sqrt[a + b]*Sqrt[Co
s[2*e] - I*Sin[2*e]]*Sin[3*e + f*x])/(a*Cos[e] + 3*b*Cos[e] + a*Cos[3*e] +
 b*Cos[3*e] + a*Cos[e + 2*f*x] + a*Cos[3*e + 2*f*x] - (3*I)*a*Sin[e] - ...
```

3.211.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4635, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sec(e+fx)^5\,(a+b\sec(e+fx)^2)^3}\,dx$$

$$\downarrow 4635$$

$$\frac{\int \frac{(1-\sin^2(e+fx))^5}{(-a\sin^2(e+fx)+a+b)^3}\,d\sin(e+fx)}{f}$$

$$\downarrow 300$$

$$\frac{\int \left(\frac{\sin^4(e+fx)}{a^3}-\frac{(2a-3b)\sin^2(e+fx)}{a^4}+\frac{a^2-3ba+6b^2}{a^5}-\frac{10a^2b^3\sin^4(e+fx)-5ab^3(4a+3b)\sin^2(e+fx)+b^3(10a^2+15ba+6b^2)}{a^5(-a\sin^2(e+fx)+a+b)^3}\right)d\sin(e+}{f}$$

$$\downarrow 2009$$

$$\frac{-\frac{b^5\sin(e+fx)}{4a^5(a+b)(-a\sin^2(e+fx)+a+b)^2}+\frac{b^4(20a+17b)\sin(e+fx)}{8a^5(a+b)^2(-a\sin^2(e+fx)+a+b)}-\frac{(2a-3b)\sin^3(e+fx)}{3a^4}+\frac{\sin^5(e+fx)}{5a^3}-\frac{b^3(80a^2+140ab+63b^2)\text{arc}}{8a^{11/2}(a+}}{f}$$

input
```
Int[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
(-1/8*(b^3*(80*a^2 + 140*a*b + 63*b^2)*ArcTanh[(Sqrt[a]*Sin[e + f*x])/Sqrt
[a + b]])/(a^(11/2)*(a + b)^(5/2)) + ((a^2 - 3*a*b + 6*b^2)*Sin[e + f*x])/
a^5 - ((2*a - 3*b)*Sin[e + f*x]^3)/(3*a^4) + Sin[e + f*x]^5/(5*a^3) - (b^5
*Sin[e + f*x])/(4*a^5*(a + b)*(a + b - a*Sin[e + f*x]^2)^2) + (b^4*(20*a +
 17*b)*Sin[e + f*x])/(8*a^5*(a + b)^2*(a + b - a*Sin[e + f*x]^2)))/f
```

3.211.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4635
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[ExpandToSum[b + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m
+ n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && In
tegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.211.4 Maple [A] (verified)

Time = 10.27 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{\frac{a^2\sin(fx+e)^5}{5}-\frac{2a^2\sin(fx+e)^3}{3}+a\sin(fx+e)^3b+\sin(fx+e)a^2-3\sin(fx+e)ab+6\sin(fx+e)b^2}{a^5}+\frac{b^3\left(\frac{-\frac{ab(20a+17b)\sin(fx+e)^3}{8\left(a^2+2ab+b^2\right)}+\frac{5(}{\left(a\sin(fx+e)^2-a\right.}\right.}{f}$
default	$\frac{\frac{a^2\sin(fx+e)^5}{5}-\frac{2a^2\sin(fx+e)^3}{3}+a\sin(fx+e)^3b+\sin(fx+e)a^2-3\sin(fx+e)ab+6\sin(fx+e)b^2}{a^5}+\frac{b^3\left(\frac{-\frac{ab(20a+17b)\sin(fx+e)^3}{8\left(a^2+2ab+b^2\right)}+\frac{5(}{\left(a\sin(fx+e)^2-a\right.}\right.}{f}$
risch	$-\frac{ie^{-3i(fx+e)}b}{8a^4f}-\frac{5ie^{3i(fx+e)}}{96a^3f}+\frac{3ie^{-i(fx+e)}b^2}{fa^5}-\frac{ie^{5i(fx+e)}}{160a^3f}-\frac{9ie^{-i(fx+e)}b}{8a^4f}+\frac{ie^{-5i(fx+e)}}{160a^3f}-\frac{ib^4\left(20a^2e^{7i(fx}\right.}{}$

input
```
int(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/a^5*(1/5*a^2*sin(f*x+e)^5-2/3*a^2*sin(f*x+e)^3+a*sin(f*x+e)^3*b+sin
(f*x+e)*a^2-3*sin(f*x+e)*a*b+6*sin(f*x+e)*b^2)+b^3/a^5*((-1/8*a*b*(20*a+17
*b)/(a^2+2*a*b+b^2)*sin(f*x+e)^3+5/8*(4*a+3*b)*b/(a+b)*sin(f*x+e))/(a*sin(
f*x+e)^2-a-b)^2-1/8*(80*a^2+140*a*b+63*b^2)/(a^2+2*a*b+b^2)/(a*(a+b))^(1/2
)*arctanh(a*sin(f*x+e)/(a*(a+b))^(1/2))))
```

3.211.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 485 vs. $2(202) = 404$.

Time = 0.35 (sec) , antiderivative size = 995, normalized size of antiderivative = 4.65

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$= \left[\frac{15\left(80\,a^2b^5+140\,ab^6+63\,b^7+\left(80\,a^4b^3+140\,a^3b^4+63\,a^2b^5\right)\cos\left(fx+e\right)^4+2\left(80\,a^3b^4+140\,a^2b^5+63\right.\right.\right.}{}\right.$$

input
```
integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[1/240*(15*(80*a^2*b^5 + 140*a*b^6 + 63*b^7 + (80*a^4*b^3 + 140*a^3*b^4 +
63*a^2*b^5)*cos(f*x + e)^4 + 2*(80*a^3*b^4 + 140*a^2*b^5 + 63*a*b^6)*cos(f
*x + e)^2)*sqrt(a^2 + a*b)*log(-(a*cos(f*x + e)^2 + 2*sqrt(a^2 + a*b)*sin(
f*x + e) - 2*a - b)/(a*cos(f*x + e)^2 + b)) + 2*(24*(a^8 + 3*a^7*b + 3*a^6
*b^2 + a^5*b^3)*cos(f*x + e)^8 + 64*a^6*b^2 - 48*a^5*b^3 + 192*a^4*b^4 + 1
774*a^3*b^5 + 2415*a^2*b^6 + 945*a*b^7 + 8*(4*a^8 + 3*a^7*b - 15*a^6*b^2 -
 23*a^5*b^3 - 9*a^4*b^4)*cos(f*x + e)^6 + 8*(8*a^8 + 2*a^7*b + 21*a^6*b^2
+ 131*a^5*b^3 + 167*a^4*b^4 + 63*a^3*b^5)*cos(f*x + e)^4 + (128*a^7*b - 64
*a^6*b^2 + 360*a^5*b^3 + 3044*a^4*b^4 + 4067*a^3*b^5 + 1575*a^2*b^6)*cos(f
*x + e)^2)*sin(f*x + e))/((a^11 + 3*a^10*b + 3*a^9*b^2 + a^8*b^3)*f*cos(f*
x + e)^4 + 2*(a^10*b + 3*a^9*b^2 + 3*a^8*b^3 + a^7*b^4)*f*cos(f*x + e)^2 +
 (a^9*b^2 + 3*a^8*b^3 + 3*a^7*b^4 + a^6*b^5)*f), 1/120*(15*(80*a^2*b^5 + 1
40*a*b^6 + 63*b^7 + (80*a^4*b^3 + 140*a^3*b^4 + 63*a^2*b^5)*cos(f*x + e)^4
 + 2*(80*a^3*b^4 + 140*a^2*b^5 + 63*a*b^6)*cos(f*x + e)^2)*sqrt(-a^2 - a*b
)*arctan(sqrt(-a^2 - a*b)*sin(f*x + e)/(a + b)) + (24*(a^8 + 3*a^7*b + 3*a
^6*b^2 + a^5*b^3)*cos(f*x + e)^8 + 64*a^6*b^2 - 48*a^5*b^3 + 192*a^4*b^4 +
 1774*a^3*b^5 + 2415*a^2*b^6 + 945*a*b^7 + 8*(4*a^8 + 3*a^7*b - 15*a^6*b^2
 - 23*a^5*b^3 - 9*a^4*b^4)*cos(f*x + e)^6 + 8*(8*a^8 + 2*a^7*b + 21*a^6*b^
2 + 131*a^5*b^3 + 167*a^4*b^4 + 63*a^3*b^5)*cos(f*x + e)^4 + (128*a^7*b -
64*a^6*b^2 + 360*a^5*b^3 + 3044*a^4*b^4 + 4067*a^3*b^5 + 1575*a^2*b^6)*...
```

3.211.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**5/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Timed out
```

3.211.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.42

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \frac{15\,(80\,a^2b^3+140\,ab^4+63\,b^5)\log\left(\frac{a\sin(fx+e)-\sqrt{(a+b)a}}{a\sin(fx+e)+\sqrt{(a+b)a}}\right)}{(a^7+2\,a^6b+a^5b^2)\sqrt{(a+b)a}} - \frac{30\left((20\,a^2b^4+17\,ab^5)\sin(fx+e)^3-5\,(4\,a^2b^4+7\,ab^5+3\,b^6)\sin(fx+e\right.}{a^9+4\,a^8b+6\,a^7b^2+4\,a^6b^3+a^5b^4+(a^9+2\,a^8b+a^7b^2)\sin(fx+e)^4-2\,(a^9+3\,a^8b+3\,a^7b^2}$$
$$240\,f$$

input
```
integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
1/240*(15*(80*a^2*b^3 + 140*a*b^4 + 63*b^5)*log((a*sin(f*x + e) - sqrt((a
+ b)*a))/(a*sin(f*x + e) + sqrt((a + b)*a)))/((a^7 + 2*a^6*b + a^5*b^2)*sq
rt((a + b)*a)) - 30*((20*a^2*b^4 + 17*a*b^5)*sin(f*x + e)^3 - 5*(4*a^2*b^4
 + 7*a*b^5 + 3*b^6)*sin(f*x + e))/(a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 +
 a^5*b^4 + (a^9 + 2*a^8*b + a^7*b^2)*sin(f*x + e)^4 - 2*(a^9 + 3*a^8*b + 3
*a^7*b^2 + a^6*b^3)*sin(f*x + e)^2) + 16*(3*a^2*sin(f*x + e)^5 - 5*(2*a^2
- 3*a*b)*sin(f*x + e)^3 + 15*(a^2 - 3*a*b + 6*b^2)*sin(f*x + e))/a^5)/f
```

3.211.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.27

$$\int \frac{\cos^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$

$$= \frac{\frac{15\left(80a^2b^3+140ab^4+63b^5\right)\arctan\left(\frac{a\sin(fx+e)}{\sqrt{-a^2-ab}}\right)}{\left(a^7+2a^6b+a^5b^2\right)\sqrt{-a^2-ab}} - \frac{15\left(20a^2b^4\sin(fx+e)^3+17ab^5\sin(fx+e)^3-20a^2b^4\sin(fx+e)-35ab^5\sin(fx+e)-15b^6\sin(fx+e)\right)}{\left(a^7+2a^6b+a^5b^2\right)\left(a\sin(fx+e)^2-a-b\right)^2}}{120f}$$

input
```
integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
1/120*(15*(80*a^2*b^3 + 140*a*b^4 + 63*b^5)*arctan(a*sin(f*x + e)/sqrt(-a^
2 - a*b))/((a^7 + 2*a^6*b + a^5*b^2)*sqrt(-a^2 - a*b)) - 15*(20*a^2*b^4*si
n(f*x + e)^3 + 17*a*b^5*sin(f*x + e)^3 - 20*a^2*b^4*sin(f*x + e) - 35*a*b^
5*sin(f*x + e) - 15*b^6*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*(a*sin(f*
x + e)^2 - a - b)^2) + 8*(3*a^12*sin(f*x + e)^5 - 10*a^12*sin(f*x + e)^3 +
 15*a^11*b*sin(f*x + e)^3 + 15*a^12*sin(f*x + e) - 45*a^11*b*sin(f*x + e)
+ 90*a^10*b^2*sin(f*x + e))/a^15)/f
```

3.211.9 Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.20

$$\int \frac{\cos^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$

$$= \frac{\frac{5\sin(e+fx)\left(3b^5+4ab^4\right)}{8(a+b)} - \frac{\sin(e+fx)^3\left(20a^2b^4+17ab^5\right)}{8(a+b)^2}}{f\left(2a^6b-\sin(e+fx)^2\left(2a^7+2ba^6\right)+a^7+a^5b^2+a^7\sin(e+fx)^4\right)} + \frac{\sin(e+fx)^5}{5a^3f}$$
$$+ \frac{\sin(e+fx)^3\left(\frac{a+b}{a^4}-\frac{5}{3a^3}\right)}{f} + \frac{\sin(e+fx)\left(\frac{10}{a^3}-\frac{3(a+b)^2}{a^5}+\frac{3(a+b)\left(\frac{3(a+b)}{a^4}-\frac{5}{a^3}\right)}{a}\right)}{f}$$
$$+ \frac{b^3\operatorname{atan}\left(\frac{\sqrt{a}\sin(e+fx)\,1i}{\sqrt{a+b}}\right)\left(80a^2+140ab+63b^2\right)1i}{8a^{11/2}f(a+b)^{5/2}}$$

input
```
int(cos(e + f*x)^5/(a + b/cos(e + f*x)^2)^3,x)
```

output

```
((5*sin(e + f*x)*(4*a*b^4 + 3*b^5))/(8*(a + b)) - (sin(e + f*x)^3*(17*a*b^
5 + 20*a^2*b^4))/(8*(a + b)^2))/(f*(2*a^6*b - sin(e + f*x)^2*(2*a^6*b + 2*
a^7) + a^7 + a^5*b^2 + a^7*sin(e + f*x)^4)) + sin(e + f*x)^5/(5*a^3*f) + (
sin(e + f*x)^3*((a + b)/a^4 - 5/(3*a^3)))/f + (sin(e + f*x)*(10/a^3 - (3*(
a + b)^2)/a^5 + (3*(a + b)*((3*(a + b))/a^4 - 5/a^3))/a))/f + (b^3*atan((a
^(1/2)*sin(e + f*x)*1i)/(a + b)^(1/2))*(140*a*b + 80*a^2 + 63*b^2)*1i)/(8*
a^(11/2)*f*(a + b)^(5/2))
```

3.212 $\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

3.212.1 Optimal result 1579
3.212.2 Mathematica [A] (verified) 1579
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3.212.8 Giac [A] (verification not implemented) 1584
3.212.9 Mupad [B] (verification not implemented) 1585

3.212.1 Optimal result

Integrand size = 23, antiderivative size = 142

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{(3a^2+8ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8b^{5/2}(a+b)^{5/2}f} - \frac{a\sec^2(e+fx)\tan(e+fx)}{4b(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{3a(a+2b)\tan(e+fx)}{8b^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}$$

output
```
1/8*(3*a^2+8*a*b+8*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/b^(5/2)/(a+
b)^(5/2)/f-1/4*a*sec(f*x+e)^2*tan(f*x+e)/b/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2-
3/8*a*(a+2b)*tan(f*x+e)/b^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)
```

3.212.2 Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.88

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{(3a^2+8ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} - \frac{a\sqrt{b}(3a^2+16ab+16b^2+3a(a+2b)\cos(2(e+fx)))\sin(2(e+fx))}{(a+b)^2(a+2b+a\cos(2(e+fx)))^2}}{8b^{5/2}f}$$

input
```
Integrate[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
(((3*a^2 + 8*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a +
 b)^(5/2) - (a*Sqrt[b]*(3*a^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cos[2*(e +
 f*x)])*Sin[2*(e + f*x)])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2))/(8
*b^(5/2)*f)
```

3.212.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4634, 315, 298, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sec(e+fx)^6}{(a+b\sec(e+fx)^2)^3}\,dx$$

$$\downarrow 4634$$

$$\frac{\int \frac{\left(\tan^2(e+fx)+1\right)^2}{(b\tan^2(e+fx)+a+b)^3}\,d\tan(e+fx)}{f}$$

$$\downarrow 315$$

$$\frac{\int \frac{(3a+4b)\tan^2(e+fx)+a+4b}{\left(b\tan^2(e+fx)+a+b\right)^2}\,d\tan(e+fx)}{4b(a+b)} - \frac{a\tan(e+fx)\left(\tan^2(e+fx)+1\right)}{4b(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\downarrow 298$$

$$\frac{\frac{\left(3a^2+8ab+8b^2\right)\int \frac{1}{b\tan^2(e+fx)+a+b}\,d\tan(e+fx)}{2b(a+b)} - \frac{3a(a+2b)\tan(e+fx)}{2b(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{4b(a+b)} - \frac{a\tan(e+fx)\left(\tan^2(e+fx)+1\right)}{4b(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\downarrow 218$$

$$\frac{\frac{\left(3a^2+8ab+8b^2\right)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2b^{3/2}(a+b)^{3/2}}-\frac{3a(a+2b)\tan(e+fx)}{2b(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{4b(a+b)}-\frac{a\tan(e+fx)\left(\tan^2(e+fx)+1\right)}{4b(a+b)(a+b\tan^2(e+fx)+b)^2}$$
$$f$$

input
```
Int[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
(-1/4*(a*Tan[e + f*x]*(1 + Tan[e + f*x]^2))/(b*(a + b)*(a + b + b*Tan[e +
f*x]^2)^2) + (((3*a^2 + 8*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[
a + b]])/(2*b^(3/2)*(a + b)^(3/2)) - (3*a*(a + 2*b)*Tan[e + f*x])/(2*b*(a
+ b)*(a + b + b*Tan[e + f*x]^2)))/(4*b*(a + b)))/f
```

3.212.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.212.4 Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{-\frac{a(5a+8b)\tan(fx+e)^3}{8b\left(a^2+2ab+b^2\right)}-\frac{(3a+8b)a\tan(fx+e)}{8b^2(a+b)}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(3a^2+8ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\left(a^2+2ab+b^2\right)b^2\sqrt{(a+b)b}}}{f}$
default	$\frac{-\frac{a(5a+8b)\tan(fx+e)^3}{8b\left(a^2+2ab+b^2\right)}-\frac{(3a+8b)a\tan(fx+e)}{8b^2(a+b)}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(3a^2+8ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\left(a^2+2ab+b^2\right)b^2\sqrt{(a+b)b}}}{f}$
risch	$-\frac{i\left(3a^3e^{6i(fx+e)}+8a^2be^{6i(fx+e)}+8ab^2e^{6i(fx+e)}+9a^3e^{4i(fx+e)}+42a^2be^{4i(fx+e)}+72ab^2e^{4i(fx+e)}+48b^3e^{4i(fx+e)}+9a\right.}{4(a+b)^2b^2f\left(ae^{4i(fx+e)}+2ae^{2i(fx+e)}+4be^{2i(fx+e)}+a\right)^2}$

input
```
int(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*((-1/8*a*(5*a+8*b)/b/(a^2+2*a*b+b^2)*tan(f*x+e)^3-1/8*(3*a+8*b)*a/b^2/
(a+b)*tan(f*x+e))/(a+b+b*tan(f*x+e)^2)^2+1/8*(3*a^2+8*a*b+8*b^2)/(a^2+2*a*
b+b^2)/b^2/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))
```

3.212.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(128) = 256$.

Time = 0.32 (sec) , antiderivative size = 722, normalized size of antiderivative = 5.08

$$\int \frac{\sec^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$

$$= \left[-\frac{\left((3\,a^4+8\,a^3b+8\,a^2b^2)\cos(fx+e)^4+3\,a^2b^2+8\,ab^3+8\,b^4+2\,(3\,a^3b+8\,a^2b^2+8\,ab^3)\cos(fx+e)^2\right)}{32\left((a^5b^3+3\,a\right.} \right.$$

$$\left. -\frac{\left((3\,a^4+8\,a^3b+8\,a^2b^2)\cos(fx+e)^4+3\,a^2b^2+8\,ab^3+8\,b^4+2\,(3\,a^3b+8\,a^2b^2+8\,ab^3)\cos(fx+e)^2\right)}{16\left((a^5b^3+3\,a^4b^4+3\,a^3b^5+a^2b^6)f\cos(fx+e\right.} \right.$$

input
```
integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[-1/32*(((3*a^4 + 8*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 3*a^2*b^2 + 8*a*b^
3 + 8*b^4 + 2*(3*a^3*b + 8*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*sqrt(-a*b -
b^2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x
 + e)^2 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b - b^2)*s
in(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) + 4*
(3*(a^4*b + 3*a^3*b^2 + 2*a^2*b^3)*cos(f*x + e)^3 + (5*a^3*b^2 + 13*a^2*b^
3 + 8*a*b^4)*cos(f*x + e))*sin(f*x + e))/((a^5*b^3 + 3*a^4*b^4 + 3*a^3*b^5
 + a^2*b^6)*f*cos(f*x + e)^4 + 2*(a^4*b^4 + 3*a^3*b^5 + 3*a^2*b^6 + a*b^7)
*f*cos(f*x + e)^2 + (a^3*b^5 + 3*a^2*b^6 + 3*a*b^7 + b^8)*f), -1/16*(((3*a
^4 + 8*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 3*a^2*b^2 + 8*a*b^3 + 8*b^4 + 2
*(3*a^3*b + 8*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*sqrt(a*b + b^2)*arctan(1/
2*((a + 2*b)*cos(f*x + e)^2 - b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e
))) + 2*(3*(a^4*b + 3*a^3*b^2 + 2*a^2*b^3)*cos(f*x + e)^3 + (5*a^3*b^2 + 1
3*a^2*b^3 + 8*a*b^4)*cos(f*x + e))*sin(f*x + e))/((a^5*b^3 + 3*a^4*b^4 + 3
*a^3*b^5 + a^2*b^6)*f*cos(f*x + e)^4 + 2*(a^4*b^4 + 3*a^3*b^5 + 3*a^2*b^6
+ a*b^7)*f*cos(f*x + e)^2 + (a^3*b^5 + 3*a^2*b^6 + 3*a*b^7 + b^8)*f)]
```

3.212.6 Sympy [F]

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

input
```
integrate(sec(f*x+e)**6/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Integral(sec(e + f*x)**6/(a + b*sec(e + f*x)**2)**3, x)
```

3.212.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.48

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \frac{\frac{(3a^2+8ab+8b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^2b^2+2ab^3+b^4)\sqrt{(a+b)b}} - \frac{(5a^2b+8ab^2)\tan(fx+e)^3+(3a^3+11a^2b+8ab^2)\tan(fx+e)}{a^4b^2+4a^3b^3+6a^2b^4+4ab^5+b^6+(a^2b^4+2ab^5+b^6)\tan(fx+e)^4+2(a^3b^3+3a^2b^4+3ab^5+b^6)\tan(fx+}}{8f}$$

input
```
integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
1/8*((3*a^2 + 8*a*b + 8*b^2)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^2*
b^2 + 2*a*b^3 + b^4)*sqrt((a + b)*b)) - ((5*a^2*b + 8*a*b^2)*tan(f*x + e)^
3 + (3*a^3 + 11*a^2*b + 8*a*b^2)*tan(f*x + e))/(a^4*b^2 + 4*a^3*b^3 + 6*a^
2*b^4 + 4*a*b^5 + b^6 + (a^2*b^4 + 2*a*b^5 + b^6)*tan(f*x + e)^4 + 2*(a^3*
b^3 + 3*a^2*b^4 + 3*a*b^5 + b^6)*tan(f*x + e)^2))/f
```

3.212.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.30

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(3a^2+8ab+8b^2)}{(a^2b^2+2ab^3+b^4)\sqrt{ab+b^2}} - \frac{5a^2b\tan(fx+e)^3+8ab^2\tan(fx+e)^3+3a^3\tan(fx+e)+11a^2b\tan(fx+e)+8}{(a^2b^2+2ab^3+b^4)\left(b\tan(fx+e)^2+a+b\right)^2}}{8f}$$

input
```
integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
1/8*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
 + b^2)))*(3*a^2 + 8*a*b + 8*b^2)/((a^2*b^2 + 2*a*b^3 + b^4)*sqrt(a*b + b^
2)) - (5*a^2*b*tan(f*x + e)^3 + 8*a*b^2*tan(f*x + e)^3 + 3*a^3*tan(f*x + e
) + 11*a^2*b*tan(f*x + e) + 8*a*b^2*tan(f*x + e))/((a^2*b^2 + 2*a*b^3 + b^
4)*(b*tan(f*x + e)^2 + a + b)^2))/f
```

3.212.9 Mupad [B] (verification not implemented)

Time = 20.04 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.05

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \frac{\operatorname{atan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)\,(3a^2+8ab+8b^2)}{8b^{5/2}f\,(a+b)^{5/2}}$$
$$- \frac{\frac{\tan(e+fx)^3\,(5a^2+8ba)}{8b\,(a+b)^2}+\frac{\tan(e+fx)\,(3a^2+8ba)}{8b^2\,(a+b)}}{f\,\left(2ab+a^2+b^2+\tan(e+fx)^2\,(2b^2+2ab)+b^2\tan(e+fx)^4\right)}$$

input
```
int(1/(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^3),x)
```

output
```
(atan((b^(1/2)*tan(e + f*x))/(a + b)^(1/2))*(8*a*b + 3*a^2 + 8*b^2))/(8*b^
(5/2)*f*(a + b)^(5/2)) - ((tan(e + f*x)^3*(8*a*b + 5*a^2))/(8*b*(a + b)^2)
 + (tan(e + f*x)*(8*a*b + 3*a^2))/(8*b^2*(a + b)))/(f*(2*a*b + a^2 + b^2 +
 tan(e + f*x)^2*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^4))
```

3.213 $\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

3.213.1 Optimal result 1586
3.213.2 Mathematica [C] (warning: unable to verify) 1586
3.213.3 Rubi [A] (verified) 1587
3.213.4 Maple [A] (verified) 1589
3.213.5 Fricas [B] (verification not implemented) 1589
3.213.6 Sympy [F] 1590
3.213.7 Maxima [A] (verification not implemented) 1590
3.213.8 Giac [A] (verification not implemented) 1591
3.213.9 Mupad [B] (verification not implemented) 1591

3.213.1 Optimal result

Integrand size = 23, antiderivative size = 123

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{(a+4b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8b^{3/2}(a+b)^{5/2}f} - \frac{a\tan(e+fx)}{4b(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{(a+4b)\tan(e+fx)}{8b(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}$$

output

```
1/8*(a+4*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/b^(3/2)/(a+b)^(5/2)/f-1
/4*a*tan(f*x+e)/b/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2+1/8*(a+4*b)*tan(f*x+e)/b/
(a+b)^2/f/(a+b+b*tan(f*x+e)^2)
```

3.213.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.13 (sec) , antiderivative size = 283, normalized size of antiderivative = 2.30

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^6(e+fx)\left(-\frac{(a+4b)\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(a+2b+a\cdots}{b\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right.}{64(a+b)^2f\,(a+b}$$

input
```
Integrate[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^6*(-(((a + 4*b)*ArcTan[(Sec[f
*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2
*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)]
)^2*(Cos[2*e] - I*Sin[2*e]))/(b*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])
) - (4*(a + b)*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/(a*(Cos[e] - Sin[e])*(
Cos[e] + Sin[e])) + ((a + 2*b + a*Cos[2*(e + f*x)])*((a + 4*b)*Sin[2*e] -
(a - 2*b)*Sin[2*f*x]))/(b*(Cos[e] - Sin[e])*(Cos[e] + Sin[e])))))/(64*(a +
b)^2*f*(a + b*Sec[e + f*x]^2)^3)
```

3.213.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4634, 298, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sec(e+fx)^4}{(a+b\sec(e+fx)^2)^3}dx$$

$$\downarrow 4634$$

$$\frac{\int \frac{\tan^2(e+fx)+1}{(b\tan^2(e+fx)+a+b)^3}d\tan(e+fx)}{f}$$

$$\downarrow 298$$

$$\frac{\frac{(a+4b)\int \frac{1}{\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{4b(a+b)} - \frac{a\tan(e+fx)}{4b(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\downarrow 215$$

$$\frac{\frac{(a+4b)\left(\frac{\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{2(a+b)} + \frac{\tan(e+fx)}{2(a+b)\left(a+b\tan^2(e+fx)+b\right)}\right)}{4b(a+b)} - \frac{a\tan(e+fx)}{4b(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\downarrow 218$$

$$\frac{(a+4b)\left(\frac{\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2\sqrt{b}(a+b)^{3/2}}+\frac{\tan(e+fx)}{2(a+b)\left(a+b\tan^2(e+fx)+b\right)}\right)}{4b(a+b)}-\frac{a\tan(e+fx)}{4b(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

input
```
Int[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
(-1/4*(a*Tan[e + f*x])/(b*(a + b)*(a + b + b*Tan[e + f*x]^2)^2) + ((a + 4*
b)*(ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]]/(2*Sqrt[b]*(a + b)^(3/2)) +
 Tan[e + f*x]/(2*(a + b)*(a + b + b*Tan[e + f*x]^2))))/(4*b*(a + b)))/f
```

3.213.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.213.4 Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{\frac{(a+4b)\tan(fx+e)^3}{8a^2+16ab+8b^2}-\frac{(a-4b)\tan(fx+e)}{8(a+b)b}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{(a+4b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\left(a^2+2ab+b^2\right)b\sqrt{(a+b)b}}}{f}$
default	$\frac{\frac{(a+4b)\tan(fx+e)^3}{8a^2+16ab+8b^2}-\frac{(a-4b)\tan(fx+e)}{8(a+b)b}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{(a+4b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\left(a^2+2ab+b^2\right)b\sqrt{(a+b)b}}}{f}$
risch	$\frac{i\left(-a^3e^{6i(fx+e)}-4a^2be^{6i(fx+e)}-3a^3e^{4i(fx+e)}-2a^2be^{4i(fx+e)}+8ab^2e^{4i(fx+e)}+16b^3e^{4i(fx+e)}-3a^3e^{2i(fx+e)}+4a^2be^{2i}\right.}{4a(a+b)^2f\left(ae^{4i(fx+e)}+2ae^{2i(fx+e)}+4be^{2i(fx+e)}+a\right)^2b}$

input
```
int(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*((1/8*(a+4*b)/(a^2+2*a*b+b^2)*tan(f*x+e)^3-1/8*(a-4*b)/(a+b)/b*tan(f*x
+e))/(a+b+b*tan(f*x+e)^2)^2+1/8*(a+4*b)/(a^2+2*a*b+b^2)/b/((a+b)*b)^(1/2)*
arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))
```

3.213.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. 2(109) = 218.

Time = 0.32 (sec) , antiderivative size = 654, normalized size of antiderivative = 5.32

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$= \left[-\frac{\left((a^3+4a^2b)\cos(fx+e)^4+ab^2+4b^3+2\left(a^2b+4ab^2\right)\cos(fx+e)^2\right)\sqrt{-ab-b^2}\log\left(\frac{(a^2+8ab+8b^2)\cos}{32\left((a^5b^2+3a^4b^3+3a^3b^4+a^2b^5)\right.\right.}\right.$$

$$\left. -\frac{\left((a^3+4a^2b)\cos(fx+e)^4+ab^2+4b^3+2\left(a^2b+4ab^2\right)\cos(fx+e)^2\right)\sqrt{ab+b^2}\arctan\left(\frac{(a+2b)\cos(fx}{2\sqrt{ab+b^2}\cos(fx+}\right.\right.}{16\left((a^5b^2+3a^4b^3+3a^3b^4+a^2b^5)f\cos(fx+e)^4+2\left(a^4b^3+3a^3b^4\right.\right.}$$

input
```
integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[-1/32*(((a^3 + 4*a^2*b)*cos(f*x + e)^4 + a*b^2 + 4*b^3 + 2*(a^2*b + 4*a*b
^2)*cos(f*x + e)^2)*sqrt(-a*b - b^2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x +
e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a + 2*b)*cos(f*x + e)^3 - b*
cos(f*x + e))*sqrt(-a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2
*a*b*cos(f*x + e)^2 + b^2)) + 4*((a^3*b - a^2*b^2 - 2*a*b^3)*cos(f*x + e)^
3 - (a^2*b^2 + 5*a*b^3 + 4*b^4)*cos(f*x + e))*sin(f*x + e))/((a^5*b^2 + 3*
a^4*b^3 + 3*a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^4 + 2*(a^4*b^3 + 3*a^3*b^4 +
 3*a^2*b^5 + a*b^6)*f*cos(f*x + e)^2 + (a^3*b^4 + 3*a^2*b^5 + 3*a*b^6 + b^
7)*f), -1/16*(((a^3 + 4*a^2*b)*cos(f*x + e)^4 + a*b^2 + 4*b^3 + 2*(a^2*b +
 4*a*b^2)*cos(f*x + e)^2)*sqrt(a*b + b^2)*arctan(1/2*((a + 2*b)*cos(f*x +
e)^2 - b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e))) + 2*((a^3*b - a^2*b
^2 - 2*a*b^3)*cos(f*x + e)^3 - (a^2*b^2 + 5*a*b^3 + 4*b^4)*cos(f*x + e))*s
in(f*x + e))/((a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^4
 + 2*(a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 + a*b^6)*f*cos(f*x + e)^2 + (a^3*b^4
 + 3*a^2*b^5 + 3*a*b^6 + b^7)*f)]
```

3.213.6 Sympy [F]

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

input
```
integrate(sec(f*x+e)**4/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Integral(sec(e + f*x)**4/(a + b*sec(e + f*x)**2)**3, x)
```

3.213.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.52

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \frac{(a+4b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^2b+2ab^2+b^3)\sqrt{(a+b)b}} + \frac{(ab+4b^2)\tan(fx+e)^3-(a^2-3ab-4b^2)\tan(fx+e)}{a^4b+4a^3b^2+6a^2b^3+4ab^4+b^5+(a^2b^3+2ab^4+b^5)\tan(fx+e)^4+2(a^3b^2+3a^2b^3+3ab^4+b^5)\tan(fx+e)^2}}{8f}$$

input
```
integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
1/8*((a + 4*b)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^2*b + 2*a*b^2 +
b^3)*sqrt((a + b)*b)) + ((a*b + 4*b^2)*tan(f*x + e)^3 - (a^2 - 3*a*b - 4*b
^2)*tan(f*x + e))/(a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5 + (a^2*b^
3 + 2*a*b^4 + b^5)*tan(f*x + e)^4 + 2*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5
)*tan(f*x + e)^2))/f
```

3.213.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.33

$$\int \frac{\sec^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$
$$= \frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(a+4b)}{(a^2b+2ab^2+b^3)\sqrt{ab+b^2}} + \frac{ab\tan(fx+e)^3+4b^2\tan(fx+e)^3-a^2\tan(fx+e)+3ab\tan(fx+e)+4b^2\tan(fx+e)}{(a^2b+2ab^2+b^3)\left(b\tan(fx+e)^2+a+b\right)^2}}{8f}$$

input
```
integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
1/8*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
 + b^2)))*(a + 4*b)/((a^2*b + 2*a*b^2 + b^3)*sqrt(a*b + b^2)) + (a*b*tan(f
*x + e)^3 + 4*b^2*tan(f*x + e)^3 - a^2*tan(f*x + e) + 3*a*b*tan(f*x + e) +
 4*b^2*tan(f*x + e))/((a^2*b + 2*a*b^2 + b^3)*(b*tan(f*x + e)^2 + a + b)^2
))/f
```

3.213.9 Mupad [B] (verification not implemented)

Time = 19.35 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.02

$$\int \frac{\sec^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$
$$= \frac{\frac{\tan(e+fx)^3\,(a+4b)}{8\,(a+b)^2} - \frac{\tan(e+fx)\,(a-4b)}{8\,b\,(a+b)}}{f\,\left(2ab+a^2+b^2+\tan(e+fx)^2\,(2b^2+2ab)+b^2\tan(e+fx)^4\right)} + \frac{\operatorname{atan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)\,(a+4b)}{8\,b^{3/2}\,f\,(a+b)^{5/2}}$$

input
```
int(1/(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^3),x)
```

output
```
((tan(e + f*x)^3*(a + 4*b))/(8*(a + b)^2) - (tan(e + f*x)*(a - 4*b))/(8*b*
(a + b)))/(f*(2*a*b + a^2 + b^2 + tan(e + f*x)^2*(2*a*b + 2*b^2) + b^2*tan
(e + f*x)^4)) + (atan((b^(1/2)*tan(e + f*x))/(a + b)^(1/2))*(a + 4*b))/(8*
b^(3/2)*f*(a + b)^(5/2))
```

3.214 $\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

3.214.1 Optimal result 1593
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3.214.1 Optimal result

Integrand size = 23, antiderivative size = 106

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{3\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8\sqrt{b}(a+b)^{5/2}f} + \frac{\tan(e+fx)}{4(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{3\tan(e+fx)}{8(a+b)^2 f\left(a+b+b\tan^2(e+fx)\right)}$$

output
```
3/8*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/(a+b)^(5/2)/f/b^(1/2)+1/4*tan(f
*x+e)/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2+3/8*tan(f*x+e)/(a+b)^2/f/(a+b+b*tan(f
*x+e)^2)
```

3.214.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.59 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.50

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^6(e+fx)\left(-\frac{3\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(a+2b+a\cos(2($$

$$64(a+b$$

input
```
Integrate[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^6*((-3*ArcTan[(Sec[f*x]*(Cos[
2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a +
 b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])^2*(Cos[
2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (4*b*(a
+ b)*Sec[2*e]*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/a^2 + ((a + 2*b + a*Cos
[2*(e + f*x)])*Sec[2*e]*(-((5*a^2 + 16*a*b + 8*b^2)*Sin[2*e]) + a*(5*a + 2
*b)*Sin[2*f*x]))/a^2))/(64*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^3)
```

3.214.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4634, 215, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sec(e+fx)^2}{(a+b\sec(e+fx)^2)^3}\,dx$$

$$\downarrow 4634$$

$$\frac{\int \frac{1}{(b\tan^2(e+fx)+a+b)^3}\,d\tan(e+fx)}{f}$$

$$\downarrow 215$$

$$\frac{\frac{3\int \frac{1}{\left(b\tan^2(e+fx)+a+b\right)^2}\,d\tan(e+fx)}{4(a+b)} + \frac{\tan(e+fx)}{4(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\downarrow 215$$

$$\frac{3\left(\frac{\int \frac{1}{b\tan^2(e+fx)+a+b}\,d\tan(e+fx)}{2(a+b)} + \frac{\tan(e+fx)}{2(a+b)\left(a+b\tan^2(e+fx)+b\right)}\right)}{4(a+b)} + \frac{\tan(e+fx)}{4(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\downarrow 218$$

$$\frac{3\left(\frac{\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2\sqrt{b}(a+b)^{3/2}}+\frac{\tan(e+fx)}{2(a+b)\left(a+b\tan^2(e+fx)+b\right)}\right)}{4(a+b)}+\frac{\tan(e+fx)}{4(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

input
```
Int[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
(Tan[e + f*x]/(4*(a + b)*(a + b + b*Tan[e + f*x]^2)^2) + (3*(ArcTan[(Sqrt[
b]*Tan[e + f*x])/Sqrt[a + b]]/(2*Sqrt[b]*(a + b)^(3/2)) + Tan[e + f*x]/(2*
(a + b)*(a + b + b*Tan[e + f*x]^2))))/(4*(a + b)))/f
```

3.214.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.214.4 Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\dfrac{\frac{\tan(fx+e)}{4(a+b)\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\frac{3\tan(fx+e)}{8(a+b)\left(a+b+b\tan(fx+e)^2\right)}+\frac{3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8(a+b)\sqrt{(a+b)b}}}{a+b}}{f}$
default	$\dfrac{\frac{\tan(fx+e)}{4(a+b)\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\frac{3\tan(fx+e)}{8(a+b)\left(a+b+b\tan(fx+e)^2\right)}+\frac{3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8(a+b)\sqrt{(a+b)b}}}{a+b}}{f}$
risch	$\dfrac{i\left(5a^3e^{6i(fx+e)}+16a^2be^{6i(fx+e)}+8ab^2e^{6i(fx+e)}+15a^3e^{4i(fx+e)}+46a^2be^{4i(fx+e)}+56ab^2e^{4i(fx+e)}+16b^3e^{4i(fx+e)}+15\right.}{4a^2(a+b)^2f\left(ae^{4i(fx+e)}+2ae^{2i(fx+e)}+4be^{2i(fx+e)}+a\right)^2}$

input
```
int(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/4*tan(f*x+e)/(a+b)/(a+b+b*tan(f*x+e)^2)^2+3/4/(a+b)*(1/2*tan(f*x+e)
/(a+b)/(a+b+b*tan(f*x+e)^2)+1/2/(a+b)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/
((a+b)*b)^(1/2))))
```

3.214.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. 2(92) = 184.

Time = 0.30 (sec) , antiderivative size = 580, normalized size of antiderivative = 5.47

$$\int \frac{\sec^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$

$$= \left[-\frac{3\left(a^2\cos(fx+e)^4+2ab\cos(fx+e)^2+b^2\right)\sqrt{-ab-b^2}\log\left(\frac{(a^2+8ab+8b^2)\cos(fx+e)^4-2(3ab+4b^2)\cos(fx+e)^2}{a^2\cos(fx+e)}\right.}{32\left((a^5b+3a^4b^2+3a^3b^3+a^2b^4)f\cos(fx+e)^4+\right.}\right.$$

$$\left. -\frac{3\left(a^2\cos(fx+e)^4+2ab\cos(fx+e)^2+b^2\right)\sqrt{ab+b^2}\arctan\left(\frac{(a+2b)\cos(fx+e)^2-b}{2\sqrt{ab+b^2}\cos(fx+e)\sin(fx+e)}\right)-2\left((5a^2b+7\right.}{16\left((a^5b+3a^4b^2+3a^3b^3+a^2b^4)f\cos(fx+e)^4+2(a^4b^2+3a^3b^3+3a^2b^4+ab^5)f\right.}\right.$$

input
```
integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[-1/32*(3*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a*b - b^
2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x +
 e)^2 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b - b^2)*sin
(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) - 4*((
5*a^2*b + 7*a*b^2 + 2*b^3)*cos(f*x + e)^3 + 3*(a*b^2 + b^3)*cos(f*x + e))*
sin(f*x + e))/((a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*f*cos(f*x + e)^4
 + 2*(a^4*b^2 + 3*a^3*b^3 + 3*a^2*b^4 + a*b^5)*f*cos(f*x + e)^2 + (a^3*b^3
 + 3*a^2*b^4 + 3*a*b^5 + b^6)*f), -1/16*(3*(a^2*cos(f*x + e)^4 + 2*a*b*cos(
f*x + e)^2 + b^2)*sqrt(a*b + b^2)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b
)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e))) - 2*((5*a^2*b + 7*a*b^2 + 2
*b^3)*cos(f*x + e)^3 + 3*(a*b^2 + b^3)*cos(f*x + e))*sin(f*x + e))/((a^5*b
 + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*f*cos(f*x + e)^4 + 2*(a^4*b^2 + 3*a^3*
b^3 + 3*a^2*b^4 + a*b^5)*f*cos(f*x + e)^2 + (a^3*b^3 + 3*a^2*b^4 + 3*a*b^5
 + b^6)*f)]
```

3.214.6 Sympy [F]

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

input
```
integrate(sec(f*x+e)**2/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Integral(sec(e + f*x)**2/(a + b*sec(e + f*x)**2)**3, x)
```

3.214.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.47

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \frac{\frac{3\,b\tan(fx+e)^3+5\,(a+b)\tan(fx+e)}{(a^2b^2+2\,ab^3+b^4)\tan(fx+e)^4+a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4+2\,(a^3b+3\,a^2b^2+3\,ab^3+b^4)\tan(fx+e)^2} + \frac{3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^2+2\,ab+b^2)\sqrt{(a+b)b}}}{8\,f}$$

input
```
integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
1/8*((3*b*tan(f*x + e)^3 + 5*(a + b)*tan(f*x + e))/((a^2*b^2 + 2*a*b^3 + b
^4)*tan(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4 + 2*(a^3*b
+ 3*a^2*b^2 + 3*a*b^3 + b^4)*tan(f*x + e)^2) + 3*arctan(b*tan(f*x + e)/sqr
t((a + b)*b))/((a^2 + 2*a*b + b^2)*sqrt((a + b)*b)))/f
```

3.214.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.17

$$\int \frac{\sec^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\frac{3\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^2+2ab+b^2)\sqrt{ab+b^2}}+\frac{3b\tan(fx+e)^3+5a\tan(fx+e)+5b\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)^2(a^2+2ab+b^2)}}{8f}$$

input
```
integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
1/8*(3*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a
*b + b^2)))/((a^2 + 2*a*b + b^2)*sqrt(a*b + b^2)) + (3*b*tan(f*x + e)^3 +
5*a*tan(f*x + e) + 5*b*tan(f*x + e))/((b*tan(f*x + e)^2 + a + b)^2*(a^2 +
2*a*b + b^2)))/f
```

3.214.9 Mupad [B] (verification not implemented)

Time = 19.07 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.06

$$\int \frac{\sec^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\frac{5\tan(e+fx)}{8(a+b)}+\frac{3b\tan(e+fx)^3}{8(a+b)^2}}{f\left(2ab+a^2+b^2+\tan(e+fx)^2\left(2b^2+2ab\right)+b^2\tan(e+fx)^4\right)}+\frac{3\operatorname{atan}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8\sqrt{b}f(a+b)^{5/2}}$$

input
```
int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^3),x)
```

output
```
((5*tan(e + f*x))/(8*(a + b)) + (3*b*tan(e + f*x)^3)/(8*(a + b)^2))/(f*(2*
a*b + a^2 + b^2 + tan(e + f*x)^2*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^4)) +
(3*atan((b^(1/2)*tan(e + f*x))/(a + b)^(1/2)))/(8*b^(1/2)*f*(a + b)^(5/2))
```

3.215 $\int \frac{1}{(a+b\sec^2(e+fx))^3}\,dx$

3.215.1 Optimal result . . . 1600
3.215.2 Mathematica [C] (warning: unable to verify) . . . 1600
3.215.3 Rubi [A] (verified) . . . 1601
3.215.4 Maple [A] (verified) . . . 1604
3.215.5 Fricas [B] (verification not implemented) . . . 1604
3.215.6 Sympy [F] . . . 1605
3.215.7 Maxima [A] (verification not implemented) . . . 1605
3.215.8 Giac [A] (verification not implemented) . . . 1606
3.215.9 Mupad [B] (verification not implemented) . . . 1606

3.215.1 Optimal result

Integrand size = 14, antiderivative size = 144

$$\int \frac{1}{(a+b\sec^2(e+fx))^3}\,dx = \frac{x}{a^3} - \frac{\sqrt{b}(15a^2+20ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{5/2}f} - \frac{b\tan(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(7a+4b)\tan(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}$$

output
```
x/a^3-1/8*(15*a^2+20*a*b+8*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(
1/2)/a^3/(a+b)^(5/2)/f-1/4*b*tan(f*x+e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2-1
/8*b*(7*a+4*b)*tan(f*x+e)/a^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)
```

3.215.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.65 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.31

$$\int \frac{1}{(a+b\sec^2(e+fx))^3}\,dx = \frac{(a+2b+a\cos(2(e+fx)))\sec^6(e+fx)\left(8x(a+2b+a\cos(2(e+fx)))^2 + \frac{b(15a^2+20ab+8b^2)\arctan\left(\frac{\sec(fx)(\cos}{}\right.}{}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(-3),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^6*(8*x*(a + 2*b + a*Cos[2*(e
+ f*x)])^2 + (b*(15*a^2 + 20*a*b + 8*b^2)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*S
in[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b
*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])^2*(Cos[2*e] - I*S
in[2*e]))/((a + b)^(5/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) - (4*b^2*((a + 2
*b)*Sin[2*e] - a*Sin[2*f*x]))/((a + b)*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e
])) + (b*(a + 2*b + a*Cos[2*(e + f*x)])*((9*a^2 + 28*a*b + 16*b^2)*Sin[2*e
] - 3*a*(3*a + 2*b)*Sin[2*f*x]))/((a + b)^2*f*(Cos[e] - Sin[e])*(Cos[e] +
Sin[e])))))/(64*a^3*(a + b*Sec[e + f*x]^2)^3)
```

3.215.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.22, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4616, 316, 402, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sec^2(e+fx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{(a+b\sec(e+fx)^2)^3}\,dx$$

$$\downarrow 4616$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^3}\,d\tan(e+fx)}{f}$$

$$\downarrow 316$$

$$\frac{\int \frac{-3b\tan^2(e+fx)+4a+b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}\,d\tan(e+fx)}{4a(a+b)} - \frac{b\tan(e+fx)}{4a(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\downarrow 402$$

$$\frac{\frac{\int \frac{8a^2+9ba+4b^2-b(7a+4b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2a(a+b)}-\frac{b(7a+4b)\tan(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{4a(a+b)}-\frac{b\tan(e+fx)}{4a(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\downarrow 397$$

$$\frac{\frac{8(a+b)^2\int\frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}-\frac{b\left(15a^2+20ab+8b^2\right)\int\frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{2a(a+b)}-\frac{b(7a+4b)\tan(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{4a(a+b)}-\frac{b\tan(e+fx)}{4a(a+b)(a+b\tan^2(e+fx)-}}{f}$$

$$\downarrow 216$$

$$\frac{\frac{8(a+b)^2\arctan(\tan(e+fx))}{a}-\frac{b\left(15a^2+20ab+8b^2\right)\int\frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{2a(a+b)}-\frac{b(7a+4b)\tan(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{4a(a+b)}-\frac{b\tan(e+fx)}{4a(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\downarrow 218$$

$$\frac{\frac{8(a+b)^2\arctan(\tan(e+fx))}{a}-\frac{\sqrt{b}\left(15a^2+20ab+8b^2\right)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{2a(a+b)}-\frac{b(7a+4b)\tan(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{4a(a+b)}-\frac{b\tan(e+fx)}{4a(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^(-3),x]
```

output
```
(-1/4*(b*Tan[e + f*x])/(a*(a + b)*(a + b + b*Tan[e + f*x]^2)^2) + (((8*(a
+ b)^2*ArcTan[Tan[e + f*x]])/a - (Sqrt[b]*(15*a^2 + 20*a*b + 8*b^2)*ArcTan
[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(2*a*(a + b)) - (b*
(7*a + 4*b)*Tan[e + f*x])/(2*a*(a + b)*(a + b + b*Tan[e + f*x]^2)))/(4*a*(
a + b)))/f
```

3.215.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

3.215.4 Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{ab(7a+4b)\tan(fx+e)^3}{8a^2+16ab+8b^2}+\frac{(9a+4b)a\tan(fx+e)}{8a+8b}\right)}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(15a^2+20ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\left(a^2+2ab+b^2\right)\sqrt{(a+b)b}}}{a^3}+\frac{\arctan(\tan(fx+e))}{a^3}}{f}$
default	$\frac{-\frac{b\left(\frac{ab(7a+4b)\tan(fx+e)^3}{8a^2+16ab+8b^2}+\frac{(9a+4b)a\tan(fx+e)}{8a+8b}\right)}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(15a^2+20ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\left(a^2+2ab+b^2\right)\sqrt{(a+b)b}}}{a^3}+\frac{\arctan(\tan(fx+e))}{a^3}}{f}$
risch	$\frac{x}{a^3}-\frac{ib\left(9a^3e^{6i(fx+e)}+28a^2be^{6i(fx+e)}+16ab^2e^{6i(fx+e)}+27a^3e^{4i(fx+e)}+90a^2be^{4i(fx+e)}+120ab^2e^{4i(fx+e)}+48b^3e^{4i}\right.}{4a^3(a+b)^2f\left(ae^{4i(fx+e)}+2ae^{2i(fx+e)}+4be^{2i(fx+}\right.$

input
```
int(1/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-b/a^3*((1/8*a*b*(7*a+4*b)/(a^2+2*a*b+b^2)*tan(f*x+e)^3+1/8*(9*a+4*b)
*a/(a+b)*tan(f*x+e))/(a+b+b*tan(f*x+e)^2)^2+1/8*(15*a^2+20*a*b+8*b^2)/(a^2
+2*a*b+b^2)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))+1/a^3*ar
ctan(tan(f*x+e)))
```

3.215.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. 2(130) = 260.

Time = 0.32 (sec) , antiderivative size = 819, normalized size of antiderivative = 5.69

$$\int \frac{1}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \left[\frac{32\left(a^4+2a^3b+a^2b^2\right)fx\cos(fx+e)^4+64\left(a^3b+2a^2b^2+ab^3\right)fx\cos(fx+e)^2+32\left(a^2b^2+2ab^3+b^4\right)\right.}{}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output

```
[1/32*(32*(a^4 + 2*a^3*b + a^2*b^2)*f*x*cos(f*x + e)^4 + 64*(a^3*b + 2*a^2
*b^2 + a*b^3)*f*x*cos(f*x + e)^2 + 32*(a^2*b^2 + 2*a*b^3 + b^4)*f*x + ((15
*a^4 + 20*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2 + 20*a*b^3 + 8*b^
4 + 2*(15*a^3*b + 20*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*sqrt(-b/(a + b))*l
og(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^
2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sq
rt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e
)^2 + b^2)) - 4*(3*(3*a^3*b + 2*a^2*b^2)*cos(f*x + e)^3 + (7*a^2*b^2 + 4*a
*b^3)*cos(f*x + e))*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*x + e
)^4 + 2*(a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*a^4*
b^3 + a^3*b^4)*f), 1/16*(16*(a^4 + 2*a^3*b + a^2*b^2)*f*x*cos(f*x + e)^4 +
 32*(a^3*b + 2*a^2*b^2 + a*b^3)*f*x*cos(f*x + e)^2 + 16*(a^2*b^2 + 2*a*b^3
 + b^4)*f*x + ((15*a^4 + 20*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2
 + 20*a*b^3 + 8*b^4 + 2*(15*a^3*b + 20*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*
sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/
(b*cos(f*x + e)*sin(f*x + e))) - 2*(3*(3*a^3*b + 2*a^2*b^2)*cos(f*x + e)^3
 + (7*a^2*b^2 + 4*a*b^3)*cos(f*x + e))*sin(f*x + e))/((a^7 + 2*a^6*b + a^5
*b^2)*f*cos(f*x + e)^4 + 2*(a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2
+ (a^5*b^2 + 2*a^4*b^3 + a^3*b^4)*f)]
```

3.215.6 Sympy [F]

$$\int \frac{1}{(a+b\sec^2(e+fx))^3}\,dx = \int \frac{1}{(a+b\sec^2(e+fx))^3}\,dx$$

input

```
integrate(1/(a+b*sec(f*x+e)**2)**3,x)
```

output

```
Integral((a + b*sec(e + f*x)**2)**(-3), x)
```

3.215.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.60

$$\int \frac{1}{(a+b\sec^2(e+fx))^3}\,dx =$$

$$-\frac{\frac{(15\,a^2b+20\,ab^2+8\,b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^5+2\,a^4b+a^3b^2)\sqrt{(a+b)b}} + \frac{(7\,ab^2+4\,b^3)\tan(fx+e)^3+(9\,a^2b+13\,ab^2+4\,b^3)\tan(fx+e)}{a^6+4\,a^5b+6\,a^4b^2+4\,a^3b^3+a^2b^4+(a^4b^2+2\,a^3b^3+a^2b^4)\tan(fx+e)^4+2\,(a^5b+3\,a^4b^2+3\,a^3b^3+a}}{8\,f}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
-1/8*((15*a^2*b + 20*a*b^2 + 8*b^3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))
/((a^5 + 2*a^4*b + a^3*b^2)*sqrt((a + b)*b)) + ((7*a*b^2 + 4*b^3)*tan(f*x
+ e)^3 + (9*a^2*b + 13*a*b^2 + 4*b^3)*tan(f*x + e))/(a^6 + 4*a^5*b + 6*a^4
*b^2 + 4*a^3*b^3 + a^2*b^4 + (a^4*b^2 + 2*a^3*b^3 + a^2*b^4)*tan(f*x + e)^
4 + 2*(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*tan(f*x + e)^2) - 8*(f*x +
 e)/a^3)/f
```

3.215.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.36

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{\frac{\left(15a^2b+20ab^2+8b^3\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{\left(a^5+2a^4b+a^3b^2\right)\sqrt{ab+b^2}} + \frac{7ab^2\tan(fx+e)^3+4b^3\tan(fx+e)^3+9a^2b\tan(fx+e)+13ab^2\tan(fx}{\left(a^4+2a^3b+a^2b^2\right)\left(b\tan(fx+e)^2+a+b\right)^2}}{8f}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
-1/8*((15*a^2*b + 20*a*b^2 + 8*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) +
 arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^5 + 2*a^4*b + a^3*b^2)*sqrt(a
*b + b^2)) + (7*a*b^2*tan(f*x + e)^3 + 4*b^3*tan(f*x + e)^3 + 9*a^2*b*tan(
f*x + e) + 13*a*b^2*tan(f*x + e) + 4*b^3*tan(f*x + e))/((a^4 + 2*a^3*b + a
^2*b^2)*(b*tan(f*x + e)^2 + a + b)^2) - 8*(f*x + e)/a^3)/f
```

3.215.9 Mupad [B] (verification not implemented)

Time = 23.59 (sec) , antiderivative size = 3271, normalized size of antiderivative = 22.72

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(a + b/cos(e + f*x)^2)^3,x)
```

output

```
atan(((((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^1
0*b^2)*1i)/(2*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)) - (tan(e
 + f*x)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a
^10*b^3 + 256*a^11*b^2))/(128*a^3*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6
*a^6*b^2)))/(2*a^3) + (tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 +
856*a^3*b^4 + 289*a^4*b^3))/(64*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a
^6*b^2)))/a^3 - ((((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)
/2 + 4*a^10*b^2)*1i)/(2*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)
) + (tan(e + f*x)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^
4 + 1536*a^10*b^3 + 256*a^11*b^2))/(128*a^3*(4*a^7*b + a^8 + a^4*b^4 + 4*a
^5*b^3 + 6*a^6*b^2)))/(2*a^3) - (tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*
a^2*b^5 + 856*a^3*b^4 + 289*a^4*b^3))/(64*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5
*b^3 + 6*a^6*b^2)))/a^3)/(((17*a*b^5)/4 + b^6 + (25*a^2*b^4)/4 + (105*a^3*
b^3)/32)/(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2) + (((((2*a^6*b
^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10*b^2)*1i)/(2*(4*
a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)) - (tan(e + f*x)*(512*a^6*
b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a^10*b^3 + 256*a^1
1*b^2))/(128*a^3*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*1i)/(
2*a^3) + (tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a^3*b^4 +
 289*a^4*b^3)*1i)/(64*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)...
```

3.216 $\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

3.216.1 Optimal result 1608
3.216.2 Mathematica [A] (verified) 1609
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3.216.9 Mupad [B] (verification not implemented) 1616

3.216.1 Optimal result

Integrand size = 23, antiderivative size = 201

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{(a-6b)x}{2a^4} + \frac{b^{3/2}(35a^2+56ab+24b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^4(a+b)^{5/2}f} + \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{b(2a+3b)\tan(e+fx)}{4a^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{b(4a+3b)(a+4b)\tan(e+fx)}{8a^3(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}$$

output

```
1/2*(a-6*b)*x/a^4+1/8*b^(3/2)*(35*a^2+56*a*b+24*b^2)*arctan(b^(1/2)*tan(f*
x+e)/(a+b)^(1/2))/a^4/(a+b)^(5/2)/f+1/2*cos(f*x+e)*sin(f*x+e)/a/f/(a+b+b*t
an(f*x+e)^2)^2+1/4*b*(2*a+3*b)*tan(f*x+e)/a^2/(a+b)/f/(a+b+b*tan(f*x+e)^2)
^2+1/8*b*(4*a+3*b)*(a+4*b)*tan(f*x+e)/a^3/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)
```

3.216.2 Mathematica [A] (verified)

Time = 4.49 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.78

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \frac{4(a-6b)(e+fx) + \frac{b^{3/2}(35a^2+56ab+24b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} + a\left(2 + \frac{13ab^2}{(a+b)^2(a+2b+a\cos(2(e+fx)))} + \frac{2b^3(3a+8b+5a\cos}{(a+b)^2(a+2b+a\cos}\right.}{8a^4 f}$$

input
```
Integrate[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
(4*(a - 6*b)*(e + f*x) + (b^(3/2)*(35*a^2 + 56*a*b + 24*b^2)*ArcTan[(Sqrt[
b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) + a*(2 + (13*a*b^2)/((a + b)^
2*(a + 2*b + a*Cos[2*(e + f*x)])) + (2*b^3*(3*a + 8*b + 5*a*Cos[2*(e + f*x
)]))/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2))*Sin[2*(e + f*x)])/(8*a^
4*f)
```

3.216.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.19, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 4634, 316, 25, 402, 27, 402, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{\sec(e+fx)^2\,(a+b\sec(e+fx)^2)^3}\,dx$$
$$\downarrow 4634$$
$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)^2(b\tan^2(e+fx)+a+b)^3}\,d\tan(e+fx)}{f}$$
$$\downarrow 316$$

$$\frac{\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)^2}-\frac{\int -\frac{5b\tan^2(e+fx)+a-b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^3}d\tan(e+fx)}{2a}}{f}$$

$$\downarrow 25$$

$$\frac{\frac{\int \frac{5b\tan^2(e+fx)+a-b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^3}d\tan(e+fx)}{2a}+\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\downarrow 402$$

$$\frac{\frac{\int \frac{2\left(2a^2-4ba-3b^2+3b(2a+3b)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{4a(a+b)}+\frac{b(2a+3b)\tan(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)^2}}{2a}+\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\downarrow 27$$

$$\frac{\frac{\int \frac{2a^2-4ba-3b^2+3b(2a+3b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{2a(a+b)}+\frac{b(2a+3b)\tan(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)^2}}{2a}+\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\downarrow 402$$

$$\frac{\frac{\frac{\int \frac{4a^3-12ba^2-25b^2a-12b^3+b(4a+3b)(a+4b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2a(a+b)}+\frac{b(4a+3b)(a+4b)\tan(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{2a(a+b)}+\frac{b(2a+3b)\tan(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)^2}}{2a}+\frac{\tan}{2a(\tan^2(e+fx)+1}}{f}$$

$$\downarrow 397$$

$$\frac{\frac{\frac{b^2\left(35a^2+56ab+24b^2\right)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}+\frac{4(a-6b)(a+b)^2\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}}{2a(a+b)}+\frac{b(4a+3b)(a+4b)\tan(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{2a(a+b)}+\frac{b(2a+3b)\tan(e+}{2a(a+b)\left(a+b\tan^2(e+}}{2a}}{f}$$

$$\downarrow 216$$

$$\frac{\frac{b^2\left(35a^2+56ab+24b^2\right)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}+\frac{4(a-6b)(a+b)^2\arctan(\tan(e+fx))}{a}+\frac{b(4a+3b)(a+4b)\tan(e+fx)}{a+b\tan^2(e+fx)+b}}{2a(a+b)}+\frac{b(2a+3b)\tan(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)^2}$$
$$2a$$
$$f$$

$$\downarrow 218$$

$$\frac{\frac{b^{3/2}\left(35a^2+56ab+24b^2\right)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}+\frac{4(a-6b)(a+b)^2\arctan(\tan(e+fx))}{a}+\frac{b(4a+3b)(a+4b)\tan(e+fx)}{a+b\tan^2(e+fx)+b}}{2a(a+b)}+\frac{b(2a+3b)\tan(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)^2}$$
$$2a$$
$$f$$

input
```
Int[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
(Tan[e + f*x]/(2*a*(1 + Tan[e + f*x]^2)*(a + b + b*Tan[e + f*x]^2)^2) + ((
b*(2*a + 3*b)*Tan[e + f*x])/(2*a*(a + b)*(a + b + b*Tan[e + f*x]^2)^2) + (
((4*(a - 6*b)*(a + b)^2*ArcTan[Tan[e + f*x]])/a + (b^(3/2)*(35*a^2 + 56*a*
b + 24*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(
2*a*(a + b)) + (b*(4*a + 3*b)*(a + 4*b)*Tan[e + f*x])/(2*a*(a + b)*(a + b
+ b*Tan[e + f*x]^2)))/(2*a*(a + b)))/(2*a))/f
```

3.216.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.216.4 Maple [A] (verified)

Time = 4.40 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{b^2\left(\frac{ab(11a+8b)\tan(fx+e)^3}{8a^2+16ab+8b^2}+\frac{(13a+8b)a\tan(fx+e)}{8a+8b}\right)}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(35a^2+56ab+24b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\left(a^2+2ab+b^2\right)\sqrt{(a+b)b}}\Bigg)}{a^4}+\frac{\frac{a\tan(fx+e)}{2+2\tan(fx+e)^2}+\frac{(a-6b)\arc}{a^4}}{f}$
default	$\frac{b^2\left(\frac{ab(11a+8b)\tan(fx+e)^3}{8a^2+16ab+8b^2}+\frac{(13a+8b)a\tan(fx+e)}{8a+8b}\right)}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(35a^2+56ab+24b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\left(a^2+2ab+b^2\right)\sqrt{(a+b)b}}\Bigg)}{a^4}+\frac{\frac{a\tan(fx+e)}{2+2\tan(fx+e)^2}+\frac{(a-6b)\arc}{a^4}}{f}$
risch	$\frac{x}{2a^3}-\frac{3xb}{a^4}-\frac{ie^{2i(fx+e)}}{8a^3f}+\frac{ie^{-2i(fx+e)}}{8a^3f}+\frac{ib^2\left(13a^3e^{6i(fx+e)}+40a^2be^{6i(fx+e)}+24ab^2e^{6i(fx+e)}+39a^3e^{4i(fx+e)}+\right.}{4a^4(}$

input
```
int(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(b^2/a^4*((1/8*a*b*(11*a+8*b)/(a^2+2*a*b+b^2)*tan(f*x+e)^3+1/8*(13*a+8
*b)*a/(a+b)*tan(f*x+e))/(a+b+b*tan(f*x+e)^2)^2+1/8*(35*a^2+56*a*b+24*b^2)/
(a^2+2*a*b+b^2)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))+1/a^
4*(1/2*a*tan(f*x+e)/(1+tan(f*x+e)^2)+1/2*(a-6*b)*arctan(tan(f*x+e))))
```

3.216.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 442 vs. 2(183) = 366.

Time = 0.33 (sec) , antiderivative size = 970, normalized size of antiderivative = 4.83

$$\int \frac{\cos^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[1/32*(16*(a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*f*x*cos(f*x + e)^4 + 32
*(a^4*b - 4*a^3*b^2 - 11*a^2*b^3 - 6*a*b^4)*f*x*cos(f*x + e)^2 + 16*(a^3*b
^2 - 4*a^2*b^3 - 11*a*b^4 - 6*b^5)*f*x + (35*a^2*b^3 + 56*a*b^4 + 24*b^5 +
 (35*a^4*b + 56*a^3*b^2 + 24*a^2*b^3)*cos(f*x + e)^4 + 2*(35*a^3*b^2 + 56*
a^2*b^3 + 24*a*b^4)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8
*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*b
+ 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f
*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) + 4*(4*(
a^5 + 2*a^4*b + a^3*b^2)*cos(f*x + e)^5 + (8*a^4*b + 29*a^3*b^2 + 18*a^2*b
^3)*cos(f*x + e)^3 + (4*a^3*b^2 + 19*a^2*b^3 + 12*a*b^4)*cos(f*x + e))*sin
(f*x + e))/((a^8 + 2*a^7*b + a^6*b^2)*f*cos(f*x + e)^4 + 2*(a^7*b + 2*a^6*
b^2 + a^5*b^3)*f*cos(f*x + e)^2 + (a^6*b^2 + 2*a^5*b^3 + a^4*b^4)*f), 1/16
*(8*(a^5 - 4*a^4*b - 11*a^3*b^2 - 6*a^2*b^3)*f*x*cos(f*x + e)^4 + 16*(a^4*
b - 4*a^3*b^2 - 11*a^2*b^3 - 6*a*b^4)*f*x*cos(f*x + e)^2 + 8*(a^3*b^2 - 4*
a^2*b^3 - 11*a*b^4 - 6*b^5)*f*x - (35*a^2*b^3 + 56*a*b^4 + 24*b^5 + (35*a^
4*b + 56*a^3*b^2 + 24*a^2*b^3)*cos(f*x + e)^4 + 2*(35*a^3*b^2 + 56*a^2*b^3
 + 24*a*b^4)*cos(f*x + e)^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x
 + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e))) + 2*(4*(a^5 +
2*a^4*b + a^3*b^2)*cos(f*x + e)^5 + (8*a^4*b + 29*a^3*b^2 + 18*a^2*b^3)*co
s(f*x + e)^3 + (4*a^3*b^2 + 19*a^2*b^3 + 12*a*b^4)*cos(f*x + e))*sin(f*...
```

3.216.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**2/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Timed out
```

3.216.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.68

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \frac{\frac{(35a^2b^2+56ab^3+24b^4)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^6+2a^5b+a^4b^2)\sqrt{(a+b)b}} + \frac{(4a^2b^2+19ab^3+12b^4)\tan(fx+e)^5+(8a^3b+37a^2b^2+56ab^3+24b^4)\tan(fx+e}{a^7+4a^6b+6a^5b^2+4a^4b^3+a^3b^4+(a^5b^2+2a^4b^3+a^3b^4)\tan(fx+e)^6+(2a^6b+7a^5b^2+8a^4b^3+3}}{8f}$$

input
```
integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
1/8*((35*a^2*b^2 + 56*a*b^3 + 24*b^4)*arctan(b*tan(f*x + e)/sqrt((a + b)*b
))/((a^6 + 2*a^5*b + a^4*b^2)*sqrt((a + b)*b)) + ((4*a^2*b^2 + 19*a*b^3 +
12*b^4)*tan(f*x + e)^5 + (8*a^3*b + 37*a^2*b^2 + 56*a*b^3 + 24*b^4)*tan(f*
x + e)^3 + (4*a^4 + 16*a^3*b + 37*a^2*b^2 + 37*a*b^3 + 12*b^4)*tan(f*x + e
))/(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4 + (a^5*b^2 + 2*a^4*b^3
 + a^3*b^4)*tan(f*x + e)^6 + (2*a^6*b + 7*a^5*b^2 + 8*a^4*b^3 + 3*a^3*b^4)
*tan(f*x + e)^4 + (a^7 + 6*a^6*b + 12*a^5*b^2 + 10*a^4*b^3 + 3*a^3*b^4)*ta
n(f*x + e)^2) + 4*(f*x + e)*(a - 6*b)/a^4)/f
```

3.216.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.13

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \frac{\frac{(35a^2b^2+56ab^3+24b^4)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^6+2a^5b+a^4b^2)\sqrt{ab+b^2}} + \frac{11ab^3\tan(fx+e)^3+8b^4\tan(fx+e)^3+13a^2b^2\tan(fx+e)+21ab^3\tan(}{(a^5+2a^4b+a^3b^2)\left(b\tan(fx+e)^2+a+b\right)^2}}{8f}$$

input
```
integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
1/8*((35*a^2*b^2 + 56*a*b^3 + 24*b^4)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b)
 + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^6 + 2*a^5*b + a^4*b^2)*sqrt
(a*b + b^2)) + (11*a*b^3*tan(f*x + e)^3 + 8*b^4*tan(f*x + e)^3 + 13*a^2*b^
2*tan(f*x + e) + 21*a*b^3*tan(f*x + e) + 8*b^4*tan(f*x + e))/((a^5 + 2*a^4
*b + a^3*b^2)*(b*tan(f*x + e)^2 + a + b)^2) + 4*(f*x + e)*(a - 6*b)/a^4 +
4*tan(f*x + e)/((tan(f*x + e)^2 + 1)*a^3))/f
```

3.216.9 Mupad [B] (verification not implemented)

Time = 24.10 (sec) , antiderivative size = 3708, normalized size of antiderivative = 18.45

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(cos(e + f*x)^2/(a + b/cos(e + f*x)^2)^3,x)
```

output
```
((tan(e + f*x)^5*(19*a*b^3 + 12*b^4 + 4*a^2*b^2))/(8*a^3*(a + b)^2) + (tan
(e + f*x)*(25*a*b^2 + 12*a^2*b + 4*a^3 + 12*b^3))/(8*a^3*(a + b)) + (b*tan
(e + f*x)^3*(56*a*b^2 + 37*a^2*b + 8*a^3 + 24*b^3))/(8*a^3*(a + b)^2))/(f*
(2*a*b + tan(e + f*x)^2*(4*a*b + a^2 + 3*b^2) + a^2 + b^2 + tan(e + f*x)^4
*(2*a*b + 3*b^2) + b^2*tan(e + f*x)^6)) + (atan((((((6*a^8*b^7 + (49*a^9*
b^6)/2 + 37*a^10*b^5 + (45*a^11*b^4)/2 + 2*a^12*b^3 - 2*a^13*b^2)/(4*a^12*
b + a^13 + a^9*b^4 + 4*a^10*b^3 + 6*a^11*b^2) - (tan(e + f*x)*(a*1i - b*6i
)*(512*a^8*b^7 + 2304*a^9*b^6 + 4096*a^10*b^5 + 3584*a^11*b^4 + 1536*a^12*
b^3 + 256*a^13*b^2))/(128*a^4*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^
8*b^2)))*(a*1i - b*6i))/(4*a^4) - (tan(e + f*x)*(4800*a*b^8 + 1152*b^9 + 7
520*a^2*b^7 + 5136*a^3*b^6 + 1129*a^4*b^5 - 128*a^5*b^4 + 16*a^6*b^3))/(32
*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)))*(a*1i - b*6i)*1i)/(4
*a^4) - (((((6*a^8*b^7 + (49*a^9*b^6)/2 + 37*a^10*b^5 + (45*a^11*b^4)/2 +
2*a^12*b^3 - 2*a^13*b^2)/(4*a^12*b + a^13 + a^9*b^4 + 4*a^10*b^3 + 6*a^11*
b^2) + (tan(e + f*x)*(a*1i - b*6i)*(512*a^8*b^7 + 2304*a^9*b^6 + 4096*a^10
*b^5 + 3584*a^11*b^4 + 1536*a^12*b^3 + 256*a^13*b^2))/(128*a^4*(4*a^9*b +
a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)))*(a*1i - b*6i))/(4*a^4) + (tan(e
+ f*x)*(4800*a*b^8 + 1152*b^9 + 7520*a^2*b^7 + 5136*a^3*b^6 + 1129*a^4*b^5
 - 128*a^5*b^4 + 16*a^6*b^3))/(32*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 +
6*a^8*b^2)))*(a*1i - b*6i)*1i)/(4*a^4))/(((405*a*b^8)/4 + 27*b^9 + (261...
```

3.217 $\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

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3.217.1 Optimal result

Integrand size = 23, antiderivative size = 269

$$\begin{aligned}\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx &= \frac{3(a^2-4ab+16b^2)\,x}{8a^5} \\ &\quad - \frac{3b^{5/2}(21a^2+36ab+16b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^5(a+b)^{5/2}f} \\ &\quad + \frac{(3a-8b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^2} \\ &\quad + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^2} \\ &\quad + \frac{b(3a^2-7ab-12b^2)\tan(e+fx)}{8a^3(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} \\ &\quad + \frac{3b(a+2b)\left(a^2-4ab-4b^2\right)\tan(e+fx)}{8a^4(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}\end{aligned}$$

output

```
3/8*(a^2-4*a*b+16*b^2)*x/a^5-3/8*b^(5/2)*(21*a^2+36*a*b+16*b^2)*arctan(b^(
1/2)*tan(f*x+e)/(a+b)^(1/2))/a^5/(a+b)^(5/2)/f+1/8*(3*a-8*b)*cos(f*x+e)*si
n(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)^2+1/4*cos(f*x+e)^3*sin(f*x+e)/a/f/(a+b
+b*tan(f*x+e)^2)^2+1/8*b*(3*a^2-7*a*b-12*b^2)*tan(f*x+e)/a^3/(a+b)/f/(a+b+
b*tan(f*x+e)^2)^2+3/8*b*(a+2*b)*(a^2-4*a*b-4*b^2)*tan(f*x+e)/a^4/(a+b)^2/f
/(a+b+b*tan(f*x+e)^2)
```

3.217.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.96 (sec) , antiderivative size = 1430, normalized size of antiderivative = 5.32

$$\int \frac{\cos^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$

$$= \frac{(21a^2+36ab+16b^2)\,(a+2b+a\cos(2e+2fx))^3\sec^6(e+fx)\left(\frac{3b^3\arctan\left(\sec(fx)\left(\frac{\cos(2e)}{2\sqrt{a+b}\sqrt{b\cos(4e)-ib\sin(4e)}}-\frac{2\sqrt{a+b}}{}\right.\right.}{64a^5\sqrt{a+}}$$

$$+\frac{(a+2b+a\cos(2e+2fx))\sec(2e)\sec^6(e+fx)\,(144a^6fx\cos(2e)+96a^5bfx\cos(2e)+912a^4b^2fx\cos(2}{}$$

input
```
Integrate[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
((21*a^2 + 36*a*b + 16*b^2)*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^
6*((3*b^3*ArcTan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*S
in[4*e]]) - ((I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]])
)*(-(a*Sin[f*x]) - 2*b*Sin[f*x] + a*Sin[2*e + f*x])]*Cos[2*e])/(64*a^5*Sqr
t[a + b]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - (((3*I)/64)*b^3*ArcTan[Sec[f
*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin
[2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b
*Sin[f*x] + a*Sin[2*e + f*x])]*Sin[2*e])/(a^5*Sqrt[a + b]*f*Sqrt[b*Cos[4*e
] - I*b*Sin[4*e]])))/((a + b)^2*(a + b*Sec[e + f*x]^2)^3) + ((a + 2*b + a*
Cos[2*e + 2*f*x])*Sec[2*e]*Sec[e + f*x]^6*(144*a^6*f*x*Cos[2*e] + 96*a^5*b
*f*x*Cos[2*e] + 912*a^4*b^2*f*x*Cos[2*e] + 6720*a^3*b^3*f*x*Cos[2*e] + 165
12*a^2*b^4*f*x*Cos[2*e] + 16896*a*b^5*f*x*Cos[2*e] + 6144*b^6*f*x*Cos[2*e]
 + 96*a^6*f*x*Cos[2*f*x] + 480*a^4*b^2*f*x*Cos[2*f*x] + 4416*a^3*b^3*f*x*C
os[2*f*x] + 6912*a^2*b^4*f*x*Cos[2*f*x] + 3072*a*b^5*f*x*Cos[2*f*x] + 96*a
^6*f*x*Cos[4*e + 2*f*x] + 480*a^4*b^2*f*x*Cos[4*e + 2*f*x] + 4416*a^3*b^3*
f*x*Cos[4*e + 2*f*x] + 6912*a^2*b^4*f*x*Cos[4*e + 2*f*x] + 3072*a*b^5*f*x*
Cos[4*e + 2*f*x] + 24*a^6*f*x*Cos[2*e + 4*f*x] - 48*a^5*b*f*x*Cos[2*e + 4*
f*x] + 216*a^4*b^2*f*x*Cos[2*e + 4*f*x] + 672*a^3*b^3*f*x*Cos[2*e + 4*f*x]
 + 384*a^2*b^4*f*x*Cos[2*e + 4*f*x] + 24*a^6*f*x*Cos[6*e + 4*f*x] - 48*a^5
*b*f*x*Cos[6*e + 4*f*x] + 216*a^4*b^2*f*x*Cos[6*e + 4*f*x] + 672*a^3*b^...
```

3.217.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.13, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 4634, 316, 25, 402, 25, 402, 27, 402, 27, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sec(e+fx)^4\,(a+b\sec(e+fx)^2)^3}dx$$

$$\downarrow 4634$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)^3(b\tan^2(e+fx)+a+b)^3}d\tan(e+fx)}{f}$$

$$\downarrow 316$$

$$\frac{\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)^2} - \frac{\int -\frac{7b\tan^2(e+fx)+3a-b}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^3}d\tan(e+fx)}{4a}}{f}$$

$$\downarrow 25$$

$$\frac{\frac{\int \frac{7b\tan^2(e+fx)+3a-b}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^3}d\tan(e+fx)}{4a} + \frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\downarrow 402$$

$$\frac{\frac{(3a-8b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^2} - \frac{\int -\frac{3a^2+3ba+8b^2+5(3a-8b)b\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^3}d\tan(e+fx)}{2a}}{4a} + \frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\downarrow 25$$

$$\frac{\frac{\int \frac{3a^2+3ba+8b^2+5(3a-8b)b\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^3}d\tan(e+fx)}{2a} + \frac{(3a-8b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^2}}{4a} + \frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\downarrow 402$$

$$\frac{\frac{\int \frac{12\left(a^3+5b^2a+4b^3+b\left(3a^2-7ba-12b^2\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{4a(a+b)}+\frac{b\left(3a^2-7ab-12b^2\right)\tan(e+fx)}{a(a+b)\left(a+b\tan^2(e+fx)+b\right)^2}}{2a}+\frac{(3a-8b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^2}}{4a}+\frac{}{4a(\tan^2(e+$$

$$f$$

$$\downarrow 27$$

$$\frac{\frac{3\int \frac{a^3+5b^2a+4b^3+b\left(3a^2-7ba-12b^2\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{a(a+b)}+\frac{b\left(3a^2-7ab-12b^2\right)\tan(e+fx)}{a(a+b)\left(a+b\tan^2(e+fx)+b\right)^2}}{2a}+\frac{(3a-8b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^2}}{4a}+\frac{}{4a(\tan^2(e+fx$$

$$f$$

$$\downarrow 402$$

$$\frac{\frac{3\left(\int \frac{2\left(a^4-ba^3+7b^2a^2+16b^3a+8b^4+b(a+2b)\left(a^2-4ba-4b^2\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2a(a+b)}+\frac{b(a+2b)\left(a^2-4ab-4b^2\right)\tan(e+fx)}{a(a+b)\left(a+b\tan^2(e+fx)+b\right)}\right)}{a(a+b)}+\frac{b\left(3a^2-7ab-12b^2\right)\tan(e+fx)}{a(a+b)\left(a+b\tan^2(e+fx)+b\right)^2}}{2a}+}{4a}$$

$$f$$

$$\downarrow 27$$

$$\frac{\frac{3\left(\int \frac{a^4-ba^3+7b^2a^2+16b^3a+8b^4+b(a+2b)\left(a^2-4ba-4b^2\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{a(a+b)}+\frac{b(a+2b)\left(a^2-4ab-4b^2\right)\tan(e+fx)}{a(a+b)\left(a+b\tan^2(e+fx)+b\right)}\right)}{a(a+b)}+\frac{b\left(3a^2-7ab-12b^2\right)\tan(e+fx)}{a(a+b)\left(a+b\tan^2(e+fx)+b\right)^2}}{2a}+\frac{2a($$

$$f$$

$$\downarrow 397$$

$$\frac{\frac{3\left(\frac{(a+b)^2\left(a^2-4ab+16b^2\right)\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}-\frac{b^3\left(21a^2+36ab+16b^2\right)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}+\frac{b(a+2b)\left(a^2-4ab-4b^2\right)\tan(e+fx)}{a(a+b)\left(a+b\tan^2(e+fx)+b\right)}\right)}{a(a+b)}+\frac{b(3$$

$$f$$

3.217. $\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

input
```
Int[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
(Tan[e + f*x]/(4*a*(1 + Tan[e + f*x]^2)^2*(a + b + b*Tan[e + f*x]^2)^2) +
(((3*a - 8*b)*Tan[e + f*x])/(2*a*(1 + Tan[e + f*x]^2)*(a + b + b*Tan[e + f
*x]^2)^2) + ((b*(3*a^2 - 7*a*b - 12*b^2)*Tan[e + f*x])/(a*(a + b)*(a + b +
 b*Tan[e + f*x]^2)^2) + (3*((((a + b)^2*(a^2 - 4*a*b + 16*b^2)*ArcTan[Tan[
e + f*x]])/a - (b^(5/2)*(21*a^2 + 36*a*b + 16*b^2)*ArcTan[(Sqrt[b]*Tan[e +
 f*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(a*(a + b)) + (b*(a + 2*b)*(a^2 - 4*
a*b - 4*b^2)*Tan[e + f*x])/(a*(a + b)*(a + b + b*Tan[e + f*x]^2))))/(a*(a
+ b)))/(2*a))/(4*a))/f
```

3.217.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.217.4 Maple [A] (verified)

Time = 6.90 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.80

method	result
derivativedivides	$\frac{\frac{\left(\frac{3}{8}a^2-\frac{3}{2}ab\right)\tan(fx+e)^3+\left(-\frac{3}{2}ab+\frac{5}{8}a^2\right)\tan(fx+e)}{\left(1+\tan(fx+e)^2\right)^2}+\frac{3\left(a^2-4ab+16b^2\right)\arctan(\tan(fx+e))}{8}}{a^5}-\frac{b^3\left(\frac{3ab(5a+4b)\tan(fx+e)^3}{8\left(a^2+2ab+b^2\right)}+\frac{(17a+}{\left(a+b+b\tan(fx+e)^2\right.}\right.}{f}$
default	$\frac{\frac{\left(\frac{3}{8}a^2-\frac{3}{2}ab\right)\tan(fx+e)^3+\left(-\frac{3}{2}ab+\frac{5}{8}a^2\right)\tan(fx+e)}{\left(1+\tan(fx+e)^2\right)^2}+\frac{3\left(a^2-4ab+16b^2\right)\arctan(\tan(fx+e))}{8}}{a^5}-\frac{b^3\left(\frac{3ab(5a+4b)\tan(fx+e)^3}{8\left(a^2+2ab+b^2\right)}+\frac{(17a+}{\left(a+b+b\tan(fx+e)^2\right.}\right.}{f}$
risch	$\frac{3x}{8a^3}-\frac{3xb}{2a^4}+\frac{6xb^2}{a^5}+\frac{3ie^{2i(fx+e)}b}{8a^4f}+\frac{ie^{-2i(fx+e)}}{8a^3f}+\frac{ie^{-4i(fx+e)}}{64a^3f}-\frac{ie^{2i(fx+e)}}{8a^3f}-\frac{3ie^{-2i(fx+e)}b}{8a^4f}-\frac{ie^{4i(fx+e)}}{64a^3f}$

input
```
int(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/a^5*(((3/8*a^2-3/2*a*b)*tan(f*x+e)^3+(-3/2*a*b+5/8*a^2)*tan(f*x+e))
/(1+tan(f*x+e)^2)^2+3/8*(a^2-4*a*b+16*b^2)*arctan(tan(f*x+e)))-b^3/a^5*((3
/8*a*b*(5*a+4*b)/(a^2+2*a*b+b^2)*tan(f*x+e)^3+1/8*(17*a+12*b)*a/(a+b)*tan(
f*x+e))/(a+b+b*tan(f*x+e)^2)^2+3/8*(21*a^2+36*a*b+16*b^2)/(a^2+2*a*b+b^2)/
((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))))
```

3.217.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 521 vs. $2(249) = 498$.

Time = 0.35 (sec) , antiderivative size = 1129, normalized size of antiderivative = 4.20

$$\int \frac{\cos^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output

```
[1/32*(12*(a^6 - 2*a^5*b + 9*a^4*b^2 + 28*a^3*b^3 + 16*a^2*b^4)*f*x*cos(f*
x + e)^4 + 24*(a^5*b - 2*a^4*b^2 + 9*a^3*b^3 + 28*a^2*b^4 + 16*a*b^5)*f*x*
cos(f*x + e)^2 + 12*(a^4*b^2 - 2*a^3*b^3 + 9*a^2*b^4 + 28*a*b^5 + 16*b^6)*
f*x + 3*(21*a^2*b^4 + 36*a*b^5 + 16*b^6 + (21*a^4*b^2 + 36*a^3*b^3 + 16*a^
2*b^4)*cos(f*x + e)^4 + 2*(21*a^3*b^3 + 36*a^2*b^4 + 16*a*b^5)*cos(f*x + e
)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b
 + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b
+ b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e
)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) + 4*(2*(a^6 + 2*a^5*b + a^4*b^2)*cos(f*
x + e)^7 + (3*a^6 - 2*a^5*b - 13*a^4*b^2 - 8*a^3*b^3)*cos(f*x + e)^5 + (6*
a^5*b - 10*a^4*b^2 - 55*a^3*b^3 - 36*a^2*b^4)*cos(f*x + e)^3 + 3*(a^4*b^2
 - 2*a^3*b^3 - 12*a^2*b^4 - 8*a*b^5)*cos(f*x + e))*sin(f*x + e))/((a^9 + 2*
a^8*b + a^7*b^2)*f*cos(f*x + e)^4 + 2*(a^8*b + 2*a^7*b^2 + a^6*b^3)*f*cos(
f*x + e)^2 + (a^7*b^2 + 2*a^6*b^3 + a^5*b^4)*f), 1/16*(6*(a^6 - 2*a^5*b +
9*a^4*b^2 + 28*a^3*b^3 + 16*a^2*b^4)*f*x*cos(f*x + e)^4 + 12*(a^5*b - 2*a^
4*b^2 + 9*a^3*b^3 + 28*a^2*b^4 + 16*a*b^5)*f*x*cos(f*x + e)^2 + 6*(a^4*b^2
 - 2*a^3*b^3 + 9*a^2*b^4 + 28*a*b^5 + 16*b^6)*f*x + 3*(21*a^2*b^4 + 36*a*b
^5 + 16*b^6 + (21*a^4*b^2 + 36*a^3*b^3 + 16*a^2*b^4)*cos(f*x + e)^4 + 2*(2
1*a^3*b^3 + 36*a^2*b^4 + 16*a*b^5)*cos(f*x + e)^2)*sqrt(b/(a + b))*arctan(
1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(...
```

3.217.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

input

```
integrate(cos(f*x+e)**4/(a+b*sec(f*x+e)**2)**3,x)
```

output

```
Timed out
```

3.217.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.72

$$\int \frac{\cos^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx =$$

$$-\frac{3\left(21a^2b^3+36ab^4+16b^5\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\left(a^7+2a^6b+a^5b^2\right)\sqrt{(a+b)b}} - \frac{3\left(a^3b^2-2a^2b^3-12ab^4-8b^5\right)\tan(fx+e)^7+\left(6a^4b-a^3b^2-73a^2b^3-144ab^4-72b^5\right)\tan(f}{\left(a^6b^2+2a^5b^3+a^4b^4\right)\tan(fx+e)^8+a^8+4a^7b+6a^6b^2+4a^5b^3+a^4b^4+2\left(a^7b+4a^6b^2+5a\right.}$$

input
```
integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
-1/8*(3*(21*a^2*b^3 + 36*a*b^4 + 16*b^5)*arctan(b*tan(f*x + e)/sqrt((a + b
)*b))/((a^7 + 2*a^6*b + a^5*b^2)*sqrt((a + b)*b)) - (3*(a^3*b^2 - 2*a^2*b^
3 - 12*a*b^4 - 8*b^5)*tan(f*x + e)^7 + (6*a^4*b - a^3*b^2 - 73*a^2*b^3 - 1
44*a*b^4 - 72*b^5)*tan(f*x + e)^5 + (3*a^5 + 10*a^4*b - 24*a^3*b^2 - 136*a
^2*b^3 - 180*a*b^4 - 72*b^5)*tan(f*x + e)^3 + (5*a^5 + 8*a^4*b - 18*a^3*b^
2 - 69*a^2*b^3 - 72*a*b^4 - 24*b^5)*tan(f*x + e))/((a^6*b^2 + 2*a^5*b^3 +
a^4*b^4)*tan(f*x + e)^8 + a^8 + 4*a^7*b + 6*a^6*b^2 + 4*a^5*b^3 + a^4*b^4
+ 2*(a^7*b + 4*a^6*b^2 + 5*a^5*b^3 + 2*a^4*b^4)*tan(f*x + e)^6 + (a^8 + 8*
a^7*b + 19*a^6*b^2 + 18*a^5*b^3 + 6*a^4*b^4)*tan(f*x + e)^4 + 2*(a^8 + 5*a
^7*b + 9*a^6*b^2 + 7*a^5*b^3 + 2*a^4*b^4)*tan(f*x + e)^2) - 3*(a^2 - 4*a*b
 + 16*b^2)*(f*x + e)/a^5)/f
```

3.217.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.72

$$\int \frac{\cos^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx =$$

$$-\frac{3\left(21a^2b^3+36ab^4+16b^5\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{\left(a^7+2a^6b+a^5b^2\right)\sqrt{ab+b^2}} - \frac{3a^3b^2\tan(fx+e)^7-6a^2b^3\tan(fx+e)^7-36ab^4\tan(fx+e)^7-24b}{}$$

input
```
integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
-1/8*(3*(21*a^2*b^3 + 36*a*b^4 + 16*b^5)*(pi*floor((f*x + e)/pi + 1/2)*sgn
(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^7 + 2*a^6*b + a^5*b^2)*s
qrt(a*b + b^2)) - (3*a^3*b^2*tan(f*x + e)^7 - 6*a^2*b^3*tan(f*x + e)^7 - 3
6*a*b^4*tan(f*x + e)^7 - 24*b^5*tan(f*x + e)^7 + 6*a^4*b*tan(f*x + e)^5 -
a^3*b^2*tan(f*x + e)^5 - 73*a^2*b^3*tan(f*x + e)^5 - 144*a*b^4*tan(f*x + e
)^5 - 72*b^5*tan(f*x + e)^5 + 3*a^5*tan(f*x + e)^3 + 10*a^4*b*tan(f*x + e)
^3 - 24*a^3*b^2*tan(f*x + e)^3 - 136*a^2*b^3*tan(f*x + e)^3 - 180*a*b^4*ta
n(f*x + e)^3 - 72*b^5*tan(f*x + e)^3 + 5*a^5*tan(f*x + e) + 8*a^4*b*tan(f*
x + e) - 18*a^3*b^2*tan(f*x + e) - 69*a^2*b^3*tan(f*x + e) - 72*a*b^4*tan(
f*x + e) - 24*b^5*tan(f*x + e))/((a^6 + 2*a^5*b + a^4*b^2)*(b*tan(f*x + e)
^4 + a*tan(f*x + e)^2 + 2*b*tan(f*x + e)^2 + a + b)^2) - 3*(a^2 - 4*a*b +
16*b^2)*(f*x + e)/a^5)/f
```

3.217.9 Mupad [B] (verification not implemented)

Time = 24.04 (sec) , antiderivative size = 4158, normalized size of antiderivative = 15.46

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(cos(e + f*x)^4/(a + b/cos(e + f*x)^2)^3,x)
```

output

```
(atan(((((tan(e + f*x)*(18432*a*b^10 + 4608*b^11 + 27360*a^2*b^9 + 17568*a
^3*b^8 + 3978*a^4*b^7 + 180*a^5*b^6 + 198*a^6*b^5 - 36*a^7*b^4 + 9*a^8*b^3
))/(32*(4*a^11*b + a^12 + a^8*b^4 + 4*a^9*b^3 + 6*a^10*b^2)) - (3*((12*a^1
0*b^8 + 48*a^11*b^7 + (141*a^12*b^6)/2 + (87*a^13*b^5)/2 + 9*a^14*b^4 + (3
*a^15*b^3)/2 + (3*a^16*b^2)/2)/(4*a^15*b + a^16 + a^12*b^4 + 4*a^13*b^3 +
6*a^14*b^2) - (3*tan(e + f*x)*(-b^5*(a + b)^5)^(1/2)*(36*a*b + 21*a^2 + 16
*b^2)*(512*a^10*b^7 + 2304*a^11*b^6 + 4096*a^12*b^5 + 3584*a^13*b^4 + 1536
*a^14*b^3 + 256*a^15*b^2))/(512*(4*a^11*b + a^12 + a^8*b^4 + 4*a^9*b^3 + 6
*a^10*b^2)*(5*a^9*b + a^10 + a^5*b^5 + 5*a^6*b^4 + 10*a^7*b^3 + 10*a^8*b^2
)))*(-b^5*(a + b)^5)^(1/2)*(36*a*b + 21*a^2 + 16*b^2))/(16*(5*a^9*b + a^10
 + a^5*b^5 + 5*a^6*b^4 + 10*a^7*b^3 + 10*a^8*b^2)))*(-b^5*(a + b)^5)^(1/2)
*(36*a*b + 21*a^2 + 16*b^2)*3i)/(16*(5*a^9*b + a^10 + a^5*b^5 + 5*a^6*b^4
+ 10*a^7*b^3 + 10*a^8*b^2)) + (((tan(e + f*x)*(18432*a*b^10 + 4608*b^11 +
27360*a^2*b^9 + 17568*a^3*b^8 + 3978*a^4*b^7 + 180*a^5*b^6 + 198*a^6*b^5 -
 36*a^7*b^4 + 9*a^8*b^3))/(32*(4*a^11*b + a^12 + a^8*b^4 + 4*a^9*b^3 + 6*a
^10*b^2)) + (3*((12*a^10*b^8 + 48*a^11*b^7 + (141*a^12*b^6)/2 + (87*a^13*b
^5)/2 + 9*a^14*b^4 + (3*a^15*b^3)/2 + (3*a^16*b^2)/2)/(4*a^15*b + a^16 + a
^12*b^4 + 4*a^13*b^3 + 6*a^14*b^2) + (3*tan(e + f*x)*(-b^5*(a + b)^5)^(1/2
)*(36*a*b + 21*a^2 + 16*b^2)*(512*a^10*b^7 + 2304*a^11*b^6 + 4096*a^12*b^5
 + 3584*a^13*b^4 + 1536*a^14*b^3 + 256*a^15*b^2))/(512*(4*a^11*b + a^12...
```

3.218 $\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

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3.218.1 Optimal result

Integrand size = 23, antiderivative size = 352

$$\begin{aligned}\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx &= \frac{(5a^3-18a^2b+48ab^2-160b^3)\,x}{16a^6}\\ &+\frac{b^{7/2}(99a^2+176ab+80b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^6(a+b)^{5/2}f}\\ &+\frac{(15a^2-34ab+80b^2)\cos(e+fx)\sin(e+fx)}{48a^3f\left(a+b+b\tan^2(e+fx)\right)^2}\\ &+\frac{5(a-2b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)^2}\\ &+\frac{\cos^5(e+fx)\sin(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)^2}\\ &+\frac{b(15a^3-29a^2b+64ab^2+120b^3)\tan(e+fx)}{48a^4(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2}\\ &+\frac{b(5a^4-8a^3b+17a^2b^2+116ab^3+80b^4)\tan(e+fx)}{16a^5(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}\end{aligned}$$

output
```
1/16*(5*a^3-18*a^2*b+48*a*b^2-160*b^3)*x/a^6+1/8*b^(7/2)*(99*a^2+176*a*b+8
0*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^6/(a+b)^(5/2)/f+1/48*(15*a
^2-34*a*b+80*b^2)*cos(f*x+e)*sin(f*x+e)/a^3/f/(a+b+b*tan(f*x+e)^2)^2+5/24*
(a-2*b)*cos(f*x+e)^3*sin(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)^2+1/6*cos(f*x+e
)^5*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)^2+1/48*b*(15*a^3-29*a^2*b+64*a*b^2
+120*b^3)*tan(f*x+e)/a^4/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2+1/16*b*(5*a^4-8*a^
3*b+17*a^2*b^2+116*a*b^3+80*b^4)*tan(f*x+e)/a^5/(a+b)^2/f/(a+b+b*tan(f*x+e
)^2)
```

3.218.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.12 (sec) , antiderivative size = 1770, normalized size of antiderivative = 5.03

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
((99*a^2 + 176*a*b + 80*b^2)*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]
^6*(-1/64*(b^4*ArcTan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] -
I*b*Sin[4*e]]) - ((I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4
*e]]))*(-(a*Sin[f*x]) - 2*b*Sin[f*x] + a*Sin[2*e + f*x])]*Cos[2*e])/(a^6*S
qrt[a + b]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) + ((I/64)*b^4*ArcTan[Sec[f*x
]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[2
*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*S
in[f*x] + a*Sin[2*e + f*x])]*Sin[2*e])/(a^6*Sqrt[a + b]*f*Sqrt[b*Cos[4*e]
- I*b*Sin[4*e]])))/((a + b)^2*(a + b*Sec[e + f*x]^2)^3) + ((a + 2*b + a*Co
s[2*e + 2*f*x])*Sec[2*e]*Sec[e + f*x]^6*(720*a^7*f*x*Cos[2*e] + 768*a^6*b*
f*x*Cos[2*e] + 1296*a^5*b^2*f*x*Cos[2*e] - 8352*a^4*b^3*f*x*Cos[2*e] - 641
28*a^3*b^4*f*x*Cos[2*e] - 158976*a^2*b^5*f*x*Cos[2*e] - 165888*a*b^6*f*x*C
os[2*e] - 61440*b^7*f*x*Cos[2*e] + 480*a^7*f*x*Cos[2*f*x] + 192*a^6*b*f*x*
Cos[2*f*x] + 96*a^5*b^2*f*x*Cos[2*f*x] - 4608*a^4*b^3*f*x*Cos[2*f*x] - 418
56*a^3*b^4*f*x*Cos[2*f*x] - 67584*a^2*b^5*f*x*Cos[2*f*x] - 30720*a*b^6*f*x
*Cos[2*f*x] + 480*a^7*f*x*Cos[4*e + 2*f*x] + 192*a^6*b*f*x*Cos[4*e + 2*f*x
] + 96*a^5*b^2*f*x*Cos[4*e + 2*f*x] - 4608*a^4*b^3*f*x*Cos[4*e + 2*f*x] -
41856*a^3*b^4*f*x*Cos[4*e + 2*f*x] - 67584*a^2*b^5*f*x*Cos[4*e + 2*f*x] -
30720*a*b^6*f*x*Cos[4*e + 2*f*x] + 120*a^7*f*x*Cos[2*e + 4*f*x] - 192*a^6*
b*f*x*Cos[2*e + 4*f*x] + 408*a^5*b^2*f*x*Cos[2*e + 4*f*x] - 1968*a^4*b^...
```

3.218.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.13, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 4634, 316, 25, 402, 25, 402, 25, 402, 27, 402, 27, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sec(e+fx)^6\,(a+b\sec(e+fx)^2)^3}dx$$

$$\downarrow 4634$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)^4(b\tan^2(e+fx)+a+b)^3}d\tan(e+fx)}{f}$$

$$\downarrow 316$$

$$\frac{\frac{\tan(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)^2} - \frac{\int -\frac{9b\tan^2(e+fx)+5a-b}{\left(\tan^2(e+fx)+1\right)^3\left(b\tan^2(e+fx)+a+b\right)^3}d\tan(e+fx)}{6a}}{f}$$

$$\downarrow 25$$

$$\frac{\frac{\int \frac{9b\tan^2(e+fx)+5a-b}{\left(\tan^2(e+fx)+1\right)^3\left(b\tan^2(e+fx)+a+b\right)^3}d\tan(e+fx)}{6a} + \frac{\tan(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\downarrow 402$$

$$\frac{\frac{5(a-2b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\left(a+b\tan^2(e+fx)+b\right)^2} - \frac{\int -\frac{15a^2+ba+10b^2+35(a-2b)b\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^3}d\tan(e+fx)}{4a}}{6a} + \frac{\tan(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\downarrow 25$$

$$\frac{\frac{\int \frac{15a^2+ba+10b^2+35(a-2b)b\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^3}d\tan(e+fx)}{4a} + \frac{5(a-2b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\left(a+b\tan^2(e+fx)+b\right)^2}}{6a} + \frac{\tan(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\downarrow 402$$

$$\frac{\frac{\left(15a^2-34ab+80b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^2}-\frac{\int -\frac{15a^3+21ba^2-26b^2a-80b^3+5b\left(15a^2-34ba+80b^2\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^3}d\tan(e+fx)}{2a}}{4a}+\frac{\frac{5(a-2b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\left(a+b\tan^2(e+fx)+b\right)^2}}{6a}}{f}$$

$$\downarrow 25$$

$$\frac{\frac{\int \frac{15a^3+21ba^2-26b^2a-80b^3+5b\left(15a^2-34ba+80b^2\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^3}d\tan(e+fx)}{2a}+\frac{\left(15a^2-34ab+80b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^2}}{4a}+\frac{\frac{5(a-2b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\left(a+b\tan^2(e+fx)+b\right)^2}}{6a}}{f}$$

$$\downarrow 402$$

$$\frac{\frac{\int \frac{12\left(5a^4+2ba^3+b^2a^2-48b^3a-40b^4+b\left(15a^3-29ba^2+64b^2a+120b^3\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{4a(a+b)}+\frac{b\left(15a^3-29a^2b+64ab^2+120b^3\right)\tan(e+fx)}{a(a+b)\left(a+b\tan^2(e+fx)+b\right)^2}}{2a}+\frac{\frac{\left(15a^2-34ab+\right.}{2a\left(\tan^2(e+fx)+1\right)}}{4a}}{6a}}{f}$$

$$\downarrow 27$$

$$\frac{\frac{3\int \frac{5a^4+2ba^3+b^2a^2-48b^3a-40b^4+b\left(15a^3-29ba^2+64b^2a+120b^3\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{a(a+b)}+\frac{b\left(15a^3-29a^2b+64ab^2+120b^3\right)\tan(e+fx)}{a(a+b)\left(a+b\tan^2(e+fx)+b\right)^2}}{2a}+\frac{\frac{\left(15a^2-34ab+80\right.}{2a\left(\tan^2(e+fx)+1\right)\left(a-\right.}}{4a}}{6a}}{f}$$

$$\downarrow 402$$

3.218. $\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

$$\frac{3\left(\frac{\int \frac{2\left(5a^5-3ba^4+9b^2a^3-65b^3a^2-156b^4a-80b^5+b\left(5a^4-8ba^3+17b^2a^2+116b^3a+80b^4\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2a(a+b)}+\frac{b\left(5a^4-8a^3b+17a^2b^2+116ab^3+80b^4\right)\tan(e+}{a(a+b)\left(a+b\tan^2(e+fx)+b\right)}\right)}{a(a+b)}$$

$$\frac{}{2a}$$

$$\frac{}{4a}$$

$$\frac{}{6a}$$

$$\downarrow 27$$

$$\frac{3\left(\frac{\int \frac{5a^5-3ba^4+9b^2a^3-65b^3a^2-156b^4a-80b^5+b\left(5a^4-8ba^3+17b^2a^2+116b^3a+80b^4\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{a(a+b)}+\frac{b\left(5a^4-8a^3b+17a^2b^2+116ab^3+80b^4\right)\tan(e+fx}{a(a+b)\left(a+b\tan^2(e+fx)+b\right)}\right)}{a(a+b)}$$

$$\frac{}{2a}$$

$$\frac{}{4a}$$

$$\frac{}{6a}$$

$$\downarrow 397$$

$$\frac{3\left(\frac{2b^4\left(99a^2+176ab+80b^2\right)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}+\frac{(a+b)^2\left(5a^3-18a^2b+48ab^2-160b^3\right)\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}+\frac{b\left(5a^4-8a^3b+17a^2b^2+116ab\right.}{a(a+b)\left(a+b\tan^2(\right.}\right)}{a(a+b)}$$

$$\frac{}{2a}$$

$$\frac{}{4a}$$

$$\frac{}{6a}$$

$$\downarrow 216$$

$$\frac{3\left(\frac{2b^4\left(99a^2+176ab+80b^2\right)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}+\frac{(a+b)^2\left(5a^3-18a^2b+48ab^2-160b^3\right)\arctan(\tan(e+fx))}{a}+\frac{b\left(5a^4-8a^3b+17a^2b^2+116ab^3+80b^4\right)t}{a(a+b)\left(a+b\tan^2(e+fx)+b\right)}\right)}{a(a+b)}$$

$$\frac{}{2a}$$

$$\frac{}{4a}$$

$$\frac{}{6a}$$

$$\downarrow 218$$

$$\frac{\left(15a^2-34ab+80b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^2}+\frac{\frac{b\left(15a^3-29a^2b+64ab^2+120b^3\right)\tan(e+fx)}{a(a+b)\left(a+b\tan^2(e+fx)+b\right)^2}+\frac{3\left(\frac{2b^{7/2}\left(99a^2+176ab+80b^2\right)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}+\frac{(a+b)^2\left(5a^3-1\right.}{a(a+b)}\right)}{2a}}{4a}}{6a}$$

input
```
Int[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
(Tan[e + f*x]/(6*a*(1 + Tan[e + f*x]^2)^3*(a + b + b*Tan[e + f*x]^2)^2) +
((5*(a - 2*b)*Tan[e + f*x])/(4*a*(1 + Tan[e + f*x]^2)^2*(a + b + b*Tan[e +
 f*x]^2)^2) + (((15*a^2 - 34*a*b + 80*b^2)*Tan[e + f*x])/(2*a*(1 + Tan[e +
 f*x]^2)*(a + b + b*Tan[e + f*x]^2)^2) + ((b*(15*a^3 - 29*a^2*b + 64*a*b^2
 + 120*b^3)*Tan[e + f*x])/(a*(a + b)*(a + b + b*Tan[e + f*x]^2)^2) + (3*((
((a + b)^2*(5*a^3 - 18*a^2*b + 48*a*b^2 - 160*b^3)*ArcTan[Tan[e + f*x]])/a
 + (2*b^(7/2)*(99*a^2 + 176*a*b + 80*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sq
rt[a + b]])/(a*Sqrt[a + b]))/(a*(a + b)) + (b*(5*a^4 - 8*a^3*b + 17*a^2*b^
2 + 116*a*b^3 + 80*b^4)*Tan[e + f*x])/(a*(a + b)*(a + b + b*Tan[e + f*x]^2
))))/(a*(a + b)))/(2*a))/(4*a))/(6*a))/f
```

3.218.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.218.4 Maple [A] (verified)

Time = 12.97 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.76

method	result
derivativedivides	$\frac{\left(\frac{5}{16}a^3-\frac{9}{8}a^2b+3ab^2\right)\tan(fx+e)^5+\left(6ab^2+\frac{5}{6}a^3-3a^2b\right)\tan(fx+e)^3+\left(-\frac{15}{8}a^2b+3ab^2+\frac{11}{16}a^3\right)\tan(fx+e)}{\left(1+\tan(fx+e)^2\right)^3}+\frac{\left(5a^3-18a^2b+48ab^2-1}{a^6}$ f
default	$\frac{\left(\frac{5}{16}a^3-\frac{9}{8}a^2b+3ab^2\right)\tan(fx+e)^5+\left(6ab^2+\frac{5}{6}a^3-3a^2b\right)\tan(fx+e)^3+\left(-\frac{15}{8}a^2b+3ab^2+\frac{11}{16}a^3\right)\tan(fx+e)}{\left(1+\tan(fx+e)^2\right)^3}+\frac{\left(5a^3-18a^2b+48ab^2-1}{a^6}$ f
risch	$\frac{5x}{16a^3}-\frac{9xb}{8a^4}+\frac{3xb^2}{a^5}-\frac{10xb^3}{a^6}+\frac{15ie^{-2i(fx+e)}}{128a^3f}+\frac{3ie^{-2i(fx+e)}b^2}{4a^5f}-\frac{15ie^{2i(fx+e)}}{128a^3f}-\frac{3ie^{2i(fx+e)}b^2}{4a^5f}+\frac{3ie^{2i(}}{8a}$

input
```
int(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/a^6*(((5/16*a^3-9/8*a^2*b+3*a*b^2)*tan(f*x+e)^5+(6*a*b^2+5/6*a^3-3*
a^2*b)*tan(f*x+e)^3+(-15/8*a^2*b+3*a*b^2+11/16*a^3)*tan(f*x+e))/(1+tan(f*x
+e)^2)^3+1/16*(5*a^3-18*a^2*b+48*a*b^2-160*b^3)*arctan(tan(f*x+e)))+b^4/a^
6*((1/8*a*b*(19*a+16*b)/(a^2+2*a*b+b^2)*tan(f*x+e)^3+1/8*(21*a+16*b)*a/(a+
b)*tan(f*x+e))/(a+b+b*tan(f*x+e)^2)^2+1/8*(99*a^2+176*a*b+80*b^2)/(a^2+2*a
*b+b^2)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))))
```

3.218.5 Fricas [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 1296, normalized size of antiderivative = 3.68

$$\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[1/96*(6*(5*a^7 - 8*a^6*b + 17*a^5*b^2 - 82*a^4*b^3 - 272*a^3*b^4 - 160*a^
2*b^5)*f*x*cos(f*x + e)^4 + 12*(5*a^6*b - 8*a^5*b^2 + 17*a^4*b^3 - 82*a^3*
b^4 - 272*a^2*b^5 - 160*a*b^6)*f*x*cos(f*x + e)^2 + 6*(5*a^5*b^2 - 8*a^4*b
^3 + 17*a^3*b^4 - 82*a^2*b^5 - 272*a*b^6 - 160*b^7)*f*x + 3*(99*a^2*b^5 +
176*a*b^6 + 80*b^7 + (99*a^4*b^3 + 176*a^3*b^4 + 80*a^2*b^5)*cos(f*x + e)^
4 + 2*(99*a^3*b^4 + 176*a^2*b^5 + 80*a*b^6)*cos(f*x + e)^2)*sqrt(-b/(a + b
))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x +
 e)^2 - 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e)
)*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x
 + e)^2 + b^2)) + 2*(8*(a^7 + 2*a^6*b + a^5*b^2)*cos(f*x + e)^9 + 10*(a^7
- 3*a^5*b^2 - 2*a^4*b^3)*cos(f*x + e)^7 + (15*a^7 - 4*a^6*b + 27*a^5*b^2 +
 126*a^4*b^3 + 80*a^3*b^4)*cos(f*x + e)^5 + 2*(15*a^6*b - 19*a^5*b^2 + 43*
a^4*b^3 + 266*a^3*b^4 + 180*a^2*b^5)*cos(f*x + e)^3 + 3*(5*a^5*b^2 - 8*a^4
*b^3 + 17*a^3*b^4 + 116*a^2*b^5 + 80*a*b^6)*cos(f*x + e))*sin(f*x + e))/((
a^10 + 2*a^9*b + a^8*b^2)*f*cos(f*x + e)^4 + 2*(a^9*b + 2*a^8*b^2 + a^7*b^
3)*f*cos(f*x + e)^2 + (a^8*b^2 + 2*a^7*b^3 + a^6*b^4)*f), 1/48*(3*(5*a^7 -
 8*a^6*b + 17*a^5*b^2 - 82*a^4*b^3 - 272*a^3*b^4 - 160*a^2*b^5)*f*x*cos(f*
x + e)^4 + 6*(5*a^6*b - 8*a^5*b^2 + 17*a^4*b^3 - 82*a^3*b^4 - 272*a^2*b^5
- 160*a*b^6)*f*x*cos(f*x + e)^2 + 3*(5*a^5*b^2 - 8*a^4*b^3 + 17*a^3*b^4 -
82*a^2*b^5 - 272*a*b^6 - 160*b^7)*f*x - 3*(99*a^2*b^5 + 176*a*b^6 + 80*...
```

3.218.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**6/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Timed out
```

3.218.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 611, normalized size of antiderivative = 1.74

$$\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$
$$= \frac{6\left(99a^2b^4+176ab^5+80b^6\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\left(a^8+2a^7b+a^6b^2\right)\sqrt{(a+b)b}} + \frac{3\left(5a^4b^2-8a^3b^3+17a^2b^4+116ab^5+80b^6\right)\tan(fx+e)^9+2\left(15a^5b+11a^4b^2-5a^3b^3+368a^2\right.}{\left(a^7b^2+2a^6b^3+a^5b^4\right)\tan(fx+e)^{10}+a^9+4a^8b+6a^7b}$$

input
```
integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
1/48*(6*(99*a^2*b^4 + 176*a*b^5 + 80*b^6)*arctan(b*tan(f*x + e)/sqrt((a +
b)*b))/((a^8 + 2*a^7*b + a^6*b^2)*sqrt((a + b)*b)) + (3*(5*a^4*b^2 - 8*a^3
*b^3 + 17*a^2*b^4 + 116*a*b^5 + 80*b^6)*tan(f*x + e)^9 + 2*(15*a^5*b + 11*
a^4*b^2 - 5*a^3*b^3 + 368*a^2*b^4 + 876*a*b^5 + 480*b^6)*tan(f*x + e)^7 +
(15*a^6 + 86*a^5*b + 3*a^4*b^2 + 240*a^3*b^3 + 1982*a^2*b^4 + 3168*a*b^5 +
 1440*b^6)*tan(f*x + e)^5 + 2*(20*a^6 + 41*a^5*b - 15*a^4*b^2 + 197*a^3*b^
3 + 980*a^2*b^4 + 1236*a*b^5 + 480*b^6)*tan(f*x + e)^3 + 3*(11*a^6 + 14*a^
5*b - 6*a^4*b^2 + 56*a^3*b^3 + 221*a^2*b^4 + 236*a*b^5 + 80*b^6)*tan(f*x +
 e))/((a^7*b^2 + 2*a^6*b^3 + a^5*b^4)*tan(f*x + e)^10 + a^9 + 4*a^8*b + 6*
a^7*b^2 + 4*a^6*b^3 + a^5*b^4 + (2*a^8*b + 9*a^7*b^2 + 12*a^6*b^3 + 5*a^5*
b^4)*tan(f*x + e)^8 + (a^9 + 10*a^8*b + 27*a^7*b^2 + 28*a^6*b^3 + 10*a^5*b
^4)*tan(f*x + e)^6 + (3*a^9 + 18*a^8*b + 37*a^7*b^2 + 32*a^6*b^3 + 10*a^5*
b^4)*tan(f*x + e)^4 + (3*a^9 + 14*a^8*b + 24*a^7*b^2 + 18*a^6*b^3 + 5*a^5*
b^4)*tan(f*x + e)^2) + 3*(5*a^3 - 18*a^2*b + 48*a*b^2 - 160*b^3)*(f*x + e)
/a^6)/f
```

3.218.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 350, normalized size of antiderivative = 0.99

$$\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$
$$= \frac{6\left(99a^2b^4+176ab^5+80b^6\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{\left(a^8+2a^7b+a^6b^2\right)\sqrt{ab+b^2}} + \frac{6\left(19ab^5\tan(fx+e)^3+16b^6\tan(fx+e)^3+21a^2b^4\tan(fx+e)+37a\right.}{\left(a^7+2a^6b+a^5b^2\right)\left(b\tan(fx+e)^2+a+\right.}$$

input
```
integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
1/48*(6*(99*a^2*b^4 + 176*a*b^5 + 80*b^6)*(pi*floor((f*x + e)/pi + 1/2)*sg
n(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^8 + 2*a^7*b + a^6*b^2)*
sqrt(a*b + b^2)) + 6*(19*a*b^5*tan(f*x + e)^3 + 16*b^6*tan(f*x + e)^3 + 21
*a^2*b^4*tan(f*x + e) + 37*a*b^5*tan(f*x + e) + 16*b^6*tan(f*x + e))/((a^7
 + 2*a^6*b + a^5*b^2)*(b*tan(f*x + e)^2 + a + b)^2) + 3*(5*a^3 - 18*a^2*b
+ 48*a*b^2 - 160*b^3)*(f*x + e)/a^6 + (15*a^2*tan(f*x + e)^5 - 54*a*b*tan(
f*x + e)^5 + 144*b^2*tan(f*x + e)^5 + 40*a^2*tan(f*x + e)^3 - 144*a*b*tan(
f*x + e)^3 + 288*b^2*tan(f*x + e)^3 + 33*a^2*tan(f*x + e) - 90*a*b*tan(f*x
 + e) + 144*b^2*tan(f*x + e))/((tan(f*x + e)^2 + 1)^3*a^5))/f
```

3.218.9 Mupad [B] (verification not implemented)

Time = 25.07 (sec) , antiderivative size = 4594, normalized size of antiderivative = 13.05

$$\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(cos(e + f*x)^6/(a + b/cos(e + f*x)^2)^3,x)
```

output
```
((tan(e + f*x)*(156*a*b^4 + 3*a^4*b + 11*a^5 + 80*b^5 + 65*a^2*b^3 - 9*a^3
*b^2))/(16*a^5*(a + b)) + (tan(e + f*x)^7*(876*a*b^5 + 15*a^5*b + 480*b^6
+ 368*a^2*b^4 - 5*a^3*b^3 + 11*a^4*b^2))/(24*a^5*(a + b)^2) + (tan(e + f*x
)^3*(1236*a*b^5 + 41*a^5*b + 20*a^6 + 480*b^6 + 980*a^2*b^4 + 197*a^3*b^3
- 15*a^4*b^2))/(24*a^5*(a + b)^2) + (tan(e + f*x)^5*(3168*a*b^5 + 86*a^5*b
 + 15*a^6 + 1440*b^6 + 1982*a^2*b^4 + 240*a^3*b^3 + 3*a^4*b^2))/(48*a^5*(a
 + b)^2) + (b*tan(e + f*x)^9*(116*a*b^4 + 5*a^4*b + 80*b^5 + 17*a^2*b^3 -
8*a^3*b^2))/(16*a^5*(a + b)^2))/(f*(2*a*b + tan(e + f*x)^6*(8*a*b + a^2 +
10*b^2) + a^2 + b^2 + tan(e + f*x)^8*(2*a*b + 5*b^2) + b^2*tan(e + f*x)^10
 + tan(e + f*x)^2*(8*a*b + 3*a^2 + 5*b^2) + tan(e + f*x)^4*(12*a*b + 3*a^2
 + 10*b^2))) - (atan(-((((((20*a^12*b^9 + 79*a^13*b^8 + (457*a^14*b^7)/4 +
 (277*a^15*b^6)/4 + (25*a^16*b^5)/2 - 2*a^17*b^4 - (7*a^18*b^3)/4 - (5*a^1
9*b^2)/4)/(4*a^18*b + a^19 + a^15*b^4 + 4*a^16*b^3 + 6*a^17*b^2) - (tan(e
+ f*x)*(a*b^2*48i - a^2*b*18i + a^3*5i - b^3*160i)*(2048*a^12*b^7 + 9216*a
^13*b^6 + 16384*a^14*b^5 + 14336*a^15*b^4 + 6144*a^16*b^3 + 1024*a^17*b^2)
)/(4096*a^6*(4*a^13*b + a^14 + a^10*b^4 + 4*a^11*b^3 + 6*a^12*b^2)))*(a*b^
2*48i - a^2*b*18i + a^3*5i - b^3*160i))/(32*a^6) - (tan(e + f*x)*(199680*a
*b^12 + 51200*b^13 + 287488*a^2*b^11 + 178560*a^3*b^10 + 39240*a^4*b^9 - 3
6*a^5*b^8 - 1119*a^6*b^7 - 1092*a^7*b^6 + 234*a^8*b^5 - 80*a^9*b^4 + 25*a^
10*b^3))/(128*(4*a^13*b + a^14 + a^10*b^4 + 4*a^11*b^3 + 6*a^12*b^2)))*...
```

3.219 $\int \frac{1}{(a+b\sec^2(c+dx))^4}\,dx$

3.219.1 Optimal result 1639
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3.219.1 Optimal result

Integrand size = 14, antiderivative size = 204

$$\begin{aligned}\int \frac{1}{(a+b\sec^2(c+dx))^4}\,dx &= \frac{x}{a^4} - \frac{\sqrt{b}(35a^3+70a^2b+56ab^2+16b^3)\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a+b}}\right)}{16a^4(a+b)^{7/2}d} \\ &- \frac{b\tan(c+dx)}{6a(a+b)d\left(a+b+b\tan^2(c+dx)\right)^3} \\ &- \frac{b(11a+6b)\tan(c+dx)}{24a^2(a+b)^2d\left(a+b+b\tan^2(c+dx)\right)^2} \\ &- \frac{b(19a^2+22ab+8b^2)\tan(c+dx)}{16a^3(a+b)^3d\left(a+b+b\tan^2(c+dx)\right)}\end{aligned}$$

output

```
x/a^4-1/16*(35*a^3+70*a^2*b+56*a*b^2+16*b^3)*arctan(b^(1/2)*tan(d*x+c)/(a+
b)^(1/2))*b^(1/2)/a^4/(a+b)^(7/2)/d-1/6*b*tan(d*x+c)/a/(a+b)/d/(a+b+b*tan(
d*x+c)^2)^3-1/24*b*(11*a+6*b)*tan(d*x+c)/a^2/(a+b)^2/d/(a+b+b*tan(d*x+c)^2
)^2-1/16*b*(19*a^2+22*a*b+8*b^2)*tan(d*x+c)/a^3/(a+b)^3/d/(a+b+b*tan(d*x+c
)^2)
```

3.219.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.44 (sec) , antiderivative size = 1411, normalized size of antiderivative = 6.92

$$\int \frac{1}{(a+b\sec^2(c+dx))^4}\,dx$$

$$= \frac{(35a^3+70a^2b+56ab^2+16b^3)\,(a+2b+a\cos(2c+2dx))^4\sec^8(c+dx)\left(\frac{b\arctan\left(\sec(dx)\left(\frac{\cos(2c)}{2\sqrt{a+b}\sqrt{b\cos(4c)-ib\sin(4c)}}\right.\right.\right.}{}$$

$$+\frac{(a+2b+a\cos(2c+2dx))\sec(2c)\sec^8(c+dx)\,(480a^6dx\cos(2c)+3168a^5bdx\cos(2c)+8928a^4b^2dx\cos}{}$$

input
```
Integrate[(a + b*Sec[c + d*x]^2)^(-4),x]
```

output
```
((35*a^3 + 70*a^2*b + 56*a*b^2 + 16*b^3)*(a + 2*b + a*Cos[2*c + 2*d*x])^4*
Sec[c + d*x]^8*((b*ArcTan[Sec[d*x]*(Cos[2*c]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*c
] - I*b*Sin[4*c]]) - ((I/2)*Sin[2*c])/(Sqrt[a + b]*Sqrt[b*Cos[4*c] - I*b*S
in[4*c]]))*(-(a*Sin[d*x]) - 2*b*Sin[d*x] + a*Sin[2*c + d*x])]*Cos[2*c])/(2
56*a^4*Sqrt[a + b]*d*Sqrt[b*Cos[4*c] - I*b*Sin[4*c]]) - ((I/256)*b*ArcTan[
Sec[d*x]*(Cos[2*c]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*c] - I*b*Sin[4*c]]) - ((I/2
)*Sin[2*c])/(Sqrt[a + b]*Sqrt[b*Cos[4*c] - I*b*Sin[4*c]]))*(-(a*Sin[d*x])
 - 2*b*Sin[d*x] + a*Sin[2*c + d*x])]*Sin[2*c])/(a^4*Sqrt[a + b]*d*Sqrt[b*Co
s[4*c] - I*b*Sin[4*c]])))/((a + b)^3*(a + b*Sec[c + d*x]^2)^4) + ((a + 2*b
 + a*Cos[2*c + 2*d*x])*Sec[2*c]*Sec[c + d*x]^8*(480*a^6*d*x*Cos[2*c] + 316
8*a^5*b*d*x*Cos[2*c] + 8928*a^4*b^2*d*x*Cos[2*c] + 14112*a^3*b^3*d*x*Cos[2
*c] + 13248*a^2*b^4*d*x*Cos[2*c] + 6912*a*b^5*d*x*Cos[2*c] + 1536*b^6*d*x*
Cos[2*c] + 360*a^6*d*x*Cos[2*d*x] + 2232*a^5*b*d*x*Cos[2*d*x] + 5688*a^4*b
^2*d*x*Cos[2*d*x] + 7272*a^3*b^3*d*x*Cos[2*d*x] + 4608*a^2*b^4*d*x*Cos[2*d
*x] + 1152*a*b^5*d*x*Cos[2*d*x] + 360*a^6*d*x*Cos[4*c + 2*d*x] + 2232*a^5*
b*d*x*Cos[4*c + 2*d*x] + 5688*a^4*b^2*d*x*Cos[4*c + 2*d*x] + 7272*a^3*b^3*
d*x*Cos[4*c + 2*d*x] + 4608*a^2*b^4*d*x*Cos[4*c + 2*d*x] + 1152*a*b^5*d*x*
Cos[4*c + 2*d*x] + 144*a^6*d*x*Cos[2*c + 4*d*x] + 720*a^5*b*d*x*Cos[2*c +
4*d*x] + 1296*a^4*b^2*d*x*Cos[2*c + 4*d*x] + 1008*a^3*b^3*d*x*Cos[2*c + 4*
d*x] + 288*a^2*b^4*d*x*Cos[2*c + 4*d*x] + 144*a^6*d*x*Cos[6*c + 4*d*x] ...
```

3.219.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.20, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {3042, 4616, 316, 402, 27, 402, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sec^2(c+dx))^4}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{(a+b\sec(c+dx)^2)^4}\,dx$$

$$\downarrow 4616$$

$$\frac{\int \frac{1}{(\tan^2(c+dx)+1)(b\tan^2(c+dx)+a+b)^4}\,d\tan(c+dx)}{d}$$

$$\downarrow 316$$

$$\frac{\int \frac{-5b\tan^2(c+dx)+6a+b}{\left(\tan^2(c+dx)+1\right)\left(b\tan^2(c+dx)+a+b\right)^3}\,d\tan(c+dx)}{6a(a+b)} - \frac{b\tan(c+dx)}{6a(a+b)(a+b\tan^2(c+dx)+b)^3}}{d}$$

$$\downarrow 402$$

$$\frac{\frac{\int \frac{3\left(8a^2+5ba+2b^2-b(11a+6b)\tan^2(c+dx)\right)}{\left(\tan^2(c+dx)+1\right)\left(b\tan^2(c+dx)+a+b\right)^2}\,d\tan(c+dx)}{4a(a+b)} - \frac{b(11a+6b)\tan(c+dx)}{4a(a+b)\left(a+b\tan^2(c+dx)+b\right)^2}}{6a(a+b)} - \frac{b\tan(c+dx)}{6a(a+b)(a+b\tan^2(c+dx)+b)^3}}{d}$$

$$\downarrow 27$$

$$\frac{\frac{3\int \frac{8a^2+5ba+2b^2-b(11a+6b)\tan^2(c+dx)}{\left(\tan^2(c+dx)+1\right)\left(b\tan^2(c+dx)+a+b\right)^2}\,d\tan(c+dx)}{4a(a+b)} - \frac{b(11a+6b)\tan(c+dx)}{4a(a+b)\left(a+b\tan^2(c+dx)+b\right)^2}}{6a(a+b)} - \frac{b\tan(c+dx)}{6a(a+b)(a+b\tan^2(c+dx)+b)^3}}{d}$$

$$\downarrow 402$$

$$\frac{3\left(\frac{\int \frac{16a^3+29ba^2+26b^2a+8b^3-b\left(19a^2+22ba+8b^2\right)\tan^2(c+dx)}{\left(\tan^2(c+dx)+1\right)\left(b\tan^2(c+dx)+a+b\right)}d\tan(c+dx)}{2a(a+b)}-\frac{b\left(19a^2+22ab+8b^2\right)\tan(c+dx)}{2a(a+b)\left(a+b\tan^2(c+dx)+b\right)}\right)}{4a(a+b)}-\frac{b(11a+6b)\tan(c+dx)}{4a(a+b)\left(a+b\tan^2(c+dx)+b\right)^2}-\frac{b}{6a(a+b)(a}}{6a(a+b)}$$
$$d$$

$$\downarrow 397$$

$$\frac{3\left(\frac{\frac{16(a+b)^3\int\frac{1}{\tan^2(c+dx)+1}d\tan(c+dx)}{a}-\frac{b\left(35a^3+70a^2b+56ab^2+16b^3\right)\int\frac{1}{b\tan^2(c+dx)+a+b}d\tan(c+dx)}{a}}{2a(a+b)}-\frac{b\left(19a^2+22ab+8b^2\right)\tan(c+dx)}{2a(a+b)\left(a+b\tan^2(c+dx)+b\right)}\right)}{4a(a+b)}-\frac{b(11a+6b)}{4a(a+b)\left(a+b\tan^2(c+dx)+b\right)^2}}{6a(a+b)}$$
$$d$$

$$\downarrow 216$$

$$\frac{3\left(\frac{\frac{16(a+b)^3\arctan(\tan(c+dx))}{a}-\frac{b\left(35a^3+70a^2b+56ab^2+16b^3\right)\int\frac{1}{b\tan^2(c+dx)+a+b}d\tan(c+dx)}{a}}{2a(a+b)}-\frac{b\left(19a^2+22ab+8b^2\right)\tan(c+dx)}{2a(a+b)\left(a+b\tan^2(c+dx)+b\right)}\right)}{4a(a+b)}-\frac{b(11a+6b)\tan(c+dx)}{4a(a+b)\left(a+b\tan^2(c+dx)+b\right)^2}}{6a(a+b)}$$
$$d$$

$$\downarrow 218$$

$$\frac{3\left(\frac{\frac{16(a+b)^3\arctan(\tan(c+dx))}{a}-\frac{\sqrt{b}\left(35a^3+70a^2b+56ab^2+16b^3\right)\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{2a(a+b)}-\frac{b\left(19a^2+22ab+8b^2\right)\tan(c+dx)}{2a(a+b)\left(a+b\tan^2(c+dx)+b\right)}\right)}{4a(a+b)}-\frac{b(11a+6b)\tan(c+dx)}{4a(a+b)\left(a+b\tan^2(c+dx)+b\right)^2}}{6a(a+b)}$$
$$d$$

input
```
Int[(a + b*Sec[c + d*x]^2)^(-4),x]
```

output
```
(-1/6*(b*Tan[c + d*x])/(a*(a + b)*(a + b + b*Tan[c + d*x]^2)^3) + (-1/4*(b
*(11*a + 6*b)*Tan[c + d*x])/(a*(a + b)*(a + b + b*Tan[c + d*x]^2)^2) + (3*
(((16*(a + b)^3*ArcTan[Tan[c + d*x]])/a - (Sqrt[b]*(35*a^3 + 70*a^2*b + 56
*a*b^2 + 16*b^3)*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a + b]])/(a*Sqrt[a + b
]))/(2*a*(a + b)) - (b*(19*a^2 + 22*a*b + 8*b^2)*Tan[c + d*x])/(2*a*(a + b
)*(a + b + b*Tan[c + d*x]^2))))/(4*a*(a + b)))/(6*a*(a + b)))/d
```

3.219. $\int \frac{1}{(a+b\sec^2(c+dx))^4}\,dx$

3.219.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

3.219.4 Maple [A] (verified)

Time = 3.25 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{\arctan(\tan(dx+c))}{a^4}-\frac{b\left(\frac{b^2a\left(19a^2+22ab+8b^2\right)\tan(dx+c)^5}{16a^3+48a^2b+48ab^2+16b^3}+\frac{\left(17a^2+18ab+6b^2\right)ab\tan(dx+c)^3}{6a^2+12ab+6b^2}+\frac{\left(29a^2+26ab+8b^2\right)a\tan(dx+c)}{16a+16b}+\frac{(}{\left(a+b+b\tan(dx+c)^2\right)^3}\right.}{a^4}}{d}$
default	$\frac{\arctan(\tan(dx+c))}{a^4}-\frac{b\left(\frac{b^2a\left(19a^2+22ab+8b^2\right)\tan(dx+c)^5}{16a^3+48a^2b+48ab^2+16b^3}+\frac{\left(17a^2+18ab+6b^2\right)ab\tan(dx+c)^3}{6a^2+12ab+6b^2}+\frac{\left(29a^2+26ab+8b^2\right)a\tan(dx+c)}{16a+16b}+\frac{(}{\left(a+b+b\tan(dx+c)^2\right)^3}\right.}{a^4}}{d}$
risch	Expression too large to display

input
```
int(1/(a+b*sec(d*x+c)^2)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/a^4*arctan(tan(d*x+c))-b/a^4*((1/16*b^2*a*(19*a^2+22*a*b+8*b^2)/(a^
3+3*a^2*b+3*a*b^2+b^3)*tan(d*x+c)^5+1/6*(17*a^2+18*a*b+6*b^2)*a*b/(a^2+2*a
*b+b^2)*tan(d*x+c)^3+1/16*(29*a^2+26*a*b+8*b^2)*a/(a+b)*tan(d*x+c))/(a+b+b
*tan(d*x+c)^2)^3+1/16*(35*a^3+70*a^2*b+56*a*b^2+16*b^3)/(a^3+3*a^2*b+3*a*b
^2+b^3)/((a+b)*b)^(1/2)*arctan(b*tan(d*x+c)/((a+b)*b)^(1/2))))
```

3.219.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 618 vs. 2(188) = 376.

Time = 0.36 (sec) , antiderivative size = 1323, normalized size of antiderivative = 6.49

$$\int \frac{1}{\left(a+b\sec^2(c+dx)\right)^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sec(d*x+c)^2)^4,x, algorithm="fricas")
```

output

```
[1/192*(192*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*x*cos(d*x + c)^6 + 576
*(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*d*x*cos(d*x + c)^4 + 576*(a^4*b
^2 + 3*a^3*b^3 + 3*a^2*b^4 + a*b^5)*d*x*cos(d*x + c)^2 + 192*(a^3*b^3 + 3*
a^2*b^4 + 3*a*b^5 + b^6)*d*x + 3*((35*a^6 + 70*a^5*b + 56*a^4*b^2 + 16*a^3
*b^3)*cos(d*x + c)^6 + 35*a^3*b^3 + 70*a^2*b^4 + 56*a*b^5 + 16*b^6 + 3*(35
*a^5*b + 70*a^4*b^2 + 56*a^3*b^3 + 16*a^2*b^4)*cos(d*x + c)^4 + 3*(35*a^4*
b^2 + 70*a^3*b^3 + 56*a^2*b^4 + 16*a*b^5)*cos(d*x + c)^2)*sqrt(-b/(a + b))
*log(((a^2 + 8*a*b + 8*b^2)*cos(d*x + c)^4 - 2*(3*a*b + 4*b^2)*cos(d*x + c
)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(d*x + c)^3 - (a*b + b^2)*cos(d*x + c))*
sqrt(-b/(a + b))*sin(d*x + c) + b^2)/(a^2*cos(d*x + c)^4 + 2*a*b*cos(d*x +
 c)^2 + b^2)) - 4*((87*a^5*b + 116*a^4*b^2 + 44*a^3*b^3)*cos(d*x + c)^5 +
2*(68*a^4*b^2 + 83*a^3*b^3 + 30*a^2*b^4)*cos(d*x + c)^3 + 3*(19*a^3*b^3 +
22*a^2*b^4 + 8*a*b^5)*cos(d*x + c))*sin(d*x + c))/((a^10 + 3*a^9*b + 3*a^8
*b^2 + a^7*b^3)*d*cos(d*x + c)^6 + 3*(a^9*b + 3*a^8*b^2 + 3*a^7*b^3 + a^6*
b^4)*d*cos(d*x + c)^4 + 3*(a^8*b^2 + 3*a^7*b^3 + 3*a^6*b^4 + a^5*b^5)*d*co
s(d*x + c)^2 + (a^7*b^3 + 3*a^6*b^4 + 3*a^5*b^5 + a^4*b^6)*d), 1/96*(96*(a
^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*x*cos(d*x + c)^6 + 288*(a^5*b + 3*a^
4*b^2 + 3*a^3*b^3 + a^2*b^4)*d*x*cos(d*x + c)^4 + 288*(a^4*b^2 + 3*a^3*b^3
 + 3*a^2*b^4 + a*b^5)*d*x*cos(d*x + c)^2 + 96*(a^3*b^3 + 3*a^2*b^4 + 3*a*b
^5 + b^6)*d*x + 3*((35*a^6 + 70*a^5*b + 56*a^4*b^2 + 16*a^3*b^3)*cos(d*...
```

3.219.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec^2(c+dx))^4}\,dx = \text{Timed out}$$

input

```
integrate(1/(a+b*sec(d*x+c)**2)**4,x)
```

output

```
Timed out
```

3.219.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 401 vs. $2(188) = 376$.

Time $= 0.29$ (sec) , antiderivative size $= 401$, normalized size of antiderivative $= 1.97$

$$\int \frac{1}{\left(a+b\sec^2(c+dx)\right)^4}\,dx =$$

$$-\frac{3\left(35a^3b+70a^2b^2+56ab^3+16b^4\right)\arctan\left(\frac{b\tan(dx+c)}{\sqrt{(a+b)b}}\right)}{\left(a^7+3a^6b+3a^5b^2+a^4b^3\right)\sqrt{(a+b)b}} + \frac{3\left(19a^2b^3+22ab^4+8b^5\right)\tan(dx+c)^5+8\left(17a^3b^2+35a^2b^3+24ab^4+6b^5\right)\tan(dx+c)^3+3\left(29a^4b+84a^3b^2+89a^2b^3+42ab^4+8b^5\right)\tan(dx+c)}{a^9+6a^8b+15a^7b^2+20a^6b^3+15a^5b^4+6a^4b^5+a^3b^6+\left(a^6b^3+3a^5b^4+3a^4b^5+a^3b^6\right)\tan(dx+c)^6+3\left(a^7b^2+4a^6b^3+6a^5b^4+4a^4b^5+a^3b^6\right)\tan(dx+c)^4+3\left(a^8b+5a^7b^2+10a^6b^3+10a^5b^4+5a^4b^5+a^3b^6\right)\tan(dx+c)^2} - \frac{48(dx+c)}{a^4}$$

input
```
integrate(1/(a+b*sec(d*x+c)^2)^4,x, algorithm="maxima")
```

output
```
-1/48*(3*(35*a^3*b + 70*a^2*b^2 + 56*a*b^3 + 16*b^4)*arctan(b*tan(d*x + c)
/sqrt((a + b)*b))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*sqrt((a + b)*b))
+ (3*(19*a^2*b^3 + 22*a*b^4 + 8*b^5)*tan(d*x + c)^5 + 8*(17*a^3*b^2 + 35*a
^2*b^3 + 24*a*b^4 + 6*b^5)*tan(d*x + c)^3 + 3*(29*a^4*b + 84*a^3*b^2 + 89*
a^2*b^3 + 42*a*b^4 + 8*b^5)*tan(d*x + c))/(a^9 + 6*a^8*b + 15*a^7*b^2 + 20
*a^6*b^3 + 15*a^5*b^4 + 6*a^4*b^5 + a^3*b^6 + (a^6*b^3 + 3*a^5*b^4 + 3*a^4
*b^5 + a^3*b^6)*tan(d*x + c)^6 + 3*(a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 + 4*a^
4*b^5 + a^3*b^6)*tan(d*x + c)^4 + 3*(a^8*b + 5*a^7*b^2 + 10*a^6*b^3 + 10*a
^5*b^4 + 5*a^4*b^5 + a^3*b^6)*tan(d*x + c)^2) - 48*(d*x + c)/a^4)/d
```

3.219.8 Giac [A] (verification not implemented)

Time $= 0.32$ (sec) , antiderivative size $= 324$, normalized size of antiderivative $= 1.59$

$$\int \frac{1}{\left(a+b\sec^2(c+dx)\right)^4}\,dx =$$

$$-\frac{3\left(35a^3b+70a^2b^2+56ab^3+16b^4\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab+b^2}}\right)\right)}{\left(a^7+3a^6b+3a^5b^2+a^4b^3\right)\sqrt{ab+b^2}} + \frac{57a^2b^3\tan(dx+c)^5+66ab^4\tan(dx+c)^5+24b^5\tan(dx+c)^5+104a^3b^2\tan(dx+c)^3+248a^2b^3\tan(dx+c)^3+192ab^4\tan(dx+c)^3+48b^5\tan(dx+c)^3+87a^4b\tan(dx+c)+252a^3b^2\tan(dx+c)+267a^2b^3\tan(dx+c)+126ab^4\tan(dx+c)+24b^5\tan(dx+c)}{3\left(a^9+6a^8b+15a^7b^2+20a^6b^3+15a^5b^4+6a^4b^5+a^3b^6\right)\left(\tan(dx+c)^2+1\right)^3}$$

input
```
integrate(1/(a+b*sec(d*x+c)^2)^4,x, algorithm="giac")
```

output
```
-1/48*(3*(35*a^3*b + 70*a^2*b^2 + 56*a*b^3 + 16*b^4)*(pi*floor((d*x + c)/p
i + 1/2)*sgn(b) + arctan(b*tan(d*x + c)/sqrt(a*b + b^2)))/((a^7 + 3*a^6*b
+ 3*a^5*b^2 + a^4*b^3)*sqrt(a*b + b^2)) + (57*a^2*b^3*tan(d*x + c)^5 + 66*
a*b^4*tan(d*x + c)^5 + 24*b^5*tan(d*x + c)^5 + 136*a^3*b^2*tan(d*x + c)^3
+ 280*a^2*b^3*tan(d*x + c)^3 + 192*a*b^4*tan(d*x + c)^3 + 48*b^5*tan(d*x +
 c)^3 + 87*a^4*b*tan(d*x + c) + 252*a^3*b^2*tan(d*x + c) + 267*a^2*b^3*tan
(d*x + c) + 126*a*b^4*tan(d*x + c) + 24*b^5*tan(d*x + c))/((a^6 + 3*a^5*b
+ 3*a^4*b^2 + a^3*b^3)*(b*tan(d*x + c)^2 + a + b)^3) - 48*(d*x + c)/a^4)/d
```

3.219.9 Mupad [B] (verification not implemented)

Time = 25.03 (sec) , antiderivative size = 4506, normalized size of antiderivative = 22.09

$$\int \frac{1}{(a+b\sec^2(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
int(1/(a + b/cos(c + d*x)^2)^4,x)
```

output
```
atan(((((2*a^8*b^8 + (25*a^9*b^7)/2 + (131*a^10*b^6)/4 + (189*a^11*b^5)/4
 + (161*a^12*b^4)/4 + (77*a^13*b^3)/4 + 4*a^14*b^2)*1i)/(2*(6*a^14*b + a^1
5 + a^9*b^6 + 6*a^10*b^5 + 15*a^11*b^4 + 20*a^12*b^3 + 15*a^13*b^2)) - (ta
n(c + d*x)*(2048*a^8*b^9 + 13312*a^9*b^8 + 36864*a^10*b^7 + 56320*a^11*b^6
 + 51200*a^12*b^5 + 27648*a^13*b^4 + 8192*a^14*b^3 + 1024*a^15*b^2))/(512*
a^4*(6*a^11*b + a^12 + a^6*b^6 + 6*a^7*b^5 + 15*a^8*b^4 + 20*a^9*b^3 + 15*
a^10*b^2)))/(2*a^4) + (tan(c + d*x)*(3328*a*b^8 + 512*b^9 + 9216*a^2*b^7 +
 14080*a^3*b^6 + 12660*a^4*b^5 + 6436*a^5*b^4 + 1481*a^6*b^3))/(256*(6*a^1
1*b + a^12 + a^6*b^6 + 6*a^7*b^5 + 15*a^8*b^4 + 20*a^9*b^3 + 15*a^10*b^2))
)/a^4 - ((((2*a^8*b^8 + (25*a^9*b^7)/2 + (131*a^10*b^6)/4 + (189*a^11*b^5)
/4 + (161*a^12*b^4)/4 + (77*a^13*b^3)/4 + 4*a^14*b^2)*1i)/(2*(6*a^14*b + a
^15 + a^9*b^6 + 6*a^10*b^5 + 15*a^11*b^4 + 20*a^12*b^3 + 15*a^13*b^2)) + (
tan(c + d*x)*(2048*a^8*b^9 + 13312*a^9*b^8 + 36864*a^10*b^7 + 56320*a^11*b
^6 + 51200*a^12*b^5 + 27648*a^13*b^4 + 8192*a^14*b^3 + 1024*a^15*b^2))/(51
2*a^4*(6*a^11*b + a^12 + a^6*b^6 + 6*a^7*b^5 + 15*a^8*b^4 + 20*a^9*b^3 + 1
5*a^10*b^2)))/(2*a^4) - (tan(c + d*x)*(3328*a*b^8 + 512*b^9 + 9216*a^2*b^7
 + 14080*a^3*b^6 + 12660*a^4*b^5 + 6436*a^5*b^4 + 1481*a^6*b^3))/(256*(6*a
^11*b + a^12 + a^6*b^6 + 6*a^7*b^5 + 15*a^8*b^4 + 20*a^9*b^3 + 15*a^10*b^2
)))/a^4)/((((((2*a^8*b^8 + (25*a^9*b^7)/2 + (131*a^10*b^6)/4 + (189*a^11*b
^5)/4 + (161*a^12*b^4)/4 + (77*a^13*b^3)/4 + 4*a^14*b^2)*1i)/(2*(6*a^14...
```

3.220 $\int (a - a\sec^2(c+dx))^{7/2}\,dx$

3.220.1 Optimal result 1648
3.220.2 Mathematica [A] (verified) 1648
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3.220.9 Mupad [F(-1)] 1653

3.220.1 Optimal result

Integrand size = 17, antiderivative size = 134

$$\int (a - a\sec^2(c+dx))^{7/2}\,dx = -\frac{a^3\cot(c+dx)\log(\cos(c+dx))\sqrt{-a\tan^2(c+dx)}}{d} - \frac{a^3\tan(c+dx)\sqrt{-a\tan^2(c+dx)}}{2d} + \frac{a^3\tan^3(c+dx)\sqrt{-a\tan^2(c+dx)}}{4d} - \frac{a^3\tan^5(c+dx)\sqrt{-a\tan^2(c+dx)}}{6d}$$

output
```
-a^3*cot(d*x+c)*ln(cos(d*x+c))*(-a*tan(d*x+c)^2)^(1/2)/d-1/2*a^3*(-a*tan(d
*x+c)^2)^(1/2)*tan(d*x+c)/d+1/4*a^3*(-a*tan(d*x+c)^2)^(1/2)*tan(d*x+c)^3/d
-1/6*a^3*(-a*tan(d*x+c)^2)^(1/2)*tan(d*x+c)^5/d
```

3.220.2 Mathematica [A] (verified)

Time = 2.54 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.52

$$\int (a - a\sec^2(c+dx))^{7/2}\,dx = \frac{\cot^7(c+dx)\,(-a\tan^2(c+dx))^{7/2}\,(12\log(\cos(c+dx)) + 6\tan^2(c+dx) - 3\tan^4(c+dx) + 2}{12d}$$

input
```
Integrate[(a - a*Sec[c + d*x]^2)^(7/2),x]
```

output
```
(Cot[c + d*x]^7*(-(a*Tan[c + d*x]^2))^(7/2)*(12*Log[Cos[c + d*x]] + 6*Tan[
c + d*x]^2 - 3*Tan[c + d*x]^4 + 2*Tan[c + d*x]^6))/(12*d)
```

3.220.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.62, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.706$, Rules used = {3042, 4609, 3042, 4141, 3042, 3954, 3042, 3954, 3042, 3954, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a - a\sec^2(c+dx)\right)^{7/2}\,dx$$

$$\downarrow 3042$$

$$\int \left(a - a\sec(c+dx)^2\right)^{7/2}dx$$

$$\downarrow 4609$$

$$\int \left(-a\tan^2(c+dx)\right)^{7/2}dx$$

$$\downarrow 3042$$

$$\int \left(-a\tan(c+dx)^2\right)^{7/2}dx$$

$$\downarrow 4141$$

$$-a^3\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\int \tan^7(c+dx)dx$$

$$\downarrow 3042$$

$$-a^3\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\int \tan(c+dx)^7dx$$

$$\downarrow 3954$$

$$-a^3\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\left(\frac{\tan^6(c+dx)}{6d} - \int \tan^5(c+dx)dx\right)$$

$$\downarrow 3042$$

$$-a^3\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\left(\frac{\tan^6(c+dx)}{6d} - \int \tan(c+dx)^5dx\right)$$

$$\downarrow 3954$$

$$-a^3\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\left(\int\tan^3(c+dx)dx+\frac{\tan^6(c+dx)}{6d}-\frac{\tan^4(c+dx)}{4d}\right)$$

$$\downarrow 3042$$

$$-a^3\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\left(\int\tan(c+dx)^3dx+\frac{\tan^6(c+dx)}{6d}-\frac{\tan^4(c+dx)}{4d}\right)$$

$$\downarrow 3954$$

$$-a^3\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\left(-\int\tan(c+dx)dx+\frac{\tan^6(c+dx)}{6d}-\frac{\tan^4(c+dx)}{4d}+\frac{\tan^2(c+dx)}{2d}\right)$$

$$\downarrow 3042$$

$$-a^3\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\left(-\int\tan(c+dx)dx+\frac{\tan^6(c+dx)}{6d}-\frac{\tan^4(c+dx)}{4d}+\frac{\tan^2(c+dx)}{2d}\right)$$

$$\downarrow 3956$$

$$-a^3\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\left(\frac{\tan^6(c+dx)}{6d}-\frac{\tan^4(c+dx)}{4d}+\frac{\tan^2(c+dx)}{2d}+\frac{\log(\cos(c+dx))}{d}\right)$$

input
```
Int[(a - a*Sec[c + d*x]^2)^(7/2),x]
```

output
```
-(a^3*Cot[c + d*x]*Sqrt[-(a*Tan[c + d*x]^2)]*(Log[Cos[c + d*x]]/d + Tan[c
+ d*x]^2/(2*d) - Tan[c + d*x]^4/(4*d) + Tan[c + d*x]^6/(6*d)))
```

3.220.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4609
```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

3.220.4 Maple [A] (verified)

Time = 5.16 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.08

method	result
default	$-\frac{\sqrt{-a\tan(dx+c)^2}\,a^3\left(12\cos(dx+c)^6\ln(-\cot(dx+c)+\csc(dx+c)-1)+12\cos(dx+c)^6\ln(-\cot(dx+c)+\csc(dx+c)+1)-12\cos(dx+c)\right.}{12d}$
risch	$\frac{a^3\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,x}{e^{2i(dx+c)}-1}-\frac{2a^3\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,(dx+c)}{\left(e^{2i(dx+c)}-1\right)d}-\frac{2ia^3\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,(9e^{10i(dx+c)}+$

input
```
int((a-a*sec(d*x+c)^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12/d*(-a*tan(d*x+c)^2)^(1/2)*a^3*(12*cos(d*x+c)^6*ln(-cot(d*x+c)+csc(d*
x+c)-1)+12*cos(d*x+c)^6*ln(-cot(d*x+c)+csc(d*x+c)+1)-12*cos(d*x+c)^6*ln(2/
(cos(d*x+c)+1))-11*cos(d*x+c)^6+18*cos(d*x+c)^4-9*cos(d*x+c)^2+2)*sec(d*x+
c)^5*csc(d*x+c)
```

3.220.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.75

$$\int \left(a - a\sec^2(c+dx)\right)^{7/2}\,dx =$$

$$-\frac{\left(12\,a^3\cos(dx+c)^6\log(-\cos(dx+c)) + 18\,a^3\cos(dx+c)^4 - 9\,a^3\cos(dx+c)^2 + 2\,a^3\right)\sqrt{\frac{a\cos(dx+c)^2-a}{\cos(dx+c)^2}}}{12\,d\cos(dx+c)^5\sin(dx+c)}$$

input
```
integrate((a-a*sec(d*x+c)^2)^(7/2),x, algorithm="fricas")
```

output
```
-1/12*(12*a^3*cos(d*x + c)^6*log(-cos(d*x + c)) + 18*a^3*cos(d*x + c)^4 -
9*a^3*cos(d*x + c)^2 + 2*a^3)*sqrt((a*cos(d*x + c)^2 - a)/cos(d*x + c)^2)/
(d*cos(d*x + c)^5*sin(d*x + c))
```

3.220.6 Sympy [F(-1)]

Timed out.

$$\int \left(a - a\sec^2(c+dx)\right)^{7/2}\,dx = \text{Timed out}$$

input
```
integrate((a-a*sec(d*x+c)**2)**(7/2),x)
```

output
```
Timed out
```

3.220.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.60

$$\int \left(a - a\sec^2(c+dx)\right)^{7/2}\,dx =$$
$$-\frac{2\sqrt{-a}a^3\tan(dx+c)^6 - 3\sqrt{-a}a^3\tan(dx+c)^4 + 6\sqrt{-a}a^3\tan(dx+c)^2 - 6\sqrt{-a}a^3\log\left(\tan(dx+c)^2+\right.}{12\,d}$$

input
```
integrate((a-a*sec(d*x+c)^2)^(7/2),x, algorithm="maxima")
```

output
```
-1/12*(2*sqrt(-a)*a^3*tan(d*x + c)^6 - 3*sqrt(-a)*a^3*tan(d*x + c)^4 + 6*s
qrt(-a)*a^3*tan(d*x + c)^2 - 6*sqrt(-a)*a^3*log(tan(d*x + c)^2 + 1))/d
```

3.220.8 Giac [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.62

$$\int \left(a - a\sec^2(c+dx)\right)^{7/2}\,dx =$$

$$6\sqrt{-a}a^3\log\left(\tan\left(\tfrac{1}{2}\,dx+\tfrac{1}{2}\,c\right)^2+\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}+2\right)-6\sqrt{-a}a^3\log\left(\tan\left(\tfrac{1}{2}\,dx+\tfrac{1}{2}\,c\right)^2+\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}-2\right)+\frac{\left(11\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}\right)^3\sqrt{-a}a^3-90\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}\right)^2\sqrt{-a}a^3+276\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}\right)\sqrt{-a}a^3-408\sqrt{-a}a^3\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}-2\right)^3}$$

input
```
integrate((a-a*sec(d*x+c)^2)^(7/2),x, algorithm="giac")
```

output
```
-1/12*(6*sqrt(-a)*a^3*log(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^
2 + 2) - 6*sqrt(-a)*a^3*log(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c
)^2 - 2) + (11*(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2)^3*sqrt(
-a)*a^3 - 90*(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2)^2*sqrt(-a
)*a^3 + 276*(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2)*sqrt(-a)*a
^3 - 408*sqrt(-a)*a^3)/(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2
- 2)^3)/d
```

3.220.9 Mupad [F(-1)]

Timed out.

$$\int \left(a - a\sec^2(c+dx)\right)^{7/2}\,dx = \int \left(a - \frac{a}{\cos\left(c+d\,x\right)^2}\right)^{7/2}\,dx$$

input
```
int((a - a/cos(c + d*x)^2)^(7/2),x)
```

output
```
int((a - a/cos(c + d*x)^2)^(7/2), x)
```

3.221 $\int (a - a\sec^2(c+dx))^{5/2}\, dx$

3.221.1 Optimal result 1654
3.221.2 Mathematica [A] (verified) 1654
3.221.3 Rubi [A] (verified) 1655
3.221.4 Maple [A] (verified) 1657
3.221.5 Fricas [A] (verification not implemented) 1657
3.221.6 Sympy [F] 1658
3.221.7 Maxima [A] (verification not implemented) 1658
3.221.8 Giac [A] (verification not implemented) 1658
3.221.9 Mupad [F(-1)] 1659

3.221.1 Optimal result

Integrand size = 17, antiderivative size = 101

$$\int (a - a\sec^2(c+dx))^{5/2}\, dx = -\frac{a^2 \cot(c+dx)\log(\cos(c+dx))\sqrt{-a\tan^2(c+dx)}}{d} - \frac{a^2\tan(c+dx)\sqrt{-a\tan^2(c+dx)}}{2d} + \frac{a^2\tan^3(c+dx)\sqrt{-a\tan^2(c+dx)}}{4d}$$

output
```
-a^2*cot(d*x+c)*ln(cos(d*x+c))*(-a*tan(d*x+c)^2)^(1/2)/d-1/2*a^2*(-a*tan(d
*x+c)^2)^(1/2)*tan(d*x+c)/d+1/4*a^2*(-a*tan(d*x+c)^2)^(1/2)*tan(d*x+c)^3/d
```

3.221.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.59

$$\int (a - a\sec^2(c+dx))^{5/2}\, dx = -\frac{\cot^5(c+dx)\,(-a\tan^2(c+dx))^{5/2}\,(4\log(\cos(c+dx)) + 2\tan^2(c+dx) - \tan^4(c+dx))}{4d}$$

input
```
Integrate[(a - a*Sec[c + d*x]^2)^(5/2),x]
```

output
```
-1/4*(Cot[c + d*x]^5*(-(a*Tan[c + d*x]^2))^(5/2)*(4*Log[Cos[c + d*x]] + 2*
Tan[c + d*x]^2 - Tan[c + d*x]^4))/d
```

3.221.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.67, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {3042, 4609, 3042, 4141, 3042, 3954, 3042, 3954, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a - a\sec^2(c+dx)\right)^{5/2}\,dx$$

$$\downarrow 3042$$

$$\int \left(a - a\sec(c+dx)^2\right)^{5/2}dx$$

$$\downarrow 4609$$

$$\int \left(-a\tan^2(c+dx)\right)^{5/2}dx$$

$$\downarrow 3042$$

$$\int \left(-a\tan(c+dx)^2\right)^{5/2}dx$$

$$\downarrow 4141$$

$$a^2\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\int \tan^5(c+dx)dx$$

$$\downarrow 3042$$

$$a^2\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\int \tan(c+dx)^5dx$$

$$\downarrow 3954$$

$$a^2\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\left(\frac{\tan^4(c+dx)}{4d} - \int \tan^3(c+dx)dx\right)$$

$$\downarrow 3042$$

$$a^2\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\left(\frac{\tan^4(c+dx)}{4d} - \int \tan(c+dx)^3dx\right)$$

$$\downarrow 3954$$

$$a^2\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\left(\int \tan(c+dx)dx + \frac{\tan^4(c+dx)}{4d} - \frac{\tan^2(c+dx)}{2d}\right)$$

$$\downarrow 3042$$

$$a^2 \cot(c+dx)\sqrt{-a\tan^2(c+dx)}\left(\int \tan(c+dx)dx + \frac{\tan^4(c+dx)}{4d} - \frac{\tan^2(c+dx)}{2d}\right)$$

$$\downarrow 3956$$

$$a^2 \cot(c+dx)\sqrt{-a\tan^2(c+dx)}\left(\frac{\tan^4(c+dx)}{4d} - \frac{\tan^2(c+dx)}{2d} - \frac{\log(\cos(c+dx))}{d}\right)$$

input
```
Int[(a - a*Sec[c + d*x]^2)^(5/2),x]
```

output
```
a^2*Cot[c + d*x]*Sqrt[-(a*Tan[c + d*x]^2)]*(-(Log[Cos[c + d*x]]/d) - Tan[c
 + d*x]^2/(2*d) + Tan[c + d*x]^4/(4*d))
```

3.221.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4609
```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

3.221.4 Maple [A] (verified)

Time = 3.37 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.34

method	result
default	$-\frac{\sqrt{-a\tan(dx+c)^2}\,a^2\left(4\cos(dx+c)^4\ln(-\cot(dx+c)+\csc(dx+c)-1)+4\cos(dx+c)^4\ln(-\cot(dx+c)+\csc(dx+c)+1)-4\cos(dx+c)^4\right.}{4d}$
risch	$\frac{a^2\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,x}{e^{2i(dx+c)}-1}-\frac{2a^2\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,(dx+c)}{\left(e^{2i(dx+c)}-1\right)d}-\frac{4ia^2\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,(e^{6i(dx+c)}+e^{4i}$

input
```
int((a-a*sec(d*x+c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/d*(-a*tan(d*x+c)^2)^(1/2)*a^2*(4*cos(d*x+c)^4*ln(-cot(d*x+c)+csc(d*x+
c)-1)+4*cos(d*x+c)^4*ln(-cot(d*x+c)+csc(d*x+c)+1)-4*cos(d*x+c)^4*ln(2/(cos
(d*x+c)+1))-3*cos(d*x+c)^4+4*cos(d*x+c)^2-1)*sec(d*x+c)^3*csc(d*x+c)
```

3.221.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.86

$$\int \left(a - a\sec^2(c+dx)\right)^{5/2}\,dx =$$
$$-\frac{\left(4\,a^2\cos(dx+c)^4\log(-\cos(dx+c)) + 4\,a^2\cos(dx+c)^2 - a^2\right)\sqrt{\frac{a\cos(dx+c)^2-a}{\cos(dx+c)^2}}}{4\,d\cos(dx+c)^3\sin(dx+c)}$$

input
```
integrate((a-a*sec(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/4*(4*a^2*cos(d*x + c)^4*log(-cos(d*x + c)) + 4*a^2*cos(d*x + c)^2 - a^2
)*sqrt((a*cos(d*x + c)^2 - a)/cos(d*x + c)^2)/(d*cos(d*x + c)^3*sin(d*x +
c))
```

3.221.6 Sympy [F]

$$\int \left(a - a\sec^2(c+dx)\right)^{5/2}\,dx = \int \left(-a\sec^2(c+dx)+a\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a-a*sec(d*x+c)**2)**(5/2),x)
```

output
```
Integral((-a*sec(c + d*x)**2 + a)**(5/2), x)
```

3.221.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.61

$$\int \left(a - a\sec^2(c+dx)\right)^{5/2}\,dx = \frac{\sqrt{-a}a^2\tan(dx+c)^4 - 2\sqrt{-a}a^2\tan(dx+c)^2 + 2\sqrt{-a}a^2\log\left(\tan(dx+c)^2+1\right)}{4\,d}$$

input
```
integrate((a-a*sec(d*x+c)^2)^(5/2),x, algorithm="maxima")
```

output
```
1/4*(sqrt(-a)*a^2*tan(d*x + c)^4 - 2*sqrt(-a)*a^2*tan(d*x + c)^2 + 2*sqrt(
-a)*a^2*log(tan(d*x + c)^2 + 1))/d
```

3.221.8 Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.80

$$\int \left(a - a\sec^2(c+dx)\right)^{5/2}\,dx =$$

$$-\frac{2\sqrt{-a}a^2\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\frac{1}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}+2\right) - 2\sqrt{-a}a^2\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\frac{1}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}-2\right)}{4\,d}$$

input
```
integrate((a-a*sec(d*x+c)^2)^(5/2),x, algorithm="giac")
```

output
```
-1/4*(2*sqrt(-a)*a^2*log(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2
 + 2) - 2*sqrt(-a)*a^2*log(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)
^2 - 2) + (3*(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2)^2*sqrt(-a
)*a^2 - 20*(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2)*sqrt(-a)*a^
2 + 44*sqrt(-a)*a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2 -
2)^2)/d
```

3.221.9 Mupad [F(-1)]

Timed out.

$$\int \left(a - a\sec^2(c+dx)\right)^{5/2}\,dx = \int \left(a - \frac{a}{\cos\left(c+d\,x\right)^2}\right)^{5/2} dx$$

input
```
int((a - a/cos(c + d*x)^2)^(5/2),x)
```

output
```
int((a - a/cos(c + d*x)^2)^(5/2), x)
```

3.222 $\int (a - a\sec^2(c+dx))^{3/2}\,dx$

3.222.1 Optimal result 1660
3.222.2 Mathematica [A] (verified) 1660
3.222.3 Rubi [A] (verified) 1661
3.222.4 Maple [A] (verified) 1663
3.222.5 Fricas [A] (verification not implemented) 1663
3.222.6 Sympy [F] 1663
3.222.7 Maxima [A] (verification not implemented) 1664
3.222.8 Giac [B] (verification not implemented) 1664
3.222.9 Mupad [F(-1)] 1664

3.222.1 Optimal result

Integrand size = 17, antiderivative size = 64

$$\int (a - a\sec^2(c+dx))^{3/2}\,dx = -\frac{a\cot(c+dx)\log(\cos(c+dx))\sqrt{-a\tan^2(c+dx)}}{d} - \frac{a\tan(c+dx)\sqrt{-a\tan^2(c+dx)}}{2d}$$

output
```
-a*cot(d*x+c)*ln(cos(d*x+c))*(-a*tan(d*x+c)^2)^(1/2)/d-1/2*a*(-a*tan(d*x+c
)^2)^(1/2)*tan(d*x+c)/d
```

3.222.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.75

$$\int (a - a\sec^2(c + dx))^{3/2}\,dx = \frac{\cot^3(c+dx)\,(-a\tan^2(c+dx))^{3/2}\,(2\log(\cos(c+dx)) + \tan^2(c+dx))}{2d}$$

input
```
Integrate[(a - a*Sec[c + d*x]^2)^(3/2),x]
```

output
```
(Cot[c + d*x]^3*(-(a*Tan[c + d*x]^2))^(3/2)*(2*Log[Cos[c + d*x]] + Tan[c +
 d*x]^2))/(2*d)
```

3.222.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.80, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {3042, 4609, 3042, 4141, 3042, 3954, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a - a\sec^2(c+dx)\right)^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int \left(a - a\sec(c+dx)^2\right)^{3/2} dx$$

$$\downarrow 4609$$

$$\int \left(-a\tan^2(c+dx)\right)^{3/2} dx$$

$$\downarrow 3042$$

$$\int \left(-a\tan(c+dx)^2\right)^{3/2} dx$$

$$\downarrow 4141$$

$$-a\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\int \tan^3(c+dx)dx$$

$$\downarrow 3042$$

$$-a\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\int \tan(c+dx)^3 dx$$

$$\downarrow 3954$$

$$-a\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\left(\frac{\tan^2(c+dx)}{2d} - \int \tan(c+dx)dx\right)$$

$$\downarrow 3042$$

$$-a\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\left(\frac{\tan^2(c+dx)}{2d} - \int \tan(c+dx)dx\right)$$

$$\downarrow 3956$$

$$-a\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\left(\frac{\tan^2(c+dx)}{2d} + \frac{\log(\cos(c+dx))}{d}\right)$$

input
```
Int[(a - a*Sec[c + d*x]^2)^(3/2),x]
```

output
```
-(a*Cot[c + d*x]*Sqrt[-(a*Tan[c + d*x]^2)]*(Log[Cos[c + d*x]]/d + Tan[c +
d*x]^2/(2*d)))
```

3.222.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4609
```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

3.222.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.53

method	result
default	$-\frac{\sqrt{-a\tan(dx+c)^2}\,a\left(-2\ln\left(\frac{2}{\cos(dx+c)+1}\right)\cot(dx+c)+2\ln(-\cot(dx+c)+\csc(dx+c)+1)\cot(dx+c)+2\ln(-\cot(dx+c)+\csc(dx+c)-\right.}{2d}$
risch	$-\frac{a\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\left(ie^{4i(dx+c)}\ln\left(e^{2i(dx+c)}+1\right)+e^{4i(dx+c)}dx+2ie^{2i(dx+c)}\ln\left(e^{2i(dx+c)}+1\right)+2e^{4i(dx+c)}c+2e^{2i(dx+c)}dx+2ie^{2i(}{\left(e^{2i(dx+c)}-1\right)\left(e^{2i(dx+c)}+1\right)d}$

input
```
int((a-a*sec(d*x+c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/d*(-a*tan(d*x+c)^2)^(1/2)*a*(-2*ln(2/(cos(d*x+c)+1))*cot(d*x+c)+2*ln(
-cot(d*x+c)+csc(d*x+c)+1)*cot(d*x+c)+2*ln(-cot(d*x+c)+csc(d*x+c)-1)*cot(d*
x+c)+tan(d*x+c))
```

3.222.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.06

$$\int \left(a - a\sec^2(c+dx)\right)^{3/2}\,dx = -\frac{\left(2\,a\cos(dx+c)^2\log(-\cos(dx+c)) + a\right)\sqrt{\frac{a\cos(dx+c)^2-a}{\cos(dx+c)^2}}}{2\,d\cos(dx+c)\sin(dx+c)}$$

input
```
integrate((a-a*sec(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(2*a*cos(d*x + c)^2*log(-cos(d*x + c)) + a)*sqrt((a*cos(d*x + c)^2 -
a)/cos(d*x + c)^2)/(d*cos(d*x + c)*sin(d*x + c))
```

3.222.6 Sympy [F]

$$\int \left(a - a\sec^2(c+dx)\right)^{3/2}\,dx = \int \left(-a\sec^2(c+dx) + a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a-a*sec(d*x+c)**2)**(3/2),x)
```

output
```
Integral((-a*sec(c + d*x)**2 + a)**(3/2), x)
```

3.222.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.62

$$\int \left(a - a\sec^2(c+dx)\right)^{3/2}\,dx = -\frac{\sqrt{-a}a\tan\left(dx+c\right)^2 - \sqrt{-a}a\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d}$$

input
```
integrate((a-a*sec(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/2*(sqrt(-a)*a*tan(d*x + c)^2 - sqrt(-a)*a*log(tan(d*x + c)^2 + 1))/d
```

3.222.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(58) = 116.

Time = 0.36 (sec) , antiderivative size = 137, normalized size of antiderivative = 2.14

$$\int \left(a - a\sec^2(c+dx)\right)^{3/2}\,dx =$$

$$-\frac{\sqrt{-a}a\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}+2\right) - \sqrt{-a}a\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\frac{1}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}-2\right)+}{2\,d}$$

input
```
integrate((a-a*sec(d*x+c)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/2*(sqrt(-a)*a*log(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2 + 2
) - sqrt(-a)*a*log(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2 - 2)
+ ((tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2)*sqrt(-a)*a - 6*sqrt
(-a)*a)/(tan(1/2*d*x + 1/2*c)^2 + 1/tan(1/2*d*x + 1/2*c)^2 - 2))/d
```

3.222.9 Mupad [F(-1)]

Timed out.

$$\int \left(a - a\sec^2(c+dx)\right)^{3/2}\,dx = \int \left(a - \frac{a}{\cos\left(c+d\,x\right)^2}\right)^{3/2}\,dx$$

input
```
int((a - a/cos(c + d*x)^2)^(3/2),x)
```

output
```
int((a - a/cos(c + d*x)^2)^(3/2), x)
```

3.223 $\int \sqrt{a - a\sec^2(c+dx)}\,dx$

3.223.1 Optimal result 1666
3.223.2 Mathematica [A] (verified) 1666
3.223.3 Rubi [A] (verified) 1667
3.223.4 Maple [A] (verified) 1668
3.223.5 Fricas [A] (verification not implemented) 1669
3.223.6 Sympy [F] 1669
3.223.7 Maxima [A] (verification not implemented) 1669
3.223.8 Giac [B] (verification not implemented) 1670
3.223.9 Mupad [F(-1)] 1670

3.223.1 Optimal result

Integrand size = 17, antiderivative size = 33

$$\int \sqrt{a - a\sec^2(c+dx)}\,dx = -\frac{\cot(c+dx)\log(\cos(c+dx))\sqrt{-a\tan^2(c+dx)}}{d}$$

output
```
-cot(d*x+c)*ln(cos(d*x+c))*(-a*tan(d*x+c)^2)^(1/2)/d
```

3.223.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \sqrt{a - a\sec^2(c+dx)}\,dx = -\frac{\cot(c+dx)\log(\cos(c+dx))\sqrt{-a\tan^2(c+dx)}}{d}$$

input
```
Integrate[Sqrt[a - a*Sec[c + d*x]^2],x]
```

output
```
-((Cot[c + d*x]*Log[Cos[c + d*x]]*Sqrt[-(a*Tan[c + d*x]^2)])/d)
```

3.223.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3042, 4609, 3042, 4141, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a - a\sec^2(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int \sqrt{a - a\sec(c+dx)^2}dx$$

$$\downarrow 4609$$

$$\int \sqrt{-a\tan^2(c+dx)}dx$$

$$\downarrow 3042$$

$$\int \sqrt{-a\tan(c+dx)^2}dx$$

$$\downarrow 4141$$

$$\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\int \tan(c+dx)dx$$

$$\downarrow 3042$$

$$\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\int \tan(c+dx)dx$$

$$\downarrow 3956$$

$$-\frac{\cot(c+dx)\sqrt{-a\tan^2(c+dx)}\log(\cos(c+dx))}{d}$$

input
```
Int[Sqrt[a - a*Sec[c + d*x]^2],x]
```

output
```
-((Cot[c + d*x]*Log[Cos[c + d*x]]*Sqrt[-(a*Tan[c + d*x]^2)])/d)
```

3.223.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4609
```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

3.223.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.15

method	result
default	$\frac{\sqrt{-a\tan(dx+c)^2}\,\ln\left(1+\tan(dx+c)^2\right)}{2d\tan(dx+c)}$
risch	$\frac{\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,\left(e^{2i(dx+c)}+1\right)x}{e^{2i(dx+c)}-1} - \frac{2\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,\left(e^{2i(dx+c)}+1\right)(dx+c)}{\left(e^{2i(dx+c)}-1\right)d} - \frac{i\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,\left(e^{2i(dx+c)}+1\right)\ln(e^{2i(dx}$

input
```
int((a-a*sec(d*x+c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*(-a*tan(d*x+c)^2)^(1/2)/tan(d*x+c)*ln(1+tan(d*x+c)^2)
```

3.223.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.61

$$\int \sqrt{a - a\sec^2(c+dx)}\,dx = -\frac{\sqrt{\frac{a\cos(dx+c)^2-a}{\cos(dx+c)^2}}\cos(dx+c)\log(-\cos(dx+c))}{d\sin(dx+c)}$$

input
```
integrate((a-a*sec(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt((a*cos(d*x + c)^2 - a)/cos(d*x + c)^2)*cos(d*x + c)*log(-cos(d*x + c
))/(d*sin(d*x + c))
```

3.223.6 Sympy [F]

$$\int \sqrt{a - a\sec^2(c+dx)}\,dx = \int \sqrt{-a\sec^2(c+dx) + a}\,dx$$

input
```
integrate((a-a*sec(d*x+c)**2)**(1/2),x)
```

output
```
Integral(sqrt(-a*sec(c + d*x)**2 + a), x)
```

3.223.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.64

$$\int \sqrt{a - a\sec^2(c+dx)}\,dx = \frac{\sqrt{-a}\log\left(\tan(dx+c)^2+1\right)}{2\,d}$$

input
```
integrate((a-a*sec(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(-a)*log(tan(d*x + c)^2 + 1)/d
```

3.223.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(31) = 62$.

Time = 0.34 (sec) , antiderivative size = 141, normalized size of antiderivative = 4.27

$$\int \sqrt{a - a\sec^2(c+dx)}\,dx =$$

$$-\frac{\left(\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)\operatorname{sgn}\left(-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)-\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right.\right.\right.$$

input
```
integrate((a-a*sec(d*x+c)^2)^(1/2),x, algorithm="giac")
```

output
```
-(log(tan(1/2*d*x + 1/2*c)^2 + 1)*sgn(-tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*
x + 1/2*c)) - log(abs(tan(1/2*d*x + 1/2*c) + 1))*sgn(-tan(1/2*d*x + 1/2*c)
^3 - tan(1/2*d*x + 1/2*c)) - log(abs(tan(1/2*d*x + 1/2*c) - 1))*sgn(-tan(1
/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c)))*sqrt(-a)*sgn(cos(d*x + c))/d
```

3.223.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a - a\sec^2(c+dx)}\,dx = \int \sqrt{a - \frac{a}{\cos\left(c+d\,x\right)^2}}\,dx$$

input
```
int((a - a/cos(c + d*x)^2)^(1/2),x)
```

output
```
int((a - a/cos(c + d*x)^2)^(1/2), x)
```

3.224 $\int \frac{1}{\sqrt{a-a\sec^2(c+dx)}}\,dx$

3.224.1 Optimal result 1671
3.224.2 Mathematica [A] (verified) 1671
3.224.3 Rubi [A] (verified) 1672
3.224.4 Maple [A] (verified) 1673
3.224.5 Fricas [A] (verification not implemented) 1674
3.224.6 Sympy [F] 1674
3.224.7 Maxima [A] (verification not implemented) 1674
3.224.8 Giac [F] 1675
3.224.9 Mupad [F(-1)] 1675

3.224.1 Optimal result

Integrand size = 17, antiderivative size = 32

$$\int \frac{1}{\sqrt{a-a\sec^2(c+dx)}}\,dx = \frac{\log(\sin(c+dx))\tan(c+dx)}{d\sqrt{-a\tan^2(c+dx)}}$$

output
```
ln(sin(d*x+c))*tan(d*x+c)/d/(-a*tan(d*x+c)^2)^(1/2)
```

3.224.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.25

$$\int \frac{1}{\sqrt{a-a\sec^2(c+dx)}}\,dx = \frac{(\log(\cos(c+dx))+\log(\tan(c+dx)))\tan(c+dx)}{d\sqrt{-a\tan^2(c+dx)}}$$

input
```
Integrate[1/Sqrt[a - a*Sec[c + d*x]^2],x]
```

output
```
((Log[Cos[c + d*x]] + Log[Tan[c + d*x]])*Tan[c + d*x])/(d*Sqrt[-(a*Tan[c +
 d*x]^2)])
```

3.224.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {3042, 4609, 3042, 4141, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a - a\sec^2(c+dx)}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{\sqrt{a - a\sec(c+dx)^2}}\,dx$$
$$\downarrow 4609$$
$$\int \frac{1}{\sqrt{-a\tan^2(c+dx)}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{\sqrt{-a\tan(c+dx)^2}}\,dx$$
$$\downarrow 4141$$
$$\frac{\tan(c+dx)\int \cot(c+dx)dx}{\sqrt{-a\tan^2(c+dx)}}$$
$$\downarrow 3042$$
$$\frac{\tan(c+dx)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx}{\sqrt{-a\tan^2(c+dx)}}$$
$$\downarrow 25$$
$$-\frac{\tan(c+dx)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx}{\sqrt{-a\tan^2(c+dx)}}$$
$$\downarrow 3956$$
$$\frac{\tan(c+dx)\log(-\sin(c+dx))}{d\sqrt{-a\tan^2(c+dx)}}$$

input
```
Int[1/Sqrt[a - a*Sec[c + d*x]^2],x]
```

output
```
(Log[-Sin[c + d*x]]*Tan[c + d*x])/(d*Sqrt[-(a*Tan[c + d*x]^2)])
```

3.224.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4609
```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

3.224.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.50

method	result	size
default	$\frac{\tan(dx+c)\left(2\ln(\tan(dx+c))-\ln\left(1+\tan(dx+c)^2\right)\right)}{2d\sqrt{-a\tan(dx+c)^2}}$	48
risch	$\frac{\left(e^{2i(dx+c)}-1\right)x}{\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\left(e^{2i(dx+c)}+1\right)}-\frac{2\left(e^{2i(dx+c)}-1\right)(dx+c)}{\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\left(e^{2i(dx+c)}+1\right)d}-\frac{i\left(e^{2i(dx+c)}-1\right)\ln\left(e^{2i(dx+c)}-1\right)}{\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\left(e^{2i(dx+c)}+1\right)d}$	194

input
```
int(1/(a-a*sec(d*x+c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*tan(d*x+c)*(2*ln(tan(d*x+c))-ln(1+tan(d*x+c)^2))/(-a*tan(d*x+c)^2)^(
1/2)
```

3.224.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.75

$$\int \frac{1}{\sqrt{a - a\sec^2(c+dx)}}\,dx = -\frac{\sqrt{\frac{a\cos(dx+c)^2 - a}{\cos(dx+c)^2}}\cos(dx+c)\log\left(\frac{1}{2}\sin(dx+c)\right)}{ad\sin(dx+c)}$$

input
```
integrate(1/(a-a*sec(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt((a*cos(d*x + c)^2 - a)/cos(d*x + c)^2)*cos(d*x + c)*log(1/2*sin(d*x
+ c))/(a*d*sin(d*x + c))
```

3.224.6 Sympy [F]

$$\int \frac{1}{\sqrt{a - a\sec^2(c+dx)}}\,dx = \int \frac{1}{\sqrt{-a\sec^2(c+dx) + a}}\,dx$$

input
```
integrate(1/(a-a*sec(d*x+c)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(-a*sec(c + d*x)**2 + a), x)
```

3.224.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

$$\int \frac{1}{\sqrt{a - a\sec^2(c+dx)}}\,dx = -\frac{\frac{\log\left(\tan(dx+c)^2+1\right)}{\sqrt{-a}} - \frac{2\log(\tan(dx+c))}{\sqrt{-a}}}{2\,d}$$

input
```
integrate(1/(a-a*sec(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*(log(tan(d*x + c)^2 + 1)/sqrt(-a) - 2*log(tan(d*x + c))/sqrt(-a))/d
```

3.224.8 Giac [F]

$$\int \frac{1}{\sqrt{a - a\sec^2(c+dx)}}\,dx = \int \frac{1}{\sqrt{-a\sec(dx+c)^2 + a}}\,dx$$

input
```
integrate(1/(a-a*sec(d*x+c)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-a*sec(d*x + c)^2 + a), x)
```

3.224.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a - a\sec^2(c+dx)}}\,dx = \int \frac{1}{\sqrt{a - \frac{a}{\cos(c+dx)^2}}}\,dx$$

input
```
int(1/(a - a/cos(c + d*x)^2)^(1/2),x)
```

output
```
int(1/(a - a/cos(c + d*x)^2)^(1/2), x)
```

3.225 $\int \frac{1}{(a-a\sec^2(c+dx))^{3/2}}\,dx$

3.225.1 Optimal result 1676
3.225.2 Mathematica [A] (verified) 1676
3.225.3 Rubi [A] (verified) 1677
3.225.4 Maple [A] (verified) 1679
3.225.5 Fricas [A] (verification not implemented) 1679
3.225.6 Sympy [F] 1680
3.225.7 Maxima [A] (verification not implemented) 1680
3.225.8 Giac [A] (verification not implemented) 1681
3.225.9 Mupad [F(-1)] 1681

3.225.1 Optimal result

Integrand size = 17, antiderivative size = 67

$$\int \frac{1}{(a-a\sec^2(c+dx))^{3/2}}\,dx = \frac{\cot(c+dx)}{2ad\sqrt{-a\tan^2(c+dx)}} + \frac{\log(\sin(c+dx))\tan(c+dx)}{ad\sqrt{-a\tan^2(c+dx)}}$$

output
```
1/2*cot(d*x+c)/a/d/(-a*tan(d*x+c)^2)^(1/2)+ln(sin(d*x+c))*tan(d*x+c)/a/d/(
-a*tan(d*x+c)^2)^(1/2)
```

3.225.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.85

$$\int \frac{1}{(a-a\sec^2(c+dx))^{3/2}}\,dx = -\frac{(\cot^2(c+dx)+2\log(\cos(c+dx))+2\log(\tan(c+dx)))\tan^3(c+dx)}{2d\left(-a\tan^2(c+dx)\right)^{3/2}}$$

input
```
Integrate[(a - a*Sec[c + d*x]^2)^(-3/2),x]
```

output
```
-1/2*((Cot[c + d*x]^2 + 2*Log[Cos[c + d*x]] + 2*Log[Tan[c + d*x]])*Tan[c +
 d*x]^3)/(d*(-(a*Tan[c + d*x]^2))^(3/2))
```

3.225.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.81, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {3042, 4609, 3042, 4141, 3042, 25, 3954, 25, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a - a\sec^2(c+dx))^{3/2}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{(a - a\sec(c+dx)^2)^{3/2}}dx$$
$$\downarrow 4609$$
$$\int \frac{1}{(-a\tan^2(c+dx))^{3/2}}dx$$
$$\downarrow 3042$$
$$\int \frac{1}{(-a\tan(c+dx)^2)^{3/2}}dx$$
$$\downarrow 4141$$
$$-\frac{\tan(c+dx)\int \cot^3(c+dx)dx}{a\sqrt{-a\tan^2(c+dx)}}$$
$$\downarrow 3042$$
$$-\frac{\tan(c+dx)\int -\tan\left(c+dx+\frac{\pi}{2}\right)^3 dx}{a\sqrt{-a\tan^2(c+dx)}}$$
$$\downarrow 25$$
$$\frac{\tan(c+dx)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)^3 dx}{a\sqrt{-a\tan^2(c+dx)}}$$
$$\downarrow 3954$$
$$\frac{\tan(c+dx)\left(\frac{\cot^2(c+dx)}{2d} - \int -\cot(c+dx)dx\right)}{a\sqrt{-a\tan^2(c+dx)}}$$
$$\downarrow 25$$

$$\frac{\tan(c+dx)\left(\int \cot(c+dx)dx+\frac{\cot^2(c+dx)}{2d}\right)}{a\sqrt{-a\tan^2(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{\tan(c+dx)\left(\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\cot^2(c+dx)}{2d}\right)}{a\sqrt{-a\tan^2(c+dx)}}$$

$$\downarrow 25$$

$$\frac{\tan(c+dx)\left(\frac{\cot^2(c+dx)}{2d}-\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx\right)}{a\sqrt{-a\tan^2(c+dx)}}$$

$$\downarrow 3956$$

$$\frac{\tan(c+dx)\left(\frac{\cot^2(c+dx)}{2d}+\frac{\log(-\sin(c+dx))}{d}\right)}{a\sqrt{-a\tan^2(c+dx)}}$$

input
```
Int[(a - a*Sec[c + d*x]^2)^(-3/2),x]
```

output
```
((Cot[c + d*x]^2/(2*d) + Log[-Sin[c + d*x]]/d)*Tan[c + d*x])/(a*Sqrt[-(a*T
an[c + d*x]^2)])
```

3.225.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4609
```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

3.225.4 Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.30

method	result
default	$\frac{4\ln(-\cot(dx+c)+\csc(dx+c))\tan(dx+c)-4\ln\left(\frac{2}{\cos(dx+c)+1}\right)\tan(dx+c)+\cot(dx+c)+\sec(dx+c)\csc(dx+c)}{4d\sqrt{-a\tan(dx+c)^2}\,a}$
risch	$-\frac{ie^{4i(dx+c)}\ln\left(e^{2i(dx+c)}-1\right)+e^{4i(dx+c)}dx-2ie^{2i(dx+c)}\ln\left(e^{2i(dx+c)}-1\right)+2e^{4i(dx+c)}c-2e^{2i(dx+c)}dx-2ie^{2i(dx+c)}+i\ln\left(e^{2i(dx+c)}-\right.}{a\left(e^{2i(dx+c)}-1\right)\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,d}$

input
```
int(1/(a-a*sec(d*x+c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d/(-a*tan(d*x+c)^2)^(1/2)/a*(4*ln(-cot(d*x+c)+csc(d*x+c))*tan(d*x+c)-4
*ln(2/(cos(d*x+c)+1))*tan(d*x+c)+cot(d*x+c)+sec(d*x+c)*csc(d*x+c))
```

3.225.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.40

$$\int \frac{1}{\left(a-a\sec^2(c+dx)\right)^{3/2}}\,dx =$$
$$-\frac{\left(2\left(\cos\left(dx+c\right)^3-\cos\left(dx+c\right)\right)\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)-\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos(dx+c)^2-a}{\cos(dx+c)^2}}}{2\left(a^2d\cos\left(dx+c\right)^2-a^2d\right)\sin\left(dx+c\right)}$$

input
```
integrate(1/(a-a*sec(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(2*(cos(d*x + c)^3 - cos(d*x + c))*log(1/2*sin(d*x + c)) - cos(d*x +
c))*sqrt((a*cos(d*x + c)^2 - a)/cos(d*x + c)^2)/((a^2*d*cos(d*x + c)^2 - a
^2*d)*sin(d*x + c))
```

3.225.6 Sympy [F]

$$\int \frac{1}{(a - a\sec^2(c + dx))^{3/2}}\,dx = \int \frac{1}{(-a\sec^2(c + dx) + a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a-a*sec(d*x+c)**2)**(3/2),x)
```

output
```
Integral((-a*sec(c + d*x)**2 + a)**(-3/2), x)
```

3.225.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.90

$$\int \frac{1}{(a - a\sec^2(c + dx))^{3/2}}\,dx = -\frac{\frac{\log\left(\tan(dx+c)^2+1\right)}{\sqrt{-a}a} - \frac{2\log(\tan(dx+c))}{\sqrt{-a}a} + \frac{\sqrt{-a}}{a^2\tan(dx+c)^2}}{2\,d}$$

input
```
integrate(1/(a-a*sec(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/2*(log(tan(d*x + c)^2 + 1)/(sqrt(-a)*a) - 2*log(tan(d*x + c))/(sqrt(-a)
*a) + sqrt(-a)/(a^2*tan(d*x + c)^2))/d
```

3.225.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.61

$$\int \frac{1}{(a - a\sec^2(c+dx))^{3/2}}\,dx = -\frac{\frac{\tan(\frac{1}{2}dx+\frac{1}{2}c)^2}{\sqrt{-a}a} - \frac{8\log\left(\tan(\frac{1}{2}dx+\frac{1}{2}c)^2+1\right)}{\sqrt{-a}a} + \frac{4\log\left(\tan(\frac{1}{2}dx+\frac{1}{2}c)^2\right)}{\sqrt{-a}a} - \frac{4\tan(\frac{1}{2}dx+\frac{1}{2}c)^2-1}{\sqrt{-a}a\tan(\frac{1}{2}dx+\frac{1}{2}c)^2}}{8d}$$

input
```
integrate(1/(a-a*sec(d*x+c)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/8*(tan(1/2*d*x + 1/2*c)^2/(sqrt(-a)*a) - 8*log(tan(1/2*d*x + 1/2*c)^2 +
 1)/(sqrt(-a)*a) + 4*log(tan(1/2*d*x + 1/2*c)^2)/(sqrt(-a)*a) - (4*tan(1/2
*d*x + 1/2*c)^2 - 1)/(sqrt(-a)*a*tan(1/2*d*x + 1/2*c)^2))/d
```

3.225.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a - a\sec^2(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a - \frac{a}{\cos(c+dx)^2}\right)^{3/2}}\,dx$$

input
```
int(1/(a - a/cos(c + d*x)^2)^(3/2),x)
```

output
```
int(1/(a - a/cos(c + d*x)^2)^(3/2), x)
```

3.226 $\int \frac{1}{(a-a\sec^2(c+dx))^{5/2}}\,dx$

3.226.1 Optimal result . . . 1682
3.226.2 Mathematica [A] (verified) . . . 1682
3.226.3 Rubi [A] (verified) . . . 1683
3.226.4 Maple [A] (verified) . . . 1685
3.226.5 Fricas [A] (verification not implemented) . . . 1686
3.226.6 Sympy [F] . . . 1686
3.226.7 Maxima [A] (verification not implemented) . . . 1687
3.226.8 Giac [A] (verification not implemented) . . . 1687
3.226.9 Mupad [F(-1)] . . . 1687

3.226.1 Optimal result

Integrand size = 17, antiderivative size = 100

$$\int \frac{1}{(a-a\sec^2(c+dx))^{5/2}}\,dx = \frac{\cot(c+dx)}{2a^2d\sqrt{-a\tan^2(c+dx)}} - \frac{\cot^3(c+dx)}{4a^2d\sqrt{-a\tan^2(c+dx)}} + \frac{\log(\sin(c+dx))\tan(c+dx)}{a^2d\sqrt{-a\tan^2(c+dx)}}$$

output
```
1/2*cot(d*x+c)/a^2/d/(-a*tan(d*x+c)^2)^(1/2)-1/4*cot(d*x+c)^3/a^2/d/(-a*ta
n(d*x+c)^2)^(1/2)+ln(sin(d*x+c))*tan(d*x+c)/a^2/d/(-a*tan(d*x+c)^2)^(1/2)
```

3.226.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.67

$$\int \frac{1}{(a-a\sec^2(c+dx))^{5/2}}\,dx = \frac{2\cot(c+dx) - \cot^3(c+dx) + 4(\log(\cos(c+dx)) + \log(\tan(c+dx)))\tan(c}{4a^2d\sqrt{-a\tan^2(c+dx)}}$$

input
```
Integrate[(a - a*Sec[c + d*x]^2)^(-5/2),x]
```

output
```
(2*Cot[c + d*x] - Cot[c + d*x]^3 + 4*(Log[Cos[c + d*x]] + Log[Tan[c + d*x]
])*Tan[c + d*x])/(4*a^2*d*Sqrt[-(a*Tan[c + d*x]^2)])
```

3.226.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.71, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.882$, Rules used = {3042, 4609, 3042, 4141, 3042, 25, 3954, 25, 3042, 25, 3954, 25, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a - a\sec^2(c+dx))^{5/2}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{(a - a\sec(c+dx)^2)^{5/2}}dx$$
$$\downarrow 4609$$
$$\int \frac{1}{(-a\tan^2(c+dx))^{5/2}}dx$$
$$\downarrow 3042$$
$$\int \frac{1}{(-a\tan(c+dx)^2)^{5/2}}dx$$
$$\downarrow 4141$$
$$\frac{\tan(c+dx)\int \cot^5(c+dx)dx}{a^2\sqrt{-a\tan^2(c+dx)}}$$
$$\downarrow 3042$$
$$\frac{\tan(c+dx)\int -\tan\left(c+dx+\frac{\pi}{2}\right)^5 dx}{a^2\sqrt{-a\tan^2(c+dx)}}$$
$$\downarrow 25$$
$$-\frac{\tan(c+dx)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)^5 dx}{a^2\sqrt{-a\tan^2(c+dx)}}$$
$$\downarrow 3954$$
$$-\frac{\tan(c+dx)\left(\frac{\cot^4(c+dx)}{4d} - \int -\cot^3(c+dx)dx\right)}{a^2\sqrt{-a\tan^2(c+dx)}}$$
$$\downarrow 25$$

$$-\frac{\tan(c+dx)\left(\int \cot^3(c+dx)dx+\frac{\cot^4(c+dx)}{4d}\right)}{a^2\sqrt{-a\tan^2(c+dx)}}$$

$$\downarrow 3042$$

$$-\frac{\tan(c+dx)\left(\int -\tan\left(c+dx+\frac{\pi}{2}\right)^3 dx+\frac{\cot^4(c+dx)}{4d}\right)}{a^2\sqrt{-a\tan^2(c+dx)}}$$

$$\downarrow 25$$

$$-\frac{\tan(c+dx)\left(\frac{\cot^4(c+dx)}{4d}-\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)^3 dx\right)}{a^2\sqrt{-a\tan^2(c+dx)}}$$

$$\downarrow 3954$$

$$-\frac{\tan(c+dx)\left(\int -\cot(c+dx)dx+\frac{\cot^4(c+dx)}{4d}-\frac{\cot^2(c+dx)}{2d}\right)}{a^2\sqrt{-a\tan^2(c+dx)}}$$

$$\downarrow 25$$

$$-\frac{\tan(c+dx)\left(-\int \cot(c+dx)dx+\frac{\cot^4(c+dx)}{4d}-\frac{\cot^2(c+dx)}{2d}\right)}{a^2\sqrt{-a\tan^2(c+dx)}}$$

$$\downarrow 3042$$

$$-\frac{\tan(c+dx)\left(-\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\cot^4(c+dx)}{4d}-\frac{\cot^2(c+dx)}{2d}\right)}{a^2\sqrt{-a\tan^2(c+dx)}}$$

$$\downarrow 25$$

$$-\frac{\tan(c+dx)\left(\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx+\frac{\cot^4(c+dx)}{4d}-\frac{\cot^2(c+dx)}{2d}\right)}{a^2\sqrt{-a\tan^2(c+dx)}}$$

$$\downarrow 3956$$

$$-\frac{\tan(c+dx)\left(\frac{\cot^4(c+dx)}{4d}-\frac{\cot^2(c+dx)}{2d}-\frac{\log(-\sin(c+dx))}{d}\right)}{a^2\sqrt{-a\tan^2(c+dx)}}$$

input
```
Int[(a - a*Sec[c + d*x]^2)^(-5/2),x]
```

output
```
-(((-1/2*Cot[c + d*x]^2/d + Cot[c + d*x]^4/(4*d) - Log[-Sin[c + d*x]]/d)*T
an[c + d*x])/(a^2*Sqrt[-(a*Tan[c + d*x]^2)]))
```

3.226.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4609
```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

3.226.4 Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.07

method	result
default	$-\frac{\left(-32\ln(-\cot(dx+c)+\csc(dx+c))\sin(dx+c)^4+32\ln\left(\frac{2}{\cos(dx+c)+1}\right)\sin(dx+c)^4+13\cos(dx+c)^4+6\cos(dx+c)^2-11\right)\sec(dx+c)\csc}{32d\sqrt{-a\tan(dx+c)^2}\,a^2}$
risch	$\frac{\left(e^{2i(dx+c)}-1\right)x}{a^2\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}}-\frac{2\left(e^{2i(dx+c)}-1\right)(dx+c)}{a^2\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,d}+\frac{4i\left(e^{6i(dx+c)}-e^{4i(dx+c)}+e^{2i(dx+c)}\right)}{a^2\left(e^{2i(dx+c)}-1\right)^3\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{a\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}}$

input
```
int(1/(a-a*sec(d*x+c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/32/d*(-32*ln(-cot(d*x+c)+csc(d*x+c))*sin(d*x+c)^4+32*ln(2/(cos(d*x+c)+1
))*sin(d*x+c)^4+13*cos(d*x+c)^4+6*cos(d*x+c)^2-11)/(-a*tan(d*x+c)^2)^(1/2)
/a^2*sec(d*x+c)*csc(d*x+c)^3
```

3.226.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.25

$$\int \frac{1}{(a - a\sec^2(c+dx))^{5/2}}\,dx = \frac{\left(4\cos(dx+c)^3 - 4\left(\cos(dx+c)^5 - 2\cos(dx+c)^3 + \cos(dx+c)\right)\log\left(\frac{1}{2}\right.\right.}{4\left(a^3 d\cos(dx+c)^4 - 2a^3 d\cos(dx+c)^2 + a^3\right.}$$

input
```
integrate(1/(a-a*sec(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/4*(4*cos(d*x + c)^3 - 4*(cos(d*x + c)^5 - 2*cos(d*x + c)^3 + cos(d*x + c
))*log(1/2*sin(d*x + c)) - 3*cos(d*x + c))*sqrt((a*cos(d*x + c)^2 - a)/cos
(d*x + c)^2)/((a^3*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^2 + a^3*d)*sin(
d*x + c))
```

3.226.6 Sympy [F]

$$\int \frac{1}{(a - a\sec^2(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(-a\sec^2(c+dx) + a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a-a*sec(d*x+c)**2)**(5/2),x)
```

output
```
Integral((-a*sec(c + d*x)**2 + a)**(-5/2), x)
```

3.226.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.79

$$\int \frac{1}{(a-a\sec^2(c+dx))^{5/2}}\,dx = -\frac{\frac{2\log\left(\tan(dx+c)^2+1\right)}{\sqrt{-a}a^2} - \frac{4\log(\tan(dx+c))}{\sqrt{-a}a^2} + \frac{2\sqrt{-a}\tan(dx+c)^2-\sqrt{-a}}{a^3\tan(dx+c)^4}}{4\,d}$$

input
```
integrate(1/(a-a*sec(d*x+c)^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/4*(2*log(tan(d*x + c)^2 + 1)/(sqrt(-a)*a^2) - 4*log(tan(d*x + c))/(sqrt
(-a)*a^2) + (2*sqrt(-a)*tan(d*x + c)^2 - sqrt(-a))/(a^3*tan(d*x + c)^4))/d
```

3.226.8 Giac [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.47

$$\int \frac{1}{(a-a\sec^2(c+dx))^{5/2}}\,dx = \frac{\frac{64\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)}{\sqrt{-a}a^2} - \frac{32\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)}{\sqrt{-a}a^2} - \frac{\sqrt{-a}a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-12\sqrt{-a}a^2\tan(}{a^5}}{64\,d}$$

input
```
integrate(1/(a-a*sec(d*x+c)^2)^(5/2),x, algorithm="giac")
```

output
```
1/64*(64*log(tan(1/2*d*x + 1/2*c)^2 + 1)/(sqrt(-a)*a^2) - 32*log(tan(1/2*d
*x + 1/2*c)^2)/(sqrt(-a)*a^2) - (sqrt(-a)*a^2*tan(1/2*d*x + 1/2*c)^4 - 12*
sqrt(-a)*a^2*tan(1/2*d*x + 1/2*c)^2)/a^5 + (48*tan(1/2*d*x + 1/2*c)^4 - 12
*tan(1/2*d*x + 1/2*c)^2 + 1)/(sqrt(-a)*a^2*tan(1/2*d*x + 1/2*c)^4))/d
```

3.226.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-a\sec^2(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(a-\frac{a}{\cos(c+dx)^2}\right)^{5/2}}\,dx$$

input
```
int(1/(a - a/cos(c + d*x)^2)^(5/2),x)
```

output
```
int(1/(a - a/cos(c + d*x)^2)^(5/2), x)
```

3.227 $\int \frac{1}{(a-a\sec^2(c+dx))^{7/2}}\,dx$

3.227.1 Optimal result . . . 1688
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3.227.1 Optimal result

Integrand size = 17, antiderivative size = 133

$$\int \frac{1}{(a-a\sec^2(c+dx))^{7/2}}\,dx = \frac{\cot(c+dx)}{2a^3d\sqrt{-a\tan^2(c+dx)}} - \frac{\cot^3(c+dx)}{4a^3d\sqrt{-a\tan^2(c+dx)}} + \frac{\cot^5(c+dx)}{6a^3d\sqrt{-a\tan^2(c+dx)}} + \frac{\log(\sin(c+dx))\tan(c+dx)}{a^3d\sqrt{-a\tan^2(c+dx)}}$$

output

```
1/2*cot(d*x+c)/a^3/d/(-a*tan(d*x+c)^2)^(1/2)-1/4*cot(d*x+c)^3/a^3/d/(-a*ta
n(d*x+c)^2)^(1/2)+1/6*cot(d*x+c)^5/a^3/d/(-a*tan(d*x+c)^2)^(1/2)+ln(sin(d*
x+c))*tan(d*x+c)/a^3/d/(-a*tan(d*x+c)^2)^(1/2)
```

3.227.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.59

$$\int \frac{1}{(a-a\sec^2(c+dx))^{7/2}}\,dx = -\frac{(6\cot^2(c+dx) - 3\cot^4(c+dx) + 2\cot^6(c+dx) + 12\log(\cos(c+dx)) + 12\log(\tan(c+dx)))\tan^7(c+d$$
$$12d\left(-a\tan^2(c+dx)\right)^{7/2}$$

input

```
Integrate[(a - a*Sec[c + d*x]^2)^(-7/2),x]
```

output
```
-1/12*((6*Cot[c + d*x]^2 - 3*Cot[c + d*x]^4 + 2*Cot[c + d*x]^6 + 12*Log[Co
s[c + d*x]] + 12*Log[Tan[c + d*x]])*Tan[c + d*x]^7)/(d*(-(a*Tan[c + d*x]^2
))^(7/2))
```

3.227.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.63, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 1.118$, Rules used = {3042, 4609, 3042, 4141, 3042, 25, 3954, 25, 3042, 25, 3954, 25, 3042, 25, 3954, 25, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a - a\sec^2(c+dx))^{7/2}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{(a - a\sec(c+dx)^2)^{7/2}}\,dx$$
$$\downarrow 4609$$
$$\int \frac{1}{(-a\tan^2(c+dx))^{7/2}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{(-a\tan(c+dx)^2)^{7/2}}\,dx$$
$$\downarrow 4141$$
$$-\frac{\tan(c+dx)\int \cot^7(c+dx)dx}{a^3\sqrt{-a\tan^2(c+dx)}}$$
$$\downarrow 3042$$
$$-\frac{\tan(c+dx)\int -\tan\left(c+dx+\frac{\pi}{2}\right)^7 dx}{a^3\sqrt{-a\tan^2(c+dx)}}$$
$$\downarrow 25$$
$$\frac{\tan(c+dx)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)^7 dx}{a^3\sqrt{-a\tan^2(c+dx)}}$$

$$\downarrow 3954$$

$$\frac{\tan(c+dx)\left(\frac{\cot^6(c+dx)}{6d}-\int -\cot^5(c+dx)dx\right)}{a^3\sqrt{-a\tan^2(c+dx)}}$$

$$\downarrow 25$$

$$\frac{\tan(c+dx)\left(\int \cot^5(c+dx)dx+\frac{\cot^6(c+dx)}{6d}\right)}{a^3\sqrt{-a\tan^2(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{\tan(c+dx)\left(\int -\tan\left(c+dx+\frac{\pi}{2}\right)^5 dx+\frac{\cot^6(c+dx)}{6d}\right)}{a^3\sqrt{-a\tan^2(c+dx)}}$$

$$\downarrow 25$$

$$\frac{\tan(c+dx)\left(\frac{\cot^6(c+dx)}{6d}-\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)^5 dx\right)}{a^3\sqrt{-a\tan^2(c+dx)}}$$

$$\downarrow 3954$$

$$\frac{\tan(c+dx)\left(\int -\cot^3(c+dx)dx+\frac{\cot^6(c+dx)}{6d}-\frac{\cot^4(c+dx)}{4d}\right)}{a^3\sqrt{-a\tan^2(c+dx)}}$$

$$\downarrow 25$$

$$\frac{\tan(c+dx)\left(-\int \cot^3(c+dx)dx+\frac{\cot^6(c+dx)}{6d}-\frac{\cot^4(c+dx)}{4d}\right)}{a^3\sqrt{-a\tan^2(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{\tan(c+dx)\left(-\int -\tan\left(c+dx+\frac{\pi}{2}\right)^3 dx+\frac{\cot^6(c+dx)}{6d}-\frac{\cot^4(c+dx)}{4d}\right)}{a^3\sqrt{-a\tan^2(c+dx)}}$$

$$\downarrow 25$$

$$\frac{\tan(c+dx)\left(\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)^3 dx+\frac{\cot^6(c+dx)}{6d}-\frac{\cot^4(c+dx)}{4d}\right)}{a^3\sqrt{-a\tan^2(c+dx)}}$$

$$\downarrow 3954$$

$$\frac{\tan(c+dx)\left(-\int -\cot(c+dx)dx+\frac{\cot^6(c+dx)}{6d}-\frac{\cot^4(c+dx)}{4d}+\frac{\cot^2(c+dx)}{2d}\right)}{a^3\sqrt{-a\tan^2(c+dx)}}$$

$$\downarrow 25$$

$$\frac{\tan(c+dx)\left(\int \cot(c+dx)dx+\frac{\cot^6(c+dx)}{6d}-\frac{\cot^4(c+dx)}{4d}+\frac{\cot^2(c+dx)}{2d}\right)}{a^3\sqrt{-a\tan^2(c+dx)}}$$

$$\downarrow 3042$$

$$\frac{\tan(c+dx)\left(\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\cot^6(c+dx)}{6d}-\frac{\cot^4(c+dx)}{4d}+\frac{\cot^2(c+dx)}{2d}\right)}{a^3\sqrt{-a\tan^2(c+dx)}}$$

$$\downarrow 25$$

$$\frac{\tan(c+dx)\left(-\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx+\frac{\cot^6(c+dx)}{6d}-\frac{\cot^4(c+dx)}{4d}+\frac{\cot^2(c+dx)}{2d}\right)}{a^3\sqrt{-a\tan^2(c+dx)}}$$

$$\downarrow 3956$$

$$\frac{\tan(c+dx)\left(\frac{\cot^6(c+dx)}{6d}-\frac{\cot^4(c+dx)}{4d}+\frac{\cot^2(c+dx)}{2d}+\frac{\log(-\sin(c+dx))}{d}\right)}{a^3\sqrt{-a\tan^2(c+dx)}}$$

input
```
Int[(a - a*Sec[c + d*x]^2)^(-7/2),x]
```

output
```
((Cot[c + d*x]^2/(2*d) - Cot[c + d*x]^4/(4*d) + Cot[c + d*x]^6/(6*d) + Log
[-Sin[c + d*x]]/d)*Tan[c + d*x])/(a^3*Sqrt[-(a*Tan[c + d*x]^2)])
```

3.227.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4609
```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

3.227.4 Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.88

method	result
default	$-\frac{\left(48\ln\left(\frac{2}{\cos(dx+c)+1}\right)\sin(dx+c)^6-48\ln(-\cot(dx+c)+\csc(dx+c))\sin(dx+c)^6-25\cos(dx+c)^6+3\cos(dx+c)^4+33\cos(dx+c)^2-19\right.}{48d\sqrt{-a\tan(dx+c)^2}\,a^3}$
risch	$\frac{(e^{2i(dx+c)}-1)x}{a^3(e^{2i(dx+c)}+1)\sqrt{\frac{a(e^{2i(dx+c)}-1)^2}{(e^{2i(dx+c)}+1)^2}}}-\frac{2(e^{2i(dx+c)}-1)(dx+c)}{a^3(e^{2i(dx+c)}+1)\sqrt{\frac{a(e^{2i(dx+c)}-1)^2}{(e^{2i(dx+c)}+1)^2}}\,d}+\frac{2i(9e^{10i(dx+c)}-18e^{8i(dx+c)}+34e^{6i(dx+c)}-18e$

input
```
int(1/(a-a*sec(d*x+c)^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/48/d*(48*ln(2/(cos(d*x+c)+1))*sin(d*x+c)^6-48*ln(-cot(d*x+c)+csc(d*x+c)
)*sin(d*x+c)^6-25*cos(d*x+c)^6+3*cos(d*x+c)^4+33*cos(d*x+c)^2-19)/(-a*tan(
d*x+c)^2)^(1/2)/a^3*sec(d*x+c)*csc(d*x+c)^5
```

3.227.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.22

$$\int \frac{1}{(a-a\sec^2(c+dx))^{7/2}}\,dx = \frac{(18\cos(dx+c)^5-27\cos(dx+c)^3-12(\cos(dx+c)^7-3\cos(dx+c)^5+}{12\left(a^4d\cos(dx+c)^6-3a^4d\cos(dx\right.}$$

input
```
integrate(1/(a-a*sec(d*x+c)^2)^(7/2),x, algorithm="fricas")
```

output
```
1/12*(18*cos(d*x + c)^5 - 27*cos(d*x + c)^3 - 12*(cos(d*x + c)^7 - 3*cos(d
*x + c)^5 + 3*cos(d*x + c)^3 - cos(d*x + c))*log(1/2*sin(d*x + c)) + 11*co
s(d*x + c))*sqrt((a*cos(d*x + c)^2 - a)/cos(d*x + c)^2)/((a^4*d*cos(d*x +
c)^6 - 3*a^4*d*cos(d*x + c)^4 + 3*a^4*d*cos(d*x + c)^2 - a^4*d)*sin(d*x +
c))
```

3.227.6 Sympy [F]

$$\int \frac{1}{(a - a\sec^2(c+dx))^{7/2}}\,dx = \int \frac{1}{(-a\sec^2(c+dx)+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a-a*sec(d*x+c)**2)**(7/2),x)
```

output
```
Integral((-a*sec(c + d*x)**2 + a)**(-7/2), x)
```

3.227.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.71

$$\int \frac{1}{(a - a\sec^2(c+dx))^{7/2}}\,dx = -\frac{\frac{6\log\left(\tan(dx+c)^2+1\right)}{\sqrt{-a}a^3} - \frac{12\log(\tan(dx+c))}{\sqrt{-a}a^3} + \frac{6\sqrt{-a}\tan(dx+c)^4 - 3\sqrt{-a}\tan(dx+c)^2 + 2\sqrt{-a}}{a^4\tan(dx+c)^6}}{12\,d}$$

input
```
integrate(1/(a-a*sec(d*x+c)^2)^(7/2),x, algorithm="maxima")
```

output
```
-1/12*(6*log(tan(d*x + c)^2 + 1)/(sqrt(-a)*a^3) - 12*log(tan(d*x + c))/(sq
rt(-a)*a^3) + (6*sqrt(-a)*tan(d*x + c)^4 - 3*sqrt(-a)*tan(d*x + c)^2 + 2*s
qrt(-a))/(a^4*tan(d*x + c)^6))/d
```

3.227.8 Giac [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.29

$$\int \frac{1}{(a-a\sec^2(c+dx))^{7/2}}\,dx = \frac{\frac{384\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)}{\sqrt{-a}a^3} - \frac{192\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)}{\sqrt{-a}a^3} + \frac{352\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-87\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+12\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1}{\sqrt{-a}a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6}}{384\,d}$$

input
```
integrate(1/(a-a*sec(d*x+c)^2)^(7/2),x, algorithm="giac")
```

output
```
1/384*(384*log(tan(1/2*d*x + 1/2*c)^2 + 1)/(sqrt(-a)*a^3) - 192*log(tan(1/
2*d*x + 1/2*c)^2)/(sqrt(-a)*a^3) + (352*tan(1/2*d*x + 1/2*c)^6 - 87*tan(1/
2*d*x + 1/2*c)^4 + 12*tan(1/2*d*x + 1/2*c)^2 - 1)/(sqrt(-a)*a^3*tan(1/2*d*
x + 1/2*c)^6) - (a^7*tan(1/2*d*x + 1/2*c)^6 - 12*a^7*tan(1/2*d*x + 1/2*c)^
4 + 87*a^7*tan(1/2*d*x + 1/2*c)^2)/(sqrt(-a)*a^10))/d
```

3.227.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-a\sec^2(c+dx))^{7/2}}\,dx = \int \frac{1}{\left(a-\frac{a}{\cos(c+dx)^2}\right)^{7/2}}\,dx$$

input
```
int(1/(a - a/cos(c + d*x)^2)^(7/2),x)
```

output
```
int(1/(a - a/cos(c + d*x)^2)^(7/2), x)
```

3.228 $\int \sec^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

3.228.1 Optimal result . . . 1695
3.228.2 Mathematica [F] . . . 1696
3.228.3 Rubi [A] (verified) . . . 1696
3.228.4 Maple [C] (warning: unable to verify) . . . 1702
3.228.5 Fricas [C] (verification not implemented) . . . 1702
3.228.6 Sympy [F] . . . 1703
3.228.7 Maxima [F] . . . 1704
3.228.8 Giac [F] . . . 1704
3.228.9 Mupad [F(-1)] . . . 1704

3.228.1 Optimal result

Integrand size = 25, antiderivative size = 372

$$
\begin{aligned}
&\int \sec^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx \\
&= -\frac{(2a^2-3ab-8b^2)\sin(e+fx)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}{15b^2f} \\
&\quad + \frac{(2a^2-3ab-8b^2)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}{15b^2f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} \\
&\quad - \frac{(a-8b)(a+b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+f}}{15bf\,(a+b-a\sin^2(e+fx))} \\
&\quad + \frac{(a+4b)\sec(e+fx)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\tan(e+fx)}{15bf} \\
&\quad + \frac{\sec^3(e+fx)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\tan(e+fx)}{5f}
\end{aligned}
$$

output

```
-1/15*(2*a^2-3*a*b-8*b^2)*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(
1/2)/b^2/f+1/15*(2*a^2-3*a*b-8*b^2)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*
(cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/b^2/f/(1-a*
sin(f*x+e)^2/(a+b))^(1/2)-1/15*(a-8*b)*(a+b)*EllipticF(sin(f*x+e),(a/(a+b)
)^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*(1
-a*sin(f*x+e)^2/(a+b))^(1/2)/b/f/(a+b-a*sin(f*x+e)^2)+1/15*(a+4*b)*sec(f*x
+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*tan(f*x+e)/b/f+1/5*sec(f*x+e
)^3*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*tan(f*x+e)/f
```

3.228.2 Mathematica [F]

$$\int \sec^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sec^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

input
```
Integrate[Sec[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
Integrate[Sec[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2], x]
```

3.228.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.05, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3042, 4636, 2057, 2058, 314, 25, 402, 25, 402, 25, 27, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \sec(e+fx)^5\sqrt{a+b\sec(e+fx)^2}dx$$

$$\downarrow 4636$$

$$\frac{\int \frac{\sqrt{a+\frac{b}{1-\sin^2(e+fx)}}}{\left(1-\sin^2(e+fx)\right)^3}d\sin(e+fx)}{f}$$

$$\downarrow 2057$$

$$\frac{\int \frac{\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}{\left(1-\sin^2(e+fx)\right)^3}d\sin(e+fx)}{f}$$

$$\downarrow 2058$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\int \frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\left(1-\sin^2(e+fx)\right)^{7/2}}d\sin(e+fx)}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\downarrow 314$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{5(1-\sin^2(e+fx))^{5/2}}-\frac{1}{5}\int-\frac{4(a+b)-3a\sin^2(e+fx)}{(1-\sin^2(e+fx))^{5/2}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\downarrow 25$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\int\frac{4(a+b)-3a\sin^2(e+fx)}{(1-\sin^2(e+fx))^{5/2}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)+\frac{\sin(e+fx)\sqrt{-a\sin^2(e+f}}{5(1-\sin^2(e+fx))^{5}}\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\downarrow 402$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\left(\frac{\int-\frac{a(a+4b)\sin^2(e+fx)+(a-8b)(a+b)}{\left(1-\sin^2(e+fx)\right)^{3/2}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{3b}+\frac{(a+4b)\sin(e+fx)\sqrt{-a\sin^2(e-}}{3b(1-\sin^2(e+fx))^{3/}}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\downarrow 25$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\left(\frac{(a+4b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3b(1-\sin^2(e+fx))^{3/2}}-\frac{\int\frac{a(a+4b)\sin^2(e+fx)+(a-8b)(a+b)}{\left(1-\sin^2(e+fx)\right)^{3/2}\sqrt{-a\sin^2(e+fx)+a+b}}d\si}{3b}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\downarrow 402$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\left(\frac{(a+4b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3b(1-\sin^2(e+fx))^{3/2}}-\frac{\int-\frac{a\left(2(a-2b)(a+b)-\left(2a^2-3ba-8b^2\right)\sin^2(e+fx\right.}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}}{b}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\downarrow 25$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\left(\frac{(a+4b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3b(1-\sin^2(e+fx))^{3/2}}-\frac{\left(2a^2-3ab-8b^2\right)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\downarrow 27$$

3.228. $\int \sec^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\left(\frac{(a+4b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3b\left(1-\sin^2(e+fx)\right)^{3/2}}-\frac{\left(2a^2-3ab-8b^2\right)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\downarrow 399$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\left(\frac{(a+4b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3b\left(1-\sin^2(e+fx)\right)^{3/2}}-\frac{\left(2a^2-3ab-8b^2\right)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}\right.\right.}{f\sqrt{-a}}$$

$$\downarrow 323$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\left(\frac{(a+4b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3b\left(1-\sin^2(e+fx)\right)^{3/2}}-\frac{\left(2a^2-3ab-8b^2\right)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}\right.\right.}{f}$$

$$\downarrow 321$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\left(\frac{(a+4b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3b\left(1-\sin^2(e+fx)\right)^{3/2}}-\frac{\left(2a^2-3ab-8b^2\right)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}\right.\right.}{f\sqrt{-a}}$$

$$\downarrow 330$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\left(\frac{(a+4b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3b\left(1-\sin^2(e+fx)\right)^{3/2}}-\frac{\left(2a^2-3ab-8b^2\right)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}\right.\right.}{f}$$

$$\downarrow 327$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\left(\frac{(a+4b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3b\left(1-\sin^2(e+fx)\right)^{3/2}}-\frac{\left(2a^2-3ab-8b^2\right)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}\right.\right.}{f\sqrt{}}$$

input

```
Int[Sec[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output

```
(Sqrt[1 - Sin[e + f*x]^2]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(1 - Sin[e + f*x
]^2)]*((Sin[e + f*x]*Sqrt[a + b - a*Sin[e + f*x]^2])/(5*(1 - Sin[e + f*x]^
2)^(5/2)) + (((a + 4*b)*Sin[e + f*x]*Sqrt[a + b - a*Sin[e + f*x]^2])/(3*b*
(1 - Sin[e + f*x]^2)^(3/2)) - (((2*a^2 - 3*a*b - 8*b^2)*Sin[e + f*x]*Sqrt[
a + b - a*Sin[e + f*x]^2])/(b*Sqrt[1 - Sin[e + f*x]^2]) - (a*(((2*a^2 - 3*
a*b - 8*b^2)*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b - a*Sin
[e + f*x]^2])/(a*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - ((a - 8*b)*b*(a +
 b)*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)
/(a + b)])/(a*Sqrt[a + b - a*Sin[e + f*x]^2])))/b)/(3*b))/5))/(f*Sqrt[a +
b - a*Sin[e + f*x]^2])
```

3.228.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 314
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] + Simp[1/(2*a*
(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*p + 3) + d
*(2*(p + q + 1) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, 2, p, q,
 x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

3.228.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 14.83 (sec) , antiderivative size = 8814, normalized size of antiderivative = 23.69

method	result	size
default	Expression too large to display	8814

input
```
int(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.228.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 887, normalized size of antiderivative = 2.38

$$\int \sec^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$= \frac{\left(2\left(-2i\,a^3+3i\,a^2b+8i\,ab^2\right)\sqrt{a}\sqrt{\frac{ab+b^2}{a^2}}\cos\left(fx+e\right)^4-\left(-2i\,a^3-i\,a^2b+14i\,ab^2+16i\,b^3\right)\sqrt{a}\cos\left(fx+e\right)\right.}{}$$

input
```
integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/30*((2*(-2*I*a^3 + 3*I*a^2*b + 8*I*a*b^2)*sqrt(a)*sqrt((a*b + b^2)/a^2)*
cos(f*x + e)^4 - (-2*I*a^3 - I*a^2*b + 14*I*a*b^2 + 16*I*b^3)*sqrt(a)*cos(
f*x + e)^4)*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_e(arcsi
n(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) + I*sin(f*x
+ e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2)
 + (2*(2*I*a^3 - 3*I*a^2*b - 8*I*a*b^2)*sqrt(a)*sqrt((a*b + b^2)/a^2)*cos(
f*x + e)^4 - (2*I*a^3 + I*a^2*b - 14*I*a*b^2 - 16*I*b^3)*sqrt(a)*cos(f*x +
 e)^4)*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_e(arcsin(sqr
t((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) - I*sin(f*x + e))
), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) - 4*
((I*a^2*b + 4*I*a*b^2)*sqrt(a)*sqrt((a*b + b^2)/a^2)*cos(f*x + e)^4 + (I*a
^3 + I*a^2*b - 4*I*a*b^2 - 4*I*b^3)*sqrt(a)*cos(f*x + e)^4)*sqrt((2*a*sqrt
((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b + b^
2)/a^2) - a - 2*b)/a)*(cos(f*x + e) + I*sin(f*x + e))), (a^2 + 8*a*b + 8*b
^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) - 4*((-I*a^2*b - 4*I*a*b^
2)*sqrt(a)*sqrt((a*b + b^2)/a^2)*cos(f*x + e)^4 + (-I*a^3 - I*a^2*b + 4*I*
a*b^2 + 4*I*b^3)*sqrt(a)*cos(f*x + e)^4)*sqrt((2*a*sqrt((a*b + b^2)/a^2) -
 a - 2*b)/a)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/
a)*(cos(f*x + e) - I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b
)*sqrt((a*b + b^2)/a^2))/a^2) - 2*((2*a^3 - 3*a^2*b - 8*a*b^2)*cos(f*x ...
```

3.228.6 Sympy [F]

$$\int \sec^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{a+b\sec^2(e+fx)}\sec^5(e+fx)\,dx$$

input
```
integrate(sec(f*x+e)**5*(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*sec(e + f*x)**5, x)
```

3.228.7 Maxima [F]

$$\int \sec^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\sec(fx+e)^5\,dx$$

input
```
integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)^5, x)
```

3.228.8 Giac [F]

$$\int \sec^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\sec(fx+e)^5\,dx$$

input
```
integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)^5, x)
```

3.228.9 Mupad [F(-1)]

Timed out.

$$\int \sec^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(e+fx)^2}}}{\cos(e+fx)^5}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x)^5,x)
```

output
```
int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x)^5, x)
```

3.229 $\int \sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

3.229.1 Optimal result 1705
3.229.2 Mathematica [F] 1706
3.229.3 Rubi [A] (verified) 1706
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3.229.9 Mupad [F(-1)] 1713

3.229.1 Optimal result

Integrand size = 25, antiderivative size = 288

$$\int \sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{(a+2b)\sin(e+fx)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}{3bf}$$
$$- \frac{(a+2b)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}{3bf\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}$$
$$+ \frac{2(a+b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\sqrt{1}}{3f\,(a+b-a\sin^2(e+fx))}$$
$$+ \frac{\sec(e+fx)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\tan(e+fx)}{3f}$$

output

```
1/3*(a+2*b)*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/b/f-1/3*(
a+2*b)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x
+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/b/f/(1-a*sin(f*x+e)^2/(a+b))^(1/2)+2/3*(
a+b)*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e
)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/f/(a+b-a*si
n(f*x+e)^2)+1/3*sec(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*tan(f
*x+e)/f
```

3.229.2 Mathematica [F]

$$\int \sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

input
```
Integrate[Sec[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
Integrate[Sec[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2], x]
```

3.229.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.06, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 4636, 2057, 2058, 314, 25, 402, 25, 27, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \sec(e+fx)^3\sqrt{a+b\sec(e+fx)^2}dx$$

$$\downarrow 4636$$

$$\frac{\int \frac{\sqrt{a+\frac{b}{1-\sin^2(e+fx)}}}{\left(1-\sin^2(e+fx)\right)^2}d\sin(e+fx)}{f}$$

$$\downarrow 2057$$

$$\frac{\int \frac{\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}{\left(1-\sin^2(e+fx)\right)^2}d\sin(e+fx)}{f}$$

$$\downarrow 2058$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\int \frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\left(1-\sin^2(e+fx)\right)^{5/2}}d\sin(e+fx)}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\downarrow 314$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3(1-\sin^2(e+fx))^{3/2}}-\frac{1}{3}\int-\frac{2(a+b)-a\sin^2(e+fx)}{(1-\sin^2(e+fx))^{3/2}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\downarrow 25$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\int\frac{2(a+b)-a\sin^2(e+fx)}{(1-\sin^2(e+fx))^{3/2}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)+\frac{\sin(e+fx)\sqrt{-a\sin^2(e+f}}{3(1-\sin^2(e+fx))^{3}}\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\downarrow 402$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\left(\frac{\int-\frac{a\left(-\left((a+2b)\sin^2(e+fx)\right)+a+b\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{b}+\frac{(a+2b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\downarrow 25$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\left(\frac{(a+2b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}-\frac{\int\frac{a\left(-\left((a+2b)\sin^2(e+fx)\right)+a+b\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e\right.}{b}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\downarrow 27$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\left(\frac{(a+2b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}-\frac{a\int\frac{-\left((a+2b)\sin^2(e+fx)\right)+a+b}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(\right.}{b}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\downarrow 399$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\left(\frac{(a+2b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}-\frac{a\left(\frac{(a+2b)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\right.\right.\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\downarrow 323$$

$$\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\left(\frac{(a+2b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}-\frac{a\left(\frac{(a+2b)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}\right.\right.\right.$$

$$f\sqrt{-a\sin^2(e+fx)+a+$$

$$\downarrow 321$$

$$\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\left(\frac{(a+2b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}-\frac{a\left(\frac{(a+2b)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}\right.\right.\right.$$

$$f\sqrt{-a\sin^2(e+fx)+a+b}$$

$$\downarrow 330$$

$$\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\left(\frac{(a+2b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}-\frac{a\left(\frac{(a+2b)\sqrt{-a\sin^2(e+fx)+a+b}\int\frac{\sqrt{1-\frac{a\sin^2(e}{a+b}}}{\sqrt{1-\sin^2(e+}}\right.\right.\right.\right.$$

$$f\sqrt{-a\sin^2(e+fx)+a}$$

$$\downarrow 327$$

$$\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\left(\frac{(a+2b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}-\frac{a\left(\frac{(a+2b)\sqrt{-a\sin^2(e+fx)+a+b}E\left(\arcsin(\sin(e+\right.\right.\right.\right.$$

$$f\sqrt{-a\sin^2(e+fx)+a+$$

input
```
Int[Sec[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2],x]
```

3.229. $\int \sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

output
```
(Sqrt[1 - Sin[e + f*x]^2]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(1 - Sin[e + f*x
]^2)]*((Sin[e + f*x]*Sqrt[a + b - a*Sin[e + f*x]^2])/(3*(1 - Sin[e + f*x]^
2)^(3/2)) + (((a + 2*b)*Sin[e + f*x]*Sqrt[a + b - a*Sin[e + f*x]^2])/(b*Sq
rt[1 - Sin[e + f*x]^2]) - (a*(((a + 2*b)*EllipticE[ArcSin[Sin[e + f*x]], a
/(a + b)]*Sqrt[a + b - a*Sin[e + f*x]^2])/(a*Sqrt[1 - (a*Sin[e + f*x]^2)/(
a + b)]) - (2*b*(a + b)*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1
- (a*Sin[e + f*x]^2)/(a + b)])/(a*Sqrt[a + b - a*Sin[e + f*x]^2])))/b)/3))
/(f*Sqrt[a + b - a*Sin[e + f*x]^2])
```

3.229.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 314
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] + Simp[1/(2*a*
(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*p + 3) + d
*(2*(p + q + 1) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, 2, p, q,
 x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

3.229.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.80 (sec) , antiderivative size = 6342, normalized size of antiderivative = 22.02

method	result	size
default	Expression too large to display	6342

input
```
int(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.229.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 782, normalized size of antiderivative = 2.72

$$\int \sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$= \frac{\left(2\,(i\,a^2+2i\,ab)\sqrt{a}\sqrt{\frac{ab+b^2}{a^2}}\cos(fx+e)^2-(i\,a^2+4i\,ab+4i\,b^2)\sqrt{a}\cos(fx+e)^2\right)\sqrt{\frac{2\,a\sqrt{\frac{ab+b^2}{a^2}}-a-2\,b}{a}}E(\text{arcs}}{}$$

input
```
integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*((2*(I*a^2 + 2*I*a*b)*sqrt(a)*sqrt((a*b + b^2)/a^2)*cos(f*x + e)^2 - (
I*a^2 + 4*I*a*b + 4*I*b^2)*sqrt(a)*cos(f*x + e)^2)*sqrt((2*a*sqrt((a*b + b
^2)/a^2) - a - 2*b)/a)*elliptic_e(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) -
 a - 2*b)/a)*(cos(f*x + e) + I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a
^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + (2*(-I*a^2 - 2*I*a*b)*sqrt(a)*sq
rt((a*b + b^2)/a^2)*cos(f*x + e)^2 - (-I*a^2 - 4*I*a*b - 4*I*b^2)*sqrt(a)*
cos(f*x + e)^2)*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_e(a
rcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) - I*sin(
f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/
a^2) - 2*(2*I*a^(3/2)*b*sqrt((a*b + b^2)/a^2)*cos(f*x + e)^2 + (-I*a^2 - 3
*I*a*b - 2*I*b^2)*sqrt(a)*cos(f*x + e)^2)*sqrt((2*a*sqrt((a*b + b^2)/a^2)
 - a - 2*b)/a)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)
/a)*(cos(f*x + e) + I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*
b)*sqrt((a*b + b^2)/a^2))/a^2) - 2*(-2*I*a^(3/2)*b*sqrt((a*b + b^2)/a^2)*c
os(f*x + e)^2 + (I*a^2 + 3*I*a*b + 2*I*b^2)*sqrt(a)*cos(f*x + e)^2)*sqrt((
2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_f(arcsin(sqrt((2*a*sqrt((
a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) - I*sin(f*x + e))), (a^2 + 8*a
*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + 2*((a^2 + 2*a*b
)*cos(f*x + e)^2 + a*b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*
x + e))/(a*b*f*cos(f*x + e)^2)
```

3.229.6 Sympy [F]

$$\int \sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{a+b\sec^2(e+fx)}\sec^3(e+fx)\,dx$$

input
```
integrate(sec(f*x+e)**3*(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*sec(e + f*x)**3, x)
```

3.229.7 Maxima [F]

$$\int \sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\sec(fx+e)^3\,dx$$

input
```
integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)^3, x)
```

3.229.8 Giac [F]

$$\int \sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\sec(fx+e)^3\,dx$$

input
```
integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)^3, x)
```

3.229.9 Mupad [F(-1)]

Timed out.

$$\int \sec^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(e+fx)^2}}}{\cos(e+fx)^3}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x)^3,x)
```

output
```
int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x)^3, x)
```

3.230 $\int \sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

3.230.1 Optimal result 1714
3.230.2 Mathematica [F] 1715
3.230.3 Rubi [A] (verified) 1715
3.230.4 Maple [C] (warning: unable to verify) 1719
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3.230.8 Giac [F] 1721
3.230.9 Mupad [F(-1)] 1721

3.230.1 Optimal result

Integrand size = 23, antiderivative size = 218

$$\int \sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\sin(e+fx)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}{f}$$
$$-\frac{\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}{f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}$$
$$+\frac{(a+b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\sqrt{1-}}{f\,(a+b-a\sin^2(e+fx))}$$

output

```
sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/f-EllipticE(sin(f*x+e
),(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2)
)^(1/2)/f/(1-a*sin(f*x+e)^2/(a+b))^(1/2)+(a+b)*EllipticF(sin(f*x+e),(a/(a+
b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*
(1-a*sin(f*x+e)^2/(a+b))^(1/2)/f/(a+b-a*sin(f*x+e)^2)
```

3.230.2 Mathematica [F]

$$\int \sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

input
```
Integrate[Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
Integrate[Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2], x]
```

3.230.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.09, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 4636, 2057, 2058, 314, 25, 27, 389, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \sec(e+fx)\sqrt{a+b\sec(e+fx)^2}dx$$

$$\downarrow 4636$$

$$\frac{\int \frac{\sqrt{a+\frac{b}{1-\sin^2(e+fx)}}}{1-\sin^2(e+fx)}d\sin(e+fx)}{f}$$

$$\downarrow 2057$$

$$\frac{\int \frac{\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}{1-\sin^2(e+fx)}d\sin(e+fx)}{f}$$

$$\downarrow 2058$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\int \frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\left(1-\sin^2(e+fx)\right)^{3/2}}d\sin(e+fx)}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\downarrow 314$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}-\int-\frac{a\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\downarrow 25$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\int\frac{a\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)+\frac{\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a-}}{\sqrt{1-\sin^2(e+fx)}}\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\downarrow 27$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(a\int\frac{\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)+\frac{\sin(e+fx)\sqrt{-a\sin^2(e+fx)+}}{\sqrt{1-\sin^2(e+fx)}}\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\downarrow 389$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(a\left(\frac{(a+b)\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{a}-\frac{\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e-\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\downarrow 323$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(a\left(\frac{(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}d\sin(e+fx)}{a\sqrt{-a\sin^2(e+fx)+a+b}}-\frac{\int\frac{\sqrt{-a\sin^2(e+f}}{\sqrt{1-\sin^2(e-}}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\downarrow 321$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(a\left(\frac{(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)}{a\sqrt{-a\sin^2(e+fx)+a+b}}-\frac{\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\downarrow 330$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(a\left(\frac{(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)}{a\sqrt{-a\sin^2(e+fx)+a+b}}-\frac{\sqrt{-a\sin^2(e+fx)+a+b}\int \frac{\sqrt{}}{a\sqrt{1-\frac{a}{}}}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\downarrow 327$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(a\left(\frac{(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)}{a\sqrt{-a\sin^2(e+fx)+a+b}}-\frac{\sqrt{-a\sin^2(e+fx)+a+b}E\left(\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

input
```
Int[Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Sqrt[1 - Sin[e + f*x]^2]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(1 - Sin[e + f*x
]^2)]*((Sin[e + f*x]*Sqrt[a + b - a*Sin[e + f*x]^2])/Sqrt[1 - Sin[e + f*x]
^2] + a*(-((EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b - a*Sin[
e + f*x]^2])/(a*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])) + ((a + b)*Elliptic
F[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(
a*Sqrt[a + b - a*Sin[e + f*x]^2])))/(f*Sqrt[a + b - a*Sin[e + f*x]^2])
```

3.230.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 314
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] + Simp[1/(2*a*
(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*p + 3) + d
*(2*(p + q + 1) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, 2, p, q,
 x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 389
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[1/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] - Simp[a/b   Int
[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0] &&  !SimplerSqrtQ[-b/a, -d/c]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

3.230.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.34 (sec) , antiderivative size = 4676, normalized size of antiderivative = 21.45

method	result	size
default	Expression too large to display	4676

input
```
int(sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f/(2*I*a^(1/2)*b^(1/2)-a+b)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*((a*
(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f
*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/((1-cos(f*x+e
))^2*csc(f*x+e)^2-1)^2)^(1/2)*(2*I*a^(3/2)*b^(1/2)*((2*I*a^(1/2)*b^(1/2)+a
-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e))+2*I*a^(1/2)*b^(3/2)*((2*I*a^(1/2)
*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e))-2*(-(2*I*a^(1/2)*b^(1/2
)*(1-cos(f*x+e))^2*csc(f*x+e)^2+a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f
*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*((2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e
))^2*csc(f*x+e)^2-a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f
*x+e)^2+a+b)/(a+b))^(1/2)*EllipticF(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6
*a*b-b^2)/(a+b)^2)^(1/2))*a*b+2*(-(2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*cs
c(f*x+e)^2+a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2
-a-b)/(a+b))^(1/2)*((2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2-a*(
1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/(a+b))^(
1/2)*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x
+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1
/2))*a*b-4*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a*b*(1-cos(f*x+e))^3*cs
c(f*x+e)^3+2*(-(2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2+a*(1-cos
(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/...
```

3.230.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 603, normalized size of antiderivative = 2.77

$$\int \sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx =$$

$$-\frac{4i\,a^{\frac{3}{2}}\sqrt{\frac{2\,a\sqrt{\frac{ab+b^2}{a^2}}-a-2\,b}{a}}\sqrt{\frac{ab+b^2}{a^2}}F(\arcsin\left(\sqrt{\frac{2\,a\sqrt{\frac{ab+b^2}{a^2}}-a-2\,b}{a}}(\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)\right)\,|\,\frac{a^2+8\,ab+8\,b^2+}{}$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(4*I*a^(3/2)*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*sqrt((a*b
+ b^2)/a^2)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a
)*(cos(f*x + e) + I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)
*sqrt((a*b + b^2)/a^2))/a^2) - 4*I*a^(3/2)*sqrt((2*a*sqrt((a*b + b^2)/a^2)
 - a - 2*b)/a)*sqrt((a*b + b^2)/a^2)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b
 + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) - I*sin(f*x + e))), (a^2 + 8*a*b
+ 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + (-2*I*a^(3/2)*sqrt
((a*b + b^2)/a^2) + sqrt(a)*(I*a + 2*I*b))*sqrt((2*a*sqrt((a*b + b^2)/a^2)
 - a - 2*b)/a)*elliptic_e(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b
)/a)*(cos(f*x + e) + I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a
*b)*sqrt((a*b + b^2)/a^2))/a^2) + (2*I*a^(3/2)*sqrt((a*b + b^2)/a^2) + sqr
t(a)*(-I*a - 2*I*b))*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*ellipti
c_e(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) - I
*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a
^2))/a^2) - 2*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/
(a*f)
```

3.230.6 Sympy [F]

$$\int \sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{a+b\sec^2\left(e+fx\right)}\sec\left(e+fx\right)dx$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*sec(e + f*x), x)
```

3.230.7 Maxima [F]

$$\int \sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\sec(fx+e)\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e), x)
```

3.230.8 Giac [F]

$$\int \sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\sec(fx+e)\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e), x)
```

3.230.9 Mupad [F(-1)]

Timed out.

$$\int \sec(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(e+fx)^2}}}{\cos(e+fx)}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x),x)
```

output
```
int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x), x)
```

3.231 $\int \cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

3.231.1 Optimal result . . . 1722
3.231.2 Mathematica [A] (verified) . . . 1722
3.231.3 Rubi [A] (verified) . . . 1723
3.231.4 Maple [C] (warning: unable to verify) . . . 1725
3.231.5 Fricas [F] . . . 1725
3.231.6 Sympy [F] . . . 1726
3.231.7 Maxima [F] . . . 1726
3.231.8 Giac [F] . . . 1726
3.231.9 Mupad [F(-1)] . . . 1727

3.231.1 Optimal result

Integrand size = 23, antiderivative size = 80

$$\int \cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}{f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}$$

output
```
EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(
a+b-a*sin(f*x+e)^2))^(1/2)/f/(1-a*sin(f*x+e)^2/(a+b))^(1/2)
```

3.231.2 Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.86

$$\int \cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{2}\cos(e+fx)E\left(e+fx\,\middle|\,\frac{a}{a+b}\right)\sqrt{a+b\sec^2(e+fx)}}{f\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}}}$$

input
```
Integrate[Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Sqrt[2]*Cos[e + f*x]*EllipticE[e + f*x, a/(a + b)]*Sqrt[a + b*Sec[e + f*x
]^2])/(f*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)])
```

3.231.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4636, 2057, 2058, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sqrt{a+b\sec(e+fx)^2}}{\sec(e+fx)}dx$$

$$\downarrow 4636$$

$$\frac{\int \sqrt{a+\frac{b}{1-\sin^2(e+fx)}}d\sin(e+fx)}{f}$$

$$\downarrow 2057$$

$$\frac{\int \sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}d\sin(e+fx)}{f}$$

$$\downarrow 2058$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\int \frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\downarrow 330$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\int \frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}$$

$$\downarrow 327$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)}{f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}$$

input
```
Int[Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[
(a + b - a*Sin[e + f*x]^2)/(1 - Sin[e + f*x]^2)])/(f*Sqrt[1 - (a*Sin[e + f
*x]^2)/(a + b)])
```

3.231.3.1 Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

3.231.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.87 (sec) , antiderivative size = 4727, normalized size of antiderivative = 59.09

method	result	size
default	Expression too large to display	4727

input
```
int(cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/(2*I*a^(1/2)*b^(1/2)-a+b)*((a
*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(
f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/((1-cos(f*x+
e))^2*csc(f*x+e)^2-1)^2)^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)*(2*I*a^(3
/2)*b^(1/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)
)+2*I*a^(1/2)*b^(3/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-
cot(f*x+e))+2*(-(2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2+a*(1-co
s(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)
*((2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2-a*(1-cos(f*x+e))^2*cs
c(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/(a+b))^(1/2)*EllipticF(((2
*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2
)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b-2*(-(2*I*
a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2+a*(1-cos(f*x+e))^2*csc(f*x+e
)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*((2*I*a^(1/2)*b^(1/2
)*(1-cos(f*x+e))^2*csc(f*x+e)^2-a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f
*x+e))^2*csc(f*x+e)^2+a+b)/(a+b))^(1/2)*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-
b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)
*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b-4*((2*I*a^(1/2)*b^(1/2)+a-b)/(
a+b))^(1/2)*a*b*(1-cos(f*x+e))^3*csc(f*x+e)^3-(-(2*I*a^(1/2)*b^(1/2)*(1-co
s(f*x+e))^2*csc(f*x+e)^2+a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e...
```

3.231.5 Fricas [F]

$$\int \cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec\left(fx+e\right)^2+a}\cos\left(fx+e\right)\,dx$$

input
```
integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e), x)
```

3.231.6 Sympy [F]

$$\int \cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{a+b\sec^2(e+fx)}\cos(e+fx)\,dx$$

input
```
integrate(cos(f*x+e)*(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*cos(e + f*x), x)
```

3.231.7 Maxima [F]

$$\int \cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\cos(fx+e)\,dx$$

input
```
integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e), x)
```

3.231.8 Giac [F]

$$\int \cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\cos(fx+e)\,dx$$

input
```
integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e), x)
```

3.231.9 Mupad [F(-1)]

Timed out.

$$\int \cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \cos(e+f\,x)\,\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}\,dx$$

input
```
int(cos(e + f*x)*(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(cos(e + f*x)*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.232 $\int \cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

3.232.1 Optimal result . . . 1728
3.232.2 Mathematica [C] (verified) . . . 1729
3.232.3 Rubi [A] (verified) . . . 1729
3.232.4 Maple [C] (warning: unable to verify) . . . 1733
3.232.5 Fricas [F] . . . 1734
3.232.6 Sympy [F(-1)] . . . 1734
3.232.7 Maxima [F] . . . 1734
3.232.8 Giac [F] . . . 1735
3.232.9 Mupad [F(-1)] . . . 1735

3.232.1 Optimal result

Integrand size = 25, antiderivative size = 246

$$\int \cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{\cos^2(e+fx)\sin(e+fx)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}{3f}$$
$$+\frac{(2a+b)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}{3af\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}$$
$$-\frac{b(a+b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\sqrt{1}}{3af\,(a+b-a\sin^2(e+fx))}$$

output

```
1/3*cos(f*x+e)^2*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/f+1/
3*(2*a+b)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(
f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/a/f/(1-a*sin(f*x+e)^2/(a+b))^(1/2)-1/
3*b*(a+b)*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(
f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/a/f/(a
+b-a*sin(f*x+e)^2)
```

3.232.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 13.40 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.63

$$\int \cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$= \cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(\frac{6\sqrt{2}\sqrt{a+2b+a\cos(2(e+fx))}E\left(e+fx\middle|\frac{a}{a+b}\right)}{\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}}} + \frac{\sqrt{-\frac{1}{b}}b\csc^2(2(e+fx))\sec(2(e+fx))\left(4i\sqrt{2}(a^2+\right.\right.$$

input
```
Integrate[Cos[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*((6*Sqrt[2]*Sqrt[a + 2*b + a*Cos[
2*(e + f*x)]]*EllipticE[e + f*x, a/(a + b)])/Sqrt[(a + 2*b + a*Cos[2*(e +
f*x)])/(a + b)] + (Sqrt[-b^(-1)]*b*Csc[2*(e + f*x)]^2*Sec[2*(e + f*x)]*((4
*I)*Sqrt[2]*(a^2 + 3*a*b + 2*b^2)*Sqrt[-((a*Cos[e + f*x]^2)/b)]*EllipticE[
I*ArcSinh[(Sqrt[-b^(-1)]*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]])/Sqrt[2]], b/(
a + b)]*Sqrt[(a*Sin[e + f*x]^2)/(a + b)] + a*(a*Sqrt[-b^(-1)]*Sqrt[a + 2*b
 + a*Cos[2*(e + f*x)]]*(-1 + Cos[4*(e + f*x)]) - (4*I)*Sqrt[2]*(a + b)*Sqr
t[-((a*Cos[e + f*x]^2)/b)]*EllipticF[I*ArcSinh[(Sqrt[-b^(-1)]*Sqrt[a + 2*b
 + a*Cos[2*(e + f*x)]])/Sqrt[2]], b/(a + b)]*Sqrt[(a*Sin[e + f*x]^2)/(a +
b)]))*Sin[4*(e + f*x)])/(2*a^2)))/(12*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]
)
```

3.232.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 4636, 2057, 2058, 319, 27, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sqrt{a+b\sec(e+fx)^2}}{\sec(e+fx)^3}dx$$

$$\downarrow 4636$$

$$\frac{\int \left(1-\sin^2(e+fx)\right)\sqrt{a+\frac{b}{1-\sin^2(e+fx)}}d\sin(e+fx)}{f}$$

$$\downarrow 2057$$

$$\frac{\int \left(1-\sin^2(e+fx)\right)\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}d\sin(e+fx)}{f}$$

$$\downarrow 2058$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\int \sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}d\sin(e+fx)}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\downarrow 319$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{2}{3}\int \frac{-((2a+b)\sin^2(e+fx))+2a+2b}{2\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)+\frac{1}{3}\sqrt{1-\sin^2(e+fx)}\sin\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\downarrow 27$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\int \frac{2(a+b)-(2a+b)\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)+\frac{1}{3}\sqrt{1-\sin^2(e+fx)}\sin\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\downarrow 399$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\left(\frac{(2a+b)\int \frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{b(a+b)\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+}}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\downarrow 323$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\left(\frac{(2a+b)\int \frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{b(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{}}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+}}$$

$$\downarrow 321$$

3.232. $\int \cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\left(\frac{(2a+b)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{b(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\arcsin(s\right.\right.\right.}{a\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$f\sqrt{-a\sin^2(e+fx)+a+b}$$

$$\downarrow 330$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\left(\frac{(2a+b)\sqrt{-a\sin^2(e+fx)+a+b}\int\frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{b(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{a\sqrt{-}}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a}}$$

$$\downarrow 327$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\left(\frac{(2a+b)\sqrt{-a\sin^2(e+fx)+a+b}E\left(\arcsin(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{b(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,E}{a\sqrt{-a\,si}}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a}}$$

input
```
Int[Cos[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Sqrt[1 - Sin[e + f*x]^2]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(1 - Sin[e + f*x
]^2)]*((Sin[e + f*x]*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^
2])/3 + (((2*a + b)*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b
 - a*Sin[e + f*x]^2])/(a*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - (b*(a + b)
*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a
 + b)])/(a*Sqrt[a + b - a*Sin[e + f*x]^2]))/3))/(f*Sqrt[a + b - a*Sin[e +
f*x]^2])
```

3.232.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 319
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[x*(a + b*x^2)^p*((c + d*x^2)^q/(2*(p + q) + 1)), x] + Simp[2/(2*(p + q) +
 1)   Int[(a + b*x^2)^(p - 1)*(c + d*x^2)^(q - 1)*Simp[a*c*(p + q) + (q*(b*
c - a*d) + a*d*(p + q))*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*
c - a*d, 0] && GtQ[q, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 2, p, q,
x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

3.232.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.23 (sec) , antiderivative size = 6300, normalized size of antiderivative = 25.61

method	result	size
default	Expression too large to display	6300

input
```
int(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.232.5 Fricas [F]

$$\int \cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\cos(fx+e)^3\,dx$$

input
```
integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^3, x)
```

3.232.6 Sympy [F(-1)]

Timed out.

$$\int \cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**3*(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Timed out
```

3.232.7 Maxima [F]

$$\int \cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\cos(fx+e)^3\,dx$$

input
```
integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^3, x)
```

3.232.8 Giac [F]

$$\int \cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\cos(fx+e)^3\,dx$$

input
```
integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^3, x)
```

3.232.9 Mupad [F(-1)]

Timed out.

$$\int \cos^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \cos(e+f\,x)^3\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}\,dx$$

input
```
int(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.233 $\int \cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

3.233.1 Optimal result . . . 1736
3.233.2 Mathematica [F] . . . 1737
3.233.3 Rubi [A] (verified) . . . 1737
3.233.4 Maple [C] (warning: unable to verify) . . . 1742
3.233.5 Fricas [F] . . . 1742
3.233.6 Sympy [F(-1)] . . . 1742
3.233.7 Maxima [F] . . . 1743
3.233.8 Giac [F] . . . 1743
3.233.9 Mupad [F(-1)] . . . 1743

3.233.1 Optimal result

Integrand size = 25, antiderivative size = 338

$$\int \cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{2(2a-b)\cos^2(e+fx)\sin(e+fx)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}{15af}$$
$$+\frac{\cos^2(e+fx)\sin(e+fx)\,(a+b-a\sin^2(e+fx))\,\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}{5af}$$
$$+\frac{(8a^2+3ab-2b^2)\,\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}{15a^2f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}$$
$$-\frac{2(2a-b)b(a+b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+}}{15a^2f\,(a+b-a\sin^2(e+fx))}$$

output

```
2/15*(2*a-b)*cos(f*x+e)^2*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(
1/2)/a/f+1/5*cos(f*x+e)^2*sin(f*x+e)*(a+b-a*sin(f*x+e)^2)*(sec(f*x+e)^2*(a
+b-a*sin(f*x+e)^2))^(1/2)/a/f+1/15*(8*a^2+3*a*b-2*b^2)*EllipticE(sin(f*x+e
),(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2)
)^(1/2)/a^2/f/(1-a*sin(f*x+e)^2/(a+b))^(1/2)-2/15*(2*a-b)*b*(a+b)*Elliptic
F(sin(f*x+e),(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a+b-a*si
n(f*x+e)^2))^(1/2)*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/a^2/f/(a+b-a*sin(f*x+e)^
2)
```

3.233.2 Mathematica [F]

$$\int \cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

input
```
Integrate[Cos[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
Integrate[Cos[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2], x]
```

3.233.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.97, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 4636, 2057, 2058, 318, 25, 403, 25, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sqrt{a+b\sec(e+fx)^2}}{\sec(e+fx)^5}dx$$

$$\downarrow 4636$$

$$\frac{\int \left(1-\sin^2(e+fx)\right)^2\sqrt{a+\frac{b}{1-\sin^2(e+fx)}}d\sin(e+fx)}{f}$$

$$\downarrow 2057$$

$$\frac{\int \left(1-\sin^2(e+fx)\right)^2\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}d\sin(e+fx)}{f}$$

$$\downarrow 2058$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\int \left(1-\sin^2(e+fx)\right)^{3/2}\sqrt{-a\sin^2(e+fx)+a+b}d\sin(e+fx)}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\downarrow 318$$

$$\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{\sin(e+fx)\sqrt{1-\sin^2(e+fx)}\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}{5a}-\frac{\int -\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(-2(2a-b)\sin\right.}{\sqrt{1-\sin^2(e+fx)}}}{5a}\right)$$

$$f\sqrt{-a\sin^2(e+fx)+a+b}$$

$$\downarrow 25$$

$$\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(-2(2a-b)\sin^2(e+fx)+4a-b\right)}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{5a}+\frac{\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{5}\right.$$

$$f\sqrt{-a\sin^2(e+fx)+a+b}$$

$$\downarrow 403$$

$$\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{\frac{2}{3}(2a-b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}-\frac{1}{3}\int -\frac{(8a-b)(a+b)-\left(8a^2+3ba-2b^2\right.}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e}}}{5a}\right)$$

$$f\sqrt{-a\sin^2(e+fx)+a+b}$$

$$\downarrow 25$$

$$\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{\frac{1}{3}\int\frac{(8a-b)(a+b)-\left(8a^2+3ba-2b^2\right)\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)+\frac{2}{3}(2a-b)\sqrt{1-\sin^2(e+fx)}\sin(e+fx)\sqrt{}}{5a}\right.$$

$$f\sqrt{-a\sin^2(e+fx)+a+b}$$

$$\downarrow 399$$

$$\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{\frac{1}{3}\left(\frac{\left(8a^2+3ab-2b^2\right)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{2b(2a-b)(a+b)\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(}}}{a}\right.\right.}{5a}$$

$$f\sqrt{-a\sin^2(e$$

$$\downarrow 323$$

$$\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\left(\frac{\left(8a^2+3ab-2b^2\right)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{2b(2a-b)(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\int\frac{}{\sqrt{1-\sin}}}{a\sqrt{-a\sin^2(e+}}\right)\right)$$

$$f\sqrt{-a\,si}$$

$$\downarrow 321$$

$$\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\left(\frac{\left(8a^2+3ab-2b^2\right)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{2b(2a-b)(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(}{a\sqrt{-a\sin^2(e+fx)+a-}}\right)\right)$$

$$5a$$

$$f\sqrt{-a\sin^2(e}$$

$$\downarrow 330$$

$$\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\left(\frac{\left(8a^2+3ab-2b^2\right)\sqrt{-a\sin^2(e+fx)+a+b}\int\frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{2b(2a-b)(a+b)\sqrt{1-\frac{a\sin^2}{}}}{a\sqrt{}}\right)\right)$$

$$f\sqrt{-a}$$

$$\downarrow 327$$

$$\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\left(\frac{\left(8a^2+3ab-2b^2\right)\sqrt{-a\sin^2(e+fx)+a+b}E\left(\arcsin(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{2b(2a-b)(a+b)\sqrt{1-\frac{a\sin^2(e+f}}{a\sqrt{-a\,si}}\right)\right)$$

$$f\sqrt{-a\,si}$$

input
```
Int[Cos[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2],x]
```

3.233. $\int \cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

output
```
(Sqrt[1 - Sin[e + f*x]^2]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(1 - Sin[e + f*x
]^2)]*((Sin[e + f*x]*Sqrt[1 - Sin[e + f*x]^2]*(a + b - a*Sin[e + f*x]^2)^(
3/2))/(5*a) + ((2*(2*a - b)*Sin[e + f*x]*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[a +
 b - a*Sin[e + f*x]^2])/3 + (((8*a^2 + 3*a*b - 2*b^2)*EllipticE[ArcSin[Sin
[e + f*x]], a/(a + b)]*Sqrt[a + b - a*Sin[e + f*x]^2])/(a*Sqrt[1 - (a*Sin[
e + f*x]^2)/(a + b)]) - (2*(2*a - b)*b*(a + b)*EllipticF[ArcSin[Sin[e + f*
x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(a*Sqrt[a + b - a*Si
n[e + f*x]^2]))/3)/(5*a)))/(f*Sqrt[a + b - a*Sin[e + f*x]^2])
```

3.233.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

3.233.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.75 (sec) , antiderivative size = 8763, normalized size of antiderivative = 25.93

method	result	size
default	Expression too large to display	8763

input
```
int(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.233.5 Fricas [F]

$$\int \cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\cos(fx+e)^5\,dx$$

input
```
integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^5, x)
```

3.233.6 Sympy [F(-1)]

Timed out.

$$\int \cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**5*(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Timed out
```

3.233.7 Maxima [F]

$$\int \cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\cos(fx+e)^5\,dx$$

input
```
integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^5, x)
```

3.233.8 Giac [F]

$$\int \cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\cos(fx+e)^5\,dx$$

input
```
integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^5, x)
```

3.233.9 Mupad [F(-1)]

Timed out.

$$\int \cos^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \cos(e+f\,x)^5\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}\,dx$$

input
```
int(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.234 $\int \sec^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

3.234.1 Optimal result . . . 1744
3.234.2 Mathematica [C] (verified) . . . 1745
3.234.3 Rubi [A] (verified) . . . 1745
3.234.4 Maple [B] (verified) . . . 1748
3.234.5 Fricas [A] (verification not implemented) . . . 1749
3.234.6 Sympy [F] . . . 1750
3.234.7 Maxima [A] (verification not implemented) . . . 1750
3.234.8 Giac [F] . . . 1751
3.234.9 Mupad [F(-1)] . . . 1751

3.234.1 Optimal result

Integrand size = 25, antiderivative size = 186

$$\begin{aligned}
&\int \sec^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx \\
&= \frac{(a+b)\left(a^2-2ab+5b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16b^{5/2}f} \\
&\quad + \frac{\left(a^2-2ab+5b^2\right)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16b^2f} \\
&\quad - \frac{(3a-5b)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{24b^2f} \\
&\quad + \frac{\sec^2(e+fx)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{6bf}
\end{aligned}$$

output

```
1/16*(a+b)*(a^2-2*a*b+5*b^2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^
2)^(1/2))/b^(5/2)/f+1/16*(a^2-2*a*b+5*b^2)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(
f*x+e)/b^2/f-1/24*(3*a-5*b)*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^(3/2)/b^2/f+1/
6*sec(f*x+e)^2*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^(3/2)/b/f
```

3.234.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 15.53 (sec) , antiderivative size = 407, normalized size of antiderivative = 2.19

$$\int \sec^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$= \frac{e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos(e+fx)\left(-\frac{i\sqrt{b}\left(-1+e^{2i(e+fx)}\right)\left(-3a^2\left(1+e^{2i(e+fx)}\right)^4+4ab\left(1+e^{2i(e+fx)}\right)^2(1\right.}{}\right.}{}$$

input
```
Integrate[Sec[e + f*x]^6*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e +
f*x))]*Cos[e + f*x]*(((-I)*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*(-3*a^2*(1 +
 E^((2*I)*(e + f*x)))^4 + 4*a*b*(1 + E^((2*I)*(e + f*x)))^2*(1 + 4*E^((2*I
)*(e + f*x)) + E^((4*I)*(e + f*x))) + b^2*(15 + 100*E^((2*I)*(e + f*x)) +
298*E^((4*I)*(e + f*x)) + 100*E^((6*I)*(e + f*x)) + 15*E^((8*I)*(e + f*x))
)))/(1 + E^((2*I)*(e + f*x)))^6 - (3*(a^3 - a^2*b + 3*a*b^2 + 5*b^3)*Log[(
-4*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*f + (4*I)*Sqrt[4*b*E^((2*I)*(e + f*x
)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)/(1 + E^((2*I)*(e + f*x)))])/Sqrt[4*
b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*Sqrt[a + b*Sec[e +
 f*x]^2])/(24*Sqrt[2]*b^(5/2)*f*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]])
```

3.234.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4634, 318, 25, 299, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \sec(e+fx)^6\sqrt{a+b\sec(e+fx)^2}dx$$

$$\downarrow 4634$$

$$\frac{\int \left(\tan^2(e+fx)+1\right)^2 \sqrt{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{f}$$

$$\downarrow 318$$

$$\frac{\int -\left(\left((3a-5b)\tan^2(e+fx)+a-5b\right)\sqrt{b\tan^2(e+fx)+a+b}\right)d\tan(e+fx)}{6b} + \frac{\tan(e+fx)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{6b}$$
$$f$$

$$\downarrow 25$$

$$\frac{\tan(e+fx)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{6b} - \frac{\int\left((3a-5b)\tan^2(e+fx)+a-5b\right)\sqrt{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{6b}$$
$$f$$

$$\downarrow 299$$

$$\frac{\tan(e+fx)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{6b} - \frac{\frac{(3a-5b)\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4b} - \frac{3\left(a^2-2ab+5b^2\right)\int\sqrt{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{4b}}{6b}$$
$$f$$

$$\downarrow 211$$

$$\frac{\tan(e+fx)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{6b} - \frac{\frac{(3a-5b)\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4b} - \frac{3\left(a^2-2ab+5b^2\right)\left(\frac{1}{2}(a+b)\int\frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\right.}{4b}}{6b}$$
$$f$$

$$\downarrow 224$$

$$\frac{\tan(e+fx)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{6b} - \frac{\frac{(3a-5b)\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4b} - \frac{3\left(a^2-2ab+5b^2\right)\left(\frac{1}{2}(a+b)\int\frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}\right.}{6b}}{f}$$

$$\downarrow 219$$

$$\frac{\tan(e+fx)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{6b} - \frac{\frac{(3a-5b)\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4b} - \frac{3\left(a^2-2ab+5b^2\right)\left(\frac{(a+b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+\right.}{2\sqrt{b}}\right.}{4b}}{6b}$$
$$f$$

input
```
Int[Sec[e + f*x]^6*Sqrt[a + b*Sec[e + f*x]^2],x]
```

3.234. $\int \sec^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

output
```
((Tan[e + f*x]*(1 + Tan[e + f*x]^2)*(a + b + b*Tan[e + f*x]^2)^(3/2))/(6*b
) - (((3*a - 5*b)*Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/2))/(4*b) - (
3*(a^2 - 2*a*b + 5*b^2)*(((a + b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a +
b + b*Tan[e + f*x]^2]])/(2*Sqrt[b]) + (Tan[e + f*x]*Sqrt[a + b + b*Tan[e +
 f*x]^2])/2))/(4*b))/(6*b))/f
```

3.234.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.234.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1374 vs. $2(166) = 332$.

Time = 17.26 (sec) , antiderivative size = 1375, normalized size of antiderivative = 7.39

method	result	size
default	Expression too large to display	1375

input
```
int(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/96/f/b^(11/2)*(a+b*sec(f*x+e)^2)^(1/2)/(1+cos(f*x+e))/((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*(30*b^(11/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*sin(f*x+e)+30*b^(11/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*tan(f*x+e)+8*b^(9/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*sin(f
*x+e)+20*b^(11/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*tan(f*x+e)*s
ec(f*x+e)+8*b^(9/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*tan(f*x+
e)-6*sin(f*x+e)*a^2*b^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+20
*b^(11/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*tan(f*x+e)*sec(f*x+e
)^2+4*b^(9/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*tan(f*x+e)*sec
(f*x+e)-6*b^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*tan(f*x+
e)+16*b^(11/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*tan(f*x+e)*sec(
f*x+e)^3+4*b^(9/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*tan(f*x+e
)*sec(f*x+e)^2+16*b^(11/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*tan
(f*x+e)*sec(f*x+e)^4+3*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b
^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(
f*x+e)*a-a-b)/(sin(f*x+e)+1))*cos(f*x+e)*a^3*b^3-3*ln(4*(((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*cos(f*x+e)*a^2*b
^4+9*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+
b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(...
```

3.234.5 Fricas [A] (verification not implemented)

Time = 1.07 (sec) , antiderivative size = 468, normalized size of antiderivative = 2.52

$$\int \sec^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$= \left[\frac{3\left(a^3-a^2b+3\,ab^2+5\,b^3\right)\sqrt{b}\cos\left(fx+e\right)^5\log\left(\frac{\left(a^2-6\,ab+b^2\right)\cos(fx+e)^4+8\left(ab-b^2\right)\cos(fx+e)^2+4\left((a-b)\cos(fx+e)^3+2\right.\right.\right.}{\cos(fx+e)^4}\right.$$

input

```
integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/192*(3*(a^3 - a^2*b + 3*a*b^2 + 5*b^3)*sqrt(b)*cos(f*x + e)^5*log(((a^2
 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)
*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*((3*a^2*b - 4*a*b^
2 - 15*b^3)*cos(f*x + e)^4 - 8*b^3 - 2*(a*b^2 + 5*b^3)*cos(f*x + e)^2)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^3*f*cos(f*x + e)
^5), 1/96*(3*(a^3 - a^2*b + 3*a*b^2 + 5*b^3)*sqrt(-b)*arctan(-1/2*((a - b)
*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e)^5 -
 2*((3*a^2*b - 4*a*b^2 - 15*b^3)*cos(f*x + e)^4 - 8*b^3 - 2*(a*b^2 + 5*b^3
)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)
)/(b^3*f*cos(f*x + e)^5)]
```

3.234.6 Sympy [F]

$$\int \sec^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{a+b\sec^2(e+fx)}\sec^6(e+fx)\,dx$$

input
```
integrate(sec(f*x+e)**6*(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*sec(e + f*x)**6, x)
```

3.234.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.70

$$\int \sec^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{8\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}\tan(fx+e)^3}{b} + \frac{3\,(a+b)^2 a\,\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{5}{2}}} + \frac{3\,(a+b)^2\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}} - \frac{12\,(a+b)a\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}}$$

input
```
integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/48*(8*(b*tan(f*x + e)^2 + a + b)^(3/2)*tan(f*x + e)^3/b + 3*(a + b)^2*a*
arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/b^(5/2) + 3*(a + b)^2*arcsinh(b*ta
n(f*x + e)/sqrt((a + b)*b))/b^(3/2) - 12*(a + b)*a*arcsinh(b*tan(f*x + e)/
sqrt((a + b)*b))/b^(3/2) - 12*(a + b)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*
b))/sqrt(b) + 24*a*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/sqrt(b) + 24*sq
rt(b)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b)) + 24*sqrt(b*tan(f*x + e)^2 +
 a + b)*tan(f*x + e) - 6*(b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)*tan(f*x
+ e)/b^2 + 3*sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^2*tan(f*x + e)/b^2 + 2
4*(b*tan(f*x + e)^2 + a + b)^(3/2)*tan(f*x + e)/b - 12*sqrt(b*tan(f*x + e)
^2 + a + b)*(a + b)*tan(f*x + e)/b)/f
```

3.234.8 Giac [F]

$$\int \sec^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\sec(fx+e)^6\,dx$$

input
```
integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)^6, x)
```

3.234.9 Mupad [F(-1)]

Timed out.

$$\int \sec^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(e+fx)^2}}}{\cos(e+fx)^6}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x)^6,x)
```

output
```
int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x)^6, x)
```

3.235 $\int \sec^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

3.235.1 Optimal result . . . 1752
3.235.2 Mathematica [C] (warning: unable to verify) . . . 1752
3.235.3 Rubi [A] (verified) . . . 1753
3.235.4 Maple [B] (verified) . . . 1755
3.235.5 Fricas [A] (verification not implemented) . . . 1756
3.235.6 Sympy [F] . . . 1757
3.235.7 Maxima [A] (verification not implemented) . . . 1757
3.235.8 Giac [F] . . . 1758
3.235.9 Mupad [F(-1)] . . . 1758

3.235.1 Optimal result

Integrand size = 25, antiderivative size = 122

$$\int \sec^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = -\frac{(a-3b)(a+b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8b^{3/2}f} - \frac{(a-3b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8bf} + \frac{\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{4bf}$$

output

```
-1/8*(a-3*b)*(a+b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/
b^(3/2)/f-1/8*(a-3*b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b/f+1/4*tan(f*
x+e)*(a+b+b*tan(f*x+e)^2)^(3/2)/b/f
```

3.235.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 7.73 (sec) , antiderivative size = 380, normalized size of antiderivative = 3.11

$$\int \sec^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx =$$

$$-\frac{b\sec^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)\tan^3(e+fx)\left(5\arcsin\left(\sqrt{-\frac{b\tan^2(e+fx)}{a+b}}\right)\left(3-\frac{2a}{}\right.\right.}{}$$

input
```
Integrate[Sec[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
-1/20*(b*Sec[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2]*(1 - (a*Sin[e + f*x]^2)
/(a + b))*Tan[e + f*x]^3*(5*ArcSin[Sqrt[-((b*Tan[e + f*x]^2)/(a + b))]]*(3
 - (2*a*Sin[e + f*x]^2)/(a + b)) - (5*Sec[e + f*x]^2*(3*b^2*(1 + Sin[e + f
*x]^2) - 2*a*b*(-3 + Sin[e + f*x]^2 + Sin[e + f*x]^4) + a^2*(3 - 5*Sin[e +
 f*x]^2 + 2*Sin[e + f*x]^4))*Sqrt[-((b*Sec[e + f*x]^2*(a + b - a*Sin[e + f
*x]^2)*Tan[e + f*x]^2)/(a + b)^2)])/(a + b)^2 + 32*Cos[e + f*x]^2*Hypergeo
metric2F1[2, 4, 7/2, -((b*Tan[e + f*x]^2)/(a + b))]*(-((b*Sec[e + f*x]^2*(
a + b - a*Sin[e + f*x]^2)*Tan[e + f*x]^2)/(a + b)^2))^(5/2)))/(Sqrt[2]*f*S
qrt[a + 2*b + a*Cos[2*e + 2*f*x]]*Sqrt[(a + b*Sec[e + f*x]^2)/(a + b)]*Sqr
t[a + b - a*Sin[e + f*x]^2]*(-((b*Tan[e + f*x]^2)/(a + b)))^(5/2))
```

3.235.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 4634, 299, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \sec(e+fx)^4\sqrt{a+b\sec(e+fx)^2}dx$$

$$\downarrow 4634$$

$$\frac{\int \left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+bd}\tan(e+fx)}{f}$$

$$\downarrow 299$$

$$\frac{\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4b}-\frac{(a-3b)\int\sqrt{b\tan^2(e+fx)+a+bd}\tan(e+fx)}{4b}}{f}$$

$$\downarrow 211$$

$$\frac{\frac{\tan(e+fx)(a+b\tan^2(e+fx)+b)^{3/2}}{4b} - \frac{(a-3b)\left(\frac{1}{2}(a+b)\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)}{4b}}{f}$$

$$\downarrow 224$$

$$\frac{\frac{\tan(e+fx)(a+b\tan^2(e+fx)+b)^{3/2}}{4b} - \frac{(a-3b)\left(\frac{1}{2}(a+b)\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)}{4b}}{f}$$

$$\downarrow 219$$

$$\frac{\frac{\tan(e+fx)(a+b\tan^2(e+fx)+b)^{3/2}}{4b} - \frac{(a-3b)\left(\frac{(a+b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2\sqrt{b}}+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)}{4b}}{f}$$

input
```
Int[Sec[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
((Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/2))/(4*b) - ((a - 3*b)*(((a +
 b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(2*Sqr
t[b]) + (Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/2))/(4*b))/f
```

3.235.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.235.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 906 vs. $2(106) = 212$.

Time = 11.17 (sec) , antiderivative size = 907, normalized size of antiderivative = 7.43

method	result	size
default	Expression too large to display	907

input
```
int(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/16/f/b^2*(a+b*sec(f*x+e)^2)^(1/2)/(1+cos(f*x+e))/((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*(3*cos(f*x+e)*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*b^(5/2)+3*cos(f*x+e)*ln(-4*(((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*b^(
5/2)+2*cos(f*x+e)*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2
)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e
)*a-a-b)/(sin(f*x+e)+1))*b^(3/2)*a+2*cos(f*x+e)*ln(-4*(((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*b^(3/2)*a-cos(f*x+
e)*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^
(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f
*x+e)+1))*b^(1/2)*a^2-cos(f*x+e)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*b^(1/2)*a^2+2*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*a*b*sin(f*x+e)+6*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*b^2*sin(f*x+e)+2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a
*b*tan(f*x+e)+6*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2*tan(f*x+e)
+4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2*tan(f*x+e)*sec(f*x+e...
```

3.235.5 Fricas [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 390, normalized size of antiderivative = 3.20

$$\int \sec^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$= \left[-\frac{(a^2-2\,ab-3\,b^2)\sqrt{b}\cos(fx+e)^3\log\left(\frac{(a^2-6\,ab+b^2)\cos(fx+e)^4+8\,(ab-b^2)\cos(fx+e)^2+4\left((a-b)\cos(fx+e)^3+2\,b\cos(fx\right.\right.}{\cos(fx+e)^4}}{32\,b^2 f\cos(f}\right.$$

$$\left. -\frac{(a^2-2\,ab-3\,b^2)\sqrt{-b}\arctan\left(-\frac{\left((a-b)\cos(fx+e)^3+2\,b\cos(fx+e)\right)\sqrt{-b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{2\left(ab\cos(fx+e)^2+b^2\right)\sin(fx+e)}\right)\cos(fx+e)^3-2\left((ab+}{16\,b^2 f\cos(fx+e)^3} \right]$$

input
```
integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/32*((a^2 - 2*a*b - 3*b^2)*sqrt(b)*cos(f*x + e)^3*log(((a^2 - 6*a*b + b
^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e
)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*((a*b + 3*b^2)*cos(f*x + e)^2
+ 2*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^2*f*
cos(f*x + e)^3), -1/16*((a^2 - 2*a*b - 3*b^2)*sqrt(-b)*arctan(-1/2*((a - b
)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/
cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e)^3
- 2*((a*b + 3*b^2)*cos(f*x + e)^2 + 2*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)*sin(f*x + e))/(b^2*f*cos(f*x + e)^3)]
```

3.235.6 Sympy [F]

$$\int \sec^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{a+b\sec^2(e+fx)}\sec^4(e+fx)\,dx$$

input
```
integrate(sec(f*x+e)**4*(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*sec(e + f*x)**4, x)
```

3.235.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.42

$$\int \sec^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx =$$

$$-\frac{\frac{(a+b)a\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}} + \frac{(a+b)\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}} - \frac{4a\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}} - 4\sqrt{b}\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right) - 4\sqrt{b\tan}}{8f}$$

input
```
integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/8*((a + b)*a*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/b^(3/2) + (a + b)*
arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/sqrt(b) - 4*a*arcsinh(b*tan(f*x +
e)/sqrt((a + b)*b))/sqrt(b) - 4*sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt((a + b
)*b)) - 4*sqrt(b*tan(f*x + e)^2 + a + b)*tan(f*x + e) - 2*(b*tan(f*x + e)^
2 + a + b)^(3/2)*tan(f*x + e)/b + sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)*t
an(f*x + e)/b)/f
```

3.235.8 Giac [F]

$$\int \sec^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\sec(fx+e)^4\,dx$$

input
```
integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)^4, x)
```

3.235.9 Mupad [F(-1)]

Timed out.

$$\int \sec^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(e+fx)^2}}}{\cos(e+fx)^4}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x)^4,x)
```

output
```
int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x)^4, x)
```

3.236 $\int \sec^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

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3.236.1 Optimal result

Integrand size = 25, antiderivative size = 76

$$\int \sec^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{(a+b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2\sqrt{b}f} + \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f}$$

output
```
1/2*(a+b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/b^(1/2)
+1/2*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f
```

3.236.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 210 vs. 2(76) = 152.

Time = 1.89 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.76

$$\int \sec^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}\left(\sqrt{2}(a+b)\operatorname{arctanh}\left(\frac{\sqrt{\frac{b\sin^2(e+fx)}{a+b}}}{\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}}\right)\cos^2(e+fx)\sqrt{\frac{a+2b+}{}}\right.}{\sqrt{2}f(a+2b+a\cos(2(e+fx)))^{3/2}\sqrt{\frac{b\sin^2(}{a}}}$$

input
```
Integrate[Sec[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2]*(Sqrt[2]*(a + b
)*ArcTanh[Sqrt[(b*Sin[e + f*x]^2)/(a + b)]/Sqrt[(a + b - a*Sin[e + f*x]^2)
/(a + b)]]*Cos[e + f*x]^2*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)] + (
a + 2*b + a*Cos[2*(e + f*x)])*Sqrt[(b*Sin[e + f*x]^2)/(a + b)])*Tan[e + f*
x])/(Sqrt[2]*f*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2)*Sqrt[(b*Sin[e + f*x]^2
)/(a + b)])
```

3.236.3 Rubi [A] (verified)

Time $= 0.25$ (sec) , antiderivative size $= 74$, normalized size of antiderivative $= 0.97$, number of steps used $= 6$, number of rules used $= 5$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used $= \{3042, 4634, 211, 224, 219\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \sec(e+fx)^2\sqrt{a+b\sec(e+fx)^2}dx$$

$$\downarrow 4634$$

$$\frac{\int \sqrt{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{f}$$

$$\downarrow 211$$

$$\frac{\frac{1}{2}(a+b)\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

$$\downarrow 224$$

$$\frac{\frac{1}{2}(a+b)\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

$$\downarrow 219$$

$$\frac{(a+b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2\sqrt{b}}+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}$$
$$f$$

input
```
Int[Sec[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(((a + b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/
(2*Sqrt[b]) + (Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/2)/f
```

3.236.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.236.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 536 vs. $2(64) = 128$.

Time = 6.68 (sec) , antiderivative size = 537, normalized size of antiderivative = 7.07

method	result
default	$\frac{\sqrt{a+b\sec(fx+e)^2}\left(b^{\frac{3}{2}}\cos(fx+e)\ln\left(-\frac{4\left(-\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{b}\cos(fx+e)+\sin(fx+e)a-\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}+a+b\right)}{\sin(fx+e)+1}\right)+b^{\frac{3}{2}}\cos(fx+e$

input
```
int(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/f/b*(a+b*sec(f*x+e)^2)^(1/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)/(1+cos(f*x+e))*(b^(3/2)*cos(f*x+e)*ln(-4*(-((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)+a+b)/(sin(f*x+e)+1))+b^(3/2)*cos(f*x+e)*ln(4*(-(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a
-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-a-b)/(sin(f*x+e)-1))+
b^(1/2)*cos(f*x+e)*ln(-4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(
1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)+a+b)/(sin(f*x+e)+1))*a+b^(1/2)*cos(f*x+e)*ln(4*(-((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-a-b)/(sin(f*x+e)-1))*a+2*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b*sin(f*x+e)+2*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*b*tan(f*x+e))
```

3.236.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(64) = 128$.

Time = 0.31 (sec) , antiderivative size = 320, normalized size of antiderivative = 4.21

$$\int \sec^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$= \left[\frac{(a+b)\sqrt{b}\cos(fx+e)\log\left(\frac{(a^2-6ab+b^2)\cos(fx+e)^4+8(ab-b^2)\cos(fx+e)^2+4\left((a-b)\cos(fx+e)^3+2b\cos(fx+e)\right)\sqrt{b}\sqrt{\frac{a\cos(fx}{\cos(fx}}}{\cos(fx+e)^4}\right.}{8bf\cos(fx+e)}\right.$$

input
```
integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*((a + b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4
 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x
+ e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8
*b^2)/cos(f*x + e)^4) + 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*si
n(f*x + e))/(b*f*cos(f*x + e)), 1/4*((a + b)*sqrt(-b)*arctan(-1/2*((a - b)
*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e) + 2
*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b*f*cos(f*x
+ e))]
```

3.236.6 Sympy [F]

$$\int \sec^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{a+b\sec^2(e+fx)}\sec^2(e+fx)\,dx$$

input
```
integrate(sec(f*x+e)**2*(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*sec(e + f*x)**2, x)
```

3.236.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.91

$$\int \sec^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\frac{a\,\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}} + \sqrt{b}\,\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right) + \sqrt{b\tan(fx+e)^2+a+b}\tan(fx+e)}{2f}$$

input
```
integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*(a*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/sqrt(b) + sqrt(b)*arcsinh(b
*tan(f*x + e)/sqrt((a + b)*b)) + sqrt(b*tan(f*x + e)^2 + a + b)*tan(f*x +
e))/f
```

3.236.8 Giac [F]

$$\int \sec^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\sec(fx+e)^2\,dx$$

input
```
integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*sec(f*x + e)^2, x)
```

3.236.9 Mupad [F(-1)]

Timed out.

$$\int \sec^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(e+fx)^2}}}{\cos(e+fx)^2}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x)^2,x)
```

output
```
int((a + b/cos(e + f*x)^2)^(1/2)/cos(e + f*x)^2, x)
```

3.237 $\int \sqrt{a+b\sec^2(e+fx)}\,dx$

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3.237.1 Optimal result

Integrand size = 16, antiderivative size = 79

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}$$

output
```
arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*a^(1/2)/f+arctanh(b^
(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f
```

3.237.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 284, normalized size of antiderivative = 3.59

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx = -\frac{i\left(1+e^{2i(e+fx)}\right)\left(2\sqrt{b}\arctan\left(\frac{\sqrt{b}\left(-1+e^{2i(e+fx)}\right)}{\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}}\right)+\sqrt{a}\operatorname{arctanh}\left(\frac{a+2b+ae^{2i(e+fx)}}{\sqrt{a}\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}}\right)-\right.}{2\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}f}$$

input
```
Integrate[Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
((-1/2*I)*(1 + E^((2*I)*(e + f*x)))*(2*Sqrt[b]*ArcTan[(Sqrt[b]*(-1 + E^((2
*I)*(e + f*x))))/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x))
)^2]] + Sqrt[a]*ArcTanh[(a + 2*b + a*E^((2*I)*(e + f*x)))/(Sqrt[a]*Sqrt[4*
b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])] - Sqrt[a]*ArcTanh
[(a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*I)*(e + f*x)))/(Sqrt[a]*Sqrt[4*b*E
^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])])*Sqrt[a + b*Sec[e +
f*x]^2])/(Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)
```

3.237.3 Rubi [A] (verified)

Time $= 0.24$ (sec) , antiderivative size $= 77$, normalized size of antiderivative $= 0.97$, number of steps used $= 8$, number of rules used $= 7$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used $= \{3042, 4616, 301, 224, 219, 291, 216\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a + b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \sqrt{a + b\sec(e+fx)^2}dx$$

$$\downarrow 4616$$

$$\frac{\int \frac{\sqrt{b\tan^2(e+fx)+a+b}}{\tan^2(e+fx)+1} d\tan(e+fx)}{f}$$

$$\downarrow 301$$

$$\frac{b\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}} d\tan(e+fx) + a\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}} d\tan(e+fx)}{f}$$

$$\downarrow 224$$

$$\frac{a\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}} d\tan(e+fx) + b\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}} d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{f}$$

$$\downarrow 219$$

$$\frac{a\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}} d\tan(e+fx) + \sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}$$

$$\downarrow 291$$

$$\frac{a\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1} d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}} + \sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}$$

$$\downarrow 216$$

$$\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right) + \sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}$$

input
```
Int[Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]] + S
qrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
```

3.237.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

3.237.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(67) = 134$.

Time = 2.11 (sec) , antiderivative size = 351, normalized size of antiderivative = 4.44

method	result
default	$\frac{\sqrt{a+b\sec(fx+e)^2}\left(\sqrt{b}\ln\left(-\frac{4\left(\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{b}\cos(fx+e)+\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-\sin(fx+e)a+a+b\right)}{\sin(fx+e)-1}\right)\sqrt{-a}+\sqrt{b}\ln\left(\frac{4\sqrt{\frac{b+a\cos(}{(1+\cos(f}\right.$ … 2

input
```
int((a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f/(-a)^(1/2)*(a+b*sec(f*x+e)^2)^(1/2)*(b^(1/2)*ln(-4*(((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*(-a)^(1/2)+b^(1
/2)*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b
^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(
f*x+e)+1))*(-a)^(1/2)+2*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)-4*sin(f*x+e)*a)*a)*cos(f*x+e)/(1+cos(f*x+e))/((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)
```

3.237.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(67) = 134$.

Time = 0.50 (sec) , antiderivative size = 1227, normalized size of antiderivative = 15.53

$$\int \sqrt{a + b\sec^2(e + fx)}\, dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)
^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 7
0*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*
x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*
(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b
^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin
(f*x + e)) + 2*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b -
b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*
x + e)^4))/f, 1/8*(4*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*co
s(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos
(f*x + e)^2 + b^2)*sin(f*x + e))) + sqrt(-a)*log(128*a^4*cos(f*x + e)^8 -
256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f
*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3
*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^
3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3
- (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)))/f, -1/4*(sqrt(a)*arctan(1/4*(8*a
^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos
(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*...
```

3.237.6 Sympy [F]

$$\int \sqrt{a + b\sec^2(e + fx)}\, dx = \int \sqrt{a + b\sec^2(e + fx)}\, dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2), x)
```

3.237.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.80 (sec) , antiderivative size = 3227, normalized size of antiderivative = 40.85

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*(2*sqrt(a)*b^(3/2)*arctan2(a*sin(2*f*x + 2*e) + (a^2*cos(4*f*x + 4*e)
^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 +
 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^
2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*c
os(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*sin(1/2*
arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4
*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)), a*cos(2*f*x + 2*e) + (a^2*cos(4*
f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x
 + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*
a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x
 + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a
)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos
(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) + a + 2*b) + a^(3/2)*sq
rt(b)*arctan2(2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2
 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin
(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2
 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos
(2*f*x + 2*e))^(1/4)*sqrt(a)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2
*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) +
a)), 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*...
```

3.237.8 Giac [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec\left(fx+e\right)^2+a}\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a), x)
```

3.237.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{a+\frac{b}{\cos(e+fx)^2}}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int((a + b/cos(e + f*x)^2)^(1/2), x)
```

3.238 $\int \cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

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3.238.1 Optimal result

Integrand size = 25, antiderivative size = 82

$$\int \cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{(a+b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2\sqrt{a}f} + \frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f}$$

output
```
1/2*(a+b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/a^(1/2)+
1/2*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/f
```

3.238.2 Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.66

$$\int \cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(2\sqrt{a+b}\arcsin\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)+\sqrt{2}\sqrt{a}\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}}\sin(e+fx)\right)}{2\sqrt{2}\sqrt{a}f\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}}}$$

input
```
Integrate[Cos[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*(2*Sqrt[a + b]*ArcSin[(Sqrt[a]*Si
n[e + f*x])/Sqrt[a + b]] + Sqrt[2]*Sqrt[a]*Sqrt[(a + 2*b + a*Cos[2*(e + f*
x)])/(a + b)]*Sin[e + f*x]))/(2*Sqrt[2]*Sqrt[a]*f*Sqrt[(a + 2*b + a*Cos[2*
(e + f*x)])/(a + b)])
```

3.238.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4634, 292, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sqrt{a+b\sec(e+fx)^2}}{\sec(e+fx)^2}dx$$

$$\downarrow 4634$$

$$\frac{\int \frac{\sqrt{b\tan^2(e+fx)+a+b}}{(\tan^2(e+fx)+1)^2}d\tan(e+fx)}{f}$$

$$\downarrow 292$$

$$\frac{\frac{1}{2}(a+b)\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx) + \frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2(\tan^2(e+fx)+1)}}{f}$$

$$\downarrow 291$$

$$\frac{\frac{1}{2}(a+b)\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}} + \frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2(\tan^2(e+fx)+1)}}{f}$$

$$\downarrow 216$$

$$\frac{\frac{(a+b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2\sqrt{a}} + \frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2(\tan^2(e+fx)+1)}}{f}$$

input
```
Int[Cos[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(((a + b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(
2*Sqrt[a]) + (Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*(1 + Tan[e +
 f*x]^2)))/f
```

3.238.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 292
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> Si
mp[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] - Simp[c*(q/(
a*(p + 1)))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1), x], x] /; FreeQ[
{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q + 1) + 1, 0] && Gt
Q[q, 0] && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.238.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 312 vs. $2(70) = 140$.

Time = 2.81 (sec) , antiderivative size = 313, normalized size of antiderivative = 3.82

method	result
default	$\frac{\left(\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{-a}\cos(fx+e)\sin(fx+e)+\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sin(fx+e)+\ln\left(4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)+4\sqrt{-a}\right.\right.}{2f\sqrt{-a}\,(1}$

input
```
int(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f/(-a)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*c
os(f*x+e)*sin(f*x+e)+(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*sin(f*x+e)+ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*c
os(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f
*x+e)*a)*a+ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos
(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x
+e)*a)*b)*(a+b*sec(f*x+e)^2)^(1/2)*cos(f*x+e)/(1+cos(f*x+e))/((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)
```

3.238.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(70) = 140$.

Time = 0.42 (sec) , antiderivative size = 499, normalized size of antiderivative = 6.09

$$\int \cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$= \left[\frac{8\,a\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)\sin(fx+e) - \sqrt{-a}(a+b)\log\left(128\,a^4\cos(fx+e)^8 - 256\,(a^4-a^3b)\cos\right.\right.}{}$$

input
```
integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*
x + e) - sqrt(-a)*(a + b)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*c
os(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 2
8*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*
b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x
+ e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b +
7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)*sin(f*x + e)))/(a*f), 1/8*(4*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)*cos(f*x + e)*sin(f*x + e) - (a + b)*sqrt(a)*arctan(1/4*(8*a^2*cos
(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x +
 e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x +
 e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))))/(a
*f)]
```

3.238.6 Sympy [F]

$$\int \cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{a+b\sec^2(e+fx)}\cos^2(e+fx)\,dx$$

input
```
integrate(cos(f*x+e)**2*(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*cos(e + f*x)**2, x)
```

3.238.7 Maxima [F]

$$\int \cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\cos(fx+e)^2\,dx$$

input
```
integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^2, x)
```

3.238.8 Giac [F]

$$\int \cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\cos(fx+e)^2\,dx$$

input
```
integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^2, x)
```

3.238.9 Mupad [F(-1)]

Timed out.

$$\int \cos^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \cos(e+f\,x)^2\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}\,dx$$

input
```
int(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.239 $\int \cos^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

3.239.1 Optimal result . . . 1778
3.239.2 Mathematica [A] (verified) . . . 1778
3.239.3 Rubi [A] (verified) . . . 1779
3.239.4 Maple [B] (verified) . . . 1781
3.239.5 Fricas [A] (verification not implemented) . . . 1782
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3.239.7 Maxima [F] . . . 1783
3.239.8 Giac [F] . . . 1784
3.239.9 Mupad [F(-1)] . . . 1784

3.239.1 Optimal result

Integrand size = 25, antiderivative size = 140

$$\begin{aligned}
&\int \cos^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx \\
&= \frac{(3a-b)(a+b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{3/2}f} \\
&\quad + \frac{(3a-b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8af} \\
&\quad + \frac{\cos^3(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{4af}
\end{aligned}$$

output

```
1/8*(3*a-b)*(a+b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^
(3/2)/f+1/8*(3*a-b)*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/a/f+1
/4*cos(f*x+e)^3*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(3/2)/a/f
```

3.239.2 Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.09

$$\begin{aligned}
&\int \cos^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx \\
&= \frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(\sqrt{2}(3a-b)\sqrt{a+b}\arcsin\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)+\sqrt{a}(4a+b+a\cos(2(e+fx))\right.}{8a^{3/2}f\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}}}
\end{aligned}$$

input
```
Integrate[Cos[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*(Sqrt[2]*(3*a - b)*Sqrt[a + b]*Ar
cSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]] + Sqrt[a]*(4*a + b + a*Cos[2*(e +
 f*x)])*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*Sin[e + f*x]))/(8*a^(
3/2)*f*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)])
```

3.239.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 4634, 296, 292, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sqrt{a+b\sec(e+fx)^2}}{\sec(e+fx)^4}dx$$

$$\downarrow 4634$$

$$\frac{\int \frac{\sqrt{b\tan^2(e+fx)+a+b}}{(\tan^2(e+fx)+1)^3}d\tan(e+fx)}{f}$$

$$\downarrow 296$$

$$\frac{\frac{(3a-b)\int \frac{\sqrt{b\tan^2(e+fx)+a+b}}{\left(\tan^2(e+fx)+1\right)^2}d\tan(e+fx)}{4a}+\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4a(\tan^2(e+fx)+1)^2}}{f}$$

$$\downarrow 292$$

$$\frac{\frac{(3a-b)\left(\frac{1}{2}(a+b)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2\left(\tan^2(e+fx)+1\right)}\right)}{4a}+\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4a(\tan^2(e+fx)+1)^2}}{f}$$

$$\downarrow 291$$

$$\frac{(3a-b)\left(\frac{1}{2}(a+b)\int\frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2\left(\tan^2(e+fx)+1\right)}\right)}{4a}+\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4a\left(\tan^2(e+fx)+1\right)^2}$$
$$f$$

$$\downarrow 216$$

$$\frac{(3a-b)\left(\frac{(a+b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2\sqrt{a}}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2\left(\tan^2(e+fx)+1\right)}\right)}{4a}+\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4a\left(\tan^2(e+fx)+1\right)^2}$$
$$f$$

input
```
Int[Cos[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
((Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/2))/(4*a*(1 + Tan[e + f*x]^2)
^2) + ((3*a - b)*((a + b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Ta
n[e + f*x]^2]])/(2*Sqrt[a]) + (Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2]
)/(2*(1 + Tan[e + f*x]^2))))/(4*a))/f
```

3.239.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 292
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> Si
mp[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] - Simp[c*(q/(
a*(p + 1)))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1), x], x] /; FreeQ[
{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q + 1) + 1, 0] && Gt
Q[q, 0] && NeQ[p, -1]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.239.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 586 vs. $2(124) = 248$.

Time = 5.10 (sec) , antiderivative size = 587, normalized size of antiderivative = 4.19

method	result
default	$\frac{\left(2\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{-a}\,a\cos(fx+e)^3\sin(fx+e)+2\cos(fx+e)^2\sin(fx+e)\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{-a}\,a+3\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{-a}\,a\cos$

input
```
int(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/f/a/(-a)^(1/2)*(2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1/
2)*a*cos(f*x+e)^3*sin(f*x+e)+2*cos(f*x+e)^2*sin(f*x+e)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*a+3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*(-a)^(1/2)*a*cos(f*x+e)*sin(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*(-a)^(1/2)*b*cos(f*x+e)*sin(f*x+e)+3*sin(f*x+e)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*a+sin(f*x+e)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*b+3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^2+2*ln(4*(-a)^(1/2)*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a*b-ln(4*(-a)^(1/2)*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*b^2)*(a+b*sec(f*x+e)^2)^(1/2)*cos(
f*x+e)/(1+cos(f*x+e))/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
```

3.239.5 Fricas [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 567, normalized size of antiderivative = 4.05

$$\int \cos^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$= \left[\frac{(3\,a^2+2\,ab-b^2)\sqrt{-a}\log\left(128\,a^4\cos(fx+e)^8-256\,(a^4-a^3b)\cos(fx+e)^6+32\,(5\,a^4-14\,a^3b+5\,a^2\right.\right.$$

$$\left.\left. -\frac{(3\,a^2+2\,ab-b^2)\sqrt{a}\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^5-8\,(a^2-ab)\cos(fx+e)^3+(a^2-6\,ab+b^2)\cos(fx+e)\right)\sqrt{a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\left(2\,a^3\cos(fx+e)^4-a^2b+ab^2-(a^3-3\,a^2b)\cos(fx+e)^2\right)\sin(fx+e)}\right)-4}{32\,a^2f} \right]$$

input
```
integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/64*((3*a^2 + 2*a*b - b^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^
4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)
^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*
a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2
*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3
 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(2*a^2*cos(f*x + e)^3 + (3*a^2 + a*b
)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/
(a^2*f), -1/32*((3*a^2 + 2*a*b - b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x +
e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sq
rt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 -
 a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(2*a^2
*cos(f*x + e)^3 + (3*a^2 + a*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/
cos(f*x + e)^2)*sin(f*x + e))/(a^2*f)]
```

3.239.6 Sympy [F]

$$\int \cos^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{a+b\sec^2(e+fx)}\cos^4(e+fx)\,dx$$

input
```
integrate(cos(f*x+e)**4*(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*cos(e + f*x)**4, x)
```

3.239.7 Maxima [F]

$$\int \cos^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\cos(fx+e)^4\,dx$$

input
```
integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^4, x)
```

3.239.8 Giac [F]

$$\int \cos^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\cos(fx+e)^4\,dx$$

input
```
integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^4, x)
```

3.239.9 Mupad [F(-1)]

Timed out.

$$\int \cos^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \cos(e+f\,x)^4\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}\,dx$$

input
```
int(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.240 $\int \cos^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

3.240.1 Optimal result . . . 1785
3.240.2 Mathematica [C] (warning: unable to verify) . . . 1786
3.240.3 Rubi [A] (verified) . . . 1786
3.240.4 Maple [B] (verified) . . . 1790
3.240.5 Fricas [A] (verification not implemented) . . . 1791
3.240.6 Sympy [F(-1)] . . . 1792
3.240.7 Maxima [F] . . . 1792
3.240.8 Giac [F] . . . 1792
3.240.9 Mupad [F(-1)] . . . 1793

3.240.1 Optimal result

Integrand size = 25, antiderivative size = 196

$$\begin{aligned}
&\int \cos^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx \\
&= \frac{(a+b)\left(5a^2-2ab+b^2\right)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{5/2}f} \\
&\quad + \frac{(3a-b)(5a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48a^2f} \\
&\quad + \frac{(5a+b)\cos^3(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24af} \\
&\quad + \frac{\cos^5(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6f}
\end{aligned}$$

output

```
1/16*(a+b)*(5*a^2-2*a*b+b^2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2
)^(1/2))/a^(5/2)/f+1/48*(3*a-b)*(5*a+3*b)*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan
(f*x+e)^2)^(1/2)/a^2/f+1/24*(5*a+b)*cos(f*x+e)^3*sin(f*x+e)*(a+b+b*tan(f*x
+e)^2)^(1/2)/a/f+1/6*cos(f*x+e)^5*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/f
```

3.240.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 16.88 (sec) , antiderivative size = 1902, normalized size of antiderivative = 9.70

$$\int \cos^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[e + f*x]^6*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(3*(a + b)*AppellF1[1/2, -2, -1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)
/(a + b)]*Cos[e + f*x]^10*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*Sqrt[a + b*Se
c[e + f*x]^2]*Sin[e + f*x])/(f*(3*(a + b)*AppellF1[1/2, -2, -1/2, 3/2, Sin
[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - (a*AppellF1[3/2, -2, 1/2, 5/2,
Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + 4*(a + b)*AppellF1[3/2, -1,
-1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)*((
3*(a + b)*AppellF1[1/2, -2, -1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/
(a + b)]*Cos[e + f*x]^5*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]])/(3*(a + b)*App
ellF1[1/2, -2, -1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - (a
*AppellF1[3/2, -2, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] +
 4*(a + b)*AppellF1[3/2, -1, -1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)
/(a + b)])*Sin[e + f*x]^2) - (12*(a + b)*AppellF1[1/2, -2, -1/2, 3/2, Sin[
e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^3*Sqrt[a + 2*b + a*Co
s[2*(e + f*x)]]*Sin[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -2, -1/2, 3/2, Si
n[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - (a*AppellF1[3/2, -2, 1/2, 5/2,
 Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + 4*(a + b)*AppellF1[3/2, -1,
 -1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2) +
 (3*(a + b)*Cos[e + f*x]^4*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*Sin[e + f*x]
*(-1/3*(a*f*AppellF1[3/2, -2, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)
/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(a + b) - (4*f*AppellF1[3/2, -1, -...
```

3.240.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4634, 314, 25, 402, 25, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sqrt{a+b\sec(e+fx)^2}}{\sec(e+fx)^6}dx$$

$$\downarrow 4634$$

$$\frac{\int \frac{\sqrt{b\tan^2(e+fx)+a+b}}{(\tan^2(e+fx)+1)^4}d\tan(e+fx)}{f}$$

$$\downarrow 314$$

$$\frac{\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{6(\tan^2(e+fx)+1)^3}-\frac{1}{6}\int -\frac{4b\tan^2(e+fx)+5(a+b)}{(\tan^2(e+fx)+1)^3\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{f}$$

$$\downarrow 25$$

$$\frac{\frac{1}{6}\int \frac{4b\tan^2(e+fx)+5(a+b)}{(\tan^2(e+fx)+1)^3\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{6(\tan^2(e+fx)+1)^3}}{f}$$

$$\downarrow 402$$

$$\frac{\frac{1}{6}\left(\frac{(5a+b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4a(\tan^2(e+fx)+1)^2}-\frac{\int -\frac{2b(5a+b)\tan^2(e+fx)+(15a-b)(a+b)}{\left(\tan^2(e+fx)+1\right)^2\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{4a}\right)+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{6(\tan^2(e+fx)+1)^3}}{f}$$

$$\downarrow 25$$

$$\frac{\frac{1}{6}\left(\frac{\int \frac{2b(5a+b)\tan^2(e+fx)+(15a-b)(a+b)}{\left(\tan^2(e+fx)+1\right)^2\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{4a}+\frac{(5a+b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4a(\tan^2(e+fx)+1)^2}\right)+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{6(\tan^2(e+fx)+1)^3}}{f}$$

$$\downarrow 402$$

$$\frac{\frac{1}{6}\left(\frac{\frac{(3a-b)(5a+3b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a\left(\tan^2(e+fx)+1\right)}-\frac{\int -\frac{3(a+b)\left(5a^2-2ba+b^2\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2a}}{4a}+\frac{(5a+b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4a(\tan^2(e+fx)+1)^2}\right)}{f}$$

$$\downarrow 27$$

$$\frac{1}{6}\left(\frac{\frac{3(a+b)\left(5a^2-2ab+b^2\right)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2a}+\frac{(3a-b)(5a+3b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a\left(\tan^2(e+fx)+1\right)}}{4a}+\frac{(5a+b)\tan(e+fx)\sqrt{a+}}{4a(\tan^2(e+fx)}\right)$$

$$f$$

$$\downarrow 291$$

$$\frac{1}{6}\left(\frac{\frac{3(a+b)\left(5a^2-2ab+b^2\right)\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{2a}+\frac{(3a-b)(5a+3b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a\left(\tan^2(e+fx)+1\right)}}{4a}+\frac{(5a+b)\tan(e+fx)\sqrt{a+b\tan}}{4a(\tan^2(e+fx)+1}\right)$$

$$f$$

$$\downarrow 216$$

$$\frac{1}{6}\left(\frac{\frac{3(a+b)\left(5a^2-2ab+b^2\right)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2a^{3/2}}+\frac{(3a-b)(5a+3b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a\left(\tan^2(e+fx)+1\right)}}{4a}+\frac{(5a+b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4a(\tan^2(e+fx)+1)^2}\right)$$

$$f$$

input
```
Int[Cos[e + f*x]^6*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
((Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(6*(1 + Tan[e + f*x]^2)^3)
+ (((5*a + b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(4*a*(1 + Tan[e
 + f*x]^2)^2) + ((3*(a + b)*(5*a^2 - 2*a*b + b^2)*ArcTan[(Sqrt[a]*Tan[e +
f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(2*a^(3/2)) + ((3*a - b)*(5*a + 3*b
)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*a*(1 + Tan[e + f*x]^2)))
/(4*a))/6)/f
```

3.240.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 314
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] + Simp[1/(2*a*
(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*p + 3) + d
*(2*(p + q + 1) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, 2, p, q,
 x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.240.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 969 vs. $2(176) = 352$.

Time = 7.74 (sec) , antiderivative size = 970, normalized size of antiderivative = 4.95

method	result	size
default	Expression too large to display	970

input
```
int(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/48/f/a^2/(-a)^(1/2)*(8*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*a^2*cos(f*x+e)^5*sin(f*x+e)+8*(-a)^(1/2)*sin(f*x+e)*cos(f*x+e)^4*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2+10*(-a)^(1/2)*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*cos(f*x+e)^3*sin(f*x+e)+2*(-a)^(1/2)*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b*cos(f*x+e)^3*sin(f*x+e)+10*(-a
)^(1/2)*sin(f*x+e)*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*a^2+2*(-a)^(1/2)*sin(f*x+e)*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*a*b+15*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*a^2*cos(f*x+e)*sin(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*a*b*cos(f*x+e)*sin(f*x+e)-3*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*b^2*cos(f*x+e)*sin(f*x+e)+15*(-a)^(1/2)*sin(f*x+e)*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2+4*(-a)^(1/2)*sin(f*x+e)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b-3*(-a)^(1/2)*sin(f*x+e)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2+15*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^3+9*ln(4*(-a)^(1/2)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^2*b-3*ln(4*(-a)^(1/2)*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a*b^2+3*ln(4*(-a)^(1/2)*((...
```

3.240.5 Fricas [A] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 641, normalized size of antiderivative = 3.27

$$\int \cos^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$= \left[-\frac{3\,(5\,a^3+3\,a^2b-ab^2+b^3)\sqrt{-a}\log\left(128\,a^4\cos\left(fx+e\right)^8-256\,(a^4-a^3b)\cos\left(fx+e\right)^6+32\,(5\,a^4-14\right.}{} \right.$$

$$-\frac{3\,(5\,a^3+3\,a^2b-ab^2+b^3)\sqrt{a}\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^5-8\,(a^2-ab)\cos(fx+e)^3+(a^2-6\,ab+b^2)\cos(fx+e)\right)\sqrt{a}\sqrt{\frac{a\cos(fx+e)^2}{\cos(fx+e)}}}{4\left(2\,a^3\cos(fx+e)^4-a^2b+ab^2-(a^3-3\,a^2b)\cos(fx+e)^2\right)\sin(fx+e)}\right.}{}$$

input
```
integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/384*(3*(5*a^3 + 3*a^2*b - a*b^2 + b^3)*sqrt(-a)*log(128*a^4*cos(f*x +
e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2
)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4
- 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 -
 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x
+ e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 8*(8*a^3*cos(f*x + e)^5
+ 2*(5*a^3 + a^2*b)*cos(f*x + e)^3 + (15*a^3 + 4*a^2*b - 3*a*b^2)*cos(f*x
+ e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^3*f), -
1/192*(3*(5*a^3 + 3*a^2*b - a*b^2 + b^3)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x
 + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))
*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^
4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(8*
a^3*cos(f*x + e)^5 + 2*(5*a^3 + a^2*b)*cos(f*x + e)^3 + (15*a^3 + 4*a^2*b
- 3*a*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f
*x + e))/(a^3*f)]
```

3.240.6 Sympy [F(-1)]

Timed out.

$$\int \cos^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**6*(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Timed out
```

3.240.7 Maxima [F]

$$\int \cos^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\cos(fx+e)^6\,dx$$

input
```
integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^6, x)
```

3.240.8 Giac [F]

$$\int \cos^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\cos(fx+e)^6\,dx$$

input
```
integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^6, x)
```

3.240.9 Mupad [F(-1)]

Timed out.

$$\int \cos^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \cos(e+f\,x)^6\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}\,dx$$

input
```
int(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.241 $\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

3.241.1 Optimal result 1794
3.241.2 Mathematica [F] 1795
3.241.3 Rubi [A] (verified) 1795
3.241.4 Maple [C] (warning: unable to verify) 1802
3.241.5 Fricas [C] (verification not implemented) 1802
3.241.6 Sympy [F] 1803
3.241.7 Maxima [F] 1804
3.241.8 Giac [F] 1804
3.241.9 Mupad [F(-1)] 1804

3.241.1 Optimal result

Integrand size = 25, antiderivative size = 450

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx =$$

$$-\frac{2(a+2b)\left(a^2-4ab-4b^2\right)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{35b^2f}$$

$$+\frac{2(a+2b)\left(a^2-4ab-4b^2\right)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\,\middle|\,\tfrac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+f\right.}}{35b^2f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}$$

$$-\frac{(a+b)\left(a^2-16ab-16b^2\right)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\tfrac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\left(a+b-a\s\right.}}{35bf\left(a+b-a\sin^2(e+fx)\right)}$$

$$+\frac{\left(a^2+11ab+8b^2\right)\sec(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{35bf}$$

$$+\frac{2(4a+3b)\sec^3(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{35f}$$

$$+\frac{b\sec^5(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{7f}$$

output
```
-2/35*(a+2*b)*(a^2-4*a*b-4*b^2)*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)
^2))^(1/2)/b^2/f+2/35*(a+2*b)*(a^2-4*a*b-4*b^2)*EllipticE(sin(f*x+e),(a/(a
+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)
/b^2/f/(1-a*sin(f*x+e)^2/(a+b))^(1/2)-1/35*(a+b)*(a^2-16*a*b-16*b^2)*Ellip
ticF(sin(f*x+e),(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a+b-a
*sin(f*x+e)^2))^(1/2)*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/b/f/(a+b-a*sin(f*x+e)
^2)+1/35*(a^2+11*a*b+8*b^2)*sec(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))
^(1/2)*tan(f*x+e)/b/f+2/35*(4*a+3*b)*sec(f*x+e)^3*(sec(f*x+e)^2*(a+b-a*sin
(f*x+e)^2))^(1/2)*tan(f*x+e)/f+1/7*b*sec(f*x+e)^5*(sec(f*x+e)^2*(a+b-a*sin
(f*x+e)^2))^(1/2)*tan(f*x+e)/f
```

3.241.2 Mathematica [F]

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$$

input
```
Integrate[Sec[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
Integrate[Sec[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2), x]
```

3.241.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.05, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.720$, Rules used = {3042, 4636, 2057, 2058, 315, 25, 402, 27, 402, 25, 402, 25, 27, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$$

$$\downarrow \text{3042}$$

$$\int \sec(e+fx)^5\left(a+b\sec(e+fx)^2\right)^{3/2}dx$$

$$\downarrow \text{4636}$$

$$\frac{\int \frac{\left(a+\frac{b}{1-\sin^2(e+fx)}\right)^{3/2}}{\left(1-\sin^2(e+fx)\right)^3} d\sin(e+fx)}{f}$$

$$\downarrow 2057$$

$$\frac{\int \frac{\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^{3/2}}{\left(1-\sin^2(e+fx)\right)^3} d\sin(e+fx)}{f}$$

$$\downarrow 2058$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\int \frac{\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}{\left(1-\sin^2(e+fx)\right)^{9/2}} d\sin(e+fx)}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\downarrow 315$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{b\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{7\left(1-\sin^2(e+fx)\right)^{7/2}} - \frac{1}{7}\int -\frac{(a+b)(7a+6b)-a(7a+5b)\sin^2(e+fx)}{\left(1-\sin^2(e+fx)\right)^{7/2}\sqrt{-a\sin^2(e+fx)+a+b}} d\sin\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\downarrow 25$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{7}\int \frac{(a+b)(7a+6b)-a(7a+5b)\sin^2(e+fx)}{\left(1-\sin^2(e+fx)\right)^{7/2}\sqrt{-a\sin^2(e+fx)+a+b}} d\sin(e+fx) + \frac{b\sin(e+fx)\sqrt{-a\sin^2(e+\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\downarrow 402$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{7}\left(\frac{\int \frac{3b\left((a+b)(9a+8b)-2a(4a+3b)\sin^2(e+fx)\right)}{\left(1-\sin^2(e+fx)\right)^{5/2}\sqrt{-a\sin^2(e+fx)+a+b}} d\sin(e+fx)}{5b} + \frac{2(4a+3b)\sin(e+fx)\sqrt{-a\sin^2(e\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\downarrow 27$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{7}\left(\frac{3}{5}\int \frac{(a+b)(9a+8b)-2a(4a+3b)\sin^2(e+fx)}{\left(1-\sin^2(e+fx)\right)^{5/2}\sqrt{-a\sin^2(e+fx)+a+b}} d\sin(e+fx) + \frac{2(4a+3b)\sin(e+fx)\sqrt{}}{5(1-\sin^2(}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\downarrow 402$$

3.241. $\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2} dx$

$$\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{7}\left(\frac{3}{5}\left(\frac{\int -\frac{a\left(a^2+11ba+8b^2\right)\sin^2(e+fx)+(a+b)\left(a^2-16ba-16b^2\right)}{\left(1-\sin^2(e+fx)\right)^{3/2}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{3b}+\frac{(a^2+11ab+8b^2}{3}\right.\right.\right.$$

$$f\sqrt{-a\sin^2(e+fx)}$$

$$\downarrow 25$$

$$\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{7}\left(\frac{3}{5}\left(\frac{(a^2+11ab+8b^2)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3b\left(1-\sin^2(e+fx)\right)^{3/2}}-\frac{\int \frac{a\left(a^2+11ba+8b^2\right)\sin^2(e+fx)+(}{\left(1-\sin^2(e+fx)\right)^{3/2}\sqrt{-a}}\right.\right.\right.$$

$$f\sqrt{-a\sin^2(e+fx)}$$

$$\downarrow 402$$

$$\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{7}\left(\frac{3}{5}\left(\frac{(a^2+11ab+8b^2)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3b\left(1-\sin^2(e+fx)\right)^{3/2}}-\frac{\int -\frac{a\left((a+b)\left(2a^2-5ba-8b^2\right)-2(a\right.}{\sqrt{1-\sin^2(e+fx)}\sqrt{}}\right.\right.\right.$$

$$\downarrow 25$$

$$\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{7}\left(\frac{3}{5}\left(\frac{(a^2+11ab+8b^2)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3b\left(1-\sin^2(e+fx)\right)^{3/2}}-\frac{2(a+2b)\left(a^2-4ab-4b^2\right)\sin(e+fx)}{b\sqrt{1-\sin^2(e+\cdot}}\right.\right.\right.$$

$$\downarrow 27$$

$$\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{7}\left(\frac{3}{5}\left(\frac{(a^2+11ab+8b^2)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3b\left(1-\sin^2(e+fx)\right)^{3/2}}-\frac{2(a+2b)\left(a^2-4ab-4b^2\right)\sin(e+fx)}{b\sqrt{1-\sin^2(e+\cdot}}\right.\right.\right.$$

$$\downarrow 399$$

$$\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{7}\left(\frac{3}{5}\left(\frac{\left(a^2+11ab+8b^2\right)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3b\left(1-\sin^2(e+fx)\right)^{3/2}}-\frac{2(a+2b)\left(a^2-4ab-4b^2\right)\sin(e+fx)}{b\sqrt{1-\sin^2(e+\right.\right.\right.$$

$$\downarrow 323$$

$$\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{7}\left(\frac{3}{5}\left(\frac{\left(a^2+11ab+8b^2\right)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3b\left(1-\sin^2(e+fx)\right)^{3/2}}-\frac{2(a+2b)\left(a^2-4ab-4b^2\right)\sin(e+fx)}{b\sqrt{1-\sin^2(e+\right.\right.\right.$$

$$\downarrow 321$$

$$\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{7}\left(\frac{3}{5}\left(\frac{\left(a^2+11ab+8b^2\right)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3b\left(1-\sin^2(e+fx)\right)^{3/2}}-\frac{2(a+2b)\left(a^2-4ab-4b^2\right)\sin(e+fx)}{b\sqrt{1-\sin^2(e+\right.\right.\right.$$

$$\downarrow 330$$

$$\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{7}\left(\frac{3}{5}\left(\frac{\left(a^2+11ab+8b^2\right)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3b\left(1-\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{2(a+2b)\left(a^2-4ab-4b^2\right)\sin(e+fx)}{b\sqrt{1-\sin^2(e+}}}\right.\right.\right.$$

$$\downarrow 327$$

$$\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{7}\left(\frac{3}{5}\left(\frac{\left(a^2+11ab+8b^2\right)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3b\left(1-\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{2(a+2b)\left(a^2-4ab-4b^2\right)\sin(e+fx)}{b\sqrt{1-\sin^2(e+}}}\right.\right.\right.$$

input

```
Int[Sec[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output

```
(Sqrt[1 - Sin[e + f*x]^2]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(1 - Sin[e + f*x
]^2)]*((b*Sin[e + f*x]*Sqrt[a + b - a*Sin[e + f*x]^2])/(7*(1 - Sin[e + f*x
]^2)^(7/2)) + ((2*(4*a + 3*b)*Sin[e + f*x]*Sqrt[a + b - a*Sin[e + f*x]^2])
/(5*(1 - Sin[e + f*x]^2)^(5/2)) + (3*(((a^2 + 11*a*b + 8*b^2)*Sin[e + f*x]
*Sqrt[a + b - a*Sin[e + f*x]^2])/(3*b*(1 - Sin[e + f*x]^2)^(3/2)) - ((2*(a
 + 2*b)*(a^2 - 4*a*b - 4*b^2)*Sin[e + f*x]*Sqrt[a + b - a*Sin[e + f*x]^2])
/(b*Sqrt[1 - Sin[e + f*x]^2]) - (a*((2*(a + 2*b)*(a^2 - 4*a*b - 4*b^2)*Ell
ipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b - a*Sin[e + f*x]^2])/(a
*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - (b*(a + b)*(a^2 - 16*a*b - 16*b^2
)*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(
a + b)])/(a*Sqrt[a + b - a*Sin[e + f*x]^2])))/b)/(3*b)))/5)/7))/(f*Sqrt[a
+ b - a*Sin[e + f*x]^2])
```

3.241. $\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

3.241.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

3.241.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 16.32 (sec) , antiderivative size = 11007, normalized size of antiderivative = 24.46

method	result	size
default	Expression too large to display	11007

input
```
int(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.241.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 981, normalized size of antiderivative = 2.18

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/35*((2*(-I*a^4 + 2*I*a^3*b + 12*I*a^2*b^2 + 8*I*a*b^3)*sqrt(a)*sqrt((a*b
 + b^2)/a^2)*cos(f*x + e)^6 - (-I*a^4 + 16*I*a^2*b^2 + 32*I*a*b^3 + 16*I*b
^4)*sqrt(a)*cos(f*x + e)^6)*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*
elliptic_e(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x +
 e) + I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b +
 b^2)/a^2))/a^2) + (2*(I*a^4 - 2*I*a^3*b - 12*I*a^2*b^2 - 8*I*a*b^3)*sqrt(
a)*sqrt((a*b + b^2)/a^2)*cos(f*x + e)^6 - (I*a^4 - 16*I*a^2*b^2 - 32*I*a*b
^3 - 16*I*b^4)*sqrt(a)*cos(f*x + e)^6)*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a
 - 2*b)/a)*elliptic_e(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)
*(cos(f*x + e) - I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*
sqrt((a*b + b^2)/a^2))/a^2) + (2*(-I*a^3*b - 11*I*a^2*b^2 - 8*I*a*b^3)*sqr
t(a)*sqrt((a*b + b^2)/a^2)*cos(f*x + e)^6 - (2*I*a^4 + I*a^3*b - 19*I*a^2*
b^2 - 34*I*a*b^3 - 16*I*b^4)*sqrt(a)*cos(f*x + e)^6)*sqrt((2*a*sqrt((a*b +
 b^2)/a^2) - a - 2*b)/a)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2)
 - a - 2*b)/a)*(cos(f*x + e) + I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*
(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + (2*(I*a^3*b + 11*I*a^2*b^2 + 8
*I*a*b^3)*sqrt(a)*sqrt((a*b + b^2)/a^2)*cos(f*x + e)^6 - (-2*I*a^4 - I*a^3
*b + 19*I*a^2*b^2 + 34*I*a*b^3 + 16*I*b^4)*sqrt(a)*cos(f*x + e)^6)*sqrt((2
*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_f(arcsin(sqrt((2*a*sqrt((a
*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) - I*sin(f*x + e))), (a^2 + 8...
```

3.241.6 Sympy [F]

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(a+b\sec^2(e+fx)\right)^{\frac{3}{2}}\sec^5(e+fx)\,dx$$

input
```
integrate(sec(f*x+e)**5*(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(3/2)*sec(e + f*x)**5, x)
```

3.241.7 Maxima [F]

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^5\,dx$$

input
```
integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^5, x)
```

3.241.8 Giac [F]

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^5\,dx$$

input
```
integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^5, x)
```

3.241.9 Mupad [F(-1)]

Timed out.

$$\int \sec^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \frac{\left(a+\frac{b}{\cos(e+fx)^2}\right)^{3/2}}{\cos\left(e+fx\right)^5}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x)^5,x)
```

output
```
int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x)^5, x)
```

3.242 $\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

3.242.1 Optimal result 1805
3.242.2 Mathematica [F] 1806
3.242.3 Rubi [A] (verified) 1806
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3.242.5 Fricas [C] (verification not implemented) 1812
3.242.6 Sympy [F] 1813
3.242.7 Maxima [F] 1813
3.242.8 Giac [F] 1813
3.242.9 Mupad [F(-1)] 1814

3.242.1 Optimal result

Integrand size = 25, antiderivative size = 371

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\left(3a^2+13ab+8b^2\right)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{15bf}$$
$$-\frac{\left(3a^2+13ab+8b^2\right)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\,\middle|\,\tfrac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{15bf\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}$$
$$+\frac{(a+b)(9a+8b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\tfrac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{15f\left(a+b-a\sin^2(e+fx)\right)}$$
$$+\frac{2(3a+2b)\sec(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{15f}$$
$$+\frac{b\sec^3(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{5f}$$

output

```
1/15*(3*a^2+13*a*b+8*b^2)*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(
1/2)/b/f-1/15*(3*a^2+13*a*b+8*b^2)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(
cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/b/f/(1-a*sin
(f*x+e)^2/(a+b))^(1/2)+1/15*(a+b)*(9*a+8*b)*EllipticF(sin(f*x+e),(a/(a+b))
^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*(1-
a*sin(f*x+e)^2/(a+b))^(1/2)/f/(a+b-a*sin(f*x+e)^2)+2/15*(3*a+2*b)*sec(f*x+
e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*tan(f*x+e)/f+1/5*b*sec(f*x+e)
^3*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*tan(f*x+e)/f
```

3.242.2 Mathematica [F]

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$$

input
```
Integrate[Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
Integrate[Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2), x]
```

3.242.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.05, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3042, 4636, 2057, 2058, 315, 25, 402, 27, 402, 25, 27, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int \sec(e+fx)^3\left(a+b\sec(e+fx)^2\right)^{3/2}\,dx$$

$$\downarrow 4636$$

$$\frac{\int \frac{\left(a+\frac{b}{1-\sin^2(e+fx)}\right)^{3/2}}{\left(1-\sin^2(e+fx)\right)^2}\,d\sin(e+fx)}{f}$$

$$\downarrow 2057$$

$$\frac{\int \frac{\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^{3/2}}{\left(1-\sin^2(e+fx)\right)^2}\,d\sin(e+fx)}{f}$$

$$\downarrow 2058$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\int \frac{\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}{\left(1-\sin^2(e+fx)\right)^{7/2}}\,d\sin(e+fx)}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\downarrow 315$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{b\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{5(1-\sin^2(e+fx))^{5/2}}-\frac{1}{5}\int-\frac{(a+b)(5a+4b)-a(5a+3b)\sin^2(e+fx)}{(1-\sin^2(e+fx))^{5/2}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\downarrow 25$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\int\frac{(a+b)(5a+4b)-a(5a+3b)\sin^2(e+fx)}{(1-\sin^2(e+fx))^{5/2}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)+\frac{b\sin(e+fx)\sqrt{-a\sin^2(e+\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\downarrow 402$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\left(\frac{\int\frac{b\left((a+b)(9a+8b)-2a(3a+2b)\sin^2(e+fx)\right)}{\left(1-\sin^2(e+fx)\right)^{3/2}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{3b}+\frac{2(3a+2b)\sin(e+fx)\sqrt{-a\sin^2(e\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\downarrow 27$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\left(\frac{1}{3}\int\frac{(a+b)(9a+8b)-2a(3a+2b)\sin^2(e+fx)}{(1-\sin^2(e+fx))^{3/2}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)+\frac{2(3a+2b)\sin(e+fx)\sqrt{}}{3(1-\sin^2(}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\downarrow 402$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{\int-\frac{a\left((a+b)(3a+4b)-\left(3a^2+13ba+8b^2\right)\sin^2(e+fx)\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{b}+\frac{\left(3a^2+13ab+8b^2\right)\sin}{b\sqrt{1}}\right.\right.\right.}{f\sqrt{-a\sin^2(e+fx)+}}$$

$$\downarrow 25$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{\left(3a^2+13ab+8b^2\right)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}-\frac{\int\frac{a\left((a+b)(3a+4b)-\left(3a^2+13ba+\right.\right.}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2}}\right.\right.\right.}{f\sqrt{-a\sin^2(e+fx)+}}$$

$$\downarrow 27$$

3.242. $\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

$$\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{\left(3a^2+13ab+8b^2\right)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}-\frac{a\int\frac{(a+b)(3a+4b)-\left(3a^2+13ba+\right.}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2}}}{b}\right.\right.\right.$$

$$f\sqrt{-a\sin^2(e+fx)+a}$$

$$\downarrow 399$$

$$\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{\left(3a^2+13ab+8b^2\right)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}-a\left(\frac{\left(3a^2+13ab+8b^2\right)\int\frac{\sqrt{-a\sin^2}}{\sqrt{1-s}}}{a}\right.\right.\right.\right.$$

$$\downarrow 323$$

$$\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{\left(3a^2+13ab+8b^2\right)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}-a\left(\frac{\left(3a^2+13ab+8b^2\right)\int\frac{\sqrt{-a\sin^2}}{\sqrt{1-s}}}{a}\right.\right.\right.\right.$$

$$\downarrow 321$$

$$\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{\left(3a^2+13ab+8b^2\right)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}-a\left(\frac{\left(3a^2+13ab+8b^2\right)\int\frac{\sqrt{-a\sin^2}}{\sqrt{1-s}}}{a}\right.\right.\right.\right.$$

$$\downarrow 330$$

$$\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{\left(3a^2+13ab+8b^2\right)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}-\frac{a\left(\frac{\left(3a^2+13ab+8b^2\right)\sqrt{-a\sin^2(e}}{a\sqrt{}}\right.\right.\right.\right.$$

$$\downarrow 327$$

$$\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{\left(3a^2+13ab+8b^2\right)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}-\frac{a\left(\frac{\left(3a^2+13ab+8b^2\right)\sqrt{-a\sin^2(e}}{a\sqrt{1}}\right.\right.\right.\right.$$

input
```
Int[Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[1 - Sin[e + f*x]^2]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(1 - Sin[e + f*x
]^2)]*((b*Sin[e + f*x]*Sqrt[a + b - a*Sin[e + f*x]^2])/(5*(1 - Sin[e + f*x
]^2)^(5/2)) + ((2*(3*a + 2*b)*Sin[e + f*x]*Sqrt[a + b - a*Sin[e + f*x]^2])
/(3*(1 - Sin[e + f*x]^2)^(3/2)) + (((3*a^2 + 13*a*b + 8*b^2)*Sin[e + f*x]*
Sqrt[a + b - a*Sin[e + f*x]^2])/(b*Sqrt[1 - Sin[e + f*x]^2]) - (a*(((3*a^2
 + 13*a*b + 8*b^2)*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b -
 a*Sin[e + f*x]^2])/(a*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - (b*(a + b)*
(9*a + 8*b)*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e +
 f*x]^2)/(a + b)])/(a*Sqrt[a + b - a*Sin[e + f*x]^2])))/b)/3)/5))/(f*Sqrt[
a + b - a*Sin[e + f*x]^2])
```

3.242.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

3.242.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 13.87 (sec) , antiderivative size = 8946, normalized size of antiderivative = 24.11

method	result	size
default	Expression too large to display	8946

input
```
int(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.242.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 890, normalized size of antiderivative = 2.40

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\left(2\left(3i\,a^3+13i\,a^2b+8i\,ab^2\right)\sqrt{a}\sqrt{\frac{ab+b^2}{a^2}}\cos\left(fx+e\right)^4-\left(3i\,a^3+19i\,a^2b+34i\,ab^2+16i\,b^3\right)\sqrt{a}\right.}{}$$

input
```
integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/30*((2*(3*I*a^3 + 13*I*a^2*b + 8*I*a*b^2)*sqrt(a)*sqrt((a*b + b^2)/a^2)*
cos(f*x + e)^4 - (3*I*a^3 + 19*I*a^2*b + 34*I*a*b^2 + 16*I*b^3)*sqrt(a)*co
s(f*x + e)^4)*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_e(arc
sin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) + I*sin(f*
x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^
2) + (2*(-3*I*a^3 - 13*I*a^2*b - 8*I*a*b^2)*sqrt(a)*sqrt((a*b + b^2)/a^2)*
cos(f*x + e)^4 - (-3*I*a^3 - 19*I*a^2*b - 34*I*a*b^2 - 16*I*b^3)*sqrt(a)*c
os(f*x + e)^4)*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_e(ar
csin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) - I*sin(f
*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a
^2) - 2*(4*(3*I*a^2*b + 2*I*a*b^2)*sqrt(a)*sqrt((a*b + b^2)/a^2)*cos(f*x +
 e)^4 + (-3*I*a^3 - 13*I*a^2*b - 18*I*a*b^2 - 8*I*b^3)*sqrt(a)*cos(f*x + e
)^4)*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_f(arcsin(sqrt(
(2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) + I*sin(f*x + e))),
 (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) - 2*(4
*(-3*I*a^2*b - 2*I*a*b^2)*sqrt(a)*sqrt((a*b + b^2)/a^2)*cos(f*x + e)^4 + (
3*I*a^3 + 13*I*a^2*b + 18*I*a*b^2 + 8*I*b^3)*sqrt(a)*cos(f*x + e)^4)*sqrt(
(2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_f(arcsin(sqrt((2*a*sqrt(
(a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) - I*sin(f*x + e))), (a^2 + 8*
a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + 2*((3*a^3 +...
```

3.242.6 Sympy [F]

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(a+b\sec^2(e+fx)\right)^{\frac{3}{2}}\sec^3(e+fx)\,dx$$

input
```
integrate(sec(f*x+e)**3*(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(3/2)*sec(e + f*x)**3, x)
```

3.242.7 Maxima [F]

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}\sec(fx+e)^3\,dx$$

input
```
integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^3, x)
```

3.242.8 Giac [F]

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}\sec(fx+e)^3\,dx$$

input
```
integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^3, x)
```

3.242.9 Mupad [F(-1)]

Timed out.

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \frac{\left(a+\frac{b}{\cos(e+fx)^2}\right)^{3/2}}{\cos(e+fx)^3}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x)^3,x)
```

output
```
int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x)^3, x)
```

3.243 $\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

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3.243.1 Optimal result

Integrand size = 23, antiderivative size = 290

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{2(2a+b)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{3f}$$
$$-\frac{2(2a+b)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{3f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}$$
$$+\frac{(a+b)(3a+2b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{3f\left(a+b-a\sin^2(e+fx)\right)}$$
$$+\frac{b\sec(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\tan(e+fx)}{3f}$$

output

```
2/3*(2*a+b)*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/f-2/3*(2*
a+b)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e
)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/f/(1-a*sin(f*x+e)^2/(a+b))^(1/2)+1/3*(a+b)
*(3*a+2*b)*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec
(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/f/(a+
b-a*sin(f*x+e)^2)+1/3*b*sec(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/
2)*tan(f*x+e)/f
```

3.243.2 Mathematica [F]

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$$

input
```
Integrate[Sec[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
Integrate[Sec[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2), x]
```

3.243.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.06, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3042, 4636, 2057, 2058, 315, 25, 402, 25, 27, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int \sec(e+fx)\left(a+b\sec(e+fx)^2\right)^{3/2}\,dx$$

$$\downarrow 4636$$

$$\frac{\int \frac{\left(a+\frac{b}{1-\sin^2(e+fx)}\right)^{3/2}}{1-\sin^2(e+fx)}\,d\sin(e+fx)}{f}$$

$$\downarrow 2057$$

$$\frac{\int \frac{\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^{3/2}}{1-\sin^2(e+fx)}\,d\sin(e+fx)}{f}$$

$$\downarrow 2058$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\int \frac{(-a\sin^2(e+fx)+a+b)^{3/2}}{(1-\sin^2(e+fx))^{5/2}}\,d\sin(e+fx)}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\downarrow 315$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{b\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3(1-\sin^2(e+fx))^{3/2}}-\frac{1}{3}\int-\frac{(a+b)(3a+2b)-a(3a+b)\sin^2(e+fx)}{(1-\sin^2(e+fx))^{3/2}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\downarrow 25$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\int\frac{(a+b)(3a+2b)-a(3a+b)\sin^2(e+fx)}{(1-\sin^2(e+fx))^{3/2}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)+\frac{b\sin(e+fx)\sqrt{-a\sin^2(e+\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\downarrow 402$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\left(\frac{\int-\frac{ab\left(-2(2a+b)\sin^2(e+fx)+a+b\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{b}+\frac{2(2a+b)\sin(e+fx)\sqrt{-a\sin^2(e+f}{\sqrt{1-\sin^2(e+fx)}}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\downarrow 25$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\left(\frac{2(2a+b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}-\frac{\int\frac{ab\left(-2(2a+b)\sin^2(e+fx)+a+b\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(}{b}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\downarrow 27$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\left(\frac{2(2a+b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}-a\int\frac{-2(2a+b)\sin^2(e+fx)+a+b}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\downarrow 399$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\left(\frac{2(2a+b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}-a\left(\frac{2(2a+b)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e-\right.\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a}}$$

$$\downarrow 323$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\left(\frac{2(2a+b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}-a\left(\frac{2(2a+b)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e-}{a}\right.\right.\right.}{f\sqrt{-a\sin^2(e+fx}}$$

$$\downarrow 321$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\left(\frac{2(2a+b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}-a\left(\frac{2(2a+b)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e-}{a}\right.\right.\right.}{f\sqrt{-a\sin^2(e+fx)+}}$$

$$\downarrow 330$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\left(\frac{2(2a+b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}-a\left(\frac{2(2a+b)\sqrt{-a\sin^2(e+fx)+a+b}\int\frac{\sqrt{1-}}{\sqrt{1-}}}{a\sqrt{1-\frac{a\sin^2(e+f}{a+b}}}\right.\right.\right.}{f\sqrt{-a\sin^2(e+}}$$

$$\downarrow 327$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\left(\frac{2(2a+b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}-a\left(\frac{2(2a+b)\sqrt{-a\sin^2(e+fx)+a+b}E\left(\text{arcs}\right.}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}\right.\right.\right.}{f\sqrt{-a\sin^2(e+}}$$

input
```
Int[Sec[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[1 - Sin[e + f*x]^2]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(1 - Sin[e + f*x
]^2)]*((b*Sin[e + f*x]*Sqrt[a + b - a*Sin[e + f*x]^2])/(3*(1 - Sin[e + f*x
]^2)^(3/2)) + ((2*(2*a + b)*Sin[e + f*x]*Sqrt[a + b - a*Sin[e + f*x]^2])/S
qrt[1 - Sin[e + f*x]^2] - a*((2*(2*a + b)*EllipticE[ArcSin[Sin[e + f*x]],
a/(a + b)]*Sqrt[a + b - a*Sin[e + f*x]^2])/(a*Sqrt[1 - (a*Sin[e + f*x]^2)/
(a + b)]) - ((a + b)*(3*a + 2*b)*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)
]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(a*Sqrt[a + b - a*Sin[e + f*x]^2])
))/3))/(f*Sqrt[a + b - a*Sin[e + f*x]^2])
```

3.243. $\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

3.243.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

3.243.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 9.32 (sec) , antiderivative size = 7055, normalized size of antiderivative = 24.33

method	result	size
default	Expression too large to display	7055

input
```
int(sec(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.243.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 798, normalized size of antiderivative = 2.75

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\left(2\left(2i\,a^2+i\,ab\right)\sqrt{a}\sqrt{\frac{ab+b^2}{a^2}}\cos\left(fx+e\right)^2-\left(2i\,a^2+5i\,ab+2i\,b^2\right)\sqrt{a}\cos\left(fx+e\right)^2\right)\sqrt{\frac{2\,a\sqrt{\frac{ab+}{a^2}}}{}}}{}$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/3*((2*(2*I*a^2 + I*a*b)*sqrt(a)*sqrt((a*b + b^2)/a^2)*cos(f*x + e)^2 - (
2*I*a^2 + 5*I*a*b + 2*I*b^2)*sqrt(a)*cos(f*x + e)^2)*sqrt((2*a*sqrt((a*b +
 b^2)/a^2) - a - 2*b)/a)*elliptic_e(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2)
 - a - 2*b)/a)*(cos(f*x + e) + I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*
(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + (2*(-2*I*a^2 - I*a*b)*sqrt(a)*
sqrt((a*b + b^2)/a^2)*cos(f*x + e)^2 - (-2*I*a^2 - 5*I*a*b - 2*I*b^2)*sqrt
(a)*cos(f*x + e)^2)*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic
_e(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) - I*
sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^
2))/a^2) + (2*(-3*I*a^2 - I*a*b)*sqrt(a)*sqrt((a*b + b^2)/a^2)*cos(f*x + e
)^2 - (-I*a^2 - 3*I*a*b - 2*I*b^2)*sqrt(a)*cos(f*x + e)^2)*sqrt((2*a*sqrt(
(a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b + b^2
)/a^2) - a - 2*b)/a)*(cos(f*x + e) + I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^
2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + (2*(3*I*a^2 + I*a*b)*sqr
t(a)*sqrt((a*b + b^2)/a^2)*cos(f*x + e)^2 - (I*a^2 + 3*I*a*b + 2*I*b^2)*sq
rt(a)*cos(f*x + e)^2)*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*ellipt
ic_f(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) -
I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/
a^2))/a^2) + (2*(2*a^2 + a*b)*cos(f*x + e)^2 + a*b)*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a*f*cos(f*x + e)^2)
```

3.243.6 Sympy [F]

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(a+b\sec^2(e+fx)\right)^{\frac{3}{2}}\sec(e+fx)\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(3/2)*sec(e + f*x), x)
```

3.243.7 Maxima [F]

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*sec(f*x + e), x)
```

3.243.8 Giac [F]

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*sec(f*x + e), x)
```

3.243.9 Mupad [F(-1)]

Timed out.

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}{\cos\left(e+f\,x\right)}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x),x)
```

output
```
int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x), x)
```

3.244 $\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

3.244.1 Optimal result 1824
3.244.2 Mathematica [F] 1825
3.244.3 Rubi [A] (verified) 1825
3.244.4 Maple [C] (warning: unable to verify) 1829
3.244.5 Fricas [F] 1829
3.244.6 Sympy [F] 1830
3.244.7 Maxima [F] 1830
3.244.8 Giac [F] 1830
3.244.9 Mupad [F(-1)] 1831

3.244.1 Optimal result

Integrand size = 23, antiderivative size = 224

$$\int \cos(e+fx)\left(a + b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{b\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{f}$$
$$+\frac{(a-b)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}$$
$$+\frac{b(a+b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\sqrt{1-}}{f\left(a+b-a\sin^2(e+fx)\right)}$$

output

```
b*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/f+(a-b)*EllipticE(s
in(f*x+e),(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f
*x+e)^2))^(1/2)/f/(1-a*sin(f*x+e)^2/(a+b))^(1/2)+b*(a+b)*EllipticF(sin(f*x
+e),(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^
2))^(1/2)*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/f/(a+b-a*sin(f*x+e)^2)
```

3.244.2 Mathematica [F]

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$$

input
```
Integrate[Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
Integrate[Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2), x]
```

3.244.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.09, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 4636, 2057, 2058, 315, 25, 27, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^{3/2}}{\sec(e+fx)}dx$$

$$\downarrow 4636$$

$$\frac{\int \left(a+\frac{b}{1-\sin^2(e+fx)}\right)^{3/2} d\sin(e+fx)}{f}$$

$$\downarrow 2057$$

$$\frac{\int \left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^{3/2} d\sin(e+fx)}{f}$$

$$\downarrow 2058$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\int \frac{\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}{\left(1-\sin^2(e+fx)\right)^{3/2}} d\sin(e+fx)}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\downarrow 315$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{b\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}-\int-\frac{a\left(-\left((a-b)\sin^2(e+fx)\right)+a+b\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\downarrow 25$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\int\frac{a\left(-\left((a-b)\sin^2(e+fx)\right)+a+b\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)+\frac{b\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a}}{\sqrt{1-\sin^2(e+fx)}}\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\downarrow 27$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(a\int\frac{-\left((a-b)\sin^2(e+fx)\right)+a+b}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)+\frac{b\sin(e+fx)\sqrt{-a\sin^2(e+fx)}}{\sqrt{1-\sin^2(e+fx)}}\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\downarrow 399$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(a\left(\frac{b(a+b)\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{a}+\frac{(a-b)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}}{a}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\downarrow 323$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(a\left(\frac{(a-b)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}+\frac{b(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{1}}}{a\sqrt{-a\sin^2(e+fx)+a}}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\downarrow 321$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(a\left(\frac{(a-b)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}+\frac{b(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\arcsin(\sin\right.\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\downarrow 330$$

3.244. $\int\cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(a\left(\frac{(a-b)\sqrt{-a\sin^2(e+fx)+a+b}\int\frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}+\frac{b(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{a\sqrt{-a}}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\downarrow 327$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(a\left(\frac{b(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)}{a\sqrt{-a\sin^2(e+fx)+a+b}}+\frac{(a-b)\sqrt{-a\sin^2(e+fx)+a}}{a\sqrt{1-}}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

input
```
Int[Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[1 - Sin[e + f*x]^2]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(1 - Sin[e + f*x
]^2)]*((b*Sin[e + f*x]*Sqrt[a + b - a*Sin[e + f*x]^2])/Sqrt[1 - Sin[e + f*
x]^2] + a*(((a - b)*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b
- a*Sin[e + f*x]^2])/(a*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) + (b*(a + b)
*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a
 + b)])/(a*Sqrt[a + b - a*Sin[e + f*x]^2])))/(f*Sqrt[a + b - a*Sin[e + f*
x]^2])
```

3.244.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

3.244.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.35 (sec) , antiderivative size = 5360, normalized size of antiderivative = 23.93

method	result	size
default	Expression too large to display	5360

input
```
int(cos(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.244.5 Fricas [F]

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)\,dx$$

input
```
integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
integral((b*cos(f*x + e)*sec(f*x + e)^2 + a*cos(f*x + e))*sqrt(b*sec(f*x +
 e)^2 + a), x)
```

3.244.6 Sympy [F]

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(a+b\sec^2(e+fx)\right)^{\frac{3}{2}}\cos(e+fx)\,dx$$

input
```
integrate(cos(f*x+e)*(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(3/2)*cos(e + f*x), x)
```

3.244.7 Maxima [F]

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}\cos(fx+e)\,dx$$

input
```
integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e), x)
```

3.244.8 Giac [F]

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}\cos(fx+e)\,dx$$

input
```
integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e), x)
```

3.244.9 Mupad [F(-1)]

Timed out.

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \cos\left(e+f\,x\right)\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}\,dx$$

input
```
int(cos(e + f*x)*(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(cos(e + f*x)*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.245 $\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

3.245.1 Optimal result 1832
3.245.2 Mathematica [A] (verified) 1833
3.245.3 Rubi [A] (verified) 1833
3.245.4 Maple [C] (verified) 1837
3.245.5 Fricas [F] 1837
3.245.6 Sympy [F(-1)] 1837
3.245.7 Maxima [F] 1838
3.245.8 Giac [F] 1838
3.245.9 Mupad [F(-1)] 1838

3.245.1 Optimal result

Integrand size = 25, antiderivative size = 241

$$\int \cos^3(e+fx)\,(a+b\sec^2(e+fx))^{3/2}\,dx = \frac{a\cos^2(e+fx)\sin(e+fx)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}{3f}$$
$$+\frac{2(a+2b)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\,\middle|\,\tfrac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}{3f\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}$$
$$-\frac{b(a+b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\tfrac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\sqrt{1-}}{3f\,(a+b-a\sin^2(e+fx))}$$

output

```
1/3*a*cos(f*x+e)^2*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/f+
2/3*(a+2*b)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(se
c(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/f/(1-a*sin(f*x+e)^2/(a+b))^(1/2)-1/
3*b*(a+b)*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(
f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/f/(a+b
-a*sin(f*x+e)^2)
```

3.245.2 Mathematica [A] (verified)

Time = 2.74 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.74

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(4\sqrt{2}(a^2+3ab+2b^2)\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}}E\left(e+fx\middle|\frac{a}{a+b}\right)-\right.}{6f(a+2b$$

input
```
Integrate[Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*(4*Sqrt[2]*(a^2 + 3*a*b + 2*b^2)*
Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*EllipticE[e + f*x, a/(a + b)]
 - 2*Sqrt[2]*b*(a + b)*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*Ellipt
icF[e + f*x, a/(a + b)] + a*(a + 2*b + a*Cos[2*(e + f*x)])*Sin[2*(e + f*x)
]))/(6*f*(a + 2*b + a*Cos[2*(e + f*x)]))
```

3.245.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4636, 2057, 2058, 318, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^{3/2}}{\sec(e+fx)^3}\,dx$$

$$\downarrow 4636$$

$$\frac{\int \left(1-\sin^2(e+fx)\right)\left(a+\frac{b}{1-\sin^2(e+fx)}\right)^{3/2} d\sin(e+fx)}{f}$$

$$\downarrow 2057$$

$$\frac{\int \left(1-\sin^2(e+fx)\right)\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^{3/2} d\sin(e+fx)}{f}$$

$$\downarrow 2058$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\int \frac{\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}{\sqrt{1-\sin^2(e+fx)}} d\sin(e+fx)}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\downarrow 318$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}a\sin(e+fx)\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}-\frac{1}{3}\int \frac{2a(a+2b)}{\sqrt{1-\sin^2}}\right.}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\downarrow 399$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\left(2(a+2b)\int \frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}} d\sin(e+fx)-b(a+b)\int \frac{}{\sqrt{1-\sin^2(e+fx)}\sqrt{}}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+}}$$

$$\downarrow 323$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\left(2(a+2b)\int \frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}} d\sin(e+fx)-\frac{b(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\int \frac{}{\sqrt{}}}{\sqrt{-a\sin}}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+}}$$

$$\downarrow 321$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\left(2(a+2b)\int \frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}} d\sin(e+fx)-\frac{b(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{Ellipt}}{\sqrt{-a\sin^2(e}}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a+}}$$

$$\downarrow 330$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\left(\frac{2(a+2b)\sqrt{-a\sin^2(e+fx)+a+b}\int \frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{\sqrt{1-\sin^2(e+fx)}} d\sin(e+fx)}{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{b(a+b)\sqrt{1-\frac{a\sin^2(e+f}}{\sqrt{-}}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+}}$$

$$\downarrow 327$$

3.245. $\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{3}\left(\frac{2(a+2b)\sqrt{-a\sin^2(e+fx)+a+b}E\left(\arcsin(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{b(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{\sqrt{-a\sin}}\right.\right.}{f\sqrt{-a\sin^2(e+fx)+a}}$$

input
```
Int[Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[1 - Sin[e + f*x]^2]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(1 - Sin[e + f*x
]^2)]*((a*Sin[e + f*x]*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x
]^2])/3 + ((2*(a + 2*b)*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a
 + b - a*Sin[e + f*x]^2])/Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)] - (b*(a + b)
*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a
 + b)])/Sqrt[a + b - a*Sin[e + f*x]^2])/3))/(f*Sqrt[a + b - a*Sin[e + f*x]
^2])
```

3.245.3.1 Defintions of rubi rules used

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

3.245.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.14 (sec) , antiderivative size = 6947, normalized size of antiderivative = 28.83

method	result	size
default	Expression too large to display	6947

input
```
int(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.245.5 Fricas [F]

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^3\,dx$$

input
```
integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
integral((b*cos(f*x + e)^3*sec(f*x + e)^2 + a*cos(f*x + e)^3)*sqrt(b*sec(f
*x + e)^2 + a), x)
```

3.245.6 Sympy [F(-1)]

Timed out.

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**3*(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

3.245.7 Maxima [F]

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^3\,dx$$

input
```
integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^3, x)
```

3.245.8 Giac [F]

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^3\,dx$$

input
```
integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^3, x)
```

3.245.9 Mupad [F(-1)]

Timed out.

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \cos\left(e+f\,x\right)^3\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}\,dx$$

input
```
int(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.246 $\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

3.246.1 Optimal result 1839
3.246.2 Mathematica [C] (verified) 1840
3.246.3 Rubi [A] (verified) 1840
3.246.4 Maple [C] (warning: unable to verify) 1844
3.246.5 Fricas [F] 1845
3.246.6 Sympy [F(-1)] 1845
3.246.7 Maxima [F] 1845
3.246.8 Giac [F] 1846
3.246.9 Mupad [F(-1)] 1846

3.246.1 Optimal result

Integrand size = 25, antiderivative size = 319

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx =$$

$$-\frac{2(a-3(a+b))\cos^2(e+fx)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{15f}$$

$$+\frac{a\cos^4(e+fx)\sin(e+fx)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{5f}$$

$$+\frac{\left(8a^2+13ab+3b^2\right)\sqrt{\cos^2(e+fx)}E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}{15af\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}$$

$$-\frac{b(a+b)(4a+3b)\sqrt{\cos^2(e+fx)}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+f}}{15af\left(a+b-a\sin^2(e+fx)\right)}$$

output

```
-2/15*(-2*a-3*b)*cos(f*x+e)^2*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2
))^(1/2)/f+1/5*a*cos(f*x+e)^4*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2
))^(1/2)/f+1/15*(8*a^2+13*a*b+3*b^2)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))
*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/a/f/(1-a*s
in(f*x+e)^2/(a+b))^(1/2)-1/15*b*(a+b)*(4*a+3*b)*EllipticF(sin(f*x+e),(a/(a
+b))^(1/2))*(cos(f*x+e)^2)^(1/2)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)
*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/a/f/(a+b-a*sin(f*x+e)^2)
```

3.246.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 18.48 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.12

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx = \frac{\cos^3(e+fx)\csc(2(e+fx))\left(a+b\sec^2(e+fx)\right)^{3/2}\left(-8i\sqrt{2}(8a^3+21a^2b+16ab^2+3b^3)\sqrt{-}\right.}{}$$

input
```
Integrate[Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Cos[e + f*x]^3*Csc[2*(e + f*x)]*(a + b*Sec[e + f*x]^2)^(3/2)*((-8*I)*Sqrt
[2]*(8*a^3 + 21*a^2*b + 16*a*b^2 + 3*b^3)*Sqrt[-((a*Cos[e + f*x]^2)/b)]*El
lipticE[I*ArcSinh[(Sqrt[-b^(-1)]*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]])/Sqrt[
2]], b/(a + b)]*Sqrt[(a*Sin[e + f*x]^2)/(a + b)] + a*((8*I)*Sqrt[2]*(8*a^2
 + 17*a*b + 9*b^2)*Sqrt[-((a*Cos[e + f*x]^2)/b)]*EllipticF[I*ArcSinh[(Sqrt
[-b^(-1)]*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]])/Sqrt[2]], b/(a + b)]*Sqrt[(a
*Sin[e + f*x]^2)/(a + b)] + a*Sqrt[-b^(-1)]*Sqrt[a + 2*b + a*Cos[2*(e + f*
x)]]*(11*a + 12*b + 3*a*Cos[2*(e + f*x)])*Sin[2*(e + f*x)]^2)))/(30*a^2*Sq
rt[-b^(-1)]*f*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2))
```

3.246.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.01, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 4636, 2057, 2058, 318, 403, 27, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^{3/2}}{\sec(e+fx)^5}dx$$

$$\downarrow 4636$$

$$\frac{\int \left(1-\sin^2(e+fx)\right)^2 \left(a+\frac{b}{1-\sin^2(e+fx)}\right)^{3/2} d\sin(e+fx)}{f}$$

$$\downarrow 2057$$

$$\frac{\int \left(1-\sin^2(e+fx)\right)^2 \left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^{3/2} d\sin(e+fx)}{f}$$

$$\downarrow 2058$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\int \sqrt{1-\sin^2(e+fx)}\left(-a\sin^2(e+fx)+a+b\right)^{3/2} d\sin(e+fx)}{f\sqrt{-a\sin^2(e+fx)+a+b}}$$

$$\downarrow 318$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}a\sin(e+fx)\left(1-\sin^2(e+fx)\right)^{3/2}\sqrt{-a\sin^2(e+fx)+a+b}-\frac{1}{5}\int \frac{\sqrt{1-}}{f\sqrt{-a\sin^2(e+fx)+a+b}}\right.$$

$$\downarrow 403$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\left(\frac{\int \frac{a\left((a+b)(8a+9b)-\left(8a^2+13ba+3b^2\right)\sin^2(e+fx)\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}} d\sin(e+fx)}{3a}-\frac{2}{3}(a-3(a+b))\sin(e\right.}{f\sqrt{-a\sin}}$$

$$\downarrow 27$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\left(\frac{1}{3}\int \frac{(a+b)(8a+9b)-\left(8a^2+13ba+3b^2\right)\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}} d\sin(e+fx)-\frac{2}{3}(a-3(a+b))\right.}{f\sqrt{-a}}$$

$$\downarrow 399$$

$$\frac{\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{\left(8a^2+13ab+3b^2\right)\int \frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}} d\sin(e+fx)}{a}-\frac{b(a+b)(4a+3b)\int \frac{}{\sqrt{1-\sin^2(e-}}\right.\right.\right.}{}$$

$$\downarrow 323$$

3.246. $\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2} dx$

$$\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{(8a^2+13ab+3b^2)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{b(a+b)(4a+3b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{a}\right.\right.\right.$$

$$\downarrow 321$$

$$\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{(8a^2+13ab+3b^2)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{b(a+b)(4a+3b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{a\sqrt{-a}}\right.\right.\right.$$

$$\downarrow 330$$

$$\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{(8a^2+13ab+3b^2)\sqrt{-a\sin^2(e+fx)+a+b}\int\frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{b(a+b)(4\right.\right.\right.\right.$$

$$\downarrow 327$$

$$\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{(8a^2+13ab+3b^2)\sqrt{-a\sin^2(e+fx)+a+b}E\left(\arcsin(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{b(a+b)(4a+\right.\right.\right.\right.$$

input

```
Int[Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output

```
(Sqrt[1 - Sin[e + f*x]^2]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(1 - Sin[e + f*x
]^2)]*((a*Sin[e + f*x]*(1 - Sin[e + f*x]^2)^(3/2)*Sqrt[a + b - a*Sin[e + f
*x]^2])/5 + ((-2*(a - 3*(a + b))*Sin[e + f*x]*Sqrt[1 - Sin[e + f*x]^2]*Sqr
t[a + b - a*Sin[e + f*x]^2])/3 + (((8*a^2 + 13*a*b + 3*b^2)*EllipticE[ArcS
in[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b - a*Sin[e + f*x]^2])/(a*Sqrt[1 - (
a*Sin[e + f*x]^2)/(a + b)]) - (b*(a + b)*(4*a + 3*b)*EllipticF[ArcSin[Sin[
e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(a*Sqrt[a + b
- a*Sin[e + f*x]^2]))/3)/5))/(f*Sqrt[a + b - a*Sin[e + f*x]^2])
```

3.246. $\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

3.246.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

3.246.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.03 (sec) , antiderivative size = 8848, normalized size of antiderivative = 27.74

method	result	size
default	Expression too large to display	8848

input
```
int(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.246.5 Fricas [F]

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^5\,dx$$

input
```
integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
integral((b*cos(f*x + e)^5*sec(f*x + e)^2 + a*cos(f*x + e)^5)*sqrt(b*sec(f
*x + e)^2 + a), x)
```

3.246.6 Sympy [F(-1)]

Timed out.

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**5*(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

3.246.7 Maxima [F]

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^5\,dx$$

input
```
integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^5, x)
```

3.246.8 Giac [F]

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^5\,dx$$

input
```
integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^5, x)
```

3.246.9 Mupad [F(-1)]

Timed out.

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \cos\left(e+f\,x\right)^5\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}\,dx$$

input
```
int(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.247 $\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

3.247.1 Optimal result 1847
3.247.2 Mathematica [C] (warning: unable to verify) 1848
3.247.3 Rubi [A] (verified) 1848
3.247.4 Maple [B] (warning: unable to verify) 1851
3.247.5 Fricas [A] (verification not implemented) 1852
3.247.6 Sympy [F] 1853
3.247.7 Maxima [A] (verification not implemented) 1853
3.247.8 Giac [F] 1854
3.247.9 Mupad [F(-1)] 1854

3.247.1 Optimal result

Integrand size = 25, antiderivative size = 243

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{(a+b)^2\left(3a^2-10ab+35b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{128b^{5/2}f} + \frac{(a+b)\left(3a^2-10ab+35b^2\right)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{128b^2f} + \frac{\left(3a^2-10ab+35b^2\right)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{192b^2f} - \frac{(3a-7b)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{48b^2f} + \frac{\sec^2(e+fx)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{8bf}$$

output

```
1/128*(a+b)^2*(3*a^2-10*a*b+35*b^2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(
f*x+e)^2)^(1/2))/b^(5/2)/f+1/128*(a+b)*(3*a^2-10*a*b+35*b^2)*(a+b+b*tan(f*
x+e)^2)^(1/2)*tan(f*x+e)/b^2/f+1/192*(3*a^2-10*a*b+35*b^2)*tan(f*x+e)*(a+b
+b*tan(f*x+e)^2)^(3/2)/b^2/f-1/48*(3*a-7*b)*tan(f*x+e)*(a+b+b*tan(f*x+e)^2
)^(5/2)/b^2/f+1/8*sec(f*x+e)^2*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^(5/2)/b/f
```

3.247.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.22 (sec) , antiderivative size = 512, normalized size of antiderivative = 2.11

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos^3(e+fx)\left(-\frac{i\sqrt{b}\left(-1+e^{2i(e+fx)}\right)\left(-9a^3\left(1+e^{2i(e+fx)}\right)^6+3a\right.\right.}{}\right.$$

input
```
Integrate[Sec[e + f*x]^6*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e +
f*x))]*Cos[e + f*x]^3*(((-I)*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*(-9*a^3*(1
 + E^((2*I)*(e + f*x)))^6 + 3*a^2*b*(1 + E^((2*I)*(e + f*x)))^4*(5 + 18*E^
((2*I)*(e + f*x)) + 5*E^((4*I)*(e + f*x))) + a*b^2*(1 + E^((2*I)*(e + f*x)
))^2*(145 + 948*E^((2*I)*(e + f*x)) + 2758*E^((4*I)*(e + f*x)) + 948*E^((6
*I)*(e + f*x)) + 145*E^((8*I)*(e + f*x))) + b^3*(105 + 910*E^((2*I)*(e + f
*x)) + 3591*E^((4*I)*(e + f*x)) + 8644*E^((6*I)*(e + f*x)) + 3591*E^((8*I)
*(e + f*x)) + 910*E^((10*I)*(e + f*x)) + 105*E^((12*I)*(e + f*x)))))/(1 +
E^((2*I)*(e + f*x)))^8 - (3*(a + b)^2*(3*a^2 - 10*a*b + 35*b^2)*Log[(-4*Sq
rt[b]*(-1 + E^((2*I)*(e + f*x)))*f + (4*I)*Sqrt[4*b*E^((2*I)*(e + f*x)) +
a*(1 + E^((2*I)*(e + f*x)))^2]*f)/(1 + E^((2*I)*(e + f*x)))])/Sqrt[4*b*E^(
(2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*(a + b*Sec[e + f*x]^2)^
(3/2))/(96*Sqrt[2]*b^(5/2)*f*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2))
```

3.247.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.89, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4634, 318, 25, 299, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int \sec(e+fx)^6 \left(a+b\sec(e+fx)^2\right)^{3/2} dx$$

$$\downarrow 4634$$

$$\frac{\int \left(\tan^2(e+fx)+1\right)^2 \left(b\tan^2(e+fx)+a+b\right)^{3/2} d\tan(e+fx)}{f}$$

$$\downarrow 318$$

$$\frac{\int -\left(\left((3a-7b)\tan^2(e+fx)+a-7b\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}\right) d\tan(e+fx)}{8b} + \frac{\tan(e+fx)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{8b}}{f}$$

$$\downarrow 25$$

$$\frac{\frac{\tan(e+fx)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{8b} - \frac{\int \left((3a-7b)\tan^2(e+fx)+a-7b\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2} d\tan(e+fx)}{8b}}{f}$$

$$\downarrow 299$$

$$\frac{\frac{\tan(e+fx)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{8b} - \frac{\frac{(3a-7b)\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{6b} - \frac{\left(3a^2-10ab+35b^2\right)\int \left(b\tan^2(e+fx)+a+b\right)^{3/2} d\tan(}{6b}}{8b}}{f}$$

$$\downarrow 211$$

$$\frac{\frac{\tan(e+fx)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{8b} - \frac{\frac{(3a-7b)\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{6b} - \frac{\left(3a^2-10ab+35b^2\right)\left(\frac{3}{4}(a+b)\int \sqrt{b\tan^2(e+fx)+a+b}\right.}{6}}{8b}}{f}$$

$$\downarrow 211$$

$$\frac{\frac{\tan(e+fx)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{8b} - \frac{\frac{(3a-7b)\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{6b} - \frac{\left(3a^2-10ab+35b^2\right)\left(\frac{3}{4}(a+b)\left(\frac{1}{2}(a+b)\int \frac{1}{\sqrt{b\tan^2(e+}\right.\right.}{}}{8b}}{f}$$

$$\downarrow 224$$

$$\frac{\frac{\tan(e+fx)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{8b} - \frac{\frac{(3a-7b)\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{6b} - \frac{\left(3a^2-10ab+35b^2\right)\left(\frac{3}{4}(a+b)\left(\frac{1}{2}(a+b)\int \frac{1}{1-\frac{b\tan}{b\tan^2(e}\right.\right.}{}}{8b}}{f}$$

$$\downarrow 219$$

$$\frac{\tan(e+fx)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{8b} - \frac{\frac{(3a-7b)\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{6b} - \frac{\left(3a^2-10ab+35b^2\right)\left(\frac{3}{4}(a+b)\left(\frac{(a+b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2\sqrt{b}} + \frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2}\right) + \frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4}\right)}{6b}}{8b}}{f}$$

input
```
Int[Sec[e + f*x]^6*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
((Tan[e + f*x]*(1 + Tan[e + f*x]^2)*(a + b + b*Tan[e + f*x]^2)^(5/2))/(8*b
) - (((3*a - 7*b)*Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(5/2))/(6*b) - (
(3*a^2 - 10*a*b + 35*b^2)*((Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/2))
/4 + (3*(a + b)*(((a + b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Ta
n[e + f*x]^2]])/(2*Sqrt[b]) + (Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2]
)/2))/4))/(6*b))/(8*b))/f
```

3.247.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.247.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1976 vs. $2(219) = 438$.

Time = 21.49 (sec) , antiderivative size = 1977, normalized size of antiderivative = 8.14

method	result	size
default	Expression too large to display	1977

input
```
int(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
1/768/f/b^(13/2)*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)/(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))*(140*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*b^(15/2)*sin(f*x+e)+105*cos(f*x+e)^3*ln(4*(((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*b^8+105*c
os(f*x+e)^3*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos
(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a
+b)/(sin(f*x+e)-1))*b^8+140*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^
(15/2)*tan(f*x+e)+290*b^(13/2)*cos(f*x+e)^2*sin(f*x+e)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*a+290*b^(13/2)*cos(f*x+e)*sin(f*x+e)*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a+30*b^(11/2)*cos(f*x+e)^2*sin(f*x+e)*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2+30*b^(11/2)*cos(f*x+e)*sin(f*x
+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2-18*b^(9/2)*cos(f*x+e)^
2*sin(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^3-18*b^(9/2)*co
s(f*x+e)*sin(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^3+144*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(13/2)*a*tan(f*x+e)*sec(f*x+e)
+144*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(13/2)*a*tan(f*x+e)*sec
(f*x+e)^2+12*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(11/2)*a^2*tan(
f*x+e)+184*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(13/2)*a*tan(f*x+
e)+96*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(15/2)*tan(f*x+e)*s...
```

3.247.5 Fricas [A] (verification not implemented)

Time = 4.13 (sec) , antiderivative size = 566, normalized size of antiderivative = 2.33

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \left[\frac{3\left(3\,a^4-4\,a^3b+18\,a^2b^2+60\,ab^3+35\,b^4\right)\sqrt{b}\cos\left(fx+e\right)^7\log\left(\frac{(a^2-6\,ab+b^2)\cos(fx+e)^4+8\,(ab-b^2}{}\right.\right.$$

input

```
integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/1536*(3*(3*a^4 - 4*a^3*b + 18*a^2*b^2 + 60*a*b^3 + 35*b^4)*sqrt(b)*cos(
f*x + e)^7*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x
 + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) -
4*((9*a^3*b - 15*a^2*b^2 - 145*a*b^3 - 105*b^4)*cos(f*x + e)^6 - 2*(3*a^2*
b^2 + 46*a*b^3 + 35*b^4)*cos(f*x + e)^4 - 48*b^4 - 8*(9*a*b^3 + 7*b^4)*cos
(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^
3*f*cos(f*x + e)^7), 1/768*(3*(3*a^4 - 4*a^3*b + 18*a^2*b^2 + 60*a*b^3 + 3
5*b^4)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sq
rt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 +
b^2)*sin(f*x + e)))*cos(f*x + e)^7 - 2*((9*a^3*b - 15*a^2*b^2 - 145*a*b^3
- 105*b^4)*cos(f*x + e)^6 - 2*(3*a^2*b^2 + 46*a*b^3 + 35*b^4)*cos(f*x + e)
^4 - 48*b^4 - 8*(9*a*b^3 + 7*b^4)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)*sin(f*x + e))/(b^3*f*cos(f*x + e)^7)]
```

3.247.6 Sympy [F]

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(a+b\sec^2(e+fx)\right)^{\frac{3}{2}}\sec^6(e+fx)\,dx$$

input
```
integrate(sec(f*x+e)**6*(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(3/2)*sec(e + f*x)**6, x)
```

3.247.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.71

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\frac{48\left(b\tan(fx+e)^2+a+b\right)^{\frac{5}{2}}\tan(fx+e)^3}{b} + \frac{9\,(a+b)^3 a\,\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{5}{2}}} + \frac{9\,(a+b)^3\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}} - \frac{48\,(a+b)^2 a\,a}{}$$

input
```
integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/384*(48*(b*tan(f*x + e)^2 + a + b)^(5/2)*tan(f*x + e)^3/b + 9*(a + b)^3*
a*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/b^(5/2) + 9*(a + b)^3*arcsinh(b*
tan(f*x + e)/sqrt((a + b)*b))/b^(3/2) - 48*(a + b)^2*a*arcsinh(b*tan(f*x +
 e)/sqrt((a + b)*b))/b^(3/2) - 48*(a + b)^2*arcsinh(b*tan(f*x + e)/sqrt((a
 + b)*b))/sqrt(b) + 144*(a + b)*a*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/
sqrt(b) + 144*(a + b)*sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b)) + 96
*(b*tan(f*x + e)^2 + a + b)^(3/2)*tan(f*x + e) + 144*sqrt(b*tan(f*x + e)^2
 + a + b)*(a + b)*tan(f*x + e) - 24*(b*tan(f*x + e)^2 + a + b)^(5/2)*(a +
b)*tan(f*x + e)/b^2 + 6*(b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)^2*tan(f*x
 + e)/b^2 + 9*sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^3*tan(f*x + e)/b^2 +
128*(b*tan(f*x + e)^2 + a + b)^(5/2)*tan(f*x + e)/b - 32*(b*tan(f*x + e)^2
 + a + b)^(3/2)*(a + b)*tan(f*x + e)/b - 48*sqrt(b*tan(f*x + e)^2 + a + b)
*(a + b)^2*tan(f*x + e)/b)/f
```

3.247.8 Giac [F]

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}\sec(fx+e)^6\,dx$$

input
```
integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^6, x)
```

3.247.9 Mupad [F(-1)]

Timed out.

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \frac{\left(a+\frac{b}{\cos(e+fx)^2}\right)^{3/2}}{\cos(e+fx)^6}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x)^6,x)
```

output
```
int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x)^6, x)
```

3.248 $\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

3.248.1 Optimal result 1855
3.248.2 Mathematica [C] (warning: unable to verify) 1856
3.248.3 Rubi [A] (verified) 1856
3.248.4 Maple [B] (warning: unable to verify) 1859
3.248.5 Fricas [A] (verification not implemented) 1860
3.248.6 Sympy [F] 1861
3.248.7 Maxima [A] (verification not implemented) 1861
3.248.8 Giac [F] 1862
3.248.9 Mupad [F(-1)] 1862

3.248.1 Optimal result

Integrand size = 25, antiderivative size = 165

$$\begin{aligned}\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx &= \\ &-\frac{(a-5b)(a+b)^2\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16b^{3/2}f} \\ &-\frac{(a-5b)(a+b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16bf} \\ &-\frac{(a-5b)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{24bf} \\ &+\frac{\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{6bf}\end{aligned}$$

output

```
-1/16*(a-5*b)*(a+b)^2*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2
))/b^(3/2)/f-1/16*(a-5*b)*(a+b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b/f-
1/24*(a-5*b)*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^(3/2)/b/f+1/6*tan(f*x+e)*(a+b
+b*tan(f*x+e)^2)^(5/2)/b/f
```

3.248.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 8.97 (sec) , antiderivative size = 450, normalized size of antiderivative = 2.73

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx =$$

$$-\frac{(a+b)^3\left(a+b\sec^2(e+fx)\right)^{3/2}\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^2\tan(e+fx)\left(5\arcsin\left(\sqrt{-\frac{b\tan^2(e+fx)}{a+b}}\right)\left(-9+\frac{6a\sin^2(e-\right.\right.}{a+b}$$

input
```
Integrate[Sec[e + f*x]^4*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
-1/60*((a + b)^3*(a + b*Sec[e + f*x]^2)^(3/2)*(1 - (a*Sin[e + f*x]^2)/(a +
 b))^2*Tan[e + f*x]*(5*ArcSin[Sqrt[-((b*Tan[e + f*x]^2)/(a + b))]]*(-9 + (
6*a*Sin[e + f*x]^2)/(a + b)) + (5*Sec[e + f*x]^4*(-3*a^3*Cos[e + f*x]^4*(-
3 + 2*Sin[e + f*x]^2) + 3*b^3*(3 + 8*Sin[e + f*x]^2 - 3*Sin[e + f*x]^4) +
a*b^2*(27 + 24*Sin[e + f*x]^2 - 49*Sin[e + f*x]^4 + 6*Sin[e + f*x]^6) + a^
2*b*(27 - 24*Sin[e + f*x]^2 - 19*Sin[e + f*x]^4 + 16*Sin[e + f*x]^6))*Sqrt
[-((b*Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)*Tan[e + f*x]^2)/(a + b)^2)
])/(a + b)^3 - (256*Hypergeometric2F1[2, 5, 7/2, -((b*Tan[e + f*x]^2)/(a +
 b))]*(a + b - a*Sin[e + f*x]^2)*(-((b*Sec[e + f*x]^2*(a + b - a*Sin[e + f
*x]^2)*Tan[e + f*x]^2)/(a + b)^2))^(5/2))/(a + b)))/(Sqrt[2]*f*(a + 2*b +
a*Cos[2*e + 2*f*x])^(3/2)*Sqrt[(a + b*Sec[e + f*x]^2)/(a + b)]*(a + b - a*
Sin[e + f*x]^2)^(3/2)*(-((b*Tan[e + f*x]^2)/(a + b)))^(3/2))
```

3.248.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.92, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4634, 299, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int \sec(e+fx)^4 \left(a+b\sec(e+fx)^2\right)^{3/2} dx$$

$$\downarrow 4634$$

$$\frac{\int \left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2} d\tan(e+fx)}{f}$$

$$\downarrow 299$$

$$\frac{\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{6b} - \frac{(a-5b)\int \left(b\tan^2(e+fx)+a+b\right)^{3/2} d\tan(e+fx)}{6b}}{f}$$

$$\downarrow 211$$

$$\frac{\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{6b} - \frac{(a-5b)\left(\frac{3}{4}(a+b)\int \sqrt{b\tan^2(e+fx)+a+b}\, d\tan(e+fx)+\frac{1}{4}\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}\right)}{6b}}{f}$$

$$\downarrow 211$$

$$\frac{\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{6b} - \frac{(a-5b)\left(\frac{3}{4}(a+b)\left(\frac{1}{2}(a+b)\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}} d\tan(e+fx)+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)+\frac{1}{4}\tan\right.}{6b}}{f}$$

$$\downarrow 224$$

$$\frac{\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{6b} - \frac{(a-5b)\left(\frac{3}{4}(a+b)\left(\frac{1}{2}(a+b)\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}} d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+\right.\right.}{6b}}{f}$$

$$\downarrow 219$$

$$\frac{\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{6b} - \frac{(a-5b)\left(\frac{3}{4}(a+b)\left(\frac{(a+b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2\sqrt{b}}+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)+\frac{1}{4}\tan(e\right.}{6b}}{f}$$

input
```
Int[Sec[e + f*x]^4*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
((Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(5/2))/(6*b) - ((a - 5*b)*((Tan[
e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/2))/4 + (3*(a + b)*(((a + b)*ArcTan
h[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(2*Sqrt[b]) + (T
an[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/2))/4))/(6*b))/f
```

3.248.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.248.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1416 vs. 2(145) = 290.

Time = 16.72 (sec) , antiderivative size = 1417, normalized size of antiderivative = 8.59

method	result	size
default	Expression too large to display	1417

input
```
int(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/96/f/b^(9/2)*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)/(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))*(-30*b^(11/2)*cos(f*x+e)^2*si
n(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-30*b^(11/2)*cos(f*x+e
)*sin(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-44*cos(f*x+e)^2*s
in(f*x+e)*a*b^(9/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-20*b^(11/2
)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*sin(f*x+e)-44*sin(f*x+e)*cos
(f*x+e)*a*b^(9/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-6*sin(f*x+e)
*cos(f*x+e)^2*a^2*b^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-20*b
^(11/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*tan(f*x+e)-28*b^(9/2)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*sin(f*x+e)-6*sin(f*x+e)*cos(
f*x+e)*a^2*b^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-16*b^(11/2)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*tan(f*x+e)*sec(f*x+e)-28*b^(9
/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*tan(f*x+e)+3*cos(f*x+e)^
3*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(
1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*
x+e)+1))*a^3*b^3-9*cos(f*x+e)^3*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*a^2*b^4-27*cos(f*x+e)^3*ln(4*(((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*a...
```

3.248.5 Fricas [A] (verification not implemented)

Time = 1.07 (sec) , antiderivative size = 470, normalized size of antiderivative = 2.85

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \left[\begin{array}{l} -\dfrac{3\left(a^3-3\,a^2b-9\,ab^2-5\,b^3\right)\sqrt{b}\cos\left(fx+e\right)^5\log\left(\frac{\left(a^2-6\,ab+b^2\right)\cos(fx+e)^4+8\left(ab-b^2\right)\cos(fx+e)^2+4}{\cdots}\right.}{\cdots} \\ \\ -\dfrac{3\left(a^3-3\,a^2b-9\,ab^2-5\,b^3\right)\sqrt{-b}\arctan\left(-\frac{\left((a-b)\cos(fx+e)^3+2\,b\cos(fx+e)\right)\sqrt{-b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{2\left(ab\cos(fx+e)^2+b^2\right)\sin(fx+e)}\right)\cos\left(fx+e\right)^5-}{96\,b^2f\cos} \end{array}\right.$$

input
```
integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/192*(3*(a^3 - 3*a^2*b - 9*a*b^2 - 5*b^3)*sqrt(b)*cos(f*x + e)^5*log(((
a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a -
 b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*((3*a^2*b + 22*
a*b^2 + 15*b^3)*cos(f*x + e)^4 + 8*b^3 + 2*(7*a*b^2 + 5*b^3)*cos(f*x + e)^
2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^2*f*cos(f*
x + e)^5), -1/96*(3*(a^3 - 3*a^2*b - 9*a*b^2 - 5*b^3)*sqrt(-b)*arctan(-1/2
*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)
^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x
 + e)^5 - 2*((3*a^2*b + 22*a*b^2 + 15*b^3)*cos(f*x + e)^4 + 8*b^3 + 2*(7*a
*b^2 + 5*b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*
sin(f*x + e))/(b^2*f*cos(f*x + e)^5)]
```

3.248.6 Sympy [F]

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(a+b\sec^2(e+fx)\right)^{\frac{3}{2}}\sec^4(e+fx)\,dx$$

input
```
integrate(sec(f*x+e)**4*(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(3/2)*sec(e + f*x)**4, x)
```

3.248.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.47

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx =$$
$$-\frac{\frac{3\,(a+b)^2 a\,\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}}+\frac{3\,(a+b)^2\,\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}}-\frac{18\,(a+b)a\,\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}}-18\,(a+b)\sqrt{b}\,\operatorname{arsinh}\left(\frac{b\tan(fx}{\sqrt{(a+b}}\right.}$$

input
```
integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/48*(3*(a + b)^2*a*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/b^(3/2) + 3*(
a + b)^2*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/sqrt(b) - 18*(a + b)*a*ar
csinh(b*tan(f*x + e)/sqrt((a + b)*b))/sqrt(b) - 18*(a + b)*sqrt(b)*arcsinh
(b*tan(f*x + e)/sqrt((a + b)*b)) - 12*(b*tan(f*x + e)^2 + a + b)^(3/2)*tan
(f*x + e) - 18*sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)*tan(f*x + e) - 8*(b*
tan(f*x + e)^2 + a + b)^(5/2)*tan(f*x + e)/b + 2*(b*tan(f*x + e)^2 + a + b
)^(3/2)*(a + b)*tan(f*x + e)/b + 3*sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^
2*tan(f*x + e)/b)/f
```

3.248.8 Giac [F]

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^4\,dx$$

input
```
integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^4, x)
```

3.248.9 Mupad [F(-1)]

Timed out.

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \frac{\left(a+\frac{b}{\cos(e+fx)^2}\right)^{3/2}}{\cos\left(e+fx\right)^4}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x)^4,x)
```

output
```
int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x)^4, x)
```

3.249 $\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

3.249.1 Optimal result 1863
3.249.2 Mathematica [C] (verified) 1863
3.249.3 Rubi [A] (verified) 1864
3.249.4 Maple [B] (verified) 1866
3.249.5 Fricas [A] (verification not implemented) 1867
3.249.6 Sympy [F] 1867
3.249.7 Maxima [A] (verification not implemented) 1868
3.249.8 Giac [F] 1868
3.249.9 Mupad [F(-1)] 1868

3.249.1 Optimal result

Integrand size = 25, antiderivative size = 111

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{3(a+b)^2\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8\sqrt{b}f} + \frac{3(a+b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} + \frac{\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{4f}$$

output
```
3/8*(a+b)^2*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/b^(1/
2)+3/8*(a+b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f+1/4*tan(f*x+e)*(a+b+b
*tan(f*x+e)^2)^(3/2)/f
```

3.249.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.46 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.76

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{(a+b)^2\,\text{Hypergeometric2F1}\left(\frac{1}{2},3,\frac{3}{2},\frac{b\sin^2(e+fx)}{a+b-a\sin^2(e+fx)}\right)\sqrt{a+b\sec^2(e+fx)}\sin(2(e+fx))}{f(a+2b+a\cos(2(e+fx)))}$$

input
```
Integrate[Sec[e + f*x]^2*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
((a + b)^2*Hypergeometric2F1[1/2, 3, 3/2, (b*Sin[e + f*x]^2)/(a + b - a*Si
n[e + f*x]^2)]*Sqrt[a + b*Sec[e + f*x]^2]*Sin[2*(e + f*x)])/(f*(a + 2*b +
a*Cos[2*(e + f*x)]))
```

3.249.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 4634, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int \sec(e+fx)^2\left(a+b\sec(e+fx)^2\right)^{3/2}dx$$

$$\downarrow 4634$$

$$\frac{\int \left(b\tan^2(e+fx)+a+b\right)^{3/2}d\tan(e+fx)}{f}$$

$$\downarrow 211$$

$$\frac{\frac{3}{4}(a+b)\int \sqrt{b\tan^2(e+fx)+a+b}d\tan(e+fx)+\frac{1}{4}\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{f}$$

$$\downarrow 211$$

$$\frac{\frac{3}{4}(a+b)\left(\frac{1}{2}(a+b)\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)+\frac{1}{4}\tan(e+fx)\left(a\right.}{f}$$

$$\downarrow 224$$

$$\frac{\frac{3}{4}(a+b)\left(\frac{1}{2}(a+b)\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)+\frac{1}{4}\tan(e+f}{f}$$

$$\downarrow 219$$

$$\frac{\frac{3}{4}(a+b)\left(\frac{(a+b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2\sqrt{b}}+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)+\frac{1}{4}\tan(e+fx)\left(a+b\tan^2\right.}{f}$$

input
```
Int[Sec[e + f*x]^2*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
((Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/2))/4 + (3*(a + b)*(((a + b)*
ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(2*Sqrt[b]
) + (Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/2))/4)/f
```

3.249.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.249.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 950 vs. 2(95) = 190.

Time = 11.80 (sec) , antiderivative size = 951, normalized size of antiderivative = 8.57

method	result	size
default	Expression too large to display	951

input
```
int(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/16/f/b^(5/2)*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)/(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))*(6*cos(f*x+e)^2*sin(f*x+e)*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(7/2)+6*cos(f*x+e)*sin(f*x+e)*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(7/2)+10*cos(f*x+e)^2*sin(f*x
+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(5/2)*a+4*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(7/2)*sin(f*x+e)+10*cos(f*x+e)*sin(f*x+e)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(5/2)*a+3*cos(f*x+e)^3*ln(4*
(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1)
)*a^2*b^2+6*cos(f*x+e)^3*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-si
n(f*x+e)*a-a-b)/(sin(f*x+e)+1))*a*b^3+3*cos(f*x+e)^3*ln(4*(((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*b^4+3*cos(f*x+
e)^3*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)
+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(si
n(f*x+e)-1))*a^2*b^2+6*cos(f*x+e)^3*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*a*b^3+3*cos(f*x+e)^3*ln(-4*(((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((...
```

3.249.5 Fricas [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 390, normalized size of antiderivative = 3.51

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \left[\frac{3\left(a^2+2\,ab+b^2\right)\sqrt{b}\cos\left(fx+e\right)^3\log\left(\frac{\left(a^2-6\,ab+b^2\right)\cos(fx+e)^4+8\left(ab-b^2\right)\cos(fx+e)^2+4\left((a-b)\cos(fx+\right.\right.\right.}{\cos(fx+e)^4}$$

input
```
integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/32*(3*(a^2 + 2*a*b + b^2)*sqrt(b)*cos(f*x + e)^3*log(((a^2 - 6*a*b + b^
2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)
^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 4*((5*a*b + 3*b^2)*cos(f*x + e)^2
 + 2*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b*f*c
os(f*x + e)^3), 1/16*(3*(a^2 + 2*a*b + b^2)*sqrt(-b)*arctan(-1/2*((a - b)*
cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e)^3 +
2*((5*a*b + 3*b^2)*cos(f*x + e)^2 + 2*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)*sin(f*x + e))/(b*f*cos(f*x + e)^3)]
```

3.249.6 Sympy [F]

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)^{\frac{3}{2}}\sec^2\left(e+fx\right)dx$$

input
```
integrate(sec(f*x+e)**2*(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(3/2)*sec(e + f*x)**2, x)
```

3.249.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.94

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\frac{3\,(a+b)a\,\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}} + 3\,(a+b)\sqrt{b}\,\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right) + 2\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}\tan($$

input
```
integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/8*(3*(a + b)*a*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/sqrt(b) + 3*(a +
b)*sqrt(b)*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b)) + 2*(b*tan(f*x + e)^2 +
 a + b)^(3/2)*tan(f*x + e) + 3*sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)*tan(
f*x + e))/f
```

3.249.8 Giac [F]

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}\sec(fx+e)^2\,dx$$

input
```
integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^2, x)
```

3.249.9 Mupad [F(-1)]

Timed out.

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \frac{\left(a+\frac{b}{\cos(e+fx)^2}\right)^{3/2}}{\cos(e+fx)^2}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x)^2,x)
```

output
```
int((a + b/cos(e + f*x)^2)^(3/2)/cos(e + f*x)^2, x)
```

3.250 $\int (a+b\sec^2(e+fx))^{3/2}\,dx$

3.250.1 Optimal result 1869
3.250.2 Mathematica [C] (warning: unable to verify) 1869
3.250.3 Rubi [A] (verified) 1870
3.250.4 Maple [B] (warning: unable to verify) 1873
3.250.5 Fricas [B] (verification not implemented) 1874
3.250.6 Sympy [F] 1874
3.250.7 Maxima [F] 1875
3.250.8 Giac [F] 1875
3.250.9 Mupad [F(-1)] 1875

3.250.1 Optimal result

Integrand size = 16, antiderivative size = 118

$$\int (a+b\sec^2(e+fx))^{3/2}\,dx = \frac{a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{\sqrt{b}(3a+b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f} + \frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f}$$

output
```
a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f+1/2*(3*a+b
)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f+1/2*b*(
a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f
```

3.250.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.14 (sec) , antiderivative size = 527, normalized size of antiderivative = 4.47

$$\int (a+b\sec^2(e+fx))^{3/2}\,dx = \frac{\sqrt{2}e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos^3(e+fx)\left(-\frac{ib\left(-1+e^{2i(e+fx)}\right)}{\left(1+e^{2i(e+fx)}\right)^2}+\frac{2a^{3/2}fx-ia^{3/2}\log(\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[2]*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*
I)*(e + f*x))]*Cos[e + f*x]^3*(((-I)*b*(-1 + E^((2*I)*(e + f*x))))/(1 + E^
((2*I)*(e + f*x)))^2 + (2*a^(3/2)*f*x - I*a^(3/2)*Log[a + 2*b + a*E^((2*I)
*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e +
f*x)))^2]] + I*a^(3/2)*Log[a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*I)*(e + f
*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^
2]] - 3*a*Sqrt[b]*Log[(-2*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*f + (2*I)*Sqr
t[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)/(b*(3*a + b)
*(1 + E^((2*I)*(e + f*x))))] - b^(3/2)*Log[(-2*Sqrt[b]*(-1 + E^((2*I)*(e +
 f*x)))*f + (2*I)*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)
))^2]*f)/(b*(3*a + b)*(1 + E^((2*I)*(e + f*x))))])/Sqrt[4*b*E^((2*I)*(e +
f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*(a + b*Sec[e + f*x]^2)^(3/2))/(f*(
a + 2*b + a*Cos[2*e + 2*f*x])^(3/2))
```

3.250.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4616, 318, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a + b\sec^2(e+fx)\right)^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int \left(a + b\sec(e+fx)^2\right)^{3/2} dx$$

$$\downarrow 4616$$

$$\frac{\int \frac{\left(b\tan^2(e+fx)+a+b\right)^{3/2}}{\tan^2(e+fx)+1} d\tan(e+fx)}{f}$$

$$\downarrow 318$$

$$\frac{\frac{1}{2}\int \frac{b(3a+b)\tan^2(e+fx)+(a+b)(2a+b)}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}} d\tan(e+fx) + \frac{1}{2}b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

$$\downarrow 398$$

$$\frac{\frac{1}{2}\left(2a^2\int\frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+b(3a+b)\int\frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)\right)+\frac{1}{2}b\tan(e+}{f}$$

$$\downarrow 224$$

$$\frac{\frac{1}{2}\left(2a^2\int\frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+b(3a+b)\int\frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}\right)+\frac{1}{2}b\tan}{f}$$

$$\downarrow 219$$

$$\frac{\frac{1}{2}\left(2a^2\int\frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+\sqrt{b}(3a+b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)\right)+\frac{1}{2}b\tan(e+fx)}{f}$$

$$\downarrow 291$$

$$\frac{\frac{1}{2}\left(2a^2\int\frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}+\sqrt{b}(3a+b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)\right)+\frac{1}{2}b\tan(e+fx)\sqrt{a+b}}{f}$$

$$\downarrow 216$$

$$\frac{\frac{1}{2}\left(2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)+\sqrt{b}(3a+b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)\right)+\frac{1}{2}b\tan(e+fx)\sqrt{a+b\tan^2(e+}}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
((2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]
+ Sqrt[b]*(3*a + b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e +
f*x]^2]])/2 + (b*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/2)/f
```

3.250.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

3.250.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 708 vs. $2(100) = 200$.

Time = 7.76 (sec) , antiderivative size = 709, normalized size of antiderivative = 6.01

method	result
default	$\frac{\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}\left(\cos(fx+e)^3 b^{\frac{5}{2}}\ln\left(\frac{-4\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{b}\cos(fx+e)+4\sin(fx+e)a-4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4a-4b}{\sin(fx+e)-1}\right)\sqrt{-a}+\cos(f\right.}{}$

input

```
int((a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
1/4/f/(-a)^(1/2)/b*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)/(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))*(cos(f*x+e)^3*b^(5/2)*ln(4
*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+
e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-a-b)/(sin(f*x+e)-
1))*(-a)^(1/2)+cos(f*x+e)^3*b^(5/2)*ln(-4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)+a+b)/(sin(f*x+e)+1))*(-a)^(1/2)+3*cos(f*x+e)^3*b^(
3/2)*ln(4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)
+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-a-b)/(si
n(f*x+e)-1))*(-a)^(1/2)*a+3*cos(f*x+e)^3*b^(3/2)*ln(-4*(-((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+a+b)/(sin(f*x+e)+1))*(-a)^(1/2)*a+2*s
in(f*x+e)*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1
/2)*b^2+4*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(
f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+
e)*a)*cos(f*x+e)^3*a^2*b+2*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*b^2*cos(f*x+e)*sin(f*x+e))
```

3.250.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(100) = 200$.

Time = 0.60 (sec) , antiderivative size = 1457, normalized size of antiderivative = 12.35

$$\int \left(a + b\sec^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(sqrt(-a)*a*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*
b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4
 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2
- a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(
f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*
b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)*sin(f*x + e)) + (3*a + b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a
*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f
*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 4*b*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)*sin(f*x + e))/(f*cos(f*x + e)), 1/8*(2*(3*a + b)*sqr
t(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f
*x + e)))*cos(f*x + e) + sqrt(-a)*a*cos(f*x + e)*log(128*a^4*cos(f*x + e)^
8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*c
os(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7
*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24
*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e
)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 4*b*sqrt((a*cos(f*x + e)...
```

3.250.6 Sympy [F]

$$\int \left(a + b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(a + b\sec^2(e+fx)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(3/2), x)
```

3.250.7 Maxima [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.250.8 Giac [F]

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.250.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int((a + b/cos(e + f*x)^2)^(3/2), x)
```

3.251 $\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

3.251.1 Optimal result 1876
3.251.2 Mathematica [C] (warning: unable to verify) 1876
3.251.3 Rubi [A] (verified) 1877
3.251.4 Maple [B] (warning: unable to verify) 1880
3.251.5 Fricas [B] (verification not implemented) 1881
3.251.6 Sympy [F(-1)] 1881
3.251.7 Maxima [F] 1882
3.251.8 Giac [F] 1882
3.251.9 Mupad [F(-1)] 1882

3.251.1 Optimal result

Integrand size = 25, antiderivative size = 124

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\sqrt{a}(a+3b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f} + \frac{b^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{a\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f}$$

output
```
b^(3/2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f+1/2*(a+3*
b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*a^(1/2)/f+1/2*a*c
os(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/f
```

3.251.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.32 (sec) , antiderivative size = 466, normalized size of antiderivative = 3.76

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{e^{-i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos^3(e+fx)\left(-ia\left(-1+e^{2i(e+fx)}\right)+\frac{2e^{2i(e+fx)}\left(2a^{3}\right.\right.}{}$$

input
```
Integrate[Cos[e + f*x]^2*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f*x))]*Cos[e + f
*x]^3*((-I)*a*(-1 + E^((2*I)*(e + f*x))) + (2*E^((2*I)*(e + f*x))*(2*a^(3/
2)*f*x + 6*Sqrt[a]*b*f*x - I*Sqrt[a]*(a + 3*b)*Log[(a + 2*b + a*E^((2*I)*(
e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*
x)))^2])/E^((2*I)*e)] + I*Sqrt[a]*(a + 3*b)*Log[(a + a*E^((2*I)*(e + f*x))
 + 2*b*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 +
 E^((2*I)*(e + f*x)))^2])/E^((2*I)*e)] - 4*b^(3/2)*Log[-1/2*(E^((3*I)*e)*(
Sqrt[b]*(-1 + E^((2*I)*(e + f*x))) - I*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1
 + E^((2*I)*(e + f*x)))^2])*f)/(b^2*(1 + E^((2*I)*(e + f*x))))])/Sqrt[4*b
*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*(a + b*Sec[e + f*x]
^2)^(3/2))/(2*Sqrt[2]*E^(I*(e + f*x))*f*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/
2))
```

3.251.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4634, 315, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^{3/2}}{\sec(e+fx)^2}dx$$

$$\downarrow 4634$$

$$\frac{\int \frac{\left(b\tan^2(e+fx)+a+b\right)^{3/2}}{\left(\tan^2(e+fx)+1\right)^2}d\tan(e+fx)}{f}$$

$$\downarrow 315$$

$$\frac{\frac{1}{2}\int \frac{2b^2\tan^2(e+fx)+(a+b)(a+2b)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+\frac{a\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2\left(\tan^2(e+fx)+1\right)}}{f}$$

$$\downarrow 398$$

$$\frac{\frac{1}{2}\left(2b^2\int\frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+a(a+3b)\int\frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)\right)+\frac{a\tan(e+fx)\sqrt{}}{2(\tan^2}}{f}$$

$$\downarrow 224$$

$$\frac{\frac{1}{2}\left(2b^2\int\frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}+a(a+3b)\int\frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)\right)+\frac{a\tan(e+}{}}{f}$$

$$\downarrow 219$$

$$\frac{\frac{1}{2}\left(a(a+3b)\int\frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+2b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)\right)+\frac{a\tan(e+fx)\sqrt{a+b}}{2(\tan^2(e+}}{f}$$

$$\downarrow 291$$

$$\frac{\frac{1}{2}\left(a(a+3b)\int\frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}+2b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)\right)+\frac{a\tan(e+fx)\sqrt{a+b\tan^2(e+f}}{2(\tan^2(e+fx)+1)}}{f}$$

$$\downarrow 216$$

$$\frac{\frac{1}{2}\left(\sqrt{a}(a+3b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)+2b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)\right)+\frac{a\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2(\tan^2(e+fx)+1)}}{f}$$

input
```
Int[Cos[e + f*x]^2*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
((Sqrt[a]*(a + 3*b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f
*x]^2]] + 2*b^(3/2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e +
f*x]^2]])/2 + (a*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*(1 + Tan[
e + f*x]^2)))/f
```

3.251. $\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

3.251.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.251.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 536 vs. $2(106) = 212$.

Time = 9.31 (sec) , antiderivative size = 537, normalized size of antiderivative = 4.33

method	result
default	$\frac{\left(\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\sqrt{-a}\,a\cos(fx+e)\sin(fx+e)+b^{\frac{3}{2}}\ln\left(-\frac{4\left(-\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\sqrt{b}\,\cos(fx+e)+\sin(fx+e)a-\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}+a+b\right.\right.}{\sin(fx+e)+1}$

input

```
int(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
1/2/f/(-a)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*a
*cos(f*x+e)*sin(f*x+e)+b^(3/2)*ln(-4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)+a+b)/(sin(f*x+e)+1))*(-a)^(1/2)+b^(3/2)*ln(4*(-((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2
)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-a-b)/(sin(f*x+e)-1))*(-a)^(1
/2)+sin(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*a+ln
(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a
)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^2+3*
ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(
-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a*b)
*(a+b*sec(f*x+e)^2)^(3/2)*cos(f*x+e)^3/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)/(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
```

3.251.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 276 vs. 2(106) = 212.

Time = 0.61 (sec) , antiderivative size = 1403, normalized size of antiderivative = 11.31

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/16*(8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*
x + e) + sqrt(-a)*(a + 3*b)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)
*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 -
 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 -
a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*
x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b
+ 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)*sin(f*x + e)) + 4*b^(3/2)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^
4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x
 + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) +
8*b^2)/cos(f*x + e)^4))/f, 1/16*(8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2)*cos(f*x + e)*sin(f*x + e) + 8*sqrt(-b)*b*arctan(-1/2*((a - b)*cos(f
*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e))) + sqrt(-a)*(a + 3*b)*l
og(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 -
 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a
*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16
*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*
b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e)
)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)))/f...
```

3.251.6 Sympy [F(-1)]

Timed out.

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**2*(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

3.251.7 Maxima [F]

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^2\,dx$$

input
```
integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^2, x)
```

3.251.8 Giac [F]

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^2\,dx$$

input
```
integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^2, x)
```

3.251.9 Mupad [F(-1)]

Timed out.

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \cos\left(e+f\,x\right)^2\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}\,dx$$

input
```
int(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.252 $\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

3.252.1 Optimal result 1883
3.252.2 Mathematica [A] (verified) 1883
3.252.3 Rubi [A] (verified) 1884
3.252.4 Maple [B] (verified) 1886
3.252.5 Fricas [B] (verification not implemented) 1887
3.252.6 Sympy [F(-1)] 1888
3.252.7 Maxima [F] 1888
3.252.8 Giac [F] 1888
3.252.9 Mupad [F(-1)] 1889

3.252.1 Optimal result

Integrand size = 25, antiderivative size = 125

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{3(a+b)^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8\sqrt{a}f} + \frac{3(a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} + \frac{\cos^3(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{4f}$$

output
```
3/8*(a+b)^2*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/a^(1/2
)+3/8*(a+b)*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/f+1/4*cos(f*x
+e)^3*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(3/2)/f
```

3.252.2 Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.53

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\cos(e+fx)\left(b+a\cos^2(e+fx)\right)\sqrt{a+b\sec^2(e+fx)}\sqrt{a+b-a\sin^2(e+fx)}\left(3(a+b)^{3/2}\text{ar}\right.}{2\sqrt{a}f(a+2b+a\cos(2(e+fx)))}$$

input
```
Integrate[Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Cos[e + f*x]*(b + a*Cos[e + f*x]^2)*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt[a + b
 - a*Sin[e + f*x]^2]*(3*(a + b)^(3/2)*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a
 + b]] + Sqrt[a]*(4*a + 5*b + a*Cos[2*(e + f*x)])*Sin[e + f*x]*Sqrt[(a + b
 - a*Sin[e + f*x]^2)/(a + b)]))/(2*Sqrt[a]*f*(a + 2*b + a*Cos[2*(e + f*x)]
)^(3/2)*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)])
```

3.252.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 4634, 292, 292, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^{3/2}}{\sec(e+fx)^4}dx$$

$$\downarrow 4634$$

$$\frac{\int \frac{\left(b\tan^2(e+fx)+a+b\right)^{3/2}}{\left(\tan^2(e+fx)+1\right)^3}d\tan(e+fx)}{f}$$

$$\downarrow 292$$

$$\frac{\frac{3}{4}(a+b)\int \frac{\sqrt{b\tan^2(e+fx)+a+b}}{\left(\tan^2(e+fx)+1\right)^2}d\tan(e+fx)+\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4\left(\tan^2(e+fx)+1\right)^2}}{f}$$

$$\downarrow 292$$

$$\frac{\frac{3}{4}(a+b)\left(\frac{1}{2}(a+b)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2\left(\tan^2(e+fx)+1\right)}\right)+\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4\left(\tan^2(e+f\right.}}{f}$$

$$\downarrow 291$$

$$\frac{\frac{3}{4}(a+b)\left(\frac{1}{2}(a+b)\int\frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2(\tan^2(e+fx)+1)}\right)+\frac{\tan(e+fx)(a+b\tan^2(e+fx)+}{4(\tan^2(e+fx)+1)^2}}{f}$$

$$\downarrow 216$$

$$\frac{\frac{3}{4}(a+b)\left(\frac{(a+b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2\sqrt{a}}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2(\tan^2(e+fx)+1)}\right)+\frac{\tan(e+fx)(a+b\tan^2(e+fx)+b)^{3/2}}{4(\tan^2(e+fx)+1)^2}}{f}$$

input
```
Int[Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
((Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/2))/(4*(1 + Tan[e + f*x]^2)^2
) + (3*(a + b)*(((a + b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[
e + f*x]^2]])/(2*Sqrt[a]) + (Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/
(2*(1 + Tan[e + f*x]^2))))/4)/f
```

3.252.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 292
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> Si
mp[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] - Simp[c*(q/(
a*(p + 1)))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1), x], x] /; FreeQ[
{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q + 1) + 1, 0] && Gt
Q[q, 0] && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.252.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 601 vs. $2(109) = 218$.

Time = 3.58 (sec) , antiderivative size = 602, normalized size of antiderivative = 4.82

method	result
default	$\left(2\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{-a}\,a\cos(fx+e)^3\sin(fx+e)+2\cos(fx+e)^2\sin(fx+e)\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{-a}\,a+3\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{-a}\,a\cos$

input
```
int(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/f/(-a)^(1/2)*(2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)
*a*cos(f*x+e)^3*sin(f*x+e)+2*cos(f*x+e)^2*sin(f*x+e)*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*a+3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*(-a)^(1/2)*a*cos(f*x+e)*sin(f*x+e)+5*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*(-a)^(1/2)*b*cos(f*x+e)*sin(f*x+e)+3*sin(f*x+e)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*a+5*sin(f*x+e)*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1/2)*b+3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^2+6*ln(4*(-a)^(1/2)*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a*b+3*ln(4*(-a)^(1/2)*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*b^2)*(a+b*sec(f*x+e)^2)^(3/2)*
cos(f*x+e)^3/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)/(b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))
```

3.252.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(109) = 218$.

Time $= 0.57$ (sec) , antiderivative size $= 563$, normalized size of antiderivative $= 4.50$

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \left[-\frac{3\left(a^2+2\,ab+b^2\right)\sqrt{-a}\log\left(128\,a^4\cos\left(fx+e\right)^8-256\left(a^4-a^3b\right)\cos\left(fx+e\right)^6+32\left(5\,a^4\right.\right.\right.}{\left.\left.\left.\right.\right.}$$

$$-\frac{3\left(a^2+2\,ab+b^2\right)\sqrt{a}\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^5-8\left(a^2-ab\right)\cos(fx+e)^3+\left(a^2-6\,ab+b^2\right)\cos(fx+e)\right)\sqrt{a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\left(2\,a^3\cos(fx+e)^4-a^2b+ab^2-\left(a^3-3\,a^2b\right)\cos(fx+e)^2\right)\sin(fx+e)}\right)-4\left(2\right.}{32\,af}$$

input
```
integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/64*(3*(a^2 + 2*a*b + b^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a
^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e
)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7
*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^
2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3
 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)*sin(f*x + e)) - 8*(2*a^2*cos(f*x + e)^3 + (3*a^2 + 5*
a*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e
))/(a*f), -1/32*(3*(a^2 + 2*a*b + b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x +
 e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*s
qrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4
- a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(2*a^
2*cos(f*x + e)^3 + (3*a^2 + 5*a*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)*sin(f*x + e))/(a*f)]
```

3.252.6 Sympy [F(-1)]

Timed out.

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**4*(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

3.252.7 Maxima [F]

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^4\,dx$$

input
```
integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^4, x)
```

3.252.8 Giac [F]

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^4\,dx$$

input
```
integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^4, x)
```

3.252.9 Mupad [F(-1)]

Timed out.

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \cos(e+fx)^4\left(a+\frac{b}{\cos(e+fx)^2}\right)^{3/2}\,dx$$

input
```
int(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.253 $\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

3.253.1 Optimal result 1890
3.253.2 Mathematica [A] (verified) 1891
3.253.3 Rubi [A] (verified) 1891
3.253.4 Maple [B] (verified) 1894
3.253.5 Fricas [A] (verification not implemented) 1895
3.253.6 Sympy [F(-1)] 1896
3.253.7 Maxima [F] 1896
3.253.8 Giac [F] 1896
3.253.9 Mupad [F(-1)] 1897

3.253.1 Optimal result

Integrand size = 25, antiderivative size = 193

$$\int \cos^6(e+fx)\,(a + b\sec^2(e+fx))^{3/2}\,dx = \frac{(5a-b)(a+b)^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{3/2}f} + \frac{(5a-b)(a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16af} + \frac{(5a-b)\cos^3(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{3/2}}{24af} + \frac{\cos^5(e+fx)\sin(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{5/2}}{6af}$$

output

```
1/16*(5*a-b)*(a+b)^2*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))
/a^(3/2)/f+1/16*(5*a-b)*(a+b)*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(
1/2)/a/f+1/24*(5*a-b)*cos(f*x+e)^3*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(3/2)/a
/f+1/6*cos(f*x+e)^5*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(5/2)/a/f
```

3.253.2 Mathematica [A] (verified)

Time = 2.22 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.85

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(\frac{3\sqrt{2}\sqrt{a+b}(5a^2+4ab-b^2)\arcsin\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)}{\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}}}+\sqrt{a}(23a^2+29ab+3\right.}{48a^{3/2}f}$$

input
```
Integrate[Cos[e + f*x]^6*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*((3*Sqrt[2]*Sqrt[a + b]*(5*a^2 +
4*a*b - b^2)*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]])/Sqrt[(a + 2*b + a
*Cos[2*(e + f*x)])/(a + b)] + Sqrt[a]*(23*a^2 + 29*a*b + 3*b^2 + a*(9*a +
7*b)*Cos[2*(e + f*x)] + a^2*Cos[4*(e + f*x)])*Sin[e + f*x]))/(48*a^(3/2)*f
)
```

3.253.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4634, 296, 292, 292, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^{3/2}}{\sec(e+fx)^6}\,dx$$

$$\downarrow 4634$$

$$\frac{\int \frac{\left(b\tan^2(e+fx)+a+b\right)^{3/2}}{\left(\tan^2(e+fx)+1\right)^4}\,d\tan(e+fx)}{f}$$

$$\downarrow 296$$

$$\frac{(5a-b)\int \frac{\left(b\tan^2(e+fx)+a+b\right)^{3/2}}{\left(\tan^2(e+fx)+1\right)^3}d\tan(e+fx)}{6a} + \frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{6a\left(\tan^2(e+fx)+1\right)^3}$$
$$f$$

$$\downarrow 292$$

$$\frac{(5a-b)\left(\frac{3}{4}(a+b)\int \frac{\sqrt{b\tan^2(e+fx)+a+b}}{\left(\tan^2(e+fx)+1\right)^2}d\tan(e+fx)+\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4\left(\tan^2(e+fx)+1\right)^2}\right)}{6a} + \frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{5/2}}{6a\left(\tan^2(e+fx)+1\right)^3}$$
$$f$$

$$\downarrow 292$$

$$\frac{(5a-b)\left(\frac{3}{4}(a+b)\left(\frac{1}{2}(a+b)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2\left(\tan^2(e+fx)+1\right)}\right)+\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4\left(\tan^2(e+fx)+1\right)^2}\right)}{6a}$$
$$f$$

$$\downarrow 291$$

$$\frac{(5a-b)\left(\frac{3}{4}(a+b)\left(\frac{1}{2}(a+b)\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2\left(\tan^2(e+fx)+1\right)}\right)+\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4\left(\tan^2(e+fx)+1\right)^2}\right)}{6a} + \frac{ta}{}$$
$$f$$

$$\downarrow 216$$

$$\frac{(5a-b)\left(\frac{3}{4}(a+b)\left(\frac{(a+b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2\sqrt{a}}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2\left(\tan^2(e+fx)+1\right)}\right)+\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}{4\left(\tan^2(e+fx)+1\right)^2}\right)}{6a} + \frac{\tan(e+fx)(a+b\tan}{6a(\tan^2(e+}$$
$$f$$

input
```
Int[Cos[e + f*x]^6*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
((Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(5/2))/(6*a*(1 + Tan[e + f*x]^2)
^3) + ((5*a - b)*((Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(3/2))/(4*(1 +
Tan[e + f*x]^2)^2) + (3*(a + b)*(((a + b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sq
rt[a + b + b*Tan[e + f*x]^2]])/(2*Sqrt[a]) + (Tan[e + f*x]*Sqrt[a + b + b*
Tan[e + f*x]^2])/(2*(1 + Tan[e + f*x]^2))))/4))/(6*a))/f
```

3.253. $\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

3.253.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 292
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> Si
mp[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] - Simp[c*(q/(
a*(p + 1)))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1), x], x] /; FreeQ[
{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q + 1) + 1, 0] && Gt
Q[q, 0] && NeQ[p, -1]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.253.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 985 vs. 2(173) = 346.

Time = 6.04 (sec) , antiderivative size = 986, normalized size of antiderivative = 5.11

method	result	size
default	Expression too large to display	986

input
```
int(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/48/f/a/(-a)^(1/2)*(8*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*a^2*cos(f*x+e)^5*sin(f*x+e)+8*(-a)^(1/2)*sin(f*x+e)*cos(f*x+e)^4*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2+10*(-a)^(1/2)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*a^2*cos(f*x+e)^3*sin(f*x+e)+14*(-a)^(1/2)*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b*cos(f*x+e)^3*sin(f*x+e)+10*(-a)
^(1/2)*sin(f*x+e)*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*a^2+14*(-a)^(1/2)*sin(f*x+e)*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*a*b+15*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*a^2*cos(f*x+e)*sin(f*x+e)+22*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*a*b*cos(f*x+e)*sin(f*x+e)+3*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*b^2*cos(f*x+e)*sin(f*x+e)+15*(-a)^(1/2)*sin(f*x+e)*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2+22*(-a)^(1/2)*sin(f*x+e)*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b+3*(-a)^(1/2)*sin(f*x+e)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2+15*ln(4*(-a)^(1/2)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^3+27*ln(4*(-a)^(1/2)*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^2*b+9*ln(4*(-a)^(1/2)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a*b^2-3*ln(4*(-a)^(1/2)...
```

3.253.5 Fricas [A] (verification not implemented)

Time = 1.19 (sec) , antiderivative size = 647, normalized size of antiderivative = 3.35

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \left[\frac{3\left(5\,a^3+9\,a^2b+3\,ab^2-b^3\right)\sqrt{-a}\log\left(128\,a^4\cos\left(fx+e\right)^8-256\left(a^4-a^3b\right)\cos\left(fx+e\right)^6+\right.\right.$$

$$\left.\left.-\frac{3\left(5\,a^3+9\,a^2b+3\,ab^2-b^3\right)\sqrt{a}\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^5-8\left(a^2-ab\right)\cos(fx+e)^3+\left(a^2-6\,ab+b^2\right)\cos(fx+e)\right)\sqrt{a}\sqrt{\frac{a\cos(fx+e)^2+}{\cos(fx+e)^2}}}{4\left(2\,a^3\cos(fx+e)^4-a^2b+ab^2-\left(a^3-3\,a^2b\right)\cos(fx+e)^2\right)\sin(fx+e)}\right.}\right.$$

input
```
integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/384*(3*(5*a^3 + 9*a^2*b + 3*a*b^2 - b^3)*sqrt(-a)*log(128*a^4*cos(f*x +
 e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^
2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4
 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7
- 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x
 + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(8*a^3*cos(f*x + e)^5
 + 2*(5*a^3 + 7*a^2*b)*cos(f*x + e)^3 + (15*a^3 + 22*a^2*b + 3*a*b^2)*cos(
f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^2*f
), -1/192*(3*(5*a^3 + 9*a^2*b + 3*a*b^2 - b^3)*sqrt(a)*arctan(1/4*(8*a^2*c
os(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x
 + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x
 + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) -
 4*(8*a^3*cos(f*x + e)^5 + 2*(5*a^3 + 7*a^2*b)*cos(f*x + e)^3 + (15*a^3 +
22*a^2*b + 3*a*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*sin(f*x + e))/(a^2*f)]
```

3.253.6 Sympy [F(-1)]

Timed out.

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**6*(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

3.253.7 Maxima [F]

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}\cos(fx+e)^6\,dx$$

input
```
integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^6, x)
```

3.253.8 Giac [F]

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}\cos(fx+e)^6\,dx$$

input
```
integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^6, x)
```

3.253.9 Mupad [F(-1)]

Timed out.

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \cos(e+fx)^6\left(a+\frac{b}{\cos(e+fx)^2}\right)^{3/2}\,dx$$

input
```
int(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.254 $\int (a+b\sec^2(c+dx))^{5/2}\,dx$

3.254.1 Optimal result 1898
3.254.2 Mathematica [C] (warning: unable to verify) 1899
3.254.3 Rubi [A] (verified) 1900
3.254.4 Maple [B] (warning: unable to verify) 1903
3.254.5 Fricas [B] (verification not implemented) 1903
3.254.6 Sympy [F] 1904
3.254.7 Maxima [F] 1905
3.254.8 Giac [F] 1905
3.254.9 Mupad [F(-1)] 1905

3.254.1 Optimal result

Integrand size = 16, antiderivative size = 166

$$\int (a+b\sec^2(c+dx))^{5/2}\,dx = \frac{a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+b+b\tan^2(c+dx)}}\right)}{d} + \frac{\sqrt{b}(15a^2+10ab+3b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a+b+b\tan^2(c+dx)}}\right)}{8d} + \frac{b(7a+3b)\tan(c+dx)\sqrt{a+b+b\tan^2(c+dx)}}{8d} + \frac{b\tan(c+dx)\left(a+b+b\tan^2(c+dx)\right)^{3/2}}{4d}$$

output

```
a^(5/2)*arctan(a^(1/2)*tan(d*x+c)/(a+b+b*tan(d*x+c)^2)^(1/2))/d+1/8*(15*a^
2+10*a*b+3*b^2)*arctanh(b^(1/2)*tan(d*x+c)/(a+b+b*tan(d*x+c)^2)^(1/2))*b^(
1/2)/d+1/8*b*(7*a+3*b)*(a+b+b*tan(d*x+c)^2)^(1/2)*tan(d*x+c)/d+1/4*b*tan(d
*x+c)*(a+b+b*tan(d*x+c)^2)^(3/2)/d
```

3.254.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.62 (sec) , antiderivative size = 706, normalized size of antiderivative = 4.25

$$\int \left(a+b\sec^2(c+dx)\right)^{5/2}\,dx = \frac{e^{i(c+dx)}\sqrt{4b+ae^{-2i(c+dx)}\left(1+e^{2i(c+dx)}\right)^2}\cos^5(c+dx)\left(-\frac{ib\left(-1+e^{2i(c+dx)}\right)\left(9a\left(1+e^{2i(c+dx)}\right)^2+b\left(3+14e^{2i(c+dx)}\right)\right)}{\left(1+e^{2i(c+dx)}\right)^4}\right.}{}$$

input
```
Integrate[(a + b*Sec[c + d*x]^2)^(5/2),x]
```

output
```
(E^(I*(c + d*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(c + d*x)))^2)/E^((2*I)*(c +
d*x))]*Cos[c + d*x]^5*(((-I)*b*(-1 + E^((2*I)*(c + d*x)))*(9*a*(1 + E^((2*
I)*(c + d*x)))^2 + b*(3 + 14*E^((2*I)*(c + d*x)) + 3*E^((4*I)*(c + d*x))))
)/(1 + E^((2*I)*(c + d*x)))^4 + (8*a^(5/2)*d*x - (4*I)*a^(5/2)*Log[a + 2*b
 + a*E^((2*I)*(c + d*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(c + d*x)) + a*(1 + E
^((2*I)*(c + d*x)))^2]] + (4*I)*a^(5/2)*Log[a + a*E^((2*I)*(c + d*x)) + 2*
b*E^((2*I)*(c + d*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(c + d*x)) + a*(1 + E^((
2*I)*(c + d*x)))^2]] - 15*a^2*Sqrt[b]*Log[(-4*Sqrt[b]*d*(-1 + E^((2*I)*(c
+ d*x))) + (4*I)*d*Sqrt[4*b*E^((2*I)*(c + d*x)) + a*(1 + E^((2*I)*(c + d*x
)))^2])/(b*(15*a^2 + 10*a*b + 3*b^2)*(1 + E^((2*I)*(c + d*x))))] - 10*a*b^
(3/2)*Log[(-4*Sqrt[b]*d*(-1 + E^((2*I)*(c + d*x))) + (4*I)*d*Sqrt[4*b*E^((
2*I)*(c + d*x)) + a*(1 + E^((2*I)*(c + d*x)))^2])/(b*(15*a^2 + 10*a*b + 3*
b^2)*(1 + E^((2*I)*(c + d*x))))] - 3*b^(5/2)*Log[(-4*Sqrt[b]*d*(-1 + E^((2
*I)*(c + d*x))) + (4*I)*d*Sqrt[4*b*E^((2*I)*(c + d*x)) + a*(1 + E^((2*I)*(
c + d*x)))^2])/(b*(15*a^2 + 10*a*b + 3*b^2)*(1 + E^((2*I)*(c + d*x))))])/S
qrt[4*b*E^((2*I)*(c + d*x)) + a*(1 + E^((2*I)*(c + d*x)))^2])*(a + b*Sec[c
 + d*x]^2)^(5/2))/(Sqrt[2]*d*(a + 2*b + a*Cos[2*c + 2*d*x])^(5/2))
```

3.254.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {3042, 4616, 318, 403, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sec^2(c+dx)\right)^{5/2}\,dx$$

$$\downarrow 3042$$

$$\int \left(a+b\sec(c+dx)^2\right)^{5/2}dx$$

$$\downarrow 4616$$

$$\frac{\int \frac{\left(b\tan^2(c+dx)+a+b\right)^{5/2}}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}$$

$$\downarrow 318$$

$$\frac{\frac{1}{4}\int \frac{\sqrt{b\tan^2(c+dx)+a+b}\left(b(7a+3b)\tan^2(c+dx)+(a+b)(4a+3b)\right)}{\tan^2(c+dx)+1}d\tan(c+dx)+\frac{1}{4}b\tan(c+dx)\left(a+b\tan^2(c+dx)+b\right)^{3/2}}{d}$$

$$\downarrow 403$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\int \frac{b\left(15a^2+10ba+3b^2\right)\tan^2(c+dx)+(a+b)\left(8a^2+7ba+3b^2\right)}{\left(\tan^2(c+dx)+1\right)\sqrt{b\tan^2(c+dx)+a+b}}d\tan(c+dx)+\frac{1}{2}b(7a+3b)\tan(c+dx)\sqrt{a+b\tan^2(c+dx)+}\right.}{d}$$

$$\downarrow 398$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(8a^3\int \frac{1}{\left(\tan^2(c+dx)+1\right)\sqrt{b\tan^2(c+dx)+a+b}}d\tan(c+dx)+b\left(15a^2+10ab+3b^2\right)\int \frac{1}{\sqrt{b\tan^2(c+dx)+a+b}}d\tan(c+dx)\right)\right.}{d}$$

$$\downarrow 224$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(8a^3\int \frac{1}{\left(\tan^2(c+dx)+1\right)\sqrt{b\tan^2(c+dx)+a+b}}d\tan(c+dx)+b\left(15a^2+10ab+3b^2\right)\int \frac{1}{1-\frac{b\tan^2(c+dx)}{b\tan^2(c+dx)+a+b}}d\frac{\tan(c+dx)}{\sqrt{b\tan^2(c+dx)+}}\right.\right.}{d}$$

$$\downarrow 219$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(8a^3\int\frac{1}{(\tan^2(c+dx)+1)\sqrt{b\tan^2(c+dx)+a+b}}d\tan(c+dx)+\sqrt{b}(15a^2+10ab+3b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a+b\tan^2(c+dx)+b}}\right)\right)+\right.}{d}$$

$$\downarrow 291$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(8a^3\int\frac{1}{\frac{a\tan^2(c+dx)}{b\tan^2(c+dx)+a+b}+1}d\frac{\tan(c+dx)}{\sqrt{b\tan^2(c+dx)+a+b}}+\sqrt{b}(15a^2+10ab+3b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a+b\tan^2(c+dx)+b}}\right)\right)+\frac{1}{2}b(7a-\right.}{d}$$

$$\downarrow 216$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(8a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+b\tan^2(c+dx)+b}}\right)+\sqrt{b}(15a^2+10ab+3b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a+b\tan^2(c+dx)+b}}\right)\right)+\frac{1}{2}b(7a+3b)\tan(\right.}{d}$$

input
```
Int[(a + b*Sec[c + d*x]^2)^(5/2),x]
```

output
```
((b*Tan[c + d*x]*(a + b + b*Tan[c + d*x]^2)^(3/2))/4 + ((8*a^(5/2)*ArcTan[
(Sqrt[a]*Tan[c + d*x])/Sqrt[a + b + b*Tan[c + d*x]^2]] + Sqrt[b]*(15*a^2 +
 10*a*b + 3*b^2)*ArcTanh[(Sqrt[b]*Tan[c + d*x])/Sqrt[a + b + b*Tan[c + d*x
]^2]])/2 + (b*(7*a + 3*b)*Tan[c + d*x]*Sqrt[a + b + b*Tan[c + d*x]^2])/2)/
4)/d
```

3.254.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

3.254.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1136 vs. 2(144) = 288.

Time = 21.08 (sec) , antiderivative size = 1137, normalized size of antiderivative = 6.85

method	result	size
default	Expression too large to display	1137

input
```
int((a+b*sec(d*x+c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/16/d/(-a)^(1/2)/b^2*(a+b*sec(d*x+c)^2)^(5/2)/((a*cos(d*x+c)^2+b)/(cos(d*
x+c)+1)^2)^(1/2)/(a*cos(d*x+c)^2+b)^2/(cos(d*x+c)+1)*(3*b^(9/2)*(-a)^(1/2)
*cos(d*x+c)^5*ln(4*(-b^(1/2)*((a*cos(d*x+c)^2+b)/(cos(d*x+c)+1)^2)^(1/2)*c
os(d*x+c)+sin(d*x+c)*a-b^(1/2)*((a*cos(d*x+c)^2+b)/(cos(d*x+c)+1)^2)^(1/2)
-a-b)/(sin(d*x+c)-1))+3*b^(9/2)*(-a)^(1/2)*cos(d*x+c)^5*ln(-4*(-b^(1/2)*((
a*cos(d*x+c)^2+b)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)+sin(d*x+c)*a-b^(1/2)*
((a*cos(d*x+c)^2+b)/(cos(d*x+c)+1)^2)^(1/2)+a+b)/(sin(d*x+c)+1))+10*b^(7/2
)*(-a)^(1/2)*cos(d*x+c)^5*ln(4*(-b^(1/2)*((a*cos(d*x+c)^2+b)/(cos(d*x+c)+1
)^2)^(1/2)*cos(d*x+c)+sin(d*x+c)*a-b^(1/2)*((a*cos(d*x+c)^2+b)/(cos(d*x+c)
+1)^2)^(1/2)-a-b)/(sin(d*x+c)-1))*a+10*b^(7/2)*(-a)^(1/2)*cos(d*x+c)^5*ln(
-4*(-b^(1/2)*((a*cos(d*x+c)^2+b)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)+sin(d*
x+c)*a-b^(1/2)*((a*cos(d*x+c)^2+b)/(cos(d*x+c)+1)^2)^(1/2)+a+b)/(sin(d*x+c
)+1))*a+15*b^(5/2)*(-a)^(1/2)*cos(d*x+c)^5*ln(4*(-b^(1/2)*((a*cos(d*x+c)^2
+b)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)+sin(d*x+c)*a-b^(1/2)*((a*cos(d*x+c)
^2+b)/(cos(d*x+c)+1)^2)^(1/2)-a-b)/(sin(d*x+c)-1))*a^2+15*b^(5/2)*(-a)^(1/
2)*cos(d*x+c)^5*ln(-4*(-b^(1/2)*((a*cos(d*x+c)^2+b)/(cos(d*x+c)+1)^2)^(1/2
)*cos(d*x+c)+sin(d*x+c)*a-b^(1/2)*((a*cos(d*x+c)^2+b)/(cos(d*x+c)+1)^2)^(1
/2)+a+b)/(sin(d*x+c)+1))*a^2+18*(-a)^(1/2)*cos(d*x+c)^4*sin(d*x+c)*((a*cos
(d*x+c)^2+b)/(cos(d*x+c)+1)^2)^(1/2)*a*b^3+6*(-a)^(1/2)*cos(d*x+c)^4*sin(d
*x+c)*((a*cos(d*x+c)^2+b)/(cos(d*x+c)+1)^2)^(1/2)*b^4+16*cos(d*x+c)^5*l...
```

3.254.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 326 vs. 2(144) = 288.

Time = 1.29 (sec) , antiderivative size = 1611, normalized size of antiderivative = 9.70

$$\int \left(a + b\sec^2(c + dx)\right)^{5/2}\, dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/32*(4*sqrt(-a)*a^2*cos(d*x + c)^3*log(128*a^4*cos(d*x + c)^8 - 256*(a^4
 - a^3*b)*cos(d*x + c)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(d*x + c)^
4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a
^2*b^2 - a*b^3)*cos(d*x + c)^2 - 8*(16*a^3*cos(d*x + c)^7 - 24*(a^3 - a^2*
b)*cos(d*x + c)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(d*x + c)^3 - (a^3 -
 7*a^2*b + 7*a*b^2 - b^3)*cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c)^2 +
b)/cos(d*x + c)^2)*sin(d*x + c)) + (15*a^2 + 10*a*b + 3*b^2)*sqrt(b)*cos(d
*x + c)^3*log(((a^2 - 6*a*b + b^2)*cos(d*x + c)^4 + 8*(a*b - b^2)*cos(d*x
+ c)^2 + 4*((a - b)*cos(d*x + c)^3 + 2*b*cos(d*x + c))*sqrt(b)*sqrt((a*cos
(d*x + c)^2 + b)/cos(d*x + c)^2)*sin(d*x + c) + 8*b^2)/cos(d*x + c)^4) + 4
*(3*(3*a*b + b^2)*cos(d*x + c)^2 + 2*b^2)*sqrt((a*cos(d*x + c)^2 + b)/cos(
d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^3), 1/16*(2*sqrt(-a)*a^2*cos(d*x
 + c)^3*log(128*a^4*cos(d*x + c)^8 - 256*(a^4 - a^3*b)*cos(d*x + c)^6 + 32
*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(d*x + c)^4 + a^4 - 28*a^3*b + 70*a^2*b
^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(d*x + c)^
2 - 8*(16*a^3*cos(d*x + c)^7 - 24*(a^3 - a^2*b)*cos(d*x + c)^5 + 2*(5*a^3
- 14*a^2*b + 5*a*b^2)*cos(d*x + c)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos
(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c)^2 + b)/cos(d*x + c)^2)*sin(d*x +
c)) + (15*a^2 + 10*a*b + 3*b^2)*sqrt(-b)*arctan(-1/2*((a - b)*cos(d*x + c)
^3 + 2*b*cos(d*x + c))*sqrt(-b)*sqrt((a*cos(d*x + c)^2 + b)/cos(d*x + c...
```

3.254.6 Sympy [F]

$$\int \left(a + b\sec^2(c + dx)\right)^{5/2}\, dx = \int \left(a + b\sec^2(c + dx)\right)^{\frac{5}{2}}\, dx$$

input
```
integrate((a+b*sec(d*x+c)**2)**(5/2),x)
```

output
```
Integral((a + b*sec(c + d*x)**2)**(5/2), x)
```

3.254.7 Maxima [F]

$$\int \left(a + b\sec^2(c + dx)\right)^{5/2}\, dx = \int \left(b\sec\left(dx + c\right)^2 + a\right)^{\frac{5}{2}}\, dx$$

input
```
integrate((a+b*sec(d*x+c)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c)^2 + a)^(5/2), x)
```

3.254.8 Giac [F]

$$\int \left(a + b\sec^2(c + dx)\right)^{5/2}\, dx = \int \left(b\sec\left(dx + c\right)^2 + a\right)^{\frac{5}{2}}\, dx$$

input
```
integrate((a+b*sec(d*x+c)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c)^2 + a)^(5/2), x)
```

3.254.9 Mupad [F(-1)]

Timed out.

$$\int \left(a + b\sec^2(c + dx)\right)^{5/2}\, dx = \int \left(a + \frac{b}{\cos\left(c + d\,x\right)^2}\right)^{5/2}\, dx$$

input
```
int((a + b/cos(c + d*x)^2)^(5/2),x)
```

output
```
int((a + b/cos(c + d*x)^2)^(5/2), x)
```

3.255 $\int \left(1+\sec^2(x)\right)^{3/2}\,dx$

3.255.1 Optimal result 1906
3.255.2 Mathematica [C] (verified) 1906
3.255.3 Rubi [A] (verified) 1907
3.255.4 Maple [B] (verified) 1909
3.255.5 Fricas [B] (verification not implemented) 1910
3.255.6 Sympy [F] 1910
3.255.7 Maxima [F] 1911
3.255.8 Giac [F] 1911
3.255.9 Mupad [F(-1)] 1911

3.255.1 Optimal result

Integrand size = 10, antiderivative size = 42

$$\int \left(1+\sec^2(x)\right)^{3/2}\,dx = 2\operatorname{arcsinh}\left(\frac{\tan(x)}{\sqrt{2}}\right) + \arctan\left(\frac{\tan(x)}{\sqrt{2+\tan^2(x)}}\right) + \frac{1}{2}\tan(x)\sqrt{2+\tan^2(x)}$$

output
```
2*arcsinh(1/2*2^(1/2)*tan(x))+arctan(tan(x)/(2+tan(x)^2)^(1/2))+1/2*(2+tan
(x)^2)^(1/2)*tan(x)
```

3.255.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.60

$$\int (1+\sec^2(x))^{3/2}\,dx = \frac{(1+\cos^2(x))\sec(x)\sqrt{1+\sec^2(x)}\left(4\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sin(x)}{\sqrt{3+\cos(2x)}}\right)\cos^2(x) - 2i\sqrt{2}\cos^2(x)\log\left(\right.\right.}{(3+\cos(2x))^{3/2}}$$

input
```
Integrate[(1 + Sec[x]^2)^(3/2),x]
```

output
```
((1 + Cos[x]^2)*Sec[x]*Sqrt[1 + Sec[x]^2]*(4*Sqrt[2]*ArcTanh[(Sqrt[2]*Sin[
x])/Sqrt[3 + Cos[2*x]]]*Cos[x]^2 - (2*I)*Sqrt[2]*Cos[x]^2*Log[Sqrt[3 + Cos
[2*x]] + I*Sqrt[2]*Sin[x]] + Sqrt[3 + Cos[2*x]]*Sin[x]))/(3 + Cos[2*x])^(3
/2)
```

3.255.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {3042, 4616, 318, 27, 398, 222, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\sec^2(x)+1\right)^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int \left(\sec(x)^2+1\right)^{3/2}\,dx$$

$$\downarrow 4616$$

$$\int \frac{\left(\tan^2(x)+2\right)^{3/2}}{\tan^2(x)+1}d\tan(x)$$

$$\downarrow 318$$

$$\frac{1}{2}\int \frac{2\left(2\tan^2(x)+3\right)}{\left(\tan^2(x)+1\right)\sqrt{\tan^2(x)+2}}d\tan(x)+\frac{1}{2}\sqrt{\tan^2(x)+2}\tan(x)$$

$$\downarrow 27$$

$$\int \frac{2\tan^2(x)+3}{\left(\tan^2(x)+1\right)\sqrt{\tan^2(x)+2}}d\tan(x)+\frac{1}{2}\sqrt{\tan^2(x)+2}\tan(x)$$

$$\downarrow 398$$

$$2\int \frac{1}{\sqrt{\tan^2(x)+2}}d\tan(x)+\int \frac{1}{\left(\tan^2(x)+1\right)\sqrt{\tan^2(x)+2}}d\tan(x)+\frac{1}{2}\sqrt{\tan^2(x)+2}\tan(x)$$

$$\downarrow 222$$

$$\int \frac{1}{\left(\tan^2(x)+1\right)\sqrt{\tan^2(x)+2}}d\tan(x)+2\text{arcsinh}\left(\frac{\tan(x)}{\sqrt{2}}\right)+\frac{1}{2}\tan(x)\sqrt{\tan^2(x)+2}$$

$$\downarrow 291$$

$$\int \frac{1}{\frac{\tan^2(x)}{\tan^2(x)+2}+1} d\frac{\tan(x)}{\sqrt{\tan^2(x)+2}} + 2\operatorname{arcsinh}\left(\frac{\tan(x)}{\sqrt{2}}\right) + \frac{1}{2}\tan(x)\sqrt{\tan^2(x)+2}$$

$$\downarrow 216$$

$$2\operatorname{arcsinh}\left(\frac{\tan(x)}{\sqrt{2}}\right) + \arctan\left(\frac{\tan(x)}{\sqrt{\tan^2(x)+2}}\right) + \frac{1}{2}\tan(x)\sqrt{\tan^2(x)+2}$$

input
```
Int[(1 + Sec[x]^2)^(3/2),x]
```

output
```
2*ArcSinh[Tan[x]/Sqrt[2]] + ArcTan[Tan[x]/Sqrt[2 + Tan[x]^2]] + (Tan[x]*Sq
rt[2 + Tan[x]^2])/2
```

3.255.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

3.255.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(35) = 70$.

Time = 8.05 (sec) , antiderivative size = 189, normalized size of antiderivative = 4.50

method	result
default	$\frac{\sqrt{2}\cos(x)\left(\sin(x)\cos(x)\sqrt{\frac{\cos(x)^2+1}{(\cos(x)+1)^2}}+2\cos(x)^2\arctan\left(\frac{\sin(x)}{(\cos(x)+1)\sqrt{\frac{\cos(x)^2+1}{(\cos(x)+1)^2}}}\right)-2\cos(x)^2\operatorname{arctanh}\left(\frac{\sin(x)-2}{(\cos(x)+1)\sqrt{\frac{\cos(x)^2+1}{(\cos(x)+1)^2}}}\right)}{4\sqrt{\frac{\cos(x)^2+1}{(\cos(x)+1)^2}}(\cos(x)+1)\left(\cos(x)^2+\right.}$

input
```
int((1+sec(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*2^(1/2)*cos(x)*(sin(x)*cos(x)*((cos(x)^2+1)/(cos(x)+1)^2)^(1/2)+2*cos(
x)^2*arctan(sin(x)/(cos(x)+1)/((cos(x)^2+1)/(cos(x)+1)^2)^(1/2))-2*cos(x)^
2*arctanh((sin(x)-2)/(cos(x)+1)/((cos(x)^2+1)/(cos(x)+1)^2)^(1/2))-2*cos(x
)^2*arctanh((sin(x)+2)/(cos(x)+1)/((cos(x)^2+1)/(cos(x)+1)^2)^(1/2))+((cos
(x)^2+1)/(cos(x)+1)^2)^(1/2)*sin(x))*(2+2*sec(x)^2)^(1/2)*(1+sec(x)^2)/((c
os(x)^2+1)/(cos(x)+1)^2)^(1/2)/(cos(x)+1)/(cos(x)^2+1)
```

3.255.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(35) = 70$.

Time $= 0.26$ (sec) , antiderivative size $= 160$, normalized size of antiderivative $= 3.81$

$$\int (1+\sec^2(x))^{3/2}\,dx = \frac{\arctan\left(\frac{\sqrt{\frac{\cos(x)^2+1}{\cos(x)^2}}\cos(x)^3\sin(x)+\cos(x)\sin(x)}{\cos(x)^4+\cos(x)^2-1}\right)\cos(x) - \arctan\left(\frac{\sin(x)}{\cos(x)}\right)\cos(x) + 2\cos(x)\log$$

input
```
integrate((1+sec(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(arctan((sqrt((cos(x)^2 + 1)/cos(x)^2)*cos(x)^3*sin(x) + cos(x)*sin(x)
)/(cos(x)^4 + cos(x)^2 - 1))*cos(x) - arctan(sin(x)/cos(x))*cos(x) + 2*cos
(x)*log(cos(x)^2 + cos(x)*sin(x) + (cos(x)^2 + cos(x)*sin(x))*sqrt((cos(x)
^2 + 1)/cos(x)^2) + 1) - 2*cos(x)*log(cos(x)^2 - cos(x)*sin(x) + (cos(x)^2
 - cos(x)*sin(x))*sqrt((cos(x)^2 + 1)/cos(x)^2) + 1) + sqrt((cos(x)^2 + 1)
/cos(x)^2)*sin(x))/cos(x)
```

3.255.6 Sympy [F]

$$\int (1+\sec^2(x))^{3/2}\,dx = \int (\sec^2(x)+1)^{\frac{3}{2}}\,dx$$

input
```
integrate((1+sec(x)**2)**(3/2),x)
```

output
```
Integral((sec(x)**2 + 1)**(3/2), x)
```

3.255.7 Maxima [F]

$$\int \left(1+\sec^2(x)\right)^{3/2}\,dx = \int \left(\sec(x)^2+1\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((1+sec(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((sec(x)^2 + 1)^(3/2), x)
```

3.255.8 Giac [F]

$$\int \left(1+\sec^2(x)\right)^{3/2}\,dx = \int \left(\sec(x)^2+1\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((1+sec(x)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((sec(x)^2 + 1)^(3/2), x)
```

3.255.9 Mupad [F(-1)]

Timed out.

$$\int \left(1+\sec^2(x)\right)^{3/2}\,dx = \int \left(\frac{1}{\cos(x)^2}+1\right)^{3/2}\,dx$$

input
```
int((1/cos(x)^2 + 1)^(3/2),x)
```

output
```
int((1/cos(x)^2 + 1)^(3/2), x)
```

3.256 $\int \sqrt{1+\sec^2(x)}\,dx$

3.256.1 Optimal result . . . 1912
3.256.2 Mathematica [B] (verified) . . . 1912
3.256.3 Rubi [A] (verified) . . . 1913
3.256.4 Maple [B] (verified) . . . 1915
3.256.5 Fricas [B] (verification not implemented) . . . 1915
3.256.6 Sympy [F] . . . 1916
3.256.7 Maxima [C] (verification not implemented) . . . 1916
3.256.8 Giac [F] . . . 1917
3.256.9 Mupad [F(-1)] . . . 1918

3.256.1 Optimal result

Integrand size = 10, antiderivative size = 24

$$\int \sqrt{1+\sec^2(x)}\,dx = \operatorname{arcsinh}\left(\frac{\tan(x)}{\sqrt{2}}\right) + \arctan\left(\frac{\tan(x)}{\sqrt{2+\tan^2(x)}}\right)$$

output
```
arcsinh(1/2*2^(1/2)*tan(x))+arctan(tan(x)/(2+tan(x)^2)^(1/2))
```

3.256.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 57 vs. $2(24) = 48$.

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.38

$$\int \sqrt{1+\sec^2(x)}\,dx = \frac{\sqrt{2}\left(\arcsin\left(\frac{\sin(x)}{\sqrt{2}}\right) + \operatorname{arctanh}\left(\frac{\sqrt{2}\sin(x)}{\sqrt{3+\cos(2x)}}\right)\right)\cos(x)\sqrt{1+\sec^2(x)}}{\sqrt{3+\cos(2x)}}$$

input
```
Integrate[Sqrt[1 + Sec[x]^2],x]
```

output
```
(Sqrt[2]*(ArcSin[Sin[x]/Sqrt[2]] + ArcTanh[(Sqrt[2]*Sin[x])/Sqrt[3 + Cos[2
*x]]])*Cos[x]*Sqrt[1 + Sec[x]^2])/Sqrt[3 + Cos[2*x]]
```

3.256.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 4616, 301, 222, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec^2(x)+1}\,dx$$

$$\downarrow 3042$$

$$\int \sqrt{\sec(x)^2+1}dx$$

$$\downarrow 4616$$

$$\int \frac{\sqrt{\tan^2(x)+2}}{\tan^2(x)+1}d\tan(x)$$

$$\downarrow 301$$

$$\int \frac{1}{\sqrt{\tan^2(x)+2}}d\tan(x) + \int \frac{1}{\left(\tan^2(x)+1\right)\sqrt{\tan^2(x)+2}}d\tan(x)$$

$$\downarrow 222$$

$$\int \frac{1}{\left(\tan^2(x)+1\right)\sqrt{\tan^2(x)+2}}d\tan(x) + \operatorname{arcsinh}\left(\frac{\tan(x)}{\sqrt{2}}\right)$$

$$\downarrow 291$$

$$\int \frac{1}{\frac{\tan^2(x)}{\tan^2(x)+2}+1}d\frac{\tan(x)}{\sqrt{\tan^2(x)+2}} + \operatorname{arcsinh}\left(\frac{\tan(x)}{\sqrt{2}}\right)$$

$$\downarrow 216$$

$$\operatorname{arcsinh}\left(\frac{\tan(x)}{\sqrt{2}}\right) + \arctan\left(\frac{\tan(x)}{\sqrt{\tan^2(x)+2}}\right)$$

input
```
Int[Sqrt[1 + Sec[x]^2],x]
```

output
```
ArcSinh[Tan[x]/Sqrt[2]] + ArcTan[Tan[x]/Sqrt[2 + Tan[x]^2]]
```

3.256.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

3.256.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(21) = 42$.

Time = 5.98 (sec) , antiderivative size = 120, normalized size of antiderivative = 5.00

method	result
default	$\frac{\sqrt{1+\sec(x)^2}\left(2\arctan\left(\frac{\sin(x)}{(\cos(x)+1)\sqrt{\frac{\cos(x)^2+1}{(\cos(x)+1)^2}}}\right)-\operatorname{arctanh}\left(\frac{\sin(x)+2}{(\cos(x)+1)\sqrt{\frac{\cos(x)^2+1}{(\cos(x)+1)^2}}}\right)-\operatorname{arctanh}\left(\frac{\sin(x)-2}{(\cos(x)+1)\sqrt{\frac{\cos(x)^2+1}{(\cos(x)+1)^2}}}\right)\right)\cos}{2(\cos(x)+1)\sqrt{\frac{\cos(x)^2+1}{(\cos(x)+1)^2}}}$

input
```
int((1+sec(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(1+sec(x)^2)^(1/2)*(2*arctan(sin(x)/(cos(x)+1)/((cos(x)^2+1)/(cos(x)+1
)^2)^(1/2))-arctanh((sin(x)+2)/(cos(x)+1)/((cos(x)^2+1)/(cos(x)+1)^2)^(1/2
))-arctanh((sin(x)-2)/(cos(x)+1)/((cos(x)^2+1)/(cos(x)+1)^2)^(1/2)))*cos(x
)/(cos(x)+1)/((cos(x)^2+1)/(cos(x)+1)^2)^(1/2)
```

3.256.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(21) = 42$.

Time = 0.26 (sec) , antiderivative size = 131, normalized size of antiderivative = 5.46

$$\int \sqrt{1+\sec^2(x)}\,dx = \frac{1}{2}\arctan\left(\frac{\sqrt{\frac{\cos(x)^2+1}{\cos(x)^2}}\cos(x)^3\sin(x)+\cos(x)\sin(x)}{\cos(x)^4+\cos(x)^2-1}\right) - \frac{1}{2}\arctan\left(\frac{\sin(x)}{\cos(x)}\right)+\frac{1}{2}\log\left(\cos(x)^2+\cos(x)\sin(x) + \left(\cos(x)^2+\cos(x)\sin(x)\right)\sqrt{\frac{\cos(x)^2+1}{\cos(x)^2}}+1\right)-\frac{1}{2}\log\left(\cos(x)^2 - \cos(x)\sin(x)+\left(\cos(x)^2-\cos(x)\sin(x)\right)\sqrt{\frac{\cos(x)^2+1}{\cos(x)^2}}+1\right)$$

input
```
integrate((1+sec(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*arctan((sqrt((cos(x)^2 + 1)/cos(x)^2)*cos(x)^3*sin(x) + cos(x)*sin(x))
/(cos(x)^4 + cos(x)^2 - 1)) - 1/2*arctan(sin(x)/cos(x)) + 1/2*log(cos(x)^2
 + cos(x)*sin(x) + (cos(x)^2 + cos(x)*sin(x))*sqrt((cos(x)^2 + 1)/cos(x)^2
) + 1) - 1/2*log(cos(x)^2 - cos(x)*sin(x) + (cos(x)^2 - cos(x)*sin(x))*sqr
t((cos(x)^2 + 1)/cos(x)^2) + 1)
```

3.256.6 Sympy [F]

$$\int \sqrt{1+\sec^2(x)}\,dx = \int \sqrt{\sec^2(x)+1}\,dx$$

input
```
integrate((1+sec(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(sec(x)**2 + 1), x)
```

3.256.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 1391, normalized size of antiderivative = 57.96

$$\int \sqrt{1+\sec^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate((1+sec(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*arctan2(2*(2*(6*cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 36*cos(2*x)^2 +
 sin(4*x)^2 + 12*sin(4*x)*sin(2*x) + 36*sin(2*x)^2 + 12*cos(2*x) + 1)^(1/4
)*sin(1/2*arctan2(sin(4*x) + 6*sin(2*x), cos(4*x) + 6*cos(2*x) + 1)), 2*(2
*(6*cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 36*cos(2*x)^2 + sin(4*x)^2 + 12*
sin(4*x)*sin(2*x) + 36*sin(2*x)^2 + 12*cos(2*x) + 1)^(1/4)*cos(1/2*arctan2
(sin(4*x) + 6*sin(2*x), cos(4*x) + 6*cos(2*x) + 1)) + 8) + 3/2*arctan2(2*(
2*(6*cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 36*cos(2*x)^2 + sin(4*x)^2 + 12
*sin(4*x)*sin(2*x) + 36*sin(2*x)^2 + 12*cos(2*x) + 1)^(1/4)*sin(1/2*arctan
2(sin(4*x) + 6*sin(2*x), cos(4*x) + 6*cos(2*x) + 1)) + 2*sin(2*x), 2*(2*(6
*cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 36*cos(2*x)^2 + sin(4*x)^2 + 12*sin
(4*x)*sin(2*x) + 36*sin(2*x)^2 + 12*cos(2*x) + 1)^(1/4)*cos(1/2*arctan2(si
n(4*x) + 6*sin(2*x), cos(4*x) + 6*cos(2*x) + 1)) + 2*cos(2*x) + 6) - arcta
n2((2*(6*cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 36*cos(2*x)^2 + sin(4*x)^2
+ 12*sin(4*x)*sin(2*x) + 36*sin(2*x)^2 + 12*cos(2*x) + 1)^(1/4)*sin(1/2*ar
ctan2(sin(4*x) + 6*sin(2*x), cos(4*x) + 6*cos(2*x) + 1)) + sin(2*x), (2*(6
*cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 36*cos(2*x)^2 + sin(4*x)^2 + 12*sin
(4*x)*sin(2*x) + 36*sin(2*x)^2 + 12*cos(2*x) + 1)^(1/4)*cos(1/2*arctan2(si
n(4*x) + 6*sin(2*x), cos(4*x) + 6*cos(2*x) + 1)) + cos(2*x) + 3) - 1/2*log
(-(2*sqrt(2)*(abs(2*e^(2*I*x) + 2)^4 + 4*cos(2*x)^4 + 4*sin(2*x)^4 - 4*(co
s(2*x)^2 - sin(2*x)^2 - 2*cos(2*x) + 1)*abs(2*e^(2*I*x) + 2)^2 - 16*cos...
```

3.256.8 Giac [F]

$$\int \sqrt{1+\sec^2(x)}\,dx = \int \sqrt{\sec(x)^2+1}\,dx$$

input
```
integrate((1+sec(x)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sec(x)^2 + 1), x)
```

3.256.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{1+\sec^2(x)}\,dx = \int \sqrt{\frac{1}{\cos(x)^2}+1}\,dx$$

input
```
int((1/cos(x)^2 + 1)^(1/2),x)
```

output
```
int((1/cos(x)^2 + 1)^(1/2), x)
```

3.257 $\int \frac{\sec^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

3.257.1 Optimal result . . . 1919
3.257.2 Mathematica [F] . . . 1920
3.257.3 Rubi [A] (verified) . . . 1920
3.257.4 Maple [C] (warning: unable to verify) . . . 1926
3.257.5 Fricas [C] (verification not implemented) . . . 1926
3.257.6 Sympy [F] . . . 1927
3.257.7 Maxima [F] . . . 1928
3.257.8 Giac [F] . . . 1928
3.257.9 Mupad [F(-1)] . . . 1928

3.257.1 Optimal result

Integrand size = 25, antiderivative size = 330

$$
\begin{aligned}
&\int \frac{\sec^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx \\
&= \frac{2(a-b)E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)(a+b-a\sin^2(e+fx))}{3b^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} \\
&\quad - \frac{(a-2b)\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{3bf\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} \\
&\quad - \frac{2(a-b)\sec(e+fx)\,(a+b-a\sin^2(e+fx))\tan(e+fx)}{3b^2f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} \\
&\quad + \frac{\sec^3(e+fx)\,(a+b-a\sin^2(e+fx))\tan(e+fx)}{3bf\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}
\end{aligned}
$$

output

```
2/3*(a-b)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(a+b-a*sin(f*x+e)^2)/b^2/f
/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/(1-a*sin(f
*x+e)^2/(a+b))^(1/2)-1/3*(a-2*b)*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*(1-
a*sin(f*x+e)^2/(a+b))^(1/2)/b/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*
sin(f*x+e)^2))^(1/2)-2/3*(a-b)*sec(f*x+e)*(a+b-a*sin(f*x+e)^2)*tan(f*x+e)/
b^2/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)+1/3*sec(f*x+e)^3*(a+b-a*si
n(f*x+e)^2)*tan(f*x+e)/b/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)
```

3.257.2 Mathematica [F]

$$\int \frac{\sec^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sec^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
Integrate[Sec[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
Integrate[Sec[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2], x]
```

3.257.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 315, normalized size of antiderivative = 0.95, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 4636, 2057, 2058, 316, 25, 402, 25, 27, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sec(e+fx)^5}{\sqrt{a+b\sec(e+fx)^2}}\,dx$$

$$\downarrow 4636$$

$$\frac{\int \frac{1}{(1-\sin^2(e+fx))^3\sqrt{a+\frac{b}{1-\sin^2(e+fx)}}}\,d\sin(e+fx)}{f}$$

$$\downarrow 2057$$

$$\frac{\int \frac{1}{(1-\sin^2(e+fx))^3\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}\,d\sin(e+fx)}{f}$$

$$\downarrow 2058$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\int \frac{1}{(1-\sin^2(e+fx))^{5/2}\sqrt{-a\sin^2(e+fx)+a+b}}\,d\sin(e+fx)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 316$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\int -\frac{a\sin^2(e+fx)+a-2b}{\left(1-\sin^2(e+fx)\right)^{3/2}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{3b}+\frac{\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3b\left(1-\sin^2(e+fx)\right)^{3/2}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 25$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3b\left(1-\sin^2(e+fx)\right)^{3/2}}-\frac{\int \frac{a\sin^2(e+fx)+a-2b}{\left(1-\sin^2(e+fx)\right)^{3/2}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{3b}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 402$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3b\left(1-\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{\int -\frac{a\left(-2(a-b)\sin^2(e+fx)+2a-b\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{b}+\frac{2(a-b)\sin(e+fx)\sqrt{-}}{b\sqrt{1-\sin^2}}}{3b}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 25$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3b\left(1-\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{2(a-b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}-\frac{\int \frac{a\left(-2(a-b)\sin^2(e+fx)+2a-b\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+}}}{b}}{3b}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 27$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3b\left(1-\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{2(a-b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}-\frac{a\int \frac{-2(a-b)\sin^2(e+fx)+2a-b}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)}}}{b}}{3b}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 399$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3b\left(1-\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{2(a-b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}-\frac{a\left(\frac{2(a-b)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d}{a}\right.}{3b}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 323$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3b\left(1-\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{2(a-b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}-\frac{a\left(\frac{2(a-b)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d}{a}\right.}{3b}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 321$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3b\left(1-\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{2(a-b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}-\frac{a\left(\frac{2(a-b)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d}{a}\right.}{3b}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 330$$

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3b\left(1-\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{2(a-b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}-a\left(\frac{2(a-b)\sqrt{-a\sin^2(e+fx)+a+b}\int\frac{\sqrt{}}{a\sqrt{1-\frac{a\sin^2}{}}}\right.\right.$$

$$f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}$$

$$\downarrow 327$$

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3b\left(1-\sin^2(e+fx)\right)^{3/2}}-\frac{\frac{2(a-b)\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}-a\left(\frac{2(a-b)\sqrt{-a\sin^2(e+fx)+a+bE\left(\right.}}{a\sqrt{1-\frac{a\sin^2(e}{a+}}}\right.\right.$$

$$f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}$$

input
```
Int[Sec[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Sqrt[a + b - a*Sin[e + f*x]^2]*((Sin[e + f*x]*Sqrt[a + b - a*Sin[e + f*x]
^2])/(3*b*(1 - Sin[e + f*x]^2)^(3/2)) - ((2*(a - b)*Sin[e + f*x]*Sqrt[a +
b - a*Sin[e + f*x]^2])/(b*Sqrt[1 - Sin[e + f*x]^2]) - (a*((2*(a - b)*Ellip
ticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b - a*Sin[e + f*x]^2])/(a*S
qrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - ((a - 2*b)*b*EllipticF[ArcSin[Sin[e
 + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(a*Sqrt[a + b -
 a*Sin[e + f*x]^2])))/b)/(3*b)))/(f*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[(a + b -
 a*Sin[e + f*x]^2)/(1 - Sin[e + f*x]^2)])
```

3.257.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

3.257.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 12.32 (sec) , antiderivative size = 6274, normalized size of antiderivative = 19.01

method	result	size
default	Expression too large to display	6274

input
```
int(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.257.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 784, normalized size of antiderivative = 2.38

$$\int \frac{\sec^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \frac{\left(2\left(-i\,a^2+i\,ab\right)\sqrt{a}\sqrt{\frac{ab+b^2}{a^2}}\cos\left(fx+e\right)^2-\left(-i\,a^2-i\,ab+2i\,b^2\right)\sqrt{a}\cos\left(fx+e\right)^2\right)\sqrt{\frac{2\,a\sqrt{\frac{ab+b^2}{a^2}}-a-2\,b}{a}}E(\text{ar}}$$

input
```
integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*((2*(-I*a^2 + I*a*b)*sqrt(a)*sqrt((a*b + b^2)/a^2)*cos(f*x + e)^2 - (-
I*a^2 - I*a*b + 2*I*b^2)*sqrt(a)*cos(f*x + e)^2)*sqrt((2*a*sqrt((a*b + b^2
)/a^2) - a - 2*b)/a)*elliptic_e(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a
 - 2*b)/a)*(cos(f*x + e) + I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2
 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + (2*(I*a^2 - I*a*b)*sqrt(a)*sqrt((a
*b + b^2)/a^2)*cos(f*x + e)^2 - (I*a^2 + I*a*b - 2*I*b^2)*sqrt(a)*cos(f*x
+ e)^2)*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_e(arcsin(sq
rt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) - I*sin(f*x + e)
)), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) - (
2*I*a^(3/2)*b*sqrt((a*b + b^2)/a^2)*cos(f*x + e)^2 + (2*I*a^2 + 3*I*a*b -
2*I*b^2)*sqrt(a)*cos(f*x + e)^2)*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b
)/a)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(
f*x + e) + I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((
a*b + b^2)/a^2))/a^2) - (-2*I*a^(3/2)*b*sqrt((a*b + b^2)/a^2)*cos(f*x + e)
^2 + (-2*I*a^2 - 3*I*a*b + 2*I*b^2)*sqrt(a)*cos(f*x + e)^2)*sqrt((2*a*sqrt
((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b + b^
2)/a^2) - a - 2*b)/a)*(cos(f*x + e) - I*sin(f*x + e))), (a^2 + 8*a*b + 8*b
^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) - (2*(a^2 - a*b)*cos(f*x
+ e)^2 - a*b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a
*b^2*f*cos(f*x + e)^2)
```

3.257.6 Sympy [F]

$$\int \frac{\sec^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sec^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
integrate(sec(f*x+e)**5/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sec(e + f*x)**5/sqrt(a + b*sec(e + f*x)**2), x)
```

3.257.7 Maxima [F]

$$\int \frac{\sec^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sec(fx+e)^5}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^5/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.257.8 Giac [F]

$$\int \frac{\sec^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sec(fx+e)^5}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.257.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{1}{\cos(e+fx)^5\sqrt{a+\frac{b}{\cos(e+fx)^2}}}\,dx$$

input
```
int(1/(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2)),x)
```

output
```
int(1/(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2)), x)
```

3.258 $\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

3.258.1 Optimal result . . . 1929
3.258.2 Mathematica [F] . . . 1929
3.258.3 Rubi [A] (verified) . . . 1930
3.258.4 Maple [C] (warning: unable to verify) . . . 1933
3.258.5 Fricas [C] (verification not implemented) . . . 1934
3.258.6 Sympy [F] . . . 1934
3.258.7 Maxima [F] . . . 1935
3.258.8 Giac [F] . . . 1935
3.258.9 Mupad [F(-1)] . . . 1935

3.258.1 Optimal result

Integrand size = 25, antiderivative size = 170

$$\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\sqrt{a}\sqrt{a+b}E\left(\arcsin\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)|\frac{a+b}{a}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{bf\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)}\,(a+b-a\sin^2(e+fx))} + \frac{\sec(e+fx)\,(a+b-a\sin^2(e+fx))\tan(e+fx)}{bf\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

output
```
-EllipticE(sin(f*x+e)*a^(1/2)/(a+b)^(1/2),((a+b)/a)^(1/2))*a^(1/2)*(a+b)^(
1/2)*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/b/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2
*(a+b-a*sin(f*x+e)^2))^(1/2)+sec(f*x+e)*(a+b-a*sin(f*x+e)^2)*tan(f*x+e)/b/
f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)
```

3.258.2 Mathematica [F]

$$\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sec^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
Integrate[Sec[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
Integrate[Sec[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2], x]
```

3.258.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.18, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4636, 2057, 2058, 316, 25, 27, 331, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sec(e+fx)^3}{\sqrt{a+b\sec(e+fx)^2}}\,dx$$

$$\downarrow 4636$$

$$\frac{\int \frac{1}{(1-\sin^2(e+fx))^2\sqrt{a+\frac{b}{1-\sin^2(e+fx)}}}\,d\sin(e+fx)}{f}$$

$$\downarrow 2057$$

$$\frac{\int \frac{1}{(1-\sin^2(e+fx))^2\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}\,d\sin(e+fx)}{f}$$

$$\downarrow 2058$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\int \frac{1}{(1-\sin^2(e+fx))^{3/2}\sqrt{-a\sin^2(e+fx)+a+b}}\,d\sin(e+fx)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 316$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\int -\frac{a\sqrt{1-\sin^2(e+fx)}}{\sqrt{-a\sin^2(e+fx)+a+b}}\,d\sin(e+fx)}{b}+\frac{\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 25$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}-\frac{\int \frac{a\sqrt{1-\sin^2(e+fx)}}{\sqrt{-a\sin^2(e+fx)+a+b}}\,d\sin(e+fx)}{b}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 27$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}-\frac{a\int\frac{\sqrt{1-\sin^2(e+fx)}}{\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{b}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 331$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}-\frac{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\int\frac{\sqrt{1-\sin^2(e+fx)}}{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}d\sin(e+fx)}{b\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 327$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{b\sqrt{1-\sin^2(e+fx)}}-\frac{\sqrt{a}\sqrt{a+b}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}E\left(\arcsin\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)|\frac{a+b}{a}\right)}{b\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

input
```
Int[Sec[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Sqrt[a + b - a*Sin[e + f*x]^2]*((Sin[e + f*x]*Sqrt[a + b - a*Sin[e + f*x]
^2])/(b*Sqrt[1 - Sin[e + f*x]^2]) - (Sqrt[a]*Sqrt[a + b]*EllipticE[ArcSin[
(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]], (a + b)/a]*Sqrt[1 - (a*Sin[e + f*x]^2
)/(a + b)])/(b*Sqrt[a + b - a*Sin[e + f*x]^2])))/(f*Sqrt[1 - Sin[e + f*x]^
2]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(1 - Sin[e + f*x]^2)])
```

3.258.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

3.258.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.89 (sec) , antiderivative size = 4103, normalized size of antiderivative = 24.14

method	result	size
default	Expression too large to display	4103

input
```
int(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/b/(2*I*a^(1/2)*b^(1/2)-a+b)*(2
*I*a^(3/2)*b^(1/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot
(f*x+e))+2*I*a^(1/2)*b^(3/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(
f*x+e)-cot(f*x+e))-4*(-(2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2+
a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b)
)^(1/2)*((2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2-a*(1-cos(f*x+e
))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/(a+b))^(1/2)*Ellipt
icF(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I
*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b+2*
(-(2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2+a*(1-cos(f*x+e))^2*cs
c(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*((2*I*a^(1/2)
*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2-a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(
1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/(a+b))^(1/2)*EllipticE(((2*I*a^(1/2)*b^(
1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*
a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b-4*((2*I*a^(1/2)*b^(1/2)
+a-b)/(a+b))^(1/2)*a*b*(1-cos(f*x+e))^3*csc(f*x+e)^3-(-(2*I*a^(1/2)*b^(1/2
)*(1-cos(f*x+e))^2*csc(f*x+e)^2+a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f
*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*((2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e
))^2*csc(f*x+e)^2-a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f
*x+e)^2+a+b)/(a+b))^(1/2)*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(...
```

3.258.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 594, normalized size of antiderivative = 3.49

$$\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx =$$

$$-\frac{2\sqrt{a}(-i\,a-2i\,b)\sqrt{\frac{2\,a\sqrt{\frac{ab+b^2}{a^2}}-a-2\,b}{a}}F(\arcsin\left(\sqrt{\frac{2\,a\sqrt{\frac{ab+b^2}{a^2}}-a-2\,b}{a}}(\cos(fx+e)+i\,\sin(fx+e))\right)\,|\,\frac{a^2+8\,ab-}{}}$$

input
```
integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(2*sqrt(a)*(-I*a - 2*I*b)*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/
a)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*
x + e) + I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*
b + b^2)/a^2))/a^2) + 2*sqrt(a)*(I*a + 2*I*b)*sqrt((2*a*sqrt((a*b + b^2)/a
^2) - a - 2*b)/a)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a -
2*b)/a)*(cos(f*x + e) - I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 +
2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + (-2*I*a^(3/2)*sqrt((a*b + b^2)/a^2) +
 sqrt(a)*(I*a + 2*I*b))*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elli
ptic_e(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e)
+ I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2
)/a^2))/a^2) + (2*I*a^(3/2)*sqrt((a*b + b^2)/a^2) + sqrt(a)*(-I*a - 2*I*b)
)*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_e(arcsin(sqrt((2*
a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) - I*sin(f*x + e))), (a
^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) - 2*a*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a*b*f)
```

3.258.6 Sympy [F]

$$\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sec^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
integrate(sec(f*x+e)**3/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sec(e + f*x)**3/sqrt(a + b*sec(e + f*x)**2), x)
```

3.258.7 Maxima [F]

$$\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sec(fx+e)^3}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^3/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.258.8 Giac [F]

$$\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sec(fx+e)^3}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.258.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{1}{\cos(e+fx)^3\sqrt{a+\frac{b}{\cos(e+fx)^2}}}\,dx$$

input
```
int(1/(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2)),x)
```

output
```
int(1/(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2)), x)
```

3.259 $\int \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

3.259.1 Optimal result 1936
3.259.2 Mathematica [A] (verified) 1936
3.259.3 Rubi [A] (verified) 1937
3.259.4 Maple [C] (verified) 1939
3.259.5 Fricas [C] (verification not implemented) 1939
3.259.6 Sympy [F] 1940
3.259.7 Maxima [F] 1940
3.259.8 Giac [F] 1941
3.259.9 Mupad [F(-1)] 1941

3.259.1 Optimal result

Integrand size = 23, antiderivative size = 80

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)}\,(a+b-a\sin^2(e+fx))}$$

output
```
EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/f/(co
s(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)
```

3.259.2 Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.86

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}}\,\text{EllipticF}\left(e+fx,\frac{a}{a+b}\right)\sec(e+fx)}{\sqrt{2}f\sqrt{a+b\sec^2(e+fx)}}$$

input
```
Integrate[Sec[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*EllipticF[e + f*x, a/(a + b)
]*Sec[e + f*x])/(Sqrt[2]*f*Sqrt[a + b*Sec[e + f*x]^2])
```

3.259.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4636, 2057, 2058, 323, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)^2}}dx$$

$$\downarrow 4636$$

$$\frac{\int \frac{1}{(1-\sin^2(e+fx))\sqrt{a+\frac{b}{1-\sin^2(e+fx)}}}d\sin(e+fx)}{f}$$

$$\downarrow 2057$$

$$\frac{\int \frac{1}{(1-\sin^2(e+fx))\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}d\sin(e+fx)}{f}$$

$$\downarrow 2058$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 323$$

$$\frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}d\sin(e+fx)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 321$$

$$\frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

input
```
Int[Sec[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a
 + b)])/(f*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(1 - S
in[e + f*x]^2)])
```

3.259.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

3.259.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.49 (sec) , antiderivative size = 243, normalized size of antiderivative = 3.04

method	result
default	$-\frac{2\sqrt{-\frac{i\left(i\cos(fx+e)a-\sqrt{a}\sqrt{b}\cos(fx+e)+ib+\sqrt{a}\sqrt{b}\right)}{(a+b)(1+\cos(fx+e))}}\sqrt{-\frac{i\left(\sqrt{a}\sqrt{b}\cos(fx+e)+i\cos(fx+e)a-\sqrt{a}\sqrt{b}+ib\right)}{(a+b)(1+\cos(fx+e))}}\,\text{EllipticF}\left(\sqrt{\frac{2i\sqrt{a}\sqrt{b}+a-b}{a+b}}(\cot($ $f\sqrt{\frac{2i\sqrt{a}\sqrt{b}+a-b}{a+b}}\sqrt{a+b\sec(fx+e)^2}$

input
```
int(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(-I/(a+b)*(I*cos(f*x+e)*a-a^(
1/2)*b^(1/2)*cos(f*x+e)+I*b+a^(1/2)*b^(1/2))/(1+cos(f*x+e)))^(1/2)*(-I/(a+
b)*(a^(1/2)*b^(1/2)*cos(f*x+e)+I*cos(f*x+e)*a-a^(1/2)*b^(1/2)+I*b)/(1+cos(
f*x+e)))^(1/2)*EllipticF(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(cot(f*x+
e)-csc(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(
a+b)^2)^(1/2))/(a+b*sec(f*x+e)^2)^(1/2)*(sec(f*x+e)+1)
```

3.259.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 303, normalized size of antiderivative = 3.79

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx =$$
$$-\frac{\left(2i\,a^{\frac{3}{2}}\sqrt{\frac{ab+b^2}{a^2}}+\sqrt{a}(i\,a+2i\,b)\right)\sqrt{\frac{2\,a\sqrt{\frac{ab+b^2}{a^2}}-a-2\,b}{a}}F(\arcsin\left(\sqrt{\frac{2\,a\sqrt{\frac{ab+b^2}{a^2}}-a-2\,b}{a}}(\cos(fx+e)+i\,\sin(fx+\right.}$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
-((2*I*a^(3/2)*sqrt((a*b + b^2)/a^2) + sqrt(a)*(I*a + 2*I*b))*sqrt((2*a*sq
rt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b +
b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) + I*sin(f*x + e))), (a^2 + 8*a*b + 8
*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + (-2*I*a^(3/2)*sqrt((a
*b + b^2)/a^2) + sqrt(a)*(-I*a - 2*I*b))*sqrt((2*a*sqrt((a*b + b^2)/a^2) -
 a - 2*b)/a)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/
a)*(cos(f*x + e) - I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b
)*sqrt((a*b + b^2)/a^2))/a^2))/(a^2*f)
```

3.259.6 Sympy [F]

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sec(e + f*x)/sqrt(a + b*sec(e + f*x)**2), x)
```

3.259.7 Maxima [F]

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sec(fx+e)}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.259.8 Giac [F]

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sec(fx+e)}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.259.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{1}{\cos(e+fx)\sqrt{a+\frac{b}{\cos(e+fx)^2}}}\,dx$$

input
```
int(1/(cos(e + f*x)*(a + b/cos(e + f*x)^2)^(1/2)),x)
```

output
```
int(1/(cos(e + f*x)*(a + b/cos(e + f*x)^2)^(1/2)), x)
```

3.260 $\int \frac{\cos(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

3.260.1 Optimal result . . . 1942
3.260.2 Mathematica [C] (verified) . . . 1942
3.260.3 Rubi [A] (verified) . . . 1943
3.260.4 Maple [C] (warning: unable to verify) . . . 1945
3.260.5 Fricas [F] . . . 1946
3.260.6 Sympy [F] . . . 1946
3.260.7 Maxima [F] . . . 1946
3.260.8 Giac [F] . . . 1947
3.260.9 Mupad [F(-1)] . . . 1947

3.260.1 Optimal result

Integrand size = 23, antiderivative size = 105

$$\int \frac{\cos(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\sqrt{a+b}E\left(\arcsin\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)|\frac{a+b}{a}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{\sqrt{a}f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)}\,(a+b-a\sin^2(e+fx))}$$

output
```
EllipticE(sin(f*x+e)*a^(1/2)/(a+b)^(1/2),((a+b)/a)^(1/2))*(a+b)^(1/2)*(1-a
*sin(f*x+e)^2/(a+b))^(1/2)/f/a^(1/2)/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a
+b-a*sin(f*x+e)^2))^(1/2)
```

3.260.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.87 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.61

$$\int \frac{\cos(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$
$$= \frac{\csc(2(e+fx))\sin(e+fx)\left(a^2\sqrt{-\frac{1}{b}}\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}}\,\text{EllipticF}\left(e+fx,\frac{a}{a+b}\right)-2i\sqrt{-\frac{a\cos^2(e+fx)}{b}}\sqrt{a+}\right.}{}$$

input
```
Integrate[Cos[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Csc[2*(e + f*x)]*Sin[e + f*x]*(a^2*Sqrt[-b^(-1)]*Sqrt[(a + 2*b + a*Cos[2*
(e + f*x)])/(a + b)]*EllipticF[e + f*x, a/(a + b)] - (2*I)*Sqrt[-((a*Cos[e
 + f*x]^2)/b)]*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*Csc[2*(e + f*x)]*(2*(a +
 b)*EllipticE[I*ArcSinh[(Sqrt[-b^(-1)]*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]])
/Sqrt[2]], b/(a + b)] - a*EllipticF[I*ArcSinh[(Sqrt[-b^(-1)]*Sqrt[a + 2*b
+ a*Cos[2*(e + f*x)]])/Sqrt[2]], b/(a + b)])*Sqrt[(a*Sin[e + f*x]^2)/(a +
b)]))/(Sqrt[2]*a^2*Sqrt[-b^(-1)]*f*Sqrt[a + b*Sec[e + f*x]^2])
```

3.260.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4636, 2057, 2058, 331, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sec(e+fx)\sqrt{a+b\sec(e+fx)^2}}dx$$

$$\downarrow 4636$$

$$\frac{\int \frac{1}{\sqrt{a+\frac{b}{1-\sin^2(e+fx)}}}d\sin(e+fx)}{f}$$

$$\downarrow 2057$$

$$\frac{\int \frac{1}{\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}d\sin(e+fx)}{f}$$

$$\downarrow 2058$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\int \frac{\sqrt{1-\sin^2(e+fx)}}{\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 331$$

$$\frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\int\frac{\sqrt{1-\sin^2(e+fx)}}{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}d\sin(e+fx)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 327$$

$$\frac{\sqrt{a+b}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}E\left(\arcsin\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)|\frac{a+b}{a}\right)}{\sqrt{a}f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

input
```
Int[Cos[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Sqrt[a + b]*EllipticE[ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]], (a + b)
/a]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(Sqrt[a]*f*Sqrt[1 - Sin[e + f*x]
^2]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(1 - Sin[e + f*x]^2)])
```

3.260.3.1 Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

3.260.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.41 (sec) , antiderivative size = 4095, normalized size of antiderivative = 39.00

method	result	size
default	Expression too large to display	4095

input
```
int(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/a/(2*I*a^(1/2)*b^(1/2)-a+b)*(
2*I*a^(3/2)*b^(1/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-co
t(f*x+e))+2*I*a^(1/2)*b^(3/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc
(f*x+e)-cot(f*x+e))+4*(-(2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2
+a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b
))^(1/2)*((2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2-a*(1-cos(f*x+
e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/(a+b))^(1/2)*Ellip
ticF(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*
I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b-2
*(-(2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2+a*(1-cos(f*x+e))^2*c
sc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*((2*I*a^(1/2
)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2-a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*
(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/(a+b))^(1/2)*EllipticE(((2*I*a^(1/2)*b^
(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I
*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))*a*b-4*((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2)*a*b*(1-cos(f*x+e))^3*csc(f*x+e)^3-(-(2*I*a^(1/2)*b^(1/
2)*(1-cos(f*x+e))^2*csc(f*x+e)^2+a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(
f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*((2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+
e))^2*csc(f*x+e)^2-a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(
f*x+e)^2+a+b)/(a+b))^(1/2)*EllipticE(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^...
```

3.260.5 Fricas [F]

$$\int \frac{\cos(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos(fx+e)}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(cos(f*x + e)/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.260.6 Sympy [F]

$$\int \frac{\cos(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
integrate(cos(f*x+e)/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(cos(e + f*x)/sqrt(a + b*sec(e + f*x)**2), x)
```

3.260.7 Maxima [F]

$$\int \frac{\cos(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos(fx+e)}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(f*x + e)/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.260.8 Giac [F]

$$\int \frac{\cos(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos(fx+e)}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.260.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos(e+fx)}{\sqrt{a+\frac{b}{\cos(e+fx)^2}}}\,dx$$

input
```
int(cos(e + f*x)/(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(cos(e + f*x)/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.261 $\int \frac{\cos^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

3.261.1 Optimal result 1948
3.261.2 Mathematica [F] 1949
3.261.3 Rubi [A] (verified) 1949
3.261.4 Maple [C] (warning: unable to verify) 1953
3.261.5 Fricas [F] 1953
3.261.6 Sympy [F(-1)] 1954
3.261.7 Maxima [F] 1954
3.261.8 Giac [F] 1954
3.261.9 Mupad [F(-1)] 1955

3.261.1 Optimal result

Integrand size = 25, antiderivative size = 255

$$\begin{aligned}
&\int \frac{\cos^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx \\
&= \frac{\sin(e+fx)\,(a+b-a\sin^2(e+fx))}{3af\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} \\
&\quad + \frac{2(a-b)E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)(a+b-a\sin^2(e+fx))}{3a^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} \\
&\quad - \frac{(a-2b)b\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{3a^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}
\end{aligned}$$

output

```
1/3*sin(f*x+e)*(a+b-a*sin(f*x+e)^2)/a/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2)
)^(1/2)+2/3*(a-b)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(a+b-a*sin(f*x+e)^
2)/a^2/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/(1
-a*sin(f*x+e)^2/(a+b))^(1/2)-1/3*(a-2*b)*b*EllipticF(sin(f*x+e),(a/(a+b))^
(1/2))*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/a^2/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+
e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)
```

3.261.2 Mathematica [F]

$$\int \frac{\cos^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
Integrate[Cos[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
Integrate[Cos[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2], x]
```

3.261.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 4636, 2057, 2058, 318, 25, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sec(e+fx)^3\sqrt{a+b\sec(e+fx)^2}}dx$$

$$\downarrow 4636$$

$$\frac{\int \frac{1-\sin^2(e+fx)}{\sqrt{a+\frac{b}{1-\sin^2(e+fx)}}}d\sin(e+fx)}{f}$$

$$\downarrow 2057$$

$$\frac{\int \frac{1-\sin^2(e+fx)}{\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}d\sin(e+fx)}{f}$$

$$\downarrow 2058$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\int \frac{\left(1-\sin^2(e+fx)\right)^{3/2}}{\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 318$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sin(e+fx)\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}{3a}-\frac{\int-\frac{-2(a-b)\sin^2(e+fx)+2a-b}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{3a}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 25$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\int\frac{-2(a-b)\sin^2(e+fx)+2a-b}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{3a}+\frac{\sqrt{1-\sin^2(e+fx)}\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3a}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 399$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{2(a-b)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{b(a-2b)\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{a}}{3a}+\frac{\sqrt{1-si}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 323$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{2(a-b)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{b(a-2b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}d\sin(e+fx}{a\sqrt{-a\sin^2(e+fx)+a+b}}}{3a}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 321$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{2(a-b)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{b(a-2b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)}{a\sqrt{-a\sin^2(e+fx)+a+b}}}{3a}+\frac{\sqrt{1-si}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 330$$

3.261. $\int\frac{\cos^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}dx$

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{2(a-b)\sqrt{-a\sin^2(e+fx)+a+b}\int\frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{b(a-2b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\right.}{a\sqrt{-a\sin^2(e+fx)+a+b}}}{3a}\right.$$

$$\left.\vphantom{\frac{1}{1}}\right)\Big/\left(f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\right)$$

$$\downarrow 327$$

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{2(a-b)\sqrt{-a\sin^2(e+fx)+a+b}E\left(\arcsin(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{b(a-2b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)}{a\sqrt{-a\sin^2(e+fx)+a+b}}}{3a}\right.$$

$$\left.\vphantom{\frac{1}{1}}\right)\Big/\left(f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\right)$$

input
```
Int[Cos[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Sqrt[a + b - a*Sin[e + f*x]^2]*((Sin[e + f*x]*Sqrt[1 - Sin[e + f*x]^2]*Sq
rt[a + b - a*Sin[e + f*x]^2])/(3*a) + ((2*(a - b)*EllipticE[ArcSin[Sin[e +
 f*x]], a/(a + b)]*Sqrt[a + b - a*Sin[e + f*x]^2])/(a*Sqrt[1 - (a*Sin[e +
f*x]^2)/(a + b)]) - ((a - 2*b)*b*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)
]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(a*Sqrt[a + b - a*Sin[e + f*x]^2])
)/(3*a)))/(f*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(1 -
 Sin[e + f*x]^2)])
```

3.261.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

3.261.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.02 (sec) , antiderivative size = 6273, normalized size of antiderivative = 24.60

method	result	size
default	Expression too large to display	6273

input
```
int(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.261.5 Fricas [F]

$$\int \frac{\cos^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos\left(fx+e\right)^3}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(cos(f*x + e)^3/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.261.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**3/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Timed out
```

3.261.7 Maxima [F]

$$\int \frac{\cos^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos\left(fx+e\right)^3}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(f*x + e)^3/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.261.8 Giac [F]

$$\int \frac{\cos^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos\left(fx+e\right)^3}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.261.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos(e+f\,x)^3}{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

input
```
int(cos(e + f*x)^3/(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(cos(e + f*x)^3/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.262 $\int \frac{\cos^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

3.262.1 Optimal result 1956
3.262.2 Mathematica [F] 1957
3.262.3 Rubi [A] (verified) 1957
3.262.4 Maple [C] (warning: unable to verify) 1962
3.262.5 Fricas [F] 1962
3.262.6 Sympy [F(-1)] 1962
3.262.7 Maxima [F] 1963
3.262.8 Giac [F] 1963
3.262.9 Mupad [F(-1)] 1963

3.262.1 Optimal result

Integrand size = 25, antiderivative size = 345

$$
\begin{aligned}
&\int \frac{\cos^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx \\
&= \frac{4(a-b)\sin(e+fx)\,(a+b-a\sin^2(e+fx))}{15a^2f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} \\
&\quad + \frac{\cos^2(e+fx)\sin(e+fx)\,(a+b-a\sin^2(e+fx))}{5af\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} \\
&\quad + \frac{(8a^2-7ab+8b^2)\,E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)(a+b-a\sin^2(e+fx))}{15a^3f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} \\
&\quad - \frac{b(4a^2-3ab+8b^2)\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{15a^3f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}
\end{aligned}
$$

output

```
4/15*(a-b)*sin(f*x+e)*(a+b-a*sin(f*x+e)^2)/a^2/f/(sec(f*x+e)^2*(a+b-a*sin(
f*x+e)^2))^(1/2)+1/5*cos(f*x+e)^2*sin(f*x+e)*(a+b-a*sin(f*x+e)^2)/a/f/(sec
(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)+1/15*(8*a^2-7*a*b+8*b^2)*EllipticE(s
in(f*x+e),(a/(a+b))^(1/2))*(a+b-a*sin(f*x+e)^2)/a^3/f/(cos(f*x+e)^2)^(1/2)
/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/(1-a*sin(f*x+e)^2/(a+b))^(1/2)-
1/15*b*(4*a^2-3*a*b+8*b^2)*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*(1-a*sin(
f*x+e)^2/(a+b))^(1/2)/a^3/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(
f*x+e)^2))^(1/2)
```

3.262.2 Mathematica [F]

$$\int \frac{\cos^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
Integrate[Cos[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
Integrate[Cos[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2], x]
```

3.262.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.98, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 4636, 2057, 2058, 318, 25, 403, 25, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sec(e+fx)^5\sqrt{a+b\sec(e+fx)^2}}dx$$

$$\downarrow 4636$$

$$\frac{\int \frac{\left(1-\sin^2(e+fx)\right)^2}{\sqrt{a+\frac{b}{1-\sin^2(e+fx)}}}d\sin(e+fx)}{f}$$

$$\downarrow 2057$$

$$\frac{\int \frac{\left(1-\sin^2(e+fx)\right)^2}{\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}d\sin(e+fx)}{f}$$

$$\downarrow 2058$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\int \frac{\left(1-\sin^2(e+fx)\right)^{5/2}}{\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 318$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sin(e+fx)\left(1-\sin^2(e+fx)\right)^{3/2}\sqrt{-a\sin^2(e+fx)+a+b}}{5a}-\frac{\int-\frac{\sqrt{1-\sin^2(e+fx)}\left(-4(a-b)\sin^2(e+fx)+4a-b\right)}{\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(}{5a}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 25$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\int\frac{\sqrt{1-\sin^2(e+fx)}\left(-4(a-b)\sin^2(e+fx)+4a-b\right)}{\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{5a}+\frac{\sin(e+fx)\left(1-\sin^2(e+fx)\right)^{3/2}\sqrt{-a\sin^2(e+fx)+}}{5a}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 403$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{4(a-b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}{3a}-\frac{\int-\frac{8a^2-3ba+4b^2-\left(8a^2-7ba+8b^2\right)\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{3a}}{5a}-\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 25$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\int\frac{8a^2-3ba+4b^2-\left(8a^2-7ba+8b^2\right)\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{3a}+\frac{4(a-b)\sqrt{1-\sin^2(e+fx)}\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3a}+\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 399$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\left(8a^2-7ab+8b^2\right)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{b\left(4a^2-3ab+8b^2\right)\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(}{a}}{3a}}{5a}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e}{1-\sin}}}$$

$$\downarrow 323$$

3.262. $\int\frac{\cos^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}dx$

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\left(8a^2-7ab+8b^2\right)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{b\left(4a^2-3ab+8b^2\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{1-\frac{a\sin}{}}}}{a\sqrt{-a\sin^2(e+fx)+a+b}}\right)$$
$$\frac{}{3a}$$
$$\frac{}{5a}$$
$$f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a}{}}$$

$$\downarrow 321$$

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\left(8a^2-7ab+8b^2\right)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{b\left(4a^2-3ab+8b^2\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\right.}{a\sqrt{-a\sin^2(e+fx)+a+b}}\right)$$
$$\frac{}{3a}$$
$$\frac{}{5a}$$
$$f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(}{1-\sin}}$$

$$\downarrow 330$$

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\left(8a^2-7ab+8b^2\right)\sqrt{-a\sin^2(e+fx)+a+b}\int\frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{b\left(4a^2-3ab+8b^2\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\right.}{a\sqrt{-a\sin^2(e+fx)+a+}}\right)$$
$$\frac{}{3a}$$
$$\frac{}{5a}$$
$$f\sqrt{1-\sin^2(e+fx)}\sqrt{}$$

$$\downarrow 327$$

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\left(8a^2-7ab+8b^2\right)\sqrt{-a\sin^2(e+fx)+a+b}E\left(\arcsin(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{b\left(4a^2-3ab+8b^2\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\arcsi\right.}{a\sqrt{-a\sin^2(e+fx)+a+b}}\right)$$
$$\frac{}{3a}$$
$$\frac{}{5a}$$
$$f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a}{}}$$

input
```
Int[Cos[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

3.262. $\int \frac{\cos^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

output
```
(Sqrt[a + b - a*Sin[e + f*x]^2]*((Sin[e + f*x]*(1 - Sin[e + f*x]^2)^(3/2)*
Sqrt[a + b - a*Sin[e + f*x]^2])/(5*a) + ((4*(a - b)*Sin[e + f*x]*Sqrt[1 -
Sin[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2])/(3*a) + (((8*a^2 - 7*a*b +
 8*b^2)*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b - a*Sin[e +
f*x]^2])/(a*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - (b*(4*a^2 - 3*a*b + 8*
b^2)*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2
)/(a + b)])/(a*Sqrt[a + b - a*Sin[e + f*x]^2]))/(3*a))/(5*a)))/(f*Sqrt[1 -
 Sin[e + f*x]^2]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(1 - Sin[e + f*x]^2)])
```

3.262.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

3.262.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.26 (sec) , antiderivative size = 8756, normalized size of antiderivative = 25.38

method	result	size
default	Expression too large to display	8756

input
```
int(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.262.5 Fricas [F]

$$\int \frac{\cos^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos\left(fx+e\right)^5}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(cos(f*x + e)^5/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.262.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**5/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Timed out
```

3.262.7 Maxima [F]

$$\int \frac{\cos^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos(fx+e)^5}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(f*x + e)^5/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.262.8 Giac [F]

$$\int \frac{\cos^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos(fx+e)^5}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.262.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos(e+f\,x)^5}{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

input
```
int(cos(e + f*x)^5/(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(cos(e + f*x)^5/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.263 $\int \frac{\sec^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

3.263.1 Optimal result 1964
3.263.2 Mathematica [C] (verified) 1964
3.263.3 Rubi [A] (verified) 1965
3.263.4 Maple [B] (verified) 1968
3.263.5 Fricas [A] (verification not implemented) 1969
3.263.6 Sympy [F] 1969
3.263.7 Maxima [A] (verification not implemented) 1970
3.263.8 Giac [F] 1970
3.263.9 Mupad [F(-1)] 1970

3.263.1 Optimal result

Integrand size = 25, antiderivative size = 137

$$\int \frac{\sec^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{(3a^2-2ab+3b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8b^{5/2}f} - \frac{3(a-b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8b^2f} + \frac{\sec^2(e+fx)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4bf}$$

output
```
1/8*(3*a^2-2*a*b+3*b^2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1
/2))/b^(5/2)/f-3/8*(a-b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b^2/f+1/4*s
ec(f*x+e)^2*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b/f
```

3.263.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.55 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.38

$$\int \frac{\sec^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \frac{e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\sqrt{a+2b+a\cos(2e+2fx)}\left(-\frac{i\sqrt{b}\left(-1+e^{2i(e+fx)}\right)\left(-3a\left(1+e^{2i(e+fx)}\right)^2+b(\right.}{\left(1+e^{2i(e+fx)}\right)^4}\right.}{8\sqrt{2}b^{5/2}f\sqrt{a+bs}}$$

input
```
Integrate[Sec[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e +
f*x))]*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]*(((-I)*Sqrt[b]*(-1 + E^((2*I)*(e
 + f*x)))*(-3*a*(1 + E^((2*I)*(e + f*x)))^2 + b*(3 + 14*E^((2*I)*(e + f*x)
) + 3*E^((4*I)*(e + f*x)))))/(1 + E^((2*I)*(e + f*x)))^4 - ((3*a^2 - 2*a*b
 + 3*b^2)*Log[(-4*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*f + (4*I)*Sqrt[4*b*E^
((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)/(1 + E^((2*I)*(e + f
*x)))])/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*Sec
[e + f*x])/(8*Sqrt[2]*b^(5/2)*f*Sqrt[a + b*Sec[e + f*x]^2])
```

3.263.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4634, 318, 25, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sec(e+fx)^6}{\sqrt{a+b\sec(e+fx)^2}}\,dx$$

$$\downarrow 4634$$

$$\frac{\int \frac{\left(\tan^2(e+fx)+1\right)^2}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{f}$$

$$\downarrow 318$$

$$\frac{\int -\frac{3(a-b)\tan^2(e+fx)+a-3b}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{4b} + \frac{\tan(e+fx)\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)+b}}{4b}$$
$$f$$

$$\downarrow 25$$

$$\frac{\tan(e+fx)\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)+b}}{4b} - \frac{\int \frac{3(a-b)\tan^2(e+fx)+a-3b}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{4b}$$
$$f$$

$$\downarrow 299$$

$$\frac{\tan(e+fx)\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)+b}}{4b} - \frac{\frac{3(a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b} - \frac{\left(3a^2-2ab+3b^2\right)\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2b}}{4b}$$
$$f$$

$$\downarrow 224$$

$$\frac{\tan(e+fx)\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)+b}}{4b} - \frac{\frac{3(a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b} - \frac{\left(3a^2-2ab+3b^2\right)\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{2b}}{4b}$$
$$f$$

$$\downarrow 219$$

$$\frac{\tan(e+fx)\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)+b}}{4b} - \frac{\frac{3(a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b} - \frac{\left(3a^2-2ab+3b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2b^{3/2}}}{4b}$$
$$f$$

input
```
Int[Sec[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
((Tan[e + f*x]*(1 + Tan[e + f*x]^2)*Sqrt[a + b + b*Tan[e + f*x]^2])/(4*b)
- (-1/2*((3*a^2 - 2*a*b + 3*b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b
 + b*Tan[e + f*x]^2]])/b^(3/2) + (3*(a - b)*Tan[e + f*x]*Sqrt[a + b + b*Ta
n[e + f*x]^2])/(2*b))/(4*b))/f
```

3.263. $\int \frac{\sec^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}dx$

3.263.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.263.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1611 vs. 2(121) = 242.

Time = 16.83 (sec) , antiderivative size = 1612, normalized size of antiderivative = 11.77

method	result	size
default	Expression too large to display	1612

input
```
int(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/16/f/b^(9/2)/(a+b*sec(f*x+e)^2)^(1/2)*(6*b^(7/2)*a*tan(f*x+e)+6*b^(9/2)*
tan(f*x+e)*sec(f*x+e)^2-6*b^(5/2)*a^2*tan(f*x+e)-2*b^(7/2)*a*tan(f*x+e)*se
c(f*x+e)^2+3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*a^2*b^2-2
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*a*b^3+3*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*b^4+3*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*c
os(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a
-a-b)/(sin(f*x+e)+1))*a^2*b^2-2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(
1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*
x+e)+1))*a*b^3+3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*b^4+4*
b^(9/2)*tan(f*x+e)*sec(f*x+e)^4+3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)...
```

3.263.5 Fricas [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 396, normalized size of antiderivative = 2.89

$$\int \frac{\sec^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \left[\frac{(3\,a^2-2\,ab+3\,b^2)\sqrt{b}\cos(fx+e)^3\log\left(\frac{(a^2-6\,ab+b^2)\cos(fx+e)^4+8\,(ab-b^2)\cos(fx+e)^2+4\left((a-b)\cos(fx+e)^3+2\,b\cos(fx+e)\right.\right.}{\cos(fx+e)^4}\right.}{32\,b^3 f\cos(fx}\right.$$

input
```
integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/32*((3*a^2 - 2*a*b + 3*b^2)*sqrt(b)*cos(f*x + e)^3*log(((a^2 - 6*a*b +
b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x +
e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^
2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*(3*(a*b - b^2)*cos(f*x + e)^2
 - 2*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^3*f
*cos(f*x + e)^3), 1/16*((3*a^2 - 2*a*b + 3*b^2)*sqrt(-b)*arctan(-1/2*((a -
 b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b
)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e)^
3 - 2*(3*(a*b - b^2)*cos(f*x + e)^2 - 2*b^2)*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)*sin(f*x + e))/(b^3*f*cos(f*x + e)^3)]
```

3.263.6 Sympy [F]

$$\int \frac{\sec^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sec^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
integrate(sec(f*x+e)**6/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sec(e + f*x)**6/sqrt(a + b*sec(e + f*x)**2), x)
```

3.263.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.17

$$\int \frac{\sec^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \frac{\frac{2\sqrt{b\tan(fx+e)^2+a+b}\tan(fx+e)^3}{b} + \frac{3(a+b)a\,\mathrm{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{5}{2}}} + \frac{3(a+b)\,\mathrm{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}} - \frac{8a\,\mathrm{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}} - \frac{3\sqrt{bt}}{8f}$$

input
```
integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/8*(2*sqrt(b*tan(f*x + e)^2 + a + b)*tan(f*x + e)^3/b + 3*(a + b)*a*arcsi
nh(b*tan(f*x + e)/sqrt((a + b)*b))/b^(5/2) + 3*(a + b)*arcsinh(b*tan(f*x +
 e)/sqrt((a + b)*b))/b^(3/2) - 8*a*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))
/b^(3/2) - 3*sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)*tan(f*x + e)/b^2 + 8*s
qrt(b*tan(f*x + e)^2 + a + b)*tan(f*x + e)/b)/f
```

3.263.8 Giac [F]

$$\int \frac{\sec^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sec(fx+e)^6}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.263.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{1}{\cos(e+fx)^6\sqrt{a+\frac{b}{\cos(e+fx)^2}}}\,dx$$

input
```
int(1/(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^(1/2)),x)
```

output
```
int(1/(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^(1/2)), x)
```

3.264 $\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

3.264.1 Optimal result 1971
3.264.2 Mathematica [C] (warning: unable to verify) 1971
3.264.3 Rubi [A] (verified) 1972
3.264.4 Maple [B] (verified) 1974
3.264.5 Fricas [B] (verification not implemented) 1975
3.264.6 Sympy [F] 1976
3.264.7 Maxima [A] (verification not implemented) 1976
3.264.8 Giac [F] 1976
3.264.9 Mupad [F(-1)] 1977

3.264.1 Optimal result

Integrand size = 25, antiderivative size = 81

$$\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{(a-b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2b^{3/2}f} + \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2bf}$$

output
```
-1/2*(a-b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b^(3/2)/
f+1/2*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b/f
```

3.264.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 7.00 (sec) , antiderivative size = 326, normalized size of antiderivative = 4.02

$$\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \frac{\sqrt{a+2b+a\cos(2e+2fx)}\,\sec^4(e+fx)\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)\tan(e+fx)\left(\frac{16b^2\left(b+a\cos^2(e+fx)\right)\text{Hypergeometric2F1}(}{}\right.}{30\sqrt{2}f\sqrt{a+b\sec^2(e+}$$

input
```
Integrate[Sec[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]*Sec[e + f*x]^4*(1 - (a*Sin[e + f*x]^2)
/(a + b))*Tan[e + f*x]*((16*b^2*(b + a*Cos[e + f*x]^2)*Hypergeometric2F1[2
, 3, 7/2, -((b*Tan[e + f*x]^2)/(a + b))]*Tan[e + f*x]^4*Sqrt[-((b*Sec[e +
f*x]^2*(a + b - a*Sin[e + f*x]^2)*Tan[e + f*x]^2)/(a + b)^2)])/(a + b)^3 +
 (15*(3*b + a*(3 - 2*Sin[e + f*x]^2))*(ArcSin[Sqrt[-((b*Tan[e + f*x]^2)/(a
 + b))]] - Sqrt[-((b*Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)*Tan[e + f*x
]^2)/(a + b)^2)]))/(a + b)))/(30*Sqrt[2]*f*Sqrt[a + b*Sec[e + f*x]^2]*Sqrt
[(a + b*Sec[e + f*x]^2)/(a + b)]*Sqrt[a + b - a*Sin[e + f*x]^2]*(-((b*Tan[
e + f*x]^2)/(a + b)))^(3/2))
```

3.264.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4634, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sec(e+fx)^4}{\sqrt{a+b\sec(e+fx)^2}}\,dx$$

$$\downarrow 4634$$

$$\frac{\int \frac{\tan^2(e+fx)+1}{\sqrt{b\tan^2(e+fx)+a+b}}\,d\tan(e+fx)}{f}$$

$$\downarrow 299$$

$$\frac{\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b} - \frac{(a-b)\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}\,d\tan(e+fx)}{2b}}{f}$$

$$\downarrow 224$$

$$\frac{\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b} - \frac{(a-b)\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}\,d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{2b}}{f}$$

$$\downarrow 219$$

$$\frac{\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b} - \frac{(a-b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2b^{3/2}}}{f}$$

input
```
Int[Sec[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(-1/2*((a - b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^
2]])/b^(3/2) + (Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*b))/f
```

3.264.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.264.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1038 vs. 2(69) = 138.

Time = 11.29 (sec) , antiderivative size = 1039, normalized size of antiderivative = 12.83

method	result	size
default	Expression too large to display	1039

input
```
int(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/f/b^(5/2)/(a+b*sec(f*x+e)^2)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f
*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b
)/(sin(f*x+e)+1))*a*b-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*b
^2+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*a*b-((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*b^2-2*b^(3/2)*a*tan(f*x+e)+((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*a*b*sec(f*x+e)-((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-s
in(f*x+e)*a-a-b)/(sin(f*x+e)+1))*b^2*sec(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/
2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f...
```

3.264.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(69) = 138$.

Time = 0.32 (sec) , antiderivative size = 324, normalized size of antiderivative = 4.00

$$\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \left[-\frac{(a-b)\sqrt{b}\cos(fx+e)\log\left(\frac{(a^2-6ab+b^2)\cos(fx+e)^4+8(ab-b^2)\cos(fx+e)^2+4\left((a-b)\cos(fx+e)^3+2b\cos(fx+e)\right)\sqrt{b}\sqrt{\frac{a\cos(}{\cos}}}{\cos(fx+e)^4}\right.}{8b^2f\cos(fx+e)}\right.$$

$$\left. -\frac{(a-b)\sqrt{-b}\arctan\left(-\frac{\left((a-b)\cos(fx+e)^3+2b\cos(fx+e)\right)\sqrt{-b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{2\left(ab\cos(fx+e)^2+b^2\right)\sin(fx+e)}\right)\cos(fx+e)-2b\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\,s}{4b^2f\cos(fx+e)}\right]$$

input
```
integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*((a - b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^
4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x
 + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) +
8*b^2)/cos(f*x + e)^4) - 4*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*s
in(f*x + e))/(b^2*f*cos(f*x + e)), -1/4*((a - b)*sqrt(-b)*arctan(-1/2*((a
- b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e)
 - 2*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(b^2*f*co
s(f*x + e))]
```

3.264.6 Sympy [F]

$$\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sec^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
integrate(sec(f*x+e)**4/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sec(e + f*x)**4/sqrt(a + b*sec(e + f*x)**2), x)
```

3.264.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.91

$$\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\frac{a\,\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}} - \frac{\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}} - \frac{\sqrt{b\tan(fx+e)^2+a+b}\tan(fx+e)}{b}}{2f}$$

input
```
integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*(a*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/b^(3/2) - arcsinh(b*tan(f*
x + e)/sqrt((a + b)*b))/sqrt(b) - sqrt(b*tan(f*x + e)^2 + a + b)*tan(f*x +
 e)/b)/f
```

3.264.8 Giac [F]

$$\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sec\left(fx+e\right)^4}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.264.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{1}{\cos(e+fx)^4\sqrt{a+\frac{b}{\cos(e+fx)^2}}}\,dx$$

input
```
int(1/(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^(1/2)),x)
```

output
```
int(1/(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^(1/2)), x)
```

3.265 $\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

3.265.1 Optimal result . . . 1978
3.265.2 Mathematica [B] (verified) . . . 1978
3.265.3 Rubi [A] (verified) . . . 1979
3.265.4 Maple [B] (warning: unable to verify) . . . 1980
3.265.5 Fricas [B] (verification not implemented) . . . 1981
3.265.6 Sympy [F] . . . 1981
3.265.7 Maxima [A] (verification not implemented) . . . 1982
3.265.8 Giac [F] . . . 1982
3.265.9 Mupad [F(-1)] . . . 1982

3.265.1 Optimal result

Integrand size = 25, antiderivative size = 39

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{b}f}$$

output
```
arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/b^(1/2)
```

3.265.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 87 vs. 2(39) = 78.

Time = 0.15 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.23

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)\sqrt{a+2b+a\cos(2e+2fx)}\sec(e+fx)}{\sqrt{2}\sqrt{b}f\sqrt{a+b\sec^2(e+fx)}}$$

input
```
Integrate[Sec[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*Sqrt[a + 2
*b + a*Cos[2*e + 2*f*x]]*Sec[e + f*x])/(Sqrt[2]*Sqrt[b]*f*Sqrt[a + b*Sec[e
 + f*x]^2])
```

3.265.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 4634, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sec(e+fx)^2}{\sqrt{a+b\sec(e+fx)^2}}\,dx$$

$$\downarrow 4634$$

$$\frac{\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}\,d\tan(e+fx)}{f}$$

$$\downarrow 224$$

$$\frac{\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}\,d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{f}$$

$$\downarrow 219$$

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}f}$$

input
```
Int[Sec[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[b]*f)
```

3.265.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.265.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(33) = 66$.

Time = 7.88 (sec) , antiderivative size = 241, normalized size of antiderivative = 6.18

method	result
default	$\frac{\left(\ln\left(-\frac{4\left(-\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{b}\cos(fx+e)+\sin(fx+e)a-\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}+a+b}\right)}{\sin(fx+e)+1}\right)+\ln\left(\frac{-4\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{b}\cos(fx+e)+4\sin(fx}{\sin(fx+e)}\right.}{2f\sqrt{b}\sqrt{a+b\sec(fx+e)^2}}$

input
```
int(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f/b^(1/2)*(ln(-4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)
*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)+a+b)/(sin(f*x+e)+1))+ln(4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)-a-b)/(sin(f*x+e)-1)))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)/(a+b*sec(f*x+e)^2)^(1/2)*(sec(f*x+e)+1)
```

3.265.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(33) = 66$.

Time = 0.30 (sec) , antiderivative size = 215, normalized size of antiderivative = 5.51

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$
$$= \left[\frac{\log\left(\frac{(a^2-6ab+b^2)\cos(fx+e)^4+8(ab-b^2)\cos(fx+e)^2+4\left((a-b)\cos(fx+e)^3+2b\cos(fx+e)\right)\sqrt{b}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin(fx+e)+8b^2}{\cos(fx+e)^4}\right)}{4\sqrt{b}f}, \sqrt{}\right.$$

input
```
integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^
2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x
+ e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4)/(sqrt(b)
*f), 1/2*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*
sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2
+ b^2)*sin(f*x + e)))/(b*f)]
```

3.265.6 Sympy [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
integrate(sec(f*x+e)**2/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sec(e + f*x)**2/sqrt(a + b*sec(e + f*x)**2), x)
```

3.265.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.59

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}f}$$

input
```
integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt(b)*f)
```

3.265.8 Giac [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\sec\left(fx+e\right)^2}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.265.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{1}{\cos\left(e+f\,x\right)^2\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

input
```
int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^(1/2)),x)
```

output
```
int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^(1/2)), x)
```

3.266 $\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

3.266.1 Optimal result 1983
3.266.2 Mathematica [B] (verified) 1983
3.266.3 Rubi [A] (verified) 1984
3.266.4 Maple [B] (verified) 1985
3.266.5 Fricas [B] (verification not implemented) 1986
3.266.6 Sympy [F] 1987
3.266.7 Maxima [B] (verification not implemented) 1987
3.266.8 Giac [F] 1988
3.266.9 Mupad [F(-1)] 1988

3.266.1 Optimal result

Integrand size = 16, antiderivative size = 39

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}f}$$

output
```
arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/a^(1/2)
```

3.266.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 87 vs. $2(39) = 78$.

Time = 0.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.23

$$\begin{aligned}&\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx \\ &= \frac{\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)\sqrt{a+2b+a\cos(2e+2fx)}\sec(e+fx)}{\sqrt{2}\sqrt{a}f\sqrt{a+b\sec^2(e+fx)}}\end{aligned}$$

input
```
Integrate[1/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*Sqrt[a + 2*
b + a*Cos[2*e + 2*f*x]]*Sec[e + f*x])/(Sqrt[2]*Sqrt[a]*f*Sqrt[a + b*Sec[e
+ f*x]^2])
```

3.266.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 4616, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)^2}}\,dx$$

$$\downarrow 4616$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}\,d\tan(e+fx)}{f}$$

$$\downarrow 291$$

$$\frac{\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}\,d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{f}$$

$$\downarrow 216$$

$$\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}f}$$

input
```
Int[1/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[a]*f)
```

3.266.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

3.266.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(33) = 66$.

Time = 3.02 (sec) , antiderivative size = 138, normalized size of antiderivative = 3.54

method	result	size
default	$\frac{\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\ln\left(4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)+4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a\right)(\sec(fx+e)+1)}{f\sqrt{-a}\sqrt{a+b\sec(fx+e)^2}}$	138

input
```
int(1/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f/(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)/(a+b*sec(f*x+e)^2)^
(1/2)*(sec(f*x+e)+1)
```

3.266.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(33) = 66$.

Time = 0.37 (sec) , antiderivative size = 408, normalized size of antiderivative = 10.46

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \left[-\frac{\sqrt{-a}\log\left(128\,a^4\cos(fx+e)^8 - 256\,(a^4-a^3b)\cos(fx+e)^6 + 32\,(5\,a^4-14\,a^3b+5\,a^2b^2)\cos(fx+e)\right.}{} \right.$$

$$\left. \left. -\frac{\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^5 - 8\,(a^2-ab)\cos(fx+e)^3 + (a^2-6\,ab+b^2)\cos(fx+e)\right)\sqrt{a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\left(2\,a^3\cos(fx+e)^4 - a^2b + ab^2 - (a^3-3\,a^2b)\cos(fx+e)^2\right)\sin(fx+e)}\right)}{4\sqrt{a}f} \right]$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)
^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 7
0*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*
x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*
(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b
^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin
(f*x + e))/(a*f), -1/4*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*co
s(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 -
 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))/(sqrt(a)*f)]
```

3.266.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*sec(e + f*x)**2), x)
```

3.266.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 992 vs. $2(33) = 66$.

Time = 0.40 (sec) , antiderivative size = 992, normalized size of antiderivative = 25.44

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*(arctan2(2*a*sin(2*f*x + 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*
f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b
)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x +
2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e)
 + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*sin(1/2*arctan2(a*sin(4
*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*
b)*cos(2*f*x + 2*e) + a)), 2*a*cos(2*f*x + 2*e) + 2*(a^2*cos(4*f*x + 4*e)^
2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 +
4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2
)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*co
s(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*cos(1/2*a
rctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*
e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) + 2*a + 4*b) - arctan2(2*(a^2*cos(
4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*
x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 +
4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f
*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt
(a)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*c
os(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)), 2*(a^2*cos(4*f*x + 4
*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*...
```

3.266.8 Giac [F]

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.266.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{\cos(e+fx)^2}}}\,dx$$

input
```
int(1/(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(1/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.267 $\int \frac{\cos^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

3.267.1 Optimal result 1989
3.267.2 Mathematica [A] (verified) 1989
3.267.3 Rubi [A] (verified) 1990
3.267.4 Maple [B] (verified) 1992
3.267.5 Fricas [B] (verification not implemented) 1992
3.267.6 Sympy [F] 1993
3.267.7 Maxima [F] 1993
3.267.8 Giac [F] 1994
3.267.9 Mupad [F(-1)] 1994

3.267.1 Optimal result

Integrand size = 25, antiderivative size = 87

$$\int \frac{\cos^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{(a-b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2a^{3/2}f} + \frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2af}$$

output
```
1/2*(a-b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f+
1/2*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/a/f
```

3.267.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.45

$$\int \frac{\cos^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\sqrt{a+2b+a\cos(2(e+fx))}\left((a-b)\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)\sec(e+fx)+\sqrt{a}\sqrt{a+b-a\sin^2(e+fx)}\right.}{2\sqrt{2}a^{3/2}f\sqrt{a+b\sec^2(e+fx)}}$$

input
```
Integrate[Cos[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*((a - b)*ArcTan[(Sqrt[a]*Sin[e + f*x])
/Sqrt[a + b - a*Sin[e + f*x]^2]]*Sec[e + f*x] + Sqrt[a]*Sqrt[a + b - a*Sin
[e + f*x]^2]*Tan[e + f*x]))/(2*Sqrt[2]*a^(3/2)*f*Sqrt[a + b*Sec[e + f*x]^2
])
```

3.267.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4634, 296, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sec(e+fx)^2\sqrt{a+b\sec(e+fx)^2}}dx$$

$$\downarrow 4634$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)^2\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{f}$$

$$\downarrow 296$$

$$\frac{\frac{(a-b)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2a}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a(\tan^2(e+fx)+1)}}{f}$$

$$\downarrow 291$$

$$\frac{\frac{(a-b)\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{2a}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a(\tan^2(e+fx)+1)}}{f}$$

$$\downarrow 216$$

$$\frac{\frac{(a-b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2a^{3/2}}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a(\tan^2(e+fx)+1)}}{f}$$

input
```
Int[Cos[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(((a - b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(
2*a^(3/2)) + (Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*a*(1 + Tan[e
 + f*x]^2)))/f
```

3.267.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.267.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 506 vs. $2(75) = 150$.

Time = 5.12 (sec) , antiderivative size = 507, normalized size of antiderivative = 5.83

method	result
default	$\frac{\sin(fx+e)\cos(fx+e)\sqrt{-a}\,a+\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\ln\left(4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\cos(fx+e)+4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a\right)a$

input
```
int(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f/(-a)^(1/2)/a/(a+b*sec(f*x+e)^2)^(1/2)*(sin(f*x+e)*cos(f*x+e)*(-a)^(1
/2)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a-((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*
sin(f*x+e)*a)*b+(-a)^(1/2)*b*tan(f*x+e)+sec(f*x+e)*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
-4*sin(f*x+e)*a)*a-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(
1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*b*sec(f*x+e))
```

3.267.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(75) = 150$.

Time = 0.40 (sec) , antiderivative size = 502, normalized size of antiderivative = 5.77

$$\int \frac{\cos^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \left[\frac{8\,a\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)\sin(fx+e)+\sqrt{-a}(a-b)\log\left(128\,a^4\cos(fx+e)^8-256\,(a^4-a^3b)\cos\right.\right.$$

input
```
integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*
x + e) + sqrt(-a)*(a - b)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*c
os(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 2
8*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*
b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x
+ e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b +
7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)*sin(f*x + e)))/(a^2*f), 1/8*(4*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)*cos(f*x + e)*sin(f*x + e) - (a - b)*sqrt(a)*arctan(1/4*(8*a^2*c
os(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x
 + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x
 + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))))/
(a^2*f)]
```

3.267.6 Sympy [F]

$$\int \frac{\cos^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
integrate(cos(f*x+e)**2/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(cos(e + f*x)**2/sqrt(a + b*sec(e + f*x)**2), x)
```

3.267.7 Maxima [F]

$$\int \frac{\cos^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos(fx+e)^2}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(f*x + e)^2/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.267.8 Giac [F]

$$\int \frac{\cos^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos(fx+e)^2}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.267.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos(e+fx)^2}{\sqrt{a+\frac{b}{\cos(e+fx)^2}}}\,dx$$

input
```
int(cos(e + f*x)^2/(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(cos(e + f*x)^2/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.268 $\int \frac{\cos^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

3.268.1 Optimal result 1995
3.268.2 Mathematica [C] (warning: unable to verify) 1995
3.268.3 Rubi [A] (verified) 1996
3.268.4 Maple [B] (verified) 1999
3.268.5 Fricas [A] (verification not implemented) 2000
3.268.6 Sympy [F] 2001
3.268.7 Maxima [F] 2001
3.268.8 Giac [F] 2002
3.268.9 Mupad [F(-1)] 2002

3.268.1 Optimal result

Integrand size = 25, antiderivative size = 143

$$\int \frac{\cos^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{(3a^2-2ab+3b^2)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{5/2}f} + \frac{3(a-b)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8a^2f} + \frac{\cos^3(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4af}$$

output
```
1/8*(3*a^2-2*a*b+3*b^2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/
2))/a^(5/2)/f+3/8*(a-b)*cos(f*x+e)*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/a
^2/f+1/4*cos(f*x+e)^3*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/a/f
```

3.268.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 16.22 (sec) , antiderivative size = 1840, normalized size of antiderivative = 12.87

$$\int \frac{\cos^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(3*(a + b)*AppellF1[1/2, -2, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/
(a + b)]*Cos[e + f*x]^8*Sin[e + f*x])/(f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)]
]*Sqrt[a + b*Sec[e + f*x]^2]*(3*(a + b)*AppellF1[1/2, -2, 1/2, 3/2, Sin[e
+ f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*AppellF1[3/2, -2, 3/2, 5/2, Sin
[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 1/2
, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)*((3*(a
 + b)*AppellF1[1/2, -2, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a +
b)]*Cos[e + f*x]^5)/(Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(3*(a + b)*AppellF
1[1/2, -2, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*Appe
llF1[3/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a
 + b)*AppellF1[3/2, -1, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a +
b)])*Sin[e + f*x]^2)) - (12*(a + b)*AppellF1[1/2, -2, 1/2, 3/2, Sin[e + f*
x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^3*Sin[e + f*x]^2)/(Sqrt[a +
 2*b + a*Cos[2*(e + f*x)]]*(3*(a + b)*AppellF1[1/2, -2, 1/2, 3/2, Sin[e +
f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*AppellF1[3/2, -2, 3/2, 5/2, Sin[e
 + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 1/2,
5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a
 + b)*Cos[e + f*x]^4*Sin[e + f*x]*((a*f*AppellF1[3/2, -2, 3/2, 5/2, Sin[e
+ f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b
)) - (4*f*AppellF1[3/2, -1, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2...
```

3.268.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4634, 316, 25, 402, 25, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sec(e+fx)^4\sqrt{a+b\sec(e+fx)^2}}dx$$

$$\downarrow 4634$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)^3\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{f}$$

$$\downarrow 316$$

$$\frac{\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4a(\tan^2(e+fx)+1)^2}-\frac{\int -\frac{2b\tan^2(e+fx)+3a-b}{(\tan^2(e+fx)+1)^2\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{4a}}{f}$$

$$\downarrow 25$$

$$\frac{\frac{\int \frac{2b\tan^2(e+fx)+3a-b}{(\tan^2(e+fx)+1)^2\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{4a}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4a(\tan^2(e+fx)+1)^2}}{f}$$

$$\downarrow 402$$

$$\frac{\frac{3(a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a\left(\tan^2(e+fx)+1\right)}-\frac{\int -\frac{3a^2-2ba+3b^2}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2a}}{4a}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4a(\tan^2(e+fx)+1)^2}}{f}$$

$$\downarrow 25$$

$$\frac{\frac{\int \frac{3a^2-2ba+3b^2}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2a}+\frac{3(a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a\left(\tan^2(e+fx)+1\right)}}{4a}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4a(\tan^2(e+fx)+1)^2}}{f}$$

$$\downarrow 27$$

$$\frac{\frac{\left(3a^2-2ab+3b^2\right)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2a}+\frac{3(a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a\left(\tan^2(e+fx)+1\right)}}{4a}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4a(\tan^2(e+fx)+1)^2}}{f}$$

$$\downarrow 291$$

$$\frac{\frac{\left(3a^2-2ab+3b^2\right)\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{2a}+\frac{3(a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a\left(\tan^2(e+fx)+1\right)}}{4a}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4a(\tan^2(e+fx)+1)^2}}{f}$$

$$\downarrow 216$$

3.268. $\int \frac{\cos^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}dx$

$$\frac{\frac{\left(3a^2-2ab+3b^2\right)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2a^{3/2}}+\frac{3(a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a\left(\tan^2(e+fx)+1\right)}}{4a}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4a(\tan^2(e+fx)+1)^2}}{f}$$

input
```
Int[Cos[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
((Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(4*a*(1 + Tan[e + f*x]^2)^2
) + (((3*a^2 - 2*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b
*Tan[e + f*x]^2]])/(2*a^(3/2)) + (3*(a - b)*Tan[e + f*x]*Sqrt[a + b + b*Ta
n[e + f*x]^2])/(2*a*(1 + Tan[e + f*x]^2)))/(4*a))/f
```

3.268.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.268.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 808 vs. $2(127) = 254$.

Time = 7.14 (sec) , antiderivative size = 809, normalized size of antiderivative = 5.66

method	result
default	$\frac{2\sin(fx+e)\cos(fx+e)^3\sqrt{-a}\,a^2+3\sin(fx+e)\cos(fx+e)\sqrt{-a}\,a^2-\cos(fx+e)\sqrt{-a}\,\sin(fx+e)ab+3\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\ln\left(4\sqrt{-a}\sqrt{\frac{b+}{(1+}\right.$

input
```
int(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/8/f/(-a)^(1/2)/a^2/(a+b*sec(f*x+e)^2)^(1/2)*(2*sin(f*x+e)*cos(f*x+e)^3*(
-a)^(1/2)*a^2+3*sin(f*x+e)*cos(f*x+e)*(-a)^(1/2)*a^2-cos(f*x+e)*(-a)^(1/2)
*sin(f*x+e)*a*b+3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1
/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^2-2*ln(4*(-a)^
(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b+3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*b^2+3*sec(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(
1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^2-2*ln(4*(-a)
^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b*sec(f*x+e)+3*ln(4*(-a)^(1/2)*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*b^2*sec(f*x+e)+3*(-a)^(1/2)*a*b*tan(f*x+e)-3*(-a)^(1/2)*b^2*
tan(f*x+e))
```

3.268.5 Fricas [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 567, normalized size of antiderivative = 3.97

$$\int \frac{\cos^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \left[-\frac{(3\,a^2-2\,ab+3\,b^2)\sqrt{-a}\log\left(128\,a^4\cos(fx+e)^8-256\,(a^4-a^3b)\cos(fx+e)^6+32\,(5\,a^4-14\,a^3b+\right.\right.$$

$$\left.\left. -\frac{(3\,a^2-2\,ab+3\,b^2)\sqrt{a}\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^5-8\,(a^2-ab)\cos(fx+e)^3+(a^2-6\,ab+b^2)\cos(fx+e)\right)\sqrt{a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\left(2\,a^3\cos(fx+e)^4-a^2b+ab^2-(a^3-3\,a^2b)\cos(fx+e)^2\right)\sin(fx+e)}\right)-}{32\,a^3f}\right.$$

input

```
integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/64*((3*a^2 - 2*a*b + 3*b^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*
(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x +
 e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b +
 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 -
a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a
^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 8*(2*a^2*cos(f*x + e)^3 + 3*(a^2 -
a*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e
))/(a^3*f), -1/32*((3*a^2 - 2*a*b + 3*b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f
*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e
))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e
)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(
2*a^2*cos(f*x + e)^3 + 3*(a^2 - a*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)*sin(f*x + e))/(a^3*f)]
```

3.268.6 Sympy [F]

$$\int \frac{\cos^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
integrate(cos(f*x+e)**4/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(cos(e + f*x)**4/sqrt(a + b*sec(e + f*x)**2), x)
```

3.268.7 Maxima [F]

$$\int \frac{\cos^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos(fx+e)^4}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(f*x + e)^4/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.268.8 Giac [F]

$$\int \frac{\cos^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos\left(fx+e\right)^4}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.268.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos\left(e+f\,x\right)^4}{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

input
```
int(cos(e + f*x)^4/(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(cos(e + f*x)^4/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.269 $\int \frac{\cos^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

3.269.1 Optimal result 2003
3.269.2 Mathematica [C] (warning: unable to verify) 2004
3.269.3 Rubi [A] (verified) 2004
3.269.4 Maple [B] (verified) 2008
3.269.5 Fricas [A] (verification not implemented) 2009
3.269.6 Sympy [F] 2010
3.269.7 Maxima [F] 2010
3.269.8 Giac [F] 2010
3.269.9 Mupad [F(-1)] 2011

3.269.1 Optimal result

Integrand size = 25, antiderivative size = 204

$$\begin{aligned}
&\int \frac{\cos^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx \\
&= \frac{(a-b)\left(5a^2+2ab+5b^2\right)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{7/2}f} \\
&\quad + \frac{\left(15a^2-14ab+15b^2\right)\cos(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48a^3f} \\
&\quad + \frac{5(a-b)\cos^3(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24a^2f} \\
&\quad + \frac{\cos^5(e+fx)\sin(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6af}
\end{aligned}$$

output

```
1/16*(a-b)*(5*a^2+2*a*b+5*b^2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)
^2)^(1/2))/a^(7/2)/f+1/48*(15*a^2-14*a*b+15*b^2)*cos(f*x+e)*sin(f*x+e)*(a+
b+b*tan(f*x+e)^2)^(1/2)/a^3/f+5/24*(a-b)*cos(f*x+e)^3*sin(f*x+e)*(a+b+b*ta
n(f*x+e)^2)^(1/2)/a^2/f+1/6*cos(f*x+e)^5*sin(f*x+e)*(a+b+b*tan(f*x+e)^2)^(
1/2)/a/f
```

3.269.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 15.44 (sec) , antiderivative size = 1739, normalized size of antiderivative = 8.52

$$\int \frac{\cos^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(3*(a + b)*AppellF1[1/2, -3, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/
(a + b)]*Cos[e + f*x]^12*Sin[e + f*x])/(f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)
]]*Sqrt[a + b*Sec[e + f*x]^2]*(3*(a + b)*AppellF1[1/2, -3, 1/2, 3/2, Sin[e
 + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*AppellF1[3/2, -3, 3/2, 5/2, Si
n[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 6*(a + b)*AppellF1[3/2, -2, 1/
2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)*((3*(
a + b)*AppellF1[1/2, -3, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a +
 b)]*Cos[e + f*x]^7)/(Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*(3*(a + b)*Appell
F1[1/2, -3, 1/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*App
ellF1[3/2, -3, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 6*(
a + b)*AppellF1[3/2, -2, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a +
 b)])*Sin[e + f*x]^2)) - (18*(a + b)*AppellF1[1/2, -3, 1/2, 3/2, Sin[e + f
*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^5*Sin[e + f*x]^2)/(Sqrt[a
+ 2*b + a*Cos[2*(e + f*x)]]*(3*(a + b)*AppellF1[1/2, -3, 1/2, 3/2, Sin[e +
 f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*AppellF1[3/2, -3, 3/2, 5/2, Sin[
e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 6*(a + b)*AppellF1[3/2, -2, 1/2,
 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(
a + b)*Cos[e + f*x]^6*Sin[e + f*x]*((a*f*AppellF1[3/2, -3, 3/2, 5/2, Sin[e
 + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]*Sin[e + f*x])/(3*(a +
b)) - 2*f*AppellF1[3/2, -2, 1/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2...
```

3.269.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4634, 316, 25, 402, 25, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sec(e+fx)^6\sqrt{a+b\sec(e+fx)^2}}dx$$

$$\downarrow 4634$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)^4\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{f}$$

$$\downarrow 316$$

$$\frac{\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{6a(\tan^2(e+fx)+1)^3}-\frac{\int -\frac{4b\tan^2(e+fx)+5a-b}{(\tan^2(e+fx)+1)^3\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{6a}}{f}$$

$$\downarrow 25$$

$$\frac{\frac{\int \frac{4b\tan^2(e+fx)+5a-b}{(\tan^2(e+fx)+1)^3\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{6a}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{6a(\tan^2(e+fx)+1)^3}}{f}$$

$$\downarrow 402$$

$$\frac{\frac{5(a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4a\left(\tan^2(e+fx)+1\right)^2}-\frac{\int -\frac{15a^2-4ba+5b^2+10(a-b)b\tan^2(e+fx)}{(\tan^2(e+fx)+1)^2\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{4a}}{6a}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{6a(\tan^2(e+fx)+1)^3}}{f}$$

$$\downarrow 25$$

$$\frac{\frac{\int \frac{15a^2-4ba+5b^2+10(a-b)b\tan^2(e+fx)}{(\tan^2(e+fx)+1)^2\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{4a}+\frac{5(a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4a\left(\tan^2(e+fx)+1\right)^2}}{6a}+\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{6a(\tan^2(e+fx)+1)^3}}{f}$$

$$\downarrow 402$$

$$\frac{\frac{\left(15a^2-14ab+15b^2\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a\left(\tan^2(e+fx)+1\right)}-\frac{\int -\frac{3(a-b)\left(5a^2+2ba+5b^2\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2a}}{4a}+\frac{5(a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4a\left(\tan^2(e+fx)+1\right)^2}}{6a}+\frac{\tan(e}{f}$$

$$\downarrow 27$$

$$\frac{\frac{3(a-b)\left(5a^2+2ab+5b^2\right)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2a}+\frac{\left(15a^2-14ab+15b^2\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a\left(\tan^2(e+fx)+1\right)}}{4a}+\frac{5(a-b)\tan(e+fx)\sqrt{a+b\tan}}{4a\left(\tan^2(e+fx)+1\right.}$$
$$\frac{}{6a}$$
$$f$$

$$\downarrow 291$$

$$\frac{\frac{3(a-b)\left(5a^2+2ab+5b^2\right)\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{2a}+\frac{\left(15a^2-14ab+15b^2\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a\left(\tan^2(e+fx)+1\right)}}{4a}+\frac{5(a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+}}{4a\left(\tan^2(e+fx)+1\right)^2}$$
$$\frac{}{6a}$$
$$f$$

$$\downarrow 216$$

$$\frac{\frac{\left(15a^2-14ab+15b^2\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2a\left(\tan^2(e+fx)+1\right)}+\frac{3(a-b)\left(5a^2+2ab+5b^2\right)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2a^{3/2}}}{4a}+\frac{5(a-b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4a\left(\tan^2(e+fx)+1\right)^2}$$
$$\frac{}{6a}+\frac{\tan(}{}$$
$$f$$

input
```
Int[Cos[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
((Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(6*a*(1 + Tan[e + f*x]^2)^3
) + ((5*(a - b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(4*a*(1 + Tan
[e + f*x]^2)^2) + ((3*(a - b)*(5*a^2 + 2*a*b + 5*b^2)*ArcTan[(Sqrt[a]*Tan[
e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/(2*a^(3/2)) + ((15*a^2 - 14*a*b
 + 15*b^2)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*a*(1 + Tan[e +
f*x]^2)))/(4*a))/(6*a))/f
```

3.269.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.269.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1135 vs. 2(184) = 368.

Time = 9.68 (sec) , antiderivative size = 1136, normalized size of antiderivative = 5.57

method	result	size
default	Expression too large to display	1136

input
```
int(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/48/f/(-a)^(1/2)/a^3/(a+b*sec(f*x+e)^2)^(1/2)*(8*cos(f*x+e)^5*sin(f*x+e)*
(-a)^(1/2)*a^3+10*cos(f*x+e)^3*sin(f*x+e)*(-a)^(1/2)*a^3-2*cos(f*x+e)^3*si
n(f*x+e)*(-a)^(1/2)*a^2*b+15*cos(f*x+e)*sin(f*x+e)*(-a)^(1/2)*a^3-4*cos(f*
x+e)*sin(f*x+e)*(-a)^(1/2)*a^2*b+5*cos(f*x+e)*sin(f*x+e)*(-a)^(1/2)*a*b^2+
15*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^3-9*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4
*sin(f*x+e)*a)*a^2*b+9*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-
a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/
2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a*b^2-15*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*b^3+15*(-a)^(1/2)*a^2*b*tan(f*x+e)-1
4*(-a)^(1/2)*a*b^2*tan(f*x+e)+15*(-a)^(1/2)*b^3*tan(f*x+e)+15*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)-4*sin(f*x+e)*a)*a^3*sec(f*x+e)-9*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1...
```

3.269.5 Fricas [A] (verification not implemented)

Time = 1.14 (sec) , antiderivative size = 643, normalized size of antiderivative = 3.15

$$\int \frac{\cos^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \left[\frac{3\left(5a^3-3a^2b+3ab^2-5b^3\right)\sqrt{-a}\log\left(128a^4\cos(fx+e)^8-256\left(a^4-a^3b\right)\cos(fx+e)^6+32\left(5a^4-\right.\right.}{} \right.$$

$$\left. -\frac{3\left(5a^3-3a^2b+3ab^2-5b^3\right)\sqrt{a}\arctan\left(\frac{\left(8a^2\cos(fx+e)^5-8\left(a^2-ab\right)\cos(fx+e)^3+\left(a^2-6ab+b^2\right)\cos(fx+e)\right)\sqrt{a}\sqrt{\frac{a\cos(fx}{\cos(f}}}{4\left(2a^3\cos(fx+e)^4-a^2b+ab^2-\left(a^3-3a^2b\right)\cos(fx+e)^2\right)\sin(fx+e)}\right.$$

input
```
integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/384*(3*(5*a^3 - 3*a^2*b + 3*a*b^2 - 5*b^3)*sqrt(-a)*log(128*a^4*cos(f*x
 + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*
b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a
^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^
7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f
*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a
*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(8*a^3*cos(f*x + e)
^5 + 10*(a^3 - a^2*b)*cos(f*x + e)^3 + (15*a^3 - 14*a^2*b + 15*a*b^2)*cos(
f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^4*f
), -1/192*(3*(5*a^3 - 3*a^2*b + 3*a*b^2 - 5*b^3)*sqrt(a)*arctan(1/4*(8*a^2
*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f
*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f
*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))
 - 4*(8*a^3*cos(f*x + e)^5 + 10*(a^3 - a^2*b)*cos(f*x + e)^3 + (15*a^3 - 1
4*a^2*b + 15*a*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)*sin(f*x + e))/(a^4*f)]
```

3.269.6 Sympy [F]

$$\int \frac{\cos^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
integrate(cos(f*x+e)**6/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(cos(e + f*x)**6/sqrt(a + b*sec(e + f*x)**2), x)
```

3.269.7 Maxima [F]

$$\int \frac{\cos^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos(fx+e)^6}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(f*x + e)^6/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.269.8 Giac [F]

$$\int \frac{\cos^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos(fx+e)^6}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.269.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cos(e+fx)^6}{\sqrt{a+\frac{b}{\cos(e+fx)^2}}}\,dx$$

input
```
int(cos(e + f*x)^6/(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(cos(e + f*x)^6/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.270 $\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

3.270.1 Optimal result 2012
3.270.2 Mathematica [F] 2013
3.270.3 Rubi [A] (verified) 2013
3.270.4 Maple [C] (warning: unable to verify) 2019
3.270.5 Fricas [C] (verification not implemented) 2019
3.270.6 Sympy [F] 2020
3.270.7 Maxima [F] 2021
3.270.8 Giac [F] 2021
3.270.9 Mupad [F(-1)] 2021

3.270.1 Optimal result

Integrand size = 25, antiderivative size = 289

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{a(2a+b)\sin(e+fx)}{b^2(a+b)f\sqrt{\sec^2(e+fx)}\,(a+b-a\sin^2(e+fx))} - \frac{(2a+b)E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)(a+b-a\sin^2(e+fx))}{b^2(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)}\,(a+b-a\sin^2(e+fx))\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} + \frac{\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{bf\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)}\,(a+b-a\sin^2(e+fx))} + \frac{\sec(e+fx)\tan(e+fx)}{bf\sqrt{\sec^2(e+fx)}\,(a+b-a\sin^2(e+fx))}$$

output

```
a*(2*a+b)*sin(f*x+e)/b^2/(a+b)/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)
-(2*a+b)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(a+b-a*sin(f*x+e)^2)/b^2/(a
+b)/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/(1-a*
sin(f*x+e)^2/(a+b))^(1/2)+EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*(1-a*sin(f
*x+e)^2/(a+b))^(1/2)/b/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x
+e)^2))^(1/2)+sec(f*x+e)*tan(f*x+e)/b/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2)
)^(1/2)
```

3.270.2 Mathematica [F]

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

input
```
Integrate[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
Integrate[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2), x]
```

3.270.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.09, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 4636, 2057, 2058, 316, 25, 27, 402, 25, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sec(e+fx)^5}{(a+b\sec(e+fx)^2)^{3/2}}\,dx$$

$$\downarrow 4636$$

$$\frac{\int \frac{1}{(1-\sin^2(e+fx))^3\left(a+\frac{b}{1-\sin^2(e+fx)}\right)^{3/2}}\,d\sin(e+fx)}{f}$$

$$\downarrow 2057$$

$$\frac{\int \frac{1}{(1-\sin^2(e+fx))^3\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^{3/2}}\,d\sin(e+fx)}{f}$$

$$\downarrow 2058$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\int \frac{1}{(1-\sin^2(e+fx))^{3/2}(-a\sin^2(e+fx)+a+b)^{3/2}}\,d\sin(e+fx)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 316$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\int -\frac{a\left(\sin^2(e+fx)+1\right)}{\sqrt{1-\sin^2(e+fx)}\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}d\sin(e+fx)}{b}+\frac{\sin(e+fx)}{b\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 25$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sin(e+fx)}{b\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}-\frac{\int \frac{a\left(\sin^2(e+fx)+1\right)}{\sqrt{1-\sin^2(e+fx)}\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}d\sin(e+fx)}{b}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 27$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sin(e+fx)}{b\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}-\frac{a\int \frac{\sin^2(e+fx)+1}{\sqrt{1-\sin^2(e+fx)}\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}d\sin(e+fx)}{b}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 402$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sin(e+fx)}{b\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}-\frac{a\left(-\frac{\int -\frac{2(a+b)-(2a+b)\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{b(a+b)}-\frac{(2a+b)}{b(a+b)}\right.\right.}{b}$$

$$f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}$$

$$\downarrow 25$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sin(e+fx)}{b\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}-\frac{a\left(\frac{\int \frac{2(a+b)-(2a+b)\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{b(a+b)}-\frac{(2a+b)\sin(}{b(a+b)\sqrt{}}\right.\right.}{b}$$

$$f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}$$

$$\downarrow 399$$

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sin(e+fx)}{b\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}-\frac{a\left(\frac{(2a+b)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{b(a+b)\int\frac{}{\sqrt{1-\sin^2(e}}}{b(a+b)}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}\right.$$

$$\downarrow 323$$

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sin(e+fx)}{b\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}-\frac{a\left(\frac{(2a+b)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{b(a+b)\sqrt{1-\frac{a\sin^2(e}{a+b}}}{b(a+b)}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}\right.$$

$$\downarrow 321$$

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sin(e+fx)}{b\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}-\frac{a\left(\frac{(2a+b)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{b(a+b)\sqrt{1-\frac{a\sin^2(e}{a+b}}}{b(a+b)}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}\right.$$

$$\downarrow 330$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sin(e+fx)}{b\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}-\frac{a\left(\frac{(2a+b)\sqrt{-a\sin^2(e+fx)+a+b}\int\frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{b(}{b(a+b)}\right.}{}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 327$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sin(e+fx)}{b\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}-\frac{a\left(\frac{(2a+b)\sqrt{-a\sin^2(e+fx)+a+b}E\left(\arcsin(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{b(a+b)}{b(a+b)}\right.}{}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

input
```
Int[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[a + b - a*Sin[e + f*x]^2]*(Sin[e + f*x]/(b*Sqrt[1 - Sin[e + f*x]^2]*
Sqrt[a + b - a*Sin[e + f*x]^2]) - (a*(-(((2*a + b)*Sin[e + f*x]*Sqrt[1 - S
in[e + f*x]^2])/(b*(a + b)*Sqrt[a + b - a*Sin[e + f*x]^2])) + (((2*a + b)*
EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b - a*Sin[e + f*x]^2])
/(a*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - (b*(a + b)*EllipticF[ArcSin[Si
n[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(a*Sqrt[a +
b - a*Sin[e + f*x]^2]))/(b*(a + b))))/b))/(f*Sqrt[1 - Sin[e + f*x]^2]*Sqrt
[(a + b - a*Sin[e + f*x]^2)/(1 - Sin[e + f*x]^2)])
```

3.270.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

3.270.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.40 (sec) , antiderivative size = 10329, normalized size of antiderivative = 35.74

method	result	size
default	Expression too large to display	10329

input
```
int(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.270.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 960, normalized size of antiderivative = 3.32

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/2*((2*(2*I*a^2*b + I*a*b^2 + (2*I*a^3 + I*a^2*b)*cos(f*x + e)^2)*sqrt(a)
*sqrt((a*b + b^2)/a^2) - (2*I*a^2*b + 5*I*a*b^2 + 2*I*b^3 + (2*I*a^3 + 5*I
*a^2*b + 2*I*a*b^2)*cos(f*x + e)^2)*sqrt(a))*sqrt((2*a*sqrt((a*b + b^2)/a^
2) - a - 2*b)/a)*elliptic_e(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2
*b)/a)*(cos(f*x + e) + I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2
*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + (2*(-2*I*a^2*b - I*a*b^2 + (-2*I*a^3 -
 I*a^2*b)*cos(f*x + e)^2)*sqrt(a)*sqrt((a*b + b^2)/a^2) - (-2*I*a^2*b - 5*
I*a*b^2 - 2*I*b^3 + (-2*I*a^3 - 5*I*a^2*b - 2*I*a*b^2)*cos(f*x + e)^2)*sqr
t(a))*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_e(arcsin(sqrt
((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) - I*sin(f*x + e)))
, (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) - 4*(
(-I*a^2*b*cos(f*x + e)^2 - I*a*b^2)*sqrt(a)*sqrt((a*b + b^2)/a^2) + (-I*a^
2*b - 3*I*a*b^2 - 2*I*b^3 + (-I*a^3 - 3*I*a^2*b - 2*I*a*b^2)*cos(f*x + e)^
2)*sqrt(a))*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_f(arcsi
n(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) + I*sin(f*x
+ e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2)
 - 4*((I*a^2*b*cos(f*x + e)^2 + I*a*b^2)*sqrt(a)*sqrt((a*b + b^2)/a^2) + (
I*a^2*b + 3*I*a*b^2 + 2*I*b^3 + (I*a^3 + 3*I*a^2*b + 2*I*a*b^2)*cos(f*x +
e)^2)*sqrt(a))*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_f(ar
csin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) - I*si...
```

3.270.6 Sympy [F]

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)**5/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(sec(e + f*x)**5/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.270.7 Maxima [F]

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sec(fx+e)^5}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^5/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.270.8 Giac [F]

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sec(fx+e)^5}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.270.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{1}{\cos(e+fx)^5\left(a+\frac{b}{\cos(e+fx)^2}\right)^{3/2}}\,dx$$

input
```
int(1/(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2)),x)
```

output
```
int(1/(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2)), x)
```

3.271 $\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

3.271.1 Optimal result . . . 2022
3.271.2 Mathematica [A] (verified) . . . 2022
3.271.3 Rubi [A] (verified) . . . 2023
3.271.4 Maple [C] (warning: unable to verify) . . . 2026
3.271.5 Fricas [C] (verification not implemented) . . . 2027
3.271.6 Sympy [F] . . . 2028
3.271.7 Maxima [F] . . . 2028
3.271.8 Giac [F] . . . 2028
3.271.9 Mupad [F(-1)] . . . 2029

3.271.1 Optimal result

Integrand size = 25, antiderivative size = 150

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{a\sin(e+fx)}{b(a+b)f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} + \frac{E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)(a+b-a\sin^2(e+fx))}{b(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}$$

output

```
-a*sin(f*x+e)/b/(a+b)/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)+Elliptic
E(sin(f*x+e),(a/(a+b))^(1/2))*(a+b-a*sin(f*x+e)^2)/b/(a+b)/f/(cos(f*x+e)^2
)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/(1-a*sin(f*x+e)^2/(a+b))
^(1/2)
```

3.271.2 Mathematica [A] (verified)

Time = 3.66 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.75

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{(a+2b+a\cos(2(e+fx)))\sec^3(e+fx)\left(\sqrt{2}(a+b)\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}}E(e\right.}{4b(a+b)f\,(a+b\sec^2(e+fx))^{3/2}}$$

input

```
Integrate[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(Sqrt[2]*(a + b)*Sqrt[(a +
2*b + a*Cos[2*(e + f*x)])/(a + b)]*EllipticE[e + f*x, a/(a + b)] - a*Sin[2
*(e + f*x)]))/(4*b*(a + b)*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

3.271.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.25, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4636, 2057, 2058, 316, 25, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sec(e+fx)^3}{(a+b\sec(e+fx)^2)^{3/2}}\,dx$$

$$\downarrow 4636$$

$$\frac{\int \frac{1}{(1-\sin^2(e+fx))^2\left(a+\frac{b}{1-\sin^2(e+fx)}\right)^{3/2}}\,d\sin(e+fx)}{f}$$

$$\downarrow 2057$$

$$\frac{\int \frac{1}{(1-\sin^2(e+fx))^2\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^{3/2}}\,d\sin(e+fx)}{f}$$

$$\downarrow 2058$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\int \frac{1}{\sqrt{1-\sin^2(e+fx)}(-a\sin^2(e+fx)+a+b)^{3/2}}\,d\sin(e+fx)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 316$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(-\frac{\int -\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}\,d\sin(e+fx)}{b(a+b)} - \frac{a\sqrt{1-\sin^2(e+fx)}\sin(e+fx)}{b(a+b)\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 25$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\int \frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b(a+b)}-\frac{a\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{b(a+b)\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 330$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sqrt{-a\sin^2(e+fx)+a+b}\int \frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{b(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{a\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{b(a+b)\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 327$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sqrt{-a\sin^2(e+fx)+a+b}E\left(\arcsin(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{b(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{a\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{b(a+b)\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

input
```
Int[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[a + b - a*Sin[e + f*x]^2]*(-((a*Sin[e + f*x]*Sqrt[1 - Sin[e + f*x]^2
])/(b*(a + b)*Sqrt[a + b - a*Sin[e + f*x]^2])) + (EllipticE[ArcSin[Sin[e +
 f*x]], a/(a + b)]*Sqrt[a + b - a*Sin[e + f*x]^2])/(b*(a + b)*Sqrt[1 - (a*
Sin[e + f*x]^2)/(a + b)])))/(f*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[(a + b - a*Si
n[e + f*x]^2)/(1 - Sin[e + f*x]^2)])
```

3.271.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

3.271.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.14 (sec) , antiderivative size = 2535, normalized size of antiderivative = 16.90

method	result	size
default	Expression too large to display	2535

input
```
int(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f/(2*I*a^(1/2)*b^(1/2)-a+b)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/(a+
b)/b*(2*I*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^(3/2)*b^(1/2)*(1-cos(f
*x+e))^3*csc(f*x+e)^3+2*I*a^(1/2)*b^(3/2)*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b)
)^(1/2)*(csc(f*x+e)-cot(f*x+e))-2*I*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2
)*a^(3/2)*b^(1/2)*(csc(f*x+e)-cot(f*x+e))+2*I*a^(3/2)*b^(1/2)*(-(2*I*a^(1/
2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2+a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b
*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*((2*I*a^(1/2)*b^(1/2)*(1-
cos(f*x+e))^2*csc(f*x+e)^2-a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e)
)^2*csc(f*x+e)^2+a+b)/(a+b))^(1/2)*EllipticF(((2*I*a^(1/2)*b^(1/2)+a-b)/(a
+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3
/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))+2*I*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(
1/2)*a^(1/2)*b^(3/2)*(1-cos(f*x+e))^3*csc(f*x+e)^3+2*I*a^(1/2)*b^(3/2)*(-(
2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2+a*(1-cos(f*x+e))^2*csc(f
*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-b)/(a+b))^(1/2)*((2*I*a^(1/2)*b^
(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2-a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-c
os(f*x+e))^2*csc(f*x+e)^2+a+b)/(a+b))^(1/2)*EllipticF(((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(
1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2))-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)*a^2*(1-cos(f*x+e))^3*csc(f*x+e)^3+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)*b^2*(1-cos(f*x+e))^3*csc(f*x+e)^3+2*(-(2*I*a^(1/2)*b^(1/2)*(1...
```

3.271.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 894, normalized size of antiderivative = 5.96

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx =$$

$$\frac{2\,a^3\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)^2\sin(fx+e) - \left(2\left(-i\,a^3\cos(fx+e)^2 - i\,a^2b\right)\sqrt{a}\sqrt{\frac{ab+b^2}{a^2}} - \left(-i\,a^2b - 2i\right.\right.}{-}$$

input
```
integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(2*a^3*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2*sin
(f*x + e) - (2*(-I*a^3*cos(f*x + e)^2 - I*a^2*b)*sqrt(a)*sqrt((a*b + b^2)/
a^2) - (-I*a^2*b - 2*I*a*b^2 + (-I*a^3 - 2*I*a^2*b)*cos(f*x + e)^2)*sqrt(a
))*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_e(arcsin(sqrt((2
*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) + I*sin(f*x + e))), (
a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) - (2*(I*
a^3*cos(f*x + e)^2 + I*a^2*b)*sqrt(a)*sqrt((a*b + b^2)/a^2) - (I*a^2*b + 2
*I*a*b^2 + (I*a^3 + 2*I*a^2*b)*cos(f*x + e)^2)*sqrt(a))*sqrt((2*a*sqrt((a*
b + b^2)/a^2) - a - 2*b)/a)*elliptic_e(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a
^2) - a - 2*b)/a)*(cos(f*x + e) - I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 +
 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + 2*(2*(I*a^2*b*cos(f*x + e)^
2 + I*a*b^2)*sqrt(a)*sqrt((a*b + b^2)/a^2) + (I*a^2*b + 3*I*a*b^2 + 2*I*b^
3 + (I*a^3 + 3*I*a^2*b + 2*I*a*b^2)*cos(f*x + e)^2)*sqrt(a))*sqrt((2*a*sqr
t((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b + b
^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) + I*sin(f*x + e))), (a^2 + 8*a*b + 8*
b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + 2*(2*(-I*a^2*b*cos(f*x
 + e)^2 - I*a*b^2)*sqrt(a)*sqrt((a*b + b^2)/a^2) + (-I*a^2*b - 3*I*a*b^2 -
 2*I*b^3 + (-I*a^3 - 3*I*a^2*b - 2*I*a*b^2)*cos(f*x + e)^2)*sqrt(a))*sqrt(
(2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_f(arcsin(sqrt((2*a*sqrt(
(a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) - I*sin(f*x + e))), (a^2 +...
```

3.271.6 Sympy [F]

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)**3/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(sec(e + f*x)**3/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.271.7 Maxima [F]

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sec(fx+e)^3}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^3/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.271.8 Giac [F]

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sec(fx+e)^3}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.271.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{1}{\cos(e+fx)^3\left(a+\frac{b}{\cos(e+fx)^2}\right)^{3/2}}\,dx$$

input
```
int(1/(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2)),x)
```

output
```
int(1/(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2)), x)
```

3.272 $\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

3.272.1 Optimal result . . . 2030
3.272.2 Mathematica [C] (verified) . . . 2031
3.272.3 Rubi [A] (verified) . . . 2032
3.272.4 Maple [C] (warning: unable to verify) . . . 2036
3.272.5 Fricas [C] (verification not implemented) . . . 2036
3.272.6 Sympy [F] . . . 2037
3.272.7 Maxima [F] . . . 2038
3.272.8 Giac [F] . . . 2038
3.272.9 Mupad [F(-1)] . . . 2038

3.272.1 Optimal result

Integrand size = 23, antiderivative size = 229

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\sin(e+fx)}{(a+b)f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} - \frac{E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)(a+b-a\sin^2(e+fx))}{a(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} + \frac{\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{af\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

output

```
sin(f*x+e)/(a+b)/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)-EllipticE(sin
(f*x+e),(a/(a+b))^(1/2))*(a+b-a*sin(f*x+e)^2)/a/(a+b)/f/(cos(f*x+e)^2)^(1/
2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/(1-a*sin(f*x+e)^2/(a+b))^(1/2
)+EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/a/f
/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)
```

3.272.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 18.39 (sec) , antiderivative size = 905, normalized size of antiderivative = 3.95

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = (a+2b+a\cos(2e+2fx))^{3/2}\sec^3(e+fx)\left(\frac{\sqrt{-\frac{1}{b}}\left((-\frac{1}{b})^{3/2}b(-a+a\cos(2e+2fx))(a-\right.\right.$$

input
```
Integrate[Sec[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
((a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sec[e + f*x]^3*((Sqrt[-b^(-1)]*((-b^
(-1))^(3/2)*b*(-a + a*Cos[2*e + 2*f*x])*(a + a*Cos[2*e + 2*f*x]) + I*(a +
b)*Sqrt[(a - a*Cos[2*e + 2*f*x])/(a + b)]*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x
]]*Sqrt[4 - (2*(a + 2*b + a*Cos[2*e + 2*f*x]))/b]*EllipticE[I*ArcSinh[(Sqr
t[-b^(-1)]*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]])/Sqrt[2]], b/(a + b)] - I*(a
 + b)*Sqrt[(a - a*Cos[2*e + 2*f*x])/(a + b)]*Sqrt[a + 2*b + a*Cos[2*e + 2*
f*x]]*Sqrt[4 - (2*(a + 2*b + a*Cos[2*e + 2*f*x]))/b]*EllipticF[I*ArcSinh[(
Sqrt[-b^(-1)]*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]])/Sqrt[2]], b/(a + b)])*Si
n[2*e + 2*f*x])/(4*a*(a + b)*f*Sqrt[((a - a*Cos[2*e + 2*f*x])*(a + a*Cos[2
*e + 2*f*x]))/a^2]*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]*Sqrt[1 - Cos[2*e + 2
*f*x]^2]) - (Sqrt[-b^(-1)]*Cos[2*(e + f*x)]*((-b^(-1))^(3/2)*b*(a + 2*b)*(
-a + a*Cos[2*e + 2*f*x])*(a + a*Cos[2*e + 2*f*x]) + I*(a^2 + 3*a*b + 2*b^2
)*Sqrt[(a - a*Cos[2*e + 2*f*x])/(a + b)]*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]
]*Sqrt[4 - (2*(a + 2*b + a*Cos[2*e + 2*f*x]))/b]*EllipticE[I*ArcSinh[(Sqrt
[-b^(-1)]*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]])/Sqrt[2]], b/(a + b)] - I*a*(
a + b)*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]*Sqrt[(4*a + 4*b - 2*(a + 2*b + a
*Cos[2*e + 2*f*x]))/(a + b)]*Sqrt[2 - (a + 2*b + a*Cos[2*e + 2*f*x])/b]*El
lipticF[I*ArcSinh[(Sqrt[-b^(-1)]*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]])/Sqrt[
2]], b/(a + b)])*Sec[2*(e + (-2*e + ArcCos[Cos[2*e + 2*f*x]])/2)]*Sin[2*e
+ 2*f*x])/(4*a^2*(a + b)*f*Sqrt[((a - a*Cos[2*e + 2*f*x])*(a + a*Cos[2*...
```

3.272.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 4636, 2057, 2058, 314, 25, 389, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx)^2)^{3/2}}\,dx$$

$$\downarrow 4636$$

$$\frac{\int \frac{1}{(1-\sin^2(e+fx))\left(a+\frac{b}{1-\sin^2(e+fx)}\right)^{3/2}}\,d\sin(e+fx)}{f}$$

$$\downarrow 2057$$

$$\frac{\int \frac{1}{(1-\sin^2(e+fx))\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^{3/2}}\,d\sin(e+fx)}{f}$$

$$\downarrow 2058$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\int \frac{\sqrt{1-\sin^2(e+fx)}}{(-a\sin^2(e+fx)+a+b)^{3/2}}\,d\sin(e+fx)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 314$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{(a+b)\sqrt{-a\sin^2(e+fx)+a+b}}-\frac{\int -\frac{\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}\,d\sin(e+fx)}{a+b}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 25$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\int \frac{\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}\,d\sin(e+fx)}{a+b}+\frac{\sqrt{1-\sin^2(e+fx)}\sin(e+fx)}{(a+b)\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 389$$

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{(a+b)\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{a}-\frac{\int \frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}}{a+b}+\frac{\sqrt{1-\sin^2(e+f}}{(a+b)\sqrt{-a\sin^2}}\right.$$

$$f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}$$

$$\downarrow 323$$

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}d\sin(e+fx)}{a\sqrt{-a\sin^2(e+fx)+a+b}}-\frac{\int \frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}}{a+b}+\frac{\sqrt{}}{(a}\right.$$

$$f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}$$

$$\downarrow 321$$

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)}{a\sqrt{-a\sin^2(e+fx)+a+b}}-\frac{\int \frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}}{a+b}+\frac{\sqrt{1-\sin^2(e+f}}{(a+b)\sqrt{-a\sin^2}}\right.$$

$$f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}$$

$$\downarrow 330$$

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)}{a\sqrt{-a\sin^2(e+fx)+a+b}}-\frac{\sqrt{-a\sin^2(e+fx)+a+b}\int \frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}}{a+b}+\right.$$

$$f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}$$

$$\downarrow 327$$

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)}{a\sqrt{-a\sin^2(e+fx)+a+b}}-\frac{\sqrt{-a\sin^2(e+fx)+a+bE\left(\arcsin(\sin(e+fx))\middle|\frac{a}{a+b}\right)}}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}}{a+b}+\frac{\sqrt{}}{(a+}\right.$$

$$\left.f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}\right)$$

input
```
Int[Sec[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[a + b - a*Sin[e + f*x]^2]*((Sin[e + f*x]*Sqrt[1 - Sin[e + f*x]^2])/(
(a + b)*Sqrt[a + b - a*Sin[e + f*x]^2]) + (-((EllipticE[ArcSin[Sin[e + f*x
]], a/(a + b)]*Sqrt[a + b - a*Sin[e + f*x]^2])/(a*Sqrt[1 - (a*Sin[e + f*x]
^2)/(a + b)])) + ((a + b)*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[
1 - (a*Sin[e + f*x]^2)/(a + b)])/(a*Sqrt[a + b - a*Sin[e + f*x]^2]))/(a +
b)))/(f*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(1 - Sin[
e + f*x]^2)])
```

3.272.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 314
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] + Simp[1/(2*a*
(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*p + 3) + d
*(2*(p + q + 1) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, 2, p, q,
 x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 389
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[1/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] - Simp[a/b   Int
[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0] &&  !SimplerSqrtQ[-b/a, -d/c]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

3.272.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.98 (sec) , antiderivative size = 2534, normalized size of antiderivative = 11.07

method	result	size
default	Expression too large to display	2534

input
```
int(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f/(2*I*a^(1/2)*b^(1/2)-a+b)/((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)/(a+b
)/a*(-2*I*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^(3/2)*b^(1/2)*(csc(f*x
+e)-cot(f*x+e))-2*I*a^(1/2)*b^(3/2)*(-(2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^
2*csc(f*x+e)^2+a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+
e)^2-a-b)/(a+b))^(1/2)*((2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2
-a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/(a+b
))^(1/2)*EllipticF(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot
(f*x+e)),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2
)^(1/2))-2*I*a^(3/2)*b^(1/2)*(-(2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f
*x+e)^2+a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2-a-
b)/(a+b))^(1/2)*((2*I*a^(1/2)*b^(1/2)*(1-cos(f*x+e))^2*csc(f*x+e)^2-a*(1-c
os(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/(a+b))^(1/2
)*EllipticF(((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e)
),(-(4*I*a^(3/2)*b^(1/2)-4*I*a^(1/2)*b^(3/2)-a^2+6*a*b-b^2)/(a+b)^2)^(1/2)
)+2*I*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^(1/2)*b^(3/2)*(1-cos(f*x+e
))^3*csc(f*x+e)^3+2*I*((2*I*a^(1/2)*b^(1/2)+a-b)/(a+b))^(1/2)*a^(3/2)*b^(1
/2)*(1-cos(f*x+e))^3*csc(f*x+e)^3+2*I*a^(1/2)*b^(3/2)*((2*I*a^(1/2)*b^(1/2
)+a-b)/(a+b))^(1/2)*(csc(f*x+e)-cot(f*x+e))-((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)*a^2*(1-cos(f*x+e))^3*csc(f*x+e)^3+((2*I*a^(1/2)*b^(1/2)+a-b)/(a+
b))^(1/2)*b^2*(1-cos(f*x+e))^3*csc(f*x+e)^3+2*(-(2*I*a^(1/2)*b^(1/2)*(1...
```

3.272.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 762, normalized size of antiderivative = 3.33

$$\int \frac{\sec(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{2\,a^2\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos\left(fx+e\right)^2\sin\left(fx+e\right)-4\left(i\,a^2\cos\left(fx+e\right)^2+i\,ab\right)\sqrt{}$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(2*a^2*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2*sin(
f*x + e) - 4*(I*a^2*cos(f*x + e)^2 + I*a*b)*sqrt(a)*sqrt((2*a*sqrt((a*b +
b^2)/a^2) - a - 2*b)/a)*sqrt((a*b + b^2)/a^2)*elliptic_f(arcsin(sqrt((2*a*
sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) + I*sin(f*x + e))), (a^2
 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) - 4*(-I*a^2
*cos(f*x + e)^2 - I*a*b)*sqrt(a)*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b
)/a)*sqrt((a*b + b^2)/a^2)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^
2) - a - 2*b)/a)*(cos(f*x + e) - I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 +
4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + (2*(I*a^2*cos(f*x + e)^2 + I
*a*b)*sqrt(a)*sqrt((a*b + b^2)/a^2) - ((I*a^2 + 2*I*a*b)*cos(f*x + e)^2 +
I*a*b + 2*I*b^2)*sqrt(a))*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*el
liptic_e(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e
) + I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b
^2)/a^2))/a^2) + (2*(-I*a^2*cos(f*x + e)^2 - I*a*b)*sqrt(a)*sqrt((a*b + b^
2)/a^2) - ((-I*a^2 - 2*I*a*b)*cos(f*x + e)^2 - I*a*b - 2*I*b^2)*sqrt(a))*s
qrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_e(arcsin(sqrt((2*a*s
qrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) - I*sin(f*x + e))), (a^2
+ 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2))/((a^4 + a^3
*b)*f*cos(f*x + e)^2 + (a^3*b + a^2*b^2)*f)
```

3.272.6 Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(sec(e + f*x)/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.272.7 Maxima [F]

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sec(fx+e)}{\left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.272.8 Giac [F]

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sec(fx+e)}{\left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.272.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{1}{\cos(e+fx)\left(a+\frac{b}{\cos(e+fx)^2}\right)^{3/2}}\,dx$$

input
```
int(1/(cos(e + f*x)*(a + b/cos(e + f*x)^2)^(3/2)),x)
```

output
```
int(1/(cos(e + f*x)*(a + b/cos(e + f*x)^2)^(3/2)), x)
```

3.273 $\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

3.273.1 Optimal result . . . 2039
3.273.2 Mathematica [F] . . . 2040
3.273.3 Rubi [A] (verified) . . . 2040
3.273.4 Maple [C] (warning: unable to verify) . . . 2044
3.273.5 Fricas [F] . . . 2044
3.273.6 Sympy [F] . . . 2045
3.273.7 Maxima [F] . . . 2045
3.273.8 Giac [F] . . . 2045
3.273.9 Mupad [F(-1)] . . . 2046

3.273.1 Optimal result

Integrand size = 23, antiderivative size = 240

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{b\sin(e+fx)}{a(a+b)f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}} + \frac{(a+2b)E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)(a+b-a\sin^2(e+fx))}{a^2(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}} - \frac{2b\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{a^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

output

```
-b*sin(f*x+e)/a/(a+b)/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)+(a+2*b)*
EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(a+b-a*sin(f*x+e)^2)/a^2/(a+b)/f/(co
s(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/(1-a*sin(f*x+e
)^2/(a+b))^(1/2)-2*b*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*(1-a*sin(f*x+e)
^2/(a+b))^(1/2)/a^2/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)
^2))^(1/2)
```

3.273.2 Mathematica [F]

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

input
```
Integrate[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
Integrate[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2), x]
```

3.273.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.09, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 4636, 2057, 2058, 315, 25, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sec(e+fx)\,(a+b\sec(e+fx)^2)^{3/2}}dx$$

$$\downarrow 4636$$

$$\frac{\int \frac{1}{\left(a+\frac{b}{1-\sin^2(e+fx)}\right)^{3/2}}d\sin(e+fx)}{f}$$

$$\downarrow 2057$$

$$\frac{\int \frac{1}{\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^{3/2}}d\sin(e+fx)}{f}$$

$$\downarrow 2058$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\int \frac{\left(1-\sin^2(e+fx)\right)^{3/2}}{\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}d\sin(e+fx)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 315$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(-\frac{\int -\frac{-\left((a+2b)\sin^2(e+fx)\right)+a+b}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{a(a+b)}-\frac{b\sqrt{1-\sin^2(e+fx)}\sin(e+fx)}{a(a+b)\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 25$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\int \frac{-\left((a+2b)\sin^2(e+fx)\right)+a+b}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{a(a+b)}-\frac{b\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{a(a+b)\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 399$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{(a+2b)\int \frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{2b(a+b)\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{a}}{a(a+b)}-\frac{b\sin(e}{a(a+b)}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 323$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{(a+2b)\int \frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{2b(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}d\sin(e+fx)}{a\sqrt{-a\sin^2(e+fx)+a+b}}}{a(a+b)}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 321$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{(a+2b)\int \frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{2b(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)}{a\sqrt{-a\sin^2(e+fx)+a+b}}}{a(a+b)}-\frac{b\sin(e}{a(a+b)}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 330$$

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{(a+2b)\sqrt{-a\sin^2(e+fx)+a+b}\int\frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{2b(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\right.}{a\sqrt{-a\sin^2(e+fx)+a+b}}\right)$$
$$\frac{}{a(a+b)}$$
$$f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}$$

$$\downarrow 327$$

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{(a+2b)\sqrt{-a\sin^2(e+fx)+a+b}E\left(\arcsin(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{2b(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)}{a\sqrt{-a\sin^2(e+fx)+a+b}}\right)$$
$$\frac{}{a(a+b)}$$
$$f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}$$

input
```
Int[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[a + b - a*Sin[e + f*x]^2]*(-((b*Sin[e + f*x]*Sqrt[1 - Sin[e + f*x]^2
])/(a*(a + b)*Sqrt[a + b - a*Sin[e + f*x]^2])) + (((a + 2*b)*EllipticE[Arc
Sin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b - a*Sin[e + f*x]^2])/(a*Sqrt[1 -
(a*Sin[e + f*x]^2)/(a + b)]) - (2*b*(a + b)*EllipticF[ArcSin[Sin[e + f*x]]
, a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(a*Sqrt[a + b - a*Sin[e
 + f*x]^2]))/(a*(a + b))))/(f*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[(a + b - a*Sin
[e + f*x]^2)/(1 - Sin[e + f*x]^2)])
```

3.273.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

3.273.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.97 (sec) , antiderivative size = 6126, normalized size of antiderivative = 25.52

method	result	size
default	Expression too large to display	6126

input
```
int(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.273.5 Fricas [F]

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos\left(fx+e\right)}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)/(b^2*sec(f*x + e)^4 + 2*a
*b*sec(f*x + e)^2 + a^2), x)
```

3.273.6 Sympy [F]

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(cos(e + f*x)/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.273.7 Maxima [F]

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos(fx+e)}{\left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(f*x + e)/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.273.8 Giac [F]

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos(fx+e)}{\left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.273.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos(e+fx)}{\left(a+\frac{b}{\cos(e+fx)^2}\right)^{3/2}}\,dx$$

input
```
int(cos(e + f*x)/(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(cos(e + f*x)/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.274 $\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

3.274.1 Optimal result . . . 2047
3.274.2 Mathematica [F] . . . 2048
3.274.3 Rubi [A] (verified) . . . 2048
3.274.4 Maple [C] (warning: unable to verify) . . . 2053
3.274.5 Fricas [F] . . . 2053
3.274.6 Sympy [F(-1)] . . . 2053
3.274.7 Maxima [F] . . . 2054
3.274.8 Giac [F] . . . 2054
3.274.9 Mupad [F(-1)] . . . 2054

3.274.1 Optimal result

Integrand size = 25, antiderivative size = 335

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{b\cos^2(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$
$$+\frac{(a+4b)\sin(e+fx)\,(a+b-a\sin^2(e+fx))}{3a^2(a+b)f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$
$$+\frac{(2a^2-3ab-8b^2)\,E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)(a+b-a\sin^2(e+fx))}{3a^3(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}$$
$$-\frac{(a-8b)b\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{3a^3f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

output

```
-b*cos(f*x+e)^2*sin(f*x+e)/a/(a+b)/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(
1/2)+1/3*(a+4*b)*sin(f*x+e)*(a+b-a*sin(f*x+e)^2)/a^2/(a+b)/f/(sec(f*x+e)^2
*(a+b-a*sin(f*x+e)^2))^(1/2)+1/3*(2*a^2-3*a*b-8*b^2)*EllipticE(sin(f*x+e),
(a/(a+b))^(1/2))*(a+b-a*sin(f*x+e)^2)/a^3/(a+b)/f/(cos(f*x+e)^2)^(1/2)/(se
c(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/(1-a*sin(f*x+e)^2/(a+b))^(1/2)-1/3*
(a-8*b)*b*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*(1-a*sin(f*x+e)^2/(a+b))^(
1/2)/a^3/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)
```

3.274.2 Mathematica [F]

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

input
```
Integrate[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
Integrate[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2), x]
```

3.274.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 4636, 2057, 2058, 315, 25, 403, 25, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sec(e+fx)^3\,(a+b\sec(e+fx)^2)^{3/2}}dx$$

$$\downarrow 4636$$

$$\frac{\int \frac{1-\sin^2(e+fx)}{\left(a+\frac{b}{1-\sin^2(e+fx)}\right)^{3/2}}d\sin(e+fx)}{f}$$

$$\downarrow 2057$$

$$\frac{\int \frac{1-\sin^2(e+fx)}{\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^{3/2}}d\sin(e+fx)}{f}$$

$$\downarrow 2058$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\int \frac{(1-\sin^2(e+fx))^{5/2}}{(-a\sin^2(e+fx)+a+b)^{3/2}}d\sin(e+fx)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 315$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(-\frac{\int -\frac{\sqrt{1-\sin^2(e+fx)}\left(-\left((a+4b)\sin^2(e+fx)\right)+a+b\right)}{\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{a(a+b)}-\frac{b\sin(e+fx)\left(1-\sin^2(e+fx)\right)^{3/2}}{a(a+b)\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 25$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\int \frac{\sqrt{1-\sin^2(e+fx)}\left(-\left((a+4b)\sin^2(e+fx)\right)+a+b\right)}{\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{a(a+b)}-\frac{b\sin(e+fx)\left(1-\sin^2(e+fx)\right)^{3/2}}{a(a+b)\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 403$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{(a+4b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}{3a}-\frac{\int -\frac{2(a-2b)(a+b)-\left(2a^2-3ba-8b^2\right)\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{3a}}{a(a+b)}-\right.$$

$$f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}$$

$$\downarrow 25$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\int \frac{2(a-2b)(a+b)-\left(2a^2-3ba-8b^2\right)\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{3a}+\frac{(a+4b)\sqrt{1-\sin^2(e+fx)}\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3a}}{a(a+b)}-\right.$$

$$f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}$$

$$\downarrow 399$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\left(2a^2-3ab-8b^2\right)\int \frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{b(a-8b)(a+b)\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+f}{a}}{3a}}{a(a+b)}\right.$$

$$f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}$$

$$\downarrow 323$$

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\left(2a^2-3ab-8b^2\right)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{b(a-8b)(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+f}}}}{a\sqrt{-a\sin^2(e+fx)+a+b}}}{3a}\right)$$
$$\frac{}{a(a+b)}$$
$$f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)}{1-\sin^2(e+f}}$$

$$\downarrow 321$$

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\left(2a^2-3ab-8b^2\right)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{b(a-8b)(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+}\right.}{a\sqrt{-a\sin^2(e+fx)+a+b}}}{3a}\right)$$
$$\frac{}{a(a+b)}$$
$$f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}$$

$$\downarrow 330$$

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\left(2a^2-3ab-8b^2\right)\sqrt{-a\sin^2(e+fx)+a+b}\int\frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{b(a-8b)(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\text{arc}\right.}{a\sqrt{-a\sin^2(e+fx)+a+b}}}{3a}\right)$$
$$\frac{}{a(a+b)}$$
$$f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+f}{1-\sin^2(e+}}$$

$$\downarrow 327$$

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\left(2a^2-3ab-8b^2\right)\sqrt{-a\sin^2(e+fx)+a+b}E\left(\arcsin(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{b(a-8b)(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\arcsin(\text{si}\right.}{a\sqrt{-a\sin^2(e+fx)+a+b}}}{3a}\right)$$
$$\frac{}{a(a+b)}$$
$$f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)}{1-\sin^2(e+f}}$$

input
```
Int[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

3.274. $\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

output
```
(Sqrt[a + b - a*Sin[e + f*x]^2]*(-((b*Sin[e + f*x]*(1 - Sin[e + f*x]^2)^(3
/2))/(a*(a + b)*Sqrt[a + b - a*Sin[e + f*x]^2])) + (((a + 4*b)*Sin[e + f*x
]*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[a + b - a*Sin[e + f*x]^2])/(3*a) + (((2*a^
2 - 3*a*b - 8*b^2)*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b -
 a*Sin[e + f*x]^2])/(a*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - ((a - 8*b)*
b*(a + b)*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f
*x]^2)/(a + b)])/(a*Sqrt[a + b - a*Sin[e + f*x]^2]))/(3*a))/(a*(a + b))))/
(f*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(1 - Sin[e + f
*x]^2)])
```

3.274.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

3.274.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.67 (sec) , antiderivative size = 15843, normalized size of antiderivative = 47.29

method	result	size
default	Expression too large to display	15843

input
```
int(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.274.5 Fricas [F]

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos\left(fx+e\right)^3}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^3/(b^2*sec(f*x + e)^4 + 2
*a*b*sec(f*x + e)^2 + a^2), x)
```

3.274.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**3/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

3.274.7 Maxima [F]

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos(fx+e)^3}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(f*x + e)^3/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.274.8 Giac [F]

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos(fx+e)^3}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.274.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos(e+f\,x)^3}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

input
```
int(cos(e + f*x)^3/(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(cos(e + f*x)^3/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.275 $\int \frac{\cos^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

3.275.1 Optimal result 2055
3.275.2 Mathematica [F] 2056
3.275.3 Rubi [A] (verified) 2056
3.275.4 Maple [C] (warning: unable to verify) 2061
3.275.5 Fricas [F] 2062
3.275.6 Sympy [F(-1)] 2062
3.275.7 Maxima [F] 2062
3.275.8 Giac [F] 2063
3.275.9 Mupad [F(-1)] 2063

3.275.1 Optimal result

Integrand size = 25, antiderivative size = 436

$$\int \frac{\cos^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = -\frac{b\cos^4(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$
$$+\frac{(4a^2-5ab-24b^2)\sin(e+fx)\,(a+b-a\sin^2(e+fx))}{15a^3(a+b)f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$
$$+\frac{(a+6b)\cos^2(e+fx)\sin(e+fx)\,(a+b-a\sin^2(e+fx))}{5a^2(a+b)f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$
$$+\frac{(8a^3-9a^2b+16ab^2+48b^3)\,E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)(a+b-a\sin^2(e+fx))}{15a^4(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}$$
$$-\frac{4b(a^2-2ab+12b^2)\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{15a^4f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

output

```
-b*cos(f*x+e)^4*sin(f*x+e)/a/(a+b)/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(
1/2)+1/15*(4*a^2-5*a*b-24*b^2)*sin(f*x+e)*(a+b-a*sin(f*x+e)^2)/a^3/(a+b)/f
/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)+1/5*(a+6*b)*cos(f*x+e)^2*sin(f*
x+e)*(a+b-a*sin(f*x+e)^2)/a^2/(a+b)/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^
(1/2)+1/15*(8*a^3-9*a^2*b+16*a*b^2+48*b^3)*EllipticE(sin(f*x+e),(a/(a+b))^
(1/2))*(a+b-a*sin(f*x+e)^2)/a^4/(a+b)/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2
*(a+b-a*sin(f*x+e)^2))^(1/2)/(1-a*sin(f*x+e)^2/(a+b))^(1/2)-4/15*b*(a^2-2*
a*b+12*b^2)*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*(1-a*sin(f*x+e)^2/(a+b))
^(1/2)/a^4/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2
)
```

3.275.2 Mathematica [F]

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

input
```
Integrate[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
Integrate[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2), x]
```

3.275.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 422, normalized size of antiderivative = 0.97, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 4636, 2057, 2058, 315, 25, 403, 25, 403, 25, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sec(e+fx)^5\,(a+b\sec(e+fx)^2)^{3/2}}dx$$

$$\downarrow 4636$$

$$\frac{\int \frac{\left(1-\sin^2(e+fx)\right)^2}{\left(a+\frac{b}{1-\sin^2(e+fx)}\right)^{3/2}}d\sin(e+fx)}{f}$$

$$\downarrow 2057$$

$$\frac{\int \frac{\left(1-\sin^2(e+fx)\right)^2}{\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^{3/2}}d\sin(e+fx)}{f}$$

$$\downarrow 2058$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\int \frac{\left(1-\sin^2(e+fx)\right)^{7/2}}{\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}d\sin(e+fx)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 315$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(-\frac{\int -\frac{\left(1-\sin^2(e+fx)\right)^{3/2}\left(-\left((a+6b)\sin^2(e+fx)\right)+a+b\right)}{\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{a(a+b)}-\frac{b\sin(e+fx)\left(1-\sin^2(e+fx)\right)^{5/2}}{a(a+b)\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 25$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\int \frac{\left(1-\sin^2(e+fx)\right)^{3/2}\left(-\left((a+6b)\sin^2(e+fx)\right)+a+b\right)}{\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{a(a+b)}-\frac{b\sin(e+fx)\left(1-\sin^2(e+fx)\right)^{5/2}}{a(a+b)\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 403$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{(a+6b)\sin(e+fx)\left(1-\sin^2(e+fx)\right)^{3/2}\sqrt{-a\sin^2(e+fx)+a+b}}{5a}-\frac{\int -\frac{\sqrt{1-\sin^2(e+fx)}\left(2(2a-3b)(a+b)-\left(4a^2-5ba-24b^2\right)\sin\right.}{\sqrt{-a\sin^2(e+fx)+a+b}}}{5a}}{a(a+b)}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 25$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\int \frac{\sqrt{1-\sin^2(e+fx)}\left(2(2a-3b)(a+b)-\left(4a^2-5ba-24b^2\right)\sin^2(e+fx)\right)}{\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{5a}+\frac{(a+6b)\sin(e+fx)\left(1-\sin^2(e+fx)\right)^{3/2}\sqrt{}}{5a}}{a(a+b)}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 403$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\left(4a^2-5ab-24b^2\right)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}{3a}-\frac{\int -\frac{(a+b)\left(8a^2-13ba+24b^2\right)-\left(8a^3-9ba^2+16b^2a+\right.}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+}}}{3a}}{5a}}{a(a+b)}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2}{1-si}}}$$

$$\downarrow 25$$

3.275. $\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\int \frac{(a+b)\left(8a^2-13ba+24b^2\right)-\left(8a^3-9ba^2+16b^2a+48b^3\right)\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{3a}+\frac{\left(4a^2-5ab-24b^2\right)\sqrt{1-\sin^2(e+fx)}\sin(e}{3a}}{5a}+\frac{}{a(a+b)}\right)$$

$$f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(}{1-\sin}}$$

$$\downarrow 399$$

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{\left(8a^3-9a^2b+16ab^2+48b^3\right)\int \frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{4b(a+b)\left(a^2-2ab+12b^2\right)\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(}}}{a}}{3a}}{5a}+\frac{}{}\right)$$

$$f\sqrt{1-}$$

$$\downarrow 323$$

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{\left(8a^3-9a^2b+16ab^2+48b^3\right)\int \frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{4b(a+b)\left(a^2-2ab+12b^2\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\int \frac{}{\sqrt{1-\sin^2}}}{a\sqrt{-a\sin^2(e+fx)+}}}{3a}}{5a}+\frac{}{}\right)$$

$$f$$

$$\downarrow 321$$

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{\left(8a^3-9a^2b+16ab^2+48b^3\right)\int \frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{4b(a+b)\left(a^2-2ab+12b^2\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(a\right.}{a\sqrt{-a\sin^2(e+fx)+a+b}}}{3a}}{5a}+\frac{}{}\right)$$

$$f\sqrt{1-}$$

$$\downarrow 330$$

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\left(8a^3-9a^2b+16ab^2+48b^3\right)\sqrt{-a\sin^2(e+fx)+a+b}\int\frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{4b(a+b)\left(a^2-2ab+12b^2\right)\sqrt{1-\frac{a\sin^2}{a}}}{a\sqrt{-a\sin}}\right)$$

$$\frac{}{3a}$$

$$\frac{}{5a}$$

$$\downarrow 327$$

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\left(4a^2-5ab-24b^2\right)\sqrt{1-\sin^2(e+fx)}\sin(e+fx)\sqrt{-a\sin^2(e+fx)+a+b}}{3a}+\frac{\left(8a^3-9a^2b+16ab^2+48b^3\right)\sqrt{-a\sin^2(e+fx)+a+bE}}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}\right)$$

$$\frac{}{5a}$$

$$f$$

input
```
Int[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[a + b - a*Sin[e + f*x]^2]*(-((b*Sin[e + f*x]*(1 - Sin[e + f*x]^2)^(5
/2))/(a*(a + b)*Sqrt[a + b - a*Sin[e + f*x]^2])) + (((a + 6*b)*Sin[e + f*x
]*(1 - Sin[e + f*x]^2)^(3/2)*Sqrt[a + b - a*Sin[e + f*x]^2])/(5*a) + (((4*
a^2 - 5*a*b - 24*b^2)*Sin[e + f*x]*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[a + b - a
*Sin[e + f*x]^2])/(3*a) + (((8*a^3 - 9*a^2*b + 16*a*b^2 + 48*b^3)*Elliptic
E[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b - a*Sin[e + f*x]^2])/(a*Sqrt
[1 - (a*Sin[e + f*x]^2)/(a + b)]) - (4*b*(a + b)*(a^2 - 2*a*b + 12*b^2)*El
lipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a +
b)])/(a*Sqrt[a + b - a*Sin[e + f*x]^2]))/(3*a))/(5*a))/(a*(a + b))))/(f*Sq
rt[1 - Sin[e + f*x]^2]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(1 - Sin[e + f*x]^2
)])
```

3.275.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

3.275.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 14.01 (sec) , antiderivative size = 18649, normalized size of antiderivative = 42.77

method	result	size
default	Expression too large to display	18649

input
```
int(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.275.5 Fricas [F]

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos\left(fx+e\right)^5}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^5/(b^2*sec(f*x + e)^4 + 2
*a*b*sec(f*x + e)^2 + a^2), x)
```

3.275.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**5/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

3.275.7 Maxima [F]

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos\left(fx+e\right)^5}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(f*x + e)^5/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.275.8 Giac [F]

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos(fx+e)^5}{\left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.275.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos(e+fx)^5}{\left(a+\frac{b}{\cos(e+fx)^2}\right)^{3/2}}\,dx$$

input
```
int(cos(e + f*x)^5/(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(cos(e + f*x)^5/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.276 $\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

3.276.1 Optimal result . . . 2064
3.276.2 Mathematica [C] (warning: unable to verify) . . . 2064
3.276.3 Rubi [A] (verified) . . . 2065
3.276.4 Maple [B] (verified) . . . 2067
3.276.5 Fricas [A] (verification not implemented) . . . 2068
3.276.6 Sympy [F] . . . 2069
3.276.7 Maxima [A] (verification not implemented) . . . 2069
3.276.8 Giac [F] . . . 2070
3.276.9 Mupad [F(-1)] . . . 2070

3.276.1 Optimal result

Integrand size = 25, antiderivative size = 138

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{(3a-b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2b^{5/2}f} - \frac{a\sec^2(e+fx)\tan(e+fx)}{b(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(3a+b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2b^2(a+b)f}$$

output

```
-1/2*(3*a-b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b^(5/2
)/f-a*sec(f*x+e)^2*tan(f*x+e)/b/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/2*(3*
a+b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b^2/(a+b)/f
```

3.276.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 3.39 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.88

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))\sec^8(e+fx)\left(-7(a+b)\cos^2(e+fx)\left(15b^2+10ab(2+\cos(2(e+fx)))+a^2(\right.\right.}{}$$

input
```
Integrate[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
-1/210*((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^8*(-7*(a + b)*Cos[e +
f*x]^2*(15*b^2 + 10*a*b*(2 + Cos[2*(e + f*x)]) + a^2*(8 + 6*Cos[2*(e + f*x
)] + Cos[4*(e + f*x)]))*Hypergeometric2F1[1, 2, 7/2, -((b*Tan[e + f*x]^2)/
(a + b))] + 4*b*(a + 2*b + a*Cos[2*(e + f*x)])^2*HypergeometricPFQ[{2, 2,
3}, {1, 9/2}, -((b*Tan[e + f*x]^2)/(a + b))]*Sin[e + f*x]^2 + 16*b*Hyperge
ometric2F1[2, 3, 9/2, -((b*Tan[e + f*x]^2)/(a + b))]*Sin[e + f*x]^2*(4*(a
+ b)^2 - 7*a*(a + b)*Sin[e + f*x]^2 + 3*a^2*Sin[e + f*x]^4))*Tan[e + f*x])
/((a + b)^4*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

3.276.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 4634, 315, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sec(e+fx)^6}{(a+b\sec(e+fx)^2)^{3/2}}\,dx$$

$$\downarrow 4634$$

$$\frac{\int \frac{(\tan^2(e+fx)+1)^2}{(b\tan^2(e+fx)+a+b)^{3/2}}\,d\tan(e+fx)}{f}$$

$$\downarrow 315$$

$$\frac{\frac{\int \frac{(3a+b)\tan^2(e+fx)+a+b}{\sqrt{b\tan^2(e+fx)+a+b}}\,d\tan(e+fx)}{b(a+b)} - \frac{a\tan(e+fx)(\tan^2(e+fx)+1)}{b(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\downarrow 299$$

$$\frac{\frac{(3a+b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b} - \frac{(3a-b)(a+b)\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}\,d\tan(e+fx)}{2b}}{b(a+b)} - \frac{a\tan(e+fx)(\tan^2(e+fx)+1)}{b(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\downarrow 224$$

$$\frac{\frac{(3a+b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{(3a-b)(a+b)\int\frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{2b}}{b(a+b)}-\frac{a\tan(e+fx)\left(\tan^2(e+fx)+1\right)}{b(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\downarrow 219$$

$$\frac{\frac{(3a+b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{(3a-b)(a+b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2b^{3/2}}}{b(a+b)}-\frac{a\tan(e+fx)\left(\tan^2(e+fx)+1\right)}{b(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

input
```
Int[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(-((a*Tan[e + f*x]*(1 + Tan[e + f*x]^2))/(b*(a + b)*Sqrt[a + b + b*Tan[e +
 f*x]^2])) + (-1/2*((3*a - b)*(a + b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[
a + b + b*Tan[e + f*x]^2]])/b^(3/2) + ((3*a + b)*Tan[e + f*x]*Sqrt[a + b +
 b*Tan[e + f*x]^2])/(2*b))/(b*(a + b)))/f
```

3.276.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.276.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3209 vs. $2(124) = 248$.

Time = 12.25 (sec) , antiderivative size = 3210, normalized size of antiderivative = 23.26

method	result	size
default	Expression too large to display	3210

input
```
int(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
-1/4/f/(a+b)/b^(7/2)/(a+b*sec(f*x+e)^2)^(3/2)*(-2*b^(7/2)*a*tan(f*x+e)*sec
(f*x+e)^4+3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*a^2*b^2*se
c(f*x+e)^2+2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*a*b^3*sec
(f*x+e)^2+3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*a^2*b^2*sec
(f*x+e)^2+2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*a*b^3*sec(f
*x+e)^2+3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*a^2*b^2*sec(
f*x+e)^3+2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*a*b^3*sec(f
*x+e)^3+3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(((b+a*cos(f...
```

3.276.5 Fricas [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 524, normalized size of antiderivative = 3.80

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \left[-\frac{\left((3\,a^3+2\,a^2b-ab^2)\cos(fx+e)^3+(3\,a^2b+2\,ab^2-b^3)\cos(fx+e)\right)\sqrt{-b}\arctan\left(-\frac{\left((a-b)\cos(fx+e)^3+2\,b\cos(fx+e)\right)\sqrt{-b}\sqrt{\frac{a\cos(fx+e)+b}{\cos(fx+e)}}}{2\left(ab\cos(fx+e)^2+b^2\right)\sin(fx+e)}\right)\sqrt{\frac{a\cos(fx+e)+b}{\cos(fx+e)}}\sqrt{\frac{\cos(fx+e)}{a+b}}}{4\left((a^2b^3+ab^4)f\cos(fx+e)^3+\right.}\right.$$

input

```
integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(((3*a^3 + 2*a^2*b - a*b^2)*cos(f*x + e)^3 + (3*a^2*b + 2*a*b^2 - b^
3)*cos(f*x + e))*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b
- b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt
(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(
f*x + e)^4) - 4*(a*b^2 + b^3 + (3*a^2*b + a*b^2)*cos(f*x + e)^2)*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^2*b^3 + a*b^4)*f*cos(
f*x + e)^3 + (a*b^4 + b^5)*f*cos(f*x + e)), -1/4*(((3*a^3 + 2*a^2*b - a*b^
2)*cos(f*x + e)^3 + (3*a^2*b + 2*a*b^2 - b^3)*cos(f*x + e))*sqrt(-b)*arcta
n(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*
x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e))) -
 2*(a*b^2 + b^3 + (3*a^2*b + a*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^2*b^3 + a*b^4)*f*cos(f*x + e)^3 +
(a*b^4 + b^5)*f*cos(f*x + e))]
```

3.276.6 Sympy [F]

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)**6/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(sec(e + f*x)**6/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.276.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.17

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\frac{\tan(fx+e)^3}{\sqrt{b\tan(fx+e)^2+a+bb}} - \frac{3\,(a+b)\,\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{5}{2}}} + \frac{4\,\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}} + \frac{2\tan(f}{\sqrt{b\tan(fx+e)^2}}}{2\,f}$$

input
```
integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/2*(tan(f*x + e)^3/(sqrt(b*tan(f*x + e)^2 + a + b)*b) - 3*(a + b)*arcsinh
(b*tan(f*x + e)/sqrt((a + b)*b))/b^(5/2) + 4*arcsinh(b*tan(f*x + e)/sqrt((
a + b)*b))/b^(3/2) + 2*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b
)) + 3*(a + b)*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*b^2) - 4*tan(f
*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*b))/f
```

3.276.8 Giac [F]

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sec(fx+e)^6}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.276.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{1}{\cos(e+fx)^6\left(a+\frac{b}{\cos(e+fx)^2}\right)^{3/2}}\,dx$$

input
```
int(1/(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^(3/2)),x)
```

output
```
int(1/(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^(3/2)), x)
```

3.277 $\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

3.277.1 Optimal result 2071
3.277.2 Mathematica [C] (warning: unable to verify) 2071
3.277.3 Rubi [A] (verified) 2072
3.277.4 Maple [B] (warning: unable to verify) 2074
3.277.5 Fricas [B] (verification not implemented) 2075
3.277.6 Sympy [F] 2075
3.277.7 Maxima [A] (verification not implemented) 2076
3.277.8 Giac [F] 2076
3.277.9 Mupad [F(-1)] 2076

3.277.1 Optimal result

Integrand size = 25, antiderivative size = 77

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{b^{3/2}f} - \frac{a\tan(e+fx)}{b(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

output
```
arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b^(3/2)/f-a*tan(f*x
+e)/b/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

3.277.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 7.31 (sec) , antiderivative size = 405, normalized size of antiderivative = 5.26

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx =$$
$$-\frac{(a+2b+a\cos(2e+2fx))^{3/2}\sec^4(e+fx)\sqrt{1-\frac{2a\sin^2(e+fx)}{2a+2b}}\tan(e+fx)\left(15\arcsin\left(\sqrt{-\frac{b\tan^2(e+fx)}{a+b}}\right)\sec\right.}{}$$

input
```
Integrate[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output

```
-1/15*((a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sec[e + f*x]^4*Sqrt[1 - (2*a*S
in[e + f*x]^2)/(2*a + 2*b)]*Tan[e + f*x]*(15*ArcSin[Sqrt[-((b*Tan[e + f*x]
^2)/(a + b))]]*Sec[e + f*x]^2*(3*b^2 + a*b*(6 - 5*Sin[e + f*x]^2) + a^2*(3
 - 5*Sin[e + f*x]^2 + 2*Sin[e + f*x]^4)) + 15*(a + b)*(-3*b + a*(-3 + 2*Si
n[e + f*x]^2))*Sqrt[-((b*Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)*Tan[e +
 f*x]^2)/(a + b)^2)] + 4*b*(a + b)*Hypergeometric2F1[2, 2, 7/2, -((b*Tan[e
 + f*x]^2)/(a + b))]*Sin[e + f*x]^2*(-((b*Sec[e + f*x]^2*(a + b - a*Sin[e
+ f*x]^2)*Tan[e + f*x]^2)/(a + b)^2))^(3/2)))/((a + b)^2*(2*a + 2*b)*f*(a
+ b*Sec[e + f*x]^2)^(3/2)*Sqrt[(a + b*Sec[e + f*x]^2)/(a + b)]*Sqrt[2*a +
2*b - 2*a*Sin[e + f*x]^2]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]*(-((b*Tan[e
 + f*x]^2)/(a + b)))^(3/2))
```

3.277.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4634, 298, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sec(e+fx)^4}{(a+b\sec(e+fx)^2)^{3/2}}\,dx$$

$$\downarrow 4634$$

$$\frac{\int \frac{\tan^2(e+fx)+1}{(b\tan^2(e+fx)+a+b)^{3/2}}\,d\tan(e+fx)}{f}$$

$$\downarrow 298$$

$$\frac{\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}\,d\tan(e+fx)}{b} - \frac{a\tan(e+fx)}{b(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\downarrow 224$$

$$\frac{\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}\,d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{b} - \frac{a\tan(e+fx)}{b(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\downarrow 219$$

$$\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{b^{3/2}} - \frac{a\tan(e+fx)}{b(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

input
```
Int[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/b^(3/2) -
(a*Tan[e + f*x])/(b*(a + b)*Sqrt[a + b + b*Tan[e + f*x]^2]))/f
```

3.277.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.277.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1512 vs. 2(69) = 138.

Time = 8.13 (sec) , antiderivative size = 1513, normalized size of antiderivative = 19.65

method	result	size
default	Expression too large to display	1513

input
```
int(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/f/(a+b)/b^(5/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*c
sc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x
+e)^2+a+b)*(-4*b^(3/2)*a*(csc(f*x+e)-cot(f*x+e))+ln(4*(a*(1-cos(f*x+e))^2*
csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+b^(1/2)*(a*(1-cos(f*x+e))^4*c
sc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e
)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(f*x+e
))+a+b)/((1-cos(f*x+e))^2*csc(f*x+e)^2-2*csc(f*x+e)+2*cot(f*x+e)+1))*(a*(1
-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x
+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*b*a+ln(4*
(a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+b^(1/2)*(
a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos
(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)-2*a*(
csc(f*x+e)-cot(f*x+e))+a+b)/((1-cos(f*x+e))^2*csc(f*x+e)^2-2*csc(f*x+e)+2*
cot(f*x+e)+1))*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x
+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+
a+b)^(1/2)*b^2+ln(4*(-a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*c
sc(f*x+e)^2+b^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*cs
c(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+
e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(f*x+e))-a-b)/((1-cos(f*x+e))^2*csc(f*x
+e)^2+2*csc(f*x+e)-2*cot(f*x+e)+1))*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*...
```

3.277.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(69) = 138$.

Time = 0.33 (sec) , antiderivative size = 410, normalized size of antiderivative = 5.32

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \left[-\frac{4\,ab\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)\sin(fx+e) - \left((a^2+ab)\cos(fx+e)^2 + \right.}{4\,(} \right.$$

$$\left. -\frac{2\,ab\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)\sin(fx+e) - \left((a^2+ab)\cos(fx+e)^2+ab+b^2\right)\sqrt{-b}\arctan\left(-\frac{\left((a-b)\cos\right.}{2\left((a^2b^2+ab^3)f\cos(fx+e)^2+(ab^3+b^4)f\right)} \right.$$

input
```
integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*(4*a*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(
f*x + e) - ((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)*sqrt(b)*log(((a^2 - 6*
a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(
f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4))/((a^2*b^2 + a*b^3)*f*cos(f
*x + e)^2 + (a*b^3 + b^4)*f), -1/2*(2*a*b*sqrt((a*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)*cos(f*x + e)*sin(f*x + e) - ((a^2 + a*b)*cos(f*x + e)^2 + a*b
+ b^2)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sq
rt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 +
b^2)*sin(f*x + e))))/((a^2*b^2 + a*b^3)*f*cos(f*x + e)^2 + (a*b^3 + b^4)*f
)]
```

3.277.6 Sympy [F]

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)**4/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(sec(e + f*x)**4/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.277.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\frac{\operatorname{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}} + \frac{\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+b}(a+b)} - \frac{\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+bb}}}{f}$$

input
```
integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
(arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/b^(3/2) + tan(f*x + e)/(sqrt(b*ta
n(f*x + e)^2 + a + b)*(a + b)) - tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a +
 b)*b))/f
```

3.277.8 Giac [F]

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sec(fx+e)^4}{\left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.277.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{1}{\cos(e+fx)^4\left(a+\frac{b}{\cos(e+fx)^2}\right)^{3/2}}\,dx$$

input
```
int(1/(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^(3/2)),x)
```

output
```
int(1/(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^(3/2)), x)
```

3.278 $\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

3.278.1 Optimal result 2077
3.278.2 Mathematica [A] (verified) 2077
3.278.3 Rubi [A] (verified) 2078
3.278.4 Maple [A] (verified) 2079
3.278.5 Fricas [B] (verification not implemented) 2079
3.278.6 Sympy [F] 2080
3.278.7 Maxima [A] (verification not implemented) 2080
3.278.8 Giac [F] 2080
3.278.9 Mupad [B] (verification not implemented) 2081

3.278.1 Optimal result

Integrand size = 25, antiderivative size = 32

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\tan(e+fx)}{(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

output
```
tan(f*x+e)/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

3.278.2 Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.78

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\tan(e+fx)}{2(a+b)f\,(a+b\sec^2(e+fx))^{3/2}}$$

input
```
Integrate[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*Tan[e + f*x])/(2*(a + b)*f*
(a + b*Sec[e + f*x]^2)^(3/2))
```

3.278.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3042, 4634, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sec(e+fx)^2}{(a+b\sec(e+fx)^2)^{3/2}}\,dx$$

$$\downarrow 4634$$

$$\frac{\int \frac{1}{(b\tan^2(e+fx)+a+b)^{3/2}}\,d\tan(e+fx)}{f}$$

$$\downarrow 208$$

$$\frac{\tan(e+fx)}{f(a+b)\sqrt{a+b\tan^2(e+fx)+b}}$$

input
```
Int[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
Tan[e + f*x]/((a + b)*f*Sqrt[a + b + b*Tan[e + f*x]^2])
```

3.278.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.278.4 Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.53

method	result	size
default	$\frac{a\tan(fx+e)+b\tan(fx+e)\sec(fx+e)^2}{f(a+b)\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}}$	49

input
```
int(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f/(a+b)/(a+b*sec(f*x+e)^2)^(3/2)*(a*tan(f*x+e)+b*tan(f*x+e)*sec(f*x+e)^2
)
```

3.278.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(30) = 60$.

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.03

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)\sin(fx+e)}{(a^2+ab)f\cos(fx+e)^2+(ab+b^2)f}$$

input
```
integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x + e)/((a^
2 + a*b)*f*cos(f*x + e)^2 + (a*b + b^2)*f)
```

3.278.6 Sympy [F]

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)**2/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(sec(e + f*x)**2/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.278.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+b(a+b)f}}$$

input
```
integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)*f)
```

3.278.8 Giac [F]

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\sec(fx+e)^2}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.278.9 Mupad [B] (verification not implemented)

Time = 20.52 (sec) , antiderivative size = 199, normalized size of antiderivative = 6.22

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\sqrt{\frac{a+2b+a\cos(2e+2fx)}{\cos(2e+2fx)+1}}\,(5a\sin(2e+2fx)+4a\sin(4$$

input
```
int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^(3/2)),x)
```

output
```
(((a + 2*b + a*cos(2*e + 2*f*x))/(cos(2*e + 2*f*x) + 1))^(1/2)*(5*a*sin(2*
e + 2*f*x) + 4*a*sin(4*e + 4*f*x) + a*sin(6*e + 6*f*x) + 8*b*sin(2*e + 2*f
*x) + 4*b*sin(4*e + 4*f*x)))/(f*(a + b)*(24*a*b + 10*a^2 + 16*b^2 + 15*a^2
*cos(2*e + 2*f*x) + 6*a^2*cos(4*e + 4*f*x) + a^2*cos(6*e + 6*f*x) + 16*b^2
*cos(2*e + 2*f*x) + 32*a*b*cos(2*e + 2*f*x) + 8*a*b*cos(4*e + 4*f*x)))
```

3.279 $\int \frac{1}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

3.279.1 Optimal result . . . 2082
3.279.2 Mathematica [B] (verified) . . . 2082
3.279.3 Rubi [A] (verified) . . . 2083
3.279.4 Maple [B] (verified) . . . 2085
3.279.5 Fricas [B] (verification not implemented) . . . 2085
3.279.6 Sympy [F] . . . 2087
3.279.7 Maxima [B] (verification not implemented) . . . 2087
3.279.8 Giac [F] . . . 2088
3.279.9 Mupad [F(-1)] . . . 2088

3.279.1 Optimal result

Integrand size = 16, antiderivative size = 77

$$\int \frac{1}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f} - \frac{b\tan(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

output
```
arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f-b*tan(f*x+
e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

3.279.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 168 vs. 2(77) = 154.

Time = 1.54 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.18

$$\int \frac{1}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{(a+2b+a\cos(2(e+fx)))\sec^3(e+fx)\left(\sqrt{a+b}\arcsin\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)(a+\right.}{4a^{3/2}(a+b)f\,(a+b\sec^2(e+fx}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(-3/2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(Sqrt[a + b]*ArcSin[(Sqrt[a
]*Sin[e + f*x])/Sqrt[a + b]]*(a + 2*b + a*Cos[2*(e + f*x)]) - Sqrt[2]*Sqrt
[a]*b*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*Sin[e + f*x]))/(4*a^(3/
2)*(a + b)*f*(a + b*Sec[e + f*x]^2)^(3/2)*Sqrt[(a + b - a*Sin[e + f*x]^2)/
(a + b)])
```

3.279.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3042, 4616, 296, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{(a+b\sec(e+fx)^2)^{3/2}}\,dx$$

$$\downarrow 4616$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^{3/2}}\,d\tan(e+fx)}{f}$$

$$\downarrow 296$$

$$\frac{\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}\,d\tan(e+fx)}{a} - \frac{b\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\downarrow 291$$

$$\frac{\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}\,d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{a} - \frac{b\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\downarrow 216$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}} - \frac{b\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^(-3/2),x]
```

output
```
(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/a^(3/2) - (
b*Tan[e + f*x])/(a*(a + b)*Sqrt[a + b + b*Tan[e + f*x]^2]))/f
```

3.279.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

3.279.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 514 vs. $2(69) = 138$.

Time = 2.54 (sec) , antiderivative size = 515, normalized size of antiderivative = 6.69

method	result
default	$-\frac{\left(b+a\cos(fx+e)^2\right)\left(-\cos(fx+e)\ln\left(4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)+4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a\right)\sqrt{\frac{b+a\cos(fx-}{(1+\cos(fx+}$

input
```
int(1/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f/(a+b)/a/(-a)^(1/2)*(b+a*cos(f*x+e)^2)*(-cos(f*x+e)*ln(4*(-a)^(1/2)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*a-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(
-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1
/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*b*cos(f*x+
e)+(-a)^(1/2)*b*sin(f*x+e)-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(
4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)
^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a-((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*b)/(a+b*sec(f*x+e)^2)^(3/2)*sec(f*x+e)^
3
```

3.279.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(69) = 138$.

Time = 0.45 (sec) , antiderivative size = 601, normalized size of antiderivative = 7.81

$$\int \frac{1}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \left[-\frac{8\,ab\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)\sin(fx+e) + \left((a^2+ab)\cos(fx+e)^2 + \right.\right.$$

$$\left. -\frac{4\,ab\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)\sin(fx+e) + \left((a^2+ab)\cos(fx+e)^2 + ab + b^2\right)\sqrt{a}\arctan\left(\frac{\left(8\,a^2\cos(fx+e)\right.}{4\left(2\right.}\right.}{4\left((a^4+a^3b)f\cos(fx+e)^2 + (a^3b+a^2b^2)f\right)}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(8*a*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(
f*x + e) + ((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)*sqrt(-a)*log(128*a^4*c
os(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b +
5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 -
 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x
 + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)
*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)))/((a^4 + a^3*b)*f
*cos(f*x + e)^2 + (a^3*b + a^2*b^2)*f), -1/4*(4*a*b*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x + e) + ((a^2 + a*b)*cos(f*x + e
)^2 + a*b + b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*
cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3
 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))))/((a^4 + a^3*b)*f*cos(f*x + e)^
2 + (a^3*b + a^2*b^2)*f)]
```

3.279.6 Sympy [F]

$$\int \frac{1}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{1}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(-3/2), x)
```

3.279.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2055 vs. 2(69) = 138.

Time = 0.54 (sec) , antiderivative size = 2055, normalized size of antiderivative = 26.69

$$\int \frac{1}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/2*(2*a*b*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2
*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))*sin(2*f*x + 2
*e) - 2*(a^2 + a*b)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2
*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^3 - 2
*(a*b*cos(2*f*x + 2*e) + (a^2 + a*b)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) +
2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x +
 2*e) + a))^2 - a^2 - 2*a*b)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2
*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) +
a)) - (a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b +
4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2
*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 +
 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2
*e))^(1/4)*(((a + b)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(
2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 +
(a + b)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e),
 a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2)*arctan2(2*a*si
n(2*f*x + 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a
^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*
sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a
^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*...
```

3.279.8 Giac [F]

$$\int \frac{1}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{1}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(-3/2), x)
```

3.279.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(e+fx)^2}\right)^{3/2}}\,dx$$

input
```
int(1/(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(1/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.280 $\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

3.280.1 Optimal result . . . 2089
3.280.2 Mathematica [C] (warning: unable to verify) . . . 2089
3.280.3 Rubi [A] (verified) . . . 2090
3.280.4 Maple [B] (verified) . . . 2093
3.280.5 Fricas [B] (verification not implemented) . . . 2094
3.280.6 Sympy [F] . . . 2095
3.280.7 Maxima [F] . . . 2095
3.280.8 Giac [F] . . . 2095
3.280.9 Mupad [F(-1)] . . . 2096

3.280.1 Optimal result

Integrand size = 25, antiderivative size = 131

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{(a-3b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2a^{5/2}f} + \frac{\cos(e+fx)\sin(e+fx)}{2af\sqrt{a+b+b\tan^2(e+fx)}} + \frac{b(a+3b)\tan(e+fx)}{2a^2(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

output

```
1/2*(a-3*b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/
f+1/2*cos(f*x+e)*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/2*b*(a+3*b)*t
an(f*x+e)/a^2/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

3.280.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 15.39 (sec) , antiderivative size = 2059, normalized size of antiderivative = 15.72

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Result too large to show}$$

input

```
Integrate[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output

```
(3*(a + b)*AppellF1[1/2, -2, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/
(a + b)]*Cos[e + f*x]^6*Sin[e + f*x])/(2*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x
)]]*(a + b*Sec[e + f*x]^2)^(3/2)*(a + b - a*Sin[e + f*x]^2)*(3*(a + b)*App
ellF1[1/2, -2, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3*
a*AppellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]
 - 4*(a + b)*AppellF1[3/2, -1, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)
/(a + b)])*Sin[e + f*x]^2)*((3*a*(a + b)*AppellF1[1/2, -2, 3/2, 3/2, Sin[e
 + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^5*Sin[e + f*x]^2)/(Sqr
t[a + 2*b + a*Cos[2*(e + f*x)]]*(a + b - a*Sin[e + f*x]^2)^2*(3*(a + b)*Ap
pellF1[1/2, -2, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3
*a*AppellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]
 - 4*(a + b)*AppellF1[3/2, -1, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2
)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*AppellF1[1/2, -2, 3/2, 3/2, Sin[
e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^5)/(2*Sqrt[a + 2*b +
a*Cos[2*(e + f*x)]]*(a + b - a*Sin[e + f*x]^2)*(3*(a + b)*AppellF1[1/2, -2
, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*AppellF1[3/
2, -2, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*A
ppellF1[3/2, -1, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Si
n[e + f*x]^2)) - (6*(a + b)*AppellF1[1/2, -2, 3/2, 3/2, Sin[e + f*x]^2, (a
*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^3*Sin[e + f*x]^2)/(Sqrt[a + 2*b ...
```

3.280.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4634, 316, 25, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sec(e+fx)^2\,(a+b\sec(e+fx)^2)^{3/2}}dx$$

$$\downarrow 4634$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)^2(b\tan^2(e+fx)+a+b)^{3/2}}d\tan(e+fx)}{f}$$

$$\downarrow 316$$

$$\frac{\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\int -\frac{2b\tan^2(e+fx)+a-b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{2a}}{f}$$

$$\downarrow 25$$

$$\frac{\frac{\int \frac{2b\tan^2(e+fx)+a-b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{2a}+\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\downarrow 402$$

$$\frac{\frac{\int \frac{(a-3b)(a+b)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a(a+b)}+\frac{b(a+3b)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{2a}+\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\downarrow 27$$

$$\frac{\frac{(a-3b)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a}+\frac{b(a+3b)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{2a}+\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\downarrow 291$$

$$\frac{\frac{(a-3b)\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{a}+\frac{b(a+3b)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{2a}+\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\downarrow 216$$

$$\frac{\frac{(a-3b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}}+\frac{b(a+3b)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{2a}+\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

input
```
Int[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Tan[e + f*x]/(2*a*(1 + Tan[e + f*x]^2)*Sqrt[a + b + b*Tan[e + f*x]^2]) +
(((a - 3*b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])
/a^(3/2) + (b*(a + 3*b)*Tan[e + f*x])/(a*(a + b)*Sqrt[a + b + b*Tan[e + f*
x]^2]))/(2*a))/f
```

3.280. $\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

3.280.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.280.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 808 vs. $2(115) = 230$.

Time = 4.13 (sec) , antiderivative size = 809, normalized size of antiderivative = 6.18

method	result
default	$\frac{\left(b+a\cos(fx+e)^2\right)\left(\cos(fx+e)^2\sin(fx+e)\sqrt{-a}\,a^2+\sqrt{-a}\,ab\cos(fx+e)^2\sin(fx+e)+\cos(fx+e)\ln\left(4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx\right.\right.$

input
```
int(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f/(a+b)/a^2/(-a)^(1/2)*(b+a*cos(f*x+e)^2)*(cos(f*x+e)^2*sin(f*x+e)*(-a
)^(1/2)*a^2+(-a)^(1/2)*a*b*cos(f*x+e)^2*sin(f*x+e)+cos(f*x+e)*ln(4*(-a)^(1
/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*a^2-2*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
a*b*cos(f*x+e)-3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*s
in(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2*cos(f*x+e)+(-
a)^(1/2)*a*b*sin(f*x+e)+3*(-a)^(1/2)*b^2*sin(f*x+e)+((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)-4*sin(f*x+e)*a)*a^2-2*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b-3*ln(4
*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^
(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2)/(a+b*sec(f*x+e)^2)^(3/2)*sec(f*x+
e)^3
```

3.280.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(115) = 230$.

Time $= 0.63$ (sec) , antiderivative size $= 699$, normalized size of antiderivative $= 5.34$

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \left[\frac{\left(a^2b-2\,ab^2-3\,b^3+\left(a^3-2\,a^2b-3\,ab^2\right)\cos\left(fx+e\right)^2\right)\sqrt{-a}\log\left(128\,a^4\,\mathrm{co}\right.}{} \right.$$

$$\left. -\frac{\left(a^2b-2\,ab^2-3\,b^3+\left(a^3-2\,a^2b-3\,ab^2\right)\cos\left(fx+e\right)^2\right)\sqrt{a}\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^5-8\left(a^2-ab\right)\cos(fx+e)^3+(a^2-6\,a\right.}{4\left(2\,a^3\cos(fx+e)^4-a^2b+ab^2-(a^3-3\,a\right.}\right)}{8\left((a^5+a^4b)f\,\mathrm{co}\right.} \right]$$

input
```
integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/16*((a^2*b - 2*a*b^2 - 3*b^3 + (a^3 - 2*a^2*b - 3*a*b^2)*cos(f*x + e)^2
)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 +
 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^
2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x +
e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a
^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*
cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x
 + e)) + 8*((a^3 + a^2*b)*cos(f*x + e)^3 + (a^2*b + 3*a*b^2)*cos(f*x + e))
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^5 + a^4*b)*
f*cos(f*x + e)^2 + (a^4*b + a^3*b^2)*f), -1/8*((a^2*b - 2*a*b^2 - 3*b^3 +
(a^3 - 2*a^2*b - 3*a*b^2)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*
x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e)
)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)
^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*((
a^3 + a^2*b)*cos(f*x + e)^3 + (a^2*b + 3*a*b^2)*cos(f*x + e))*sqrt((a*cos(
f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^5 + a^4*b)*f*cos(f*x + e
)^2 + (a^4*b + a^3*b^2)*f)]
```

3.280.6 Sympy [F]

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)**2/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(cos(e + f*x)**2/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.280.7 Maxima [F]

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos(fx+e)^2}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.280.8 Giac [F]

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos(fx+e)^2}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.280.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos(e+fx)^2}{\left(a+\frac{b}{\cos(e+fx)^2}\right)^{3/2}}\,dx$$

input
```
int(cos(e + f*x)^2/(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(cos(e + f*x)^2/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.281 $\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

3.281.1 Optimal result 2097
3.281.2 Mathematica [C] (warning: unable to verify) 2097
3.281.3 Rubi [A] (verified) 2098
3.281.4 Maple [B] (verified) 2101
3.281.5 Fricas [A] (verification not implemented) 2102
3.281.6 Sympy [F] 2103
3.281.7 Maxima [F] 2104
3.281.8 Giac [F] 2104
3.281.9 Mupad [F(-1)] 2104

3.281.1 Optimal result

Integrand size = 25, antiderivative size = 194

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{3(a^2-2ab+5b^2)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{7/2}f} + \frac{(3a-5b)\cos(e+fx)\sin(e+fx)}{8a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(a-3b)b(3a+5b)\tan(e+fx)}{8a^3(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

output

```
3/8*(a^2-2*a*b+5*b^2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2)
)/a^(7/2)/f+1/8*(3*a-5*b)*cos(f*x+e)*sin(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)
^(1/2)+1/4*cos(f*x+e)^3*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/8*(a-3
*b)*b*(3*a+5*b)*tan(f*x+e)/a^3/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

3.281.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 16.18 (sec) , antiderivative size = 2046, normalized size of antiderivative = 10.55

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
((a + b)*AppellF1[1/2, -3, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a
 + b)]*Cos[e + f*x]^10*Sin[e + f*x])/(2*f*Sqrt[a + 2*b + a*Cos[2*(e + f*x)
]]*(a + b*Sec[e + f*x]^2)^(3/2)*(a + b - a*Sin[e + f*x]^2)*((a + b)*Appell
F1[1/2, -3, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*App
ellF1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 2*(
a + b)*AppellF1[3/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a +
 b)])*Sin[e + f*x]^2)*((a*(a + b)*AppellF1[1/2, -3, 3/2, 3/2, Sin[e + f*x]
^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^7*Sin[e + f*x]^2)/(Sqrt[a + 2
*b + a*Cos[2*(e + f*x)]]*(a + b - a*Sin[e + f*x]^2)^2*((a + b)*AppellF1[1/
2, -3, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*AppellF1
[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 2*(a + b
)*AppellF1[3/2, -2, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])
*Sin[e + f*x]^2)) + ((a + b)*AppellF1[1/2, -3, 3/2, 3/2, Sin[e + f*x]^2, (
a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^7)/(2*Sqrt[a + 2*b + a*Cos[2*(e +
f*x)]]*(a + b - a*Sin[e + f*x]^2)*((a + b)*AppellF1[1/2, -3, 3/2, 3/2, Sin
[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a*AppellF1[3/2, -3, 5/2, 5/2,
Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 2*(a + b)*AppellF1[3/2, -2,
3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) -
(3*(a + b)*AppellF1[1/2, -3, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/
(a + b)]*Cos[e + f*x]^5*Sin[e + f*x]^2)/(Sqrt[a + 2*b + a*Cos[2*(e + f*...
```

3.281.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4634, 316, 25, 402, 25, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$
$$\downarrow \text{3042}$$
$$\int \frac{1}{\sec(e+fx)^4\left(a+b\sec(e+fx)^2\right)^{3/2}}dx$$
$$\downarrow \text{4634}$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)^3 (b\tan^2(e+fx)+a+b)^{3/2}} d\tan(e+fx)}{f}$$

$$\downarrow 316$$

$$\frac{\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2\sqrt{a+b\tan^2(e+fx)+b}} - \frac{\int -\frac{4b\tan^2(e+fx)+3a-b}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^{3/2}} d\tan(e+fx)}{4a}}{f}$$

$$\downarrow 25$$

$$\frac{\frac{\int \frac{4b\tan^2(e+fx)+3a-b}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^{3/2}} d\tan(e+fx)}{4a} + \frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\downarrow 402$$

$$\frac{\frac{(3a-5b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)+b}} - \frac{\int -\frac{3a^2+5b^2+2(3a-5b)b\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}} d\tan(e+fx)}{2a}}{4a} + \frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\downarrow 25$$

$$\frac{\frac{\int \frac{3a^2+5b^2+2(3a-5b)b\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}} d\tan(e+fx)}{2a} + \frac{(3a-5b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)+b}}}{4a} + \frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\downarrow 402$$

$$\frac{\frac{\int \frac{3(a+b)\left(a^2-2ba+5b^2\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}} d\tan(e+fx)}{a(a+b)} + \frac{b(a-3b)(3a+5b)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{2a} + \frac{(3a-5b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)+b}}}{4a} + \frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2\sqrt{a+}}}{f}$$

$$\downarrow 27$$

$$\frac{\frac{3\left(a^2-2ab+5b^2\right)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}} d\tan(e+fx)}{a} + \frac{b(a-3b)(3a+5b)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{2a} + \frac{(3a-5b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)+b}}}{4a} + \frac{}{4a(\tan^2(e+}}{f}$$

3.281. $\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\, dx$

$$\downarrow 291$$

$$\frac{3\left(a^2-2ab+5b^2\right)\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}\,d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{2a}+\frac{\frac{b(a-3b)(3a+5b)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}+\frac{(3a-5b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)+b}}}{4a}+\frac{\tan(e+fx)}{4a(\tan^2(e+fx)}$$

$$f$$

$$\downarrow 216$$

$$\frac{\frac{3\left(a^2-2ab+5b^2\right)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}}}{2a}+\frac{\frac{b(a-3b)(3a+5b)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}+\frac{(3a-5b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)+b}}}{4a}+\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2\sqrt{a+b\tan}}$$

$$f$$

input
```
Int[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Tan[e + f*x]/(4*a*(1 + Tan[e + f*x]^2)^2*Sqrt[a + b + b*Tan[e + f*x]^2])
+ (((3*a - 5*b)*Tan[e + f*x])/(2*a*(1 + Tan[e + f*x]^2)*Sqrt[a + b + b*Tan
[e + f*x]^2]) + ((3*(a^2 - 2*a*b + 5*b^2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sq
rt[a + b + b*Tan[e + f*x]^2]])/a^(3/2) + ((a - 3*b)*b*(3*a + 5*b)*Tan[e +
f*x])/(a*(a + b)*Sqrt[a + b + b*Tan[e + f*x]^2]))/(2*a))/(4*a))/f
```

3.281.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.281.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1142 vs. $2(174) = 348$.

Time = 6.40 (sec) , antiderivative size = 1143, normalized size of antiderivative = 5.89

method	result	size
default	Expression too large to display	1143

input
```
int(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/f/(a+b)/a^3/(-a)^(1/2)*(b+a*cos(f*x+e)^2)*(2*(-a)^(1/2)*a^3*cos(f*x+e)
^4*sin(f*x+e)+2*(-a)^(1/2)*a^2*b*cos(f*x+e)^4*sin(f*x+e)+3*(-a)^(1/2)*a^3*
cos(f*x+e)^2*sin(f*x+e)-2*(-a)^(1/2)*a^2*b*cos(f*x+e)^2*sin(f*x+e)-5*(-a)^
(1/2)*a*b^2*cos(f*x+e)^2*sin(f*x+e)+3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*a^3*cos(f*x+e)-3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*b*cos(f*x
+e)+9*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+
e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a
)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b^2*cos(f*x+e)+15*ln(4*(-a
)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2
)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^3*cos(f*x+e)+3*(-a)^(1/2)*a^2*b*sin(f*x+
e)-4*(-a)^(1/2)*a*b^2*sin(f*x+e)-15*(-a)^(1/2)*b^3*sin(f*x+e)+3*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^3-3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f...
```

3.281.5 Fricas [A] (verification not implemented)

Time = 1.48 (sec) , antiderivative size = 811, normalized size of antiderivative = 4.18

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \left[-\frac{3\left(a^3b-a^2b^2+3\,ab^3+5\,b^4+\left(a^4-a^3b+3\,a^2b^2+5\,ab^3\right)\cos\left(fx+e\right)^2\right)\sqrt{}}{} \right.$$

$$\left. -\frac{3\left(a^3b-a^2b^2+3\,ab^3+5\,b^4+\left(a^4-a^3b+3\,a^2b^2+5\,ab^3\right)\cos\left(fx+e\right)^2\right)\sqrt{a}\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^5-8\left(a^2-ab\right.\right.\right.}{4\left(2\,a^3\cos(fx+e\right.}\right.$$

input
```
integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/64*(3*(a^3*b - a^2*b^2 + 3*a*b^3 + 5*b^4 + (a^4 - a^3*b + 3*a^2*b^2 +
5*a*b^3)*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 -
a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 +
 a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*
b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*
cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*
a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/
cos(f*x + e)^2)*sin(f*x + e)) - 8*(2*(a^4 + a^3*b)*cos(f*x + e)^5 + (3*a^4
 - 2*a^3*b - 5*a^2*b^2)*cos(f*x + e)^3 + (3*a^3*b - 4*a^2*b^2 - 15*a*b^3)*
cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((
a^6 + a^5*b)*f*cos(f*x + e)^2 + (a^5*b + a^4*b^2)*f), -1/32*(3*(a^3*b - a^
2*b^2 + 3*a*b^3 + 5*b^4 + (a^4 - a^3*b + 3*a^2*b^2 + 5*a*b^3)*cos(f*x + e)
^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^
3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/
cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*c
os(f*x + e)^2)*sin(f*x + e))) - 4*(2*(a^4 + a^3*b)*cos(f*x + e)^5 + (3*a^4
 - 2*a^3*b - 5*a^2*b^2)*cos(f*x + e)^3 + (3*a^3*b - 4*a^2*b^2 - 15*a*b^3)*
cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((
a^6 + a^5*b)*f*cos(f*x + e)^2 + (a^5*b + a^4*b^2)*f)]
```

3.281.6 Sympy [F]

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)**4/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(cos(e + f*x)**4/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.281.7 Maxima [F]

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos(fx+e)^4}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.281.8 Giac [F]

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos(fx+e)^4}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.281.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos(e+fx)^4}{\left(a+\frac{b}{\cos(e+fx)^2}\right)^{3/2}}\,dx$$

input
```
int(cos(e + f*x)^4/(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(cos(e + f*x)^4/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.282 $\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx$

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3.282.1 Optimal result

Integrand size = 25, antiderivative size = 271

$$\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \frac{(5a^3-9a^2b+15ab^2-35b^3)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{9/2}f} + \frac{(15a^2-22ab+35b^2)\cos(e+fx)\sin(e+fx)}{48a^3f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(5a-7b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{\cos^5(e+fx)\sin(e+fx)}{6af\sqrt{a+b+b\tan^2(e+fx)}} + \frac{b(15a^3-17a^2b+25ab^2+105b^3)\tan(e+fx)}{48a^4(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

output

```
1/16*(5*a^3-9*a^2*b+15*a*b^2-35*b^3)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(
f*x+e)^2)^(1/2))/a^(9/2)/f+1/48*(15*a^2-22*a*b+35*b^2)*cos(f*x+e)*sin(f*x+
e)/a^3/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/24*(5*a-7*b)*cos(f*x+e)^3*sin(f*x+e)
/a^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/6*cos(f*x+e)^5*sin(f*x+e)/a/f/(a+b+b*t
an(f*x+e)^2)^(1/2)+1/48*b*(15*a^3-17*a^2*b+25*a*b^2+105*b^3)*tan(f*x+e)/a^
4/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

3.282.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 17.72 (sec) , antiderivative size = 2068, normalized size of antiderivative = 7.63

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(3*(a + b)*AppellF1[1/2, -4, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/
(a + b)]*Cos[e + f*x]^14*Sin[e + f*x])/(2*f*Sqrt[a + 2*b + a*Cos[2*(e + f*
x)]]*(a + b*Sec[e + f*x]^2)^(3/2)*(a + b - a*Sin[e + f*x]^2)*(3*(a + b)*Ap
pellF1[1/2, -4, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3
*a*AppellF1[3/2, -4, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]
 - 8*(a + b)*AppellF1[3/2, -3, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2
)/(a + b)])*Sin[e + f*x]^2)*((3*a*(a + b)*AppellF1[1/2, -4, 3/2, 3/2, Sin[
e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^9*Sin[e + f*x]^2)/(Sq
rt[a + 2*b + a*Cos[2*(e + f*x)]]*(a + b - a*Sin[e + f*x]^2)^2*(3*(a + b)*A
ppellF1[1/2, -4, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (
3*a*AppellF1[3/2, -4, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)
] - 8*(a + b)*AppellF1[3/2, -3, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^
2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)*AppellF1[1/2, -4, 3/2, 3/2, Sin
[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^9)/(2*Sqrt[a + 2*b +
 a*Cos[2*(e + f*x)]]*(a + b - a*Sin[e + f*x]^2)*(3*(a + b)*AppellF1[1/2, -
4, 3/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (3*a*AppellF1[3
/2, -4, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 8*(a + b)*
AppellF1[3/2, -3, 3/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*S
in[e + f*x]^2)) - (12*(a + b)*AppellF1[1/2, -4, 3/2, 3/2, Sin[e + f*x]^2,
(a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^7*Sin[e + f*x]^2)/(Sqrt[a + 2*...
```

3.282.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 4634, 316, 25, 402, 25, 402, 25, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sec(e+fx)^6\,(a+b\sec(e+fx)^2)^{3/2}}dx$$

$$\downarrow 4634$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)^4(b\tan^2(e+fx)+a+b)^{3/2}}d\tan(e+fx)}{f}$$

$$\downarrow 316$$

$$\frac{\frac{\tan(e+fx)}{6a(\tan^2(e+fx)+1)^3\sqrt{a+b\tan^2(e+fx)+b}} - \frac{\int -\frac{6b\tan^2(e+fx)+5a-b}{\left(\tan^2(e+fx)+1\right)^3\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{6a}}{f}$$

$$\downarrow 25$$

$$\frac{\frac{\int \frac{6b\tan^2(e+fx)+5a-b}{\left(\tan^2(e+fx)+1\right)^3\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{6a} + \frac{\tan(e+fx)}{6a(\tan^2(e+fx)+1)^3\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\downarrow 402$$

$$\frac{\frac{(5a-7b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\sqrt{a+b\tan^2(e+fx)+b}} - \frac{\int -\frac{15a^2-2ba+7b^2+4(5a-7b)b\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{4a}}{6a} + \frac{\tan(e+fx)}{6a(\tan^2(e+fx)+1)^3\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\downarrow 25$$

$$\frac{\frac{\int \frac{15a^2-2ba+7b^2+4(5a-7b)b\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{4a} + \frac{(5a-7b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\sqrt{a+b\tan^2(e+fx)+b}}}{6a} + \frac{\tan(e+fx)}{6a(\tan^2(e+fx)+1)^3\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\downarrow 402$$

$$\frac{\frac{\left(15a^2-22ab+35b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)+b}} - \frac{\int -\frac{15a^3+3ba^2+b^2a-35b^3+2b\left(15a^2-22ba+35b^2\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{2a}}{4a} + \frac{\frac{(5a-7b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\sqrt{a+b\tan^2(e+fx)+}}}{6a}}{f}$$

3.282. $\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

$$\downarrow 25$$

$$\frac{\int \frac{15a^3+3ba^2+b^2a-35b^3+2b\left(15a^2-22ba+35b^2\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{2a}+\frac{\left(15a^2-22ab+35b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)+b}}+\frac{(5a-7b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\sqrt{a+b\tan^2(e+fx)+b}}}{4a}$$
$$6a$$
$$f$$

$$\downarrow 402$$

$$\frac{\int \frac{3(a+b)\left(5a^3-9ba^2+15b^2a-35b^3\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a(a+b)}+\frac{b\left(15a^3-17a^2b+25ab^2+105b^3\right)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{2a}+\frac{\left(15a^2-22ab+35b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)+b}}+\frac{(5a-7}{4a\left(\tan^2(e+fx)+1\right.}}{4a}$$
$$6a$$
$$f$$

$$\downarrow 27$$

$$\frac{3\left(5a^3-9a^2b+15ab^2-35b^3\right)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a}+\frac{b\left(15a^3-17a^2b+25ab^2+105b^3\right)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{2a}+\frac{\left(15a^2-22ab+35b^2\right)\tan(e+f}{2a\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e-}}{4a}$$
$$6a$$
$$f$$

$$\downarrow 291$$

$$\frac{3\left(5a^3-9a^2b+15ab^2-35b^3\right)\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{a}+\frac{b\left(15a^3-17a^2b+25ab^2+105b^3\right)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{2a}+\frac{\left(15a^2-22ab+35b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)}}}{4a}$$
$$6a$$
$$f$$

$$\downarrow 216$$

$$\frac{\left(15a^2-22ab+35b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\sqrt{a+b\tan^2(e+fx)+b}}+\frac{\frac{b\left(15a^3-17a^2b+25ab^2+105b^3\right)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}+\frac{3\left(5a^3-9a^2b+15ab^2-35b^3\right)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}}}{2a}}{4a}+\frac{}{4a\left(\tan^2(e-\right.}$$
$$6a$$
$$f$$

3.282. $\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

input
```
Int[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Tan[e + f*x]/(6*a*(1 + Tan[e + f*x]^2)^3*Sqrt[a + b + b*Tan[e + f*x]^2])
+ (((5*a - 7*b)*Tan[e + f*x])/(4*a*(1 + Tan[e + f*x]^2)^2*Sqrt[a + b + b*T
an[e + f*x]^2]) + (((15*a^2 - 22*a*b + 35*b^2)*Tan[e + f*x])/(2*a*(1 + Tan
[e + f*x]^2)*Sqrt[a + b + b*Tan[e + f*x]^2]) + ((3*(5*a^3 - 9*a^2*b + 15*a
*b^2 - 35*b^3)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2
]])/a^(3/2) + (b*(15*a^3 - 17*a^2*b + 25*a*b^2 + 105*b^3)*Tan[e + f*x])/(a
*(a + b)*Sqrt[a + b + b*Tan[e + f*x]^2]))/(2*a))/(4*a))/(6*a))/f
```

3.282.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.282.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1498 vs. 2(247) = 494.

Time = 9.78 (sec) , antiderivative size = 1499, normalized size of antiderivative = 5.53

method	result	size
default	Expression too large to display	1499

input
```
int(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/48/f/a^4/(-a)^(1/2)/(a+b)*(b+a*cos(f*x+e)^2)*(8*(-a)^(1/2)*a^4*cos(f*x+e
)^6*sin(f*x+e)+8*(-a)^(1/2)*a^3*b*cos(f*x+e)^6*sin(f*x+e)+10*(-a)^(1/2)*a^
4*cos(f*x+e)^4*sin(f*x+e)-4*(-a)^(1/2)*a^3*b*cos(f*x+e)^4*sin(f*x+e)-14*(-
a)^(1/2)*a^2*b^2*cos(f*x+e)^4*sin(f*x+e)+15*(-a)^(1/2)*a^4*cos(f*x+e)^2*si
n(f*x+e)-7*(-a)^(1/2)*a^3*b*cos(f*x+e)^2*sin(f*x+e)+13*(-a)^(1/2)*a^2*b^2*
cos(f*x+e)^2*sin(f*x+e)+35*(-a)^(1/2)*a*b^3*cos(f*x+e)^2*sin(f*x+e)+15*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^4*cos(f*x+e)-12*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)-4*sin(f*x+e)*a)*a^3*b*cos(f*x+e)+18*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos
(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x
+e)*a)*a^2*b^2*cos(f*x+e)-60*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*l
n(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-
a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a*b^3
*cos(f*x+e)-105*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2
)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*b^4*cos(f*x+e)+...
```

3.282.5 Fricas [A] (verification not implemented)

Time = 4.39 (sec) , antiderivative size = 941, normalized size of antiderivative = 3.47

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/384*(3*(5*a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 20*a*b^4 - 35*b^5 + (5*a^5 -
4*a^4*b + 6*a^3*b^2 - 20*a^2*b^3 - 35*a*b^4)*cos(f*x + e)^2)*sqrt(-a)*log(
128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14
*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^
3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^
3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b +
 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*s
qrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(8*(
a^5 + a^4*b)*cos(f*x + e)^7 + 2*(5*a^5 - 2*a^4*b - 7*a^3*b^2)*cos(f*x + e)
^5 + (15*a^5 - 7*a^4*b + 13*a^3*b^2 + 35*a^2*b^3)*cos(f*x + e)^3 + (15*a^4
*b - 17*a^3*b^2 + 25*a^2*b^3 + 105*a*b^4)*cos(f*x + e))*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^7 + a^6*b)*f*cos(f*x + e)^2 +
(a^6*b + a^5*b^2)*f), -1/192*(3*(5*a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 20*a*b^
4 - 35*b^5 + (5*a^5 - 4*a^4*b + 6*a^3*b^2 - 20*a^2*b^3 - 35*a*b^4)*cos(f*x
 + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x
+ e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2
*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(8*(a^5 + a^4*b)*cos(f*x + e)^7 + 2
*(5*a^5 - 2*a^4*b - 7*a^3*b^2)*cos(f*x + e)^5 + (15*a^5 - 7*a^4*b + 13*a^3
*b^2 + 35*a^2*b^3)*cos(f*x + e)^3 + (15*a^4*b - 17*a^3*b^2 + 25*a^2*b^3...
```

3.282.6 Sympy [F]

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)**6/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(cos(e + f*x)**6/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.282.7 Maxima [F]

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos(fx+e)^6}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.282.8 Giac [F]

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos(fx+e)^6}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.282.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cos(e+fx)^6}{\left(a+\frac{b}{\cos(e+fx)^2}\right)^{3/2}}\,dx$$

input
```
int(cos(e + f*x)^6/(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(cos(e + f*x)^6/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.283 $\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

3.283.1 Optimal result 2114
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3.283.9 Mupad [F(-1)] 2122

3.283.1 Optimal result

Integrand size = 25, antiderivative size = 321

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{2a(a+2b)\sin(e+fx)}{3b^2(a+b)^2 f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$
$$-\frac{a\sin(e+fx)}{3b(a+b)f\,(a+b-a\sin^2(e+fx))\,\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$
$$+\frac{2(a+2b)E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)(a+b-a\sin^2(e+fx))}{3b^2(a+b)^2 f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}$$
$$-\frac{\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{3b(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

output

```
-2/3*a*(a+2*b)*sin(f*x+e)/b^2/(a+b)^2/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2)
)^(1/2)-1/3*a*sin(f*x+e)/b/(a+b)/f/(a+b-a*sin(f*x+e)^2)/(sec(f*x+e)^2*(a+b
-a*sin(f*x+e)^2))^(1/2)+2/3*(a+2*b)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*
(a+b-a*sin(f*x+e)^2)/b^2/(a+b)^2/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b
-a*sin(f*x+e)^2))^(1/2)/(1-a*sin(f*x+e)^2/(a+b))^(1/2)-1/3*EllipticF(sin(f
*x+e),(a/(a+b))^(1/2))*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/b/(a+b)/f/(cos(f*x+e
)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)
```

3.283.2 Mathematica [A] (verified)

Time = 4.18 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.52

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{(a+2b+a\cos(2(e+fx)))\sec^5(e+fx)\left(\sqrt{2}(a+b)^2\left(\frac{a+2b+a\cos(2(e+fx))}{a+b}\right)^{3/}\right.}{}$$

input
```
Integrate[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^5*(Sqrt[2]*(a + b)^2*((a + 2*
b + a*Cos[2*(e + f*x)])/(a + b))^(3/2)*(2*(a + 2*b)*EllipticE[e + f*x, a/(
a + b)] - b*EllipticF[e + f*x, a/(a + b)]) - 2*a*(a^2 + 5*a*b + 5*b^2 + a*
(a + 2*b)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)]))/(24*b^2*(a + b)^2*f*(a + b*
Sec[e + f*x]^2)^(5/2))
```

3.283.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.02, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 4636, 2057, 2058, 316, 25, 402, 25, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\sec(e+fx)^5}{(a+b\sec(e+fx)^2)^{5/2}}\,dx$$
$$\downarrow 4636$$
$$\frac{\int \frac{1}{(1-\sin^2(e+fx))^3\left(a+\frac{b}{1-\sin^2(e+fx)}\right)^{5/2}}\,d\sin(e+fx)}{f}$$
$$\downarrow 2057$$
$$\frac{\int \frac{1}{(1-\sin^2(e+fx))^3\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^{5/2}}\,d\sin(e+fx)}{f}$$

$$\downarrow 2058$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\left(-a\sin^2(e+fx)+a+b\right)^{5/2}}d\sin(e+fx)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 316$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(-\frac{\int -\frac{a\sin^2(e+fx)+a+3b}{\sqrt{1-\sin^2(e+fx)}\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}d\sin(e+fx)}{3b(a+b)}-\frac{a\sqrt{1-\sin^2(e+fx)}\sin(e+fx)}{3b(a+b)\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 25$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\int \frac{a\sin^2(e+fx)+a+3b}{\sqrt{1-\sin^2(e+fx)}\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}d\sin(e+fx)}{3b(a+b)}-\frac{a\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{3b(a+b)\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 402$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{-\frac{\int -\frac{(a+b)(2a+3b)-2a(a+2b)\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{b(a+b)}-\frac{2a(a+2b)\sqrt{1-\sin^2(e+fx)}\sin(e+fx)}{b(a+b)\sqrt{-a\sin^2(e+fx)+a+b}}}{3b(a+b)}-\frac{a\sin(e+fx)\sqrt{1}}{3b(a+b)(-a\sin^2(e}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 25$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{\int \frac{(a+b)(2a+3b)-2a(a+2b)\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{b(a+b)}-\frac{2a(a+2b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{b(a+b)\sqrt{-a\sin^2(e+fx)+a+b}}}{3b(a+b)}-\frac{a\sin(e+fx)\sqrt{1-si}}{3b(a+b)(-a\sin^2(e+f}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 399$$

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{2(a+2b)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)-b(a+b)\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{b(a+b)}-\frac{2a(a+2b)\sin}{b(a+b)\sqrt{}}\right)$$
$$\Big/\left(3b(a+b)\right)$$
$$f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}$$

$$\downarrow 323$$

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{2(a+2b)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)-\frac{b(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}d\sin(e+fx)}{\sqrt{-a\sin^2(e+fx)+a+b}}}{b(a+b)}\right)$$
$$\Big/\left(3b(a+b)\right)$$
$$f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}$$

$$\downarrow 321$$

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{2(a+2b)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)-\frac{b(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)}{\sqrt{-a\sin^2(e+fx)+a+b}}}{b(a+b)}-\frac{2a(a+2b)\,s}{b(a+b)}\right)$$
$$\Big/\left(3b(a+b)\right)$$
$$f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}$$

$$\downarrow 330$$

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{2(a+2b)\sqrt{-a\sin^2(e+fx)+a+b}\int\frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{b(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\right.}{\sqrt{-a\sin^2(e+fx)+a+b}}}{b(a+b)}\right)$$
$$\Big/\left(3b(a+b)\right)$$
$$f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}$$

$$\downarrow 327$$

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{2(a+2b)\sqrt{-a\sin^2(e+fx)+a+b}E\left(\arcsin(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{b(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)}{\sqrt{-a\sin^2(e+fx)+a+b}}}{b(a+b)}\right.$$
$$\left.\frac{}{3b(a+b)}\right)$$
$$f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}$$

input
```
Int[Sec[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(Sqrt[a + b - a*Sin[e + f*x]^2]*(-1/3*(a*Sin[e + f*x]*Sqrt[1 - Sin[e + f*x
]^2])/(b*(a + b)*(a + b - a*Sin[e + f*x]^2)^(3/2)) + ((-2*a*(a + 2*b)*Sin[
e + f*x]*Sqrt[1 - Sin[e + f*x]^2])/(b*(a + b)*Sqrt[a + b - a*Sin[e + f*x]^
2]) + ((2*(a + 2*b)*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b
 - a*Sin[e + f*x]^2])/Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)] - (b*(a + b)*Ell
ipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b
)])/Sqrt[a + b - a*Sin[e + f*x]^2])/(b*(a + b)))/(3*b*(a + b))))/(f*Sqrt[1
 - Sin[e + f*x]^2]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(1 - Sin[e + f*x]^2)])
```

3.283.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

3.283.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.01 (sec) , antiderivative size = 16966, normalized size of antiderivative = 52.85

method	result	size
default	Expression too large to display	16966

input
```
int(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.283.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 1374, normalized size of antiderivative = 4.28

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/3*((2*(-I*a^3*b^2 - 2*I*a^2*b^3 + (-I*a^5 - 2*I*a^4*b)*cos(f*x + e)^4 -
2*(I*a^4*b + 2*I*a^3*b^2)*cos(f*x + e)^2)*sqrt(a)*sqrt((a*b + b^2)/a^2) -
(-I*a^3*b^2 - 4*I*a^2*b^3 - 4*I*a*b^4 + (-I*a^5 - 4*I*a^4*b - 4*I*a^3*b^2)
*cos(f*x + e)^4 + 2*(-I*a^4*b - 4*I*a^3*b^2 - 4*I*a^2*b^3)*cos(f*x + e)^2)
*sqrt(a))*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_e(arcsin(
sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) + I*sin(f*x +
e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) +
 (2*(I*a^3*b^2 + 2*I*a^2*b^3 + (I*a^5 + 2*I*a^4*b)*cos(f*x + e)^4 - 2*(-I*
a^4*b - 2*I*a^3*b^2)*cos(f*x + e)^2)*sqrt(a)*sqrt((a*b + b^2)/a^2) - (I*a^
3*b^2 + 4*I*a^2*b^3 + 4*I*a*b^4 + (I*a^5 + 4*I*a^4*b + 4*I*a^3*b^2)*cos(f*
x + e)^4 + 2*(I*a^4*b + 4*I*a^3*b^2 + 4*I*a^2*b^3)*cos(f*x + e)^2)*sqrt(a)
)*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_e(arcsin(sqrt((2*
a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) - I*sin(f*x + e))), (a
^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + (2*(-I*
a^2*b^3 - 3*I*a*b^4 + (-I*a^4*b - 3*I*a^3*b^2)*cos(f*x + e)^4 - 2*(I*a^3*b
^2 + 3*I*a^2*b^3)*cos(f*x + e)^2)*sqrt(a)*sqrt((a*b + b^2)/a^2) - (2*I*a^3
*b^2 + 9*I*a^2*b^3 + 13*I*a*b^4 + 6*I*b^5 + (2*I*a^5 + 9*I*a^4*b + 13*I*a^
3*b^2 + 6*I*a^2*b^3)*cos(f*x + e)^4 + 2*(2*I*a^4*b + 9*I*a^3*b^2 + 13*I*a^
2*b^3 + 6*I*a*b^4)*cos(f*x + e)^2)*sqrt(a))*sqrt((2*a*sqrt((a*b + b^2)/a^2
) - a - 2*b)/a)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a -...
```

3.283.6 Sympy [F]

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)**5/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral(sec(e + f*x)**5/(a + b*sec(e + f*x)**2)**(5/2), x)
```

3.283.7 Maxima [F]

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sec(fx+e)^5}{(b\sec(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^5/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

3.283.8 Giac [F]

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sec(fx+e)^5}{(b\sec(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.283.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{1}{\cos(e+fx)^5\left(a+\frac{b}{\cos(e+fx)^2}\right)^{5/2}}\,dx$$

input
```
int(1/(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^(5/2)),x)
```

output
```
int(1/(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^(5/2)), x)
```

3.284 $\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

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3.284.1 Optimal result

Integrand size = 25, antiderivative size = 319

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{(a-b)\sin(e+fx)}{3b(a+b)^2 f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$
$$+\frac{\sin(e+fx)}{3(a+b)f\,(a+b-a\sin^2(e+fx))\,\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$
$$+\frac{(a-b)E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)(a+b-a\sin^2(e+fx))}{3ab(a+b)^2 f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}$$
$$+\frac{\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{3a(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

output

```
-1/3*(a-b)*sin(f*x+e)/b/(a+b)^2/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2
)+1/3*sin(f*x+e)/(a+b)/f/(a+b-a*sin(f*x+e)^2)/(sec(f*x+e)^2*(a+b-a*sin(f*x
+e)^2))^(1/2)+1/3*(a-b)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(a+b-a*sin(f
*x+e)^2)/a/b/(a+b)^2/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e
)^2))^(1/2)/(1-a*sin(f*x+e)^2/(a+b))^(1/2)+1/3*EllipticF(sin(f*x+e),(a/(a+
b))^(1/2))*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/a/(a+b)/f/(cos(f*x+e)^2)^(1/2)/(
sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)
```

3.284.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 22.18 (sec) , antiderivative size = 1204, normalized size of antiderivative = 3.77

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = (a+2b+a\cos(2e+2fx))^{5/2}\sec^5(e+fx)\left(-\frac{\left(-2\sqrt{-\frac{1}{b}}(-a-a\cos(2e+2fx))(2a^2(a-\right.\right.$$

input
```
Integrate[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
((a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^5*(-1/24*((-2*Sqrt[-b^(
-1)]*(-a - a*Cos[2*e + 2*f*x])*(2*a^2*(a + 3*b + a*Cos[2*e + 2*f*x]) + b*(
2*b^2 + 3*b*(a + 2*b + a*Cos[2*e + 2*f*x]) - 2*(a + 2*b + a*Cos[2*e + 2*f*
x])^2) + a*(4*b^2 + 5*b*(a + 2*b + a*Cos[2*e + 2*f*x]) - (a + 2*b + a*Cos[
2*e + 2*f*x])^2)) + (2*I)*(a^2 + 3*a*b + 2*b^2)*Sqrt[(a - a*Cos[2*e + 2*f*
x])/(a + b)]*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sqrt[4 - (2*(a + 2*b + a
*Cos[2*e + 2*f*x]))/b]*EllipticE[I*ArcSinh[(Sqrt[-b^(-1)]*Sqrt[a + 2*b + a
*Cos[2*e + 2*f*x]])/Sqrt[2]], b/(a + b)] - I*(2*a^2 + 5*a*b + 3*b^2)*(a +
2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sqrt[(4*a + 4*b - 2*(a + 2*b + a*Cos[2*e +
 2*f*x]))/(a + b)]*Sqrt[2 - (a + 2*b + a*Cos[2*e + 2*f*x])/b]*EllipticF[I*
ArcSinh[(Sqrt[-b^(-1)]*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]])/Sqrt[2]], b/(a
+ b)])*Sin[2*e + 2*f*x])/(a*Sqrt[-b^(-1)]*b^2*(a + b)^2*f*Sqrt[((a - a*Cos
[2*e + 2*f*x])*(a + a*Cos[2*e + 2*f*x]))/a^2]*(a + 2*b + a*Cos[2*e + 2*f*x
])^(3/2)*Sqrt[1 - Cos[2*e + 2*f*x]^2]) + (Cos[2*(e + f*x)]*(-2*Sqrt[-b^(-1
)]*(-a - a*Cos[2*e + 2*f*x])*(4*b^4 - b^2*(a + 2*b + a*Cos[2*e + 2*f*x])^2
 + 2*a^3*(a + 3*b + a*Cos[2*e + 2*f*x]) + a*b*(10*b^2 + b*(a + 2*b + a*Cos
[2*e + 2*f*x]) - (a + 2*b + a*Cos[2*e + 2*f*x])^2) + a^2*(8*b^2 + 3*b*(a +
 2*b + a*Cos[2*e + 2*f*x]) - (a + 2*b + a*Cos[2*e + 2*f*x])^2)) + (2*I)*(a
^3 + 2*a^2*b + 2*a*b^2 + b^3)*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sqrt[(4
*a + 4*b - 2*(a + 2*b + a*Cos[2*e + 2*f*x]))/(a + b)]*Sqrt[2 - (a + 2*b...
```

3.284.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 4636, 2057, 2058, 314, 25, 402, 25, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sec(e+fx)^3}{(a+b\sec(e+fx)^2)^{5/2}}\,dx$$

$$\downarrow 4636$$

$$\frac{\int \frac{1}{(1-\sin^2(e+fx))^2\left(a+\frac{b}{1-\sin^2(e+fx)}\right)^{5/2}}\,d\sin(e+fx)}{f}$$

$$\downarrow 2057$$

$$\frac{\int \frac{1}{(1-\sin^2(e+fx))^2\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^{5/2}}\,d\sin(e+fx)}{f}$$

$$\downarrow 2058$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\int \frac{\sqrt{1-\sin^2(e+fx)}}{(-a\sin^2(e+fx)+a+b)^{5/2}}\,d\sin(e+fx)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 314$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{3(a+b)(-a\sin^2(e+fx)+a+b)^{3/2}}-\frac{\int -\frac{2-\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}\,d\sin(e+fx)}{3(a+b)}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 25$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\int\frac{2-\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}d\sin(e+fx)}{3(a+b)}+\frac{\sqrt{1-\sin^2(e+fx)}\sin(e+fx)}{3(a+b)\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 402$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{-\frac{\int-\frac{-\left((a-b)\sin^2(e+fx)\right)+a+b}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{b(a+b)}-\frac{(a-b)\sqrt{1-\sin^2(e+fx)}\sin(e+fx)}{b(a+b)\sqrt{-a\sin^2(e+fx)+a+b}}}{3(a+b)}+\frac{\sqrt{1-\sin^2(e+fx)}\,si}{3(a+b)\left(-a\sin^2(e+fx\right.}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 25$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{\int\frac{-\left((a-b)\sin^2(e+fx)\right)+a+b}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{b(a+b)}-\frac{(a-b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{b(a+b)\sqrt{-a\sin^2(e+fx)+a+b}}}{3(a+b)}+\frac{\sqrt{1-\sin^2(e+fx)}\sin(e\right.}{3(a+b)\left(-a\sin^2(e+fx)+\right.}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 399$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{b(a+b)\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{a}+\frac{(a-b)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}}{b(a+b)}-\frac{(a-b)\sin(e+\right.}{b(a+b)\sqrt{-}}\right.}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 323$$

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{(a-b)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}+\frac{b(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}d\sin(e+fx)}{a\sqrt{-a\sin^2(e+fx)+a+b}}}{b(a+b)}\right.$$
$$\left.\frac{}{3(a+b)}\right)$$
$$f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}$$

$$\downarrow 321$$

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{(a-b)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}+\frac{b(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)}{a\sqrt{-a\sin^2(e+fx)+a+b}}}{b(a+b)}\right.$$
$$\left.-\frac{(a-b)\sin(e+\right.$$
$$\left.\frac{}{3(a+b)}\right)$$
$$f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}$$

$$\downarrow 330$$

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{(a-b)\sqrt{-a\sin^2(e+fx)+a+b}\int\frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}+\frac{b(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+\right.}}{a\sqrt{-a\sin^2(e+fx)+a+b}}}{b(a+b)}\right.$$
$$\left.\frac{}{3(a+b)}\right)$$
$$f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}$$

$$\downarrow 327$$

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{b(a+b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)}{a\sqrt{-a\sin^2(e+fx)+a+b}}+\frac{(a-b)\sqrt{-a\sin^2(e+fx)+a+b}E\left(\arcsin(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}}{b(a+b)}\right.$$
$$\left.-\right.$$
$$\left.\frac{}{3(a+b)}\right)$$
$$f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}$$

3.284. $\int\frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}dx$

input
```
Int[Sec[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(Sqrt[a + b - a*Sin[e + f*x]^2]*((Sin[e + f*x]*Sqrt[1 - Sin[e + f*x]^2])/(
3*(a + b)*(a + b - a*Sin[e + f*x]^2)^(3/2)) + (-(((a - b)*Sin[e + f*x]*Sqr
t[1 - Sin[e + f*x]^2])/(b*(a + b)*Sqrt[a + b - a*Sin[e + f*x]^2])) + (((a
- b)*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b - a*Sin[e + f*x
]^2])/(a*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) + (b*(a + b)*EllipticF[ArcS
in[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(a*Sqrt
[a + b - a*Sin[e + f*x]^2]))/(b*(a + b)))/(3*(a + b))))/(f*Sqrt[1 - Sin[e
+ f*x]^2]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(1 - Sin[e + f*x]^2)])
```

3.284.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 314
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] + Simp[1/(2*a*
(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*p + 3) + d
*(2*(p + q + 1) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, 2, p, q,
 x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

3.284.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.09 (sec) , antiderivative size = 12063, normalized size of antiderivative = 37.82

method	result	size
default	Expression too large to display	12063

input
```
int(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.284.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 1244, normalized size of antiderivative = 3.90

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/6*((2*((-I*a^4 + I*a^3*b)*cos(f*x + e)^4 - I*a^2*b^2 + I*a*b^3 - 2*(I*a^
3*b - I*a^2*b^2)*cos(f*x + e)^2)*sqrt(a)*sqrt((a*b + b^2)/a^2) - ((-I*a^4
- I*a^3*b + 2*I*a^2*b^2)*cos(f*x + e)^4 - I*a^2*b^2 - I*a*b^3 + 2*I*b^4 +
2*(-I*a^3*b - I*a^2*b^2 + 2*I*a*b^3)*cos(f*x + e)^2)*sqrt(a))*sqrt((2*a*sq
rt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_e(arcsin(sqrt((2*a*sqrt((a*b +
b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) + I*sin(f*x + e))), (a^2 + 8*a*b + 8
*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + (2*((I*a^4 - I*a^3*b)
*cos(f*x + e)^4 + I*a^2*b^2 - I*a*b^3 - 2*(-I*a^3*b + I*a^2*b^2)*cos(f*x +
 e)^2)*sqrt(a)*sqrt((a*b + b^2)/a^2) - ((I*a^4 + I*a^3*b - 2*I*a^2*b^2)*co
s(f*x + e)^4 + I*a^2*b^2 + I*a*b^3 - 2*I*b^4 + 2*(I*a^3*b + I*a^2*b^2 - 2*
I*a*b^3)*cos(f*x + e)^2)*sqrt(a))*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*
b)/a)*elliptic_e(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos
(f*x + e) - I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt(
(a*b + b^2)/a^2))/a^2) - 2*(4*(I*a^3*b*cos(f*x + e)^4 + 2*I*a^2*b^2*cos(f*
x + e)^2 + I*a*b^3)*sqrt(a)*sqrt((a*b + b^2)/a^2) + ((I*a^4 + 3*I*a^3*b +
2*I*a^2*b^2)*cos(f*x + e)^4 + I*a^2*b^2 + 3*I*a*b^3 + 2*I*b^4 + 2*(I*a^3*b
 + 3*I*a^2*b^2 + 2*I*a*b^3)*cos(f*x + e)^2)*sqrt(a))*sqrt((2*a*sqrt((a*b +
 b^2)/a^2) - a - 2*b)/a)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a^2)
 - a - 2*b)/a)*(cos(f*x + e) + I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^2 + 4*
(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) - 2*(4*(-I*a^3*b*cos(f*x + e)...
```

3.284.6 Sympy [F]

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)**3/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral(sec(e + f*x)**3/(a + b*sec(e + f*x)**2)**(5/2), x)
```

3.284.7 Maxima [F]

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sec(fx+e)^3}{(b\sec(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^3/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

3.284.8 Giac [F]

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sec(fx+e)^3}{(b\sec(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.284.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{1}{\cos(e+fx)^3\left(a+\frac{b}{\cos(e+fx)^2}\right)^{5/2}}\,dx$$

input
```
int(1/(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^(5/2)),x)
```

output
```
int(1/(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^(5/2)), x)
```

3.285 $\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

3.285.1 Optimal result 2133
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3.285.9 Mupad [F(-1)] 2141

3.285.1 Optimal result

Integrand size = 23, antiderivative size = 327

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{2(2a+b)\sin(e+fx)}{3a(a+b)^2 f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$
$$- \frac{b\sin(e+fx)}{3a(a+b)f\,(a+b-a\sin^2(e+fx))\,\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$
$$- \frac{2(2a+b)E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)(a+b-a\sin^2(e+fx))}{3a^2(a+b)^2 f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}$$
$$+ \frac{(3a+2b)\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{3a^2(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

output

```
2/3*(2*a+b)*sin(f*x+e)/a/(a+b)^2/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/
2)-1/3*b*sin(f*x+e)/a/(a+b)/f/(a+b-a*sin(f*x+e)^2)/(sec(f*x+e)^2*(a+b-a*si
n(f*x+e)^2))^(1/2)-2/3*(2*a+b)*EllipticE(sin(f*x+e),(a/(a+b))^(1/2))*(a+b-
a*sin(f*x+e)^2)/a^2/(a+b)^2/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*si
n(f*x+e)^2))^(1/2)/(1-a*sin(f*x+e)^2/(a+b))^(1/2)+1/3*(3*a+2*b)*EllipticF(
sin(f*x+e),(a/(a+b))^(1/2))*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/a^2/(a+b)/f/(co
s(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)
```

3.285.2 Mathematica [F]

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

input
```
Integrate[Sec[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
Integrate[Sec[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2), x]
```

3.285.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.02, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 4636, 2057, 2058, 315, 25, 402, 27, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx)^2)^{5/2}}\,dx$$

$$\downarrow 4636$$

$$\frac{\int \frac{1}{(1-\sin^2(e+fx))\left(a+\frac{b}{1-\sin^2(e+fx)}\right)^{5/2}}\,d\sin(e+fx)}{f}$$

$$\downarrow 2057$$

$$\frac{\int \frac{1}{(1-\sin^2(e+fx))\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^{5/2}}\,d\sin(e+fx)}{f}$$

$$\downarrow 2058$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\int \frac{(1-\sin^2(e+fx))^{3/2}}{(-a\sin^2(e+fx)+a+b)^{5/2}}\,d\sin(e+fx)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 315$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(-\frac{\int -\frac{-\left((3a+2b)\sin^2(e+fx)\right)+3a+b}{\sqrt{1-\sin^2(e+fx)}\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}d\sin(e+fx)}{3a(a+b)}-\frac{b\sqrt{1-\sin^2(e+fx)}\sin(e+fx)}{3a(a+b)\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 25$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\int \frac{-\left((3a+2b)\sin^2(e+fx)\right)+3a+b}{\sqrt{1-\sin^2(e+fx)}\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}d\sin(e+fx)}{3a(a+b)}-\frac{b\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{3a(a+b)\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 402$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{2(2a+b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{(a+b)\sqrt{-a\sin^2(e+fx)+a+b}}-\frac{\int \frac{b\left(-2(2a+b)\sin^2(e+fx)+a+b\right)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{b(a+b)}}{3a(a+b)}-\frac{b\sin(e+fx)\sqrt{1-\sin}\right.}{3a(a+b)\left(-a\sin^2(e+fx\right.}$$

$$\frac{}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 27$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{2(2a+b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{(a+b)\sqrt{-a\sin^2(e+fx)+a+b}}-\frac{\int \frac{-2(2a+b)\sin^2(e+fx)+a+b}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{a+b}}{3a(a+b)}-\frac{b\sin(e+fx)\sqrt{1-\sin}\right.}{3a(a+b)\left(-a\sin^2(e+fx\right.}$$

$$\frac{}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 399$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{2(2a+b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{(a+b)\sqrt{-a\sin^2(e+fx)+a+b}}-\frac{\frac{2(2a+b)\int \frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{(a+b)(3a+2b)\int \frac{}{\sqrt{1-\sin^2(e+fx)}\sqrt{-}}}{a}}{a+b}\right.}{3a(a+b)}$$

$$\frac{}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

3.285. $\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

$\downarrow$ 323

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{2(2a+b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{(a+b)\sqrt{-a\sin^2(e+fx)+a+b}}-\frac{\frac{2(2a+b)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{(a+b)(3a+2b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\int -}{a\sqrt{-a}}}{a+b}\right)$$
$$\overline{3a(a+b)}$$
$$\overline{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\downarrow$ 321

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{2(2a+b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{(a+b)\sqrt{-a\sin^2(e+fx)+a+b}}-\frac{\frac{2(2a+b)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{(a+b)(3a+2b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\mathrm{Ell}}{a\sqrt{-a\sin^2(e+}}}{a+b}\right)$$
$$\overline{3a(a+b)}$$
$$\overline{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\downarrow$ 330

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{2(2a+b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{(a+b)\sqrt{-a\sin^2(e+fx)+a+b}}-\frac{\frac{2(2a+b)\sqrt{-a\sin^2(e+fx)+a+b}\int\frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{(a+b)(3a+2b)\sqrt{1}}{a+b}\right)$$
$$\overline{3a(a+b)}$$
$$\overline{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$\downarrow$ 327

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{2(2a+b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{(a+b)\sqrt{-a\sin^2(e+fx)+a+b}}-\frac{\frac{2(2a+b)\sqrt{-a\sin^2(e+fx)+a+b}E\left(\arcsin(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{(a+b)(3a+2b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}F\left(\arcsin(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{a\sqrt{-a\sin^2(e+fx)+a+b}}}{a+b}}{3a(a+b)}\right)$$

$$f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}$$

input
```
Int[Sec[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(Sqrt[a + b - a*Sin[e + f*x]^2]*(-1/3*(b*Sin[e + f*x]*Sqrt[1 - Sin[e + f*x
]^2])/(a*(a + b)*(a + b - a*Sin[e + f*x]^2)^(3/2)) + ((2*(2*a + b)*Sin[e +
 f*x]*Sqrt[1 - Sin[e + f*x]^2])/((a + b)*Sqrt[a + b - a*Sin[e + f*x]^2]) -
 ((2*(2*a + b)*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b - a*S
in[e + f*x]^2])/(a*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - ((a + b)*(3*a +
 2*b)*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e + f*x]^
2)/(a + b)])/(a*Sqrt[a + b - a*Sin[e + f*x]^2]))/(a + b))/(3*a*(a + b))))/
(f*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[(a + b - a*Sin[e + f*x]^2)/(1 - Sin[e + f
*x]^2)])
```

3.285.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

3.285.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.02 (sec) , antiderivative size = 16966, normalized size of antiderivative = 51.88

method	result	size
default	Expression too large to display	16966

input
```
int(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.285.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 1293, normalized size of antiderivative = 3.95

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/3*((2*((2*I*a^4 + I*a^3*b)*cos(f*x + e)^4 + 2*I*a^2*b^2 + I*a*b^3 - 2*(-
2*I*a^3*b - I*a^2*b^2)*cos(f*x + e)^2)*sqrt(a)*sqrt((a*b + b^2)/a^2) - ((2
*I*a^4 + 5*I*a^3*b + 2*I*a^2*b^2)*cos(f*x + e)^4 + 2*I*a^2*b^2 + 5*I*a*b^3
 + 2*I*b^4 + 2*(2*I*a^3*b + 5*I*a^2*b^2 + 2*I*a*b^3)*cos(f*x + e)^2)*sqrt(
a))*sqrt((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_e(arcsin(sqrt((
2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) + I*sin(f*x + e))),
(a^2 + 8*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + (2*((
-2*I*a^4 - I*a^3*b)*cos(f*x + e)^4 - 2*I*a^2*b^2 - I*a*b^3 - 2*(2*I*a^3*b
+ I*a^2*b^2)*cos(f*x + e)^2)*sqrt(a)*sqrt((a*b + b^2)/a^2) - ((-2*I*a^4 -
5*I*a^3*b - 2*I*a^2*b^2)*cos(f*x + e)^4 - 2*I*a^2*b^2 - 5*I*a*b^3 - 2*I*b^
4 + 2*(-2*I*a^3*b - 5*I*a^2*b^2 - 2*I*a*b^3)*cos(f*x + e)^2)*sqrt(a))*sqrt
((2*a*sqrt((a*b + b^2)/a^2) - a - 2*b)/a)*elliptic_e(arcsin(sqrt((2*a*sqrt
((a*b + b^2)/a^2) - a - 2*b)/a)*(cos(f*x + e) - I*sin(f*x + e))), (a^2 + 8
*a*b + 8*b^2 + 4*(a^2 + 2*a*b)*sqrt((a*b + b^2)/a^2))/a^2) + (2*((-3*I*a^4
 - I*a^3*b)*cos(f*x + e)^4 - 3*I*a^2*b^2 - I*a*b^3 - 2*(3*I*a^3*b + I*a^2*
b^2)*cos(f*x + e)^2)*sqrt(a)*sqrt((a*b + b^2)/a^2) - ((-I*a^4 - 3*I*a^3*b
- 2*I*a^2*b^2)*cos(f*x + e)^4 - I*a^2*b^2 - 3*I*a*b^3 - 2*I*b^4 + 2*(-I*a^
3*b - 3*I*a^2*b^2 - 2*I*a*b^3)*cos(f*x + e)^2)*sqrt(a))*sqrt((2*a*sqrt((a*
b + b^2)/a^2) - a - 2*b)/a)*elliptic_f(arcsin(sqrt((2*a*sqrt((a*b + b^2)/a
^2) - a - 2*b)/a)*(cos(f*x + e) + I*sin(f*x + e))), (a^2 + 8*a*b + 8*b^...
```

3.285.6 Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral(sec(e + f*x)/(a + b*sec(e + f*x)**2)**(5/2), x)
```

3.285.7 Maxima [F]

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sec(fx+e)}{\left(b\sec(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

3.285.8 Giac [F]

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sec(fx+e)}{\left(b\sec(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

3.285.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{1}{\cos(e+fx)\left(a+\frac{b}{\cos(e+fx)^2}\right)^{5/2}}\,dx$$

input
```
int(1/(cos(e + f*x)*(a + b/cos(e + f*x)^2)^(5/2)),x)
```

output
```
int(1/(cos(e + f*x)*(a + b/cos(e + f*x)^2)^(5/2)), x)
```

3.286 $\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

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3.286.1 Optimal result

Integrand size = 23, antiderivative size = 349

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{2b(3a+2b)\sin(e+fx)}{3a^2(a+b)^2 f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$
$$-\frac{b\cos^2(e+fx)\sin(e+fx)}{3a(a+b)f\,(a+b-a\sin^2(e+fx))\,\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$
$$+\frac{(3a^2+13ab+8b^2)\,E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)(a+b-a\sin^2(e+fx))}{3a^3(a+b)^2 f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}$$
$$-\frac{b(9a+8b)\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{3a^3(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

output

```
-2/3*b*(3*a+2*b)*sin(f*x+e)/a^2/(a+b)^2/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^
2))^(1/2)-1/3*b*cos(f*x+e)^2*sin(f*x+e)/a/(a+b)/f/(a+b-a*sin(f*x+e)^2)/(se
c(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)+1/3*(3*a^2+13*a*b+8*b^2)*EllipticE(
sin(f*x+e),(a/(a+b))^(1/2))*(a+b-a*sin(f*x+e)^2)/a^3/(a+b)^2/f/(cos(f*x+e)
^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/(1-a*sin(f*x+e)^2/(a+b
))^(1/2)-1/3*b*(9*a+8*b)*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*(1-a*sin(f*
x+e)^2/(a+b))^(1/2)/a^3/(a+b)/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*
sin(f*x+e)^2))^(1/2)
```

3.286.2 Mathematica [F]

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

input
```
Integrate[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
Integrate[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2), x]
```

3.286.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.01, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 4636, 2057, 2058, 315, 25, 401, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sec(e+fx)\,(a+b\sec(e+fx)^2)^{5/2}}dx$$

$$\downarrow 4636$$

$$\frac{\int \frac{1}{\left(a+\frac{b}{1-\sin^2(e+fx)}\right)^{5/2}}d\sin(e+fx)}{f}$$

$$\downarrow 2057$$

$$\frac{\int \frac{1}{\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^{5/2}}d\sin(e+fx)}{f}$$

$$\downarrow 2058$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\int \frac{\left(1-\sin^2(e+fx)\right)^{5/2}}{\left(-a\sin^2(e+fx)+a+b\right)^{5/2}}d\sin(e+fx)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 315$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(-\frac{\int -\frac{\sqrt{1-\sin^2(e+fx)}\left(-\left((3a+4b)\sin^2(e+fx)\right)+3a+b\right)}{\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}d\sin(e+fx)}{3a(a+b)}-\frac{b\sin(e+fx)\left(1-\sin^2(e+fx)\right)^{3/2}}{3a(a+b)\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 25$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\int \frac{\sqrt{1-\sin^2(e+fx)}\left(-\left((3a+4b)\sin^2(e+fx)\right)+3a+b\right)}{\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}d\sin(e+fx)}{3a(a+b)}-\frac{b\sin(e+fx)\left(1-\sin^2(e+fx)\right)^{3/2}}{3a(a+b)\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 401$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\int \frac{(a+b)(3a+4b)-\left(3a^2+13ba+8b^2\right)\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{a(a+b)}-\frac{2b(3a+2b)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}}{a(a+b)\sqrt{-a\sin^2(e+fx)+a+b}}}{3a(a+b)}-\frac{b\sin(e+fx)(1}{3a(a+b)(-a\sin}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 399$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\left(3a^2+13ab+8b^2\right)\int \frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{b(a+b)(9a+8b)\int \frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e-}{a}}{a(a+b)}}{3a(a+b)}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 323$$

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\left(3a^2+13ab+8b^2\right)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{b(a+b)(9a+8b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{1-\frac{a\sin^2}{a}}}}{a\sqrt{-a\sin^2(e+fx)+a+b}}\right)$$
$$\frac{a(a+b)}{3a(a+b)}$$
$$f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}$$

$$\downarrow 321$$

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\left(3a^2+13ab+8b^2\right)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{b(a+b)(9a+8b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\operatorname{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{}{a}\right.}{a\sqrt{-a\sin^2(e+fx)+a+b}}\right)$$
$$\frac{a(a+b)}{3a(a+b)}$$
$$f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}$$

$$\downarrow 330$$

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\left(3a^2+13ab+8b^2\right)\sqrt{-a\sin^2(e+fx)+a+b}\int\frac{\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{b(a+b)(9a+8b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\operatorname{EllipticF}\left(a\right.}{a\sqrt{-a\sin^2(e+fx)+a+}}\right)$$
$$\frac{a(a+b)}{3a(a+b)}$$
$$f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+}{1-\sin^2(e+fx)}}$$

$$\downarrow 327$$

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\left(3a^2+13ab+8b^2\right)\sqrt{-a\sin^2(e+fx)+a+b}E\left(\arcsin(\sin(e+fx))\middle|\frac{a}{a+b}\right)}{a\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}-\frac{b(a+b)(9a+8b)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\operatorname{EllipticF}\left(\arcsin(\right.}{a\sqrt{-a\sin^2(e+fx)+a+b}}\right)$$
$$\frac{a(a+b)}{3a(a+b)}$$
$$f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}$$

3.286. $\int\frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}dx$

input
```
Int[Cos[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(Sqrt[a + b - a*Sin[e + f*x]^2]*(-1/3*(b*Sin[e + f*x]*(1 - Sin[e + f*x]^2)
^(3/2))/(a*(a + b)*(a + b - a*Sin[e + f*x]^2)^(3/2)) + ((-2*b*(3*a + 2*b)*
Sin[e + f*x]*Sqrt[1 - Sin[e + f*x]^2])/(a*(a + b)*Sqrt[a + b - a*Sin[e + f
*x]^2]) + (((3*a^2 + 13*a*b + 8*b^2)*EllipticE[ArcSin[Sin[e + f*x]], a/(a
+ b)]*Sqrt[a + b - a*Sin[e + f*x]^2])/(a*Sqrt[1 - (a*Sin[e + f*x]^2)/(a +
b)]) - (b*(a + b)*(9*a + 8*b)*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*S
qrt[1 - (a*Sin[e + f*x]^2)/(a + b)])/(a*Sqrt[a + b - a*Sin[e + f*x]^2]))/(
a*(a + b)))/(3*a*(a + b))))/(f*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[(a + b - a*Si
n[e + f*x]^2)/(1 - Sin[e + f*x]^2)])
```

3.286.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

3.286.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.61 (sec) , antiderivative size = 20403, normalized size of antiderivative = 58.46

method	result	size
default	Expression too large to display	20403

input
```
int(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.286.5 Fricas [F]

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos(fx+e)}{\left(b\sec(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)/(b^3*sec(f*x + e)^6 + 3*a
*b^2*sec(f*x + e)^4 + 3*a^2*b*sec(f*x + e)^2 + a^3), x)
```

3.286.6 Sympy [F]

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral(cos(e + f*x)/(a + b*sec(e + f*x)**2)**(5/2), x)
```

3.286.7 Maxima [F]

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos(fx+e)}{\left(b\sec(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(f*x + e)/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

3.286.8 Giac [F]

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos(fx+e)}{\left(b\sec(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.286.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos(e+f\,x)}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

input
```
int(cos(e + f*x)/(a + b/cos(e + f*x)^2)^(5/2),x)
```

output
```
int(cos(e + f*x)/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.287 $\int \frac{\cos^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

3.287.1 Optimal result 2150
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3.287.3 Rubi [A] (verified) 2151
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3.287.8 Giac [F] 2159
3.287.9 Mupad [F(-1)] 2160

3.287.1 Optimal result

Integrand size = 25, antiderivative size = 441

$$\int \frac{\cos^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = -\frac{2b(4a+3b)\cos^2(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$
$$-\frac{b\cos^4(e+fx)\sin(e+fx)}{3a(a+b)f\,(a+b-a\sin^2(e+fx))\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$
$$+\frac{(a^2+11ab+8b^2)\sin(e+fx)\,(a+b-a\sin^2(e+fx))}{3a^3(a+b)^2f\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$
$$+\frac{2(a+2b)\,(a^2-4ab-4b^2)\,E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)(a+b-a\sin^2(e+fx))}{3a^4(a+b)^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}$$
$$-\frac{b(a^2-16ab-16b^2)\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{3a^4(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\,(a+b-a\sin^2(e+fx))}}$$

output

```
-2/3*b*(4*a+3*b)*cos(f*x+e)^2*sin(f*x+e)/a^2/(a+b)^2/f/(sec(f*x+e)^2*(a+b-
a*sin(f*x+e)^2))^(1/2)-1/3*b*cos(f*x+e)^4*sin(f*x+e)/a/(a+b)/f/(a+b-a*sin(
f*x+e)^2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)+1/3*(a^2+11*a*b+8*b^2)
*sin(f*x+e)*(a+b-a*sin(f*x+e)^2)/a^3/(a+b)^2/f/(sec(f*x+e)^2*(a+b-a*sin(f*
x+e)^2))^(1/2)+2/3*(a+2*b)*(a^2-4*a*b-4*b^2)*EllipticE(sin(f*x+e),(a/(a+b)
)^(1/2))*(a+b-a*sin(f*x+e)^2)/a^4/(a+b)^2/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+
e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/(1-a*sin(f*x+e)^2/(a+b))^(1/2)-1/3*b*(a^2
-16*a*b-16*b^2)*EllipticF(sin(f*x+e),(a/(a+b))^(1/2))*(1-a*sin(f*x+e)^2/(a
+b))^(1/2)/a^4/(a+b)/f/(cos(f*x+e)^2)^(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e
)^2))^(1/2)
```

3.287.2 Mathematica [F]

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

input
```
Integrate[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
Integrate[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2), x]
```

3.287.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 430, normalized size of antiderivative = 0.98, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 4636, 2057, 2058, 315, 25, 401, 27, 403, 25, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sec(e+fx)^3\,(a+b\sec(e+fx)^2)^{5/2}}dx$$

$$\downarrow 4636$$

$$\frac{\int \frac{1-\sin^2(e+fx)}{\left(a+\frac{b}{1-\sin^2(e+fx)}\right)^{5/2}}d\sin(e+fx)}{f}$$

$$\downarrow 2057$$

$$\frac{\int \frac{1-\sin^2(e+fx)}{\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^{5/2}}d\sin(e+fx)}{f}$$

$$\downarrow 2058$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\int \frac{\left(1-\sin^2(e+fx)\right)^{7/2}}{(-a\sin^2(e+fx)+a+b)^{5/2}}d\sin(e+fx)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 315$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(-\frac{\int -\frac{\left(1-\sin^2(e+fx)\right)^{3/2}\left(-3(a+2b)\sin^2(e+fx)+3a+b\right)}{\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}d\sin(e+fx)}{3a(a+b)}-\frac{b\sin(e+fx)\left(1-\sin^2(e+fx)\right)^{5/2}}{3a(a+b)\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 25$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\int \frac{\left(1-\sin^2(e+fx)\right)^{3/2}\left(-3(a+2b)\sin^2(e+fx)+3a+b\right)}{\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}d\sin(e+fx)}{3a(a+b)}-\frac{b\sin(e+fx)\left(1-\sin^2(e+fx)\right)^{5/2}}{3a(a+b)\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 401$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\int \frac{3\sqrt{1-\sin^2(e+fx)}\left((a+b)(a+2b)-\left(a^2+11ba+8b^2\right)\sin^2(e+fx)\right)}{\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{a(a+b)}-\frac{2b(4a+3b)\sin(e+fx)\left(1-\sin^2(e+fx)\right)^{3/2}}{a(a+b)\sqrt{-a\sin^2(e+fx)+a+b}}}{3a(a+b)}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 27$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{3\int \frac{\sqrt{1-\sin^2(e+fx)}\left((a+b)(a+2b)-\left(a^2+11ba+8b^2\right)\sin^2(e+fx)\right)}{\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{a(a+b)}-\frac{2b(4a+3b)\sin(e+fx)\left(1-\sin^2(e+fx)\right)^{3/2}}{a(a+b)\sqrt{-a\sin^2(e+fx)+a+b}}}{3a(a+b)}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 403$$

3.287. $\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{3\left(\frac{\left(a^2+11ab+8b^2\right)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}{3a}-\frac{\int -\frac{(a+b)\left(2a^2-5ba-8b^2\right)-2(a+2b)\left(a^2-4ba-4b^2\right.}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a}}}{3a}\right.}{a(a+b)}\right.$$

$$\left.\frac{}{3a(a+b)}\right)$$

$$f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)-}{1-\sin^2(e+f}}$$

$$\downarrow 25$$

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{3\left(\frac{\int\frac{(a+b)\left(2a^2-5ba-8b^2\right)-2(a+2b)\left(a^2-4ba-4b^2\right)\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{3a}+\frac{\left(a^2+11ab+8b^2\right)\sqrt{1-\sin^2(e+fx)}\sin(e+fx)}{3a}\right.}{a(a+b)}\right.$$

$$\left.\frac{}{3a(a+b)}\right)$$

$$f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+}{1-\sin^2(e+fx}}$$

$$\downarrow 399$$

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{3\left(\frac{2(a+2b)\left(a^2-4ab-4b^2\right)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{b(a+b)\left(a^2-16ab-16b^2\right)\int\frac{1}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e}}}{a}\right.}{3a}\right.$$

$$\left.\frac{}{a(a+b)}\right)$$

$$f\sqrt{1-\sin^2}$$

$$\downarrow 323$$

3.287. $\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{3\left(\frac{2(a+2b)\left(a^2-4ab-4b^2\right)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{b(a+b)\left(a^2-16ab-16b^2\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\int\frac{}{\sqrt{1-\sin^2}}}{a\sqrt{-a\sin^2(e+fx)+}}\right)}{3a}\right)$$

$$\frac{}{a(a+b)}$$

$$f\sqrt{1-}$$

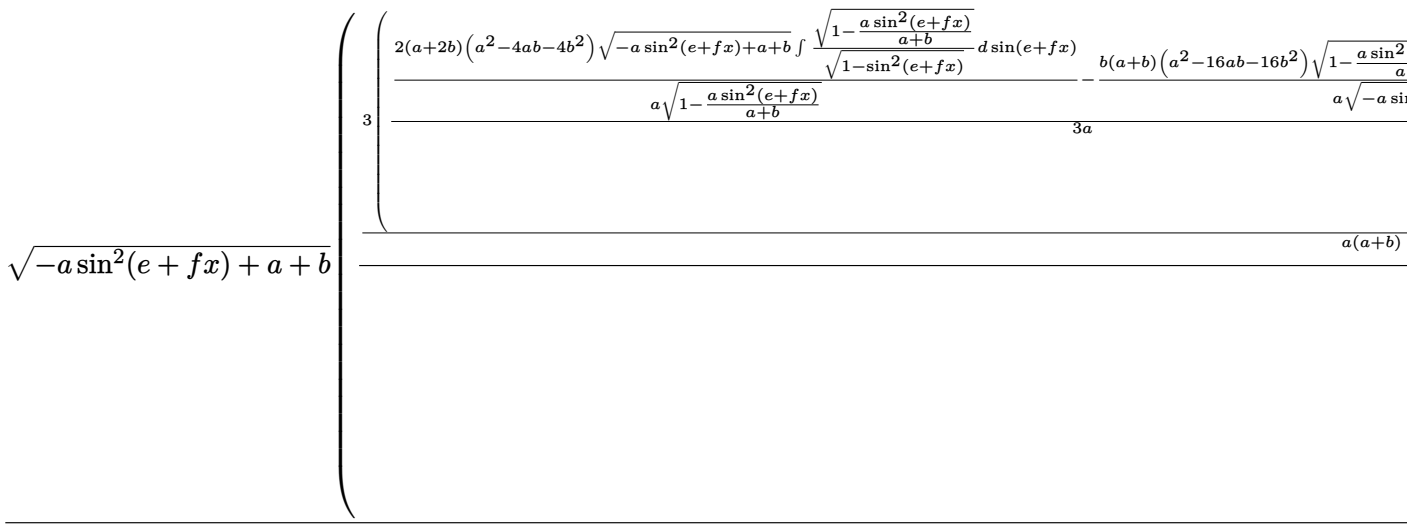
$$\downarrow 321$$

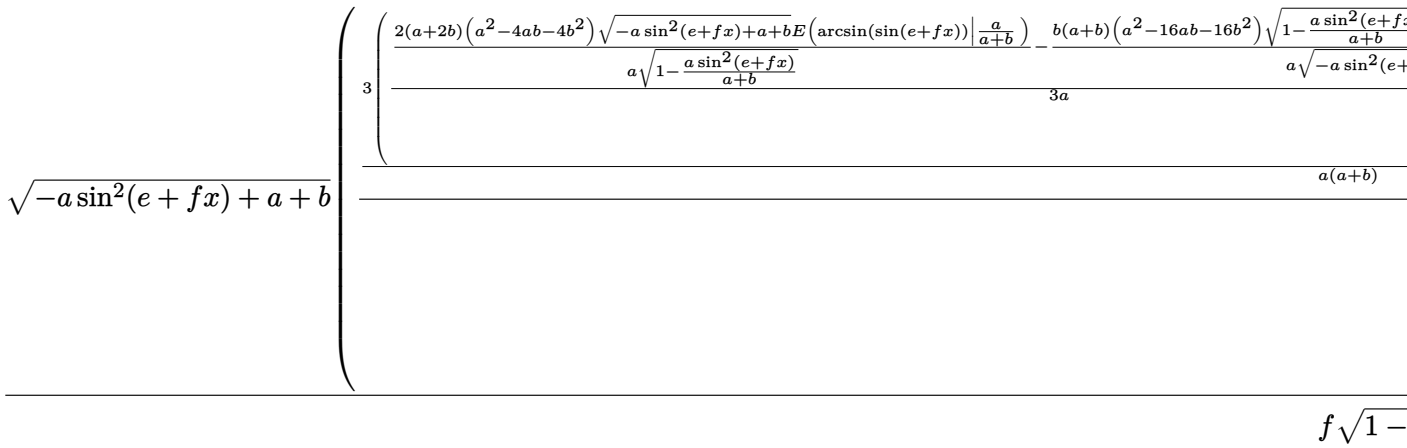
$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{3\left(\frac{2(a+2b)\left(a^2-4ab-4b^2\right)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{b(a+b)\left(a^2-16ab-16b^2\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}\,\mathrm{EllipticF}\left(a\right.}{a\sqrt{-a\sin^2(e+fx)+a+b}}\right)}{3a}\right)$$

$$\frac{}{a(a+b)}$$

$$f\sqrt{1-\sin^2}$$

$$\downarrow 330$$





input
```
Int[Cos[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(Sqrt[a + b - a*Sin[e + f*x]^2]*(-1/3*(b*Sin[e + f*x]*(1 - Sin[e + f*x]^2)
^(5/2))/(a*(a + b)*(a + b - a*Sin[e + f*x]^2)^(3/2)) + ((-2*b*(4*a + 3*b)*
Sin[e + f*x]*(1 - Sin[e + f*x]^2)^(3/2))/(a*(a + b)*Sqrt[a + b - a*Sin[e +
 f*x]^2]) + (3*(((a^2 + 11*a*b + 8*b^2)*Sin[e + f*x]*Sqrt[1 - Sin[e + f*x]
^2]*Sqrt[a + b - a*Sin[e + f*x]^2])/(3*a) + ((2*(a + 2*b)*(a^2 - 4*a*b - 4
*b^2)*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b - a*Sin[e + f*
x]^2])/(a*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - (b*(a + b)*(a^2 - 16*a*b
 - 16*b^2)*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1 - (a*Sin[e +
f*x]^2)/(a + b)])/(a*Sqrt[a + b - a*Sin[e + f*x]^2]))/(3*a)))/(a*(a + b)))
/(3*a*(a + b))))/(f*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[(a + b - a*Sin[e + f*x]^
2)/(1 - Sin[e + f*x]^2)])
```

3.287.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

3.287. $\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

3.287.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.93 (sec) , antiderivative size = 24449, normalized size of antiderivative = 55.44

method	result	size
default	Expression too large to display	24449

input
```
int(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.287.5 Fricas [F]

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos(fx+e)^3}{(b\sec(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^3/(b^3*sec(f*x + e)^6 + 3
*a*b^2*sec(f*x + e)^4 + 3*a^2*b*sec(f*x + e)^2 + a^3), x)
```

3.287.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**3/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Timed out
```

3.287.7 Maxima [F]

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos(fx+e)^3}{\left(b\sec(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(f*x + e)^3/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

3.287.8 Giac [F]

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos(fx+e)^3}{\left(b\sec(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.287.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos(e+fx)^3}{\left(a+\frac{b}{\cos(e+fx)^2}\right)^{5/2}}\,dx$$

input
```
int(cos(e + f*x)^3/(a + b/cos(e + f*x)^2)^(5/2),x)
```

output
```
int(cos(e + f*x)^3/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.288 $\int \frac{\cos^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

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3.288.1 Optimal result

Integrand size = 25, antiderivative size = 559

$$\int \frac{\cos^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = -\frac{2b(5a+4b)\cos^4(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$
$$-\frac{b\cos^6(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b-a\sin^2(e+fx)\right)\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$
$$+\frac{2(2a^3-3a^2b-42ab^2-32b^3)\sin(e+fx)\left(a+b-a\sin^2(e+fx)\right)}{15a^4(a+b)^2f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$
$$+\frac{(3a^2+61ab+48b^2)\cos^2(e+fx)\sin(e+fx)\left(a+b-a\sin^2(e+fx)\right)}{15a^3(a+b)^2f\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$
$$+\frac{(8a^4-11a^3b+27a^2b^2+184ab^3+128b^4)\,E\left(\arcsin(\sin(e+fx))\,\middle|\,\frac{a}{a+b}\right)\left(a+b-a\sin^2(e+fx)\right)}{15a^5(a+b)^2f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}$$
$$-\frac{b(4a^3-9a^2b+120ab^2+128b^3)\,\text{EllipticF}\left(\arcsin(\sin(e+fx)),\frac{a}{a+b}\right)\sqrt{1-\frac{a\sin^2(e+fx)}{a+b}}}{15a^5(a+b)f\sqrt{\cos^2(e+fx)}\sqrt{\sec^2(e+fx)\left(a+b-a\sin^2(e+fx)\right)}}$$

output
```
-2/3*b*(5*a+4*b)*cos(f*x+e)^4*sin(f*x+e)/a^2/(a+b)^2/f/(sec(f*x+e)^2*(a+b-
a*sin(f*x+e)^2))^(1/2)-1/3*b*cos(f*x+e)^6*sin(f*x+e)/a/(a+b)/f/(a+b-a*sin(
f*x+e)^2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)+2/15*(2*a^3-3*a^2*b-42
*a*b^2-32*b^3)*sin(f*x+e)*(a+b-a*sin(f*x+e)^2)/a^4/(a+b)^2/f/(sec(f*x+e)^2
*(a+b-a*sin(f*x+e)^2))^(1/2)+1/15*(3*a^2+61*a*b+48*b^2)*cos(f*x+e)^2*sin(f
*x+e)*(a+b-a*sin(f*x+e)^2)/a^3/(a+b)^2/f/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2
))^(1/2)+1/15*(8*a^4-11*a^3*b+27*a^2*b^2+184*a*b^3+128*b^4)*EllipticE(sin(
f*x+e),(a/(a+b))^(1/2))*(a+b-a*sin(f*x+e)^2)/a^5/(a+b)^2/f/(cos(f*x+e)^2)^
(1/2)/(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)/(1-a*sin(f*x+e)^2/(a+b))^(
1/2)-1/15*b*(4*a^3-9*a^2*b+120*a*b^2+128*b^3)*EllipticF(sin(f*x+e),(a/(a+b
))^(1/2))*(1-a*sin(f*x+e)^2/(a+b))^(1/2)/a^5/(a+b)/f/(cos(f*x+e)^2)^(1/2)/
(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^(1/2)
```

3.288.2 Mathematica [F]

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

input
```
Integrate[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
Integrate[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2), x]
```

3.288.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 531, normalized size of antiderivative = 0.95, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3042, 4636, 2057, 2058, 315, 25, 401, 403, 27, 403, 25, 399, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$
$$\downarrow \text{3042}$$
$$\int \frac{1}{\sec(e+fx)^5\left(a+b\sec(e+fx)^2\right)^{5/2}}dx$$

$$\downarrow 4636$$

$$\frac{\int \frac{\left(1-\sin^2(e+fx)\right)^2}{\left(a+\frac{b}{1-\sin^2(e+fx)}\right)^{5/2}} d\sin(e+fx)}{f}$$

$$\downarrow 2057$$

$$\frac{\int \frac{\left(1-\sin^2(e+fx)\right)^2}{\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^{5/2}} d\sin(e+fx)}{f}$$

$$\downarrow 2058$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\int \frac{\left(1-\sin^2(e+fx)\right)^{9/2}}{\left(-a\sin^2(e+fx)+a+b\right)^{5/2}} d\sin(e+fx)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 315$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(-\frac{\int -\frac{\left(1-\sin^2(e+fx)\right)^{5/2}\left(-\left((3a+8b)\sin^2(e+fx)\right)+3a+b\right)}{\left(-a\sin^2(e+fx)+a+b\right)^{3/2}} d\sin(e+fx)}{3a(a+b)} - \frac{b\sin(e+fx)\left(1-\sin^2(e+fx)\right)^{7/2}}{3a(a+b)\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 25$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\int \frac{\left(1-\sin^2(e+fx)\right)^{5/2}\left(-\left((3a+8b)\sin^2(e+fx)\right)+3a+b\right)}{\left(-a\sin^2(e+fx)+a+b\right)^{3/2}} d\sin(e+fx)}{3a(a+b)} - \frac{b\sin(e+fx)\left(1-\sin^2(e+fx)\right)^{7/2}}{3a(a+b)\left(-a\sin^2(e+fx)+a+b\right)^{3/2}}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 401$$

$$\frac{\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\int \frac{\left(1-\sin^2(e+fx)\right)^{3/2}\left((a+b)(3a+8b)-\left(3a^2+61ba+48b^2\right)\sin^2(e+fx)\right)}{\sqrt{-a\sin^2(e+fx)+a+b}} d\sin(e+fx)}{a(a+b)} - \frac{2b(5a+4b)\sin(e+fx)\left(1-\sin^2(e+fx\right.}{3a(a+b)}\right)}{f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}}}$$

$$\downarrow 403$$

3.288. $\int \frac{\cos^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{\left(3a^2+61ab+48b^2\right)\sin(e+fx)\left(1-\sin^2(e+fx)\right)^{3/2}\sqrt{-a\sin^2(e+fx)+a+b}}{5a}-\frac{\int -\frac{3\sqrt{1-\sin^2(e+fx)}\left((a+b)\left(4a^2-7ba-16b^2\right)\right.}{\sqrt{-a\sin^2(e}}}{a(a+b)}}{3a(a+b)}\right.$$

$$f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a}{}}$$

$$\downarrow 27$$

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{\frac{3\int\frac{\sqrt{1-\sin^2(e+fx)}\left((a+b)\left(4a^2-7ba-16b^2\right)-2\left(2a^3-3ba^2-42b^2a-32b^3\right)\sin^2(e+fx)\right)}{\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{5a}}{a(a+b)}+\frac{\left(3a^2+61ab+48b^2\right)}{3a(a+b)}\right.$$

$$f\sqrt{1-\sin^2(e+fx)}\sqrt{\frac{-a\,s}{1}}$$

$$\downarrow 403$$

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{3\left(\frac{2\left(2a^3-3a^2b-42ab^2-32b^3\right)\sin(e+fx)\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}{3a}-\frac{\int -\frac{(a+b)\left(8a^3-15ba^2+36b^2a+64b^3\right)}{\sqrt{1-\sin^2(e}}}{}\right.\right.$$

$$\downarrow 25$$

3.288. $\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{3\left(\frac{\int\frac{(a+b)\left(8a^3-15ba^2+36b^2a+64b^3\right)-\left(8a^4-11ba^3+27b^2a^2+184b^3a+128b^4\right)\sin^2(e+fx)}{\sqrt{1-\sin^2(e+fx)}\sqrt{-a\sin^2(e+fx)+a+b}}d\sin(e+fx)}{3a}+\frac{2\left(2a^3-3a^2b-4\right.}{5a}\right.}{}\right.$$

$$\downarrow 399$$

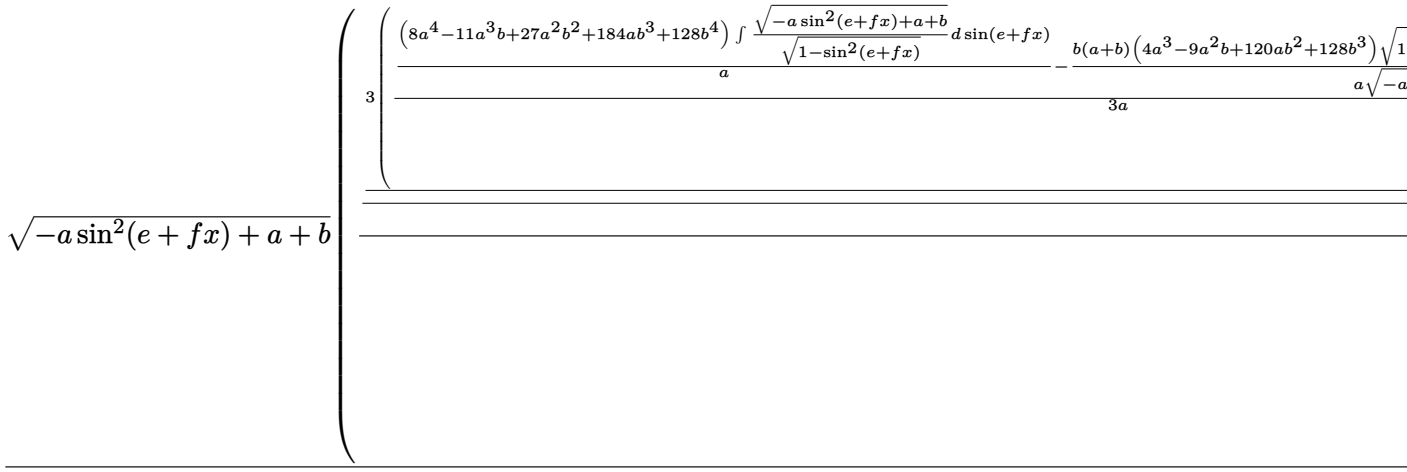
$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{3\left(\frac{\left(8a^4-11a^3b+27a^2b^2+184ab^3+128b^4\right)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{b(a+b)\left(4a^3-9a^2b+120ab^2+128b^3\right)\int-}{3a}\right.}{\sqrt{}}\right.$$

$$\downarrow 323$$

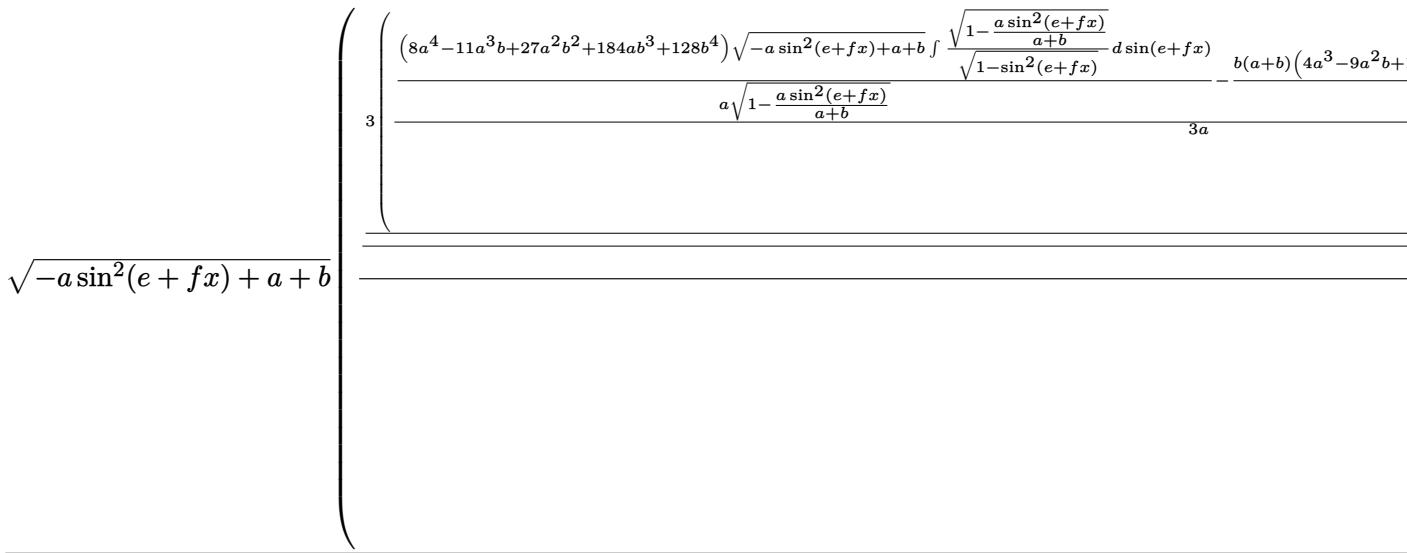
$$\sqrt{-a\sin^2(e+fx)+a+b}\left(\frac{3\left(\frac{\left(8a^4-11a^3b+27a^2b^2+184ab^3+128b^4\right)\int\frac{\sqrt{-a\sin^2(e+fx)+a+b}}{\sqrt{1-\sin^2(e+fx)}}d\sin(e+fx)}{a}-\frac{b(a+b)\left(4a^3-9a^2b+120ab^2+128b^3\right)\sqrt{1}}{3a}\right.}{}\right.$$

$$\downarrow 321$$

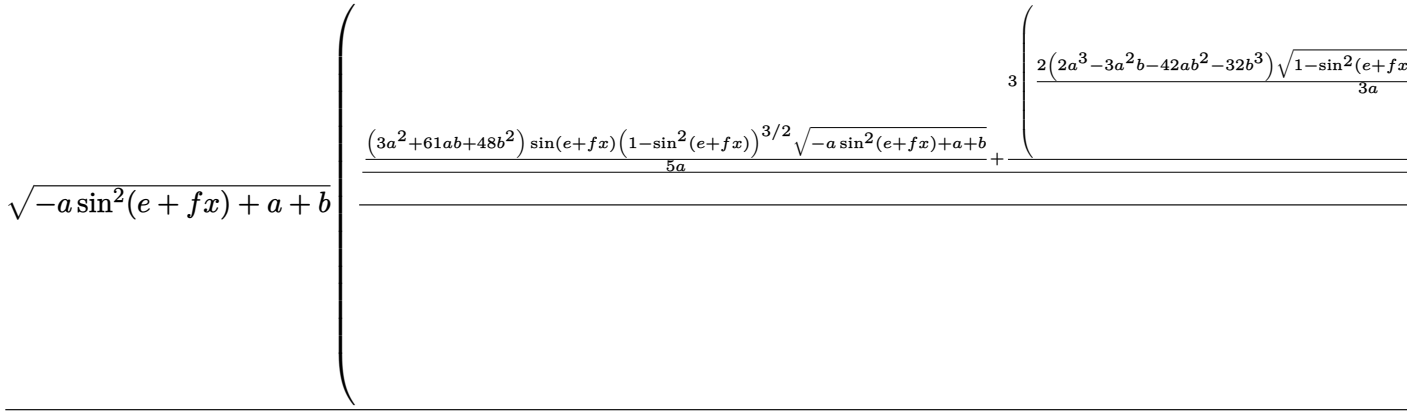
3.288. $\int\frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}dx$



↓ 330



↓ 327



input
```
Int[Cos[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(Sqrt[a + b - a*Sin[e + f*x]^2]*(-1/3*(b*Sin[e + f*x]*(1 - Sin[e + f*x]^2)
^(7/2))/(a*(a + b)*(a + b - a*Sin[e + f*x]^2)^(3/2)) + ((-2*b*(5*a + 4*b)*
Sin[e + f*x]*(1 - Sin[e + f*x]^2)^(5/2))/(a*(a + b)*Sqrt[a + b - a*Sin[e +
 f*x]^2]) + (((3*a^2 + 61*a*b + 48*b^2)*Sin[e + f*x]*(1 - Sin[e + f*x]^2)^
(3/2)*Sqrt[a + b - a*Sin[e + f*x]^2])/(5*a) + (3*((2*(2*a^3 - 3*a^2*b - 42
*a*b^2 - 32*b^3)*Sin[e + f*x]*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[a + b - a*Sin[
e + f*x]^2])/(3*a) + (((8*a^4 - 11*a^3*b + 27*a^2*b^2 + 184*a*b^3 + 128*b^
4)*EllipticE[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[a + b - a*Sin[e + f*x]^
2])/(a*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)]) - (b*(a + b)*(4*a^3 - 9*a^2*b
 + 120*a*b^2 + 128*b^3)*EllipticF[ArcSin[Sin[e + f*x]], a/(a + b)]*Sqrt[1
 - (a*Sin[e + f*x]^2)/(a + b)])/(a*Sqrt[a + b - a*Sin[e + f*x]^2]))/(3*a)))
/(5*a))/(a*(a + b)))/(3*a*(a + b))))/(f*Sqrt[1 - Sin[e + f*x]^2]*Sqrt[(a +
 b - a*Sin[e + f*x]^2)/(1 - Sin[e + f*x]^2)])
```

3.288.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

3.288.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 13.44 (sec) , antiderivative size = 31577, normalized size of antiderivative = 56.49

method	result	size
default	Expression too large to display	31577

input
```
int(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.288.5 Fricas [F]

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos\left(fx+e\right)^5}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(f*x + e)^2 + a)*cos(f*x + e)^5/(b^3*sec(f*x + e)^6 + 3
*a*b^2*sec(f*x + e)^4 + 3*a^2*b*sec(f*x + e)^2 + a^3), x)
```

3.288.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**5/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Timed out
```

3.288.7 Maxima [F]

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos(fx+e)^5}{(b\sec(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(f*x + e)^5/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

3.288.8 Giac [F]

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos(fx+e)^5}{(b\sec(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.288.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos(e+fx)^5}{\left(a+\frac{b}{\cos(e+fx)^2}\right)^{5/2}}\,dx$$

input
```
int(cos(e + f*x)^5/(a + b/cos(e + f*x)^2)^(5/2),x)
```

output
```
int(cos(e + f*x)^5/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.289 $\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

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3.289.2 Mathematica [C] (warning: unable to verify) . . . 2172
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3.289.1 Optimal result

Integrand size = 25, antiderivative size = 133

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{b^{5/2}f} - \frac{a\sec^2(e+fx)\tan(e+fx)}{3b(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{a(3a+5b)\tan(e+fx)}{3b^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

output
```
arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b^(5/2)/f-1/3*a*(3*
a+5*b)*tan(f*x+e)/b^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/3*a*sec(f*x+e
)^2*tan(f*x+e)/b/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(3/2)
```

3.289.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.35 (sec) , antiderivative size = 592, normalized size of antiderivative = 4.45

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{(a+2b+a\cos(2e+2fx))^{5/2}\sec^6(e+fx)\sqrt{1-\frac{2a\sin^2(e+fx)}{2a+2b}}\tan(e+fx)\left(-\right.}{}$$

input
```
Integrate[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
((a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^6*Sqrt[1 - (2*a*Sin[e +
 f*x]^2)/(2*a + 2*b)]*Tan[e + f*x]*((-24*b*Cos[e + f*x]^2*HypergeometricPF
Q[{2, 2, 2}, {1, 9/2}, -((b*Tan[e + f*x]^2)/(a + b))]*Sin[e + f*x]^2*(-((b
*Sec[e + f*x]^2*(a + b - a*Sin[e + f*x]^2)*Tan[e + f*x]^2)/(a + b)^2))^(5/
2))/(a + b) - (Sec[e + f*x]^6*(24*b^3*Hypergeometric2F1[2, 2, 9/2, -((b*Ta
n[e + f*x]^2)/(a + b))]*Sin[e + f*x]^6*(4*b^2 + a*b*(8 - 7*Sin[e + f*x]^2)
 + a^2*(4 - 7*Sin[e + f*x]^2 + 3*Sin[e + f*x]^4))*Sqrt[-((b*Sec[e + f*x]^2
*(a + b - a*Sin[e + f*x]^2)*Tan[e + f*x]^2)/(a + b)^2)] + 35*(a + b)*Cos[e
 + f*x]^2*(15*b^2 + 10*a*b*(3 - 2*Sin[e + f*x]^2) + a^2*(15 - 20*Sin[e + f
*x]^2 + 8*Sin[e + f*x]^4))*(-3*ArcSin[Sqrt[-((b*Tan[e + f*x]^2)/(a + b))]]
*(a + b - a*Sin[e + f*x]^2)^2 - (a + b)*Cos[e + f*x]^2*(-3*a*Cos[e + f*x]^
2 - b*(3 + Sin[e + f*x]^2))*Sqrt[-((b*Sec[e + f*x]^2*(a + b - a*Sin[e + f*
x]^2)*Tan[e + f*x]^2)/(a + b)^2)]))/(a + b)^5))/(315*(2*a + 2*b)^2*f*(a +
 b*Sec[e + f*x]^2)^(5/2)*Sqrt[(a + b*Sec[e + f*x]^2)/(a + b)]*Sqrt[2*a + 2
*b - 2*a*Sin[e + f*x]^2]*(1 - (a*Sin[e + f*x]^2)/(a + b))^(3/2)*(-((b*Tan[
e + f*x]^2)/(a + b)))^(5/2))
```

3.289.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 4634, 315, 298, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sec(e+fx)^6}{(a+b\sec(e+fx)^2)^{5/2}}dx$$

$$\downarrow 4634$$

$$\frac{\int \frac{\left(\tan^2(e+fx)+1\right)^2}{(b\tan^2(e+fx)+a+b)^{5/2}}d\tan(e+fx)}{f}$$

$$\downarrow 315$$

$$\frac{\frac{\int \frac{3(a+b)\tan^2(e+fx)+a+3b}{\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{3b(a+b)}-\frac{a\tan(e+fx)\left(\tan^2(e+fx)+1\right)}{3b(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\downarrow 298$$

$$\frac{\frac{3(a+b)\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{b}-\frac{a(3a+5b)\tan(e+fx)}{b(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3b(a+b)}-\frac{a\tan(e+fx)\left(\tan^2(e+fx)+1\right)}{3b(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\downarrow 224$$

$$\frac{\frac{3(a+b)\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{b}-\frac{a(3a+5b)\tan(e+fx)}{b(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3b(a+b)}-\frac{a\tan(e+fx)\left(\tan^2(e+fx)+1\right)}{3b(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\downarrow 219$$

$$\frac{\frac{3(a+b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{b^{3/2}}-\frac{a(3a+5b)\tan(e+fx)}{b(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3b(a+b)}-\frac{a\tan(e+fx)\left(\tan^2(e+fx)+1\right)}{3b(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

input
```
Int[Sec[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(-1/3*(a*Tan[e + f*x]*(1 + Tan[e + f*x]^2))/(b*(a + b)*(a + b + b*Tan[e +
f*x]^2)^(3/2)) + ((3*(a + b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b
*Tan[e + f*x]^2]])/b^(3/2) - (a*(3*a + 5*b)*Tan[e + f*x])/(b*(a + b)*Sqrt[
a + b + b*Tan[e + f*x]^2]))/(3*b*(a + b)))/f
```

3.289.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.289.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2354 vs. $2(119) = 238$.

Time = 6.33 (sec) , antiderivative size = 2355, normalized size of antiderivative = 17.71

method	result	size
default	Expression too large to display	2355

input
```
int(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6/f/(a+b)^2/b^(7/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4
*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f
*x+e)^2+a+b)*(-36*b^(5/2)*a^2*(1-cos(f*x+e))^5*csc(f*x+e)^5-24*b^(7/2)*a*(
1-cos(f*x+e))^5*csc(f*x+e)^5-12*b^(3/2)*a^3*(1-cos(f*x+e))^5*csc(f*x+e)^5+
8*b^(5/2)*a^2*(1-cos(f*x+e))^3*csc(f*x+e)^3-48*(1-cos(f*x+e))^3*b^(7/2)*a*
csc(f*x+e)^3+24*b^(3/2)*a^3*(1-cos(f*x+e))^3*csc(f*x+e)^3-36*b^(5/2)*a^2*(
csc(f*x+e)-cot(f*x+e))-24*b^(7/2)*a*(csc(f*x+e)-cot(f*x+e))+3*ln(4*(-a*(1-
cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2+b^(1/2)*(a*(1-c
os(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e
))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)-2*a*(csc(f*
x+e)-cot(f*x+e))-a-b)/((1-cos(f*x+e))^2*csc(f*x+e)^2+2*csc(f*x+e)-2*cot(f*
x+e)+1))*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-
2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(
3/2)*a^2*b+6*ln(4*(-a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc
(f*x+e)^2+b^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(
f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)
^2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(f*x+e))-a-b)/((1-cos(f*x+e))^2*csc(f*x+e
)^2+2*csc(f*x+e)-2*cot(f*x+e)+1))*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-co
s(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+
e))^2*csc(f*x+e)^2+a+b)^(3/2)*a*b^2+3*ln(4*(-a*(1-cos(f*x+e))^2*csc(f*x...
```

3.289.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 328 vs. $2(119) = 238$.

Time $= 0.50$ (sec) , antiderivative size $= 688$, normalized size of antiderivative $= 5.17$

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \left[\frac{3\left(\left(a^4+2\,a^3b+a^2b^2\right)\cos\left(fx+e\right)^4+a^2b^2+2\,ab^3+b^4+2\left(a^3b+2\,a^2b^2+\right.\right.}{}\right.$$

input
```
integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/12*(3*((a^4 + 2*a^3*b + a^2*b^2)*cos(f*x + e)^4 + a^2*b^2 + 2*a*b^3 + b
^4 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*cos(f*x + e)^2)*sqrt(b)*log(((a^2 - 6*a
*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f
*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*((3*a^3*b + 5*a^2*b^2)*c
os(f*x + e)^3 + 2*(2*a^2*b^2 + 3*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)
^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*f*c
os(f*x + e)^4 + 2*(a^3*b^4 + 2*a^2*b^5 + a*b^6)*f*cos(f*x + e)^2 + (a^2*b^
5 + 2*a*b^6 + b^7)*f), 1/6*(3*((a^4 + 2*a^3*b + a^2*b^2)*cos(f*x + e)^4 +
a^2*b^2 + 2*a*b^3 + b^4 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*cos(f*x + e)^2)*sq
rt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(
f*x + e))) - 2*((3*a^3*b + 5*a^2*b^2)*cos(f*x + e)^3 + 2*(2*a^2*b^2 + 3*a*
b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e
))/((a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^4 + 2*(a^3*b^4 + 2*a^2*
b^5 + a*b^6)*f*cos(f*x + e)^2 + (a^2*b^5 + 2*a*b^6 + b^7)*f)]
```

3.289.6 Sympy [F]

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)**6/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral(sec(e + f*x)**6/(a + b*sec(e + f*x)**2)**(5/2), x)
```

3.289.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. 2(119) = 238.

Time = 0.20 (sec) , antiderivative size = 275, normalized size of antiderivative = 2.07

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx =$$
$$-\frac{\left(\frac{3\tan(fx+e)^2}{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}b}+\frac{2a}{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}b^2}+\frac{2}{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}b}\right)\tan(fx+e)-\frac{3\,\mathrm{arsinh}\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{5}{2}}}-\frac{}{\sqrt{b}}$$

input
```
integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*((3*tan(f*x + e)^2/((b*tan(f*x + e)^2 + a + b)^(3/2)*b) + 2*a/((b*tan
(f*x + e)^2 + a + b)^(3/2)*b^2) + 2/((b*tan(f*x + e)^2 + a + b)^(3/2)*b))*
tan(f*x + e) - 3*arcsinh(b*tan(f*x + e)/sqrt((a + b)*b))/b^(5/2) - 2*tan(f
*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^2) - tan(f*x + e)/((b*tan(
f*x + e)^2 + a + b)^(3/2)*(a + b)) + 3*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2
 + a + b)*b^2) - 2*a*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)*
b^2) + 2*tan(f*x + e)/((b*tan(f*x + e)^2 + a + b)^(3/2)*b) - 4*tan(f*x + e
)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)*b))/f
```

3.289.8 Giac [F]

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sec\left(fx+e\right)^6}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.289.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{1}{\cos\left(e+f\,x\right)^6\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

input
```
int(1/(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^(5/2)),x)
```

output
```
int(1/(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^(5/2)), x)
```

3.290 $\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

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3.290.1 Optimal result

Integrand size = 25, antiderivative size = 79

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\sec^2(e+fx)\tan(e+fx)}{3(a+b)f\,(a+b+b\tan^2(e+fx))^{3/2}} + \frac{2\tan(e+fx)}{3(a+b)^2 f\sqrt{a+b+b\tan^2(e+fx)}}$$

output
```
2/3*tan(f*x+e)/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/3*sec(f*x+e)^2*tan(f
*x+e)/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(3/2)
```

3.290.2 Mathematica [A] (verified)

Time = 5.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.94

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{(a+2b+a\cos(2(e+fx)))(2a+3b+a\cos(2(e+fx)))\sec^4(e+fx)\tan(e}{6(a+b)^2 f\,(a+b\sec^2(e+fx))^{5/2}}$$

input
```
Integrate[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*(2*a + 3*b + a*Cos[2*(e + f*x)])*Sec[e + f
*x]^4*Tan[e + f*x])/(6*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

3.290.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 4634, 292, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sec(e+fx)^4}{(a+b\sec(e+fx)^2)^{5/2}}\,dx$$

$$\downarrow 4634$$

$$\frac{\int \frac{\tan^2(e+fx)+1}{(b\tan^2(e+fx)+a+b)^{5/2}}\,d\tan(e+fx)}{f}$$

$$\downarrow 292$$

$$\frac{2\int \frac{1}{\left(b\tan^2(e+fx)+a+b\right)^{3/2}}\,d\tan(e+fx)}{3(a+b)} + \frac{\tan(e+fx)\left(\tan^2(e+fx)+1\right)}{3(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\downarrow 208$$

$$\frac{\frac{2\tan(e+fx)}{3(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}} + \frac{\left(\tan^2(e+fx)+1\right)\tan(e+fx)}{3(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

input
```
Int[Sec[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
((Tan[e + f*x]*(1 + Tan[e + f*x]^2))/(3*(a + b)*(a + b + b*Tan[e + f*x]^2)
^(3/2)) + (2*Tan[e + f*x])/(3*(a + b)^2*Sqrt[a + b + b*Tan[e + f*x]^2]))/f
```

3.290.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 292
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> Si
mp[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] - Simp[c*(q/(
a*(p + 1)))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1), x], x] /; FreeQ[
{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q + 1) + 1, 0] && Gt
Q[q, 0] && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.290.4 Maple [A] (verified)

Time = 3.14 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{\left(b+a\cos(fx+e)^2\right)\left(2a\cos(fx+e)^2+a+3b\right)\tan(fx+e)\sec(fx+e)^4}{3f(a^2+2ab+b^2)\left(a+b\sec(fx+e)^2\right)^{\frac{5}{2}}}$	75

input
```
int(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/f/(a^2+2*a*b+b^2)*(b+a*cos(f*x+e)^2)*(2*a*cos(f*x+e)^2+a+3*b)/(a+b*sec
(f*x+e)^2)^(5/2)*tan(f*x+e)*sec(f*x+e)^4
```

3.290.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.70

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\left(2\,a\cos(fx+e)^3+(a+3\,b)\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin}{3\left((a^4+2\,a^3b+a^2b^2)f\cos(fx+e)^4+2\,(a^3b+2\,a^2b^2+ab^3)f\cos(fx+e)^2\right.}$$

input
```
integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(2*a*cos(f*x + e)^3 + (a + 3*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)*sin(f*x + e)/((a^4 + 2*a^3*b + a^2*b^2)*f*cos(f*x + e)
^4 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*f*cos(f*x + e)^2 + (a^2*b^2 + 2*a*b^3 +
 b^4)*f)
```

3.290.6 Sympy [F]

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)**4/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral(sec(e + f*x)**4/(a + b*sec(e + f*x)**2)**(5/2), x)
```

3.290.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.48

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\frac{2\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+b}(a+b)^2}+\frac{\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}(a+b)}-\frac{\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}b}+\frac{}{\sqrt{b\tan}}}{3\,f}$$

input
```
integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
1/3*(2*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^2) + tan(f*x +
 e)/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)) - tan(f*x + e)/((b*tan(f*x
+ e)^2 + a + b)^(3/2)*b) + tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a
 + b)*b))/f
```

3.290.8 Giac [F]

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sec(fx+e)^4}{\left(b\sec(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.290.9 Mupad [B] (verification not implemented)

Time = 29.44 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.94

$$\int \frac{\sec^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{2\left(e^{e\,4i+f\,x\,4i}-1\right)\sqrt{a+\frac{b}{\left(\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}\right)^2}}\left(a\,1i+a\,e^{e\,2i+f\,x\,2i}\,4i+a\,e^{e\,4i+f\,x\,4i}\,1i+b\,e^{e\,2i+f\,x\,2i}\,6i\right)}{3\,f\,(a+b)^2\left(a+2\,a\,e^{e\,2i+f\,x\,2i}+a\,e^{e\,4i+f\,x\,4i}+4\,b\,e^{e\,2i+f\,x\,2i}\right)^2}$$

input
```
int(1/(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^(5/2)),x)
```

output
```
-(2*(exp(e*4i + f*x*4i) - 1)*(a + b/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f
*x*1i)/2)^2)^(1/2)*(a*1i + a*exp(e*2i + f*x*2i)*4i + a*exp(e*4i + f*x*4i)*
1i + b*exp(e*2i + f*x*2i)*6i))/(3*f*(a + b)^2*(a + 2*a*exp(e*2i + f*x*2i)
+ a*exp(e*4i + f*x*4i) + 4*b*exp(e*2i + f*x*2i))^2)
```

3.291 $\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

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3.291.1 Optimal result

Integrand size = 25, antiderivative size = 71

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\tan(e+fx)}{3(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{2\tan(e+fx)}{3(a+b)^2 f\sqrt{a+b+b\tan^2(e+fx)}}$$

output
```
2/3*tan(f*x+e)/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/3*tan(f*x+e)/(a+b)/f
/(a+b+b*tan(f*x+e)^2)^(3/2)
```

3.291.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 215 vs. 2(71) = 142.

Time = 6.14 (sec) , antiderivative size = 215, normalized size of antiderivative = 3.03

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{(3a+b)(a+2b+a\cos(2e+2fx))^{5/2}\sec^5(e+fx)\left(\frac{\sqrt{2}\sin(e+fx)}{(a+b)(a+b-a\sin^2(e+fx))^{3/2}}\right.}{48af\left(a+b\sec^2(e+fx)\right)^{5/2}} - \frac{(a+2b+a\cos(2e+2fx))^{5/2}\sec^4(e+fx)\tan(e+fx)}{8\sqrt{2}af\left(a+b\sec^2(e+fx)\right)^{5/2}(a+b-a\sin^2(e+fx))^{3/2}}$$

input
```
Integrate[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
((3*a + b)*(a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^5*((Sqrt[2]*S
in[e + f*x])/((a + b)*(a + b - a*Sin[e + f*x]^2)^(3/2)) + (2*Sqrt[2]*Sin[e
 + f*x])/((a + b)^2*Sqrt[a + b - a*Sin[e + f*x]^2])))/(48*a*f*(a + b*Sec[e
 + f*x]^2)^(5/2)) - ((a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^4*T
an[e + f*x])/(8*Sqrt[2]*a*f*(a + b*Sec[e + f*x]^2)^(5/2)*(a + b - a*Sin[e
+ f*x]^2)^(3/2))
```

3.291.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 4634, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sec(e+fx)^2}{(a+b\sec(e+fx)^2)^{5/2}}\,dx$$

$$\downarrow 4634$$

$$\frac{\int \frac{1}{(b\tan^2(e+fx)+a+b)^{5/2}}\,d\tan(e+fx)}{f}$$

$$\downarrow 209$$

$$\frac{2\int \frac{1}{\left(b\tan^2(e+fx)+a+b\right)^{3/2}}\,d\tan(e+fx)}{3(a+b)} + \frac{\tan(e+fx)}{3(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\downarrow 208$$

$$\frac{\frac{2\tan(e+fx)}{3(a+b)^2\sqrt{a+b\tan^2(e+fx)+b}} + \frac{\tan(e+fx)}{3(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

input
```
Int[Sec[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(Tan[e + f*x]/(3*(a + b)*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (2*Tan[e + f*
x])/(3*(a + b)^2*Sqrt[a + b + b*Tan[e + f*x]^2]))/f
```

3.291.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.291.4 Maple [A] (verified)

Time = 3.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.18

method	result	size
default	$\frac{\left(3a\cos(fx+e)^2+\cos(fx+e)^2b+2b\right)\left(b+a\cos(fx+e)^2\right)\tan(fx+e)\sec(fx+e)^4}{3f(a^2+2ab+b^2)\left(a+b\sec(fx+e)^2\right)^{\frac{5}{2}}}$	84

input
```
int(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/f/(a^2+2*a*b+b^2)*(3*a*cos(f*x+e)^2+cos(f*x+e)^2*b+2*b)*(b+a*cos(f*x+e
)^2)/(a+b*sec(f*x+e)^2)^(5/2)*tan(f*x+e)*sec(f*x+e)^4
```

3.291.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(63) = 126$.

Time = 0.38 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.89

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\left((3\,a+b)\cos(fx+e)^3+2\,b\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\sin}{3\left((a^4+2\,a^3b+a^2b^2)f\cos(fx+e)^4+2\,(a^3b+2\,a^2b^2+ab^3)f\cos(fx+e)^2\right.}$$

input
```
integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/3*((3*a + b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)*sin(f*x + e)/((a^4 + 2*a^3*b + a^2*b^2)*f*cos(f*x + e)
^4 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*f*cos(f*x + e)^2 + (a^2*b^2 + 2*a*b^3 +
 b^4)*f)
```

3.291.6 Sympy [F]

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)**2/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral(sec(e + f*x)**2/(a + b*sec(e + f*x)**2)**(5/2), x)
```

3.291.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.86

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\frac{2\tan(fx+e)}{\sqrt{b\tan(fx+e)^2+a+b(a+b)^2}}+\frac{\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)^{\frac{3}{2}}(a+b)}}{3\,f}$$

input
```
integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
1/3*(2*tan(f*x + e)/(sqrt(b*tan(f*x + e)^2 + a + b)*(a + b)^2) + tan(f*x +
 e)/((b*tan(f*x + e)^2 + a + b)^(3/2)*(a + b)))/f
```

3.291.8 Giac [F]

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\sec(fx+e)^2}{\left(b\sec(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.291.9 Mupad [B] (verification not implemented)

Time = 30.12 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.42

$$\int \frac{\sec^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx =$$
$$-\frac{\left(e^{e\,4i+f\,x\,4i}-1\right)\sqrt{a+\frac{b}{\left(\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}\right)^2}}\left(a\,3i+b\,1i+a\,e^{e\,2i+f\,x\,2i}\,6i+a\,e^{e\,4i+f\,x\,4i}\,3i+b\,e^{e\,2i+f\,x\,2i}\,10i+\right.}{3\,f\,(a+b)^2\left(a+2\,a\,e^{e\,2i+f\,x\,2i}+a\,e^{e\,4i+f\,x\,4i}+4\,b\,e^{e\,2i+f\,x\,2i}\right)^2}$$

input
```
int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^(5/2)),x)
```

output
```
-((exp(e*4i + f*x*4i) - 1)*(a + b/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x
*1i)/2)^2)^(1/2)*(a*3i + b*1i + a*exp(e*2i + f*x*2i)*6i + a*exp(e*4i + f*x
*4i)*3i + b*exp(e*2i + f*x*2i)*10i + b*exp(e*4i + f*x*4i)*1i))/(3*f*(a + b
)^2*(a + 2*a*exp(e*2i + f*x*2i) + a*exp(e*4i + f*x*4i) + 4*b*exp(e*2i + f*
x*2i))^2)
```

3.292 $\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

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3.292.1 Optimal result

Integrand size = 16, antiderivative size = 125

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} - \frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(5a+3b)\tan(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

output
```
arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/f-1/3*b*(5*a
+3*b)*tan(f*x+e)/a^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/3*b*tan(f*x+e)
/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(3/2)
```

3.292.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 6.50 (sec) , antiderivative size = 1927, normalized size of antiderivative = 15.42

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(-5/2),x]
```

output
```
(3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/
(a + b)]*Cos[e + f*x]^4*Sin[e + f*x])/(4*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^
(5/2)*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2, 5/2, 3
/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -2, 7
/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[
3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*
x]^2)*((15*a*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e
+ f*x]^2)/(a + b)]*Cos[e + f*x]^5*Sin[e + f*x]^2)/(4*Sqrt[2]*(a + b - a*Si
n[e + f*x]^2)^(7/2)*(3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2,
 (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*
x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2,
Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)
*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*C
os[e + f*x]^5)/(4*Sqrt[2]*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*Appe
llF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a
*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] -
 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/
(a + b)])*Sin[e + f*x]^2)) - (3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e
+ f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^3*Sin[e + f*x]^2)/(Sqrt
[2]*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2, 5/2, ...
```

3.292.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {3042, 4616, 316, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{(a+b\sec(e+fx)^2)^{5/2}}dx$$
$$\downarrow 4616$$
$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^{5/2}}d\tan(e+fx)}{f}$$
$$\downarrow 316$$

$$\frac{\int \frac{-2b\tan^2(e+fx)+3a+b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}\,d\tan(e+fx)}{3a(a+b)} - \frac{b\tan(e+fx)}{3a(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}$$

$$\Big\downarrow f$$

$$\Big\downarrow 402$$

$$\frac{\int \frac{3(a+b)^2}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}\,d\tan(e+fx)}{a(a+b)} - \frac{b(5a+3b)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}} - \frac{b\tan(e+fx)}{3a(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}$$

$$\Big\downarrow f$$

$$\Big\downarrow 27$$

$$\frac{3(a+b)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}\,d\tan(e+fx)}{a} - \frac{b(5a+3b)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}} - \frac{b\tan(e+fx)}{3a(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}$$

$$\Big\downarrow f$$

$$\Big\downarrow 291$$

$$\frac{3(a+b)\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}\,d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{a} - \frac{b(5a+3b)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}} - \frac{b\tan(e+fx)}{3a(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}$$

$$\Big\downarrow f$$

$$\Big\downarrow 216$$

$$\frac{3(a+b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}} - \frac{b(5a+3b)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}} - \frac{b\tan(e+fx)}{3a(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}$$

$$\Big\downarrow f$$

input

```
Int[(a + b*Sec[e + f*x]^2)^(-5/2),x]
```

output

```
(-1/3*(b*Tan[e + f*x])/(a*(a + b)*(a + b + b*Tan[e + f*x]^2)^(3/2)) + ((3*
(a + b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/a^(
3/2) - (b*(5*a + 3*b)*Tan[e + f*x])/(a*(a + b)*Sqrt[a + b + b*Tan[e + f*x]
^2]))/(3*a*(a + b)))/f
```

3.292. $\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

3.292.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

3.292.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1262 vs. 2(111) = 222.

Time = 3.76 (sec) , antiderivative size = 1263, normalized size of antiderivative = 10.10

method	result	size
default	Expression too large to display	1263

input
```
int(1/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/f/(a+b)^2/a^2/(-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x
+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2
*csc(f*x+e)^2+a+b)*(12*(-a)^(1/2)*a^2*b*(1-cos(f*x+e))^5*csc(f*x+e)^5+18*(
-a)^(1/2)*a*b^2*(1-cos(f*x+e))^5*csc(f*x+e)^5+6*(-a)^(1/2)*b^3*(1-cos(f*x+
e))^5*csc(f*x+e)^5-24*(1-cos(f*x+e))^3*a^2*(-a)^(1/2)*b*csc(f*x+e)^3+4*(-a
)^(1/2)*a*b^2*(1-cos(f*x+e))^3*csc(f*x+e)^3+12*(-a)^(1/2)*b^3*(1-cos(f*x+e
))^3*csc(f*x+e)^3-3*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*cs
c(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+
e)^2+a+b)^(3/2)*ln(4*((-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos
(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e
))^2*csc(f*x+e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(f*x+e)))/((1-cos(f*x+e))^
2*csc(f*x+e)^2+1))*a^2-6*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))
^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc
(f*x+e)^2+a+b)^(3/2)*ln(4*((-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(
1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(
f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(f*x+e)))/((1-cos(f*x
+e))^2*csc(f*x+e)^2+1))*a*b-3*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*
x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^
2*csc(f*x+e)^2+a+b)^(3/2)*ln(4*((-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^
4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b...
```

3.292.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(111) = 222$.

Time $= 0.73$ (sec) , antiderivative size $= 881$, normalized size of antiderivative $= 7.05$

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \left[-\frac{3\left(\left(a^4+2\,a^3b+a^2b^2\right)\cos\left(fx+e\right)^4+a^2b^2+2\,ab^3+b^4+2\left(a^3b+2\,a^2b^2\right.\right.}{}\right.$$

$$\left. -\frac{3\left(\left(a^4+2\,a^3b+a^2b^2\right)\cos\left(fx+e\right)^4+a^2b^2+2\,ab^3+b^4+2\left(a^3b+2\,a^2b^2+ab^3\right)\cos\left(fx+e\right)^2\right)\sqrt{a}\arctan\left(\right.}{12\left(\left(a^7+2\,a^6b+a^5b^2\right).\right.}\right.$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[-1/24*(3*((a^4 + 2*a^3*b + a^2*b^2)*cos(f*x + e)^4 + a^2*b^2 + 2*a*b^3 +
b^4 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*c
os(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b +
5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 -
 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x
 + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)
*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(2*(3*a^3*b +
 2*a^2*b^2)*cos(f*x + e)^3 + (5*a^2*b^2 + 3*a*b^3)*cos(f*x + e))*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2
)*f*cos(f*x + e)^4 + 2*(a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a
^5*b^2 + 2*a^4*b^3 + a^3*b^4)*f), -1/12*(3*((a^4 + 2*a^3*b + a^2*b^2)*cos(
f*x + e)^4 + a^2*b^2 + 2*a*b^3 + b^4 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*cos(f
*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*
x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a
^2*b)*cos(f*x + e)^2)*sin(f*x + e))) + 4*(2*(3*a^3*b + 2*a^2*b^2)*cos(f*x
+ e)^3 + (5*a^2*b^2 + 3*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*x + e)^4 +
 2*(a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*a^4*b^...
```

3.292.6 Sympy [F]

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{1}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(-5/2), x)
```

3.292.7 Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.292.8 Giac [F]

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{1}{(b\sec(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(-5/2), x)
```

3.292.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(e+fx)^2}\right)^{5/2}}\,dx$$

input
```
int(1/(a + b/cos(e + f*x)^2)^(5/2),x)
```

output
```
int(1/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.293 $\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

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3.293.1 Optimal result

Integrand size = 25, antiderivative size = 187

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{(a-5b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2a^{7/2}f} + \frac{\cos(e+fx)\sin(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b(3a+5b)\tan(e+fx)}{6a^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b(3a^2+22ab+15b^2)\tan(e+fx)}{6a^3(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

output

```
1/2*(a-5*b)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(7/2)/
f+1/6*b*(3*a^2+22*a*b+15*b^2)*tan(f*x+e)/a^3/(a+b)^2/f/(a+b+b*tan(f*x+e)^2
)^(1/2)+1/2*cos(f*x+e)*sin(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)^(3/2)+1/6*b*(3*
a+5*b)*tan(f*x+e)/a^2/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(3/2)
```

3.293.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 16.74 (sec) , antiderivative size = 1775, normalized size of antiderivative = 9.49

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(3*(a + b)*AppellF1[1/2, -3, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/
(a + b)]*Cos[e + f*x]^8*Sin[e + f*x])/(4*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^
(5/2)*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -3, 5/2, 3
/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -3, 7
/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 6*(a + b)*AppellF1[
3/2, -2, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*
x]^2)*((15*a*(a + b)*AppellF1[1/2, -3, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e
+ f*x]^2)/(a + b)]*Cos[e + f*x]^7*Sin[e + f*x]^2)/(4*Sqrt[2]*(a + b - a*Si
n[e + f*x]^2)^(7/2)*(3*(a + b)*AppellF1[1/2, -3, 5/2, 3/2, Sin[e + f*x]^2,
 (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -3, 7/2, 5/2, Sin[e + f*
x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 6*(a + b)*AppellF1[3/2, -2, 5/2, 5/2,
Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)
*AppellF1[1/2, -3, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*C
os[e + f*x]^7)/(4*Sqrt[2]*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*Appe
llF1[1/2, -3, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a
*AppellF1[3/2, -3, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] -
 6*(a + b)*AppellF1[3/2, -2, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/
(a + b)])*Sin[e + f*x]^2)) - (9*(a + b)*AppellF1[1/2, -3, 5/2, 3/2, Sin[e
+ f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^5*Sin[e + f*x]^2)/(2*Sq
rt[2]*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -3, 5/2...
```

3.293.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4634, 316, 25, 402, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$
$$\downarrow \text{3042}$$
$$\int \frac{1}{\sec(e+fx)^2\,(a+b\sec(e+fx)^2)^{5/2}}dx$$
$$\downarrow \text{4634}$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)^2(b\tan^2(e+fx)+a+b)^{5/2}}d\tan(e+fx)}{f}$$

$$\downarrow 316$$

$$\frac{\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)^{3/2}}-\frac{\int -\frac{4b\tan^2(e+fx)+a-b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{5/2}}d\tan(e+fx)}{2a}}{f}$$

$$\downarrow 25$$

$$\frac{\frac{\int \frac{4b\tan^2(e+fx)+a-b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{5/2}}d\tan(e+fx)}{2a}+\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\downarrow 402$$

$$\frac{\frac{\int \frac{3a^2-6ba-5b^2+2b(3a+5b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{3a(a+b)}+\frac{b(3a+5b)\tan(e+fx)}{3a(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}}{2a}+\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\downarrow 402$$

$$\frac{\frac{\int \frac{3(a-5b)(a+b)^2}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a(a+b)}+\frac{b\left(3a^2+22ab+15b^2\right)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a(a+b)}+\frac{b(3a+5b)\tan(e+fx)}{3a(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}}{2a}+\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+$$

$$\downarrow 27$$

$$\frac{\frac{3(a-5b)(a+b)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a}+\frac{b\left(3a^2+22ab+15b^2\right)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a(a+b)}+\frac{b(3a+5b)\tan(e+fx)}{3a(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}}{2a}+\frac{ta}{2a(\tan^2(e+fx)+1}$$

$$\downarrow 291$$

$$\frac{\frac{3(a-5b)(a+b)\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{a}+\frac{b\left(3a^2+22ab+15b^2\right)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a(a+b)}+\frac{b(3a+5b)\tan(e+fx)}{3a(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}}{2a}+\frac{\tan(e}{2a(\tan^2(e+fx)+1)(a-}$$

$$f$$

3.293. $\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

$$\downarrow 216$$

$$\frac{\frac{3(a-5b)(a+b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}}+\frac{b\left(3a^2+22ab+15b^2\right)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a(a+b)}+\frac{b(3a+5b)\tan(e+fx)}{3a(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}}{2a}+\frac{\frac{\tan(e+fx)}{2a(\tan^2(e+fx)+1)(a+b\tan^2(e+fx)}}{f}$$

input
```
Int[Cos[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(Tan[e + f*x]/(2*a*(1 + Tan[e + f*x]^2)*(a + b + b*Tan[e + f*x]^2)^(3/2))
+ ((b*(3*a + 5*b)*Tan[e + f*x])/(3*a*(a + b)*(a + b + b*Tan[e + f*x]^2)^(3
/2)) + ((3*(a - 5*b)*(a + b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*
Tan[e + f*x]^2]])/a^(3/2) + (b*(3*a^2 + 22*a*b + 15*b^2)*Tan[e + f*x])/(a*
(a + b)*Sqrt[a + b + b*Tan[e + f*x]^2]))/(3*a*(a + b)))/(2*a))/f
```

3.293.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.293.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2142 vs. $2(167) = 334$.

Time = 5.35 (sec) , antiderivative size = 2143, normalized size of antiderivative = 11.46

method	result	size
default	Expression too large to display	2143

input
```
int(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/6/f/(a+b)^2/a^3/(-a)^(1/2)*(b+a*cos(f*x+e)^2)*(-15*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)-4*sin(f*x+e)*a)*a*b^3*cos(f*x+e)^2+6*(-a)^(1/2)*a^3*b*cos(f*x+e)^2*sin(
f*x+e)+30*(-a)^(1/2)*a^2*b^2*cos(f*x+e)^2*sin(f*x+e)+20*(-a)^(1/2)*a*b^3*c
os(f*x+e)^2*sin(f*x+e)+3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*
(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(
1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^3*b*cos
(f*x+e)-9*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^2*b^2*cos(f*x+e)-27*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a*b^3*cos(f*x+e)+6*(-a)^(1/2)*a^3*b*
cos(f*x+e)^4*sin(f*x+e)+3*(-a)^(1/2)*a^2*b^2*cos(f*x+e)^4*sin(f*x+e)-9*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^3*b*cos(f*x+e)^3-27*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e...
```

3.293.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 451 vs. $2(167) = 334$.

Time $= 1.78$ (sec) , antiderivative size $= 1023$, normalized size of antiderivative $= 5.47$

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/48*(3*(a^3*b^2 - 3*a^2*b^3 - 9*a*b^4 - 5*b^5 + (a^5 - 3*a^4*b - 9*a^3*b
^2 - 5*a^2*b^3)*cos(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2 - 9*a^2*b^3 - 5*a*b^
4)*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)
*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 -
 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 -
a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*
x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b
+ 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)*sin(f*x + e)) + 8*(3*(a^5 + 2*a^4*b + a^3*b^2)*cos(f*x + e)^5 +
2*(3*a^4*b + 15*a^3*b^2 + 10*a^2*b^3)*cos(f*x + e)^3 + (3*a^3*b^2 + 22*a^2
*b^3 + 15*a*b^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
*sin(f*x + e))/((a^8 + 2*a^7*b + a^6*b^2)*f*cos(f*x + e)^4 + 2*(a^7*b + 2*
a^6*b^2 + a^5*b^3)*f*cos(f*x + e)^2 + (a^6*b^2 + 2*a^5*b^3 + a^4*b^4)*f),
-1/24*(3*(a^3*b^2 - 3*a^2*b^3 - 9*a*b^4 - 5*b^5 + (a^5 - 3*a^4*b - 9*a^3*b
^2 - 5*a^2*b^3)*cos(f*x + e)^4 + 2*(a^4*b - 3*a^3*b^2 - 9*a^2*b^3 - 5*a*b^
4)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b
)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a
^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(3*(a^5 + 2*a^4*b + a^3*b
^2)*cos(f*x + e)^5 + 2*(3*a^4*b + 15*a^3*b^2 + 10*a^2*b^3)*cos(f*x + e)...
```

3.293.6 Sympy [F]

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)**2/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral(cos(e + f*x)**2/(a + b*sec(e + f*x)**2)**(5/2), x)
```

3.293.7 Maxima [F]

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos(fx+e)^2}{\left(b\sec(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

3.293.8 Giac [F]

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos(fx+e)^2}{\left(b\sec(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.293.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos(e+fx)^2}{\left(a+\frac{b}{\cos(e+fx)^2}\right)^{5/2}}\,dx$$

input
```
int(cos(e + f*x)^2/(a + b/cos(e + f*x)^2)^(5/2),x)
```

output
```
int(cos(e + f*x)^2/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.294 $\int \frac{\cos^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

3.294.1 Optimal result . . . 2205
3.294.2 Mathematica [C] (warning: unable to verify) . . . 2206
3.294.3 Rubi [A] (verified) . . . 2206
3.294.4 Maple [B] (verified) . . . 2210
3.294.5 Fricas [B] (verification not implemented) . . . 2211
3.294.6 Sympy [F] . . . 2212
3.294.7 Maxima [F] . . . 2213
3.294.8 Giac [F] . . . 2213
3.294.9 Mupad [F(-1)] . . . 2213

3.294.1 Optimal result

Integrand size = 25, antiderivative size = 261

$$\int \frac{\cos^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{(3a^2-10ab+35b^2)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8a^{9/2}f} + \frac{(3a-7b)\cos(e+fx)\sin(e+fx)}{8a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^3(e+fx)\sin(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b(9a^2-18ab-35b^2)\tan(e+fx)}{24a^3(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b(9a^3-15a^2b-145ab^2-105b^3)\tan(e+fx)}{24a^4(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

output

```
1/8*(3*a^2-10*a*b+35*b^2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(
1/2))/a^(9/2)/f+1/24*b*(9*a^3-15*a^2*b-145*a*b^2-105*b^3)*tan(f*x+e)/a^4/(
a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/8*(3*a-7*b)*cos(f*x+e)*sin(f*x+e)/a^
2/f/(a+b+b*tan(f*x+e)^2)^(3/2)+1/4*cos(f*x+e)^3*sin(f*x+e)/a/f/(a+b+b*tan(
f*x+e)^2)^(3/2)+1/24*b*(9*a^2-18*a*b-35*b^2)*tan(f*x+e)/a^3/(a+b)/f/(a+b+b
*tan(f*x+e)^2)^(3/2)
```

3.294.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 18.42 (sec) , antiderivative size = 1777, normalized size of antiderivative = 6.81

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(3*(a + b)*AppellF1[1/2, -4, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/
(a + b)]*Cos[e + f*x]^12*Sin[e + f*x])/(4*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)
^(5/2)*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -4, 5/2,
3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -4,
7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 8*(a + b)*AppellF1
[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f
*x]^2)*((15*a*(a + b)*AppellF1[1/2, -4, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e
 + f*x]^2)/(a + b)]*Cos[e + f*x]^9*Sin[e + f*x]^2)/(4*Sqrt[2]*(a + b - a*S
in[e + f*x]^2)^(7/2)*(3*(a + b)*AppellF1[1/2, -4, 5/2, 3/2, Sin[e + f*x]^2
, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -4, 7/2, 5/2, Sin[e + f
*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 8*(a + b)*AppellF1[3/2, -3, 5/2, 5/2,
 Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b
)*AppellF1[1/2, -4, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*
Cos[e + f*x]^9)/(4*Sqrt[2]*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*App
ellF1[1/2, -4, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*
a*AppellF1[3/2, -4, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]
- 8*(a + b)*AppellF1[3/2, -3, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)
/(a + b)])*Sin[e + f*x]^2)) - (3*Sqrt[2]*(a + b)*AppellF1[1/2, -4, 5/2, 3/
2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^7*Sin[e + f*x]
^2)/((a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -4, 5/2,...
```

3.294.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.10, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 4634, 316, 25, 402, 25, 402, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sec(e+fx)^4\,(a+b\sec(e+fx)^2)^{5/2}}dx$$

$$\downarrow 4634$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)^3(b\tan^2(e+fx)+a+b)^{5/2}}d\tan(e+fx)}{f}$$

$$\downarrow 316$$

$$\frac{\frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)^{3/2}} - \frac{\int -\frac{6b\tan^2(e+fx)+3a-b}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^{5/2}}d\tan(e+fx)}{4a}}{f}$$

$$\downarrow 25$$

$$\frac{\frac{\int \frac{6b\tan^2(e+fx)+3a-b}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^{5/2}}d\tan(e+fx)}{4a} + \frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\downarrow 402$$

$$\frac{\frac{(3a-7b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{3/2}} - \frac{\int -\frac{3a^2+2ba+7b^2+4(3a-7b)b\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{5/2}}d\tan(e+fx)}{2a}}{4a} + \frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\downarrow 25$$

$$\frac{\frac{\int \frac{3a^2+2ba+7b^2+4(3a-7b)b\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{5/2}}d\tan(e+fx)}{2a} + \frac{(3a-7b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}}{4a} + \frac{\tan(e+fx)}{4a(\tan^2(e+fx)+1)^2(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\downarrow 402$$

$$\frac{\frac{\int \frac{9a^3-3ba^2+39b^2a+35b^3+2b\left(9a^2-18ba-35b^2\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{3a(a+b)} + \frac{b\left(9a^2-18ab-35b^2\right)\tan(e+fx)}{3a(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}}{2a} + \frac{(3a-7b)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}}{4a} + \cdots$$

$$f$$

3.294. $\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

$$\downarrow 402$$

$$\frac{\frac{\int \frac{3(a+b)^2\left(3a^2-10ba+35b^2\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a(a+b)}+\frac{b\left(9a^3-15a^2b-145ab^2-105b^3\right)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a(a+b)}+\frac{b\left(9a^2-18ab-35b^2\right)\tan(e+fx)}{3a(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}}{2a}+\frac{(3a-7b)\tan}{2a\left(\tan^2(e+fx)+1\right)(a+b}$$
$$4a$$
$$f$$

$$\downarrow 27$$

$$\frac{\frac{3(a+b)\left(3a^2-10ab+35b^2\right)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a}+\frac{b\left(9a^3-15a^2b-145ab^2-105b^3\right)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a(a+b)}+\frac{b\left(9a^2-18ab-35b^2\right)\tan(e+fx)}{3a(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}}{2a}+$$
$$4a$$
$$f$$

$$\downarrow 291$$

$$\frac{\frac{3(a+b)\left(3a^2-10ab+35b^2\right)\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{a}+\frac{b\left(9a^3-15a^2b-145ab^2-105b^3\right)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a(a+b)}+\frac{b\left(9a^2-18ab-35b^2\right)\tan(e+fx)}{3a(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}}{2a}+\frac{}{2a(}$$
$$4a$$
$$f$$

$$\downarrow 216$$

$$\frac{\frac{b\left(9a^2-18ab-35b^2\right)\tan(e+fx)}{3a(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}+\frac{\frac{3(a+b)\left(3a^2-10ab+35b^2\right)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}}+\frac{b\left(9a^3-15a^2b-145ab^2-105b^3\right)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a(a+b)}}{2a}+\frac{(3a-7b}{2a\left(\tan^2(e+fx)+1\right)}$$
$$4a$$
$$f$$

input
```
Int[Cos[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

3.294. $\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

output
```
(Tan[e + f*x]/(4*a*(1 + Tan[e + f*x]^2)^2*(a + b + b*Tan[e + f*x]^2)^(3/2)
) + (((3*a - 7*b)*Tan[e + f*x])/(2*a*(1 + Tan[e + f*x]^2)*(a + b + b*Tan[e
 + f*x]^2)^(3/2)) + ((b*(9*a^2 - 18*a*b - 35*b^2)*Tan[e + f*x])/(3*a*(a +
b)*(a + b + b*Tan[e + f*x]^2)^(3/2)) + ((3*(a + b)*(3*a^2 - 10*a*b + 35*b^
2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/a^(3/2)
+ (b*(9*a^3 - 15*a^2*b - 145*a*b^2 - 105*b^3)*Tan[e + f*x])/(a*(a + b)*Sqr
t[a + b + b*Tan[e + f*x]^2]))/(3*a*(a + b)))/(2*a))/(4*a))/f
```

3.294.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.294.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2769 vs. $2(237) = 474$.

Time = 8.14 (sec) , antiderivative size = 2770, normalized size of antiderivative = 10.61

method	result	size
default	Expression too large to display	2770

input
```
int(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/24/f/a^4/(-a)^(1/2)/(a+b)^2*(b+a*cos(f*x+e)^2)*(-105*(-a)^(1/2)*b^5*sin(
f*x+e)+180*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos
(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x
+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*b^3*cos(f*x+e)^2+10
5*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4
*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b^4*cos(f*x+e)^2+18*(-a)^(1/2)
*a^4*b*cos(f*x+e)^2*sin(f*x+e)-24*(-a)^(1/2)*a^3*b^2*cos(f*x+e)^2*sin(f*x+
e)-198*(-a)^(1/2)*a^2*b^3*cos(f*x+e)^2*sin(f*x+e)-140*(-a)^(1/2)*a*b^4*cos
(f*x+e)^2*sin(f*x+e)+9*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^4*b*cos(f
*x+e)-12*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f
*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e
)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^3*b^2*cos(f*x+e)+54*ln(
4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)
^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*b^3*cos(f*x+e)+180*ln(4*(-a)^(1/2
)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e...
```

3.294.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 533 vs. $2(237) = 474$.

Time = 5.42 (sec) , antiderivative size = 1187, normalized size of antiderivative = 4.55

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[-1/192*(3*(3*a^4*b^2 - 4*a^3*b^3 + 18*a^2*b^4 + 60*a*b^5 + 35*b^6 + (3*a^
6 - 4*a^5*b + 18*a^4*b^2 + 60*a^3*b^3 + 35*a^2*b^4)*cos(f*x + e)^4 + 2*(3*
a^5*b - 4*a^4*b^2 + 18*a^3*b^3 + 60*a^2*b^4 + 35*a*b^5)*cos(f*x + e)^2)*sq
rt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*
(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^
2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2
 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 -
 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(
f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e
)) - 8*(6*(a^6 + 2*a^5*b + a^4*b^2)*cos(f*x + e)^7 + 3*(3*a^6 - a^5*b - 11
*a^4*b^2 - 7*a^3*b^3)*cos(f*x + e)^5 + 2*(9*a^5*b - 12*a^4*b^2 - 99*a^3*b^
3 - 70*a^2*b^4)*cos(f*x + e)^3 + (9*a^4*b^2 - 15*a^3*b^3 - 145*a^2*b^4 - 1
05*a*b^5)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*
x + e))/((a^9 + 2*a^8*b + a^7*b^2)*f*cos(f*x + e)^4 + 2*(a^8*b + 2*a^7*b^2
 + a^6*b^3)*f*cos(f*x + e)^2 + (a^7*b^2 + 2*a^6*b^3 + a^5*b^4)*f), -1/96*(
3*(3*a^4*b^2 - 4*a^3*b^3 + 18*a^2*b^4 + 60*a*b^5 + 35*b^6 + (3*a^6 - 4*a^5
*b + 18*a^4*b^2 + 60*a^3*b^3 + 35*a^2*b^4)*cos(f*x + e)^4 + 2*(3*a^5*b - 4
*a^4*b^2 + 18*a^3*b^3 + 60*a^2*b^4 + 35*a*b^5)*cos(f*x + e)^2)*sqrt(a)*arc
tan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*
b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)...
```

3.294.6 Sympy [F]

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)**4/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral(cos(e + f*x)**4/(a + b*sec(e + f*x)**2)**(5/2), x)
```

3.294.7 Maxima [F]

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos(fx+e)^4}{(b\sec(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

3.294.8 Giac [F]

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos(fx+e)^4}{(b\sec(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.294.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos(e+fx)^4}{\left(a+\frac{b}{\cos(e+fx)^2}\right)^{5/2}}\,dx$$

input
```
int(cos(e + f*x)^4/(a + b/cos(e + f*x)^2)^(5/2),x)
```

output
```
int(cos(e + f*x)^4/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.295 $\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

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3.295.1 Optimal result

Integrand size = 25, antiderivative size = 332

$$\int \frac{\cos^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{5(a-3b)\left(a^2+7b^2\right)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16a^{11/2}f} + \frac{\left(5a^2-10ab+21b^2\right)\cos(e+fx)\sin(e+fx)}{16a^3f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(5a-9b)\cos^3(e+fx)\sin(e+fx)}{24a^2f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\cos^5(e+fx)\sin(e+fx)}{6af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b(15a^3-25a^2b+49ab^2+105b^3)\tan(e+fx)}{48a^4(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{b(15a^4-20a^3b+38a^2b^2+420ab^3+315b^4)\tan(e+fx)}{48a^5(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

output

```
5/16*(a-3*b)*(a^2+7*b^2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1
/2))/a^(11/2)/f+1/48*b*(15*a^4-20*a^3*b+38*a^2*b^2+420*a*b^3+315*b^4)*tan(
f*x+e)/a^5/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/16*(5*a^2-10*a*b+21*b^2)
*cos(f*x+e)*sin(f*x+e)/a^3/f/(a+b+b*tan(f*x+e)^2)^(3/2)+1/24*(5*a-9*b)*cos
(f*x+e)^3*sin(f*x+e)/a^2/f/(a+b+b*tan(f*x+e)^2)^(3/2)+1/6*cos(f*x+e)^5*sin
(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)^(3/2)+1/48*b*(15*a^3-25*a^2*b+49*a*b^2+10
5*b^3)*tan(f*x+e)/a^4/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(3/2)
```

3.295.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 21.31 (sec) , antiderivative size = 1776, normalized size of antiderivative = 5.35

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(3*(a + b)*AppellF1[1/2, -5, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/
(a + b)]*Cos[e + f*x]^16*Sin[e + f*x])/(4*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)
^(5/2)*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -5, 5/2,
3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + 5*(a*AppellF1[3/2, -5,
7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 2*(a + b)*AppellF1
[3/2, -4, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f
*x]^2)*((15*a*(a + b)*AppellF1[1/2, -5, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e
 + f*x]^2)/(a + b)]*Cos[e + f*x]^11*Sin[e + f*x]^2)/(4*Sqrt[2]*(a + b - a*
Sin[e + f*x]^2)^(7/2)*(3*(a + b)*AppellF1[1/2, -5, 5/2, 3/2, Sin[e + f*x]^
2, (a*Sin[e + f*x]^2)/(a + b)] + 5*(a*AppellF1[3/2, -5, 7/2, 5/2, Sin[e +
f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 2*(a + b)*AppellF1[3/2, -4, 5/2, 5/2
, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a +
b)*AppellF1[1/2, -5, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]
*Cos[e + f*x]^11)/(4*Sqrt[2]*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*A
ppellF1[1/2, -5, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + 5
*(a*AppellF1[3/2, -5, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)
] - 2*(a + b)*AppellF1[3/2, -4, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^
2)/(a + b)])*Sin[e + f*x]^2)) - (15*(a + b)*AppellF1[1/2, -5, 5/2, 3/2, Si
n[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^9*Sin[e + f*x]^2)/(
2*Sqrt[2]*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -5,...
```

3.295.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.11, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 4634, 316, 25, 402, 27, 402, 25, 402, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sec(e+fx)^6\,(a+b\sec(e+fx)^2)^{5/2}}dx$$

$$\downarrow 4634$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)^4(b\tan^2(e+fx)+a+b)^{5/2}}d\tan(e+fx)}{f}$$

$$\downarrow 316$$

$$\frac{\frac{\tan(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)^{3/2}} - \frac{\int -\frac{8b\tan^2(e+fx)+5a-b}{\left(\tan^2(e+fx)+1\right)^3\left(b\tan^2(e+fx)+a+b\right)^{5/2}}d\tan(e+fx)}{6a}}{f}$$

$$\downarrow 25$$

$$\frac{\frac{\int \frac{8b\tan^2(e+fx)+5a-b}{\left(\tan^2(e+fx)+1\right)^3\left(b\tan^2(e+fx)+a+b\right)^{5/2}}d\tan(e+fx)}{6a} + \frac{\tan(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\downarrow 402$$

$$\frac{\frac{(5a-9b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\left(a+b\tan^2(e+fx)+b\right)^{3/2}} - \frac{\int -\frac{3\left(5a^2+3b^2+2(5a-9b)b\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^{5/2}}d\tan(e+fx)}{4a}}{6a} + \frac{\tan(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)^{3/}}}{f}$$

$$\downarrow 27$$

$$\frac{\frac{3\int \frac{5a^2+3b^2+2(5a-9b)b\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^{5/2}}d\tan(e+fx)}{4a} + \frac{(5a-9b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\left(a+b\tan^2(e+fx)+b\right)^{3/2}}}{6a} + \frac{\tan(e+fx)}{6a(\tan^2(e+fx)+1)^3(a+b\tan^2(e+fx)+b)^{3/}}}{f}$$

$$\downarrow 402$$

$$\frac{3\left(\frac{\left(5a^2-10ab+21b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}-\frac{\int -\frac{5a^3+5ba^2-5b^2a-21b^3+4b\left(5a^2-10ba+21b^2\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{5/2}}d\tan(e+fx)}{2a}\right)}{4a}+\frac{(5a-9b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\left(a+b\tan^2(e+fx)+b\right)^{3/2}}}{6a}$$

$$f$$

$$\downarrow 25$$

$$\frac{3\left(\frac{\int \frac{5a^3+5ba^2-5b^2a-21b^3+4b\left(5a^2-10ba+21b^2\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{5/2}}d\tan(e+fx)}{2a}+\frac{\left(5a^2-10ab+21b^2\right)\tan(e+fx)}{2a\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}\right)}{4a}+\frac{(5a-9b)\tan(e+fx)}{4a\left(\tan^2(e+fx)+1\right)^2\left(a+b\tan^2(e+fx)+b\right)^{3/2}}}{6a}$$

$$f$$

$$\downarrow 402$$

$$\frac{3\left(\frac{\int \frac{15a^4+10b^2a^2-112b^3a-105b^4+2b\left(15a^3-25ba^2+49b^2a+105b^3\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{3a(a+b)}+\frac{b\left(15a^3-25a^2b+49ab^2+105b^3\right)\tan(e+fx)}{3a(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}}{2a}+\frac{\left(5a^2-10ab+\right.}{2a\left(\tan^2(e+fx)+1\right)\left(\right.}\right)}{4a}$$

$$\frac{}{6a}$$

$$f$$

$$\downarrow 402$$

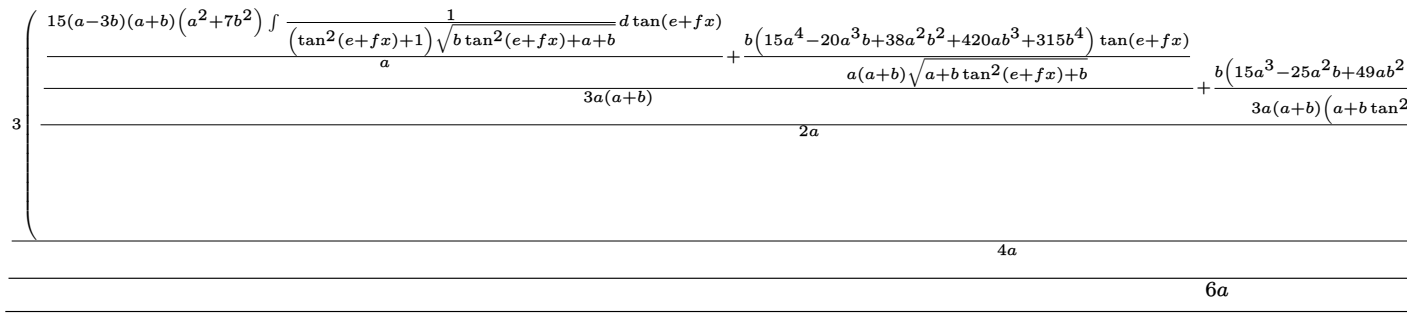
$$\frac{3\left(\frac{\int \frac{15(a-3b)(a+b)^2\left(a^2+7b^2\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a(a+b)}+\frac{b\left(15a^4-20a^3b+38a^2b^2+420ab^3+315b^4\right)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a(a+b)}+\frac{b\left(15a^3-25a^2b+49ab^2+105b^3\right)\tan(e+fx)}{3a(a+b)\left(a+b\tan^2(e+fx)+b\right)^{3/2}}}{2a}+\frac{}{2a\left(\right.}\right)}{4a}$$

$$\frac{}{6a}$$

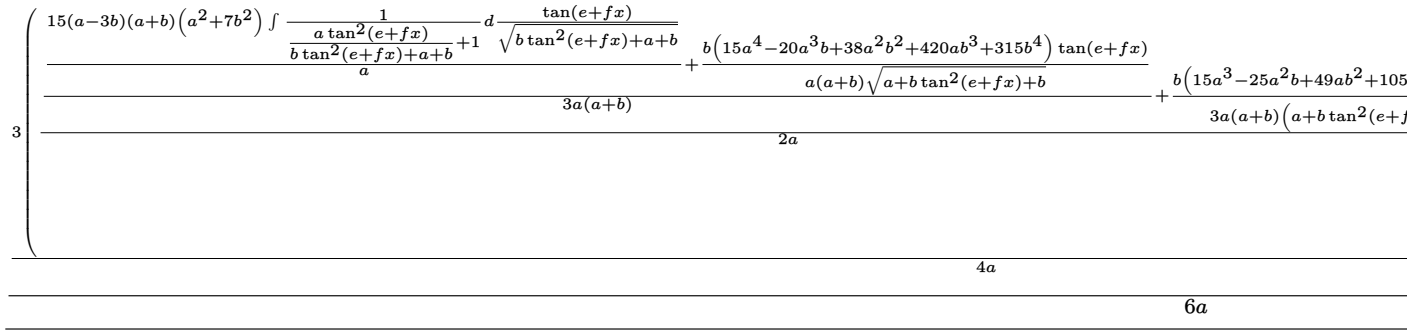
$$f$$

$$\downarrow 27$$

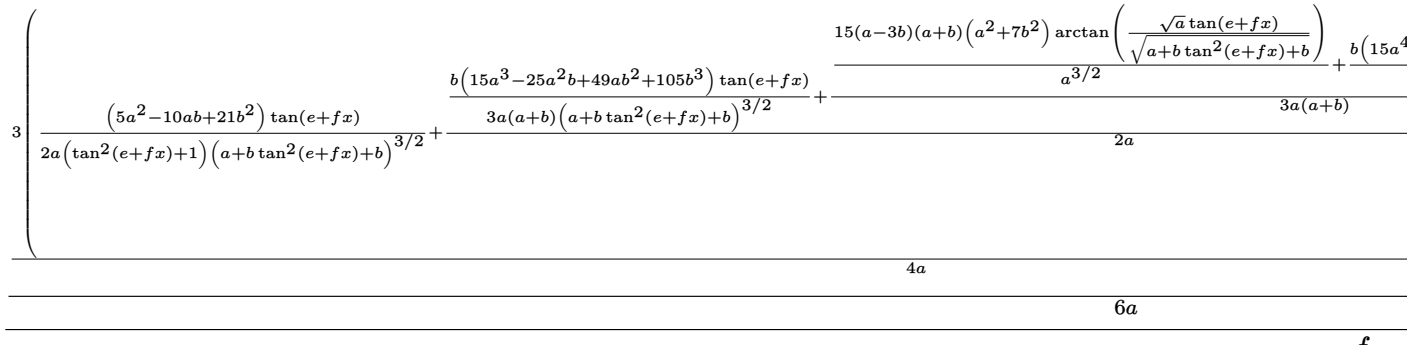
3.295. $\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$



$$\downarrow 291$$



$$\downarrow 216$$



input `Int[Cos[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]`

3.295. $\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

output
```
(Tan[e + f*x]/(6*a*(1 + Tan[e + f*x]^2)^3*(a + b + b*Tan[e + f*x]^2)^(3/2)
) + (((5*a - 9*b)*Tan[e + f*x])/(4*a*(1 + Tan[e + f*x]^2)^2*(a + b + b*Tan
[e + f*x]^2)^(3/2)) + (3*(((5*a^2 - 10*a*b + 21*b^2)*Tan[e + f*x])/(2*a*(1
 + Tan[e + f*x]^2)*(a + b + b*Tan[e + f*x]^2)^(3/2)) + ((b*(15*a^3 - 25*a^
2*b + 49*a*b^2 + 105*b^3)*Tan[e + f*x])/(3*a*(a + b)*(a + b + b*Tan[e + f*
x]^2)^(3/2)) + ((15*(a - 3*b)*(a + b)*(a^2 + 7*b^2)*ArcTan[(Sqrt[a]*Tan[e
 + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/a^(3/2) + (b*(15*a^4 - 20*a^3*b +
 38*a^2*b^2 + 420*a*b^3 + 315*b^4)*Tan[e + f*x])/(a*(a + b)*Sqrt[a + b + b
*Tan[e + f*x]^2]))/(3*a*(a + b)))/(2*a)))/(4*a))/(6*a))/f
```

3.295.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.295.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3398 vs. $2(304) = 608$.

Time = 10.81 (sec) , antiderivative size = 3399, normalized size of antiderivative = 10.24

method	result	size
default	Expression too large to display	3399

input
```
int(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/48/f/a^5/(-a)^(1/2)/(a+b)^2*(b+a*cos(f*x+e)^2)*(315*(-a)^(1/2)*b^6*sin(f
*x+e)-315*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*b^6+420*(-a)^(1/2)*a*b^5
*cos(f*x+e)^2*sin(f*x+e)-15*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln
(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a
)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^5*b*
cos(f*x+e)^2+30*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2
)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^4*b^2*cos(f*x+e)
^2-150*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^3*b^3*cos(f*x+e)^2-525*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a^2*b^4*cos(f*x+e)^2-315*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)-4*sin(f*x+e)*a)*a*b^5*cos(f*x+e)^2+15*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)))...
```

3.295.5 Fricas [A] (verification not implemented)

Time = 16.21 (sec) , antiderivative size = 1337, normalized size of antiderivative = 4.03

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output

```
[1/384*(15*(a^5*b^2 - a^4*b^3 + 2*a^3*b^4 - 10*a^2*b^5 - 35*a*b^6 - 21*b^7
 + (a^7 - a^6*b + 2*a^5*b^2 - 10*a^4*b^3 - 35*a^3*b^4 - 21*a^2*b^5)*cos(f*
x + e)^4 + 2*(a^6*b - a^5*b^2 + 2*a^4*b^3 - 10*a^3*b^4 - 35*a^2*b^5 - 21*a
*b^6)*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3
*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^
4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2
 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos
(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2
*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)*sin(f*x + e)) + 8*(8*(a^7 + 2*a^6*b + a^5*b^2)*cos(f*x + e)^9
 + 2*(5*a^7 + a^6*b - 13*a^5*b^2 - 9*a^4*b^3)*cos(f*x + e)^7 + 3*(5*a^7 +
6*a^5*b^2 + 32*a^4*b^3 + 21*a^3*b^4)*cos(f*x + e)^5 + 2*(15*a^6*b - 15*a^5
*b^2 + 31*a^4*b^3 + 287*a^3*b^4 + 210*a^2*b^5)*cos(f*x + e)^3 + (15*a^5*b^
2 - 20*a^4*b^3 + 38*a^3*b^4 + 420*a^2*b^5 + 315*a*b^6)*cos(f*x + e))*sqrt(
(a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^10 + 2*a^9*b + a^
8*b^2)*f*cos(f*x + e)^4 + 2*(a^9*b + 2*a^8*b^2 + a^7*b^3)*f*cos(f*x + e)^2
 + (a^8*b^2 + 2*a^7*b^3 + a^6*b^4)*f), -1/192*(15*(a^5*b^2 - a^4*b^3 + 2*a
^3*b^4 - 10*a^2*b^5 - 35*a*b^6 - 21*b^7 + (a^7 - a^6*b + 2*a^5*b^2 - 10*a^
4*b^3 - 35*a^3*b^4 - 21*a^2*b^5)*cos(f*x + e)^4 + 2*(a^6*b - a^5*b^2 + 2*a
^4*b^3 - 10*a^3*b^4 - 35*a^2*b^5 - 21*a*b^6)*cos(f*x + e)^2)*sqrt(a)*ar...
```

3.295.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input

```
integrate(cos(f*x+e)**6/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output

```
Timed out
```

3.295.7 Maxima [F]

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos(fx+e)^6}{\left(b\sec(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

3.295.8 Giac [F]

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos(fx+e)^6}{\left(b\sec(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.295.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cos(e+fx)^6}{\left(a+\frac{b}{\cos(e+fx)^2}\right)^{5/2}}\,dx$$

input
```
int(cos(e + f*x)^6/(a + b/cos(e + f*x)^2)^(5/2),x)
```

output
```
int(cos(e + f*x)^6/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.296 $\int \frac{1}{(a+b\sec^2(c+dx))^{7/2}}\,dx$

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3.296.1 Optimal result

Integrand size = 16, antiderivative size = 179

$$\int \frac{1}{(a+b\sec^2(c+dx))^{7/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+b+b\tan^2(c+dx)}}\right)}{a^{7/2}d} - \frac{b\tan(c+dx)}{5a(a+b)d\,(a+b+b\tan^2(c+dx))^{5/2}} - \frac{b(9a+5b)\tan(c+dx)}{15a^2(a+b)^2d\,(a+b+b\tan^2(c+dx))^{3/2}} - \frac{b(33a^2+40ab+15b^2)\tan(c+dx)}{15a^3(a+b)^3d\sqrt{a+b+b\tan^2(c+dx)}}$$

output

```
arctan(a^(1/2)*tan(d*x+c)/(a+b+b*tan(d*x+c)^2)^(1/2))/a^(7/2)/d-1/15*b*(33
*a^2+40*a*b+15*b^2)*tan(d*x+c)/a^3/(a+b)^3/d/(a+b+b*tan(d*x+c)^2)^(1/2)-1/
5*b*tan(d*x+c)/a/(a+b)/d/(a+b+b*tan(d*x+c)^2)^(5/2)-1/15*b*(9*a+5*b)*tan(d
*x+c)/a^2/(a+b)^2/d/(a+b+b*tan(d*x+c)^2)^(3/2)
```

3.296.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 16.81 (sec) , antiderivative size = 1777, normalized size of antiderivative = 9.93

$$\int \frac{1}{(a+b\sec^2(c+dx))^{7/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*Sec[c + d*x]^2)^(-7/2),x]
```

output
```
(3*(a + b)*AppellF1[1/2, -3, 7/2, 3/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/
(a + b)]*Cos[c + d*x]^6*Sin[c + d*x])/(8*Sqrt[2]*d*(a + b*Sec[c + d*x]^2)^
(7/2)*(a + b - a*Sin[c + d*x]^2)^(7/2)*(3*(a + b)*AppellF1[1/2, -3, 7/2, 3
/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)] + (7*a*AppellF1[3/2, -3, 9
/2, 5/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)] - 6*(a + b)*AppellF1[
3/2, -2, 7/2, 5/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)])*Sin[c + d*
x]^2)*((21*a*(a + b)*AppellF1[1/2, -3, 7/2, 3/2, Sin[c + d*x]^2, (a*Sin[c
+ d*x]^2)/(a + b)]*Cos[c + d*x]^7*Sin[c + d*x]^2)/(8*Sqrt[2]*(a + b - a*Si
n[c + d*x]^2)^(9/2)*(3*(a + b)*AppellF1[1/2, -3, 7/2, 3/2, Sin[c + d*x]^2,
 (a*Sin[c + d*x]^2)/(a + b)] + (7*a*AppellF1[3/2, -3, 9/2, 5/2, Sin[c + d*
x]^2, (a*Sin[c + d*x]^2)/(a + b)] - 6*(a + b)*AppellF1[3/2, -2, 7/2, 5/2,
Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)])*Sin[c + d*x]^2)) + (3*(a + b)
*AppellF1[1/2, -3, 7/2, 3/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)]*C
os[c + d*x]^7)/(8*Sqrt[2]*(a + b - a*Sin[c + d*x]^2)^(7/2)*(3*(a + b)*Appe
llF1[1/2, -3, 7/2, 3/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)] + (7*a
*AppellF1[3/2, -3, 9/2, 5/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/(a + b)] -
 6*(a + b)*AppellF1[3/2, -2, 7/2, 5/2, Sin[c + d*x]^2, (a*Sin[c + d*x]^2)/
(a + b)])*Sin[c + d*x]^2)) - (9*(a + b)*AppellF1[1/2, -3, 7/2, 3/2, Sin[c
+ d*x]^2, (a*Sin[c + d*x]^2)/(a + b)]*Cos[c + d*x]^5*Sin[c + d*x]^2)/(4*Sq
rt[2]*(a + b - a*Sin[c + d*x]^2)^(7/2)*(3*(a + b)*AppellF1[1/2, -3, 7/2...
```

3.296.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4616, 316, 402, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sec^2(c+dx))^{7/2}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{(a+b\sec(c+dx)^2)^{7/2}}dx$$
$$\downarrow 4616$$

$$\frac{\int \frac{1}{(\tan^2(c+dx)+1)(b\tan^2(c+dx)+a+b)^{7/2}}\,d\tan(c+dx)}{d}$$

$$\downarrow 316$$

$$\frac{\int \frac{-4b\tan^2(c+dx)+5a+b}{\left(\tan^2(c+dx)+1\right)\left(b\tan^2(c+dx)+a+b\right)^{5/2}}\,d\tan(c+dx)}{5a(a+b)} - \frac{b\tan(c+dx)}{5a(a+b)(a+b\tan^2(c+dx)+b)^{5/2}}}{d}$$

$$\downarrow 402$$

$$\frac{\frac{\int \frac{15a^2+12ba+5b^2-2b(9a+5b)\tan^2(c+dx)}{\left(\tan^2(c+dx)+1\right)\left(b\tan^2(c+dx)+a+b\right)^{3/2}}\,d\tan(c+dx)}{3a(a+b)} - \frac{b(9a+5b)\tan(c+dx)}{3a(a+b)\left(a+b\tan^2(c+dx)+b\right)^{3/2}}}{5a(a+b)} - \frac{b\tan(c+dx)}{5a(a+b)(a+b\tan^2(c+dx)+b)^{5/2}}}{d}$$

$$\downarrow 402$$

$$\frac{\frac{\frac{\int \frac{15(a+b)^3}{\left(\tan^2(c+dx)+1\right)\sqrt{b\tan^2(c+dx)+a+b}}\,d\tan(c+dx)}{a(a+b)} - \frac{b\left(33a^2+40ab+15b^2\right)\tan(c+dx)}{a(a+b)\sqrt{a+b\tan^2(c+dx)+b}}}{3a(a+b)} - \frac{b(9a+5b)\tan(c+dx)}{3a(a+b)\left(a+b\tan^2(c+dx)+b\right)^{3/2}}}{5a(a+b)} - \frac{b\tan(c+dx)}{5a(a+b)(a+b\tan^2(c+dx)+b)^5}}{d}$$

$$\downarrow 27$$

$$\frac{\frac{\frac{15(a+b)^2\int \frac{1}{\left(\tan^2(c+dx)+1\right)\sqrt{b\tan^2(c+dx)+a+b}}\,d\tan(c+dx)}{a} - \frac{b\left(33a^2+40ab+15b^2\right)\tan(c+dx)}{a(a+b)\sqrt{a+b\tan^2(c+dx)+b}}}{3a(a+b)} - \frac{b(9a+5b)\tan(c+dx)}{3a(a+b)\left(a+b\tan^2(c+dx)+b\right)^{3/2}}}{5a(a+b)} - \frac{b\tan(c+dx)}{5a(a+b)(a+b\tan^2(c+$$

$$\downarrow 291$$

$$\frac{\frac{\frac{15(a+b)^2\int \frac{1}{\frac{a\tan^2(c+dx)}{b\tan^2(c+dx)+a+b}+1}\,d\frac{\tan(c+dx)}{\sqrt{b\tan^2(c+dx)+a+b}}}{a} - \frac{b\left(33a^2+40ab+15b^2\right)\tan(c+dx)}{a(a+b)\sqrt{a+b\tan^2(c+dx)+b}}}{3a(a+b)} - \frac{b(9a+5b)\tan(c+dx)}{3a(a+b)\left(a+b\tan^2(c+dx)+b\right)^{3/2}}}{5a(a+b)} - \frac{b\tan(c+dx)}{5a(a+b)(a+b\tan^2(c+dx)}}{d}$$

$$\downarrow 216$$

$$\frac{\frac{15(a+b)^2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+b\tan^2(c+dx)+b}}\right)}{a^{3/2}}-\frac{b\left(33a^2+40ab+15b^2\right)\tan(c+dx)}{a(a+b)\sqrt{a+b\tan^2(c+dx)+b}}}{3a(a+b)}-\frac{\frac{b(9a+5b)\tan(c+dx)}{3a(a+b)\left(a+b\tan^2(c+dx)+b\right)^{3/2}}}{5a(a+b)}-\frac{b\tan(c+dx)}{5a(a+b)(a+b\tan^2(c+dx)+b)^{5/2}}}{d}$$

input
```
Int[(a + b*Sec[c + d*x]^2)^(-7/2),x]
```

output
```
(-1/5*(b*Tan[c + d*x])/(a*(a + b)*(a + b + b*Tan[c + d*x]^2)^(5/2)) + (-1/
3*(b*(9*a + 5*b)*Tan[c + d*x])/(a*(a + b)*(a + b + b*Tan[c + d*x]^2)^(3/2)
) + ((15*(a + b)^2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + b + b*Tan[c + d*
x]^2]])/a^(3/2) - (b*(33*a^2 + 40*a*b + 15*b^2)*Tan[c + d*x])/(a*(a + b)*S
qrt[a + b + b*Tan[c + d*x]^2]))/(3*a*(a + b)))/(5*a*(a + b)))/d
```

3.296.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

3.296.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2017 vs. $2(161) = 322$.

Time = 5.46 (sec) , antiderivative size = 2018, normalized size of antiderivative = 11.27

method	result	size
default	Expression too large to display	2018

input
```
int(1/(a+b*sec(d*x+c)^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15/d/(a+b)^3/a^3/(-a)^(1/2)*(a*(1-cos(d*x+c))^4*csc(d*x+c)^4+b*(1-cos(d
*x+c))^4*csc(d*x+c)^4-2*a*(1-cos(d*x+c))^2*csc(d*x+c)^2+2*b*(1-cos(d*x+c))
^2*csc(d*x+c)^2+a+b)*(-30*(-a)^(1/2)*b^5*(-cot(d*x+c)+csc(d*x+c))+15*ln(4*
((-a)^(1/2)*(a*(1-cos(d*x+c))^4*csc(d*x+c)^4+b*(1-cos(d*x+c))^4*csc(d*x+c)
^4-2*a*(1-cos(d*x+c))^2*csc(d*x+c)^2+2*b*(1-cos(d*x+c))^2*csc(d*x+c)^2+a+b
)^(1/2)-2*a*(-cot(d*x+c)+csc(d*x+c)))/((1-cos(d*x+c))^2*csc(d*x+c)^2+1))*(
a*(1-cos(d*x+c))^4*csc(d*x+c)^4+b*(1-cos(d*x+c))^4*csc(d*x+c)^4-2*a*(1-cos
(d*x+c))^2*csc(d*x+c)^2+2*b*(1-cos(d*x+c))^2*csc(d*x+c)^2+a+b)^(5/2)*a^3+1
5*ln(4*((-a)^(1/2)*(a*(1-cos(d*x+c))^4*csc(d*x+c)^4+b*(1-cos(d*x+c))^4*csc
(d*x+c)^4-2*a*(1-cos(d*x+c))^2*csc(d*x+c)^2+2*b*(1-cos(d*x+c))^2*csc(d*x+c
)^2+a+b)^(1/2)-2*a*(-cot(d*x+c)+csc(d*x+c)))/((1-cos(d*x+c))^2*csc(d*x+c)^
2+1))*(a*(1-cos(d*x+c))^4*csc(d*x+c)^4+b*(1-cos(d*x+c))^4*csc(d*x+c)^4-2*a
*(1-cos(d*x+c))^2*csc(d*x+c)^2+2*b*(1-cos(d*x+c))^2*csc(d*x+c)^2+a+b)^(5/2
)*b^3-80*(-a)^(1/2)*a^2*b^3*(1-cos(d*x+c))^7*csc(d*x+c)^7-320*(-a)^(1/2)*a
*b^4*(1-cos(d*x+c))^7*csc(d*x+c)^7-340*(-a)^(1/2)*a*b^4*(1-cos(d*x+c))^5*c
sc(d*x+c)^5+480*(-a)^(1/2)*a^3*b^2*(1-cos(d*x+c))^3*csc(d*x+c)^3-80*(-a)^(
1/2)*a^2*b^3*(1-cos(d*x+c))^3*csc(d*x+c)^3+360*(-a)^(1/2)*a^4*b*(1-cos(d*x
+c))^3*csc(d*x+c)^3-320*(-a)^(1/2)*a*b^4*(1-cos(d*x+c))^3*csc(d*x+c)^3-300
*(-a)^(1/2)*a^2*b^3*(1-cos(d*x+c))^9*csc(d*x+c)^9-150*(-a)^(1/2)*a*b^4*(1-
cos(d*x+c))^9*csc(d*x+c)^9+480*(-a)^(1/2)*a^3*b^2*(1-cos(d*x+c))^7*csc(...
```

3.296.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 560 vs. $2(161) = 322$.

Time = 2.12 (sec) , antiderivative size = 1241, normalized size of antiderivative = 6.93

$$\int \frac{1}{(a + b\sec^2(c + dx))^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sec(d*x+c)^2)^(7/2),x, algorithm="fricas")
```

output
```
[-1/120*(15*((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*cos(d*x + c)^6 + a^3*b^
3 + 3*a^2*b^4 + 3*a*b^5 + b^6 + 3*(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4
)*cos(d*x + c)^4 + 3*(a^4*b^2 + 3*a^3*b^3 + 3*a^2*b^4 + a*b^5)*cos(d*x + c
)^2)*sqrt(-a)*log(128*a^4*cos(d*x + c)^8 - 256*(a^4 - a^3*b)*cos(d*x + c)^
6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(d*x + c)^4 + a^4 - 28*a^3*b + 70
*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(d*x
 + c)^2 + 8*(16*a^3*cos(d*x + c)^7 - 24*(a^3 - a^2*b)*cos(d*x + c)^5 + 2*(
5*a^3 - 14*a^2*b + 5*a*b^2)*cos(d*x + c)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^
3)*cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c)^2 + b)/cos(d*x + c)^2)*sin(
d*x + c)) + 8*((45*a^5*b + 60*a^4*b^2 + 23*a^3*b^3)*cos(d*x + c)^5 + (75*a
^4*b^2 + 94*a^3*b^3 + 35*a^2*b^4)*cos(d*x + c)^3 + (33*a^3*b^3 + 40*a^2*b^
4 + 15*a*b^5)*cos(d*x + c))*sqrt((a*cos(d*x + c)^2 + b)/cos(d*x + c)^2)*si
n(d*x + c))/((a^10 + 3*a^9*b + 3*a^8*b^2 + a^7*b^3)*d*cos(d*x + c)^6 + 3*(
a^9*b + 3*a^8*b^2 + 3*a^7*b^3 + a^6*b^4)*d*cos(d*x + c)^4 + 3*(a^8*b^2 + 3
*a^7*b^3 + 3*a^6*b^4 + a^5*b^5)*d*cos(d*x + c)^2 + (a^7*b^3 + 3*a^6*b^4 +
3*a^5*b^5 + a^4*b^6)*d), -1/60*(15*((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*
cos(d*x + c)^6 + a^3*b^3 + 3*a^2*b^4 + 3*a*b^5 + b^6 + 3*(a^5*b + 3*a^4*b^
2 + 3*a^3*b^3 + a^2*b^4)*cos(d*x + c)^4 + 3*(a^4*b^2 + 3*a^3*b^3 + 3*a^2*b
^4 + a*b^5)*cos(d*x + c)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(d*x + c)^5 - 8*(
a^2 - a*b)*cos(d*x + c)^3 + (a^2 - 6*a*b + b^2)*cos(d*x + c))*sqrt(a)*s...
```

3.296.6 Sympy [F]

$$\int \frac{1}{(a+b\sec^2(c+dx))^{7/2}}\,dx = \int \frac{1}{(a+b\sec^2(c+dx))^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c)**2)**(7/2),x)
```

output
```
Integral((a + b*sec(c + d*x)**2)**(-7/2), x)
```

3.296.7 Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec^2(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*sec(d*x+c)^2)^(7/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.296.8 Giac [F]

$$\int \frac{1}{(a+b\sec^2(c+dx))^{7/2}}\,dx = \int \frac{1}{\left(b\sec\left(dx+c\right)^2+a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c)^2)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c)^2 + a)^(-7/2), x)
```

3.296.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec^2(c+dx))^{7/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)^2}\right)^{7/2}}\,dx$$

input
```
int(1/(a + b/cos(c + d*x)^2)^(7/2),x)
```

output
```
int(1/(a + b/cos(c + d*x)^2)^(7/2), x)
```

3.297 $\int \frac{1}{\sqrt{1+\sec^2(x)}}\,dx$

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3.297.1 Optimal result

Integrand size = 10, antiderivative size = 14

$$\int \frac{1}{\sqrt{1+\sec^2(x)}}\,dx = \arctan\left(\frac{\tan(x)}{\sqrt{2+\tan^2(x)}}\right)$$

output
```
arctan(tan(x)/(2+tan(x)^2)^(1/2))
```

3.297.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 37 vs. 2(14) = 28.

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.64

$$\int \frac{1}{\sqrt{1+\sec^2(x)}}\,dx = \frac{\arcsin\left(\frac{\sin(x)}{\sqrt{2}}\right)\sqrt{3+\cos(2x)}\sec(x)}{\sqrt{2}\sqrt{1+\sec^2(x)}}$$

input
```
Integrate[1/Sqrt[1 + Sec[x]^2],x]
```

output
```
(ArcSin[Sin[x]/Sqrt[2]]*Sqrt[3 + Cos[2*x]]*Sec[x])/(Sqrt[2]*Sqrt[1 + Sec[x
]^2])
```

3.297.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4616, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sec^2(x)+1}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{\sqrt{\sec(x)^2+1}}\,dx$$
$$\downarrow 4616$$
$$\int \frac{1}{\left(\tan^2(x)+1\right)\sqrt{\tan^2(x)+2}}\,d\tan(x)$$
$$\downarrow 291$$
$$\int \frac{1}{\frac{\tan^2(x)}{\tan^2(x)+2}+1}\,d\frac{\tan(x)}{\sqrt{\tan^2(x)+2}}$$
$$\downarrow 216$$
$$\arctan\left(\frac{\tan(x)}{\sqrt{\tan^2(x)+2}}\right)$$

input
```
Int[1/Sqrt[1 + Sec[x]^2],x]
```

output
```
ArcTan[Tan[x]/Sqrt[2 + Tan[x]^2]]
```

3.297.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

3.297.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(12) = 24$.

Time = 0.87 (sec) , antiderivative size = 59, normalized size of antiderivative = 4.21

method	result	size
default	$\frac{\sqrt{2}\sqrt{\frac{\cos(x)^2+1}{(\cos(x)+1)^2}}\arctan\left(\frac{\sin(x)}{(\cos(x)+1)\sqrt{\frac{\cos(x)^2+1}{(\cos(x)+1)^2}}}\right)(1+\sec(x))}{\sqrt{2+2\sec(x)^2}}$	59

input
```
int(1/(1+sec(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2^(1/2)*((cos(x)^2+1)/(cos(x)+1)^2)^(1/2)*arctan(sin(x)/(cos(x)+1)/((cos(x
)^2+1)/(cos(x)+1)^2)^(1/2))/(2+2*sec(x)^2)^(1/2)*(1+sec(x))
```

3.297.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(12) = 24$.

Time $= 0.25$ (sec) , antiderivative size $= 53$, normalized size of antiderivative $= 3.79$

$$\int \frac{1}{\sqrt{1+\sec^2(x)}}\,dx = \frac{1}{2}\arctan\left(\frac{\sqrt{\frac{\cos(x)^2+1}{\cos(x)^2}}\cos(x)^3\sin(x)+\cos(x)\sin(x)}{\cos(x)^4+\cos(x)^2-1}\right) - \frac{1}{2}\arctan\left(\frac{\sin(x)}{\cos(x)}\right)$$

input
```
integrate(1/(1+sec(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*arctan((sqrt((cos(x)^2 + 1)/cos(x)^2)*cos(x)^3*sin(x) + cos(x)*sin(x))
/(cos(x)^4 + cos(x)^2 - 1)) - 1/2*arctan(sin(x)/cos(x))
```

3.297.6 Sympy [F]

$$\int \frac{1}{\sqrt{1+\sec^2(x)}}\,dx = \int \frac{1}{\sqrt{\sec^2(x)+1}}\,dx$$

input
```
integrate(1/(1+sec(x)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(sec(x)**2 + 1), x)
```

3.297.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(12) = 24$.

Time = 0.38 (sec) , antiderivative size = 388, normalized size of antiderivative = 27.71

$$\int \frac{1}{\sqrt{1+\sec^2(x)}}\,dx =$$

$$-\frac{1}{2}\arctan\left(2\left(2\left(6\cos\left(2x\right)+1\right)\cos\left(4x\right)+\cos\left(4x\right)^2+36\cos\left(2x\right)^2+\sin\left(4x\right)^2+12\sin\left(4x\right)\sin\left(2x\right)+8\right)\right)$$

$$+\frac{1}{2}\arctan\left(2\left(2\left(6\cos\left(2x\right)+1\right)\cos\left(4x\right)+\cos\left(4x\right)^2+36\cos\left(2x\right)^2+\sin\left(4x\right)^2+12\sin\left(4x\right)\sin\left(2x\right)+2\sin\left(2x\right),2\left(2\left(6\cos\left(2x\right)+1\right)\cos\left(4x\right)+\cos\left(4x\right)^2+36\cos\left(2x\right)^2+\sin\left(4x\right)^2+12\sin\left(4x\right)\sin\left(2x\right)+2\cos\left(2x\right)+6\right)\right)$$

input
```
integrate(1/(1+sec(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*arctan2(2*(2*(6*cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 36*cos(2*x)^2 +
 sin(4*x)^2 + 12*sin(4*x)*sin(2*x) + 36*sin(2*x)^2 + 12*cos(2*x) + 1)^(1/4
)*sin(1/2*arctan2(sin(4*x) + 6*sin(2*x), cos(4*x) + 6*cos(2*x) + 1)), 2*(2
*(6*cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 36*cos(2*x)^2 + sin(4*x)^2 + 12*
sin(4*x)*sin(2*x) + 36*sin(2*x)^2 + 12*cos(2*x) + 1)^(1/4)*cos(1/2*arctan2
(sin(4*x) + 6*sin(2*x), cos(4*x) + 6*cos(2*x) + 1)) + 8) + 1/2*arctan2(2*(
2*(6*cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 36*cos(2*x)^2 + sin(4*x)^2 + 12
*sin(4*x)*sin(2*x) + 36*sin(2*x)^2 + 12*cos(2*x) + 1)^(1/4)*sin(1/2*arctan
2(sin(4*x) + 6*sin(2*x), cos(4*x) + 6*cos(2*x) + 1)) + 2*sin(2*x), 2*(2*(6
*cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 36*cos(2*x)^2 + sin(4*x)^2 + 12*sin
(4*x)*sin(2*x) + 36*sin(2*x)^2 + 12*cos(2*x) + 1)^(1/4)*cos(1/2*arctan2(si
n(4*x) + 6*sin(2*x), cos(4*x) + 6*cos(2*x) + 1)) + 2*cos(2*x) + 6)
```

3.297.8 Giac [F]

$$\int \frac{1}{\sqrt{1+\sec^2(x)}}\,dx = \int \frac{1}{\sqrt{\sec\left(x\right)^2+1}}\,dx$$

input
```
integrate(1/(1+sec(x)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(sec(x)^2 + 1), x)
```

3.297.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1+\sec^2(x)}}\,dx = \int \frac{1}{\sqrt{\frac{1}{\cos(x)^2}+1}}\,dx$$

input
```
int(1/(1/cos(x)^2 + 1)^(1/2),x)
```

output
```
int(1/(1/cos(x)^2 + 1)^(1/2), x)
```

3.298 $\int (d\sec(e+fx))^m (a+b\sec^2(e+fx))^p \, dx$

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3.298.1 Optimal result

Integrand size = 25, antiderivative size = 111

$$\int (d\sec(e+fx))^m (a+b\sec^2(e+fx))^p \, dx$$
$$= \frac{\text{AppellF1}\left(\frac{m}{2}, \frac{1}{2}, -p, \frac{2+m}{2}, \sec^2(e+fx), -\frac{b\sec^2(e+fx)}{a}\right) \cot(e+fx)(d\sec(e+fx))^m (a+b\sec^2(e+fx))^p \left($$
$$fm$$

output
```
AppellF1(1/2*m,1/2,-p,1+1/2*m,sec(f*x+e)^2,-b*sec(f*x+e)^2/a)*cos(f*x+e)*(
d*sec(f*x+e))^m*(a+b*sec(f*x+e)^2)^p*(-tan(f*x+e)^2)^(1/2)/f/m/((1+b*sec(f
*x+e)^2/a)^p)/sin(f*x+e)
```

3.298.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2195 vs. 2(111) = 222.

Time = 20.08 (sec) , antiderivative size = 2195, normalized size of antiderivative = 19.77

$$\int (d\sec(e+fx))^m (a+b\sec^2(e+fx))^p \, dx = \text{Result too large to show}$$

input
```
Integrate[(d*Sec[e + f*x])^m*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
(3*(a + b)*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f
*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(d*Sec[e + f*x])^m*(Sec[
e + f*x]^2)^(-1 + m/2 + p)*(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x])/(f*(3*(a
 + b)*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2
)/(a + b))] + (2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e + f*x]^2, -
((b*Tan[e + f*x]^2)/(a + b))] + (a + b)*(-2 + m)*AppellF1[3/2, 2 - m/2, -p
, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2)*((
3*(a + b)*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*
x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(m/2 + p
))/(3*(a + b)*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e
+ f*x]^2)/(a + b))] + (2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e + f
*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (a + b)*(-2 + m)*AppellF1[3/2, 2 -
 m/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*
x]^2) - (6*a*(a + b)*p*AppellF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -(
(b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*(Sec[
e + f*x]^2)^(-1 + m/2 + p)*Sin[2*(e + f*x)]*Tan[e + f*x])/(3*(a + b)*Appel
lF1[1/2, 1 - m/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
 + (2*b*p*AppellF1[3/2, 1 - m/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e +
 f*x]^2)/(a + b))] + (a + b)*(-2 + m)*AppellF1[3/2, 2 - m/2, -p, 5/2, -Tan
[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) + (6*(a + ...
```

3.298.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\sec(e+fx))^m \left(a+b\sec^2(e+fx)\right)^p \, dx$$

$$\downarrow 3042$$

$$\int (d\sec(e+fx))^m \left(a+b\sec(e+fx)^2\right)^p \, dx$$

$$\downarrow 4638$$

$$\int (d\sec(e+fx))^m \left(a+b\sec^2(e+fx)\right)^p \, dx$$

input
```
Int[(d*Sec[e + f*x])^m*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
$Aborted
```

3.298.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4638
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*sec[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Unintegrable[(d*Sec[e + f*x])^m*(a +
 b*(c*Sec[e + f*x])^n)^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

3.298.4 Maple [F]

$$\int (d\sec(fx+e))^m \left(a+b\sec(fx+e)^2\right)^p dx$$

input
```
int((d*sec(f*x+e))^m*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int((d*sec(f*x+e))^m*(a+b*sec(f*x+e)^2)^p,x)
```

3.298.5 Fricas [F]

$$\int (d\sec(e+fx))^m \left(a+b\sec^2(e+fx)\right)^p dx = \int \left(b\sec(fx+e)^2+a\right)^p (d\sec(fx+e))^m dx$$

input
```
integrate((d*sec(f*x+e))^m*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*(d*sec(f*x + e))^m, x)
```

3.298.6 Sympy [F]

$$\int (d\sec(e+fx))^m \left(a+b\sec^2(e+fx)\right)^p dx = \int (d\sec(e+fx))^m \left(a+b\sec^2(e+fx)\right)^p dx$$

input
```
integrate((d*sec(f*x+e))**m*(a+b*sec(f*x+e)**2)**p,x)
```

output
```
Integral((d*sec(e + f*x))**m*(a + b*sec(e + f*x)**2)**p, x)
```

3.298.7 Maxima [F]

$$\int (d\sec(e+fx))^m \left(a+b\sec^2(e+fx)\right)^p \, dx = \int \left(b\sec(fx+e)^2+a\right)^p (d\sec(fx+e))^m \, dx$$

input
```
integrate((d*sec(f*x+e))^m*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*(d*sec(f*x + e))^m, x)
```

3.298.8 Giac [F]

$$\int (d\sec(e+fx))^m \left(a+b\sec^2(e+fx)\right)^p \, dx = \int \left(b\sec(fx+e)^2+a\right)^p (d\sec(fx+e))^m \, dx$$

input
```
integrate((d*sec(f*x+e))^m*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*(d*sec(f*x + e))^m, x)
```

3.298.9 Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^m \left(a+b\sec^2(e+fx)\right)^p \, dx = \int \left(a+\frac{b}{\cos(e+fx)^2}\right)^p \left(\frac{d}{\cos(e+fx)}\right)^m dx$$

input
```
int((a + b/cos(e + f*x)^2)^p*(d/cos(e + f*x))^m,x)
```

output
```
int((a + b/cos(e + f*x)^2)^p*(d/cos(e + f*x))^m, x)
```

3.299 $\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

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3.299.8 Giac [F] 2246
3.299.9 Mupad [F(-1)] 2247

3.299.1 Optimal result

Integrand size = 23, antiderivative size = 103

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$
$$= \frac{\text{AppellF1}\left(\frac{1}{2},2+p,-p,\frac{3}{2},\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\cos^2(e+fx)^p\sin(e+fx)\left(\sec^2(e+fx)\left(a+b-a\sin\right.\right.}{f}$$

output
```
AppellF1(1/2,2+p,-p,3/2,sin(f*x+e)^2,a*sin(f*x+e)^2/(a+b))*(cos(f*x+e)^2)^
p*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^p/f/((1-a*sin(f*x+e)^2/(a
+b))^p)
```

3.299.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1989 vs. 2(103) = 206.

Time = 17.99 (sec) , antiderivative size = 1989, normalized size of antiderivative = 19.31

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \text{Too large to display}$$

input
```
Integrate[Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2)^p,x]
```

output

```
(3*(a + b)*AppellF1[1/2, -1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]
^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*Sec[e + f*x]^3*(Sec[e + f*x
]^2)^(1/2 + p)*(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x])/(f*(3*(a + b)*Appell
F1[1/2, -1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (
2*b*p*AppellF1[3/2, -1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2
)/(a + b))] + (a + b)*AppellF1[3/2, 1/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Ta
n[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2)*((3*(a + b)*AppellF1[1/2, -1/2, -
p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2
*(e + f*x)])^p*(Sec[e + f*x]^2)^(3/2 + p))/(3*(a + b)*AppellF1[1/2, -1/2,
-p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (2*b*p*AppellF1
[3/2, -1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] +
(a + b)*AppellF1[3/2, 1/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/
(a + b))])*Tan[e + f*x]^2) - (6*a*(a + b)*p*AppellF1[1/2, -1/2, -p, 3/2, -
Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x
)])^(-1 + p)*(Sec[e + f*x]^2)^(1/2 + p)*Sin[2*(e + f*x)]*Tan[e + f*x])/(3*
(a + b)*AppellF1[1/2, -1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)
/(a + b))] + (2*b*p*AppellF1[3/2, -1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*
Tan[e + f*x]^2)/(a + b))] + (a + b)*AppellF1[3/2, 1/2, -p, 5/2, -Tan[e + f
*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) + (6*(a + b)*(1/2 +
 p)*AppellF1[1/2, -1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/...
```

3.299.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4636, 2057, 2058, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$

$$\downarrow 3042$$

$$\int \sec(e+fx)^3\left(a+b\sec(e+fx)^2\right)^p\,dx$$

$$\downarrow 4636$$

$$\frac{\int \frac{\left(a+\frac{b}{1-\sin^2(e+fx)}\right)^p}{\left(1-\sin^2(e+fx)\right)^2}d\sin(e+fx)}{f}$$

$$\downarrow 2057$$

$$\frac{\int \frac{\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^p}{\left(1-\sin^2(e+fx)\right)^2} d\sin(e+fx)}{f}$$

$$\downarrow 2058$$

$$\frac{\left(1-\sin^2(e+fx)\right)^p \left(-a\sin^2(e+fx)+a+b\right)^{-p} \left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^p \int \left(1-\sin^2(e+fx)\right)^{-p-2} \left(-a\sin^2(e+fx)\right.}{f}$$

$$\downarrow 334$$

$$\frac{\left(1-\sin^2(e+fx)\right)^p \left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^p \left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^{-p} \int \left(1-\sin^2(e+fx)\right)^{-p-2} \left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^p d\sin(e}{f}$$

$$\downarrow 333$$

$$\frac{\sin(e+fx)\left(1-\sin^2(e+fx)\right)^p \left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^p \left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^{-p} \text{AppellF1}\left(\frac{1}{2}, p+2, -p, \frac{3}{2}, \sin^2(e+fx),\right.}{f}$$

input
```
Int[Sec[e + f*x]^3*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
(AppellF1[1/2, 2 + p, -p, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]
*Sin[e + f*x]*(1 - Sin[e + f*x]^2)^p*((a + b - a*Sin[e + f*x]^2)/(1 - Sin[
e + f*x]^2))^p)/(f*(1 - (a*Sin[e + f*x]^2)/(a + b))^p)
```

3.299.3.1 Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

3.299.4 Maple [F]

$$\int \sec(fx+e)^3 \left(a+b\sec(fx+e)^2\right)^p dx$$

input
```
int(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x)
```

3.299.5 Fricas [F]

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p dx = \int \left(b\sec(fx+e)^2+a\right)^p \sec(fx+e)^3 dx$$

input
```
integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^3, x)
```

3.299.6 Sympy [F]

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)^p \sec^3\left(e+fx\right)\,dx$$

input
```
integrate(sec(f*x+e)**3*(a+b*sec(f*x+e)**2)**p,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**p*sec(e + f*x)**3, x)
```

3.299.7 Maxima [F]

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p \sec\left(fx+e\right)^3\,dx$$

input
```
integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^3, x)
```

3.299.8 Giac [F]

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p \sec\left(fx+e\right)^3\,dx$$

input
```
integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^3, x)
```

3.299.9 Mupad [F(-1)]

Timed out.

$$\int \sec^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \frac{\left(a+\frac{b}{\cos(e+fx)^2}\right)^p}{\cos\left(e+fx\right)^3}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^p/cos(e + f*x)^3,x)
```

output
```
int((a + b/cos(e + f*x)^2)^p/cos(e + f*x)^3, x)
```

3.300 $\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

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3.300.8 Giac [F] 2252
3.300.9 Mupad [F(-1)] 2253

3.300.1 Optimal result

Integrand size = 21, antiderivative size = 103

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$
$$= \frac{\text{AppellF1}\left(\frac{1}{2},1+p,-p,\frac{3}{2},\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\cos^2(e+fx)^p\sin(e+fx)\left(\sec^2(e+fx)\left(a+b-a\sin\right.\right.}{f}$$

output
```
AppellF1(1/2,p+1,-p,3/2,sin(f*x+e)^2,a*sin(f*x+e)^2/(a+b))*(cos(f*x+e)^2)^
p*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^p/f/((1-a*sin(f*x+e)^2/(a
+b))^p)
```

3.300.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1995 vs. $2(103) = 206$.

Time = 17.42 (sec) , antiderivative size = 1995, normalized size of antiderivative = 19.37

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \text{Too large to display}$$

input
```
Integrate[Sec[e + f*x]*(a + b*Sec[e + f*x]^2)^p,x]
```

output

```
(3*(a + b)*AppellF1[1/2, 1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^
2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*Sec[e + f*x]*(Sec[e + f*x]^2
)^(-1/2 + p)*(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x])/(f*(3*(a + b)*AppellF1
[1/2, 1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (2*b
*p*AppellF1[3/2, 1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a
 + b))] - (a + b)*AppellF1[3/2, 3/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e
+ f*x]^2)/(a + b))])*Tan[e + f*x]^2)*((3*(a + b)*AppellF1[1/2, 1/2, -p, 3/
2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e +
 f*x)])^p*(Sec[e + f*x]^2)^(1/2 + p))/(3*(a + b)*AppellF1[1/2, 1/2, -p, 3/
2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (2*b*p*AppellF1[3/2,
1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - (a + b)
*AppellF1[3/2, 3/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b)
)])*Tan[e + f*x]^2) - (6*a*(a + b)*p*AppellF1[1/2, 1/2, -p, 3/2, -Tan[e +
f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^(-1
+ p)*(Sec[e + f*x]^2)^(-1/2 + p)*Sin[2*(e + f*x)]*Tan[e + f*x])/(3*(a + b)
*AppellF1[1/2, 1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b)
)] + (2*b*p*AppellF1[3/2, 1/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f
*x]^2)/(a + b))] - (a + b)*AppellF1[3/2, 3/2, -p, 5/2, -Tan[e + f*x]^2, -(
(b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) + (6*(a + b)*(-1/2 + p)*Appe
llF1[1/2, 1/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]...
```

3.300.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4636, 2057, 2058, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$

$$\downarrow 3042$$

$$\int \sec(e+fx)\left(a+b\sec(e+fx)^2\right)^p dx$$

$$\downarrow 4636$$

$$\frac{\int \frac{\left(a+\frac{b}{1-\sin^2(e+fx)}\right)^p}{1-\sin^2(e+fx)} d\sin(e+fx)}{f}$$

$$\downarrow 2057$$

$$\frac{\int \frac{\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^p}{1-\sin^2(e+fx)} d\sin(e+fx)}{f}$$

$$\downarrow 2058$$

$$\frac{\left(1-\sin^2(e+fx)\right)^p \left(-a\sin^2(e+fx)+a+b\right)^{-p} \left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^p \int \left(1-\sin^2(e+fx)\right)^{-p-1} \left(-a\sin^2(e+fx)\right.}{f}$$

$$\downarrow 334$$

$$\frac{\left(1-\sin^2(e+fx)\right)^p \left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^p \left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^{-p} \int \left(1-\sin^2(e+fx)\right)^{-p-1} \left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^p d\sin(e}{f}$$

$$\downarrow 333$$

$$\frac{\sin(e+fx)\left(1-\sin^2(e+fx)\right)^p \left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^p \left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^{-p} \text{AppellF1}\left(\frac{1}{2}, p+1, -p, \frac{3}{2}, \sin^2(e+fx),\right.}{f}$$

input
```
Int[Sec[e + f*x]*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
(AppellF1[1/2, 1 + p, -p, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]
*Sin[e + f*x]*(1 - Sin[e + f*x]^2)^p*((a + b - a*Sin[e + f*x]^2)/(1 - Sin[
e + f*x]^2))^p)/(f*(1 - (a*Sin[e + f*x]^2)/(a + b))^p)
```

3.300.3.1 Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

3.300.4 Maple [F]

$$\int \sec(fx+e)\left(a+b\sec(fx+e)^2\right)^p dx$$

input
```
int(sec(f*x+e)*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int(sec(f*x+e)*(a+b*sec(f*x+e)^2)^p,x)
```

3.300.5 Fricas [F]

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^p dx = \int \left(b\sec(fx+e)^2+a\right)^p \sec(fx+e)\, dx$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*sec(f*x + e), x)
```

3.300.6 Sympy [F]

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)^p \sec\left(e+fx\right)\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e)**2)**p,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**p*sec(e + f*x), x)
```

3.300.7 Maxima [F]

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p \sec\left(fx+e\right)\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e), x)
```

3.300.8 Giac [F]

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p \sec\left(fx+e\right)\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e), x)
```

3.300.9 Mupad [F(-1)]

Timed out.

$$\int \sec(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \frac{\left(a+\frac{b}{\cos(e+fx)^2}\right)^p}{\cos(e+fx)}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^p/cos(e + f*x),x)
```

output
```
int((a + b/cos(e + f*x)^2)^p/cos(e + f*x), x)
```

3.301 $\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

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3.301.2 Mathematica [B] (warning: unable to verify) . . . 2254
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3.301.7 Maxima [F] . . . 2258
3.301.8 Giac [F] . . . 2258
3.301.9 Mupad [F(-1)] . . . 2259

3.301.1 Optimal result

Integrand size = 21, antiderivative size = 101

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$
$$= \frac{\text{AppellF1}\left(\frac{1}{2},p,-p,\frac{3}{2},\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\cos^2(e+fx)^p\sin(e+fx)\left(\sec^2(e+fx)\left(a+b-a\sin^2(e\right.}{f}$$

output
```
AppellF1(1/2,p,-p,3/2,sin(f*x+e)^2,a*sin(f*x+e)^2/(a+b))*(cos(f*x+e)^2)^p*
sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^p/f/((1-a*sin(f*x+e)^2/(a+b
))^p)
```

3.301.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1983 vs. 2(101) = 202.

Time = 17.08 (sec) , antiderivative size = 1983, normalized size of antiderivative = 19.63

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^p,x]
```

output

```
(-3*(a + b)*AppellF1[1/2, 3/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]
^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-3/2 + p)
*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x])/(f*(-3*(a + b)*AppellF1[1/2, 3/2,
-p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (-2*b*p*AppellF
1[3/2, 3/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] +
3*(a + b)*AppellF1[3/2, 5/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2
)/(a + b))])*Tan[e + f*x]^2)*((-3*(a + b)*AppellF1[1/2, 3/2, -p, 3/2, -Tan
[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])
^p*(Sec[e + f*x]^2)^(-1/2 + p))/(-3*(a + b)*AppellF1[1/2, 3/2, -p, 3/2, -T
an[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (-2*b*p*AppellF1[3/2, 3/2,
 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 3*(a + b)*A
ppellF1[3/2, 5/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
)*Tan[e + f*x]^2) + (6*a*(a + b)*p*AppellF1[1/2, 3/2, -p, 3/2, -Tan[e + f*
x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 +
p)*(Sec[e + f*x]^2)^(-3/2 + p)*Sin[2*(e + f*x)]*Tan[e + f*x])/(-3*(a + b)*
AppellF1[1/2, 3/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))
] + (-2*b*p*AppellF1[3/2, 3/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f
*x]^2)/(a + b))] + 3*(a + b)*AppellF1[3/2, 5/2, -p, 5/2, -Tan[e + f*x]^2,
-((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (6*(a + b)*(-3/2 + p)*Ap
pellF1[1/2, 3/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b)...
```

3.301.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4636, 2057, 2058, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^p}{\sec(e+fx)}dx$$

$$\downarrow 4636$$

$$\frac{\int \left(a+\frac{b}{1-\sin^2(e+fx)}\right)^p d\sin(e+fx)}{f}$$

$$\downarrow 2057$$

$$\frac{\int \left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^p d\sin(e+fx)}{f}$$

$$\downarrow 2058$$

$$\frac{(1-\sin^2(e+fx))^p\left(-a\sin^2(e+fx)+a+b\right)^{-p}\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^p \int (1-\sin^2(e+fx))^{-p}\left(-a\sin^2(e+fx)+\right.}{f}$$

$$\downarrow 334$$

$$\frac{(1-\sin^2(e+fx))^p\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^p\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^{-p}\int (1-\sin^2(e+fx))^{-p}\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^p d\sin(e+}{f}$$

$$\downarrow 333$$

$$\frac{\sin(e+fx)\,(1-\sin^2(e+fx))^p\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^p\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},p,-p,\frac{3}{2},\sin^2(e+fx),\frac{a\sin}{}\right.}{f}$$

input
```
Int[Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
(AppellF1[1/2, p, -p, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Sin
[e + f*x]*(1 - Sin[e + f*x]^2)^p*((a + b - a*Sin[e + f*x]^2)/(1 - Sin[e +
f*x]^2))^p)/(f*(1 - (a*Sin[e + f*x]^2)/(a + b))^p)
```

3.301.3.1 Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

3.301.4 Maple [F]

$$\int \cos(fx+e)\left(a+b\sec(fx+e)^2\right)^p dx$$

input
```
int(cos(f*x+e)*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int(cos(f*x+e)*(a+b*sec(f*x+e)^2)^p,x)
```

3.301.5 Fricas [F]

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p dx = \int \left(b\sec(fx+e)^2+a\right)^p \cos(fx+e)\, dx$$

input
```
integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*cos(f*x + e), x)
```

3.301.6 Sympy [F]

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(a+b\sec^2(e+fx)\right)^p\cos(e+fx)\,dx$$

input
```
integrate(cos(f*x+e)*(a+b*sec(f*x+e)**2)**p,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**p*cos(e + f*x), x)
```

3.301.7 Maxima [F]

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec(fx+e)^2+a\right)^p\cos(fx+e)\,dx$$

input
```
integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e), x)
```

3.301.8 Giac [F]

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec(fx+e)^2+a\right)^p\cos(fx+e)\,dx$$

input
```
integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e), x)
```

3.301.9 Mupad [F(-1)]

Timed out.

$$\int \cos(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \cos\left(e+f\,x\right)\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p dx$$

input
```
int(cos(e + f*x)*(a + b/cos(e + f*x)^2)^p,x)
```

output
```
int(cos(e + f*x)*(a + b/cos(e + f*x)^2)^p, x)
```

3.302 $\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

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3.302.7 Maxima [F] . . . 2264
3.302.8 Giac [F] . . . 2264
3.302.9 Mupad [F(-1)] . . . 2265

3.302.1 Optimal result

Integrand size = 23, antiderivative size = 103

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$
$$= \frac{\text{AppellF1}\left(\frac{1}{2},-1+p,-p,\frac{3}{2},\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\cos^2(e+fx)^p\sin(e+fx)\left(\sec^2(e+fx)\left(a+b-a\s\right.\right.}{f}$$

output
```
AppellF1(1/2,-1+p,-p,3/2,sin(f*x+e)^2,a*sin(f*x+e)^2/(a+b))*(cos(f*x+e)^2)
^p*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^p/f/((1-a*sin(f*x+e)^2/(
a+b))^p)
```

3.302.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1987 vs. 2(103) = 206.

Time = 17.43 (sec) , antiderivative size = 1987, normalized size of antiderivative = 19.29

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^p,x]
```

output

```
(-3*(a + b)*AppellF1[1/2, 5/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]
^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-7/2 + p)
*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x])/(f*(-3*(a + b)*AppellF1[1/2, 5/2,
-p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (-2*b*p*AppellF
1[3/2, 5/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] +
5*(a + b)*AppellF1[3/2, 7/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2
)/(a + b))])*Tan[e + f*x]^2)*((-3*(a + b)*AppellF1[1/2, 5/2, -p, 3/2, -Tan
[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])
^p*(Sec[e + f*x]^2)^(-3/2 + p))/(-3*(a + b)*AppellF1[1/2, 5/2, -p, 3/2, -T
an[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (-2*b*p*AppellF1[3/2, 5/2,
 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 5*(a + b)*A
ppellF1[3/2, 7/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
)*Tan[e + f*x]^2) + (6*a*(a + b)*p*AppellF1[1/2, 5/2, -p, 3/2, -Tan[e + f*
x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 +
p)*(Sec[e + f*x]^2)^(-5/2 + p)*Sin[2*(e + f*x)]*Tan[e + f*x])/(-3*(a + b)*
AppellF1[1/2, 5/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))
] + (-2*b*p*AppellF1[3/2, 5/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f
*x]^2)/(a + b))] + 5*(a + b)*AppellF1[3/2, 7/2, -p, 5/2, -Tan[e + f*x]^2,
-((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (6*(a + b)*(-5/2 + p)*Ap
pellF1[1/2, 5/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b)...
```

3.302.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4636, 2057, 2058, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^p}{\sec(e+fx)^3}dx$$

$$\downarrow 4636$$

$$\frac{\int \left(1-\sin^2(e+fx)\right)\left(a+\frac{b}{1-\sin^2(e+fx)}\right)^p d\sin(e+fx)}{f}$$

$$\downarrow 2057$$

$$\frac{\int \left(1-\sin^2(e+fx)\right)\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^p d\sin(e+fx)}{f}$$

$$\downarrow 2058$$

$$\frac{\left(1-\sin^2(e+fx)\right)^p\left(-a\sin^2(e+fx)+a+b\right)^{-p}\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^p\int\left(1-\sin^2(e+fx)\right)^{1-p}\left(-a\sin^2(e+fx)+\right.}{f}$$

$$\downarrow 334$$

$$\frac{\left(1-\sin^2(e+fx)\right)^p\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^p\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^{-p}\int\left(1-\sin^2(e+fx)\right)^{1-p}\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^p d\sin(e+\right.}{f}$$

$$\downarrow 333$$

$$\frac{\sin(e+fx)\left(1-\sin^2(e+fx)\right)^p\left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^p\left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},p-1,-p,\frac{3}{2},\sin^2(e+fx),\right.}{f}$$

input
```
Int[Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
(AppellF1[1/2, -1 + p, -p, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)
]*Sin[e + f*x]*(1 - Sin[e + f*x]^2)^p*((a + b - a*Sin[e + f*x]^2)/(1 - Sin
[e + f*x]^2))^p)/(f*(1 - (a*Sin[e + f*x]^2)/(a + b))^p)
```

3.302.3.1 Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

3.302.4 Maple [F]

$$\int \cos(fx+e)^3 \left(a+b\sec(fx+e)^2\right)^p dx$$

input
```
int(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x)
```

3.302.5 Fricas [F]

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p dx = \int \left(b\sec(fx+e)^2+a\right)^p \cos(fx+e)^3 dx$$

input
```
integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^3, x)
```

3.302.6 Sympy [F(-1)]

Timed out.

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**3*(a+b*sec(f*x+e)**2)**p,x)
```

output
```
Timed out
```

3.302.7 Maxima [F]

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)^3\,dx$$

input
```
integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^3, x)
```

3.302.8 Giac [F]

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)^3\,dx$$

input
```
integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^3, x)
```

3.302.9 Mupad [F(-1)]

Timed out.

$$\int \cos^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \cos\left(e+f\,x\right)^3\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p dx$$

input
```
int(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^p,x)
```

output
```
int(cos(e + f*x)^3*(a + b/cos(e + f*x)^2)^p, x)
```

3.303 $\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

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3.303.1 Optimal result

Integrand size = 23, antiderivative size = 103

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$
$$= \frac{\text{AppellF1}\left(\frac{1}{2},-2+p,-p,\frac{3}{2},\sin^2(e+fx),\frac{a\sin^2(e+fx)}{a+b}\right)\cos^2(e+fx)^p\sin(e+fx)\left(\sec^2(e+fx)\left(a+b-a\s\right.\right.}{f}$$

output
```
AppellF1(1/2,-2+p,-p,3/2,sin(f*x+e)^2,a*sin(f*x+e)^2/(a+b))*(cos(f*x+e)^2)
^p*sin(f*x+e)*(sec(f*x+e)^2*(a+b-a*sin(f*x+e)^2))^p/f/((1-a*sin(f*x+e)^2/(
a+b))^p)
```

3.303.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1997 vs. 2(103) = 206.

Time = 17.60 (sec) , antiderivative size = 1997, normalized size of antiderivative = 19.39

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2)^p,x]
```

output

```
(-3*(a + b)*AppellF1[1/2, 7/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]
^2)/(a + b))]*Cos[e + f*x]^4*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x
]^2)^(-7/2 + p)*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x])/(f*(-3*(a + b)*Appe
llF1[1/2, 7/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] +
(-2*b*p*AppellF1[3/2, 7/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^
2)/(a + b))] + 7*(a + b)*AppellF1[3/2, 9/2, -p, 5/2, -Tan[e + f*x]^2, -((b
*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2)*((-3*(a + b)*AppellF1[1/2, 7/2
, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Co
s[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(-5/2 + p))/(-3*(a + b)*AppellF1[1/2, 7
/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + (-2*b*p*App
ellF1[3/2, 7/2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))
] + 7*(a + b)*AppellF1[3/2, 9/2, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*
x]^2)/(a + b))])*Tan[e + f*x]^2) + (6*a*(a + b)*p*AppellF1[1/2, 7/2, -p, 3
/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e
+ f*x)])^(-1 + p)*(Sec[e + f*x]^2)^(-7/2 + p)*Sin[2*(e + f*x)]*Tan[e + f*x
])/(-3*(a + b)*AppellF1[1/2, 7/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f
*x]^2)/(a + b))] + (-2*b*p*AppellF1[3/2, 7/2, 1 - p, 5/2, -Tan[e + f*x]^2,
 -((b*Tan[e + f*x]^2)/(a + b))] + 7*(a + b)*AppellF1[3/2, 9/2, -p, 5/2, -T
an[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x]^2) - (6*(a + b
)*(-7/2 + p)*AppellF1[1/2, 7/2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + ...
```

3.303.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4636, 2057, 2058, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^p}{\sec(e+fx)^5}dx$$

$$\downarrow 4636$$

$$\frac{\int \left(1-\sin^2(e+fx)\right)^2\left(a+\frac{b}{1-\sin^2(e+fx)}\right)^p d\sin(e+fx)}{f}$$

$$\downarrow 2057$$

$$\frac{\int \left(1-\sin^2(e+fx)\right)^2 \left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^p d\sin(e+fx)}{f}$$

$$\downarrow 2058$$

$$\frac{\left(1-\sin^2(e+fx)\right)^p \left(-a\sin^2(e+fx)+a+b\right)^{-p} \left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^p \int \left(1-\sin^2(e+fx)\right)^{2-p} \left(-a\sin^2(e+fx)+\right.}{f}$$

$$\downarrow 334$$

$$\frac{\left(1-\sin^2(e+fx)\right)^p \left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^p \left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^{-p} \int \left(1-\sin^2(e+fx)\right)^{2-p} \left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^p d\sin(e+\right.}{f}$$

$$\downarrow 333$$

$$\frac{\sin(e+fx)\left(1-\sin^2(e+fx)\right)^p \left(\frac{-a\sin^2(e+fx)+a+b}{1-\sin^2(e+fx)}\right)^p \left(1-\frac{a\sin^2(e+fx)}{a+b}\right)^{-p} \text{AppellF1}\left(\frac{1}{2},p-2,-p,\frac{3}{2},\sin^2(e+fx),\right.}{f}$$

input
```
Int[Cos[e + f*x]^5*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
(AppellF1[1/2, -2 + p, -p, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)
]*Sin[e + f*x]*(1 - Sin[e + f*x]^2)^p*((a + b - a*Sin[e + f*x]^2)/(1 - Sin
[e + f*x]^2))^p)/(f*(1 - (a*Sin[e + f*x]^2)/(a + b))^p)
```

3.303.3.1 Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4636
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
 Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x
, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
 && IntegerQ[n/2] &&  !IntegerQ[p]
```

3.303.4 Maple [F]

$$\int \cos(fx+e)^5 \left(a+b\sec(fx+e)^2\right)^p dx$$

input
```
int(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^p,x)
```

3.303.5 Fricas [F]

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^p dx = \int \left(b\sec(fx+e)^2+a\right)^p \cos(fx+e)^5 dx$$

input
```
integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^5, x)
```

3.303.6 Sympy [F(-1)]

Timed out.

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**5*(a+b*sec(f*x+e)**2)**p,x)
```

output
```
Timed out
```

3.303.7 Maxima [F]

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)^5\,dx$$

input
```
integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^5, x)
```

3.303.8 Giac [F]

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)^5\,dx$$

input
```
integrate(cos(f*x+e)^5*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^5, x)
```

3.303.9 Mupad [F(-1)]

Timed out.

$$\int \cos^5(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \cos(e+f\,x)^5\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p dx$$

input
```
int(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^p,x)
```

output
```
int(cos(e + f*x)^5*(a + b/cos(e + f*x)^2)^p, x)
```

3.304 $\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

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3.304.1 Optimal result

Integrand size = 23, antiderivative size = 216

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$
$$= -\frac{(3a-2b(2+p))\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{b^2 f(3+2p)(5+2p)}$$
$$+\frac{\sec^2(e+fx)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{bf(5+2p)}$$
$$+\frac{(3a^2-4ab(1+p)+4b^2(2+3p+p^2))\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\,(a+b}{b^2 f(3+2p)(5+2p)}$$

output

```
-(3*a-2*b*(2+p))*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^(p+1)/b^2/f/(4*p^2+16*p+1
5)+sec(f*x+e)^2*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^(p+1)/b/f/(5+2*p)+(3*a^2-4
*a*b*(p+1)+4*b^2*(p^2+3*p+2))*hypergeom([1/2, -p],[3/2],-b*tan(f*x+e)^2/(a
+b))*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^p/b^2/f/(4*p^2+16*p+15)/((1+b*tan(f*x
+e)^2/(a+b))^p)
```

3.304.2 Mathematica [A] (verified)

Time = 2.16 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.69

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$
$$= \frac{\left(a+b\sec^2(e+fx)\right)^p \tan(e+fx)\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\left(15\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)+\right.}{}$$

input
```
Integrate[Sec[e + f*x]^6*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
((a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]*(15*Hypergeometric2F1[1/2, -p, 3/2,
 -((b*Tan[e + f*x]^2)/(a + b))] + 10*Hypergeometric2F1[3/2, -p, 5/2, -((b*
Tan[e + f*x]^2)/(a + b))]*Tan[e + f*x]^2 + 3*Hypergeometric2F1[5/2, -p, 7/
2, -((b*Tan[e + f*x]^2)/(a + b))]*Tan[e + f*x]^4))/(15*f*(1 + (b*Tan[e + f
*x]^2)/(a + b))^p)
```

3.304.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4634, 318, 25, 299, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$
$$\downarrow 3042$$
$$\int \sec(e+fx)^6\left(a+b\sec(e+fx)^2\right)^p\,dx$$
$$\downarrow 4634$$
$$\frac{\int \left(\tan^2(e+fx)+1\right)^2\left(b\tan^2(e+fx)+a+b\right)^p\,d\tan(e+fx)}{f}$$
$$\downarrow 318$$
$$\frac{\int -\left(b\tan^2(e+fx)+a+b\right)^p\left((3a-2b(p+2))\tan^2(e+fx)+a-2b(p+2)\right)d\tan(e+fx)}{b(2p+5)} + \frac{\tan(e+fx)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{p+1}}{b(2p+5)}$$
$$f$$

$$\downarrow 25$$

$$\frac{\tan(e+fx)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{p+1}}{b(2p+5)} - \frac{\int \left(b\tan^2(e+fx)+a+b\right)^p\left((3a-2b(p+2))\tan^2(e+fx)+a-2b(p+2)\right)d\tan(e+fx)}{b(2p+5)}$$
$$f$$

$$\downarrow 299$$

$$\frac{\tan(e+fx)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{p+1}}{b(2p+5)} - \frac{\frac{(3a-2b(p+2))\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{p+1}}{b(2p+3)} - \frac{\left(3a^2-4ab(p+1)+4b^2\left(p^2+3p+2\right)\right)\int\left(b\tan^2(\right.}{b(2p+3)}}{b(2p+5)}$$
$$f$$

$$\downarrow 238$$

$$\frac{\tan(e+fx)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{p+1}}{b(2p+5)} - \frac{\frac{(3a-2b(p+2))\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{p+1}}{b(2p+3)} - \frac{\left(3a^2-4ab(p+1)+4b^2\left(p^2+3p+2\right)\right)\left(a+b\tan^2\right.}{b(2p+5)}}{f}$$

$$\downarrow 237$$

$$\frac{\tan(e+fx)\left(\tan^2(e+fx)+1\right)\left(a+b\tan^2(e+fx)+b\right)^{p+1}}{b(2p+5)} - \frac{\frac{(3a-2b(p+2))\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{p+1}}{b(2p+3)} - \frac{\left(3a^2-4ab(p+1)+4b^2\left(p^2+3p+2\right)\right)\tan(e+fx}{b(2p+}}{f}$$

input
```
Int[Sec[e + f*x]^6*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
((Tan[e + f*x]*(1 + Tan[e + f*x]^2)*(a + b + b*Tan[e + f*x]^2)^(1 + p))/(b
*(5 + 2*p)) - (((3*a - 2*b*(2 + p))*Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2
)^(1 + p))/(b*(3 + 2*p)) - ((3*a^2 - 4*a*b*(1 + p) + 4*b^2*(2 + 3*p + p^2)
)*Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(a + b))]*Tan[e + f
*x]*(a + b + b*Tan[e + f*x]^2)^p)/(b*(3 + 2*p)*(1 + (b*Tan[e + f*x]^2)/(a
+ b))^p))/(b*(5 + 2*p)))/f
```

3.304.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.304.4 Maple [F]

$$\int \sec(fx+e)^6 \left(a+b\sec(fx+e)^2\right)^p dx$$

input
```
int(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x)
```

3.304.5 Fricas [F]

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p dx = \int \left(b\sec(fx+e)^2+a\right)^p \sec(fx+e)^6 dx$$

input
```
integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^6, x)
```

3.304.6 Sympy [F(-1)]

Timed out.

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)**6*(a+b*sec(f*x+e)**2)**p,x)
```

output
```
Timed out
```

3.304.7 Maxima [F]

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p \sec\left(fx+e\right)^6\,dx$$

input
```
integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^6, x)
```

3.304.8 Giac [F]

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p \sec\left(fx+e\right)^6\,dx$$

input
```
integrate(sec(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^6, x)
```

3.304.9 Mupad [F(-1)]

Timed out.

$$\int \sec^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p}{\cos\left(e+f\,x\right)^6}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^p/cos(e + f*x)^6,x)
```

output
```
int((a + b/cos(e + f*x)^2)^p/cos(e + f*x)^6, x)
```

3.305 $\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

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3.305.9 Mupad [F(-1)] 2282

3.305.1 Optimal result

Integrand size = 23, antiderivative size = 129

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^{1+p}}{bf(3+2p)} - \frac{(a-2b(1+p))\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^p\left(1-\right.}{bf(3+2p)}$$

output

```
tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^(p+1)/b/f/(3+2*p)-(a-2*b*(p+1))*hypergeom(
[1/2, -p],[3/2],-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)*(a+b+b*tan(f*x+e)^2)^p/b
/f/(3+2*p)/((1+b*tan(f*x+e)^2/(a+b))^p)
```

3.305.2 Mathematica [A] (verified)

Time = 2.14 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.98

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\left(a+b\sec^2(e+fx)\right)^p\tan(e+fx)\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}\left((-a+2b(1+p))\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\right.\right.}{bf(3+2p)}$$

input

```
Integrate[Sec[e + f*x]^4*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
((a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]*((-a + 2*b*(1 + p))*Hypergeometric2
F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(a + b))] + (a + b + b*Tan[e + f*x]^
2)*(1 + (b*Tan[e + f*x]^2)/(a + b))^p))/(b*f*(3 + 2*p)*(1 + (b*Tan[e + f*x
]^2)/(a + b))^p)
```

3.305.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4634, 299, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$

$$\downarrow 3042$$

$$\int \sec(e+fx)^4\left(a+b\sec(e+fx)^2\right)^p\,dx$$

$$\downarrow 4634$$

$$\frac{\int \left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^p\,d\tan(e+fx)}{f}$$

$$\downarrow 299$$

$$\frac{\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{p+1}}{b(2p+3)}-\frac{(a-2b(p+1))\int\left(b\tan^2(e+fx)+a+b\right)^p d\tan(e+fx)}{b(2p+3)}}{f}$$

$$\downarrow 238$$

$$\frac{\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{p+1}}{b(2p+3)}-\frac{(a-2b(p+1))\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\int\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^p d\tan(e+fx)}{b(2p+3)}}{f}$$

$$\downarrow 237$$

$$\frac{\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^{p+1}}{b(2p+3)}-\frac{(a-2b(p+1))\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{b\tan}{}\right.}{b(2p+3)}}{f}$$

input
```
Int[Sec[e + f*x]^4*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
((Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^(1 + p))/(b*(3 + 2*p)) - ((a - 2
*b*(1 + p))*Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(a + b))]
*Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(b*(3 + 2*p)*(1 + (b*Tan[e + f
*x]^2)/(a + b))^p))/f
```

3.305.3.1 Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.305.4 Maple [F]

$$\int \sec(fx+e)^4 \left(a+b\sec(fx+e)^2\right)^p dx$$

input
```
int(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x)
```

3.305.5 Fricas [F]

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p dx = \int \left(b\sec(fx+e)^2+a\right)^p \sec(fx+e)^4 dx$$

input
```
integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^4, x)
```

3.305.6 Sympy [F(-1)]

Timed out.

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)**4*(a+b*sec(f*x+e)**2)**p,x)
```

output
```
Timed out
```

3.305.7 Maxima [F]

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p \sec\left(fx+e\right)^4\,dx$$

input
```
integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^4, x)
```

3.305.8 Giac [F]

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p \sec\left(fx+e\right)^4\,dx$$

input
```
integrate(sec(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^4, x)
```

3.305.9 Mupad [F(-1)]

Timed out.

$$\int \sec^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \frac{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p}{\cos\left(e+f\,x\right)^4}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^p/cos(e + f*x)^4,x)
```

output
```
int((a + b/cos(e + f*x)^2)^p/cos(e + f*x)^4, x)
```

3.306 $\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

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3.306.1 Optimal result

Integrand size = 23, antiderivative size = 72

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}}{f}$$

output
```
hypergeom([1/2, -p],[3/2],-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)*(a+b+b*tan(f*x
+e)^2)^p/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

3.306.2 Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.99

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \frac{\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)\left(a+b\sec^2(e+fx)\right)^p\tan(e+fx)\left(1+\frac{b\tan^2(e+fx)}{a+b}\right)^{-p}}{f}$$

input
```
Integrate[Sec[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
(Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + b*Sec
[e + f*x]^2)^p*Tan[e + f*x])/(f*(1 + (b*Tan[e + f*x]^2)/(a + b))^p)
```

3.306.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4634, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$

$$\downarrow 3042$$

$$\int \sec(e+fx)^2\left(a+b\sec(e+fx)^2\right)^p\,dx$$

$$\downarrow 4634$$

$$\frac{\int \left(b\tan^2(e+fx)+a+b\right)^p\,d\tan(e+fx)}{f}$$

$$\downarrow 238$$

$$\frac{(a+b\tan^2(e+fx)+b)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\int \left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^p\,d\tan(e+fx)}{f}$$

$$\downarrow 237$$

$$\frac{\tan(e+fx)\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

input
```
Int[Sec[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
(Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(a + b))]*Tan[e + f*
x]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a + b))^p)
```

3.306.3.1 Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.306.4 Maple [F]

$$\int \sec(fx+e)^2 \left(a+b\sec(fx+e)^2\right)^p dx$$

input
```
int(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x)
```

3.306.5 Fricas [F]

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p dx = \int \left(b\sec(fx+e)^2+a\right)^p \sec(fx+e)^2 dx$$

input
```
integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^2, x)
```

3.306.6 Sympy [F]

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)^p \sec^2\left(e+fx\right)\,dx$$

input
```
integrate(sec(f*x+e)**2*(a+b*sec(f*x+e)**2)**p,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**p*sec(e + f*x)**2, x)
```

3.306.7 Maxima [F]

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p \sec\left(fx+e\right)^2\,dx$$

input
```
integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^2, x)
```

3.306.8 Giac [F]

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p \sec\left(fx+e\right)^2\,dx$$

input
```
integrate(sec(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*sec(f*x + e)^2, x)
```

3.306.9 Mupad [F(-1)]

Timed out.

$$\int \sec^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \frac{\left(a+\frac{b}{\cos(e+fx)^2}\right)^p}{\cos\left(e+fx\right)^2}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^p/cos(e + f*x)^2,x)
```

output
```
int((a + b/cos(e + f*x)^2)^p/cos(e + f*x)^2, x)
```

3.307 $\int (a+b\sec^2(e+fx))^p\,dx$

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3.307.9 Mupad [F(-1)] 2292

3.307.1 Optimal result

Integrand size = 14, antiderivative size = 83

$$\int (a+b\sec^2(e+fx))^p\,dx$$
$$= \frac{\text{AppellF1}\left(\frac{1}{2},1,-p,\frac{3}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\,(a+b+b\tan^2(e+fx))^p\left(1+\frac{b\tan^2(e+f}{a+b}\right.}{f}$$

output
```
AppellF1(1/2,1,-p,3/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)*(a+b
+b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

3.307.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2137 vs. 2(83) = 166.

Time = 6.25 (sec) , antiderivative size = 2137, normalized size of antiderivative = 25.75

$$\int (a+b\sec^2(e+fx))^p\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^p,x]
```

output

```
(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e
 + f*x]^2]*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^
p*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x])/(f*(3*(a + b)*AppellF1[1/2, -p, 1
, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3
/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a +
b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^
2])*Tan[e + f*x]^2)*((3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x
]^2)/(a + b)), -Tan[e + f*x]^2]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e +
f*x]^2)^(-1 + p))/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2
)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Ta
n[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/
2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) - (3*(
a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f
*x]^2]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]^2)
/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[
e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a
+ b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f
*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) + (6*(a + b)*p*AppellF1
[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*(a + 2*b
 + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]^2)/(3*(a + b)*...
```

3.307.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4616, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + b\sec^2(e + fx))^p \, dx$$

$$\downarrow 3042$$

$$\int (a + b\sec(e + fx)^2)^p \, dx$$

$$\downarrow 4616$$

$$\frac{\int \frac{(b\tan^2(e+fx)+a+b)^p}{\tan^2(e+fx)+1} d\tan(e + fx)}{f}$$

$$\downarrow 334$$

$$\frac{(a+b\tan^2(e+fx)+b)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\int\frac{\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^p}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\downarrow 333$$

$$\frac{\tan(e+fx)\,(a+b\tan^2(e+fx)+b)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},1,-p,\frac{3}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
(AppellF1[1/2, 1, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
*Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a
+ b))^p)
```

3.307.3.1 Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

3.307.4 Maple [F]

$$\int (a + b \sec (fx + e)^2)^p \, dx$$

input
```
int((a+b*sec(f*x+e)^2)^p,x)
```

output
```
int((a+b*sec(f*x+e)^2)^p,x)
```

3.307.5 Fricas [F]

$$\int (a + b \sec^2(e + fx))^p \, dx = \int (b \sec (fx + e)^2 + a)^p \, dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p, x)
```

3.307.6 Sympy [F]

$$\int (a + b \sec^2(e + fx))^p \, dx = \int (a + b \sec^2 (e + fx))^p \, dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**p,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**p, x)
```

3.307.7 Maxima [F]

$$\int (a + b \sec^2(e + fx))^p \, dx = \int (b \sec(fx + e)^2 + a)^p \, dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p, x)
```

3.307.8 Giac [F]

$$\int (a + b \sec^2(e + fx))^p \, dx = \int (b \sec(fx + e)^2 + a)^p \, dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p, x)
```

3.307.9 Mupad [F(-1)]

Timed out.

$$\int (a + b \sec^2(e + fx))^p \, dx = \int \left(a + \frac{b}{\cos(e + fx)^2}\right)^p dx$$

input
```
int((a + b/cos(e + f*x)^2)^p,x)
```

output
```
int((a + b/cos(e + f*x)^2)^p, x)
```

3.308 $\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

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3.308.9 Mupad [F(-1)] . . . 2297

3.308.1 Optimal result

Integrand size = 23, antiderivative size = 83

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$
$$= \frac{\text{AppellF1}\left(\frac{1}{2},2,-p,\frac{3}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+f}{a+b}\right.}{f}$$

output
```
AppellF1(1/2,2,-p,3/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)*(a+b
+b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

3.308.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1914 vs. 2(83) = 166.

Time = 16.55 (sec) , antiderivative size = 1914, normalized size of antiderivative = 23.06

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p,x]
```

output

```
(3*(a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)
/(a + b))]*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^
(-2 + p)*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x])/(f*(3*(a + b)*AppellF1[1/2
, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*App
ellF1[3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
 - 2*(a + b)*AppellF1[3/2, 3, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2
)/(a + b))])*Tan[e + f*x]^2)*((3*(a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e
 + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^p*
(Sec[e + f*x]^2)^(-1 + p))/(3*(a + b)*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f
*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 2, 1 - p, 5/2
, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 2*(a + b)*AppellF1[3/2
, 3, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f*x
]^2) - (6*a*(a + b)*p*AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[
e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*(Sec[e + f*x
]^2)^(-2 + p)*Sin[2*(e + f*x)]*Tan[e + f*x])/(3*(a + b)*AppellF1[1/2, 2, -
p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF1[
3/2, 2, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 2*(a
 + b)*AppellF1[3/2, 3, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a +
 b))])*Tan[e + f*x]^2) + (6*(a + b)*(-2 + p)*AppellF1[1/2, 2, -p, 3/2, -Ta
n[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*...
```

3.308.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4634, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^p}{\sec(e+fx)^2}dx$$

$$\downarrow 4634$$

$$\frac{\int \frac{\left(b\tan^2(e+fx)+a+b\right)^p}{\left(\tan^2(e+fx)+1\right)^2}d\tan(e+fx)}{f}$$

$$\downarrow 334$$

$$\frac{(a+b\tan^2(e+fx)+b)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\int\frac{\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^p}{(\tan^2(e+fx)+1)^2}d\tan(e+fx)}{f}$$

$$\downarrow 333$$

$$\frac{\tan(e+fx)\,(a+b\tan^2(e+fx)+b)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},2,-p,\frac{3}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

input
```
Int[Cos[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
(AppellF1[1/2, 2, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
*Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a
+ b))^p)
```

3.308.3.1 Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.308.4 Maple [F]

$$\int \cos(fx+e)^2 \left(a+b\sec(fx+e)^2\right)^p dx$$

input
```
int(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x)
```

3.308.5 Fricas [F]

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p dx = \int \left(b\sec(fx+e)^2+a\right)^p \cos(fx+e)^2 dx$$

input
```
integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^2, x)
```

3.308.6 Sympy [F(-1)]

Timed out.

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**2*(a+b*sec(f*x+e)**2)**p,x)
```

output
```
Timed out
```

3.308.7 Maxima [F]

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec(fx+e)^2+a\right)^p\cos(fx+e)^2\,dx$$

input
```
integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^2, x)
```

3.308.8 Giac [F]

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec(fx+e)^2+a\right)^p\cos(fx+e)^2\,dx$$

input
```
integrate(cos(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^2, x)
```

3.308.9 Mupad [F(-1)]

Timed out.

$$\int \cos^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \cos(e+fx)^2\left(a+\frac{b}{\cos(e+fx)^2}\right)^p dx$$

input
```
int(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^p,x)
```

output
```
int(cos(e + f*x)^2*(a + b/cos(e + f*x)^2)^p, x)
```

3.309 $\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

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3.309.1 Optimal result

Integrand size = 23, antiderivative size = 83

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$
$$= \frac{\text{AppellF1}\left(\frac{1}{2},3,-p,\frac{3}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+f}{a+b}\right.}{f}$$

output
```
AppellF1(1/2,3,-p,3/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)*(a+b
+b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

3.309.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1912 vs. 2(83) = 166.

Time = 16.25 (sec) , antiderivative size = 1912, normalized size of antiderivative = 23.04

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^p,x]
```

output

```
(3*(a + b)*AppellF1[1/2, 3, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)
/(a + b))]*Cos[e + f*x]^3*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2
)^(-3 + p)*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x])/(f*(3*(a + b)*AppellF1[1
/2, 3, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*A
ppellF1[3/2, 3, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))
] - 3*(a + b)*AppellF1[3/2, 4, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]
^2)/(a + b))])*Tan[e + f*x]^2)*((3*(a + b)*AppellF1[1/2, 3, -p, 3/2, -Tan[
e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^
p*(Sec[e + f*x]^2)^(-2 + p))/(3*(a + b)*AppellF1[1/2, 3, -p, 3/2, -Tan[e +
 f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 3, 1 - p, 5
/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 3*(a + b)*AppellF1[3
/2, 4, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f
*x]^2) - (6*a*(a + b)*p*AppellF1[1/2, 3, -p, 3/2, -Tan[e + f*x]^2, -((b*Ta
n[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*(Sec[e + f
*x]^2)^(-3 + p)*Sin[2*(e + f*x)]*Tan[e + f*x])/(3*(a + b)*AppellF1[1/2, 3,
 -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF
1[3/2, 3, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 3*
(a + b)*AppellF1[3/2, 4, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a
 + b))])*Tan[e + f*x]^2) + (6*(a + b)*(-3 + p)*AppellF1[1/2, 3, -p, 3/2, -
Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + ...
```

3.309.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4634, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^p}{\sec(e+fx)^4}dx$$

$$\downarrow 4634$$

$$\frac{\int \frac{\left(b\tan^2(e+fx)+a+b\right)^p}{\left(\tan^2(e+fx)+1\right)^3}d\tan(e+fx)}{f}$$

$$\downarrow 334$$

$$\frac{(a+b\tan^2(e+fx)+b)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\int\frac{\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^p}{(\tan^2(e+fx)+1)^3}d\tan(e+fx)}{f}$$

$$\downarrow 333$$

$$\frac{\tan(e+fx)\,(a+b\tan^2(e+fx)+b)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},3,-p,\frac{3}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

input
```
Int[Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
(AppellF1[1/2, 3, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
*Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a
+ b))^p)
```

3.309.3.1 Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.309.4 Maple [F]

$$\int \cos(fx+e)^4 \left(a+b\sec(fx+e)^2\right)^p dx$$

input
```
int(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x)
```

3.309.5 Fricas [F]

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p dx = \int \left(b\sec(fx+e)^2+a\right)^p \cos(fx+e)^4 dx$$

input
```
integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^4, x)
```

3.309.6 Sympy [F(-1)]

Timed out.

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**4*(a+b*sec(f*x+e)**2)**p,x)
```

output
```
Timed out
```

3.309.7 Maxima [F]

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec(fx+e)^2+a\right)^p\cos(fx+e)^4\,dx$$

input
```
integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^4, x)
```

3.309.8 Giac [F]

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec(fx+e)^2+a\right)^p\cos(fx+e)^4\,dx$$

input
```
integrate(cos(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^4, x)
```

3.309.9 Mupad [F(-1)]

Timed out.

$$\int \cos^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \cos(e+f\,x)^4\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p\,dx$$

input
```
int(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^p,x)
```

output
```
int(cos(e + f*x)^4*(a + b/cos(e + f*x)^2)^p, x)
```

3.310 $\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

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3.310.1 Optimal result

Integrand size = 23, antiderivative size = 83

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$
$$= \frac{\text{AppellF1}\left(\frac{1}{2},4,-p,\frac{3}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2(e+f}{a+b}\right.}{f}$$

output
```
AppellF1(1/2,4,-p,3/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)*(a+b
+b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

3.310.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1914 vs. 2(83) = 166.

Time = 16.86 (sec) , antiderivative size = 1914, normalized size of antiderivative = 23.06

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[e + f*x]^6*(a + b*Sec[e + f*x]^2)^p,x]
```

output

```
(3*(a + b)*AppellF1[1/2, 4, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)
/(a + b))]*Cos[e + f*x]^5*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2
)^(-4 + p)*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x])/(f*(3*(a + b)*AppellF1[1
/2, 4, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*A
ppellF1[3/2, 4, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))
] - 4*(a + b)*AppellF1[3/2, 5, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]
^2)/(a + b))])*Tan[e + f*x]^2)*((3*(a + b)*AppellF1[1/2, 4, -p, 3/2, -Tan[
e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^
p*(Sec[e + f*x]^2)^(-3 + p))/(3*(a + b)*AppellF1[1/2, 4, -p, 3/2, -Tan[e +
 f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF1[3/2, 4, 1 - p, 5
/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 4*(a + b)*AppellF1[3
/2, 5, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))])*Tan[e + f
*x]^2) - (6*a*(a + b)*p*AppellF1[1/2, 4, -p, 3/2, -Tan[e + f*x]^2, -((b*Ta
n[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*(Sec[e + f
*x]^2)^(-4 + p)*Sin[2*(e + f*x)]*Tan[e + f*x])/(3*(a + b)*AppellF1[1/2, 4,
 -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] + 2*(b*p*AppellF
1[3/2, 4, 1 - p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))] - 4*
(a + b)*AppellF1[3/2, 5, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a
 + b))])*Tan[e + f*x]^2) + (6*(a + b)*(-4 + p)*AppellF1[1/2, 4, -p, 3/2, -
Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]*(a + 2*b + a*Cos[2*(e + ...
```

3.310.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4634, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^p}{\sec(e+fx)^6}dx$$

$$\downarrow 4634$$

$$\frac{\int \frac{\left(b\tan^2(e+fx)+a+b\right)^p}{\left(\tan^2(e+fx)+1\right)^4}d\tan(e+fx)}{f}$$

$$\downarrow 334$$

$$\frac{(a+b\tan^2(e+fx)+b)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\int\frac{\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^p}{(\tan^2(e+fx)+1)^4}d\tan(e+fx)}{f}$$

$$\downarrow 333$$

$$\frac{\tan(e+fx)\,(a+b\tan^2(e+fx)+b)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},4,-p,\frac{3}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

input
```
Int[Cos[e + f*x]^6*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
(AppellF1[1/2, 4, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
*Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a
+ b))^p)
```

3.310.3.1 Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.310.4 Maple [F]

$$\int \cos(fx+e)^6 \left(a+b\sec(fx+e)^2\right)^p dx$$

input
```
int(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x)
```

3.310.5 Fricas [F]

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p dx = \int \left(b\sec(fx+e)^2+a\right)^p \cos(fx+e)^6 dx$$

input
```
integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^6, x)
```

3.310.6 Sympy [F(-1)]

Timed out.

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**6*(a+b*sec(f*x+e)**2)**p,x)
```

output
```
Timed out
```

3.310.7 Maxima [F]

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec(fx+e)^2+a\right)^p\cos(fx+e)^6\,dx$$

input
```
integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^6, x)
```

3.310.8 Giac [F]

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec(fx+e)^2+a\right)^p\cos(fx+e)^6\,dx$$

input
```
integrate(cos(f*x+e)^6*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*cos(f*x + e)^6, x)
```

3.310.9 Mupad [F(-1)]

Timed out.

$$\int \cos^6(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \cos(e+fx)^6\left(a+\frac{b}{\cos(e+fx)^2}\right)^p dx$$

input
```
int(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^p,x)
```

output
```
int(cos(e + f*x)^6*(a + b/cos(e + f*x)^2)^p, x)
```

3.311 $\int (a + b\sec^2(e+fx))\tan^5(e+fx)\,dx$

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3.311.1 Optimal result

Integrand size = 21, antiderivative size = 72

$$\int (a + b\sec^2(e+fx))\tan^5(e+fx)\,dx = -\frac{a\log(\cos(e+fx))}{f} - \frac{(2a-b)\sec^2(e+fx)}{2f} + \frac{(a-2b)\sec^4(e+fx)}{4f} + \frac{b\sec^6(e+fx)}{6f}$$

output
```
-a*ln(cos(f*x+e))/f-1/2*(2*a-b)*sec(f*x+e)^2/f+1/4*(a-2*b)*sec(f*x+e)^4/f+
1/6*b*sec(f*x+e)^6/f
```

3.311.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.76

$$\int (a + b\sec^2(e+fx))\tan^5(e+fx)\,dx = \frac{b\tan^6(e+fx)}{6f} - \frac{a(4\log(\cos(e+fx)) + 2\tan^2(e+fx) - \tan^4(e+fx))}{4f}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^5,x]
```

output
```
(b*Tan[e + f*x]^6)/(6*f) - (a*(4*Log[Cos[e + f*x]] + 2*Tan[e + f*x]^2 - Ta
n[e + f*x]^4))/(4*f)
```

3.311.3 Rubi [A] (warning: unable to verify)

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4626, 354, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \tan(e+fx)^5\left(a+b\sec(e+fx)^2\right)dx$$

$$\downarrow 4626$$

$$-\frac{\int \left(1-\cos^2(e+fx)\right)^2\left(a\cos^2(e+fx)+b\right)\sec^7(e+fx)d\cos(e+fx)}{f}$$

$$\downarrow 354$$

$$-\frac{\int \left(1-\cos^2(e+fx)\right)^2\left(a\cos^2(e+fx)+b\right)\sec^4(e+fx)d\cos^2(e+fx)}{2f}$$

$$\downarrow 85$$

$$-\frac{\int \left(b\sec^4(e+fx)+(a-2b)\sec^3(e+fx)+(b-2a)\sec^2(e+fx)+a\sec(e+fx)\right)d\cos^2(e+fx)}{2f}$$

$$\downarrow 2009$$

$$-\frac{-\frac{1}{2}(a-2b)\sec^2(e+fx)+(2a-b)\sec(e+fx)+a\log\left(\cos^2(e+fx)\right)-\frac{1}{3}b\sec^3(e+fx)}{2f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^5,x]
```

output
```
-1/2*(a*Log[Cos[e + f*x]^2] + (2*a - b)*Sec[e + f*x] - ((a - 2*b)*Sec[e +
f*x]^2)/2 - (b*Sec[e + f*x]^3)/3)/f
```

3.311.3.1 Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.311.4 Maple [A] (verified)

Time = 1.63 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.76

method	result
parts	$\frac{a\left(\frac{\tan(fx+e)^4}{4}-\frac{\tan(fx+e)^2}{2}+\frac{\ln\left(1+\tan(fx+e)^2\right)}{2}\right)}{f}+\frac{b\tan(fx+e)^6}{6f}$
derivativedivides	$\frac{\frac{\sec(fx+e)^6 b}{6}+\frac{\sec(fx+e)^4 a}{4}-\frac{\sec(fx+e)^4 b}{2}-\sec(fx+e)^2 a+\frac{b\sec(fx+e)^2}{2}+a\ln(\sec(fx+e))}{f}$
default	$\frac{\frac{\sec(fx+e)^6 b}{6}+\frac{\sec(fx+e)^4 a}{4}-\frac{\sec(fx+e)^4 b}{2}-\sec(fx+e)^2 a+\frac{b\sec(fx+e)^2}{2}+a\ln(\sec(fx+e))}{f}$
risch	$iax+\frac{2iae}{f}+\frac{-4a\,e^{10i(fx+e)}+2b\,e^{10i(fx+e)}-12a\,e^{8i(fx+e)}-16a\,e^{6i(fx+e)}+\frac{20b\,e^{6i(fx+e)}}{3}-12a\,e^{4i(fx+e)}-4a\,e^{2i(fx}}{f\left(e^{2i(fx+e)}+1\right)^6}$

input
```
int((a+b*sec(f*x+e)^2)*tan(f*x+e)^5,x,method=_RETURNVERBOSE)
```

output
```
a/f*(1/4*tan(f*x+e)^4-1/2*tan(f*x+e)^2+1/2*ln(1+tan(f*x+e)^2))+1/6*b/f*tan
(f*x+e)^6
```

3.311.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.96

$$\int \left(a+b\sec^2(e+fx)\right)\tan^5(e+fx)\,dx =$$
$$-\frac{12\,a\cos\left(fx+e\right)^6\log\left(-\cos\left(fx+e\right)\right)+6\left(2\,a-b\right)\cos\left(fx+e\right)^4-3\left(a-2\,b\right)\cos\left(fx+e\right)^2-2\,b}{12\,f\cos\left(fx+e\right)^6}$$

input
```
integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^5,x, algorithm="fricas")
```

output
```
-1/12*(12*a*cos(f*x + e)^6*log(-cos(f*x + e)) + 6*(2*a - b)*cos(f*x + e)^4
 - 3*(a - 2*b)*cos(f*x + e)^2 - 2*b)/(f*cos(f*x + e)^6)
```

3.311.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(56) = 112$.

Time = 0.55 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.61

$$\int \left(a + b\sec^2(e+fx)\right)\tan^5(e+fx)\,dx$$
$$= \begin{cases} \frac{a\log\left(\tan^2(e+fx)+1\right)}{2f} + \frac{a\tan^4(e+fx)}{4f} - \frac{a\tan^2(e+fx)}{2f} + \frac{b\tan^4(e+fx)\sec^2(e+fx)}{6f} - \frac{b\tan^2(e+fx)\sec^2(e+fx)}{6f} + \frac{b\sec^2(e+fx)}{6f} \\ x(a+b\sec^2(e))\tan^5(e) \end{cases}$$

input
```
integrate((a+b*sec(f*x+e)**2)*tan(f*x+e)**5,x)
```

output
```
Piecewise((a*log(tan(e + f*x)**2 + 1)/(2*f) + a*tan(e + f*x)**4/(4*f) - a*
tan(e + f*x)**2/(2*f) + b*tan(e + f*x)**4*sec(e + f*x)**2/(6*f) - b*tan(e
+ f*x)**2*sec(e + f*x)**2/(6*f) + b*sec(e + f*x)**2/(6*f), Ne(f, 0)), (x*(
a + b*sec(e)**2)*tan(e)**5, True))
```

3.311.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.32

$$\int \left(a + b\sec^2(e+fx)\right)\tan^5(e+fx)\,dx$$
$$= -\frac{6\,a\log\left(\sin(fx+e)^2-1\right) - \frac{6\,(2\,a-b)\sin(fx+e)^4 - 3\,(7\,a-2\,b)\sin(fx+e)^2 + 9\,a - 2\,b}{\sin(fx+e)^6 - 3\,\sin(fx+e)^4 + 3\,\sin(fx+e)^2 - 1}}{12\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^5,x, algorithm="maxima")
```

output
```
-1/12*(6*a*log(sin(f*x + e)^2 - 1) - (6*(2*a - b)*sin(f*x + e)^4 - 3*(7*a
- 2*b)*sin(f*x + e)^2 + 9*a - 2*b)/(sin(f*x + e)^6 - 3*sin(f*x + e)^4 + 3*
sin(f*x + e)^2 - 1))/f
```

3.311.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(66) = 132$.

Time = 1.59 (sec) , antiderivative size = 280, normalized size of antiderivative = 3.89

$$\int \left(a + b\sec^2(e+fx)\right)\tan^5(e+fx)\,dx$$

$$= \frac{6\,a\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1} - \frac{\cos(fx+e)-1}{\cos(fx+e)+1} + 2\right|\right) - 6\,a\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1} - \frac{\cos(fx+e)-1}{\cos(fx+e)+1} - 2\right|\right) + \frac{11\,a\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(}{\cos(}\right.}{12\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^5,x, algorithm="giac")
```

output
```
1/12*(6*a*log(abs(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f*x + e) -
 1)/(cos(f*x + e) + 1) + 2)) - 6*a*log(abs(-(cos(f*x + e) + 1)/(cos(f*x +
e) - 1) - (cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2)) + (11*a*((cos(f*x +
e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1))^3 + 90
*a*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x +
e) + 1))^2 + 276*a*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e)
- 1)/(cos(f*x + e) + 1)) + 280*a - 128*b)/((cos(f*x + e) + 1)/(cos(f*x + e
) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 2)^3)/f
```

3.311.9 Mupad [B] (verification not implemented)

Time = 19.69 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.72

$$\int \left(a + b\sec^2(e+fx)\right)\tan^5(e+fx)\,dx$$

$$= \frac{\frac{a\,\ln\left(\tan(e+fx)^2+1\right)}{2} - \frac{a\tan(e+fx)^2}{2} + \frac{a\tan(e+fx)^4}{4} + \frac{b\tan(e+fx)^6}{6}}{f}$$

input
```
int(tan(e + f*x)^5*(a + b/cos(e + f*x)^2),x)
```

output
```
((a*log(tan(e + f*x)^2 + 1))/2 - (a*tan(e + f*x)^2)/2 + (a*tan(e + f*x)^4)
/4 + (b*tan(e + f*x)^6)/6)/f
```

3.312 $\int (a + b\sec^2(e+fx))\tan^3(e+fx)\,dx$

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3.312.1 Optimal result

Integrand size = 21, antiderivative size = 49

$$\int (a + b\sec^2(e+fx))\tan^3(e+fx)\,dx = \frac{a\log(\cos(e+fx))}{f} + \frac{(a-b)\sec^2(e+fx)}{2f} + \frac{b\sec^4(e+fx)}{4f}$$

output
```
a*ln(cos(f*x+e))/f+1/2*(a-b)*sec(f*x+e)^2/f+1/4*b*sec(f*x+e)^4/f
```

3.312.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int (a + b\sec^2(e+fx))\tan^3(e+fx)\,dx = \frac{b\tan^4(e+fx)}{4f} + \frac{a(2\log(\cos(e+fx)) + \tan^2(e+fx))}{2f}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^3,x]
```

output
```
(b*Tan[e + f*x]^4)/(4*f) + (a*(2*Log[Cos[e + f*x]] + Tan[e + f*x]^2))/(2*f
)
```

3.312.3 Rubi [A] (warning: unable to verify)

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4626, 354, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \tan(e+fx)^3\left(a+b\sec(e+fx)^2\right)dx$$

$$\downarrow 4626$$

$$-\frac{\int \left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)\sec^5(e+fx)d\cos(e+fx)}{f}$$

$$\downarrow 354$$

$$-\frac{\int \left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)\sec^3(e+fx)d\cos^2(e+fx)}{2f}$$

$$\downarrow 85$$

$$-\frac{\int \left(b\sec^3(e+fx)+(a-b)\sec^2(e+fx)-a\sec(e+fx)\right)d\cos^2(e+fx)}{2f}$$

$$\downarrow 2009$$

$$-\frac{-(a-b)\sec(e+fx)-a\log\left(\cos^2(e+fx)\right)-\frac{1}{2}b\sec^2(e+fx)}{2f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^3,x]
```

output
```
-1/2*(-(a*Log[Cos[e + f*x]^2]) - (a - b)*Sec[e + f*x] - (b*Sec[e + f*x]^2)
/2)/f
```

3.312.3.1 Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.312.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.92

method	result	siz
parts	$\frac{a\left(\frac{\tan(fx+e)^2}{2}-\frac{\ln\left(1+\tan(fx+e)^2\right)}{2}\right)}{f}+\frac{b\tan(fx+e)^4}{4f}$	45
derivativedivides	$\frac{\frac{\sec(fx+e)^4 b}{4}+\frac{\sec(fx+e)^2 a}{2}-\frac{b\sec(fx+e)^2}{2}-a\ln(\sec(fx+e))}{f}$	49
default	$\frac{\frac{\sec(fx+e)^4 b}{4}+\frac{\sec(fx+e)^2 a}{2}-\frac{b\sec(fx+e)^2}{2}-a\ln(\sec(fx+e))}{f}$	49
risch	$-iax-\frac{2iae}{f}-\frac{2\left(-a\,\mathrm{e}^{6i(fx+e)}+b\,\mathrm{e}^{6i(fx+e)}-2a\,\mathrm{e}^{4i(fx+e)}-a\,\mathrm{e}^{2i(fx+e)}+b\,\mathrm{e}^{2i(fx+e)}\right)}{f\left(\mathrm{e}^{2i(fx+e)}+1\right)^4}+\frac{a\ln\left(\mathrm{e}^{2i(fx+e)}+1\right)}{f}$	10

input
```
int((a+b*sec(f*x+e)^2)*tan(f*x+e)^3,x,method=_RETURNVERBOSE)
```

output
```
a/f*(1/2*tan(f*x+e)^2-1/2*ln(1+tan(f*x+e)^2))+1/4*b/f*tan(f*x+e)^4
```

3.312.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02

$$\int \left(a+b\sec^2(e+fx)\right)\tan^3(e+fx)\,dx$$
$$=\frac{4\,a\cos\left(fx+e\right)^4\log\left(-\cos\left(fx+e\right)\right)+2\left(a-b\right)\cos\left(fx+e\right)^2+b}{4\,f\cos\left(fx+e\right)^4}$$

input
```
integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^3,x, algorithm="fricas")
```

output
```
1/4*(4*a*cos(f*x + e)^4*log(-cos(f*x + e)) + 2*(a - b)*cos(f*x + e)^2 + b)
/(f*cos(f*x + e)^4)
```

3.312.6 Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.63

$$\int \left(a+b\sec^2(e+fx)\right)\tan^3(e+fx)\,dx$$
$$=\begin{cases}-\frac{a\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{a\tan^2\left(e+fx\right)}{2f}+\frac{b\tan^2\left(e+fx\right)\sec^2\left(e+fx\right)}{4f}-\frac{b\sec^2\left(e+fx\right)}{4f} & \text{for } f\neq 0\\ x\left(a+b\sec^2\left(e\right)\right)\tan^3\left(e\right) & \text{otherwise}\end{cases}$$

input
```
integrate((a+b*sec(f*x+e)**2)*tan(f*x+e)**3,x)
```

output
```
Piecewise((-a*log(tan(e + f*x)**2 + 1)/(2*f) + a*tan(e + f*x)**2/(2*f) + b
*tan(e + f*x)**2*sec(e + f*x)**2/(4*f) - b*sec(e + f*x)**2/(4*f), Ne(f, 0)
), (x*(a + b*sec(e)**2)*tan(e)**3, True))
```

3.312.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.31

$$\int \left(a+b\sec^2(e+fx)\right)\tan^3(e+fx)\,dx = \frac{2\,a\log\left(\sin\left(fx+e\right)^2-1\right)-\frac{2\,(a-b)\sin(fx+e)^2-2\,a+b}{\sin(fx+e)^4-2\,\sin(fx+e)^2+1}}{4\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^3,x, algorithm="maxima")
```

output
```
1/4*(2*a*log(sin(f*x + e)^2 - 1) - (2*(a - b)*sin(f*x + e)^2 - 2*a + b)/(s
in(f*x + e)^4 - 2*sin(f*x + e)^2 + 1))/f
```

3.312.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(45) = 90$.

Time = 0.66 (sec) , antiderivative size = 236, normalized size of antiderivative = 4.82

$$\int \left(a+b\sec^2(e+fx)\right)\tan^3(e+fx)\,dx =$$

$$-\frac{2\,a\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+2\right|\right)-2\,a\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}-2\right|\right)+\frac{3\,a\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+2\right)+20\,a+16\,b}{\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+2\right)^2}+28\,a}{4\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^3,x, algorithm="giac")
```

output
```
-1/4*(2*a*log(abs(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f*x + e) -
 1)/(cos(f*x + e) + 1) + 2)) - 2*a*log(abs(-(cos(f*x + e) + 1)/(cos(f*x +
e) - 1) - (cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2)) + (3*a*((cos(f*x + e
) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1))^2 + 20*
a*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e
) + 1)) + 28*a - 16*b)/((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x +
 e) - 1)/(cos(f*x + e) + 1) + 2)^2)/f
```

3.312.9 Mupad [B] (verification not implemented)

Time = 19.54 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int (a + b\sec^2(e+fx))\tan^3(e+fx)\,dx = \frac{a\tan(e+fx)^2}{2f} - \frac{a\,\ln\left(\tan(e+fx)^2+1\right)}{2f} + \frac{b\tan(e+fx)^4}{4f}$$

input
```
int(tan(e + f*x)^3*(a + b/cos(e + f*x)^2),x)
```

output
```
(a*tan(e + f*x)^2)/(2*f) - (a*log(tan(e + f*x)^2 + 1))/(2*f) + (b*tan(e +
f*x)^4)/(4*f)
```

3.313 $\int (a + b\sec^2(e + fx))\tan(e + fx)\,dx$

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3.313.2 Mathematica [A] (verified) . . . 2320
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3.313.1 Optimal result

Integrand size = 19, antiderivative size = 30

$$\int (a + b\sec^2(e + fx))\tan(e + fx)\,dx = -\frac{a\log(\cos(e + fx))}{f} + \frac{b\sec^2(e + fx)}{2f}$$

output
```
-a*ln(cos(f*x+e))/f+1/2*b*sec(f*x+e)^2/f
```

3.313.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (a + b\sec^2(e + fx))\tan(e + fx)\,dx = -\frac{a\log(\cos(e + fx))}{f} + \frac{b\sec^2(e + fx)}{2f}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)*Tan[e + f*x],x]
```

output
```
-((a*Log[Cos[e + f*x]])/f) + (b*Sec[e + f*x]^2)/(2*f)
```

3.313.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 4626, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \tan(e+fx)\left(a+b\sec(e+fx)^2\right)\,dx$$

$$\downarrow 4626$$

$$-\frac{\int \left(a\cos^2(e+fx)+b\right)\sec^3(e+fx)d\cos(e+fx)}{f}$$

$$\downarrow 244$$

$$-\frac{\int \left(b\sec^3(e+fx)+a\sec(e+fx)\right)d\cos(e+fx)}{f}$$

$$\downarrow 2009$$

$$-\frac{a\log(\cos(e+fx))-\frac{1}{2}b\sec^2(e+fx)}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)*Tan[e + f*x],x]
```

output
```
-((a*Log[Cos[e + f*x]] - (b*Sec[e + f*x]^2)/2)/f)
```

3.313.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.313.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

method	result	size
derivativedivides	$\frac{\frac{b\sec(fx+e)^2}{2}+a\ln(\sec(fx+e))}{f}$	26
default	$\frac{\frac{b\sec(fx+e)^2}{2}+a\ln(\sec(fx+e))}{f}$	26
parts	$\frac{a\ln\left(1+\tan(fx+e)^2\right)}{2f}+\frac{b\sec(fx+e)^2}{2f}$	33
risch	$iax+\frac{2iae}{f}+\frac{2b\mathrm{e}^{2i(fx+e)}}{f\left(\mathrm{e}^{2i(fx+e)}+1\right)^2}-\frac{a\ln\left(\mathrm{e}^{2i(fx+e)}+1\right)}{f}$	61

input
```
int((a+b*sec(f*x+e)^2)*tan(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/2*b*sec(f*x+e)^2+a*ln(sec(f*x+e)))
```

3.313.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.23

$$\int \left(a+b\sec^2(e+fx)\right)\tan(e+fx)\,dx = -\frac{2\,a\cos\left(fx+e\right)^2\log\left(-\cos\left(fx+e\right)\right)-b}{2\,f\cos\left(fx+e\right)^2}$$

input
```
integrate((a+b*sec(f*x+e)^2)*tan(f*x+e),x, algorithm="fricas")
```

output
```
-1/2*(2*a*cos(f*x + e)^2*log(-cos(f*x + e)) - b)/(f*cos(f*x + e)^2)
```

3.313.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.40

$$\int \left(a + b\sec^2(e+fx)\right)\tan(e+fx)\,dx = \begin{cases} \frac{a\log\left(\tan^2(e+fx)+1\right)}{2f} + \frac{b\sec^2(e+fx)}{2f} & \text{for } f \neq 0 \\ x(a+b\sec^2(e))\tan(e) & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*sec(f*x+e)**2)*tan(f*x+e),x)
```

output
```
Piecewise((a*log(tan(e + f*x)**2 + 1)/(2*f) + b*sec(e + f*x)**2/(2*f), Ne(
f, 0)), (x*(a + b*sec(e)**2)*tan(e), True))
```

3.313.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.10

$$\int \left(a + b\sec^2(e+fx)\right)\tan(e+fx)\,dx = -\frac{a\log\left(\sin(fx+e)^2-1\right) + \frac{b}{\sin(fx+e)^2-1}}{2f}$$

input
```
integrate((a+b*sec(f*x+e)^2)*tan(f*x+e),x, algorithm="maxima")
```

output
```
-1/2*(a*log(sin(f*x + e)^2 - 1) + b/(sin(f*x + e)^2 - 1))/f
```

3.313.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. 2(28) = 56.

Time = 0.33 (sec) , antiderivative size = 190, normalized size of antiderivative = 6.33

$$\int \left(a + b\sec^2(e+fx)\right)\tan(e+fx)\,dx$$
$$= \frac{a\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1} - \frac{\cos(fx+e)-1}{\cos(fx+e)+1} + 2\right|\right) - a\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1} - \frac{\cos(fx+e)-1}{\cos(fx+e)+1} - 2\right|\right) + \frac{a\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1} + \frac{\cos(fx+e)-}{\cos(fx+e)+}\right.}{\frac{\cos(fx+e)+1}{\cos(fx+e)-1} + \frac{\cos(fx+e}{\cos(fx+e}}$$
$$2f$$

input
```
integrate((a+b*sec(f*x+e)^2)*tan(f*x+e),x, algorithm="giac")
```

output
```
1/2*(a*log(abs(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f*x + e) - 1)
/(cos(f*x + e) + 1) + 2)) - a*log(abs(-(cos(f*x + e) + 1)/(cos(f*x + e) -
1) - (cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2)) + (a*((cos(f*x + e) + 1)/
(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) + 2*a - 4*b)/(
(cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) +
 1) + 2))/f
```

3.313.9 Mupad [B] (verification not implemented)

Time = 18.98 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \left(a + b\sec^2(e+fx)\right)\tan(e+fx)\,dx = \frac{a\,\ln\left(\tan(e+f\,x)^2+1\right)}{2\,f} + \frac{b\tan(e+f\,x)^2}{2\,f}$$

input
```
int(tan(e + f*x)*(a + b/cos(e + f*x)^2),x)
```

output
```
(a*log(tan(e + f*x)^2 + 1))/(2*f) + (b*tan(e + f*x)^2)/(2*f)
```

3.314 $\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

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3.314.2 Mathematica [A] (verified) . . . 2325
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3.314.1 Optimal result

Integrand size = 19, antiderivative size = 28

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{b\log(\cos(e+fx))}{f}+\frac{(a+b)\log(\sin(e+fx))}{f}$$

output
```
-b*ln(cos(f*x+e))/f+(a+b)*ln(sin(f*x+e))/f
```

3.314.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.79

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{a\log(\cos(e+fx))}{f}-\frac{b\log(\cos(e+fx))}{f}+\frac{b\log(\sin(e+fx))}{f}+\frac{a\log(\tan(e+fx))}{f}$$

input
```
Integrate[Cot[e + f*x]*(a + b*Sec[e + f*x]^2),x]
```

output
```
(a*Log[Cos[e + f*x]])/f - (b*Log[Cos[e + f*x]])/f + (b*Log[Sin[e + f*x]])/
f + (a*Log[Tan[e + f*x]])/f
```

3.314.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.32, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 4626, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \frac{a+b\sec(e+fx)^2}{\tan(e+fx)}dx$$

$$\downarrow 4626$$

$$-\frac{\int \frac{\left(a\cos^2(e+fx)+b\right)\sec(e+fx)}{1-\cos^2(e+fx)}d\cos(e+fx)}{f}$$

$$\downarrow 354$$

$$-\frac{\int \frac{\left(a\cos^2(e+fx)+b\right)\sec(e+fx)}{1-\cos^2(e+fx)}d\cos^2(e+fx)}{2f}$$

$$\downarrow 86$$

$$-\frac{\int \left(\frac{-a-b}{\cos^2(e+fx)-1}+b\sec(e+fx)\right)d\cos^2(e+fx)}{2f}$$

$$\downarrow 2009$$

$$-\frac{b\log\left(\cos^2(e+fx)\right)-(a+b)\log\left(1-\cos^2(e+fx)\right)}{2f}$$

input
```
Int[Cot[e + f*x]*(a + b*Sec[e + f*x]^2),x]
```

output
```
-1/2*(b*Log[Cos[e + f*x]^2] - (a + b)*Log[1 - Cos[e + f*x]^2])/f
```

3.314.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.314.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{a\ln(\sin(fx+e))+b\ln(\tan(fx+e))}{f}$	24
default	$\frac{a\ln(\sin(fx+e))+b\ln(\tan(fx+e))}{f}$	24
risch	$-iax-\frac{2iae}{f}+\frac{\ln\left(e^{2i(fx+e)}-1\right)a}{f}+\frac{\ln\left(e^{2i(fx+e)}-1\right)b}{f}-\frac{b\ln\left(e^{2i(fx+e)}+1\right)}{f}$	67

input
```
int(cot(f*x+e)*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(a*ln(sin(f*x+e))+b*ln(tan(f*x+e)))
```

3.314.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.25

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{b\log\left(\cos\left(fx+e\right)^2\right)-(a+b)\log\left(-\frac{1}{4}\cos\left(fx+e\right)^2+\frac{1}{4}\right)}{2\,f}$$

input
```
integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
-1/2*(b*log(cos(f*x + e)^2) - (a + b)*log(-1/4*cos(f*x + e)^2 + 1/4))/f
```

3.314.6 Sympy [F]

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)\cot\left(e+fx\right)dx$$

input
```
integrate(cot(f*x+e)*(a+b*sec(f*x+e)**2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)*cot(e + f*x), x)
```

3.314.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.18

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{b\log\left(\sin\left(fx+e\right)^2-1\right)-(a+b)\log\left(\sin\left(fx+e\right)^2\right)}{2\,f}$$

input
```
integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
-1/2*(b*log(sin(f*x + e)^2 - 1) - (a + b)*log(sin(f*x + e)^2))/f
```

3.314.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(28) = 56$.

Time $= 0.28$ (sec) , antiderivative size $= 98$, normalized size of antiderivative $= 3.50$

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{a\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+2\right|\right)+b\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}-2\right|\right)}{2f}$$

input
```
integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
-1/2*(a*log(abs(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f*x + e) - 1
)/(cos(f*x + e) + 1) + 2)) + b*log(abs(-(cos(f*x + e) + 1)/(cos(f*x + e) -
 1) - (cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2)))/f
```

3.314.9 Mupad [B] (verification not implemented)

Time $= 19.68$ (sec) , antiderivative size $= 32$, normalized size of antiderivative $= 1.14$

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{\ln\left(\tan(e+fx)\right)\,(a+b)}{f}-\frac{a\,\ln\left(\tan(e+fx)^2+1\right)}{2f}$$

input
```
int(cot(e + f*x)*(a + b/cos(e + f*x)^2),x)
```

output
```
(log(tan(e + f*x))*(a + b))/f - (a*log(tan(e + f*x)^2 + 1))/(2*f)
```

3.315 $\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

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3.315.1 Optimal result

Integrand size = 21, antiderivative size = 32

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{(a+b)\csc^2(e+fx)}{2f} - \frac{a\log(\sin(e+fx))}{f}$$

output
```
-1/2*(a+b)*csc(f*x+e)^2/f-a*ln(sin(f*x+e))/f
```

3.315.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.62

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{b\csc^2(e+fx)}{2f} - \frac{a(\cot^2(e+fx)+2\log(\cos(e+fx))+2\log(\tan(e+fx)))}{2f}$$

input
```
Integrate[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^2),x]
```

output
```
-1/2*(b*Csc[e + f*x]^2)/f - (a*(Cot[e + f*x]^2 + 2*Log[Cos[e + f*x]] + 2*L
og[Tan[e + f*x]]))/(2*f)
```

3.315.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.28, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4626, 353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \frac{a+b\sec(e+fx)^2}{\tan(e+fx)^3}dx$$

$$\downarrow 4626$$

$$-\frac{\int \frac{\cos(e+fx)(a\cos^2(e+fx)+b)}{(1-\cos^2(e+fx))^2}d\cos(e+fx)}{f}$$

$$\downarrow 353$$

$$-\frac{\int \frac{a\cos^2(e+fx)+b}{(1-\cos^2(e+fx))^2}d\cos^2(e+fx)}{2f}$$

$$\downarrow 49$$

$$-\frac{\int \left(\frac{a}{\cos^2(e+fx)-1}+\frac{a+b}{(\cos^2(e+fx)-1)^2}\right)d\cos^2(e+fx)}{2f}$$

$$\downarrow 2009$$

$$-\frac{\frac{a+b}{1-\cos^2(e+fx)}+a\log\left(1-\cos^2(e+fx)\right)}{2f}$$

input
```
Int[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^2),x]
```

output
```
-1/2*((a + b)/(1 - Cos[e + f*x]^2) + a*Log[1 - Cos[e + f*x]^2])/f
```

3.315.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.315.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.22

method	result	size
derivativedivides	$\frac{a\left(-\frac{\cot(fx+e)^2}{2}-\ln(\sin(fx+e))\right)-\frac{b}{2\sin(fx+e)^2}}{f}$	39
default	$\frac{a\left(-\frac{\cot(fx+e)^2}{2}-\ln(\sin(fx+e))\right)-\frac{b}{2\sin(fx+e)^2}}{f}$	39
risch	$iax+\frac{2iae}{f}+\frac{2(a+b)e^{2i(fx+e)}}{f\left(e^{2i(fx+e)}-1\right)^2}-\frac{\ln\left(e^{2i(fx+e)}-1\right)a}{f}$	63

input
```
int(cot(f*x+e)^3*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(a*(-1/2*cot(f*x+e)^2-ln(sin(f*x+e)))-1/2*b/sin(f*x+e)^2)
```

3.315.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.56

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{2\left(a\cos\left(fx+e\right)^2-a\right)\log\left(\frac{1}{2}\sin\left(fx+e\right)\right)-a-b}{2\left(f\cos\left(fx+e\right)^2-f\right)}$$

input
```
integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
-1/2*(2*(a*cos(f*x + e)^2 - a)*log(1/2*sin(f*x + e)) - a - b)/(f*cos(f*x +
 e)^2 - f)
```

3.315.6 Sympy [F]

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)\cot^3\left(e+fx\right)dx$$

input
```
integrate(cot(f*x+e)**3*(a+b*sec(f*x+e)**2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)*cot(e + f*x)**3, x)
```

3.315.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{a\log\left(\sin\left(fx+e\right)^2\right)+\frac{a+b}{\sin(fx+e)^2}}{2f}$$

input
```
integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
-1/2*(a*log(sin(f*x + e)^2) + (a + b)/sin(f*x + e)^2)/f
```

3.315.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(30) = 60$.

Time $= 0.31$ (sec) , antiderivative size $= 146$, normalized size of antiderivative $= 4.56$

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx =$$

$$-\frac{4\,a\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)-8\,a\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)-\frac{\left(a+b+\frac{4\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}\right)(\cos(fx+e)+1)}{\cos(fx+e)-1}-\frac{a(\cos(fx+e)-1)}{\cos(fx+e)+1}}{8\,f}$$

input
```
integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
-1/8*(4*a*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1)) - 8*a*log(abs(
-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1)) - (a + b + 4*a*(cos(f*x + e)
- 1)/(cos(f*x + e) + 1))*(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - a*(cos(f*
x + e) - 1)/(cos(f*x + e) + 1) - b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1))/
f
```

3.315.9 Mupad [B] (verification not implemented)

Time $= 19.27$ (sec) , antiderivative size $= 51$, normalized size of antiderivative $= 1.59$

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{a\,\ln\left(\tan(e+f\,x)^2+1\right)}{2\,f} - \frac{a\,\ln\left(\tan(e+f\,x)\right)}{f} - \frac{\cot(e+f\,x)^2\left(\frac{a}{2}+\frac{b}{2}\right)}{f}$$

input
```
int(cot(e + f*x)^3*(a + b/cos(e + f*x)^2),x)
```

output
```
(a*log(tan(e + f*x)^2 + 1))/(2*f) - (a*log(tan(e + f*x)))/f - (cot(e + f*x
)^2*(a/2 + b/2))/f
```

3.316 $\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

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3.316.1 Optimal result

Integrand size = 21, antiderivative size = 51

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{(2a+b)\csc^2(e+fx)}{2f} - \frac{(a+b)\csc^4(e+fx)}{4f} + \frac{a\log(\sin(e+fx))}{f}$$

output
```
1/2*(2*a+b)*csc(f*x+e)^2/f-1/4*(a+b)*csc(f*x+e)^4/f+a*ln(sin(f*x+e))/f
```

3.316.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.43

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{a\cot^2(e+fx)}{2f} - \frac{a\cot^4(e+fx)}{4f} - \frac{b\cot^4(e+fx)}{4f} + \frac{a\log(\cos(e+fx))}{f} + \frac{a\log(\tan(e+fx))}{f}$$

input
```
Integrate[Cot[e + f*x]^5*(a + b*Sec[e + f*x]^2),x]
```

output
```
(a*Cot[e + f*x]^2)/(2*f) - (a*Cot[e + f*x]^4)/(4*f) - (b*Cot[e + f*x]^4)/(
4*f) + (a*Log[Cos[e + f*x]])/f + (a*Log[Tan[e + f*x]])/f
```

3.316.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.29, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4626, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \frac{a+b\sec(e+fx)^2}{\tan(e+fx)^5}dx$$

$$\downarrow 4626$$

$$-\frac{\int \frac{\cos^3(e+fx)(a\cos^2(e+fx)+b)}{(1-\cos^2(e+fx))^3}d\cos(e+fx)}{f}$$

$$\downarrow 354$$

$$-\frac{\int \frac{\cos^2(e+fx)(a\cos^2(e+fx)+b)}{(1-\cos^2(e+fx))^3}d\cos^2(e+fx)}{2f}$$

$$\downarrow 86$$

$$-\frac{\int \left(-\frac{a}{\cos^2(e+fx)-1}+\frac{-2a-b}{(\cos^2(e+fx)-1)^2}+\frac{-a-b}{(\cos^2(e+fx)-1)^3}\right)d\cos^2(e+fx)}{2f}$$

$$\downarrow 2009$$

$$-\frac{\frac{a+b}{2(1-\cos^2(e+fx))^2}-\frac{2a+b}{1-\cos^2(e+fx)}-a\log\left(1-\cos^2(e+fx)\right)}{2f}$$

input
```
Int[Cot[e + f*x]^5*(a + b*Sec[e + f*x]^2),x]
```

output
```
-1/2*((a + b)/(2*(1 - Cos[e + f*x]^2)^2) - (2*a + b)/(1 - Cos[e + f*x]^2)
- a*Log[1 - Cos[e + f*x]^2])/f
```

3.316.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.316.4 Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.08

method	result	siz
derivativedivides	$\frac{a\left(-\frac{\cot(fx+e)^4}{4}+\frac{\cot(fx+e)^2}{2}+\ln(\sin(fx+e))\right)-\frac{b\cos(fx+e)^4}{4\sin(fx+e)^4}}{f}$	55
default	$\frac{a\left(-\frac{\cot(fx+e)^4}{4}+\frac{\cot(fx+e)^2}{2}+\ln(\sin(fx+e))\right)-\frac{b\cos(fx+e)^4}{4\sin(fx+e)^4}}{f}$	55
risch	$-iax-\frac{2iae}{f}-\frac{2\left(2a\,e^{6i(fx+e)}+b\,e^{6i(fx+e)}-2a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+b\,e^{2i(fx+e)}\right)}{f\left(e^{2i(fx+e)}-1\right)^4}+\frac{\ln\left(e^{2i(fx+e)}-1\right)a}{f}$	10

input
```
int(cot(f*x+e)^5*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(a*(-1/4*cot(f*x+e)^4+1/2*cot(f*x+e)^2+ln(sin(f*x+e)))-1/4*b/sin(f*x+e
)^4*cos(f*x+e)^4)
```

3.316.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.63

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx =$$
$$-\frac{2\left(2\,a+b\right)\cos\left(fx+e\right)^2-4\left(a\cos\left(fx+e\right)^4-2\,a\cos\left(fx+e\right)^2+a\right)\log\left(\frac{1}{2}\,\sin\left(fx+e\right)\right)-3\,a-b}{4\left(f\cos\left(fx+e\right)^4-2\,f\cos\left(fx+e\right)^2+f\right)}$$

input
```
integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
-1/4*(2*(2*a + b)*cos(f*x + e)^2 - 4*(a*cos(f*x + e)^4 - 2*a*cos(f*x + e)^
2 + a)*log(1/2*sin(f*x + e)) - 3*a - b)/(f*cos(f*x + e)^4 - 2*f*cos(f*x +
e)^2 + f)
```

3.316.6 Sympy [F]

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)\cot^5\left(e+fx\right)dx$$

input
```
integrate(cot(f*x+e)**5*(a+b*sec(f*x+e)**2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)*cot(e + f*x)**5, x)
```

3.316.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{2\,a\log\left(\sin\left(fx+e\right)^2\right)+\frac{2\,(2\,a+b)\sin(fx+e)^2-a-b}{\sin(fx+e)^4}}{4\,f}$$

input
```
integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/4*(2*a*log(sin(f*x + e)^2) + (2*(2*a + b)*sin(f*x + e)^2 - a - b)/sin(f*
x + e)^4)/f
```

3.316.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(47) = 94.

Time = 0.35 (sec) , antiderivative size = 238, normalized size of antiderivative = 4.67

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$= \frac{32\,a\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)-64\,a\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)-\frac{\left(a+b+\frac{12\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{4\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{48\,a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right.}{(\cos(fx+e)-1)^2}}{64\,f}$$

input
```
integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/64*(32*a*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1)) - 64*a*log(ab
s(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1)) - (a + b + 12*a*(cos(f*x +
e) - 1)/(cos(f*x + e) + 1) + 4*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 4
8*a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)^2/(cos(f
*x + e) - 1)^2 - 12*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 4*b*(cos(f*x
 + e) - 1)/(cos(f*x + e) + 1) - a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^
2 - b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/f
```

3.316.9 Mupad [B] (verification not implemented)

Time = 19.88 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.20

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{a\,\ln\left(\tan(e+f\,x)\right)}{f} - \frac{a\,\ln\left(\tan(e+f\,x)^2+1\right)}{2\,f} - \frac{-\frac{a\,\tan(e+f\,x)^2}{2}+\frac{a}{4}+\frac{b}{4}}{f\,\tan\left(e+f\,x\right)^4}$$

input
```
int(cot(e + f*x)^5*(a + b/cos(e + f*x)^2),x)
```

output
```
(a*log(tan(e + f*x)))/f - (a*log(tan(e + f*x)^2 + 1))/(2*f) - (a/4 + b/4 -
 (a*tan(e + f*x)^2)/2)/(f*tan(e + f*x)^4)
```

3.317 $\int (a + b\sec^2(e+fx))\tan^6(e+fx)\,dx$

3.317.1 Optimal result 2341
3.317.2 Mathematica [A] (verified) 2341
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3.317.9 Mupad [B] (verification not implemented) 2346

3.317.1 Optimal result

Integrand size = 21, antiderivative size = 64

$$\int (a + b\sec^2(e+fx))\tan^6(e+fx)\,dx = -ax + \frac{a\tan(e+fx)}{f} - \frac{a\tan^3(e+fx)}{3f} + \frac{a\tan^5(e+fx)}{5f} + \frac{b\tan^7(e+fx)}{7f}$$

output
```
-a*x+a*tan(f*x+e)/f-1/3*a*tan(f*x+e)^3/f+1/5*a*tan(f*x+e)^5/f+1/7*b*tan(f*
x+e)^7/f
```

3.317.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.14

$$\int (a + b\sec^2(e+fx))\tan^6(e+fx)\,dx = -\frac{a\arctan(\tan(e+fx))}{f} + \frac{a\tan(e+fx)}{f} - \frac{a\tan^3(e+fx)}{3f} + \frac{a\tan^5(e+fx)}{5f} + \frac{b\tan^7(e+fx)}{7f}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^6,x]
```

output
```
-((a*ArcTan[Tan[e + f*x]])/f) + (a*Tan[e + f*x])/f - (a*Tan[e + f*x]^3)/(3
*f) + (a*Tan[e + f*x]^5)/(5*f) + (b*Tan[e + f*x]^7)/(7*f)
```

3.317.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4629, 2075, 363, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \tan(e+fx)^6\left(a+b\sec(e+fx)^2\right)dx$$

$$\downarrow 4629$$

$$\frac{\int \frac{\tan^6(e+fx)(a+b(\tan^2(e+fx)+1))}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\tan^6(e+fx)(b\tan^2(e+fx)+a+b)}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\downarrow 363$$

$$\frac{a\int \frac{\tan^6(e+fx)}{\tan^2(e+fx)+1}d\tan(e+fx)+\frac{1}{7}b\tan^7(e+fx)}{f}$$

$$\downarrow 254$$

$$\frac{a\int \left(\tan^4(e+fx)-\tan^2(e+fx)-\frac{1}{\tan^2(e+fx)+1}+1\right)d\tan(e+fx)+\frac{1}{7}b\tan^7(e+fx)}{f}$$

$$\downarrow 2009$$

$$\frac{a\left(-\arctan(\tan(e+fx))+\frac{1}{5}\tan^5(e+fx)-\frac{1}{3}\tan^3(e+fx)+\tan(e+fx)\right)+\frac{1}{7}b\tan^7(e+fx)}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^6,x]
```

output
```
((b*Tan[e + f*x]^7)/7 + a*(-ArcTan[Tan[e + f*x]] + Tan[e + f*x] - Tan[e +
f*x]^3/3 + Tan[e + f*x]^5/5))/f
```

3.317.3.1 Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.317.4 Maple [A] (verified)

Time = 2.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.89

method	result
parts	$\frac{a\left(\frac{\tan(fx+e)^5}{5}-\frac{\tan(fx+e)^3}{3}+\tan(fx+e)-\arctan(\tan(fx+e))\right)}{f}+\frac{b\tan(fx+e)^7}{7f}$
derivativedivides	$\frac{a\left(\frac{\tan(fx+e)^5}{5}-\frac{\tan(fx+e)^3}{3}+\tan(fx+e)-fx-e\right)+\frac{b\sin(fx+e)^7}{7\cos(fx+e)^7}}{f}$
default	$\frac{a\left(\frac{\tan(fx+e)^5}{5}-\frac{\tan(fx+e)^3}{3}+\tan(fx+e)-fx-e\right)+\frac{b\sin(fx+e)^7}{7\cos(fx+e)^7}}{f}$
risch	$-ax-\frac{2i\left(-315a\,e^{12i(fx+e)}+105b\,e^{12i(fx+e)}-1260a\,e^{10i(fx+e)}-2555a\,e^{8i(fx+e)}+525b\,e^{8i(fx+e)}-3080a\,e^{6i(fx+e)}-\right.}{105f\left(e^{2i(fx+e)}+1\right)^7}$

input
```
int((a+b*sec(f*x+e)^2)*tan(f*x+e)^6,x,method=_RETURNVERBOSE)
```

output
```
a/f*(1/5*tan(f*x+e)^5-1/3*tan(f*x+e)^3+tan(f*x+e)-arctan(tan(f*x+e)))+1/7*
b*tan(f*x+e)^7/f
```

3.317.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.39

$$\int \left(a+b\sec^2(e+fx)\right)\tan^6(e+fx)\,dx =$$
$$-\frac{105\,afx\cos\left(fx+e\right)^7-\left(\left(161\,a-15\,b\right)\cos\left(fx+e\right)^6-\left(77\,a-45\,b\right)\cos\left(fx+e\right)^4+3\left(7\,a-15\,b\right)\cos\left(\right.\right.}{105\,f\cos\left(fx+e\right)^7}$$

input
```
integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^6,x, algorithm="fricas")
```

output
```
-1/105*(105*a*f*x*cos(f*x + e)^7 - ((161*a - 15*b)*cos(f*x + e)^6 - (77*a
- 45*b)*cos(f*x + e)^4 + 3*(7*a - 15*b)*cos(f*x + e)^2 + 15*b)*sin(f*x + e
))/(f*cos(f*x + e)^7)
```

3.317.6 Sympy [A] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.03

$$\begin{aligned}
&\int \left(a + b\sec^2(e+fx)\right)\tan^6(e+fx)\,dx \\
&= a\left(\begin{cases} -x + \frac{\tan^5(e+fx)}{5f} - \frac{\tan^3(e+fx)}{3f} + \frac{\tan(e+fx)}{f} & \text{for } f \neq 0 \\ x\tan^6(e) & \text{otherwise} \end{cases}\right) \\
&\quad + b\left(\begin{cases} x\tan^6(e)\sec^2(e) & \text{for } f = 0 \\ \frac{\tan^7(e+fx)}{7f} & \text{otherwise} \end{cases}\right)
\end{aligned}$$

input
```
integrate((a+b*sec(f*x+e)**2)*tan(f*x+e)**6,x)
```

output
```
a*Piecewise((-x + tan(e + f*x)**5/(5*f) - tan(e + f*x)**3/(3*f) + tan(e +
f*x)/f, Ne(f, 0)), (x*tan(e)**6, True)) + b*Piecewise((x*tan(e)**6*sec(e)*
*2, Eq(f, 0)), (tan(e + f*x)**7/(7*f), True))
```

3.317.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\begin{aligned}
&\int \left(a + b\sec^2(e+fx)\right)\tan^6(e+fx)\,dx \\
&= \frac{15\,b\tan(fx+e)^7 + 21\,a\tan(fx+e)^5 - 35\,a\tan(fx+e)^3 - 105\,(fx+e)a + 105\,a\tan(fx+e)}{105\,f}
\end{aligned}$$

input
```
integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^6,x, algorithm="maxima")
```

output
```
1/105*(15*b*tan(f*x + e)^7 + 21*a*tan(f*x + e)^5 - 35*a*tan(f*x + e)^3 - 1
05*(f*x + e)*a + 105*a*tan(f*x + e))/f
```

3.317.8 Giac [A] (verification not implemented)

Time = 2.18 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int \left(a + b\sec^2(e+fx)\right)\tan^6(e+fx)\,dx$$
$$= \frac{15\,b\tan(fx+e)^7 + 21\,a\tan(fx+e)^5 - 35\,a\tan(fx+e)^3 - 105\,(fx+e)a + 105\,a\tan(fx+e)}{105\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^6,x, algorithm="giac")
```

output
```
1/105*(15*b*tan(f*x + e)^7 + 21*a*tan(f*x + e)^5 - 35*a*tan(f*x + e)^3 - 1
05*(f*x + e)*a + 105*a*tan(f*x + e))/f
```

3.317.9 Mupad [B] (verification not implemented)

Time = 18.99 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.80

$$\int \left(a + b\sec^2(e+fx)\right)\tan^6(e+fx)\,dx$$
$$= \frac{\frac{b\tan(e+fx)^7}{7} + \frac{a\tan(e+fx)^5}{5} - \frac{a\tan(e+fx)^3}{3} + a\tan(e+fx) - afx}{f}$$

input
```
int(tan(e + f*x)^6*(a + b/cos(e + f*x)^2),x)
```

output
```
(a*tan(e + f*x) - (a*tan(e + f*x)^3)/3 + (a*tan(e + f*x)^5)/5 + (b*tan(e +
 f*x)^7)/7 - a*f*x)/f
```

3.318 $\int (a + b\sec^2(e+fx))\tan^4(e+fx)\,dx$

3.318.1 Optimal result 2347
3.318.2 Mathematica [A] (verified) 2347
3.318.3 Rubi [A] (verified) 2348
3.318.4 Maple [A] (verified) 2350
3.318.5 Fricas [A] (verification not implemented) 2350
3.318.6 Sympy [A] (verification not implemented) 2351
3.318.7 Maxima [A] (verification not implemented) 2351
3.318.8 Giac [A] (verification not implemented) 2352
3.318.9 Mupad [B] (verification not implemented) 2352

3.318.1 Optimal result

Integrand size = 21, antiderivative size = 48

$$\int (a+b\sec^2(e+fx))\tan^4(e+fx)\,dx = ax - \frac{a\tan(e+fx)}{f} + \frac{a\tan^3(e+fx)}{3f} + \frac{b\tan^5(e+fx)}{5f}$$

output
```
a*x-a*tan(f*x+e)/f+1/3*a*tan(f*x+e)^3/f+1/5*b*tan(f*x+e)^5/f
```

3.318.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int (a + b\sec^2(e+fx))\tan^4(e+fx)\,dx = \frac{a\arctan(\tan(e+fx))}{f} - \frac{a\tan(e+fx)}{f} + \frac{a\tan^3(e+fx)}{3f} + \frac{b\tan^5(e+fx)}{5f}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^4,x]
```

output
```
(a*ArcTan[Tan[e + f*x]])/f - (a*Tan[e + f*x])/f + (a*Tan[e + f*x]^3)/(3*f)
 + (b*Tan[e + f*x]^5)/(5*f)
```

3.318.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4629, 2075, 363, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \tan(e+fx)^4\left(a+b\sec(e+fx)^2\right)dx$$

$$\downarrow 4629$$

$$\frac{\int \frac{\tan^4(e+fx)(a+b(\tan^2(e+fx)+1))}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\tan^4(e+fx)(b\tan^2(e+fx)+a+b)}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\downarrow 363$$

$$\frac{a\int \frac{\tan^4(e+fx)}{\tan^2(e+fx)+1}d\tan(e+fx)+\frac{1}{5}b\tan^5(e+fx)}{f}$$

$$\downarrow 254$$

$$\frac{a\int \left(\tan^2(e+fx)+\frac{1}{\tan^2(e+fx)+1}-1\right)d\tan(e+fx)+\frac{1}{5}b\tan^5(e+fx)}{f}$$

$$\downarrow 2009$$

$$\frac{a\left(\arctan(\tan(e+fx))+\frac{1}{3}\tan^3(e+fx)-\tan(e+fx)\right)+\frac{1}{5}b\tan^5(e+fx)}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^4,x]
```

output
```
((b*Tan[e + f*x]^5)/5 + a*(ArcTan[Tan[e + f*x]] - Tan[e + f*x] + Tan[e + f
*x]^3/3))/f
```

3.318.3.1 Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.318.4 Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.98

method	result	si
parts	$\frac{a\left(\frac{\tan(fx+e)^3}{3}-\tan(fx+e)+\arctan(\tan(fx+e))\right)}{f}+\frac{b\tan(fx+e)^5}{5f}$	4
derivativedivides	$\frac{a\left(\frac{\tan(fx+e)^3}{3}-\tan(fx+e)+fx+e\right)+\frac{b\sin(fx+e)^5}{5\cos(fx+e)^5}}{f}$	5
default	$\frac{a\left(\frac{\tan(fx+e)^3}{3}-\tan(fx+e)+fx+e\right)+\frac{b\sin(fx+e)^5}{5\cos(fx+e)^5}}{f}$	5
risch	$ax+\frac{2i\left(-30a\,e^{8i(fx+e)}+15b\,e^{8i(fx+e)}-90a\,e^{6i(fx+e)}-110a\,e^{4i(fx+e)}+30b\,e^{4i(fx+e)}-70a\,e^{2i(fx+e)}-20a+3b\right)}{15f\left(e^{2i(fx+e)}+1\right)^5}$	1

input
```
int((a+b*sec(f*x+e)^2)*tan(f*x+e)^4,x,method=_RETURNVERBOSE)
```

output
```
a/f*(1/3*tan(f*x+e)^3-tan(f*x+e)+arctan(tan(f*x+e)))+1/5*b*tan(f*x+e)^5/f
```

3.318.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.50

$$\int \left(a+b\sec^2(e+fx)\right)\tan^4(e+fx)\,dx$$
$$=\frac{15\,afx\cos\left(fx+e\right)^5-\left(\left(20\,a-3\,b\right)\cos\left(fx+e\right)^4-\left(5\,a-6\,b\right)\cos\left(fx+e\right)^2-3\,b\right)\sin\left(fx+e\right)}{15\,f\cos\left(fx+e\right)^5}$$

input
```
integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^4,x, algorithm="fricas")
```

output
```
1/15*(15*a*f*x*cos(f*x + e)^5 - ((20*a - 3*b)*cos(f*x + e)^4 - (5*a - 6*b)
*cos(f*x + e)^2 - 3*b)*sin(f*x + e))/(f*cos(f*x + e)^5)
```

3.318.6 Sympy [A] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.12

$$\int \left(a + b\sec^2(e+fx)\right)\tan^4(e+fx)\,dx = a\left(\begin{cases} x + \frac{\tan^3(e+fx)}{3f} - \frac{\tan(e+fx)}{f} & \text{for } f \neq 0 \\ x\tan^4(e) & \text{otherwise} \end{cases}\right) + b\left(\begin{cases} x\tan^4(e)\sec^2(e) & \text{for } f = 0 \\ \frac{\tan^5(e+fx)}{5f} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((a+b*sec(f*x+e)**2)*tan(f*x+e)**4,x)
```

output
```
a*Piecewise((x + tan(e + f*x)**3/(3*f) - tan(e + f*x)/f, Ne(f, 0)), (x*tan
(e)**4, True)) + b*Piecewise((x*tan(e)**4*sec(e)**2, Eq(f, 0)), (tan(e + f
*x)**5/(5*f), True))
```

3.318.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

$$\int \left(a + b\sec^2(e+fx)\right)\tan^4(e+fx)\,dx = \frac{3\,b\tan(fx+e)^5 + 5\,a\tan(fx+e)^3 + 15\,(fx+e)a - 15\,a\tan(fx+e)}{15\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^4,x, algorithm="maxima")
```

output
```
1/15*(3*b*tan(f*x + e)^5 + 5*a*tan(f*x + e)^3 + 15*(f*x + e)*a - 15*a*tan(
f*x + e))/f
```

3.318.8 Giac [A] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

$$\int \left(a + b\sec^2(e+fx)\right)\tan^4(e+fx)\,dx$$
$$= \frac{3\,b\tan(fx+e)^5 + 5\,a\tan(fx+e)^3 + 15\,(fx+e)a - 15\,a\tan(fx+e)}{15\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^4,x, algorithm="giac")
```

output
```
1/15*(3*b*tan(f*x + e)^5 + 5*a*tan(f*x + e)^3 + 15*(f*x + e)*a - 15*a*tan(
f*x + e))/f
```

3.318.9 Mupad [B] (verification not implemented)

Time = 19.46 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.83

$$\int \left(a + b\sec^2(e+fx)\right)\tan^4(e+fx)\,dx = \frac{\frac{b\tan(e+f\,x)^5}{5} + \frac{a\tan(e+f\,x)^3}{3} - a\tan(e+f\,x) + a\,f\,x}{f}$$

input
```
int(tan(e + f*x)^4*(a + b/cos(e + f*x)^2),x)
```

output
```
((a*tan(e + f*x)^3)/3 - a*tan(e + f*x) + (b*tan(e + f*x)^5)/5 + a*f*x)/f
```

3.319 $\int (a + b\sec^2(e+fx))\tan^2(e+fx)\,dx$

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3.319.2 Mathematica [A] (verified) . . . 2353
3.319.3 Rubi [A] (verified) . . . 2354
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3.319.9 Mupad [B] (verification not implemented) . . . 2358

3.319.1 Optimal result

Integrand size = 21, antiderivative size = 32

$$\int (a + b\sec^2(e+fx))\tan^2(e+fx)\,dx = -ax + \frac{a\tan(e+fx)}{f} + \frac{b\tan^3(e+fx)}{3f}$$

output
```
-a*x+a*tan(f*x+e)/f+1/3*b*tan(f*x+e)^3/f
```

3.319.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.28

$$\int (a + b\sec^2(e+fx))\tan^2(e+fx)\,dx = -\frac{a\arctan(\tan(e+fx))}{f} + \frac{a\tan(e+fx)}{f} + \frac{b\tan^3(e+fx)}{3f}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^2,x]
```

output
```
-((a*ArcTan[Tan[e + f*x]])/f) + (a*Tan[e + f*x])/f + (b*Tan[e + f*x]^3)/(3
*f)
```

3.319.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4629, 2075, 363, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \tan(e+fx)^2\left(a+b\sec(e+fx)^2\right)dx$$

$$\downarrow 4629$$

$$\frac{\int \frac{\tan^2(e+fx)(a+b(\tan^2(e+fx)+1))}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\tan^2(e+fx)(b\tan^2(e+fx)+a+b)}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\downarrow 363$$

$$\frac{a\int \frac{\tan^2(e+fx)}{\tan^2(e+fx)+1}d\tan(e+fx)+\frac{1}{3}b\tan^3(e+fx)}{f}$$

$$\downarrow 262$$

$$\frac{a\left(\tan(e+fx)-\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)\right)+\frac{1}{3}b\tan^3(e+fx)}{f}$$

$$\downarrow 216$$

$$\frac{a(\tan(e+fx)-\arctan(\tan(e+fx)))+\frac{1}{3}b\tan^3(e+fx)}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)*Tan[e + f*x]^2,x]
```

output
```
((b*Tan[e + f*x]^3)/3 + a*(-ArcTan[Tan[e + f*x]] + Tan[e + f*x]))/f
```

3.319.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.319.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

method	result	size
parts	$\frac{a(\tan(fx+e)-\arctan(\tan(fx+e)))}{f}+\frac{b\tan(fx+e)^3}{3f}$	37
derivativedivides	$\frac{a(\tan(fx+e)-fx-e)+\frac{b\sin(fx+e)^3}{3\cos(fx+e)^3}}{f}$	41
default	$\frac{a(\tan(fx+e)-fx-e)+\frac{b\sin(fx+e)^3}{3\cos(fx+e)^3}}{f}$	41
risch	$-ax-\frac{2i\left(-3a\,e^{4i(fx+e)}+3b\,e^{4i(fx+e)}-6a\,e^{2i(fx+e)}-3a+b\right)}{3f\left(e^{2i(fx+e)}+1\right)^3}$	66

input
```
int((a+b*sec(f*x+e)^2)*tan(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
a/f*(tan(f*x+e)-arctan(tan(f*x+e)))+1/3*b*tan(f*x+e)^3/f
```

3.319.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.66

$$\int \left(a+b\sec^2(e+fx)\right)\tan^2(e+fx)\,dx$$
$$=-\frac{3\,afx\cos\left(fx+e\right)^3-\left(\left(3\,a-b\right)\cos\left(fx+e\right)^2+b\right)\sin\left(fx+e\right)}{3\,f\cos\left(fx+e\right)^3}$$

input
```
integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^2,x, algorithm="fricas")
```

output
```
-1/3*(3*a*f*x*cos(f*x + e)^3 - ((3*a - b)*cos(f*x + e)^2 + b)*sin(f*x + e)
)/(f*cos(f*x + e)^3)
```

3.319.6 Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.31

$$\int \left(a + b\sec^2(e+fx)\right)\tan^2(e+fx)\,dx = a\left(\begin{cases} -x + \frac{\tan(e+fx)}{f} & \text{for } f \neq 0 \\ x\tan^2(e) & \text{otherwise} \end{cases}\right) + b\left(\begin{cases} x\tan^2(e)\sec^2(e) & \text{for } f = 0 \\ \frac{\tan^3(e+fx)}{3f} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((a+b*sec(f*x+e)**2)*tan(f*x+e)**2,x)
```

output
```
a*Piecewise((-x + tan(e + f*x)/f, Ne(f, 0)), (x*tan(e)**2, True)) + b*Piec
ewise((x*tan(e)**2*sec(e)**2, Eq(f, 0)), (tan(e + f*x)**3/(3*f), True))
```

3.319.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$\int \left(a + b\sec^2(e+fx)\right)\tan^2(e+fx)\,dx = \frac{b\tan(fx+e)^3 - 3(fx+e)a + 3a\tan(fx+e)}{3f}$$

input
```
integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^2,x, algorithm="maxima")
```

output
```
1/3*(b*tan(f*x + e)^3 - 3*(f*x + e)*a + 3*a*tan(f*x + e))/f
```

3.319.8 Giac [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$\int \left(a + b\sec^2(e+fx)\right)\tan^2(e+fx)\,dx = \frac{b\tan(fx+e)^3 - 3(fx+e)a + 3a\tan(fx+e)}{3f}$$

input
```
integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^2,x, algorithm="giac")
```

output
```
1/3*(b*tan(f*x + e)^3 - 3*(f*x + e)*a + 3*a*tan(f*x + e))/f
```

3.319.9 Mupad [B] (verification not implemented)

Time = 19.43 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \left(a + b\sec^2(e+fx)\right)\tan^2(e+fx)\,dx = \frac{\frac{b\tan(e+fx)^3}{3} + a\tan(e+fx) - afx}{f}$$

input
```
int(tan(e + f*x)^2*(a + b/cos(e + f*x)^2),x)
```

output
```
(a*tan(e + f*x) + (b*tan(e + f*x)^3)/3 - a*f*x)/f
```

3.320 $\int (a + b \sec^2(e + fx))\, dx$

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3.320.7 Maxima [A] (verification not implemented) 2361
3.320.8 Giac [A] (verification not implemented) 2362
3.320.9 Mupad [B] (verification not implemented) 2362

3.320.1 Optimal result

Integrand size = 12, antiderivative size = 15

$$\int (a + b \sec^2(e + fx))\, dx = ax + \frac{b \tan(e + fx)}{f}$$

output
```
a*x+b*tan(f*x+e)/f
```

3.320.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int (a + b \sec^2(e + fx))\, dx = ax + \frac{b \tan(e + fx)}{f}$$

input
```
Integrate[a + b*Sec[e + f*x]^2,x]
```

output
```
a*x + (b*Tan[e + f*x])/f
```

3.320.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + b\sec^2(e+fx))\,dx$$

$$\downarrow \text{2009}$$

$$ax + \frac{b\tan(e+fx)}{f}$$

input
```
Int[a + b*Sec[e + f*x]^2,x]
```

output
```
a*x + (b*Tan[e + f*x])/f
```

3.320.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.320.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result	size
default	$ax + \frac{b\tan(fx+e)}{f}$	16
parts	$ax + \frac{b\tan(fx+e)}{f}$	16
derivativedivides	$\frac{(fx+e)a+b\tan(fx+e)}{f}$	21
risch	$ax + \frac{2ib}{f(e^{2i(fx+e)}+1)}$	25
parallelrisch	$-\frac{2b\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)} + ax$	35
norman	$\frac{ax\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2 - ax - \frac{2b\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1}$	51

input
```
int(a+b*sec(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
a*x+b*tan(f*x+e)/f
```

3.320.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(15) = 30$.

Time $= 0.25$ (sec) , antiderivative size $= 31$, normalized size of antiderivative $= 2.07$

$$\int \left(a + b\sec^2(e + fx)\right)\, dx = \frac{afx\cos\left(fx + e\right) + b\sin\left(fx + e\right)}{f\cos\left(fx + e\right)}$$

input
```
integrate(a+b*sec(f*x+e)^2,x, algorithm="fricas")
```

output
```
(a*f*x*cos(f*x + e) + b*sin(f*x + e))/(f*cos(f*x + e))
```

3.320.6 Sympy [F]

$$\int \left(a + b\sec^2(e + fx)\right)\, dx = \int \left(a + b\sec^2\left(e + fx\right)\right)\, dx$$

input
```
integrate(a+b*sec(f*x+e)**2,x)
```

output
```
Integral(a + b*sec(e + f*x)**2, x)
```

3.320.7 Maxima [A] (verification not implemented)

Time $= 0.18$ (sec) , antiderivative size $= 15$, normalized size of antiderivative $= 1.00$

$$\int \left(a + b\sec^2(e + fx)\right)\, dx = ax + \frac{b\tan\left(fx + e\right)}{f}$$

input
```
integrate(a+b*sec(f*x+e)^2,x, algorithm="maxima")
```

output
```
a*x + b*tan(f*x + e)/f
```

3.320.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \left(a + b\sec^2(e + fx)\right)\, dx = ax + \frac{b\tan\left(fx + e\right)}{f}$$

input
```
integrate(a+b*sec(f*x+e)^2,x, algorithm="giac")
```

output
```
a*x + b*tan(f*x + e)/f
```

3.320.9 Mupad [B] (verification not implemented)

Time = 19.59 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \left(a + b\sec^2(e + fx)\right)\, dx = \frac{b\tan(e + f\,x) + a\,f\,x}{f}$$

input
```
int(a + b/cos(e + f*x)^2,x)
```

output
```
(b*tan(e + f*x) + a*f*x)/f
```

3.321 $\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

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3.321.2 Mathematica [C] (verified) . . . 2363
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3.321.4 Maple [A] (verified) . . . 2365
3.321.5 Fricas [A] (verification not implemented) . . . 2366
3.321.6 Sympy [F] . . . 2366
3.321.7 Maxima [A] (verification not implemented) . . . 2366
3.321.8 Giac [B] (verification not implemented) . . . 2367
3.321.9 Mupad [B] (verification not implemented) . . . 2367

3.321.1 Optimal result

Integrand size = 21, antiderivative size = 19

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -ax - \frac{(a+b)\cot(e+fx)}{f}$$

output
```
-a*x-(a+b)*cot(f*x+e)/f
```

3.321.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.26

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$= -\frac{b\cot(e+fx)}{f} - \frac{a\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(e+fx)\right)}{f}$$

input
```
Integrate[Cot[e + f*x]^2*(a + b*Sec[e + f*x]^2),x]
```

output
```
-((b*Cot[e + f*x])/f) - (a*Cot[e + f*x]*Hypergeometric2F1[-1/2, 1, 1/2, -T
an[e + f*x]^2])/f
```

3.321.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4629, 2075, 359, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \frac{a+b\sec(e+fx)^2}{\tan(e+fx)^2}dx$$

$$\downarrow 4629$$

$$\frac{\int \frac{\cot^2(e+fx)(a+b(\tan^2(e+fx)+1))}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\cot^2(e+fx)(b\tan^2(e+fx)+a+b)}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\downarrow 359$$

$$\frac{-a\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)-((a+b)\cot(e+fx))}{f}$$

$$\downarrow 216$$

$$\frac{-a\arctan(\tan(e+fx))-(a+b)\cot(e+fx)}{f}$$

input
```
Int[Cot[e + f*x]^2*(a + b*Sec[e + f*x]^2),x]
```

output
```
(-(a*ArcTan[Tan[e + f*x]]) - (a + b)*Cot[e + f*x])/f
```

3.321.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.321.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.74

method	result	size
derivativedivides	$\frac{a(-\cot(fx+e)-fx-e)-b\cot(fx+e)}{f}$	33
default	$\frac{a(-\cot(fx+e)-fx-e)-b\cot(fx+e)}{f}$	33
risch	$-ax-\frac{2ia}{f(e^{2i(fx+e)}-1)}-\frac{2ib}{f(e^{2i(fx+e)}-1)}$	46

input
```
int(cot(f*x+e)^2*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(a*(-cot(f*x+e)-f*x-e)-b*cot(f*x+e))
```

3.321.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.79

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{afx\sin\left(fx+e\right)+(a+b)\cos\left(fx+e\right)}{f\sin\left(fx+e\right)}$$

input
```
integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
-(a*f*x*sin(f*x + e) + (a + b)*cos(f*x + e))/(f*sin(f*x + e))
```

3.321.6 Sympy [F]

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)\cot^2\left(e+fx\right)dx$$

input
```
integrate(cot(f*x+e)**2*(a+b*sec(f*x+e)**2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)*cot(e + f*x)**2, x)
```

3.321.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.32

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{(fx+e)a+\frac{a+b}{\tan(fx+e)}}{f}$$

input
```
integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
-((f*x + e)*a + (a + b)/tan(f*x + e))/f
```

3.321.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(19) = 38$.

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.79

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{2\,(fx+e)a - a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right) - b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right) + \frac{a+b}{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}}{2\,f}$$

input
```
integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
-1/2*(2*(f*x + e)*a - a*tan(1/2*f*x + 1/2*e) - b*tan(1/2*f*x + 1/2*e) + (a
 + b)/tan(1/2*f*x + 1/2*e))/f
```

3.321.9 Mupad [B] (verification not implemented)

Time = 19.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -a\,x - \frac{\cot(e+f\,x)\,(a+b)}{f}$$

input
```
int(cot(e + f*x)^2*(a + b/cos(e + f*x)^2),x)
```

output
```
- a*x - (cot(e + f*x)*(a + b))/f
```

3.322 $\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

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3.322.1 Optimal result

Integrand size = 21, antiderivative size = 33

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = ax + \frac{a\cot(e+fx)}{f} - \frac{(a+b)\cot^3(e+fx)}{3f}$$

output
```
a*x+a*cot(f*x+e)/f-1/3*(a+b)*cot(f*x+e)^3/f
```

3.322.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.55

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$= -\frac{b\cot^3(e+fx)}{3f} - \frac{a\cot^3(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2(e+fx)\right)}{3f}$$

input
```
Integrate[Cot[e + f*x]^4*(a + b*Sec[e + f*x]^2),x]
```

output
```
-1/3*(b*Cot[e + f*x]^3)/f - (a*Cot[e + f*x]^3*Hypergeometric2F1[-3/2, 1, -
1/2, -Tan[e + f*x]^2])/(3*f)
```

3.322.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.24, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4629, 2075, 359, 264, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \frac{a+b\sec(e+fx)^2}{\tan(e+fx)^4}dx$$

$$\downarrow 4629$$

$$\frac{\int \frac{\cot^4(e+fx)(a+b(\tan^2(e+fx)+1))}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\cot^4(e+fx)(b\tan^2(e+fx)+a+b)}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\downarrow 359$$

$$\frac{-a\int \frac{\cot^2(e+fx)}{\tan^2(e+fx)+1}d\tan(e+fx)-\frac{1}{3}(a+b)\cot^3(e+fx)}{f}$$

$$\downarrow 264$$

$$\frac{-a\left(-\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)-\cot(e+fx)\right)-\frac{1}{3}(a+b)\cot^3(e+fx)}{f}$$

$$\downarrow 216$$

$$\frac{-a(-\arctan(\tan(e+fx))-\cot(e+fx))-\frac{1}{3}(a+b)\cot^3(e+fx)}{f}$$

input
```
Int[Cot[e + f*x]^4*(a + b*Sec[e + f*x]^2),x]
```

output
```
(-(a*(-ArcTan[Tan[e + f*x]] - Cot[e + f*x])) - ((a + b)*Cot[e + f*x]^3)/3)
/f
```

3.322.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.322.4 Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.45

method	result	size
derivativedivides	$\frac{a\left(-\frac{\cot(fx+e)^3}{3}+\cot(fx+e)+fx+e\right)-\frac{b\cos(fx+e)^3}{3\sin(fx+e)^3}}{f}$	48
default	$\frac{a\left(-\frac{\cot(fx+e)^3}{3}+\cot(fx+e)+fx+e\right)-\frac{b\cos(fx+e)^3}{3\sin(fx+e)^3}}{f}$	48
risch	$ax+\frac{2i\left(6a\,e^{4i(fx+e)}+3b\,e^{4i(fx+e)}-6a\,e^{2i(fx+e)}+4a+b\right)}{3f\left(e^{2i(fx+e)}-1\right)^3}$	65

input
```
int(cot(f*x+e)^4*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(a*(-1/3*cot(f*x+e)^3+cot(f*x+e)+f*x+e)-1/3*b/sin(f*x+e)^3*cos(f*x+e)^
3)
```

3.322.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(31) = 62.

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.30

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$=\frac{(4\,a+b)\cos(fx+e)^3-3\,a\cos(fx+e)+3\left(afx\cos(fx+e)^2-afx\right)\sin(fx+e)}{3\left(f\cos(fx+e)^2-f\right)\sin(fx+e)}$$

input
```
integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/3*((4*a + b)*cos(f*x + e)^3 - 3*a*cos(f*x + e) + 3*(a*f*x*cos(f*x + e)^2
 - a*f*x)*sin(f*x + e))/((f*cos(f*x + e)^2 - f)*sin(f*x + e))
```

3.322.6 Sympy [F]

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \int \left(a+b\sec^2(e+fx)\right)\cot^4(e+fx)\,dx$$

input
```
integrate(cot(f*x+e)**4*(a+b*sec(f*x+e)**2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)*cot(e + f*x)**4, x)
```

3.322.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.24

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \frac{3\,(fx+e)a + \frac{3\,a\tan(fx+e)^2 - a - b}{\tan(fx+e)^3}}{3\,f}$$

input
```
integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/3*(3*(f*x + e)*a + (3*a*tan(f*x + e)^2 - a - b)/tan(f*x + e)^3)/f
```

3.322.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(31) = 62.

Time = 0.31 (sec) , antiderivative size = 111, normalized size of antiderivative = 3.36

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$= \frac{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 + b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 + 24\,(fx+e)a - 15\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right) - 3\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right) +}{24\,f}$$

input
```
integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/24*(a*tan(1/2*f*x + 1/2*e)^3 + b*tan(1/2*f*x + 1/2*e)^3 + 24*(f*x + e)*a
 - 15*a*tan(1/2*f*x + 1/2*e) - 3*b*tan(1/2*f*x + 1/2*e) + (15*a*tan(1/2*f*
x + 1/2*e)^2 + 3*b*tan(1/2*f*x + 1/2*e)^2 - a - b)/tan(1/2*f*x + 1/2*e)^3)
/f
```

3.322.9 Mupad [B] (verification not implemented)

Time = 19.44 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = a\,x - \frac{-a\tan(e+f\,x)^2 + \frac{a}{3} + \frac{b}{3}}{f\tan(e+f\,x)^3}$$

input
```
int(cot(e + f*x)^4*(a + b/cos(e + f*x)^2),x)
```

output
```
a*x - (a/3 + b/3 - a*tan(e + f*x)^2)/(f*tan(e + f*x)^3)
```

3.323 $\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$

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3.323.1 Optimal result

Integrand size = 21, antiderivative size = 51

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -ax - \frac{a\cot(e+fx)}{f} + \frac{a\cot^3(e+fx)}{3f} - \frac{(a+b)\cot^5(e+fx)}{5f}$$

output
```
-a*x-a*cot(f*x+e)/f+1/3*a*cot(f*x+e)^3/f-1/5*(a+b)*cot(f*x+e)^5/f
```

3.323.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{b\cot^5(e+fx)}{5f} - \frac{a\cot^5(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-\tan^2(e+fx)\right)}{5f}$$

input
```
Integrate[Cot[e + f*x]^6*(a + b*Sec[e + f*x]^2),x]
```

output
```
-1/5*(b*Cot[e + f*x]^5)/f - (a*Cot[e + f*x]^5*Hypergeometric2F1[-5/2, 1, -
3/2, -Tan[e + f*x]^2])/(5*f)
```

3.323.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4629, 2075, 359, 264, 264, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \frac{a+b\sec(e+fx)^2}{\tan(e+fx)^6}dx$$

$$\downarrow 4629$$

$$\frac{\int \frac{\cot^6(e+fx)(a+b(\tan^2(e+fx)+1))}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\cot^6(e+fx)(b\tan^2(e+fx)+a+b)}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\downarrow 359$$

$$\frac{-a\int \frac{\cot^4(e+fx)}{\tan^2(e+fx)+1}d\tan(e+fx)-\frac{1}{5}(a+b)\cot^5(e+fx)}{f}$$

$$\downarrow 264$$

$$\frac{-a\left(-\int \frac{\cot^2(e+fx)}{\tan^2(e+fx)+1}d\tan(e+fx)-\frac{1}{3}\cot^3(e+fx)\right)-\frac{1}{5}(a+b)\cot^5(e+fx)}{f}$$

$$\downarrow 264$$

$$\frac{-a\left(\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)-\frac{1}{3}\cot^3(e+fx)+\cot(e+fx)\right)-\frac{1}{5}(a+b)\cot^5(e+fx)}{f}$$

$$\downarrow 216$$

$$\frac{-a\left(\arctan(\tan(e+fx))-\frac{1}{3}\cot^3(e+fx)+\cot(e+fx)\right)-\frac{1}{5}(a+b)\cot^5(e+fx)}{f}$$

input
```
Int[Cot[e + f*x]^6*(a + b*Sec[e + f*x]^2),x]
```

output
```
(-1/5*((a + b)*Cot[e + f*x]^5) - a*(ArcTan[Tan[e + f*x]] + Cot[e + f*x] -
Cot[e + f*x]^3/3))/f
```

3.323.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.323.4 Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.24

method	result	s
derivativedivides	$\frac{a\left(-\frac{\cot(fx+e)^5}{5}+\frac{\cot(fx+e)^3}{3}-\cot(fx+e)-fx-e\right)-\frac{b\cos(fx+e)^5}{5\sin(fx+e)^5}}{f}$	6
default	$\frac{a\left(-\frac{\cot(fx+e)^5}{5}+\frac{\cot(fx+e)^3}{3}-\cot(fx+e)-fx-e\right)-\frac{b\cos(fx+e)^5}{5\sin(fx+e)^5}}{f}$	6
risch	$-ax-\frac{2i\left(45a\,e^{8i(fx+e)}+15b\,e^{8i(fx+e)}-90a\,e^{6i(fx+e)}+140a\,e^{4i(fx+e)}+30b\,e^{4i(fx+e)}-70a\,e^{2i(fx+e)}+23a+3b\right)}{15f\left(e^{2i(fx+e)}-1\right)^5}$	1

input
```
int(cot(f*x+e)^6*(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(a*(-1/5*cot(f*x+e)^5+1/3*cot(f*x+e)^3-cot(f*x+e)-f*x-e)-1/5*b/sin(f*x
+e)^5*cos(f*x+e)^5)
```

3.323.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(47) = 94.

Time = 0.24 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.16

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx =$$

$$-\frac{(23\,a+3\,b)\cos(fx+e)^5-35\,a\cos(fx+e)^3+15\,a\cos(fx+e)+15\left(afx\cos(fx+e)^4-2\,afx\cos(\right.}{15\left(f\cos(fx+e)^4-2\,f\cos(fx+e)^2+f\right)\sin(fx+e)}$$

input
```
integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
-1/15*((23*a + 3*b)*cos(f*x + e)^5 - 35*a*cos(f*x + e)^3 + 15*a*cos(f*x +
e) + 15*(a*f*x*cos(f*x + e)^4 - 2*a*f*x*cos(f*x + e)^2 + a*f*x)*sin(f*x +
e))/((f*cos(f*x + e)^4 - 2*f*cos(f*x + e)^2 + f)*sin(f*x + e))
```

3.323.6 Sympy [F]

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = \int \left(a+b\sec^2(e+fx)\right)\cot^6(e+fx)\,dx$$

input
```
integrate(cot(f*x+e)**6*(a+b*sec(f*x+e)**2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)*cot(e + f*x)**6, x)
```

3.323.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -\frac{15\,(fx+e)a + \frac{15\,a\tan(fx+e)^4 - 5\,a\tan(fx+e)^2 + 3\,a + 3\,b}{\tan(fx+e)^5}}{15\,f}$$

input
```
integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
-1/15*(15*(f*x + e)*a + (15*a*tan(f*x + e)^4 - 5*a*tan(f*x + e)^2 + 3*a +
3*b)/tan(f*x + e)^5)/f
```

3.323.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(47) = 94.

Time = 0.34 (sec) , antiderivative size = 170, normalized size of antiderivative = 3.33

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx$$
$$= \frac{3\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5 + 3\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5 - 35\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 - 15\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 - 480\,(fx+}{}$$

input
```
integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/480*(3*a*tan(1/2*f*x + 1/2*e)^5 + 3*b*tan(1/2*f*x + 1/2*e)^5 - 35*a*tan(
1/2*f*x + 1/2*e)^3 - 15*b*tan(1/2*f*x + 1/2*e)^3 - 480*(f*x + e)*a + 330*a
*tan(1/2*f*x + 1/2*e) + 30*b*tan(1/2*f*x + 1/2*e) - (330*a*tan(1/2*f*x + 1
/2*e)^4 + 30*b*tan(1/2*f*x + 1/2*e)^4 - 35*a*tan(1/2*f*x + 1/2*e)^2 - 15*b
*tan(1/2*f*x + 1/2*e)^2 + 3*a + 3*b)/tan(1/2*f*x + 1/2*e)^5)/f
```

3.323.9 Mupad [B] (verification not implemented)

Time = 19.80 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)\,dx = -a\,x - \frac{a\tan(e+f\,x)^4 - \frac{a\tan(e+f\,x)^2}{3} + \frac{a}{5} + \frac{b}{5}}{f\tan\left(e+f\,x\right)^5}$$

input
```
int(cot(e + f*x)^6*(a + b/cos(e + f*x)^2),x)
```

output
```
- a*x - (a/5 + b/5 - (a*tan(e + f*x)^2)/3 + a*tan(e + f*x)^4)/(f*tan(e + f
*x)^5)
```

3.324 $\int (a+b\sec^2(e+fx))^2 \tan^5(e+fx)\,dx$

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3.324.1 Optimal result

Integrand size = 23, antiderivative size = 100

$$\int (a+b\sec^2(e+fx))^2 \tan^5(e+fx)\,dx = -\frac{a^2\log(\cos(e+fx))}{f} - \frac{a(a-b)\sec^2(e+fx)}{f} + \frac{(a^2-4ab+b^2)\sec^4(e+fx)}{4f} + \frac{(a-b)b\sec^6(e+fx)}{3f} + \frac{b^2\sec^8(e+fx)}{8f}$$

output
```
-a^2*ln(cos(f*x+e))/f-a*(a-b)*sec(f*x+e)^2/f+1/4*(a^2-4*a*b+b^2)*sec(f*x+e
)^4/f+1/3*(a-b)*b*sec(f*x+e)^6/f+1/8*b^2*sec(f*x+e)^8/f
```

3.324.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.26

$$\int (a+b\sec^2(e+fx))^2 \tan^5(e+fx)\,dx = -\frac{\cos^4(e+fx)\,(a+b\sec^2(e+fx))^2\,(24a^2\log(\cos(e+fx)) + 24a(a-b)\sec^2(e+fx) - 6(a^2-4ab+b^2)}{6f(a+2b+a\cos(2e+2fx))^2}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^5,x]
```

output
```
-1/6*(Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2*(24*a^2*Log[Cos[e + f*x]] +
24*a*(a - b)*Sec[e + f*x]^2 - 6*(a^2 - 4*a*b + b^2)*Sec[e + f*x]^4 - 8*(a
- b)*b*Sec[e + f*x]^6 - 3*b^2*Sec[e + f*x]^8))/(f*(a + 2*b + a*Cos[2*e + 2
*f*x])^2)
```

3.324.3 Rubi [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4626, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int \tan(e+fx)^5\left(a+b\sec(e+fx)^2\right)^2 dx$$

$$\downarrow 4626$$

$$-\frac{\int \left(1-\cos^2(e+fx)\right)^2\left(a\cos^2(e+fx)+b\right)^2\sec^9(e+fx)d\cos(e+fx)}{f}$$

$$\downarrow 354$$

$$-\frac{\int \left(1-\cos^2(e+fx)\right)^2\left(a\cos^2(e+fx)+b\right)^2\sec^5(e+fx)d\cos^2(e+fx)}{2f}$$

$$\downarrow 99$$

$$-\frac{\int \left(b^2\sec^5(e+fx)+2(a-b)b\sec^4(e+fx)+\left(a^2-4ba+b^2\right)\sec^3(e+fx)-2a(a-b)\sec^2(e+fx)+a^2\sec(e+\right.}{2f}$$

$$\downarrow 2009$$

$$-\frac{-\frac{1}{2}\left(a^2-4ab+b^2\right)\sec^2(e+fx)+a^2\log\left(\cos^2(e+fx)\right)-\frac{2}{3}b(a-b)\sec^3(e+fx)+2a(a-b)\sec(e+fx)-\frac{1}{4}b^2}{2f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^5,x]
```

output
```
-1/2*(a^2*Log[Cos[e + f*x]^2] + 2*a*(a - b)*Sec[e + f*x] - ((a^2 - 4*a*b +
 b^2)*Sec[e + f*x]^2)/2 - (2*(a - b)*b*Sec[e + f*x]^3)/3 - (b^2*Sec[e + f*
x]^4)/4)/f
```

3.324.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.324.4 Maple [A] (verified)

Time = 9.06 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.86

method	result
parts	$\frac{a^2\left(\frac{\tan(fx+e)^4}{4}-\frac{\tan(fx+e)^2}{2}+\frac{\ln\left(1+\tan(fx+e)^2\right)}{2}\right)}{f}+\frac{b^2\left(\frac{\tan(fx+e)^8}{8}+\frac{\tan(fx+e)^6}{6}\right)}{f}+\frac{ab\tan(fx+e)^6}{3f}$
derivativedivides	$\frac{\frac{b^2\sec(fx+e)^8}{8}+\frac{b\sec(fx+e)^6a}{3}-\frac{b^2\sec(fx+e)^6}{3}+\frac{a^2\sec(fx+e)^4}{4}-ab\sec(fx+e)^4+\frac{\sec(fx+e)^4b^2}{4}-a^2\sec(fx+e)^2+\sec(fx+e)^2}{f}$
default	$\frac{\frac{b^2\sec(fx+e)^8}{8}+\frac{b\sec(fx+e)^6a}{3}-\frac{b^2\sec(fx+e)^6}{3}+\frac{a^2\sec(fx+e)^4}{4}-ab\sec(fx+e)^4+\frac{\sec(fx+e)^4b^2}{4}-a^2\sec(fx+e)^2+\sec(fx+e)^2}{f}$
risch	$ia^2x+\frac{2ia^2e}{f}+\frac{-4a^2e^{14i(fx+e)}+4abe^{14i(fx+e)}-20a^2e^{12i(fx+e)}+8abe^{12i(fx+e)}+4b^2e^{12i(fx+e)}-44a^2e^{10i(fx+e)}+$

input
```
int((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^5,x,method=_RETURNVERBOSE)
```

output
```
a^2/f*(1/4*tan(f*x+e)^4-1/2*tan(f*x+e)^2+1/2*ln(1+tan(f*x+e)^2))+b^2/f*(1/
8*tan(f*x+e)^8+1/6*tan(f*x+e)^6)+1/3*a*b/f*tan(f*x+e)^6
```

3.324.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.99

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^5(e+fx)\,dx =$$
$$-\frac{24\,a^2\cos(fx+e)^8\log(-\cos(fx+e))+24\,(a^2-ab)\cos(fx+e)^6-6\,(a^2-4\,ab+b^2)\cos(fx+e)^4-}{24\,f\cos(fx+e)^8}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^5,x, algorithm="fricas")
```

output
```
-1/24*(24*a^2*cos(f*x + e)^8*log(-cos(f*x + e)) + 24*(a^2 - a*b)*cos(f*x +
 e)^6 - 6*(a^2 - 4*a*b + b^2)*cos(f*x + e)^4 - 8*(a*b - b^2)*cos(f*x + e)^
2 - 3*b^2)/(f*cos(f*x + e)^8)
```

3.324.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. 2(85) = 170.

Time = 1.13 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.90

$$\int \left(a+b\sec^2(e+fx)\right)^2 \tan^5(e+fx)\,dx$$
$$= \begin{cases} \frac{a^2\log\left(\tan^2(e+fx)+1\right)}{2f} + \frac{a^2\tan^4(e+fx)}{4f} - \frac{a^2\tan^2(e+fx)}{2f} + \frac{ab\tan^4(e+fx)\sec^2(e+fx)}{3f} - \frac{ab\tan^2(e+fx)\sec^2(e+fx)}{3f} + \frac{ab\sec^2(e} \\ x\left(a+b\sec^2(e)\right)^2\tan^5(e) \end{cases}$$

input
```
integrate((a+b*sec(f*x+e)**2)**2*tan(f*x+e)**5,x)
```

output
```
Piecewise((a**2*log(tan(e + f*x)**2 + 1)/(2*f) + a**2*tan(e + f*x)**4/(4*f
) - a**2*tan(e + f*x)**2/(2*f) + a*b*tan(e + f*x)**4*sec(e + f*x)**2/(3*f)
 - a*b*tan(e + f*x)**2*sec(e + f*x)**2/(3*f) + a*b*sec(e + f*x)**2/(3*f) +
 b**2*tan(e + f*x)**4*sec(e + f*x)**4/(8*f) - b**2*tan(e + f*x)**2*sec(e +
 f*x)**4/(12*f) + b**2*sec(e + f*x)**4/(24*f), Ne(f, 0)), (x*(a + b*sec(e)
**2)**2*tan(e)**5, True))
```

3.324.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.47

$$\int \left(a+b\sec^2(e+fx)\right)^2 \tan^5(e+fx)\,dx =$$
$$-\frac{12\,a^2\log\left(\sin\left(fx+e\right)^2-1\right) - \frac{24\left(a^2-ab\right)\sin(fx+e)^6-6\left(11\,a^2-8\,ab-b^2\right)\sin(fx+e)^4+4\left(15\,a^2-8\,ab-b^2\right)\sin(fx+e)^2-18\,a^2+8}{\sin(fx+e)^8-4\sin(fx+e)^6+6\sin(fx+e)^4-4\sin(fx+e)^2+1}}{24\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^5,x, algorithm="maxima")
```

output
```
-1/24*(12*a^2*log(sin(f*x + e)^2 - 1) - (24*(a^2 - a*b)*sin(f*x + e)^6 - 6
*(11*a^2 - 8*a*b - b^2)*sin(f*x + e)^4 + 4*(15*a^2 - 8*a*b - b^2)*sin(f*x
 + e)^2 - 18*a^2 + 8*a*b + b^2)/(sin(f*x + e)^8 - 4*sin(f*x + e)^6 + 6*sin(
f*x + e)^4 - 4*sin(f*x + e)^2 + 1))/f
```

3.324.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 431 vs. $2(94) = 188$.

Time = 1.89 (sec) , antiderivative size = 431, normalized size of antiderivative = 4.31

$$\int \left(a + b\sec^2(e+fx)\right)^2 \tan^5(e+fx)\,dx$$

$$= \frac{12\,a^2 \log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1} - \frac{\cos(fx+e)-1}{\cos(fx+e)+1} + 2\right|\right) - 12\,a^2 \log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1} - \frac{\cos(fx+e)-1}{\cos(fx+e)+1} - 2\right|\right) + \frac{25\,a^2\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-}\right.}{}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^5,x, algorithm="giac")
```

output
```
1/24*(12*a^2*log(abs(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f*x + e
) - 1)/(cos(f*x + e) + 1) + 2)) - 12*a^2*log(abs(-(cos(f*x + e) + 1)/(cos(
f*x + e) - 1) - (cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2)) + (25*a^2*((co
s(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)
)^4 + 248*a^2*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/
(cos(f*x + e) + 1))^3 + 984*a^2*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (
cos(f*x + e) - 1)/(cos(f*x + e) + 1))^2 + 1760*a^2*((cos(f*x + e) + 1)/(co
s(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) - 512*a*b*((cos(f
*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) -
 256*b^2*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(
f*x + e) + 1)) + 1168*a^2 - 1024*a*b + 256*b^2)/((cos(f*x + e) + 1)/(cos(f
*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 2)^4)/f
```

3.324.9 Mupad [B] (verification not implemented)

Time = 19.52 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.24

$$\int \left(a + b\sec^2(e+fx)\right)^2 \tan^5(e+fx)\,dx = \frac{\tan(e+f\,x)^4\left(\frac{(a+b)^2}{4} + \frac{b^2}{4} - \frac{b\,(a+b)}{2}\right)}{f} - \frac{\tan(e+f\,x)^2\left(\frac{(a+b)^2}{2} + \frac{b^2}{2} - b\,(a+b)\right)}{f} - \frac{\tan(e+f\,x)^6\left(\frac{b^2}{6} - \frac{b\,(a+b)}{3}\right)}{f} + \frac{a^2\,\ln\left(\tan(e+f\,x)^2+1\right)}{2\,f} + \frac{b^2\tan(e+f\,x)^8}{8\,f}$$

input
```
int(tan(e + f*x)^5*(a + b/cos(e + f*x)^2)^2,x)
```

output
```
(tan(e + f*x)^4*((a + b)^2/4 + b^2/4 - (b*(a + b))/2))/f - (tan(e + f*x)^2
*((a + b)^2/2 + b^2/2 - b*(a + b)))/f - (tan(e + f*x)^6*(b^2/6 - (b*(a + b
))/3))/f + (a^2*log(tan(e + f*x)^2 + 1))/(2*f) + (b^2*tan(e + f*x)^8)/(8*f
)
```

3.325 $\int (a+b\sec^2(e+fx))^2 \tan^3(e+fx)\,dx$

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3.325.1 Optimal result

Integrand size = 23, antiderivative size = 77

$$\int (a+b\sec^2(e+fx))^2 \tan^3(e+fx)\,dx = \frac{a^2\log(\cos(e+fx))}{f} + \frac{a(a-2b)\sec^2(e+fx)}{2f} + \frac{(2a-b)b\sec^4(e+fx)}{4f} + \frac{b^2\sec^6(e+fx)}{6f}$$

output
```
a^2*ln(cos(f*x+e))/f+1/2*a*(a-2*b)*sec(f*x+e)^2/f+1/4*(2*a-b)*b*sec(f*x+e)
^4/f+1/6*b^2*sec(f*x+e)^6/f
```

3.325.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.39

$$\int (a+b\sec^2(e+fx))^2 \tan^3(e+fx)\,dx = \frac{\cos^4(e+fx)\,(a+b\sec^2(e+fx))^2\,(12a^2\log(\cos(e+fx)) + 6a(a-2b)\sec^2(e+fx) + 3(2a-b)b\sec^4(e+}{3f(a+2b+a\cos(2e+2fx))^2}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^3,x]
```

output
```
(Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2*(12*a^2*Log[Cos[e + f*x]] + 6*a*(
a - 2*b)*Sec[e + f*x]^2 + 3*(2*a - b)*b*Sec[e + f*x]^4 + 2*b^2*Sec[e + f*x
]^6))/(3*f*(a + 2*b + a*Cos[2*e + 2*f*x])^2)
```

3.325.3 Rubi [A] (warning: unable to verify)

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4626, 354, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$

$$\downarrow 3042$$

$$\int \tan(e+fx)^3\left(a+b\sec(e+fx)^2\right)^2\,dx$$

$$\downarrow 4626$$

$$-\frac{\int \left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)^2\sec^7(e+fx)d\cos(e+fx)}{f}$$

$$\downarrow 354$$

$$-\frac{\int \left(1-\cos^2(e+fx)\right)\left(a\cos^2(e+fx)+b\right)^2\sec^4(e+fx)d\cos^2(e+fx)}{2f}$$

$$\downarrow 85$$

$$-\frac{\int \left(b^2\sec^4(e+fx)+(2a-b)b\sec^3(e+fx)+a(a-2b)\sec^2(e+fx)-a^2\sec(e+fx)\right)d\cos^2(e+fx)}{2f}$$

$$\downarrow 2009$$

$$-\frac{-a^2\log\left(\cos^2(e+fx)\right)-\frac{1}{2}b(2a-b)\sec^2(e+fx)-a(a-2b)\sec(e+fx)-\frac{1}{3}b^2\sec^3(e+fx)}{2f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^3,x]
```

output
```
-1/2*(-(a^2*Log[Cos[e + f*x]^2]) - a*(a - 2*b)*Sec[e + f*x] - ((2*a - b)*b
*Sec[e + f*x]^2)/2 - (b^2*Sec[e + f*x]^3)/3)/f
```

3.325.3.1 Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.325.4 Maple [A] (verified)

Time = 4.57 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

method	result
parts	$\frac{a^2\left(\frac{\tan(fx+e)^2}{2}-\frac{\ln\left(1+\tan(fx+e)^2\right)}{2}\right)}{f}+\frac{b^2\left(\frac{\sec(fx+e)^6}{6}-\frac{\sec(fx+e)^4}{4}\right)}{f}+\frac{ab\tan(fx+e)^4}{2f}$
derivativedivides	$\frac{\frac{b^2\sec(fx+e)^6}{6}+\frac{ab\sec(fx+e)^4}{2}-\frac{\sec(fx+e)^4b^2}{4}+\frac{a^2\sec(fx+e)^2}{2}-\sec(fx+e)^2ab-a^2\ln(\sec(fx+e))}{f}$
default	$\frac{\frac{b^2\sec(fx+e)^6}{6}+\frac{ab\sec(fx+e)^4}{2}-\frac{\sec(fx+e)^4b^2}{4}+\frac{a^2\sec(fx+e)^2}{2}-\sec(fx+e)^2ab-a^2\ln(\sec(fx+e))}{f}$
risch	$-ia^2x-\frac{2ia^2e}{f}-\frac{2\left(-3a^2e^{10i(fx+e)}+6abe^{10i(fx+e)}-12a^2e^{8i(fx+e)}+12abe^{8i(fx+e)}+6b^2e^{8i(fx+e)}-18a^2e^{6i(fx+e}\right.$ $3f$

input
```
int((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^3,x,method=_RETURNVERBOSE)
```

output
```
a^2/f*(1/2*tan(f*x+e)^2-1/2*ln(1+tan(f*x+e)^2))+b^2/f*(1/6*sec(f*x+e)^6-1/
4*sec(f*x+e)^4)+1/2*a*b/f*tan(f*x+e)^4
```

3.325.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.03

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^3(e+fx)\,dx$$
$$=\frac{12\,a^2\cos(fx+e)^6\log(-\cos(fx+e))+6\,(a^2-2\,ab)\cos(fx+e)^4+3\,(2\,ab-b^2)\cos(fx+e)^2+2\,b^2}{12\,f\cos(fx+e)^6}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^3,x, algorithm="fricas")
```

output
```
1/12*(12*a^2*cos(f*x + e)^6*log(-cos(f*x + e)) + 6*(a^2 - 2*a*b)*cos(f*x +
 e)^4 + 3*(2*a*b - b^2)*cos(f*x + e)^2 + 2*b^2)/(f*cos(f*x + e)^6)
```

3.325.6 Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.66

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^3(e+fx)\,dx$$
$$=\begin{cases}-\frac{a^2\log\left(\tan^2(e+fx)+1\right)}{2f}+\frac{a^2\tan^2(e+fx)}{2f}+\frac{ab\tan^2(e+fx)\sec^2(e+fx)}{2f}-\frac{ab\sec^2(e+fx)}{2f}+\frac{b^2\tan^2(e+fx)\sec^4(e+fx)}{6f}-\frac{b^2\sec^4}{12} \\ x(a+b\sec^2(e))^2\tan^3(e)\end{cases}$$

input
```
integrate((a+b*sec(f*x+e)**2)**2*tan(f*x+e)**3,x)
```

output
```
Piecewise((-a**2*log(tan(e + f*x)**2 + 1)/(2*f) + a**2*tan(e + f*x)**2/(2*
f) + a*b*tan(e + f*x)**2*sec(e + f*x)**2/(2*f) - a*b*sec(e + f*x)**2/(2*f)
 + b**2*tan(e + f*x)**2*sec(e + f*x)**4/(6*f) - b**2*sec(e + f*x)**4/(12*f
), Ne(f, 0)), (x*(a + b*sec(e)**2)**2*tan(e)**3, True))
```

3.325.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.48

$$\int \left(a+b\sec^2(e+fx)\right)^2 \tan^3(e+fx)\,dx$$
$$= \frac{6\,a^2\log\left(\sin\left(fx+e\right)^2-1\right) - \frac{6\,(a^2-2\,ab)\sin(fx+e)^4 - 3\,(4\,a^2-6\,ab-b^2)\sin(fx+e)^2 + 6\,a^2-6\,ab-b^2}{\sin(fx+e)^6-3\sin(fx+e)^4+3\sin(fx+e)^2-1}}{12\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^3,x, algorithm="maxima")
```

output
```
1/12*(6*a^2*log(sin(f*x + e)^2 - 1) - (6*(a^2 - 2*a*b)*sin(f*x + e)^4 - 3*
(4*a^2 - 6*a*b - b^2)*sin(f*x + e)^2 + 6*a^2 - 6*a*b - b^2)/(sin(f*x + e)^
6 - 3*sin(f*x + e)^4 + 3*sin(f*x + e)^2 - 1))/f
```

3.325.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. 2(71) = 142.

Time = 0.88 (sec) , antiderivative size = 385, normalized size of antiderivative = 5.00

$$\int \left(a+b\sec^2(e+fx)\right)^2 \tan^3(e+fx)\,dx =$$
$$-\frac{6\,a^2\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+2\right|\right) - 6\,a^2\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}-2\right|\right) + \frac{11\,a^2\left(\frac{\cos(fx+e)+}{\cos(fx+e)-}\right.}{}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^3,x, algorithm="giac")
```

output
```
-1/12*(6*a^2*log(abs(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f*x + e
) - 1)/(cos(f*x + e) + 1) + 2)) - 6*a^2*log(abs(-(cos(f*x + e) + 1)/(cos(f
*x + e) - 1) - (cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2)) + (11*a^2*((cos
(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1))
^3 + 90*a^2*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(c
os(f*x + e) + 1))^2 + 228*a^2*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (co
s(f*x + e) - 1)/(cos(f*x + e) + 1)) - 96*a*b*((cos(f*x + e) + 1)/(cos(f*x
+ e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) - 48*b^2*((cos(f*x + e)
 + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) + 184*a^
2 - 192*a*b + 32*b^2)/((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x +
e) - 1)/(cos(f*x + e) + 1) + 2)^3)/f
```

3.325.9 Mupad [B] (verification not implemented)

Time = 20.18 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.19

$$\int \left(a+b\sec^2(e+fx)\right)^2 \tan^3(e+fx)\,dx = \frac{\tan(e+fx)^2\left(\frac{(a+b)^2}{2}+\frac{b^2}{2}-b\,(a+b)\right)}{f} - \frac{\tan(e+fx)^4\left(\frac{b^2}{4}-\frac{b\,(a+b)}{2}\right)}{f} - \frac{a^2\,\ln\left(\tan(e+fx)^2+1\right)}{2\,f}+\frac{b^2\tan(e+fx)^6}{6\,f}$$

input
```
int(tan(e + f*x)^3*(a + b/cos(e + f*x)^2)^2,x)
```

output
```
(tan(e + f*x)^2*((a + b)^2/2 + b^2/2 - b*(a + b)))/f - (tan(e + f*x)^4*(b^
2/4 - (b*(a + b))/2))/f - (a^2*log(tan(e + f*x)^2 + 1))/(2*f) + (b^2*tan(e
 + f*x)^6)/(6*f)
```

3.326 $\int (a + b\sec^2(e+fx))^2 \tan(e+fx)\,dx$

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3.326.1 Optimal result

Integrand size = 21, antiderivative size = 48

$$\int (a + b\sec^2(e+fx))^2 \tan(e+fx)\,dx = -\frac{a^2\log(\cos(e+fx))}{f} + \frac{ab\sec^2(e+fx)}{f} + \frac{b^2\sec^4(e+fx)}{4f}$$

output
```
-a^2*ln(cos(f*x+e))/f+a*b*sec(f*x+e)^2/f+1/4*b^2*sec(f*x+e)^4/f
```

3.326.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.73

$$\int (a + b\sec^2(e+fx))^2 \tan(e+fx)\,dx = -\frac{\cos^4(e+fx)\,(a+b\sec^2(e+fx))^2\,(4a^2\log(\cos(e+fx)) - 4ab\sec^2(e+fx) - b^2\sec^4(e+fx))}{f(a+2b+a\cos(2e+2fx))^2}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x],x]
```

output
```
-((Cos[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2*(4*a^2*Log[Cos[e + f*x]] - 4*a*
b*Sec[e + f*x]^2 - b^2*Sec[e + f*x]^4))/(f*(a + 2*b + a*Cos[2*e + 2*f*x])^
2))
```

3.326.3 Rubi [A] (warning: unable to verify)

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4626, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int \tan(e+fx)\left(a+b\sec(e+fx)^2\right)^2 dx$$

$$\downarrow 4626$$

$$-\frac{\int \left(a\cos^2(e+fx)+b\right)^2 \sec^5(e+fx)d\cos(e+fx)}{f}$$

$$\downarrow 243$$

$$-\frac{\int \left(a\cos^2(e+fx)+b\right)^2 \sec^3(e+fx)d\cos^2(e+fx)}{2f}$$

$$\downarrow 49$$

$$-\frac{\int \left(b^2\sec^3(e+fx)+2ab\sec^2(e+fx)+a^2\sec(e+fx)\right)d\cos^2(e+fx)}{2f}$$

$$\downarrow 2009$$

$$-\frac{a^2\log\left(\cos^2(e+fx)\right)-2ab\sec(e+fx)-\frac{1}{2}b^2\sec^2(e+fx)}{2f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x],x]
```

output
```
-1/2*(a^2*Log[Cos[e + f*x]^2] - 2*a*b*Sec[e + f*x] - (b^2*Sec[e + f*x]^2)/
2)/f
```

3.326.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.326.4 Maple [A] (verified)

Time = 2.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{\frac{\sec(fx+e)^4 b^2}{4}+\sec(fx+e)^2 ab+a^2\ln(\sec(fx+e))}{f}$	41
default	$\frac{\frac{\sec(fx+e)^4 b^2}{4}+\sec(fx+e)^2 ab+a^2\ln(\sec(fx+e))}{f}$	41
parts	$\frac{a^2\ln\left(1+\tan(fx+e)^2\right)}{2f}+\frac{b^2\sec(fx+e)^4}{4f}+\frac{ab\sec(fx+e)^2}{f}$	51
risch	$ia^2x+\frac{2ia^2e}{f}+\frac{4b\left(a\,\mathrm{e}^{6i(fx+e)}+2a\,\mathrm{e}^{4i(fx+e)}+b\,\mathrm{e}^{4i(fx+e)}+a\,\mathrm{e}^{2i(fx+e)}\right)}{f\left(\mathrm{e}^{2i(fx+e)}+1\right)^4}-\frac{a^2\ln\left(\mathrm{e}^{2i(fx+e)}+1\right)}{f}$	104

input
```
int((a+b*sec(f*x+e)^2)^2*tan(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/4*sec(f*x+e)^4*b^2+sec(f*x+e)^2*a*b+a^2*ln(sec(f*x+e)))
```

3.326.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

$$\int \left(a + b\sec^2(e+fx)\right)^2 \tan(e+fx)\,dx$$
$$= -\frac{4\,a^2\cos(fx+e)^4\log(-\cos(fx+e)) - 4\,ab\cos(fx+e)^2 - b^2}{4\,f\cos(fx+e)^4}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e),x, algorithm="fricas")
```

output
```
-1/4*(4*a^2*cos(f*x + e)^4*log(-cos(f*x + e)) - 4*a*b*cos(f*x + e)^2 - b^2
)/(f*cos(f*x + e)^4)
```

3.326.6 Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.27

$$\int \left(a + b\sec^2(e+fx)\right)^2 \tan(e+fx)\,dx$$
$$= \begin{cases} \frac{a^2\log\left(\tan^2(e+fx)+1\right)}{2f} + \frac{ab\sec^2(e+fx)}{f} + \frac{b^2\sec^4(e+fx)}{4f} & \text{for } f \neq 0 \\ x(a+b\sec^2(e))^2\tan(e) & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*sec(f*x+e)**2)**2*tan(f*x+e),x)
```

output
```
Piecewise((a**2*log(tan(e + f*x)**2 + 1)/(2*f) + a*b*sec(e + f*x)**2/f + b
**2*sec(e + f*x)**4/(4*f), Ne(f, 0)), (x*(a + b*sec(e)**2)**2*tan(e), True
))
```

3.326.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.40

$$\int \left(a+b\sec^2(e+fx)\right)^2 \tan(e+fx)\,dx = -\frac{2\,a^2\log\left(\sin\left(fx+e\right)^2-1\right)+\frac{4\,ab\sin(fx+e)^2-4\,ab-b^2}{\sin(fx+e)^4-2\,\sin(fx+e)^2+1}}{4\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e),x, algorithm="maxima")
```

output
```
-1/4*(2*a^2*log(sin(f*x + e)^2 - 1) + (4*a*b*sin(f*x + e)^2 - 4*a*b - b^2)
/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1))/f
```

3.326.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. 2(46) = 92.

Time = 0.42 (sec) , antiderivative size = 334, normalized size of antiderivative = 6.96

$$\int \left(a+b\sec^2(e+fx)\right)^2 \tan(e+fx)\,dx$$

$$= \frac{2\,a^2\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+2\right|\right)-2\,a^2\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}-2\right|\right)+\frac{3\,a^2\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{c}{c}\right.}{4\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e),x, algorithm="giac")
```

output
```
1/4*(2*a^2*log(abs(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f*x + e)
- 1)/(cos(f*x + e) + 1) + 2)) - 2*a^2*log(abs(-(cos(f*x + e) + 1)/(cos(f*x
 + e) - 1) - (cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2)) + (3*a^2*((cos(f*
x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1))^2
+ 12*a^2*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(
f*x + e) + 1)) - 16*a*b*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x
+ e) - 1)/(cos(f*x + e) + 1)) - 8*b^2*((cos(f*x + e) + 1)/(cos(f*x + e) -
1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) + 12*a^2 - 32*a*b)/((cos(f*x +
 e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 2)^2
)/f
```

3.326.9 Mupad [B] (verification not implemented)

Time = 19.69 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.27

$$\int \left(a+b\sec^2(e+fx)\right)^2 \tan(e+fx)\,dx = \frac{a^2\,\ln\left(\tan(e+f\,x)^2+1\right)}{2\,f} - \frac{\tan(e+f\,x)^2\left(\frac{b^2}{2}-b\,(a+b)\right)}{f} + \frac{b^2\tan(e+f\,x)^4}{4\,f}$$

input
```
int(tan(e + f*x)*(a + b/cos(e + f*x)^2)^2,x)
```

output
```
(a^2*log(tan(e + f*x)^2 + 1))/(2*f) - (tan(e + f*x)^2*(b^2/2 - b*(a + b)))
/f + (b^2*tan(e + f*x)^4)/(4*f)
```

3.327 $\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

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3.327.1 Optimal result

Integrand size = 21, antiderivative size = 53

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{b(2a+b)\log(\cos(e+fx))}{f} + \frac{(a+b)^2\log(\sin(e+fx))}{f} + \frac{b^2\sec^2(e+fx)}{2f}$$

output
```
-b*(2*a+b)*ln(cos(f*x+e))/f+(a+b)^2*ln(sin(f*x+e))/f+1/2*b^2*sec(f*x+e)^2/
f
```

3.327.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.58

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{2(b^2+2\cos^2(e+fx)\left(-b(2a+b)\log(\cos(e+fx))+(a+b)^2\log(\sin(e+fx))\right))\left(a\cos(e+fx)+b\sec(e+fx)\right)^2}{f(a+2b+a\cos(2(e+fx)))^2}$$

input
```
Integrate[Cot[e + f*x]*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(2*(b^2 + 2*Cos[e + f*x]^2*(-(b*(2*a + b)*Log[Cos[e + f*x]]) + (a + b)^2*L
og[Sin[e + f*x]]))*(a*Cos[e + f*x] + b*Sec[e + f*x])^2)/(f*(a + 2*b + a*Co
s[2*(e + f*x)])^2)
```

3.327.3 Rubi [A] (warning: unable to verify)

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4626, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^2}{\tan(e+fx)}dx$$

$$\downarrow 4626$$

$$-\frac{\int \frac{\left(a\cos^2(e+fx)+b\right)^2\sec^3(e+fx)}{1-\cos^2(e+fx)}d\cos(e+fx)}{f}$$

$$\downarrow 354$$

$$-\frac{\int \frac{\left(a\cos^2(e+fx)+b\right)^2\sec^2(e+fx)}{1-\cos^2(e+fx)}d\cos^2(e+fx)}{2f}$$

$$\downarrow 99$$

$$-\frac{\int \left(-\frac{(a+b)^2}{\cos^2(e+fx)-1}+b^2\sec^2(e+fx)+b(2a+b)\sec(e+fx)\right)d\cos^2(e+fx)}{2f}$$

$$\downarrow 2009$$

$$-\frac{b(2a+b)\log\left(\cos^2(e+fx)\right)-(a+b)^2\log\left(1-\cos^2(e+fx)\right)+b^2(-\sec(e+fx))}{2f}$$

input
```
Int[Cot[e + f*x]*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
-1/2*(b*(2*a + b)*Log[Cos[e + f*x]^2] - (a + b)^2*Log[1 - Cos[e + f*x]^2]
- b^2*Sec[e + f*x])/f
```

3.327.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.327.4 Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{a^2\ln(\sin(fx+e))+2ab\ln(\tan(fx+e))+b^2\left(\frac{1}{2\cos(fx+e)^2}+\ln(\tan(fx+e))\right)}{f}$
default	$\frac{a^2\ln(\sin(fx+e))+2ab\ln(\tan(fx+e))+b^2\left(\frac{1}{2\cos(fx+e)^2}+\ln(\tan(fx+e))\right)}{f}$
risch	$-ia^2x-\frac{2ia^2e}{f}+\frac{2b^2e^{2i(fx+e)}}{f\left(e^{2i(fx+e)}+1\right)^2}-\frac{2b\ln\left(e^{2i(fx+e)}+1\right)a}{f}-\frac{b^2\ln\left(e^{2i(fx+e)}+1\right)}{f}+\frac{\ln\left(e^{2i(fx+e)}-1\right)a^2}{f}+\frac{2}{}$

input
```
int(cot(f*x+e)*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(a^2*ln(sin(f*x+e))+2*a*b*ln(tan(f*x+e))+b^2*(1/2/cos(f*x+e)^2+ln(tan(
f*x+e))))
```

3.327.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.49

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx =$$
$$-\frac{(2\,ab+b^2)\cos(fx+e)^2\log\left(\cos(fx+e)^2\right)-(a^2+2\,ab+b^2)\cos(fx+e)^2\log\left(-\frac{1}{4}\cos(fx+e)^2+\frac{1}{4}\right)}{2\,f\cos(fx+e)^2}$$

input
```
integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
-1/2*((2*a*b + b^2)*cos(f*x + e)^2*log(cos(f*x + e)^2) - (a^2 + 2*a*b + b^
2)*cos(f*x + e)^2*log(-1/4*cos(f*x + e)^2 + 1/4) - b^2)/(f*cos(f*x + e)^2)
```

3.327.6 Sympy [F]

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \int \left(a+b\sec^2(e+fx)\right)^2\cot(e+fx)\,dx$$

input
```
integrate(cot(f*x+e)*(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**2*cot(e + f*x), x)
```

3.327.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.21

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=-\frac{(2\,ab+b^2)\log\left(\sin(fx+e)^2-1\right)-(a^2+2\,ab+b^2)\log\left(\sin(fx+e)^2\right)+\frac{b^2}{\sin(fx+e)^2-1}}{2\,f}$$

input
```
integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/2*((2*a*b + b^2)*log(sin(f*x + e)^2 - 1) - (a^2 + 2*a*b + b^2)*log(sin(
f*x + e)^2) + b^2/(sin(f*x + e)^2 - 1))/f
```

3.327.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(51) = 102$.

Time $= 0.32$ (sec) , antiderivative size $= 247$, normalized size of antiderivative $= 4.66$

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{a^2\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+2\right|\right)+(2\,ab+b^2)\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}-2\right|\right)-\frac{2\,ab\left(\frac{\cos(f}{\cos(f}\right.}{2\,f}$$

input
```
integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-1/2*(a^2*log(abs(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f*x + e) -
 1)/(cos(f*x + e) + 1) + 2)) + (2*a*b + b^2)*log(abs(-(cos(f*x + e) + 1)/(
cos(f*x + e) - 1) - (cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2)) - (2*a*b*(
(cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) +
 1)) + b^2*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(co
s(f*x + e) + 1)) + 4*a*b - 2*b^2)/((cos(f*x + e) + 1)/(cos(f*x + e) - 1) +
 (cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 2))/f
```

3.327.9 Mupad [B] (verification not implemented)

Time $= 20.39$ (sec) , antiderivative size $= 58$, normalized size of antiderivative $= 1.09$

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{\ln\left(\tan(e+f\,x)\right)\,\left(a^2+2\,a\,b+b^2\right)}{f} - \frac{a^2\,\ln\left(\tan(e+f\,x)^2+1\right)}{2\,f}+\frac{b^2\tan(e+f\,x)^2}{2\,f}$$

input
```
int(cot(e + f*x)*(a + b/cos(e + f*x)^2)^2,x)
```

output
```
(log(tan(e + f*x))*(2*a*b + a^2 + b^2))/f - (a^2*log(tan(e + f*x)^2 + 1))/
(2*f) + (b^2*tan(e + f*x)^2)/(2*f)
```

3.328 $\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

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3.328.2 Mathematica [A] (verified) . . . 2405
3.328.3 Rubi [A] (verified) . . . 2406
3.328.4 Maple [A] (verified) . . . 2407
3.328.5 Fricas [A] (verification not implemented) . . . 2408
3.328.6 Sympy [F] . . . 2408
3.328.7 Maxima [A] (verification not implemented) . . . 2408
3.328.8 Giac [B] (verification not implemented) . . . 2409
3.328.9 Mupad [B] (verification not implemented) . . . 2409

3.328.1 Optimal result

Integrand size = 23, antiderivative size = 57

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{(a+b)^2\csc^2(e+fx)}{2f} - \frac{b^2\log(\cos(e+fx))}{f} - \frac{(a^2-b^2)\log(\sin(e+fx))}{f}$$

output
```
-1/2*(a+b)^2*csc(f*x+e)^2/f-b^2*ln(cos(f*x+e))/f-(a^2-b^2)*ln(sin(f*x+e))/
f
```

3.328.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.42

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{2(b+a\cos^2(e+fx))^2\left((a+b)^2\csc^2(e+fx)+2b^2\log(\cos(e+fx))+2(a^2-b^2)\log(\sin(e+fx))\right)}{f(a+2b+a\cos(2(e+fx)))^2}$$

input
```
Integrate[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(-2*(b + a*Cos[e + f*x]^2)^2*((a + b)^2*Csc[e + f*x]^2 + 2*b^2*Log[Cos[e +
 f*x]] + 2*(a^2 - b^2)*Log[Sin[e + f*x]]))/(f*(a + 2*b + a*Cos[2*(e + f*x)
])^2)
```

3.328.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4626, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^2}{\tan(e+fx)^3} dx$$

$$\downarrow 4626$$

$$-\frac{\int \frac{\left(a\cos^2(e+fx)+b\right)^2 \sec(e+fx)}{\left(1-\cos^2(e+fx)\right)^2} d\cos(e+fx)}{f}$$

$$\downarrow 354$$

$$-\frac{\int \frac{\left(a\cos^2(e+fx)+b\right)^2 \sec(e+fx)}{\left(1-\cos^2(e+fx)\right)^2} d\cos^2(e+fx)}{2f}$$

$$\downarrow 99$$

$$-\frac{\int \left(\sec(e+fx)b^2 + \frac{a^2-b^2}{\cos^2(e+fx)-1} + \frac{(a+b)^2}{\left(\cos^2(e+fx)-1\right)^2}\right) d\cos^2(e+fx)}{2f}$$

$$\downarrow 2009$$

$$-\frac{\left(a^2-b^2\right)\log\left(1-\cos^2(e+fx)\right) + \frac{(a+b)^2}{1-\cos^2(e+fx)} + b^2\log\left(\cos^2(e+fx)\right)}{2f}$$

input
```
Int[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
-1/2*((a + b)^2/(1 - Cos[e + f*x]^2) + b^2*Log[Cos[e + f*x]^2] + (a^2 - b^
2)*Log[1 - Cos[e + f*x]^2])/f
```

3.328.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.328.4 Maple [A] (verified)

Time = 2.62 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cot(fx+e)^2}{2}-\ln(\sin(fx+e))\right)-\frac{ab}{\sin(fx+e)^2}+b^2\left(-\frac{1}{2\sin(fx+e)^2}+\ln(\tan(fx+e))\right)}{f}$
default	$\frac{a^2\left(-\frac{\cot(fx+e)^2}{2}-\ln(\sin(fx+e))\right)-\frac{ab}{\sin(fx+e)^2}+b^2\left(-\frac{1}{2\sin(fx+e)^2}+\ln(\tan(fx+e))\right)}{f}$
risch	$ia^2x+\frac{2ia^2e}{f}+\frac{2(a^2+2ab+b^2)e^{2i(fx+e)}}{f\left(e^{2i(fx+e)}-1\right)^2}-\frac{\ln\left(e^{2i(fx+e)}-1\right)a^2}{f}+\frac{\ln\left(e^{2i(fx+e)}-1\right)b^2}{f}-\frac{b^2\ln\left(e^{2i(fx+e)}+1\right)}{f}$

input
```
int(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(a^2*(-1/2*cot(f*x+e)^2-ln(sin(f*x+e)))-a*b/sin(f*x+e)^2+b^2*(-1/2/sin
(f*x+e)^2+ln(tan(f*x+e))))
```

3.328.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.75

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{a^2+2\,ab+b^2-\left(b^2\cos\left(fx+e\right)^2-b^2\right)\log\left(\cos\left(fx+e\right)^2\right)-\left(\left(a^2-b^2\right)\cos\left(fx+e\right)^2-a^2+b^2\right)\log\left(-\frac{1}{4}\right.}{2\left(f\cos\left(fx+e\right)^2-f\right)}$$

input
```
integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/2*(a^2 + 2*a*b + b^2 - (b^2*cos(f*x + e)^2 - b^2)*log(cos(f*x + e)^2) -
((a^2 - b^2)*cos(f*x + e)^2 - a^2 + b^2)*log(-1/4*cos(f*x + e)^2 + 1/4))/(
f*cos(f*x + e)^2 - f)
```

3.328.6 Sympy [F]

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx=\int \left(a+b\sec^2\left(e+fx\right)\right)^2\cot^3\left(e+fx\right)dx$$

input
```
integrate(cot(f*x+e)**3*(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**2*cot(e + f*x)**3, x)
```

3.328.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.05

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=-\frac{b^2\log\left(\sin\left(fx+e\right)^2-1\right)+\left(a^2-b^2\right)\log\left(\sin\left(fx+e\right)^2\right)+\frac{a^2+2\,ab+b^2}{\sin(fx+e)^2}}{2\,f}$$

input
```
integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/2*(b^2*log(sin(f*x + e)^2 - 1) + (a^2 - b^2)*log(sin(f*x + e)^2) + (a^2
 + 2*a*b + b^2)/sin(f*x + e)^2)/f
```

3.328.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(55) = 110$.

Time $= 0.33$ (sec) , antiderivative size $= 233$, normalized size of antiderivative $= 4.09$

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$= \frac{a^2\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)+2\,ab\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)+b^2\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)+4\,a^2\log\left(\right.}{8\,f}$$

input
```
integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/8*(a^2*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(
f*x + e) + 1)) + 2*a*b*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x +
 e) - 1)/(cos(f*x + e) + 1)) + b^2*((cos(f*x + e) + 1)/(cos(f*x + e) - 1)
+ (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) + 4*a^2*log(abs(-(cos(f*x + e) +
1)/(cos(f*x + e) - 1) - (cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 2)) - 4*b^
2*log(abs(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f*x + e) - 1)/(cos
(f*x + e) + 1) - 2)))/f
```

3.328.9 Mupad [B] (verification not implemented)

Time $= 20.28$ (sec) , antiderivative size $= 68$, normalized size of antiderivative $= 1.19$

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{a^2\,\ln\left(\tan(e+f\,x)^2+1\right)}{2\,f} - \frac{\ln\left(\tan(e+f\,x)\right)\,\left(a^2-b^2\right)}{f} - \frac{\cot(e+f\,x)^2\left(\frac{a^2}{2}+a\,b+\frac{b^2}{2}\right)}{f}$$

input
```
int(cot(e + f*x)^3*(a + b/cos(e + f*x)^2)^2,x)
```

output
```
(a^2*log(tan(e + f*x)^2 + 1))/(2*f) - (log(tan(e + f*x))*(a^2 - b^2))/f -
(cot(e + f*x)^2*(a*b + a^2/2 + b^2/2))/f
```

3.329 $\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

3.329.1 Optimal result 2411
3.329.2 Mathematica [A] (verified) 2411
3.329.3 Rubi [A] (verified) 2412
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3.329.5 Fricas [A] (verification not implemented) 2414
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3.329.9 Mupad [B] (verification not implemented) 2415

3.329.1 Optimal result

Integrand size = 23, antiderivative size = 51

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{a(a+b)\csc^2(e+fx)}{f} - \frac{(a+b)^2\csc^4(e+fx)}{4f} + \frac{a^2\log(\sin(e+fx))}{f}$$

output
```
a*(a+b)*csc(f*x+e)^2/f-1/4*(a+b)^2*csc(f*x+e)^4/f+a^2*ln(sin(f*x+e))/f
```

3.329.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.51

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{\left(b+a\cos^2(e+fx)\right)^2\left(-4a(a+b)\csc^2(e+fx)+(a+b)^2\csc^4(e+fx)-4a^2\log(\sin(e+fx))\right)}{f(a+2b+a\cos(2(e+fx)))^2}$$

input
```
Integrate[Cot[e + f*x]^5*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
-(((b + a*Cos[e + f*x]^2)^2*(-4*a*(a + b)*Csc[e + f*x]^2 + (a + b)^2*Csc[e
 + f*x]^4 - 4*a^2*Log[Sin[e + f*x]]))/(f*(a + 2*b + a*Cos[2*(e + f*x)])^2)
)
```

3.329.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.35, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4626, 353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^2}{\tan(e+fx)^5} dx$$

$$\downarrow 4626$$

$$-\frac{\int \frac{\cos(e+fx)\left(a\cos^2(e+fx)+b\right)^2}{\left(1-\cos^2(e+fx)\right)^3} d\cos(e+fx)}{f}$$

$$\downarrow 353$$

$$-\frac{\int \frac{\left(a\cos^2(e+fx)+b\right)^2}{\left(1-\cos^2(e+fx)\right)^3} d\cos^2(e+fx)}{2f}$$

$$\downarrow 49$$

$$-\frac{\int \left(-\frac{a^2}{\cos^2(e+fx)-1}-\frac{2(a+b)a}{\left(\cos^2(e+fx)-1\right)^2}-\frac{(a+b)^2}{\left(\cos^2(e+fx)-1\right)^3}\right) d\cos^2(e+fx)}{2f}$$

$$\downarrow 2009$$

$$-\frac{a^2\left(-\log\left(1-\cos^2(e+fx)\right)\right)-\frac{2a(a+b)}{1-\cos^2(e+fx)}+\frac{(a+b)^2}{2\left(1-\cos^2(e+fx)\right)^2}}{2f}$$

input
```
Int[Cot[e + f*x]^5*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
-1/2*((a + b)^2/(2*(1 - Cos[e + f*x]^2)^2) - (2*a*(a + b))/(1 - Cos[e + f*
x]^2) - a^2*Log[1 - Cos[e + f*x]^2])/f
```

3.329.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.329.4 Maple [A] (verified)

Time = 5.40 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.39

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cot(fx+e)^4}{4}+\frac{\cot(fx+e)^2}{2}+\ln(\sin(fx+e))\right)-\frac{ab\cos(fx+e)^4}{2\sin(fx+e)^4}-\frac{b^2}{4\sin(fx+e)^4}}{f}$
default	$\frac{a^2\left(-\frac{\cot(fx+e)^4}{4}+\frac{\cot(fx+e)^2}{2}+\ln(\sin(fx+e))\right)-\frac{ab\cos(fx+e)^4}{2\sin(fx+e)^4}-\frac{b^2}{4\sin(fx+e)^4}}{f}$
risch	$-ia^2x-\frac{2ia^2e}{f}-\frac{4\left(a^2e^{6i(fx+e)}+ab\,e^{6i(fx+e)}-a^2e^{4i(fx+e)}+b^2e^{4i(fx+e)}+a^2e^{2i(fx+e)}+ab\,e^{2i(fx+e)}\right)}{f\left(e^{2i(fx+e)}-1\right)^4}+\frac{\ln\left(e^{2i(f}$

input
```
int(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(a^2*(-1/4*cot(f*x+e)^4+1/2*cot(f*x+e)^2+ln(sin(f*x+e)))-1/2*a*b/sin(f
*x+e)^4*cos(f*x+e)^4-1/4*b^2/sin(f*x+e)^4)
```

3.329.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.90

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx =$$
$$-\frac{4\left(a^2+ab\right)\cos\left(fx+e\right)^2-3\,a^2-2\,ab+b^2-4\left(a^2\cos\left(fx+e\right)^4-2\,a^2\cos\left(fx+e\right)^2+a^2\right)\log\left(\frac{1}{2}\sin\left(f\right.\right.}{4\left(f\cos\left(fx+e\right)^4-2\,f\cos\left(fx+e\right)^2+f\right)}$$

input
```
integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
-1/4*(4*(a^2 + a*b)*cos(f*x + e)^2 - 3*a^2 - 2*a*b + b^2 - 4*(a^2*cos(f*x
+ e)^4 - 2*a^2*cos(f*x + e)^2 + a^2)*log(1/2*sin(f*x + e)))/(f*cos(f*x + e
)^4 - 2*f*cos(f*x + e)^2 + f)
```

3.329.6 Sympy [F]

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)^2\cot^5\left(e+fx\right)dx$$

input
```
integrate(cot(f*x+e)**5*(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**2*cot(e + f*x)**5, x)
```

3.329.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.20

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{2\,a^2\log\left(\sin\left(fx+e\right)^2\right)+\frac{4\left(a^2+ab\right)\sin(fx+e)^2-a^2-2\,ab-b^2}{\sin(fx+e)^4}}{4\,f}$$

input
```
integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/4*(2*a^2*log(sin(f*x + e)^2) + (4*(a^2 + a*b)*sin(f*x + e)^2 - a^2 - 2*a
*b - b^2)/sin(f*x + e)^4)/f
```

3.329.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 332 vs. $2(49) = 98$.

Time $= 0.36$ (sec) , antiderivative size $= 332$, normalized size of antiderivative $= 6.51$

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$

$$= \frac{32\,a^2\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right) - 64\,a^2\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right) - \frac{12\,a^2(\cos(fx+e)-1)}{\cos(fx+e)+1} - \frac{8\,ab(\cos(fx+e)-1)}{\cos(fx+e)+1} + \frac{4\,b^2(\cos(fx+}{\cos(fx+e}$$

input
```
integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/64*(32*a^2*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1)) - 64*a^2*lo
g(abs(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1)) - 12*a^2*(cos(f*x + e)
- 1)/(cos(f*x + e) + 1) - 8*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 4*
b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - a^2*(cos(f*x + e) - 1)^2/(cos(
f*x + e) + 1)^2 - 2*a*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - b^2*(c
os(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - (a^2 + 2*a*b + b^2 + 12*a^2*(cos
(f*x + e) - 1)/(cos(f*x + e) + 1) + 8*a*b*(cos(f*x + e) - 1)/(cos(f*x + e)
 + 1) - 4*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 48*a^2*(cos(f*x + e)
 - 1)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)^2/(cos(f*x + e) - 1)^2)/f
```

3.329.9 Mupad [B] (verification not implemented)

Time $= 20.71$ (sec) , antiderivative size $= 83$, normalized size of antiderivative $= 1.63$

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{a^2\,\ln\left(\tan(e+f\,x)\right)}{f} - \frac{\frac{a\,b}{2}+\frac{a^2}{4}+\frac{b^2}{4}-\tan(e+f\,x)^2\left(\frac{a^2}{2}-\frac{b^2}{2}\right)}{f\tan\left(e+f\,x\right)^4} - \frac{a^2\,\ln\left(\tan(e+f\,x)^2+1\right)}{2\,f}$$

input
```
int(cot(e + f*x)^5*(a + b/cos(e + f*x)^2)^2,x)
```

output
```
(a^2*log(tan(e + f*x)))/f - ((a*b)/2 + a^2/4 + b^2/4 - tan(e + f*x)^2*(a^2
/2 - b^2/2))/(f*tan(e + f*x)^4) - (a^2*log(tan(e + f*x)^2 + 1))/(2*f)
```

3.330 $\int (a+b\sec^2(e+fx))^2 \tan^6(e+fx)\,dx$

3.330.1 Optimal result 2417
3.330.2 Mathematica [B] (verified) 2417
3.330.3 Rubi [A] (verified) 2418
3.330.4 Maple [A] (verified) 2420
3.330.5 Fricas [A] (verification not implemented) 2420
3.330.6 Sympy [F] 2421
3.330.7 Maxima [A] (verification not implemented) 2421
3.330.8 Giac [A] (verification not implemented) 2421
3.330.9 Mupad [B] (verification not implemented) 2422

3.330.1 Optimal result

Integrand size = 23, antiderivative size = 95

$$\int (a+b\sec^2(e+fx))^2 \tan^6(e+fx)\,dx = -a^2x + \frac{a^2\tan(e+fx)}{f} - \frac{a^2\tan^3(e+fx)}{3f} + \frac{a^2\tan^5(e+fx)}{5f} + \frac{b(2a+b)\tan^7(e+fx)}{7f} + \frac{b^2\tan^9(e+fx)}{9f}$$

output
```
-a^2*x+a^2*tan(f*x+e)/f-1/3*a^2*tan(f*x+e)^3/f+1/5*a^2*tan(f*x+e)^5/f+1/7*
b*(2*a+b)*tan(f*x+e)^7/f+1/9*b^2*tan(f*x+e)^9/f
```

3.330.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 275 vs. 2(95) = 190.

Time = 3.71 (sec) , antiderivative size = 275, normalized size of antiderivative = 2.89

$$\int (a+b\sec^2(e+fx))^2 \tan^6(e+fx)\,dx =$$
$$-\frac{4(b+a\cos^2(e+fx))^2\sec^9(e+fx)\left(315a^2fx\cos^9(e+fx) - 35b^2\sec(e)\sin(fx) - 5(18a-19b)b\cos^2(e\right.$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^6,x]
```

output
```
(-4*(b + a*Cos[e + f*x]^2)^2*Sec[e + f*x]^9*(315*a^2*f*x*Cos[e + f*x]^9 -
35*b^2*Sec[e]*Sin[f*x] - 5*(18*a - 19*b)*b*Cos[e + f*x]^2*Sec[e]*Sin[f*x]
- 3*(21*a^2 - 90*a*b + 25*b^2)*Cos[e + f*x]^4*Sec[e]*Sin[f*x] + (231*a^2 -
 270*a*b + 5*b^2)*Cos[e + f*x]^6*Sec[e]*Sin[f*x] - (483*a^2 - 90*a*b - 10*
b^2)*Cos[e + f*x]^8*Sec[e]*Sin[f*x] - 35*b^2*Cos[e + f*x]*Tan[e] - 5*(18*a
 - 19*b)*b*Cos[e + f*x]^3*Tan[e] - 3*(21*a^2 - 90*a*b + 25*b^2)*Cos[e + f*
x]^5*Tan[e] + (231*a^2 - 270*a*b + 5*b^2)*Cos[e + f*x]^7*Tan[e]))/(315*f*(
a + 2*b + a*Cos[2*(e + f*x)])^2)
```

3.330.3 Rubi [A] (verified)

Time $= 0.32$ (sec) , antiderivative size $= 90$, normalized size of antiderivative $= 0.95$, number of steps used $= 6$, number of rules used $= 5$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used $= \{3042, 4629, 2075, 364, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int \tan(e+fx)^6\left(a+b\sec(e+fx)^2\right)^2 dx$$

$$\downarrow 4629$$

$$\frac{\int \frac{\tan^6(e+fx)\left(a+b\left(\tan^2(e+fx)+1\right)\right)^2}{\tan^2(e+fx)+1} d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\tan^6(e+fx)\left(b\tan^2(e+fx)+a+b\right)^2}{\tan^2(e+fx)+1} d\tan(e+fx)}{f}$$

$$\downarrow 364$$

$$\frac{\int \left(b^2\tan^8(e+fx)+b(2a+b)\tan^6(e+fx)+a^2\tan^4(e+fx)-a^2\tan^2(e+fx)+a^2-\frac{a^2}{\tan^2(e+fx)+1}\right) d\tan(e+}{f}$$

$$\downarrow 2009$$

$$\frac{-a^2 \arctan(\tan(e+fx)) + \frac{1}{5}a^2 \tan^5(e+fx) - \frac{1}{3}a^2 \tan^3(e+fx) + a^2 \tan(e+fx) + \frac{1}{7}b(2a+b)\tan^7(e+fx) + \frac{1}{9}$$
$$f$$

input
```
Int[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^6,x]
```

output
```
(-(a^2*ArcTan[Tan[e + f*x]]) + a^2*Tan[e + f*x] - (a^2*Tan[e + f*x]^3)/3 +
 (a^2*Tan[e + f*x]^5)/5 + (b*(2*a + b)*Tan[e + f*x]^7)/7 + (b^2*Tan[e + f*
x]^9)/9)/f
```

3.330.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.330.4 Maple [A] (verified)

Time = 12.31 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.93

method	result
parts	$\frac{a^2\left(\frac{\tan(fx+e)^5}{5}-\frac{\tan(fx+e)^3}{3}+\tan(fx+e)-\arctan(\tan(fx+e))\right)}{f}+\frac{b^2\left(\frac{\tan(fx+e)^9}{9}+\frac{\tan(fx+e)^7}{7}\right)}{f}+\frac{2ab\tan(fx+e)^7}{7f}$
derivativedivides	$\frac{a^2\left(\frac{\tan(fx+e)^5}{5}-\frac{\tan(fx+e)^3}{3}+\tan(fx+e)-fx-e\right)+\frac{2ab\sin(fx+e)^7}{7\cos(fx+e)^7}+b^2\left(\frac{\sin(fx+e)^7}{9\cos(fx+e)^9}+\frac{2\sin(fx+e)^7}{63\cos(fx+e)^7}\right)}{f}$
default	$\frac{a^2\left(\frac{\tan(fx+e)^5}{5}-\frac{\tan(fx+e)^3}{3}+\tan(fx+e)-fx-e\right)+\frac{2ab\sin(fx+e)^7}{7\cos(fx+e)^7}+b^2\left(\frac{\sin(fx+e)^7}{9\cos(fx+e)^9}+\frac{2\sin(fx+e)^7}{63\cos(fx+e)^7}\right)}{f}$
risch	$-a^2x-\frac{2i\left(90ab-483a^2+10b^2-28350a^2e^{10i(fx+e)}+5040abe^{8i(fx+e)}+3780abe^{6i(fx+e)}+1980abe^{4i(fx+e)}+180abe\right.$

input
```
int((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^6,x,method=_RETURNVERBOSE)
```

output
```
a^2/f*(1/5*tan(f*x+e)^5-1/3*tan(f*x+e)^3+tan(f*x+e)-arctan(tan(f*x+e)))+b^
2/f*(1/9*tan(f*x+e)^9+1/7*tan(f*x+e)^7)+2/7*a*b/f*tan(f*x+e)^7
```

3.330.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.44

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^6(e+fx)\,dx =$$
$$-\frac{315\,a^2fx\cos(fx+e)^9-\left((483\,a^2-90\,ab-10\,b^2)\cos(fx+e)^8-(231\,a^2-270\,ab+5\,b^2)\cos(fx+e)\right.}{315\,f\cos(fx}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^6,x, algorithm="fricas")
```

output
```
-1/315*(315*a^2*f*x*cos(f*x + e)^9 - ((483*a^2 - 90*a*b - 10*b^2)*cos(f*x
+ e)^8 - (231*a^2 - 270*a*b + 5*b^2)*cos(f*x + e)^6 + 3*(21*a^2 - 90*a*b +
 25*b^2)*cos(f*x + e)^4 + 5*(18*a*b - 19*b^2)*cos(f*x + e)^2 + 35*b^2)*sin
(f*x + e))/(f*cos(f*x + e)^9)
```

3.330.6 Sympy [F]

$$\int \left(a+b\sec^2(e+fx)\right)^2 \tan^6(e+fx)\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)^2 \tan^6\left(e+fx\right)\,dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**2*tan(f*x+e)**6,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**2*tan(e + f*x)**6, x)
```

3.330.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.88

$$\int \left(a+b\sec^2(e+fx)\right)^2 \tan^6(e+fx)\,dx$$
$$= \frac{35\,b^2\tan\left(fx+e\right)^9 + 45\left(2\,ab+b^2\right)\tan\left(fx+e\right)^7 + 63\,a^2\tan\left(fx+e\right)^5 - 105\,a^2\tan\left(fx+e\right)^3 - 315\left(fx+\right.}{315\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^6,x, algorithm="maxima")
```

output
```
1/315*(35*b^2*tan(f*x + e)^9 + 45*(2*a*b + b^2)*tan(f*x + e)^7 + 63*a^2*ta
n(f*x + e)^5 - 105*a^2*tan(f*x + e)^3 - 315*(f*x + e)*a^2 + 315*a^2*tan(f*
x + e))/f
```

3.330.8 Giac [A] (verification not implemented)

Time = 2.52 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.96

$$\int \left(a+b\sec^2(e+fx)\right)^2 \tan^6(e+fx)\,dx$$
$$= \frac{35\,b^2\tan\left(fx+e\right)^9 + 90\,ab\tan\left(fx+e\right)^7 + 45\,b^2\tan\left(fx+e\right)^7 + 63\,a^2\tan\left(fx+e\right)^5 - 105\,a^2\tan\left(fx+e\right)}{315\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^6,x, algorithm="giac")
```

output
```
1/315*(35*b^2*tan(f*x + e)^9 + 90*a*b*tan(f*x + e)^7 + 45*b^2*tan(f*x + e)
^7 + 63*a^2*tan(f*x + e)^5 - 105*a^2*tan(f*x + e)^3 - 315*(f*x + e)*a^2 +
315*a^2*tan(f*x + e))/f
```

3.330.9 Mupad [B] (verification not implemented)

Time = 19.69 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.33

$$\int \left(a+b\sec^2(e+fx)\right)^2 \tan^6(e+fx)\,dx$$
$$= \frac{\tan(e+f\,x)\,\left((a+b)^2+b^2-2\,b\,(a+b)\right)-\tan(e+f\,x)^3\left(\frac{(a+b)^2}{3}+\frac{b^2}{3}-\frac{2\,b\,(a+b)}{3}\right)+\tan(e+f\,x)^5\left(\frac{(a+b)^2}{5}\right.}{f}$$

input
```
int(tan(e + f*x)^6*(a + b/cos(e + f*x)^2)^2,x)
```

output
```
(tan(e + f*x)*((a + b)^2 + b^2 - 2*b*(a + b)) - tan(e + f*x)^3*((a + b)^2/
3 + b^2/3 - (2*b*(a + b))/3) + tan(e + f*x)^5*((a + b)^2/5 + b^2/5 - (2*b*
(a + b))/5) - tan(e + f*x)^7*(b^2/7 - (2*b*(a + b))/7) + (b^2*tan(e + f*x)
^9)/9 - a^2*f*x)/f
```

3.331 $\int (a + b\sec^2(e+fx))^2 \tan^4(e+fx)\,dx$

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3.331.1 Optimal result

Integrand size = 23, antiderivative size = 77

$$\int (a + b\sec^2(e+fx))^2 \tan^4(e+fx)\,dx = a^2x - \frac{a^2\tan(e+fx)}{f} + \frac{a^2\tan^3(e+fx)}{3f} + \frac{b(2a+b)\tan^5(e+fx)}{5f} + \frac{b^2\tan^7(e+fx)}{7f}$$

output
```
a^2*x-a^2*tan(f*x+e)/f+1/3*a^2*tan(f*x+e)^3/f+1/5*b*(2*a+b)*tan(f*x+e)^5/f
+1/7*b^2*tan(f*x+e)^7/f
```

3.331.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 395 vs. 2(77) = 154.

Time = 2.30 (sec) , antiderivative size = 395, normalized size of antiderivative = 5.13

$$\int (a + b\sec^2(e+fx))^2 \tan^4(e+fx)\,dx$$
$$= \frac{\sec(e)\sec^7(e+fx)\,(3675a^2fx\cos(fx) + 3675a^2fx\cos(2e+fx) + 2205a^2fx\cos(2e+3fx) + 2205a^2fx\,c$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^4,x]
```

output
```
(Sec[e]*Sec[e + f*x]^7*(3675*a^2*f*x*Cos[f*x] + 3675*a^2*f*x*Cos[2*e + f*x
] + 2205*a^2*f*x*Cos[2*e + 3*f*x] + 2205*a^2*f*x*Cos[4*e + 3*f*x] + 735*a^
2*f*x*Cos[4*e + 5*f*x] + 735*a^2*f*x*Cos[6*e + 5*f*x] + 105*a^2*f*x*Cos[6*
e + 7*f*x] + 105*a^2*f*x*Cos[8*e + 7*f*x] - 5320*a^2*Sin[f*x] + 1680*a*b*S
in[f*x] + 840*b^2*Sin[f*x] + 4480*a^2*Sin[2*e + f*x] - 1260*a*b*Sin[2*e +
f*x] + 420*b^2*Sin[2*e + f*x] - 3780*a^2*Sin[2*e + 3*f*x] + 924*a*b*Sin[2*
e + 3*f*x] - 168*b^2*Sin[2*e + 3*f*x] + 2100*a^2*Sin[4*e + 3*f*x] - 840*a*
b*Sin[4*e + 3*f*x] - 420*b^2*Sin[4*e + 3*f*x] - 1540*a^2*Sin[4*e + 5*f*x]
+ 168*a*b*Sin[4*e + 5*f*x] + 84*b^2*Sin[4*e + 5*f*x] + 420*a^2*Sin[6*e + 5
*f*x] - 420*a*b*Sin[6*e + 5*f*x] - 280*a^2*Sin[6*e + 7*f*x] + 84*a*b*Sin[6
*e + 7*f*x] + 12*b^2*Sin[6*e + 7*f*x]))/(13440*f)
```

3.331.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4629, 2075, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int \tan(e+fx)^4\left(a+b\sec(e+fx)^2\right)^2 dx$$

$$\downarrow 4629$$

$$\frac{\int \frac{\tan^4(e+fx)\left(a+b\left(\tan^2(e+fx)+1\right)\right)^2}{\tan^2(e+fx)+1} d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\tan^4(e+fx)\left(b\tan^2(e+fx)+a+b\right)^2}{\tan^2(e+fx)+1} d\tan(e+fx)}{f}$$

$$\downarrow 364$$

$$\frac{\int \left(b^2\tan^6(e+fx)+b(2a+b)\tan^4(e+fx)+a^2\tan^2(e+fx)-a^2+\frac{a^2}{\tan^2(e+fx)+1}\right) d\tan(e+fx)}{f}$$

$$\downarrow 2009$$

$$\frac{a^2 \arctan(\tan(e+fx)) + \frac{1}{3}a^2 \tan^3(e+fx) - a^2 \tan(e+fx) + \frac{1}{5}b(2a+b)\tan^5(e+fx) + \frac{1}{7}b^2 \tan^7(e+fx)}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^4,x]
```

output
```
(a^2*ArcTan[Tan[e + f*x]] - a^2*Tan[e + f*x] + (a^2*Tan[e + f*x]^3)/3 + (b
*(2*a + b)*Tan[e + f*x]^5)/5 + (b^2*Tan[e + f*x]^7)/7)/f
```

3.331.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.331.4 Maple [A] (verified)

Time = 6.53 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01

method	result
parts	$\frac{a^2\left(\frac{\tan(fx+e)^3}{3}-\tan(fx+e)+\arctan(\tan(fx+e))\right)}{f}+\frac{b^2\left(\frac{\tan(fx+e)^7}{7}+\frac{\tan(fx+e)^5}{5}\right)}{f}+\frac{2ab\tan(fx+e)^5}{5f}$
derivativedivides	$\frac{a^2\left(\frac{\tan(fx+e)^3}{3}-\tan(fx+e)+fx+e\right)+\frac{2ab\sin(fx+e)^5}{5\cos(fx+e)^5}+b^2\left(\frac{\sin(fx+e)^5}{7\cos(fx+e)^7}+\frac{2\sin(fx+e)^5}{35\cos(fx+e)^5}\right)}{f}$
default	$\frac{a^2\left(\frac{\tan(fx+e)^3}{3}-\tan(fx+e)+fx+e\right)+\frac{2ab\sin(fx+e)^5}{5\cos(fx+e)^5}+b^2\left(\frac{\sin(fx+e)^5}{7\cos(fx+e)^7}+\frac{2\sin(fx+e)^5}{35\cos(fx+e)^5}\right)}{f}$
risch	$a^2x+\frac{4i\left(-105a^2e^{12i(fx+e)}+105ab\,e^{12i(fx+e)}-525a^2e^{10i(fx+e)}+210ab\,e^{10i(fx+e)}+105b^2e^{10i(fx+e)}-1120a^2e^{8i(fx+$

input
```
int((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^4,x,method=_RETURNVERBOSE)
```

output
```
a^2/f*(1/3*tan(f*x+e)^3-tan(f*x+e)+arctan(tan(f*x+e)))+b^2/f*(1/7*tan(f*x+
e)^7+1/5*tan(f*x+e)^5)+2/5*a*b/f*tan(f*x+e)^5
```

3.331.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.47

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^4(e+fx)\,dx$$
$$=\frac{105\,a^2fx\cos\left(fx+e\right)^7-\left(2\left(70\,a^2-21\,ab-3\,b^2\right)\cos\left(fx+e\right)^6-\left(35\,a^2-84\,ab+3\,b^2\right)\cos\left(fx+e\right)^4-6\right.}{105\,f\cos\left(fx+e\right)^7}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^4,x, algorithm="fricas")
```

output
```
1/105*(105*a^2*f*x*cos(f*x + e)^7 - (2*(70*a^2 - 21*a*b - 3*b^2)*cos(f*x +
 e)^6 - (35*a^2 - 84*a*b + 3*b^2)*cos(f*x + e)^4 - 6*(7*a*b - 4*b^2)*cos(f
*x + e)^2 - 15*b^2)*sin(f*x + e))/(f*cos(f*x + e)^7)
```

3.331.6 Sympy [F]

$$\int \left(a + b\sec^2(e+fx)\right)^2 \tan^4(e+fx)\,dx = \int \left(a + b\sec^2(e+fx)\right)^2 \tan^4(e+fx)\,dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**2*tan(f*x+e)**4,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**2*tan(e + f*x)**4, x)
```

3.331.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.92

$$\int \left(a + b\sec^2(e+fx)\right)^2 \tan^4(e+fx)\,dx$$
$$= \frac{15\,b^2\tan(fx+e)^7 + 21\,(2\,ab+b^2)\tan(fx+e)^5 + 35\,a^2\tan(fx+e)^3 + 105\,(fx+e)a^2 - 105\,a^2\tan(fx}{105\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^4,x, algorithm="maxima")
```

output
```
1/105*(15*b^2*tan(f*x + e)^7 + 21*(2*a*b + b^2)*tan(f*x + e)^5 + 35*a^2*ta
n(f*x + e)^3 + 105*(f*x + e)*a^2 - 105*a^2*tan(f*x + e))/f
```

3.331.8 Giac [A] (verification not implemented)

Time = 1.20 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01

$$\int \left(a + b\sec^2(e+fx)\right)^2 \tan^4(e+fx)\,dx$$
$$= \frac{15\,b^2\tan(fx+e)^7 + 42\,ab\tan(fx+e)^5 + 21\,b^2\tan(fx+e)^5 + 35\,a^2\tan(fx+e)^3 + 105\,(fx+e)a^2 - 1}{105\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^4,x, algorithm="giac")
```

output
```
1/105*(15*b^2*tan(f*x + e)^7 + 42*a*b*tan(f*x + e)^5 + 21*b^2*tan(f*x + e)
^5 + 35*a^2*tan(f*x + e)^3 + 105*(f*x + e)*a^2 - 105*a^2*tan(f*x + e))/f
```

3.331.9 Mupad [B] (verification not implemented)

Time = 19.43 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.26

$$\int \left(a+b\sec^2(e+fx)\right)^2 \tan^4(e+fx)\,dx$$
$$= \frac{\tan(e+fx)^3\left(\frac{(a+b)^2}{3}+\frac{b^2}{3}-\frac{2b(a+b)}{3}\right)-\tan(e+fx)\left((a+b)^2+b^2-2b(a+b)\right)-\tan(e+fx)^5\left(\frac{b^2}{5}-\frac{2}{}\right.}{f}$$

input
```
int(tan(e + f*x)^4*(a + b/cos(e + f*x)^2)^2,x)
```

output
```
(tan(e + f*x)^3*((a + b)^2/3 + b^2/3 - (2*b*(a + b))/3) - tan(e + f*x)*((a
 + b)^2 + b^2 - 2*b*(a + b)) - tan(e + f*x)^5*(b^2/5 - (2*b*(a + b))/5) +
(b^2*tan(e + f*x)^7)/7 + a^2*f*x)/f
```

3.332 $\int (a + b\sec^2(e+fx))^2 \tan^2(e+fx)\,dx$

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3.332.2 Mathematica [B] (verified) . . . 2429
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3.332.7 Maxima [A] (verification not implemented) . . . 2433
3.332.8 Giac [A] (verification not implemented) . . . 2433
3.332.9 Mupad [B] (verification not implemented) . . . 2434

3.332.1 Optimal result

Integrand size = 23, antiderivative size = 59

$$\int (a + b\sec^2(e+fx))^2 \tan^2(e+fx)\,dx = -a^2x + \frac{a^2\tan(e+fx)}{f} + \frac{b(2a+b)\tan^3(e+fx)}{3f} + \frac{b^2\tan^5(e+fx)}{5f}$$

output
```
-a^2*x+a^2*tan(f*x+e)/f+1/3*b*(2*a+b)*tan(f*x+e)^3/f+1/5*b^2*tan(f*x+e)^5/
f
```

3.332.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 281 vs. 2(59) = 118.

Time = 1.71 (sec) , antiderivative size = 281, normalized size of antiderivative = 4.76

$$\int (a + b\sec^2(e+fx))^2 \tan^2(e+fx)\,dx = -\frac{\sec(e)\sec^5(e+fx)\,(150a^2fx\cos(fx) + 150a^2fx\cos(2e+fx) + 75a^2fx\cos(2e+3fx) + 75a^2fx\cos(4$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^2,x]
```

output
```
-1/480*(Sec[e]*Sec[e + f*x]^5*(150*a^2*f*x*Cos[f*x] + 150*a^2*f*x*Cos[2*e
+ f*x] + 75*a^2*f*x*Cos[2*e + 3*f*x] + 75*a^2*f*x*Cos[4*e + 3*f*x] + 15*a^
2*f*x*Cos[4*e + 5*f*x] + 15*a^2*f*x*Cos[6*e + 5*f*x] - 180*a^2*Sin[f*x] +
80*a*b*Sin[f*x] - 20*b^2*Sin[f*x] + 120*a^2*Sin[2*e + f*x] - 120*a*b*Sin[2
*e + f*x] - 60*b^2*Sin[2*e + f*x] - 120*a^2*Sin[2*e + 3*f*x] + 40*a*b*Sin[
2*e + 3*f*x] + 20*b^2*Sin[2*e + 3*f*x] + 30*a^2*Sin[4*e + 3*f*x] - 60*a*b*
Sin[4*e + 3*f*x] - 30*a^2*Sin[4*e + 5*f*x] + 20*a*b*Sin[4*e + 5*f*x] + 4*b
^2*Sin[4*e + 5*f*x]))/f
```

3.332.3 Rubi [A] (verified)

Time $= 0.30$ (sec) , antiderivative size $= 60$, normalized size of antiderivative $= 1.02$, number of steps used $= 6$, number of rules used $= 5$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used $= \{3042, 4629, 2075, 364, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int \tan(e+fx)^2\left(a+b\sec(e+fx)^2\right)^2 dx$$

$$\downarrow 4629$$

$$\frac{\int \frac{\tan^2(e+fx)\left(a+b\left(\tan^2(e+fx)+1\right)\right)^2}{\tan^2(e+fx)+1} d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\tan^2(e+fx)\left(b\tan^2(e+fx)+a+b\right)^2}{\tan^2(e+fx)+1} d\tan(e+fx)}{f}$$

$$\downarrow 364$$

$$\frac{\int \left(b^2\tan^4(e+fx)+b(2a+b)\tan^2(e+fx)+a^2-\frac{a^2}{\tan^2(e+fx)+1}\right) d\tan(e+fx)}{f}$$

$$\downarrow 2009$$

$$\frac{-a^2\arctan(\tan(e+fx))+a^2\tan(e+fx)+\frac{1}{3}b(2a+b)\tan^3(e+fx)+\frac{1}{5}b^2\tan^5(e+fx)}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^2*Tan[e + f*x]^2,x]
```

output
```
(-(a^2*ArcTan[Tan[e + f*x]]) + a^2*Tan[e + f*x] + (b*(2*a + b)*Tan[e + f*x
]^3)/3 + (b^2*Tan[e + f*x]^5)/5)/f
```

3.332.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.332.4 Maple [A] (verified)

Time = 3.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.15

method	result
parts	$\frac{a^2(\tan(fx+e)-\arctan(\tan(fx+e)))}{f}+\frac{b^2\left(\frac{\tan(fx+e)^5}{5}+\frac{\tan(fx+e)^3}{3}\right)}{f}+\frac{2ab\tan(fx+e)^3}{3f}$
derivativedivides	$\frac{a^2(\tan(fx+e)-fx-e)+\frac{2ab\sin(fx+e)^3}{3\cos(fx+e)^3}+b^2\left(\frac{\sin(fx+e)^3}{5\cos(fx+e)^5}+\frac{2\sin(fx+e)^3}{15\cos(fx+e)^3}\right)}{f}$
default	$\frac{a^2(\tan(fx+e)-fx-e)+\frac{2ab\sin(fx+e)^3}{3\cos(fx+e)^3}+b^2\left(\frac{\sin(fx+e)^3}{5\cos(fx+e)^5}+\frac{2\sin(fx+e)^3}{15\cos(fx+e)^3}\right)}{f}$
risch	$-a^2x-\frac{2i\left(-15a^2e^{8i(fx+e)}+30abe^{8i(fx+e)}-60a^2e^{6i(fx+e)}+60abe^{6i(fx+e)}+30b^2e^{6i(fx+e)}-90a^2e^{4i(fx+e)}+40abe\right.}{15f\left(e^{2i(fx+e)}+1\right.}$

input
```
int((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2/f*(tan(f*x+e)-arctan(tan(f*x+e)))+b^2/f*(1/5*tan(f*x+e)^5+1/3*tan(f*x+
e)^3)+2/3*a*b/f*tan(f*x+e)^3
```

3.332.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.46

$$\int \left(a+b\sec^2(e+fx)\right)^2\tan^2(e+fx)\,dx =$$
$$-\frac{15\,a^2fx\cos\left(fx+e\right)^5-\left(\left(15\,a^2-10\,ab-2\,b^2\right)\cos\left(fx+e\right)^4+\left(10\,ab-b^2\right)\cos\left(fx+e\right)^2+3\,b^2\right)\sin\left(fx\right.}{15\,f\cos\left(fx+e\right)^5}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^2,x, algorithm="fricas")
```

output
```
-1/15*(15*a^2*f*x*cos(f*x + e)^5 - ((15*a^2 - 10*a*b - 2*b^2)*cos(f*x + e)
^4 + (10*a*b - b^2)*cos(f*x + e)^2 + 3*b^2)*sin(f*x + e))/(f*cos(f*x + e)^
5)
```

3.332.6 Sympy [F]

$$\int \left(a+b\sec^2(e+fx)\right)^2 \tan^2(e+fx)\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)^2 \tan^2\left(e+fx\right)\,dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**2*tan(f*x+e)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**2*tan(e + f*x)**2, x)
```

3.332.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.98

$$\int \left(a+b\sec^2(e+fx)\right)^2 \tan^2(e+fx)\,dx$$
$$= \frac{3\,b^2\tan\left(fx+e\right)^5 + 5\left(2\,ab+b^2\right)\tan\left(fx+e\right)^3 - 15\left(fx+e\right)a^2 + 15\,a^2\tan\left(fx+e\right)}{15\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^2,x, algorithm="maxima")
```

output
```
1/15*(3*b^2*tan(f*x + e)^5 + 5*(2*a*b + b^2)*tan(f*x + e)^3 - 15*(f*x + e)
*a^2 + 15*a^2*tan(f*x + e))/f
```

3.332.8 Giac [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.10

$$\int \left(a+b\sec^2(e+fx)\right)^2 \tan^2(e+fx)\,dx$$
$$= \frac{3\,b^2\tan\left(fx+e\right)^5 + 10\,ab\tan\left(fx+e\right)^3 + 5\,b^2\tan\left(fx+e\right)^3 - 15\left(fx+e\right)a^2 + 15\,a^2\tan\left(fx+e\right)}{15\,f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^2,x, algorithm="giac")
```

output
```
1/15*(3*b^2*tan(f*x + e)^5 + 10*a*b*tan(f*x + e)^3 + 5*b^2*tan(f*x + e)^3
- 15*(f*x + e)*a^2 + 15*a^2*tan(f*x + e))/f
```

3.332.9 Mupad [B] (verification not implemented)

Time = 19.41 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.17

$$\int \left(a+b\sec^2(e+fx)\right)^2 \tan^2(e+fx)\,dx$$
$$= \frac{\tan(e+f\,x)\,\left((a+b)^2+b^2-2\,b\,(a+b)\right)-\tan(e+f\,x)^3\left(\frac{b^2}{3}-\frac{2\,b\,(a+b)}{3}\right)+\frac{b^2\tan(e+f\,x)^5}{5}-a^2\,f\,x}{f}$$

input
```
int(tan(e + f*x)^2*(a + b/cos(e + f*x)^2)^2,x)
```

output
```
(tan(e + f*x)*((a + b)^2 + b^2 - 2*b*(a + b)) - tan(e + f*x)^3*(b^2/3 - (2
*b*(a + b))/3) + (b^2*tan(e + f*x)^5)/5 - a^2*f*x)/f
```

3.333 $\int (a + b\sec^2(e + fx))^2\, dx$

3.333.1 Optimal result . . . 2435
3.333.2 Mathematica [A] (verified) . . . 2435
3.333.3 Rubi [A] (verified) . . . 2436
3.333.4 Maple [A] (verified) . . . 2437
3.333.5 Fricas [A] (verification not implemented) . . . 2438
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3.333.7 Maxima [A] (verification not implemented) . . . 2438
3.333.8 Giac [A] (verification not implemented) . . . 2439
3.333.9 Mupad [B] (verification not implemented) . . . 2439

3.333.1 Optimal result

Integrand size = 14, antiderivative size = 40

$$\int (a + b\sec^2(e + fx))^2\, dx = a^2 x + \frac{b(2a + b)\tan(e + fx)}{f} + \frac{b^2\tan^3(e + fx)}{3f}$$

output
```
a^2*x+b*(2*a+b)*tan(f*x+e)/f+1/3*b^2*tan(f*x+e)^3/f
```

3.333.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int (a + b\sec^2(e + fx))^2\, dx = \frac{3a^2 fx + 3b(2a + b)\tan(e + fx) + b^2\tan^3(e + fx)}{3f}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(3*a^2*f*x + 3*b*(2*a + b)*Tan[e + f*x] + b^2*Tan[e + f*x]^3)/(3*f)
```

3.333.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4616, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + b\sec^2(e+fx))^2\,dx$$

$$\downarrow 3042$$

$$\int (a + b\sec(e+fx)^2)^2\,dx$$

$$\downarrow 4616$$

$$\frac{\int \frac{(b\tan^2(e+fx)+a+b)^2}{\tan^2(e+fx)+1} d\tan(e+fx)}{f}$$

$$\downarrow 300$$

$$\frac{\int \left(\frac{a^2}{\tan^2(e+fx)+1} + b^2\tan^2(e+fx) + b(2a+b)\right) d\tan(e+fx)}{f}$$

$$\downarrow 2009$$

$$\frac{a^2\arctan(\tan(e+fx)) + b(2a+b)\tan(e+fx) + \frac{1}{3}b^2\tan^3(e+fx)}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(a^2*ArcTan[Tan[e + f*x]] + b*(2*a + b)*Tan[e + f*x] + (b^2*Tan[e + f*x]^3
)/3)/f
```

3.333.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f  Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

3.333.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

method	result
parts	$a^2x-\frac{b^2\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)}{f}+\frac{2ab\tan(fx+e)}{f}$
derivativedivides	$\frac{a^2(fx+e)+2ab\tan(fx+e)-b^2\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)}{f}$
default	$\frac{a^2(fx+e)+2ab\tan(fx+e)-b^2\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)}{f}$
risch	$a^2x+\frac{4ib\left(3a\,\mathrm{e}^{4i(fx+e)}+6a\,\mathrm{e}^{2i(fx+e)}+3b\,\mathrm{e}^{2i(fx+e)}+3a+b\right)}{3f\left(\mathrm{e}^{2i(fx+e)}+1\right)^3}$
norman	$\frac{a^2x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6-a^2x+3a^2x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-3a^2x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4-\frac{2b(2a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{2b(2a+b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{f}+\frac{4b(6a}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3}$
parallelrisch	$\frac{3x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6a^2f+(-12ab-6b^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5-9x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4a^2f+(24ab+4b^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3+9x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2$

input
```
int((a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x-b^2/f*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+2*a*b/f*tan(f*x+e)
```

3.333.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.45

$$\int \left(a + b\sec^2(e+fx)\right)^2 dx$$
$$= \frac{3\,a^2 f x \cos\left(fx+e\right)^3 + \left(2\left(3\,ab+b^2\right)\cos\left(fx+e\right)^2 + b^2\right)\sin\left(fx+e\right)}{3\,f\cos\left(fx+e\right)^3}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/3*(3*a^2*f*x*cos(f*x + e)^3 + (2*(3*a*b + b^2)*cos(f*x + e)^2 + b^2)*sin
(f*x + e))/(f*cos(f*x + e)^3)
```

3.333.6 Sympy [F]

$$\int \left(a + b\sec^2(e+fx)\right)^2 dx = \int \left(a + b\sec^2\left(e+fx\right)\right)^2 dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**2, x)
```

3.333.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10

$$\int \left(a + b\sec^2(e+fx)\right)^2 dx = a^2 x + \frac{\left(\tan\left(fx+e\right)^3 + 3\,\tan\left(fx+e\right)\right)b^2}{3\,f} + \frac{2\,ab\tan\left(fx+e\right)}{f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
a^2*x + 1/3*(tan(f*x + e)^3 + 3*tan(f*x + e))*b^2/f + 2*a*b*tan(f*x + e)/f
```

3.333.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.22

$$\int \left(a + b\sec^2(e+fx)\right)^2 dx = \frac{b^2\tan(fx+e)^3 + 3(fx+e)a^2 + 6ab\tan(fx+e) + 3b^2\tan(fx+e)}{3f}$$

input
```
integrate((a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/3*(b^2*tan(f*x + e)^3 + 3*(f*x + e)*a^2 + 6*a*b*tan(f*x + e) + 3*b^2*tan
(f*x + e))/f
```

3.333.9 Mupad [B] (verification not implemented)

Time = 19.57 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \left(a + b\sec^2(e+fx)\right)^2 dx = \frac{\frac{b^2\tan(e+fx)^3}{3} - \tan(e+fx)\left(b^2 - 2b(a+b)\right) + a^2 fx}{f}$$

input
```
int((a + b/cos(e + f*x)^2)^2,x)
```

output
```
((b^2*tan(e + f*x)^3)/3 - tan(e + f*x)*(b^2 - 2*b*(a + b)) + a^2*f*x)/f
```

3.334 $\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

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3.334.2 Mathematica [B] (verified) . . . 2440
3.334.3 Rubi [A] (verified) . . . 2441
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3.334.5 Fricas [A] (verification not implemented) . . . 2443
3.334.6 Sympy [F] . . . 2443
3.334.7 Maxima [A] (verification not implemented) . . . 2443
3.334.8 Giac [A] (verification not implemented) . . . 2444
3.334.9 Mupad [B] (verification not implemented) . . . 2444

3.334.1 Optimal result

Integrand size = 23, antiderivative size = 36

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -a^2x - \frac{(a+b)^2\cot(e+fx)}{f} + \frac{b^2\tan(e+fx)}{f}$$

output
```
-a^2*x-(a+b)^2*cot(f*x+e)/f+b^2*tan(f*x+e)/f
```

3.334.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 82 vs. 2(36) = 72.

Time = 3.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.28

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{4(b+a\cos^2(e+fx))^2\sec(e+fx)\left(a^2fx\cos(e+fx)-\left((a+b)^2\cot(e+fx)\csc(e)+b^2\sec(e)\right)\sin(fx\right.}{f(a+2b+a\cos(2(e+fx)))^2}$$

input
```
Integrate[Cot[e + f*x]^2*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(-4*(b + a*Cos[e + f*x]^2)^2*Sec[e + f*x]*(a^2*f*x*Cos[e + f*x] - ((a + b)
^2*Cot[e + f*x]*Csc[e] + b^2*Sec[e])*Sin[f*x]))/(f*(a + 2*b + a*Cos[2*(e +
 f*x)])^2)
```

3.334.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4629, 2075, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^2}{\tan(e+fx)^2} dx$$

$$\downarrow 4629$$

$$\frac{\int \frac{\cot^2(e+fx)\left(a+b\left(\tan^2(e+fx)+1\right)\right)^2}{\tan^2(e+fx)+1} d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\cot^2(e+fx)\left(b\tan^2(e+fx)+a+b\right)^2}{\tan^2(e+fx)+1} d\tan(e+fx)}{f}$$

$$\downarrow 364$$

$$\frac{\int \left(-\frac{a^2}{\tan^2(e+fx)+1}+b^2+(a+b)^2\cot^2(e+fx)\right) d\tan(e+fx)}{f}$$

$$\downarrow 2009$$

$$\frac{a^2(-\arctan(\tan(e+fx)))-(a+b)^2\cot(e+fx)+b^2\tan(e+fx)}{f}$$

input
```
Int[Cot[e + f*x]^2*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(-(a^2*ArcTan[Tan[e + f*x]]) - (a + b)^2*Cot[e + f*x] + b^2*Tan[e + f*x])/
f
```

3.334.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.334.4 Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.83

method	result	size
derivativedivides	$\frac{a^2(-\cot(fx+e)-fx-e)-2ab\cot(fx+e)+b^2\left(\frac{1}{\sin(fx+e)\cos(fx+e)}-2\cot(fx+e)\right)}{f}$	66
default	$\frac{a^2(-\cot(fx+e)-fx-e)-2ab\cot(fx+e)+b^2\left(\frac{1}{\sin(fx+e)\cos(fx+e)}-2\cot(fx+e)\right)}{f}$	66
risch	$-a^2x-\frac{2i\left(a^2e^{2i(fx+e)}+2ab\,e^{2i(fx+e)}+a^2+2ab+2b^2\right)}{f\left(e^{2i(fx+e)}-1\right)\left(e^{2i(fx+e)}+1\right)}$	79

input
```
int(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(a^2*(-cot(f*x+e)-f*x-e)-2*a*b*cot(f*x+e)+b^2*(1/sin(f*x+e)/cos(f*x+e)
-2*cot(f*x+e)))
```

3.334.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.86

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$= -\frac{a^2 fx\cos(fx+e)\sin(fx+e)+(a^2+2ab+2b^2)\cos(fx+e)^2-b^2}{f\cos(fx+e)\sin(fx+e)}$$

input
```
integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
-(a^2*f*x*cos(f*x + e)*sin(f*x + e) + (a^2 + 2*a*b + 2*b^2)*cos(f*x + e)^2
 - b^2)/(f*cos(f*x + e)*sin(f*x + e))
```

3.334.6 Sympy [F]

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \int \left(a+b\sec^2(e+fx)\right)^2\cot^2(e+fx)\,dx$$

input
```
integrate(cot(f*x+e)**2*(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**2*cot(e + f*x)**2, x)
```

3.334.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.28

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{(fx+e)a^2-b^2\tan(fx+e)+\frac{a^2+2ab+b^2}{\tan(fx+e)}}{f}$$

input
```
integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-((f*x + e)*a^2 - b^2*tan(f*x + e) + (a^2 + 2*a*b + b^2)/tan(f*x + e))/f
```

3.334.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.28

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{(fx+e)a^2 - b^2\tan(fx+e) + \frac{a^2+2ab+b^2}{\tan(fx+e)}}{f}$$

input
```
integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-((f*x + e)*a^2 - b^2*tan(f*x + e) + (a^2 + 2*a*b + b^2)/tan(f*x + e))/f
```

3.334.9 Mupad [B] (verification not implemented)

Time = 19.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{b^2\tan(e+f\,x)}{f} - a^2\,x - \frac{a^2+2\,a\,b+b^2}{f\tan(e+f\,x)}$$

input
```
int(cot(e + f*x)^2*(a + b/cos(e + f*x)^2)^2,x)
```

output
```
(b^2*tan(e + f*x))/f - a^2*x - (2*a*b + a^2 + b^2)/(f*tan(e + f*x))
```

3.335 $\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

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3.335.1 Optimal result

Integrand size = 23, antiderivative size = 45

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = a^2x+\frac{(a^2-b^2)\cot(e+fx)}{f}-\frac{(a+b)^2\cot^3(e+fx)}{3f}$$

output
```
a^2*x+(a^2-b^2)*cot(f*x+e)/f-1/3*(a+b)^2*cot(f*x+e)^3/f
```

3.335.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(45) = 90.

Time = 1.57 (sec) , antiderivative size = 160, normalized size of antiderivative = 3.56

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$=\frac{\csc(e)\csc^3(e+fx)\left(9a^2fx\cos(fx)-9a^2fx\cos(2e+fx)-3a^2fx\cos(2e+3fx)+3a^2fx\cos(4e+3fx)+\right.}{}$$

input
```
Integrate[Cot[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(Csc[e]*Csc[e + f*x]^3*(9*a^2*f*x*Cos[f*x] - 9*a^2*f*x*Cos[2*e + f*x] - 3*
a^2*f*x*Cos[2*e + 3*f*x] + 3*a^2*f*x*Cos[4*e + 3*f*x] - 12*a^2*Sin[f*x] +
12*b^2*Sin[f*x] - 12*a^2*Sin[2*e + f*x] - 12*a*b*Sin[2*e + f*x] + 8*a^2*Si
n[2*e + 3*f*x] + 4*a*b*Sin[2*e + 3*f*x] - 4*b^2*Sin[2*e + 3*f*x]))/(24*f)
```

3.335.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4629, 2075, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^2}{\tan(e+fx)^4} dx$$

$$\downarrow 4629$$

$$\frac{\int \frac{\cot^4(e+fx)\left(a+b\left(\tan^2(e+fx)+1\right)\right)^2}{\tan^2(e+fx)+1} d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\cot^4(e+fx)\left(b\tan^2(e+fx)+a+b\right)^2}{\tan^2(e+fx)+1} d\tan(e+fx)}{f}$$

$$\downarrow 364$$

$$\frac{\int \left((a+b)^2\cot^4(e+fx)+\left(b^2-a^2\right)\cot^2(e+fx)+\frac{a^2}{\tan^2(e+fx)+1}\right) d\tan(e+fx)}{f}$$

$$\downarrow 2009$$

$$\frac{a^2\arctan(\tan(e+fx))+\left(a^2-b^2\right)\cot(e+fx)-\frac{1}{3}(a+b)^2\cot^3(e+fx)}{f}$$

input
```
Int[Cot[e + f*x]^4*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(a^2*ArcTan[Tan[e + f*x]] + (a^2 - b^2)*Cot[e + f*x] - ((a + b)^2*Cot[e +
f*x]^3)/3)/f
```

3.335.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.335.4 Maple [A] (verified)

Time = 3.91 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.62

method	result	size
derivativedivides	$\frac{a^2\left(-\frac{\cot(fx+e)^3}{3}+\cot(fx+e)+fx+e\right)-\frac{2ab\cos(fx+e)^3}{3\sin(fx+e)^3}+b^2\left(-\frac{2}{3}-\frac{\csc(fx+e)^2}{3}\right)\cot(fx+e)}{f}$	73
default	$\frac{a^2\left(-\frac{\cot(fx+e)^3}{3}+\cot(fx+e)+fx+e\right)-\frac{2ab\cos(fx+e)^3}{3\sin(fx+e)^3}+b^2\left(-\frac{2}{3}-\frac{\csc(fx+e)^2}{3}\right)\cot(fx+e)}{f}$	73
risch	$a^2x+\frac{4i\left(3a^2e^{4i(fx+e)}+3ab\,e^{4i(fx+e)}-3a^2e^{2i(fx+e)}+3b^2e^{2i(fx+e)}+2a^2+ab-b^2\right)}{3f\left(e^{2i(fx+e)}-1\right)^3}$	95

input
```
int(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(a^2*(-1/3*cot(f*x+e)^3+cot(f*x+e)+f*x+e)-2/3*a*b/sin(f*x+e)^3*cos(f*x
+e)^3+b^2*(-2/3-1/3*csc(f*x+e)^2)*cot(f*x+e))
```

3.335.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(43) = 86$.

Time $= 0.25$ (sec) , antiderivative size $= 98$, normalized size of antiderivative $= 2.18$

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$= \frac{2\left(2a^2+ab-b^2\right)\cos\left(fx+e\right)^3 - 3\left(a^2-b^2\right)\cos\left(fx+e\right) + 3\left(a^2 fx\cos\left(fx+e\right)^2 - a^2 fx\right)\sin\left(fx+e\right)}{3\left(f\cos\left(fx+e\right)^2 - f\right)\sin\left(fx+e\right)}$$

input
```
integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/3*(2*(2*a^2 + a*b - b^2)*cos(f*x + e)^3 - 3*(a^2 - b^2)*cos(f*x + e) + 3
*(a^2*f*x*cos(f*x + e)^2 - a^2*f*x)*sin(f*x + e))/((f*cos(f*x + e)^2 - f)*
sin(f*x + e))
```

3.335.6 Sympy [F]

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)^2\cot^4\left(e+fx\right)dx$$

input
```
integrate(cot(f*x+e)**4*(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**2*cot(e + f*x)**4, x)
```

3.335.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.31

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \frac{3\,(fx+e)a^2+\frac{3\,(a^2-b^2)\tan(fx+e)^2-a^2-2\,ab-b^2}{\tan(fx+e)^3}}{3\,f}$$

input
```
integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/3*(3*(f*x + e)*a^2 + (3*(a^2 - b^2)*tan(f*x + e)^2 - a^2 - 2*a*b - b^2)/
tan(f*x + e)^3)/f
```

3.335.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. 2(43) = 86.

Time = 0.32 (sec) , antiderivative size = 176, normalized size of antiderivative = 3.91

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$= \frac{a^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+2\,ab\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+b^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+24\,(fx+e)a^2-15\,a^2\tan\left(\frac{1}{2}fx+\frac{1}{2}\right.}{}$$

input
```
integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/24*(a^2*tan(1/2*f*x + 1/2*e)^3 + 2*a*b*tan(1/2*f*x + 1/2*e)^3 + b^2*tan(
1/2*f*x + 1/2*e)^3 + 24*(f*x + e)*a^2 - 15*a^2*tan(1/2*f*x + 1/2*e) - 6*a*
b*tan(1/2*f*x + 1/2*e) + 9*b^2*tan(1/2*f*x + 1/2*e) + (15*a^2*tan(1/2*f*x
+ 1/2*e)^2 + 6*a*b*tan(1/2*f*x + 1/2*e)^2 - 9*b^2*tan(1/2*f*x + 1/2*e)^2 -
 a^2 - 2*a*b - b^2)/tan(1/2*f*x + 1/2*e)^3)/f
```

3.335.9 Mupad [B] (verification not implemented)

Time = 19.97 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = a^2\,x - \frac{\frac{2\,a\,b}{3} - \tan(e+f\,x)^2\,(a^2-b^2) + \frac{a^2}{3} + \frac{b^2}{3}}{f\tan\left(e+f\,x\right)^3}$$

input
```
int(cot(e + f*x)^4*(a + b/cos(e + f*x)^2)^2,x)
```

output
```
a^2*x - ((2*a*b)/3 - tan(e + f*x)^2*(a^2 - b^2) + a^2/3 + b^2/3)/(f*tan(e
+ f*x)^3)
```

3.336 $\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$

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3.336.1 Optimal result

Integrand size = 23, antiderivative size = 65

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -a^2x - \frac{a^2\cot(e+fx)}{f} + \frac{(a^2-b^2)\cot^3(e+fx)}{3f} - \frac{(a+b)^2\cot^5(e+fx)}{5f}$$

output
```
-a^2*x-a^2*cot(f*x+e)/f+1/3*(a^2-b^2)*cot(f*x+e)^3/f-1/5*(a+b)^2*cot(f*x+e
)^5/f
```

3.336.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 256 vs. 2(65) = 130.

Time = 3.89 (sec) , antiderivative size = 256, normalized size of antiderivative = 3.94

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$= \frac{\csc(e)\csc^5(e+fx)\left(-150a^2fx\cos(fx)+150a^2fx\cos(2e+fx)+75a^2fx\cos(2e+3fx)-75a^2fx\cos(4e\right.}{}$$

input
```
Integrate[Cot[e + f*x]^6*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(Csc[e]*Csc[e + f*x]^5*(-150*a^2*f*x*Cos[f*x] + 150*a^2*f*x*Cos[2*e + f*x]
 + 75*a^2*f*x*Cos[2*e + 3*f*x] - 75*a^2*f*x*Cos[4*e + 3*f*x] - 15*a^2*f*x*
Cos[4*e + 5*f*x] + 15*a^2*f*x*Cos[6*e + 5*f*x] + 280*a^2*Sin[f*x] + 120*a*
b*Sin[f*x] + 20*b^2*Sin[f*x] + 180*a^2*Sin[2*e + f*x] - 60*b^2*Sin[2*e + f
*x] - 140*a^2*Sin[2*e + 3*f*x] + 20*b^2*Sin[2*e + 3*f*x] - 90*a^2*Sin[4*e
 + 3*f*x] - 60*a*b*Sin[4*e + 3*f*x] + 46*a^2*Sin[4*e + 5*f*x] + 12*a*b*Sin[
4*e + 5*f*x] - 4*b^2*Sin[4*e + 5*f*x]))/(480*f)
```

3.336.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4629, 2075, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^2}{\tan(e+fx)^6}dx$$

$$\downarrow 4629$$

$$\frac{\int \frac{\cot^6(e+fx)(a+b(\tan^2(e+fx)+1))^2}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\cot^6(e+fx)(b\tan^2(e+fx)+a+b)^2}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\downarrow 364$$

$$\frac{\int \left((a+b)^2\cot^6(e+fx)+\left(b^2-a^2\right)\cot^4(e+fx)+a^2\cot^2(e+fx)-\frac{a^2}{\tan^2(e+fx)+1}\right)d\tan(e+fx)}{f}$$

$$\downarrow 2009$$

$$\frac{-a^2\arctan(\tan(e+fx))+\frac{1}{3}\left(a^2-b^2\right)\cot^3(e+fx)-a^2\cot(e+fx)-\frac{1}{5}(a+b)^2\cot^5(e+fx)}{f}$$

input
```
Int[Cot[e + f*x]^6*(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(-(a^2*ArcTan[Tan[e + f*x]]) - a^2*Cot[e + f*x] + ((a^2 - b^2)*Cot[e + f*x
]^3)/3 - ((a + b)^2*Cot[e + f*x]^5)/5)/f
```

3.336.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.336.4 Maple [A] (verified)

Time = 7.66 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.65

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cot(fx+e)^5}{5}+\frac{\cot(fx+e)^3}{3}-\cot(fx+e)-fx-e\right)-\frac{2ab\cos(fx+e)^5}{5\sin(fx+e)^5}+b^2\left(-\frac{\cos(fx+e)^3}{5\sin(fx+e)^5}-\frac{2\cos(fx+e)^3}{15\sin(fx+e)^3}\right)}{f}$
default	$\frac{a^2\left(-\frac{\cot(fx+e)^5}{5}+\frac{\cot(fx+e)^3}{3}-\cot(fx+e)-fx-e\right)-\frac{2ab\cos(fx+e)^5}{5\sin(fx+e)^5}+b^2\left(-\frac{\cos(fx+e)^3}{5\sin(fx+e)^5}-\frac{2\cos(fx+e)^3}{15\sin(fx+e)^3}\right)}{f}$
risch	$-a^2x-\frac{2i\left(45a^2e^{8i(fx+e)}+30ab\,e^{8i(fx+e)}-90a^2e^{6i(fx+e)}+30b^2e^{6i(fx+e)}+140a^2e^{4i(fx+e)}+60ab\,e^{4i(fx+e)}+10b^2e^{4}\right.}{15f\left(e^{2i(fx+e)}-1\right)^5}$

input
```
int(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(a^2*(-1/5*cot(f*x+e)^5+1/3*cot(f*x+e)^3-cot(f*x+e)-f*x-e)-2/5*a*b/sin
(f*x+e)^5*cos(f*x+e)^5+b^2*(-1/5/sin(f*x+e)^5*cos(f*x+e)^3-2/15*cos(f*x+e)
^3/sin(f*x+e)^3))
```

3.336.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(61) = 122.

Time = 0.27 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.09

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx =$$
$$-\frac{(23\,a^2+6\,ab-2\,b^2)\cos(fx+e)^5-5\,(7\,a^2-b^2)\cos(fx+e)^3+15\,a^2\cos(fx+e)+15\,\left(a^2fx\cos(fx\right.}{15\left(f\cos(fx+e)^4-2\,f\cos(fx+e)^2+f\right)\sin(fx+e}$$

input
```
integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
-1/15*((23*a^2 + 6*a*b - 2*b^2)*cos(f*x + e)^5 - 5*(7*a^2 - b^2)*cos(f*x +
 e)^3 + 15*a^2*cos(f*x + e) + 15*(a^2*f*x*cos(f*x + e)^4 - 2*a^2*f*x*cos(f
*x + e)^2 + a^2*f*x)*sin(f*x + e))/((f*cos(f*x + e)^4 - 2*f*cos(f*x + e)^2
 + f)*sin(f*x + e))
```

3.336.6 Sympy [F]

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = \int \left(a+b\sec^2(e+fx)\right)^2\cot^6(e+fx)\,dx$$

input
```
integrate(cot(f*x+e)**6*(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**2*cot(e + f*x)**6, x)
```

3.336.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.11

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx = -\frac{15\,(fx+e)a^2+\frac{15\,a^2\tan(fx+e)^4-5\,(a^2-b^2)\tan(fx+e)^2+3\,a^2+6\,ab+3\,b^2}{\tan(fx+e)^5}}{15\,f}$$

input
```
integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/15*(15*(f*x + e)*a^2 + (15*a^2*tan(f*x + e)^4 - 5*(a^2 - b^2)*tan(f*x +
 e)^2 + 3*a^2 + 6*a*b + 3*b^2)/tan(f*x + e)^5)/f
```

3.336.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 273 vs. 2(61) = 122.

Time = 0.35 (sec) , antiderivative size = 273, normalized size of antiderivative = 4.20

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$

$$= \frac{3\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+6\,ab\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+3\,b^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-35\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-30\,ab\,\text{ta}}{}$$

input
```
integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/480*(3*a^2*tan(1/2*f*x + 1/2*e)^5 + 6*a*b*tan(1/2*f*x + 1/2*e)^5 + 3*b^2
*tan(1/2*f*x + 1/2*e)^5 - 35*a^2*tan(1/2*f*x + 1/2*e)^3 - 30*a*b*tan(1/2*f
*x + 1/2*e)^3 + 5*b^2*tan(1/2*f*x + 1/2*e)^3 - 480*(f*x + e)*a^2 + 330*a^2
*tan(1/2*f*x + 1/2*e) + 60*a*b*tan(1/2*f*x + 1/2*e) - 30*b^2*tan(1/2*f*x +
 1/2*e) - (330*a^2*tan(1/2*f*x + 1/2*e)^4 + 60*a*b*tan(1/2*f*x + 1/2*e)^4
- 30*b^2*tan(1/2*f*x + 1/2*e)^4 - 35*a^2*tan(1/2*f*x + 1/2*e)^2 - 30*a*b*t
an(1/2*f*x + 1/2*e)^2 + 5*b^2*tan(1/2*f*x + 1/2*e)^2 + 3*a^2 + 6*a*b + 3*b
^2)/tan(1/2*f*x + 1/2*e)^5)/f
```

3.336.9 Mupad [B] (verification not implemented)

Time = 19.66 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.05

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^2\,dx$$
$$= -a^2 x - \frac{\frac{2ab}{5} + \frac{a^2}{5} + \frac{b^2}{5} - \tan(e+fx)^2\left(\frac{a^2}{3} - \frac{b^2}{3}\right) + a^2\tan(e+fx)^4}{f\tan(e+fx)^5}$$

input
```
int(cot(e + f*x)^6*(a + b/cos(e + f*x)^2)^2,x)
```

output
```
- a^2*x - ((2*a*b)/5 + a^2/5 + b^2/5 - tan(e + f*x)^2*(a^2/3 - b^2/3) + a^
2*tan(e + f*x)^4)/(f*tan(e + f*x)^5)
```

3.337 $\int \frac{\tan^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$

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3.337.2 Mathematica [A] (verified) . . . 2457
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3.337.5 Fricas [A] (verification not implemented) . . . 2460
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3.337.9 Mupad [B] (verification not implemented) . . . 2462

3.337.1 Optimal result

Integrand size = 23, antiderivative size = 69

$$\int \frac{\tan^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{(a+2b)\log(\cos(e+fx))}{b^2 f} - \frac{(a+b)^2\log\left(b+a\cos^2(e+fx)\right)}{2ab^2 f} + \frac{\sec^2(e+fx)}{2bf}$$

output
```
(a+2*b)*ln(cos(f*x+e))/b^2/f-1/2*(a+b)^2*ln(b+a*cos(f*x+e)^2)/a/b^2/f+1/2*
sec(f*x+e)^2/b/f
```

3.337.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.43

$$\int \frac{\tan^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(2a(a+2b)\log(\cos(e+fx)) - (a+b)^2\log\left(a+b-a\sin^2(e+f\right.\right.}{4ab^2 f\left(a+b\sec^2(e+fx)\right)}$$

input
```
Integrate[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(2*a*(a + 2*b)*Log[Cos[e +
f*x]] - (a + b)^2*Log[a + b - a*Sin[e + f*x]^2] + a*b*Sec[e + f*x]^2))/(4*
a*b^2*f*(a + b*Sec[e + f*x]^2))
```

3.337.3 Rubi [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4626, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\tan(e+fx)^5}{a+b\sec(e+fx)^2}\,dx$$

$$\downarrow 4626$$

$$-\frac{\int \frac{\left(1-\cos^2(e+fx)\right)^2\sec^3(e+fx)}{a\cos^2(e+fx)+b}\,d\cos(e+fx)}{f}$$

$$\downarrow 354$$

$$-\frac{\int \frac{\left(1-\cos^2(e+fx)\right)^2\sec^2(e+fx)}{a\cos^2(e+fx)+b}\,d\cos^2(e+fx)}{2f}$$

$$\downarrow 99$$

$$-\frac{\int \left(\frac{(a+b)^2}{b^2(a\cos^2(e+fx)+b)}+\frac{\sec^2(e+fx)}{b}+\frac{(-a-2b)\sec(e+fx)}{b^2}\right)d\cos^2(e+fx)}{2f}$$

$$\downarrow 2009$$

$$-\frac{\frac{(a+b)^2\log(a\cos^2(e+fx)+b)}{ab^2}-\frac{(a+2b)\log(\cos^2(e+fx))}{b^2}-\frac{\sec(e+fx)}{b}}{2f}$$

input
```
Int[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

output
```
-1/2*(-(((a + 2*b)*Log[Cos[e + f*x]^2])/b^2) + ((a + b)^2*Log[b + a*Cos[e
+ f*x]^2])/(a*b^2) - Sec[e + f*x]/b)/f
```

3.337.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.337.4 Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{\frac{(a+2b)\ln(\cos(fx+e))}{b^2}+\frac{1}{2b\cos(fx+e)^2}-\frac{\left(a^2+2ab+b^2\right)\ln\left(b+a\cos(fx+e)^2\right)}{2b^2a}}{f}$
default	$\frac{\frac{(a+2b)\ln(\cos(fx+e))}{b^2}+\frac{1}{2b\cos(fx+e)^2}-\frac{\left(a^2+2ab+b^2\right)\ln\left(b+a\cos(fx+e)^2\right)}{2b^2a}}{f}$
risch	$\frac{ix}{a}+\frac{2ie}{af}+\frac{2e^{2i(fx+e)}}{fb\left(e^{2i(fx+e)}+1\right)^2}+\frac{\ln\left(e^{2i(fx+e)}+1\right)a}{b^2f}+\frac{2\ln\left(e^{2i(fx+e)}+1\right)}{bf}-\frac{a\ln\left(e^{4i(fx+e)}+\frac{2(a+2b)e^{2i(fx+e)}}{a}+1\right.}{2b^2f}$

input
```
int(tan(f*x+e)^5/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*((a+2*b)/b^2*ln(cos(f*x+e))+1/2/b/cos(f*x+e)^2-1/2*(a^2+2*a*b+b^2)/b^2
/a*ln(b+a*cos(f*x+e)^2))
```

3.337.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.22

$$\int \frac{\tan^5(e+fx)}{a+b\sec^2(e+fx)}\,dx =$$
$$-\frac{(a^2+2\,ab+b^2)\cos(fx+e)^2\log\left(a\cos(fx+e)^2+b\right)-2\,(a^2+2\,ab)\cos(fx+e)^2\log(-\cos(fx+e))}{2\,ab^2f\cos(fx+e)^2}$$

input
```
integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
-1/2*((a^2 + 2*a*b + b^2)*cos(f*x + e)^2*log(a*cos(f*x + e)^2 + b) - 2*(a^
2 + 2*a*b)*cos(f*x + e)^2*log(-cos(f*x + e)) - a*b)/(a*b^2*f*cos(f*x + e)^
2)
```

3.337.6 Sympy [F]

$$\int \frac{\tan^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\tan^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

input
```
integrate(tan(f*x+e)**5/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(tan(e + f*x)**5/(a + b*sec(e + f*x)**2), x)
```

3.337.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.17

$$\int \frac{\tan^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{(a+2b)\log\left(\sin(fx+e)^2-1\right)}{b^2} - \frac{1}{b\sin(fx+e)^2-b} - \frac{(a^2+2ab+b^2)\log\left(a\sin(fx+e)^2-a-b\right)}{ab^2}}{2f}$$

input
```
integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/2*((a + 2*b)*log(sin(f*x + e)^2 - 1)/b^2 - 1/(b*sin(f*x + e)^2 - b) - (a
^2 + 2*a*b + b^2)*log(a*sin(f*x + e)^2 - a - b)/(a*b^2))/f
```

3.337.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 373 vs. 2(65) = 130.

Time = 1.52 (sec) , antiderivative size = 373, normalized size of antiderivative = 5.41

$$\int \frac{\tan^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{(a^3+3a^2b+3ab^2+b^3)\log\left(\left|-a\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)-b\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)-2a+2b\right|\right)}{a^2b^2+ab^3} - \frac{\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-}{\cos(fx+e)+}\right.\right.}{a}}{2f}$$

input
```
integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
-1/2*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*log(abs(-a*((cos(f*x + e) + 1)/(cos(
f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) - b*((cos(f*x + e)
+ 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) - 2*a + 2
*b))/(a^2*b^2 + a*b^3) - log(abs(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) -
(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 2))/a - (a + 2*b)*log(abs(-(cos(f*
x + e) + 1)/(cos(f*x + e) - 1) - (cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2
))/b^2 + (a*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(c
os(f*x + e) + 1)) + 2*b*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x
+ e) - 1)/(cos(f*x + e) + 1)) + 2*a + 8*b)/(b^2*((cos(f*x + e) + 1)/(cos(f
*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 2)))/f
```

3.337.9 Mupad [B] (verification not implemented)

Time = 19.46 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.49

$$\int \frac{\tan^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,a\,f} - \frac{\ln\left(b\tan(e+f\,x)^2+a+b\right)}{b\,f} - \frac{\ln\left(b\tan(e+f\,x)^2+a+b\right)}{2\,a\,f} + \frac{\tan(e+f\,x)^2}{2\,b\,f} - \frac{a\,\ln\left(b\tan(e+f\,x)^2+a+b\right)}{2\,b^2\,f}$$

input
```
int(tan(e + f*x)^5/(a + b/cos(e + f*x)^2),x)
```

output
```
log(tan(e + f*x)^2 + 1)/(2*a*f) - log(a + b + b*tan(e + f*x)^2)/(b*f) - lo
g(a + b + b*tan(e + f*x)^2)/(2*a*f) + tan(e + f*x)^2/(2*b*f) - (a*log(a +
b + b*tan(e + f*x)^2))/(2*b^2*f)
```

3.338 $\int \frac{\tan^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$

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3.338.2 Mathematica [A] (verified) . . . 2463
3.338.3 Rubi [A] (verified) . . . 2464
3.338.4 Maple [A] (verified) . . . 2465
3.338.5 Fricas [A] (verification not implemented) . . . 2466
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3.338.7 Maxima [A] (verification not implemented) . . . 2466
3.338.8 Giac [B] (verification not implemented) . . . 2467
3.338.9 Mupad [B] (verification not implemented) . . . 2467

3.338.1 Optimal result

Integrand size = 23, antiderivative size = 45

$$\int \frac{\tan^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\log(\cos(e+fx))}{bf} + \frac{(a+b)\log\left(b+a\cos^2(e+fx)\right)}{2abf}$$

output
```
-ln(cos(f*x+e))/b/f+1/2*(a+b)*ln(b+a*cos(f*x+e)^2)/a/b/f
```

3.338.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \frac{\tan^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{-2a\log(\cos(e+fx)) + (a+b)\log\left(b+a\cos^2(e+fx)\right)}{2abf}$$

input
```
Integrate[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

output
```
(-2*a*Log[Cos[e + f*x]] + (a + b)*Log[b + a*Cos[e + f*x]^2])/(2*a*b*f)
```

3.338.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4626, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\tan(e+fx)^3}{a+b\sec(e+fx)^2}\,dx$$

$$\downarrow 4626$$

$$-\frac{\int \frac{\left(1-\cos^2(e+fx)\right)\sec(e+fx)}{a\cos^2(e+fx)+b}\,d\cos(e+fx)}{f}$$

$$\downarrow 354$$

$$-\frac{\int \frac{\left(1-\cos^2(e+fx)\right)\sec(e+fx)}{a\cos^2(e+fx)+b}\,d\cos^2(e+fx)}{2f}$$

$$\downarrow 86$$

$$-\frac{\int \left(\frac{-a-b}{b(a\cos^2(e+fx)+b)}+\frac{\sec(e+fx)}{b}\right)d\cos^2(e+fx)}{2f}$$

$$\downarrow 2009$$

$$-\frac{\frac{\log\left(\cos^2(e+fx)\right)}{b}-\frac{(a+b)\log\left(a\cos^2(e+fx)+b\right)}{ab}}{2f}$$

input
```
Int[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

output
```
-1/2*(Log[Cos[e + f*x]^2]/b - ((a + b)*Log[b + a*Cos[e + f*x]^2])/(a*b))/f
```

3.338.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.338.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{\frac{(a+b)\ln\left(b+a\cos(fx+e)^2\right)}{2ba}-\frac{\ln(\cos(fx+e))}{b}}{f}$
default	$\frac{\frac{(a+b)\ln\left(b+a\cos(fx+e)^2\right)}{2ba}-\frac{\ln(\cos(fx+e))}{b}}{f}$
risch	$-\frac{ix}{a}-\frac{2ie}{af}-\frac{\ln\left(e^{2i(fx+e)}+1\right)}{bf}+\frac{\ln\left(e^{4i(fx+e)}+\frac{2(a+2b)e^{2i(fx+e)}}{a}+1\right)}{2bf}+\frac{\ln\left(e^{4i(fx+e)}+\frac{2(a+2b)e^{2i(fx+e)}}{a}+1\right)}{2af}$

input
```
int(tan(f*x+e)^3/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/2*(a+b)/b/a*ln(b+a*cos(f*x+e)^2)-1/b*ln(cos(f*x+e)))
```

3.338.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \frac{\tan^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{(a+b)\log\left(a\cos\left(fx+e\right)^2+b\right)-2\,a\log\left(-\cos\left(fx+e\right)\right)}{2\,abf}$$

input
```
integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/2*((a + b)*log(a*cos(f*x + e)^2 + b) - 2*a*log(-cos(f*x + e)))/(a*b*f)
```

3.338.6 Sympy [F]

$$\int \frac{\tan^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\tan^3\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

input
```
integrate(tan(f*x+e)**3/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(tan(e + f*x)**3/(a + b*sec(e + f*x)**2), x)
```

3.338.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11

$$\int \frac{\tan^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{(a+b)\log\left(a\sin(fx+e)^2-a-b\right)}{ab}-\frac{\log\left(\sin(fx+e)^2-1\right)}{b}}{2\,f}$$

input
```
integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/2*((a + b)*log(a*sin(f*x + e)^2 - a - b)/(a*b) - log(sin(f*x + e)^2 - 1)
/b)/f
```

3.338.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(43) = 86$.

Time = 0.61 (sec) , antiderivative size = 222, normalized size of antiderivative = 4.93

$$\int \frac{\tan^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{\frac{(a^2+2ab+b^2)\log\left(\left|-a\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)-b\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)-2a+2b\right|\right)}{a^2b+ab^2} - \frac{\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+2\right|\right)}{a} -$$
$$2f$$

input
```
integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/2*((a^2 + 2*a*b + b^2)*log(abs(-a*((cos(f*x + e) + 1)/(cos(f*x + e) - 1)
 + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) - b*((cos(f*x + e) + 1)/(cos(f*x
 + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) - 2*a + 2*b))/(a^2*b +
 a*b^2) - log(abs(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (cos(f*x + e) -
 1)/(cos(f*x + e) + 1) + 2))/a - log(abs(-(cos(f*x + e) + 1)/(cos(f*x + e)
 - 1) - (cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2))/b)/f
```

3.338.9 Mupad [B] (verification not implemented)

Time = 20.16 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.42

$$\int \frac{\tan^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\ln\left(b\tan(e+fx)^2+a+b\right)}{2af} + \frac{\ln\left(b\tan(e+fx)^2+a+b\right)}{2bf} - \frac{\ln\left(\tan(e+fx)^2+1\right)}{2af}$$

input
```
int(tan(e + f*x)^3/(a + b/cos(e + f*x)^2),x)
```

output
```
log(a + b + b*tan(e + f*x)^2)/(2*a*f) + log(a + b + b*tan(e + f*x)^2)/(2*b
*f) - log(tan(e + f*x)^2 + 1)/(2*a*f)
```

3.339 $\int \frac{\tan(e+fx)}{a+b\sec^2(e+fx)}\,dx$

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3.339.1 Optimal result

Integrand size = 21, antiderivative size = 23

$$\int \frac{\tan(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\log\left(b+a\cos^2(e+fx)\right)}{2af}$$

output
```
-1/2*ln(b+a*cos(f*x+e)^2)/a/f
```

3.339.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{\tan(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\log(a+2b+a\cos(2(e+fx)))}{2af}$$

input
```
Integrate[Tan[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

output
```
-1/2*Log[a + 2*b + a*Cos[2*(e + f*x)]]/(a*f)
```

3.339.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4626, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\tan(e+fx)}{a+b\sec(e+fx)^2}\,dx$$
$$\downarrow 4626$$
$$-\frac{\int \frac{\cos(e+fx)}{a\cos^2(e+fx)+b}\,d\cos(e+fx)}{f}$$
$$\downarrow 240$$
$$-\frac{\log\left(a\cos^2(e+fx)+b\right)}{2af}$$

input
```
Int[Tan[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

output
```
-1/2*Log[b + a*Cos[e + f*x]^2]/(a*f)
```

3.339.3.1 Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)  Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.339.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.52

method	result	size
derivativedivides	$\frac{-\frac{\ln\left(a+b\sec(fx+e)^2\right)}{2a}+\frac{\ln(\sec(fx+e))}{a}}{f}$	35
default	$\frac{-\frac{\ln\left(a+b\sec(fx+e)^2\right)}{2a}+\frac{\ln(\sec(fx+e))}{a}}{f}$	35
risch	$\frac{ix}{a}+\frac{2ie}{af}-\frac{\ln\left(e^{4i(fx+e)}+\frac{2(a+2b)e^{2i(fx+e)}}{a}+1\right)}{2af}$	58

input
```
int(tan(f*x+e)/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/2/a*ln(a+b*sec(f*x+e)^2)+1/a*ln(sec(f*x+e)))
```

3.339.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\tan(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\log\left(a\cos\left(fx+e\right)^2+b\right)}{2\,af}$$

input
```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
-1/2*log(a*cos(f*x + e)^2 + b)/(a*f)
```

3.339.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(19) = 38$.

Time = 5.56 (sec) , antiderivative size = 117, normalized size of antiderivative = 5.09

$$\int \frac{\tan(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$= \begin{cases} \frac{\tilde{\infty} x \tan(e)}{\sec^2(e)} & \text{for } a=0 \wedge b=0 \wedge f=0 \\ -\frac{1}{2bf\sec^2(e+fx)} & \text{for } a=0 \\ \frac{\log\left(\tan^2(e+fx)+1\right)}{2af} & \text{for } b=0 \\ \frac{x\tan(e)}{a+b\sec^2(e)} & \text{for } f=0 \\ -\frac{\log\left(-\sqrt{-\frac{a}{b}}+\sec(e+fx)\right)}{2af} - \frac{\log\left(\sqrt{-\frac{a}{b}}+\sec(e+fx)\right)}{2af} + \frac{\log\left(\tan^2(e+fx)+1\right)}{2af} & \text{otherwise} \end{cases}$$

input
```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)**2),x)
```

output
```
Piecewise((zoo*x*tan(e)/sec(e)**2, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (-1/(2
*b*f*sec(e + f*x)**2), Eq(a, 0)), (log(tan(e + f*x)**2 + 1)/(2*a*f), Eq(b,
 0)), (x*tan(e)/(a + b*sec(e)**2), Eq(f, 0)), (-log(-sqrt(-a/b) + sec(e +
f*x))/(2*a*f) - log(sqrt(-a/b) + sec(e + f*x))/(2*a*f) + log(tan(e + f*x)*
*2 + 1)/(2*a*f), True))
```

3.339.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{\tan(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\log\left(a\sin(fx+e)^2 - a - b\right)}{2af}$$

input
```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
-1/2*log(a*sin(f*x + e)^2 - a - b)/(a*f)
```

3.339.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(21) = 42$.

Time = 0.33 (sec) , antiderivative size = 129, normalized size of antiderivative = 5.61

$$\int \frac{\tan(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{\log\left(a+b+\frac{2\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{a}-\frac{2\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)}{a}}{2\,f}$$

input
```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
-1/2*(log(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(cos(f*x
 + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^
2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/a - 2*log(abs(-(cos(f*x +
 e) - 1)/(cos(f*x + e) + 1) + 1))/a)/f
```

3.339.9 Mupad [B] (verification not implemented)

Time = 19.83 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.74

$$\int \frac{\tan(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\operatorname{atanh}\left(\frac{a}{2\left(\frac{3a}{2}+2b+\frac{a\cos(2e+2fx)}{2}\right)}-\frac{a\cos(2e+2fx)}{2\left(\frac{3a}{2}+2b+\frac{a\cos(2e+2fx)}{2}\right)}\right)}{af}$$

input
```
int(tan(e + f*x)/(a + b/cos(e + f*x)^2),x)
```

output
```
atanh(a/(2*((3*a)/2 + 2*b + (a*cos(2*e + 2*f*x))/2)) - (a*cos(2*e + 2*f*x)
)/(2*((3*a)/2 + 2*b + (a*cos(2*e + 2*f*x))/2)))/(a*f)
```

3.340 $\int \frac{\cot(e+fx)}{a+b\sec^2(e+fx)}\,dx$

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3.340.1 Optimal result

Integrand size = 21, antiderivative size = 46

$$\int \frac{\cot(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{b\log\left(b+a\cos^2(e+fx)\right)}{2a(a+b)f} + \frac{\log(\sin(e+fx))}{(a+b)f}$$

output
```
1/2*b*ln(b+a*cos(f*x+e)^2)/a/(a+b)/f+ln(sin(f*x+e))/(a+b)/f
```

3.340.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \frac{\cot(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{2a\log(\sin(e+fx)) + b\log\left(a+b-a\sin^2(e+fx)\right)}{2a^2f+2abf}$$

input
```
Integrate[Cot[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

output
```
(2*a*Log[Sin[e + f*x]] + b*Log[a + b - a*Sin[e + f*x]^2])/(2*a^2*f + 2*a*b
*f)
```

3.340.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4626, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\tan(e+fx)\left(a+b\sec(e+fx)^2\right)}dx$$

$$\downarrow 4626$$

$$-\frac{\int \frac{\cos^3(e+fx)}{(1-\cos^2(e+fx))(a\cos^2(e+fx)+b)}d\cos(e+fx)}{f}$$

$$\downarrow 354$$

$$-\frac{\int \frac{\cos^2(e+fx)}{(1-\cos^2(e+fx))(a\cos^2(e+fx)+b)}d\cos^2(e+fx)}{2f}$$

$$\downarrow 86$$

$$-\frac{\int \left(\frac{1}{(-a-b)(\cos^2(e+fx)-1)}-\frac{b}{(a+b)(a\cos^2(e+fx)+b)}\right)d\cos^2(e+fx)}{2f}$$

$$\downarrow 2009$$

$$-\frac{-\frac{\log(1-\cos^2(e+fx))}{a+b}-\frac{b\log(a\cos^2(e+fx)+b)}{a(a+b)}}{2f}$$

input
```
Int[Cot[e + f*x]/(a + b*Sec[e + f*x]^2),x]
```

output
```
-1/2*(-(Log[1 - Cos[e + f*x]^2]/(a + b)) - (b*Log[b + a*Cos[e + f*x]^2])/(
a*(a + b)))/f
```

3.340.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.340.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.48

method	result	siz
derivativedivides	$\frac{\frac{b\ln\left(b+a\cos(fx+e)^2\right)}{2(a+b)a}+\frac{\ln(1+\cos(fx+e))}{2a+2b}+\frac{\ln(-1+\cos(fx+e))}{2a+2b}}{f}$	68
default	$\frac{\frac{b\ln\left(b+a\cos(fx+e)^2\right)}{2(a+b)a}+\frac{\ln(1+\cos(fx+e))}{2a+2b}+\frac{\ln(-1+\cos(fx+e))}{2a+2b}}{f}$	68
risch	$\frac{ix}{a}-\frac{2ix}{a+b}-\frac{2ie}{f(a+b)}-\frac{2ibx}{a(a+b)}-\frac{2ibe}{af(a+b)}+\frac{\ln\left(e^{2i(fx+e)}-1\right)}{f(a+b)}+\frac{b\ln\left(e^{4i(fx+e)}+\frac{2(a+2b)e^{2i(fx+e)}}{a}+1\right)}{2af(a+b)}$	12

input
```
int(cot(f*x+e)/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/2/(a+b)*b/a*ln(b+a*cos(f*x+e)^2)+1/(2*a+2*b)*ln(1+cos(f*x+e))+1/(2*
a+2*b)*ln(-1+cos(f*x+e)))
```

3.340.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \frac{\cot(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{b\log\left(a\cos\left(fx+e\right)^2+b\right)+2\,a\log\left(\frac{1}{2}\,\sin\left(fx+e\right)\right)}{2\left(a^2+ab\right)f}$$

input
```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/2*(b*log(a*cos(f*x + e)^2 + b) + 2*a*log(1/2*sin(f*x + e)))/((a^2 + a*b)
*f)
```

3.340.6 Sympy [F]

$$\int \frac{\cot(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\cot\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

input
```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(cot(e + f*x)/(a + b*sec(e + f*x)**2), x)
```

3.340.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09

$$\int \frac{\cot(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{b\log\left(a\sin(fx+e)^2-a-b\right)}{a^2+ab}+\frac{\log\left(\sin(fx+e)^2\right)}{a+b}}{2\,f}$$

input
```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/2*(b*log(a*sin(f*x + e)^2 - a - b)/(a^2 + a*b) + log(sin(f*x + e)^2)/(a
+ b))/f
```

3.340.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(44) = 88$.

Time = 0.30 (sec) , antiderivative size = 166, normalized size of antiderivative = 3.61

$$\int \frac{\cot(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{b\log\left(a+b+\frac{2\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{a^2+ab}+\frac{\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a+b}-\frac{2\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)}{a}}{2f}$$

input
```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/2*(b*log(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(cos(f*
x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)
^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/(a^2 + a*b) + log(abs(-c
os(f*x + e) + 1)/abs(cos(f*x + e) + 1))/(a + b) - 2*log(abs(-(cos(f*x + e)
 - 1)/(cos(f*x + e) + 1) + 1))/a)/f
```

3.340.9 Mupad [B] (verification not implemented)

Time = 20.64 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.41

$$\int \frac{\cot(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\ln\left(\tan(e+f\,x)\right)}{f\,\left(a+b\right)}-\frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,a\,f}+\frac{b\,\ln\left(b\tan(e+f\,x)^2+a+b\right)}{2\,f\,\left(a^2+b\,a\right)}$$

input
```
int(cot(e + f*x)/(a + b/cos(e + f*x)^2),x)
```

output
```
log(tan(e + f*x))/(f*(a + b)) - log(tan(e + f*x)^2 + 1)/(2*a*f) + (b*log(a
 + b + b*tan(e + f*x)^2))/(2*f*(a*b + a^2))
```

3.341 $\int \frac{\cot^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$

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3.341.1 Optimal result

Integrand size = 23, antiderivative size = 74

$$\int \frac{\cot^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\csc^2(e+fx)}{2(a+b)f} - \frac{b^2\log\left(b+a\cos^2(e+fx)\right)}{2a(a+b)^2f} - \frac{(a+2b)\log(\sin(e+fx))}{(a+b)^2f}$$

output
```
-1/2*csc(f*x+e)^2/(a+b)/f-1/2*b^2*ln(b+a*cos(f*x+e)^2)/a/(a+b)^2/f-(a+2*b)
*ln(sin(f*x+e))/(a+b)^2/f
```

3.341.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.35

$$\int \frac{\cot^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))\left(a(a+b)\csc^2(e+fx)+2a(a+2b)\log(\sin(e+fx))+b^2\log\left(a+b-a\sin\right.\right.}{4a(a+b)^2f\left(a+b\sec^2(e+fx)\right)}$$

input
```
Integrate[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

output
```
-1/4*((a + 2*b + a*Cos[2*(e + f*x)])*(a*(a + b)*Csc[e + f*x]^2 + 2*a*(a +
2*b)*Log[Sin[e + f*x]] + b^2*Log[a + b - a*Sin[e + f*x]^2])*Sec[e + f*x]^2
)/(a*(a + b)^2*f*(a + b*Sec[e + f*x]^2))
```

3.341.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4626, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\tan(e+fx)^3\,(a+b\sec(e+fx)^2)}dx$$

$$\downarrow 4626$$

$$-\frac{\int \frac{\cos^5(e+fx)}{(1-\cos^2(e+fx))^2(a\cos^2(e+fx)+b)}d\cos(e+fx)}{f}$$

$$\downarrow 354$$

$$-\frac{\int \frac{\cos^4(e+fx)}{(1-\cos^2(e+fx))^2(a\cos^2(e+fx)+b)}d\cos^2(e+fx)}{2f}$$

$$\downarrow 99$$

$$-\frac{\int \left(\frac{b^2}{(a+b)^2(a\cos^2(e+fx)+b)}+\frac{a+2b}{(a+b)^2(\cos^2(e+fx)-1)}+\frac{1}{(a+b)(\cos^2(e+fx)-1)^2}\right)d\cos^2(e+fx)}{2f}$$

$$\downarrow 2009$$

$$-\frac{\frac{b^2\log(a\cos^2(e+fx)+b)}{a(a+b)^2}+\frac{1}{(a+b)(1-\cos^2(e+fx))}+\frac{(a+2b)\log(1-\cos^2(e+fx))}{(a+b)^2}}{2f}$$

input
```
Int[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2),x]
```

output
```
-1/2*(1/((a + b)*(1 - Cos[e + f*x]^2)) + ((a + 2*b)*Log[1 - Cos[e + f*x]^2
])/(a + b)^2 + (b^2*Log[b + a*Cos[e + f*x]^2])/(a*(a + b)^2))/f
```

3.341.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.341.4 Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.61

method	result
derivativedivides	$\frac{-\frac{1}{(4a+4b)(1+\cos(fx+e))}+\frac{(-a-2b)\ln(1+\cos(fx+e))}{2(a+b)^2}-\frac{b^2\ln\left(b+a\cos(fx+e)^2\right)}{2(a+b)^2a}+\frac{1}{(4a+4b)(-1+\cos(fx+e))}+\frac{(-a-2b)\ln(-1+\cos(f}{f}$
default	$\frac{-\frac{1}{(4a+4b)(1+\cos(fx+e))}+\frac{(-a-2b)\ln(1+\cos(fx+e))}{2(a+b)^2}-\frac{b^2\ln\left(b+a\cos(fx+e)^2\right)}{2(a+b)^2a}+\frac{1}{(4a+4b)(-1+\cos(fx+e))}+\frac{(-a-2b)\ln(-1+\cos(f}{f}$
risch	$-\frac{ix}{a}+\frac{2iax}{a^2+2ab+b^2}+\frac{2iae}{f(a^2+2ab+b^2)}+\frac{4ibx}{a^2+2ab+b^2}+\frac{4ibe}{f(a^2+2ab+b^2)}+\frac{2ib^2x}{a(a^2+2ab+b^2)}+\frac{2ib^2e}{af(a^2+2ab+b^2)}$

input
```
int(cot(f*x+e)^3/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/(4*a+4*b)/(1+cos(f*x+e))+1/2*(-a-2*b)/(a+b)^2*ln(1+cos(f*x+e))-1/2
*b^2/(a+b)^2/a*ln(b+a*cos(f*x+e)^2)+1/(4*a+4*b)/(-1+cos(f*x+e))+1/2*(-a-2*
b)/(a+b)^2*ln(-1+cos(f*x+e)))
```

3.341.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.70

$$\int \frac{\cot^3(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{a^2+ab-\left(b^2\cos\left(fx+e\right)^2-b^2\right)\log\left(a\cos\left(fx+e\right)^2+b\right)-2\left(\left(a^2+2\,ab\right)\cos\left(fx+e\right)^2-a^2-2\,ab\right)\log(}{2\left(\left(a^3+2\,a^2b+ab^2\right)f\cos\left(fx+e\right)^2-\left(a^3+2\,a^2b+ab^2\right)f\right)}$$

input
```
integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/2*(a^2 + a*b - (b^2*cos(f*x + e)^2 - b^2)*log(a*cos(f*x + e)^2 + b) - 2*
((a^2 + 2*a*b)*cos(f*x + e)^2 - a^2 - 2*a*b)*log(1/2*sin(f*x + e)))/((a^3
+ 2*a^2*b + a*b^2)*f*cos(f*x + e)^2 - (a^3 + 2*a^2*b + a*b^2)*f)
```

3.341.6 Sympy [F]

$$\int \frac{\cot^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\cot^3\left(e+fx\right)}{a+b\sec^2\left(e+fx\right)}\,dx$$

input
```
integrate(cot(f*x+e)**3/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(cot(e + f*x)**3/(a + b*sec(e + f*x)**2), x)
```

3.341.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.18

$$\int \frac{\cot^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{b^2\log\left(a\sin(fx+e)^2-a-b\right)}{a^3+2a^2b+ab^2}+\frac{(a+2b)\log\left(\sin(fx+e)^2\right)}{a^2+2ab+b^2}+\frac{1}{(a+b)\sin(fx+e)^2}}{2f}$$

input
```
integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
-1/2*(b^2*log(a*sin(f*x + e)^2 - a - b)/(a^3 + 2*a^2*b + a*b^2) + (a + 2*b
)*log(sin(f*x + e)^2)/(a^2 + 2*a*b + b^2) + 1/((a + b)*sin(f*x + e)^2))/f
```

3.341.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(70) = 140.

Time = 0.34 (sec) , antiderivative size = 294, normalized size of antiderivative = 3.97

$$\int \frac{\cot^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{4b^2\log\left(a+b+\frac{2a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{a^3+2a^2b+ab^2}+\frac{4(a+2b)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a^2+2ab+b^2}-\frac{\left(a+b+\frac{4a(\cos}{8f}$$

input
```
integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
-1/8*(4*b^2*log(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(c
os(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e)
 + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/(a^3 + 2*a^2*b + a*
b^2) + 4*(a + 2*b)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1))/(a^2
+ 2*a*b + b^2) - (a + b + 4*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 8*b*
(cos(f*x + e) - 1)/(cos(f*x + e) + 1))*(cos(f*x + e) + 1)/((a^2 + 2*a*b +
b^2)*(cos(f*x + e) - 1)) - 8*log(abs(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1
) + 1))/a - (cos(f*x + e) - 1)/((a + b)*(cos(f*x + e) + 1)))/f
```

3.341.9 Mupad [B] (verification not implemented)

Time = 20.76 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.32

$$\int \frac{\cot^3(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,a\,f} - \frac{\cot(e+f\,x)^2}{2\,f\,(a+b)} - \frac{\ln\left(\tan(e+f\,x)\right)\,(a+2\,b)}{f\,(a^2+2\,a\,b+b^2)} - \frac{b^2\,\ln\left(b\tan(e+f\,x)^2+a+b\right)}{2\,a\,f\,(a+b)^2}$$

input
```
int(cot(e + f*x)^3/(a + b/cos(e + f*x)^2),x)
```

output
```
log(tan(e + f*x)^2 + 1)/(2*a*f) - cot(e + f*x)^2/(2*f*(a + b)) - (log(tan(
e + f*x))*(a + 2*b))/(f*(2*a*b + a^2 + b^2)) - (b^2*log(a + b + b*tan(e +
f*x)^2))/(2*a*f*(a + b)^2)
```

3.342 $\int \frac{\cot^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$

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3.342.1 Optimal result

Integrand size = 23, antiderivative size = 108

$$\int \frac{\cot^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{(2a+3b)\csc^2(e+fx)}{2(a+b)^2 f} - \frac{\csc^4(e+fx)}{4(a+b)f} + \frac{b^3\log\left(b+a\cos^2(e+fx)\right)}{2a(a+b)^3 f} + \frac{(a^2+3ab+3b^2)\log(\sin(e+fx))}{(a+b)^3 f}$$

output
```
1/2*(2*a+3*b)*csc(f*x+e)^2/(a+b)^2/f-1/4*csc(f*x+e)^4/(a+b)/f+1/2*b^3*ln(b
+a*cos(f*x+e)^2)/a/(a+b)^3/f+(a^2+3*a*b+3*b^2)*ln(sin(f*x+e))/(a+b)^3/f
```

3.342.2 Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.28

$$\int \frac{\cot^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{(a+2b+a\cos(2e+2fx))\left(\frac{2(2a+3b)\csc^2(e+fx)}{(a+b)^2} - \frac{\csc^4(e+fx)}{a+b} + \frac{4(a^2+3ab+3b^2)\log(\sin(e+fx))}{(a+b)^3} + \frac{2b^3\log(a+b-a\sin^2(e+fx}{a(a+b)^3}\right.}{8f\left(a+b\sec^2(e+fx)\right)}$$

input
```
Integrate[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

output
```
((a + 2*b + a*Cos[2*e + 2*f*x])*((2*(2*a + 3*b)*Csc[e + f*x]^2)/(a + b)^2
- Csc[e + f*x]^4/(a + b) + (4*(a^2 + 3*a*b + 3*b^2)*Log[Sin[e + f*x]])/(a
+ b)^3 + (2*b^3*Log[a + b - a*Sin[e + f*x]^2])/(a*(a + b)^3))*Sec[e + f*x]
^2)/(8*f*(a + b*Sec[e + f*x]^2))
```

3.342.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4626, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\tan(e+fx)^5\,(a+b\sec(e+fx)^2)}dx$$

$$\downarrow 4626$$

$$-\frac{\int \frac{\cos^7(e+fx)}{(1-\cos^2(e+fx))^3(a\cos^2(e+fx)+b)}d\cos(e+fx)}{f}$$

$$\downarrow 354$$

$$-\frac{\int \frac{\cos^6(e+fx)}{(1-\cos^2(e+fx))^3(a\cos^2(e+fx)+b)}d\cos^2(e+fx)}{2f}$$

$$\downarrow 99$$

$$-\frac{\int \left(-\frac{b^3}{(a+b)^3(a\cos^2(e+fx)+b)}+\frac{-a^2-3ba-3b^2}{(a+b)^3(\cos^2(e+fx)-1)}+\frac{-2a-3b}{(a+b)^2(\cos^2(e+fx)-1)^2}-\frac{1}{(a+b)(\cos^2(e+fx)-1)^3}\right)d\cos^2(e+fx)}{2f}$$

$$\downarrow 2009$$

$$-\frac{-\frac{(a^2+3ab+3b^2)\log(1-\cos^2(e+fx))}{(a+b)^3}-\frac{b^3\log(a\cos^2(e+fx)+b)}{a(a+b)^3}-\frac{2a+3b}{(a+b)^2(1-\cos^2(e+fx))}+\frac{1}{2(a+b)(1-\cos^2(e+fx))^2}}{2f}$$

input
```
Int[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2),x]
```

output
```
-1/2*(1/(2*(a + b)*(1 - Cos[e + f*x]^2)^2) - (2*a + 3*b)/((a + b)^2*(1 - C
os[e + f*x]^2)) - ((a^2 + 3*a*b + 3*b^2)*Log[1 - Cos[e + f*x]^2])/(a + b)^
3 - (b^3*Log[b + a*Cos[e + f*x]^2])/(a*(a + b)^3))/f
```

3.342.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.342.4 Maple [A] (verified)

Time = 2.61 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.67

method	result
derivativedivides	$\frac{\frac{b^3\ln\left(b+a\cos(fx+e)^2\right)}{2(a+b)^3a}-\frac{1}{2(8a+8b)(-1+\cos(fx+e))^2}-\frac{7a+11b}{16(a+b)^2(-1+\cos(fx+e))}+\frac{\left(a^2+3ab+3b^2\right)\ln(-1+\cos(fx+e))}{2(a+b)^3}-\frac{}{2(8a+8b)(}}{f}$
default	$\frac{\frac{b^3\ln\left(b+a\cos(fx+e)^2\right)}{2(a+b)^3a}-\frac{1}{2(8a+8b)(-1+\cos(fx+e))^2}-\frac{7a+11b}{16(a+b)^2(-1+\cos(fx+e))}+\frac{\left(a^2+3ab+3b^2\right)\ln(-1+\cos(fx+e))}{2(a+b)^3}-\frac{}{2(8a+8b)(}}{f}$
risch	$\frac{ix}{a}-\frac{2ia^2x}{a^3+3a^2b+3ab^2+b^3}-\frac{2ia^2e}{f(a^3+3a^2b+3ab^2+b^3)}-\frac{6iabx}{a^3+3a^2b+3ab^2+b^3}-\frac{6iabe}{f(a^3+3a^2b+3ab^2+b^3)}-\frac{6}{a^3+3a^2b}$

input
```
int(cot(f*x+e)^5/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/2*b^3/(a+b)^3/a*ln(b+a*cos(f*x+e)^2)-1/2/(8*a+8*b)/(-1+cos(f*x+e))^
2-1/16*(7*a+11*b)/(a+b)^2/(-1+cos(f*x+e))+1/2*(a^2+3*a*b+3*b^2)/(a+b)^3*ln
(-1+cos(f*x+e))-1/2/(8*a+8*b)/(1+cos(f*x+e))^2-1/16*(-7*a-11*b)/(a+b)^2/(1
+cos(f*x+e))+1/2*(a^2+3*a*b+3*b^2)/(a+b)^3*ln(1+cos(f*x+e)))
```

3.342.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. 2(102) = 204.

Time = 0.46 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.45

$$\int \frac{\cot^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{3\,a^3+8\,a^2b+5\,ab^2-2\left(2\,a^3+5\,a^2b+3\,ab^2\right)\cos\left(fx+e\right)^2+2\left(b^3\cos\left(fx+e\right)^4-2\,b^3\cos\left(fx+e\right)^2+b^3\right)}{4\left(\left(a^4+3\,a^3b+3\,a^2b^2+ab^3\right)f\cos\left(fx+e\right)\right.}$$

input
```
integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
1/4*(3*a^3 + 8*a^2*b + 5*a*b^2 - 2*(2*a^3 + 5*a^2*b + 3*a*b^2)*cos(f*x + e
)^2 + 2*(b^3*cos(f*x + e)^4 - 2*b^3*cos(f*x + e)^2 + b^3)*log(a*cos(f*x +
e)^2 + b) + 4*((a^3 + 3*a^2*b + 3*a*b^2)*cos(f*x + e)^4 + a^3 + 3*a^2*b +
3*a*b^2 - 2*(a^3 + 3*a^2*b + 3*a*b^2)*cos(f*x + e)^2)*log(1/2*sin(f*x + e)
))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^4 - 2*(a^4 + 3*a^3*
b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^2 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b
^3)*f)
```

3.342.6 Sympy [F]

$$\int \frac{\cot^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\cot^5(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

input
```
integrate(cot(f*x+e)**5/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(cot(e + f*x)**5/(a + b*sec(e + f*x)**2), x)
```

3.342.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.34

$$\int \frac{\cot^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{2b^3\log\left(a\sin(fx+e)^2-a-b\right)}{a^4+3a^3b+3a^2b^2+ab^3} + \frac{2(a^2+3ab+3b^2)\log\left(\sin(fx+e)^2\right)}{a^3+3a^2b+3ab^2+b^3} + \frac{2(2a+3b)\sin(fx+e)^2-a-b}{(a^2+2ab+b^2)\sin(fx+e)^4}}{4f}$$

input
```
integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/4*(2*b^3*log(a*sin(f*x + e)^2 - a - b)/(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^
3) + 2*(a^2 + 3*a*b + 3*b^2)*log(sin(f*x + e)^2)/(a^3 + 3*a^2*b + 3*a*b^2
+ b^3) + (2*(2*a + 3*b)*sin(f*x + e)^2 - a - b)/((a^2 + 2*a*b + b^2)*sin(f
*x + e)^4))/f
```

3.342.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 513 vs. 2(102) = 204.

Time = 0.40 (sec) , antiderivative size = 513, normalized size of antiderivative = 4.75

$$\int \frac{\cot^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{32b^3\log\left(a+b+\frac{2a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{a^4+3a^3b+3a^2b^2+ab^3} + \frac{32(a^2+3ab+3b^2)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a^3+3a^2b+3ab^2+b^3} - \frac{12a(\cos}{}$$

input
```
integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/64*(32*b^3*log(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(
cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e
) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/(a^4 + 3*a^3*b + 3
*a^2*b^2 + a*b^3) + 32*(a^2 + 3*a*b + 3*b^2)*log(abs(-cos(f*x + e) + 1)/ab
s(cos(f*x + e) + 1))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (12*a*(cos(f*x + e)
 - 1)/(cos(f*x + e) + 1) + 20*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*
(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*
x + e) + 1)^2)/(a^2 + 2*a*b + b^2) - (a^2 + 2*a*b + b^2 + 12*a^2*(cos(f*x
+ e) - 1)/(cos(f*x + e) + 1) + 32*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1
) + 20*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 48*a^2*(cos(f*x + e) -
1)^2/(cos(f*x + e) + 1)^2 + 144*a*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1
)^2 + 144*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1
)^2/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(cos(f*x + e) - 1)^2) - 64*log(abs(-(
cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1))/a)/f
```

3.342.9 Mupad [B] (verification not implemented)

Time = 20.66 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.48

$$\int \frac{\cot^5(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\ln\left(\tan(e+f\,x)\right)\,\left(a^2+3\,a\,b+3\,b^2\right)}{f\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)} - \frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,a\,f} - \frac{\ln\left(b\tan(e+f\,x)^2+a+b\right)\,\left(\frac{b}{2\,(a+b)^2}+\frac{1}{2\,(a+b)}+\frac{b^2}{2\,(a+b)^3}-\frac{1}{2\,a}\right)}{f} - \frac{\cot(e+f\,x)^4\,\left(\frac{1}{4\,(a+b)}-\frac{\tan(e+f\,x)^2\,(a+2\,b)}{2\,(a+b)^2}\right)}{f}$$

input
```
int(cot(e + f*x)^5/(a + b/cos(e + f*x)^2),x)
```

output
```
(log(tan(e + f*x))*(3*a*b + a^2 + 3*b^2))/(f*(3*a*b^2 + 3*a^2*b + a^3 + b^
3)) - log(tan(e + f*x)^2 + 1)/(2*a*f) - (log(a + b + b*tan(e + f*x)^2)*(b/
(2*(a + b)^2) + 1/(2*(a + b)) + b^2/(2*(a + b)^3) - 1/(2*a)))/f - (cot(e +
 f*x)^4*(1/(4*(a + b)) - (tan(e + f*x)^2*(a + 2*b))/(2*(a + b)^2)))/f
```

3.343 $\int \frac{\tan^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$

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3.343.1 Optimal result

Integrand size = 23, antiderivative size = 83

$$\int \frac{\tan^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{x}{a} + \frac{(a+b)^{5/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{ab^{5/2}f} - \frac{(a+2b)\tan(e+fx)}{b^2 f} + \frac{\tan^3(e+fx)}{3bf}$$

output

```
-x/a+(a+b)^(5/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a/b^(5/2)/f-(a+2*b
)*tan(f*x+e)/b^2/f+1/3*tan(f*x+e)^3/b/f
```

3.343.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.95 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.76

$$\int \frac{\tan^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(-\frac{3x}{a} - \frac{3(a+b)^{5/2}\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)}{ab^2 f\sqrt{b(\cos(e)-i\sin(e))^4}}\right)}{6\left(a+b\sec^2(e+fx)\right)}$$

input
```
Integrate[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*((-3*x)/a - (3*(a + b)^(5/2
)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[
2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*
Sin[2*e]))/(a*b^2*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) - ((3*a + 7*b)*Sec[e]*S
ec[e + f*x]*Sin[f*x])/(b^2*f) + (Sec[e]*Sec[e + f*x]^3*Sin[f*x])/(b*f) + (
Sec[e + f*x]^2*Tan[e])/(b*f)))/(6*(a + b*Sec[e + f*x]^2))
```

3.343.3 Rubi [A] (verified)

Time $= 0.39$ (sec) , antiderivative size $= 98$, normalized size of antiderivative $= 1.18$, number of steps used $= 10$, number of rules used $= 9$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used $= \{3042, 4629, 2075, 381, 27, 444, 397, 216, 218\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\tan(e+fx)^6}{a+b\sec(e+fx)^2}\,dx$$

$$\downarrow 4629$$

$$\frac{\int \frac{\tan^6(e+fx)}{(\tan^2(e+fx)+1)(a+b(\tan^2(e+fx)+1))}\,d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\tan^6(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)}\,d\tan(e+fx)}{f}$$

$$\downarrow 381$$

$$\frac{\frac{\tan^3(e+fx)}{3b} - \frac{\int \frac{3\tan^2(e+fx)\left((a+2b)\tan^2(e+fx)+a+b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}\,d\tan(e+fx)}{3b}}{f}$$

$$\downarrow 27$$

$$\frac{\frac{\tan^3(e+fx)}{3b}-\frac{\int\frac{\tan^2(e+fx)\left((a+2b)\tan^2(e+fx)+a+b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{b}}{f}$$

$$\downarrow 444$$

$$\frac{\frac{\tan^3(e+fx)}{3b}-\frac{\frac{(a+2b)\tan(e+fx)}{b}-\frac{\int\frac{\left(a^2+3ba+3b^2\right)\tan^2(e+fx)+(a+b)(a+2b)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{b}}{b}}{f}$$

$$\downarrow 397$$

$$\frac{\frac{\tan^3(e+fx)}{3b}-\frac{\frac{(a+2b)\tan(e+fx)}{b}-\frac{\frac{(a+b)^3\int\frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}-b^2\int\frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}}{b}}{b}}{f}$$

$$\downarrow 216$$

$$\frac{\frac{\tan^3(e+fx)}{3b}-\frac{\frac{(a+2b)\tan(e+fx)}{b}-\frac{\frac{(a+b)^3\int\frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}-b^2\arctan(\tan(e+fx))}{a}}{b}}{b}}{f}$$

$$\downarrow 218$$

$$\frac{\frac{\tan^3(e+fx)}{3b}-\frac{\frac{(a+2b)\tan(e+fx)}{b}-\frac{\frac{(a+b)^{5/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{b}}-b^2\arctan(\tan(e+fx))}{a}}{b}}{b}}{f}$$

input
```
Int[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

output
```
(Tan[e + f*x]^3/(3*b) - (-((-((b^2*ArcTan[Tan[e + f*x]])/a) + ((a + b)^(5/
2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*Sqrt[b]))/b) + ((a + 2*b
)*Tan[e + f*x])/b)/b)/f
```

3.343.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 381
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
+ 1)/(b*d*(m + 2*(p + q) + 1))), x] - Simp[e^4/(b*d*(m + 2*(p + q) + 1))
Int[(e*x)^(m - 4)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*c*(m - 3) + (a*d*(m +
2*q - 1) + b*c*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 3] && IntBinomialQ[a, b, c, d, e, m, 2
, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f  Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.343.4 Maple [A] (verified)

Time = 2.32 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.22

method	result
derivativedivides	$\frac{-\frac{-\frac{b\tan(fx+e)^3}{3}+a\tan(fx+e)+2b\tan(fx+e)}{b^2}+\frac{\left(a^3+3a^2b+3ab^2+b^3\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^2a\sqrt{(a+b)b}}-\frac{\arctan(\tan(fx+e))}{a}}{f}$
default	$\frac{-\frac{-\frac{b\tan(fx+e)^3}{3}+a\tan(fx+e)+2b\tan(fx+e)}{b^2}+\frac{\left(a^3+3a^2b+3ab^2+b^3\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{b^2a\sqrt{(a+b)b}}-\frac{\arctan(\tan(fx+e))}{a}}{f}$
risch	$-\frac{x}{a}-\frac{2i\left(3a\,e^{4i(fx+e)}+9b\,e^{4i(fx+e)}+6a\,e^{2i(fx+e)}+12b\,e^{2i(fx+e)}+3a+7b\right)}{3fb^2\left(e^{2i(fx+e)}+1\right)^3}-\frac{\sqrt{-(a+b)b}\,a\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-(a+b}}{a}\right.}{2b^3f}$

input

```
int(tan(f*x+e)^6/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output

```
1/f*(-1/b^2*(-1/3*b*tan(f*x+e)^3+a*tan(f*x+e)+2*b*tan(f*x+e))+1/b^2*(a^3+3
*a^2*b+3*a*b^2+b^3)/a/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))
-1/a*arctan(tan(f*x+e)))
```

3.343.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(73) = 146.

Time = 0.31 (sec) , antiderivative size = 373, normalized size of antiderivative = 4.49

$$\int \frac{\tan^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$= \left[-\frac{12\,b^2 fx\cos(fx+e)^3 - 3\,(a^2+2\,ab+b^2)\sqrt{-\frac{a+b}{b}}\cos(fx+e)^3\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4 - 2\,(3\,ab+4\,b^2)\cos}{a^2}\right.}{12\,ab^2 f}\right.$$

$$\left. -\frac{6\,b^2 fx\cos(fx+e)^3 + 3\,(a^2+2\,ab+b^2)\sqrt{\frac{a+b}{b}}\arctan\left(\frac{\left((a+2\,b)\cos(fx+e)^2-b\right)\sqrt{\frac{a+b}{b}}}{2\,(a+b)\cos(fx+e)\sin(fx+e)}\right)\cos(fx+e)^3 + 2\,((3}{6\,ab^2 f\cos(fx+e)^3} \right]$$

input
```
integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[-1/12*(12*b^2*f*x*cos(f*x + e)^3 - 3*(a^2 + 2*a*b + b^2)*sqrt(-(a + b)/b)
*cos(f*x + e)^3*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b
^2)*cos(f*x + e)^2 - 4*((a*b + 2*b^2)*cos(f*x + e)^3 - b^2*cos(f*x + e))*s
qrt(-(a + b)/b)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x +
e)^2 + b^2)) + 4*((3*a^2 + 7*a*b)*cos(f*x + e)^2 - a*b)*sin(f*x + e))/(a*b
^2*f*cos(f*x + e)^3), -1/6*(6*b^2*f*x*cos(f*x + e)^3 + 3*(a^2 + 2*a*b + b^
2)*sqrt((a + b)/b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt((a + b)/
b)/((a + b)*cos(f*x + e)*sin(f*x + e)))*cos(f*x + e)^3 + 2*((3*a^2 + 7*a*b
)*cos(f*x + e)^2 - a*b)*sin(f*x + e))/(a*b^2*f*cos(f*x + e)^3)]
```

3.343.6 Sympy [F]

$$\int \frac{\tan^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\tan^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

input
```
integrate(tan(f*x+e)**6/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(tan(e + f*x)**6/(a + b*sec(e + f*x)**2), x)
```

3.343.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.14

$$\int \frac{\tan^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= -\frac{\frac{3(fx+e)}{a} - \frac{b\tan(fx+e)^3 - 3(a+2b)\tan(fx+e)}{b^2} - \frac{3(a^3+3a^2b+3ab^2+b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}ab^2}}{3f}$$

input
```
integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
-1/3*(3*(f*x + e)/a - (b*tan(f*x + e)^3 - 3*(a + 2*b)*tan(f*x + e))/b^2 -
3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(
sqrt((a + b)*b)*a*b^2))/f
```

3.343.8 Giac [A] (verification not implemented)

Time = 2.09 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.52

$$\int \frac{\tan^6(e+fx)}{a+b\sec^2(e+fx)}\,dx =$$
$$-\frac{\frac{3(fx+e)}{a} - \frac{3(a^3+3a^2b+3ab^2+b^3)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{\sqrt{ab+b^2}ab^2} - \frac{b^2\tan(fx+e)^3 - 3ab\tan(fx+e) - 6b^2\tan(fx+e)}{b^3}}{3f}$$

input
```
integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
-1/3*(3*(f*x + e)/a - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(pi*floor((f*x + e
)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/(sqrt(a*b + b
^2)*a*b^2) - (b^2*tan(f*x + e)^3 - 3*a*b*tan(f*x + e) - 6*b^2*tan(f*x + e)
)/b^3)/f
```

3.343.9 Mupad [B] (verification not implemented)

Time = 19.88 (sec) , antiderivative size = 1109, normalized size of antiderivative = 13.36

$$\int \frac{\tan^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\tan(e+f\,x)^3}{3\,b\,f}$$

$$-\frac{\operatorname{atan}\left(\frac{40\,a^2\tan(e+f\,x)}{30\,a\,b+40\,a^2+10\,b^2+\frac{30\,a^3}{b}+\frac{12\,a^4}{b^2}+\frac{2\,a^5}{b^3}}+\frac{30\,a^3\tan(e+f\,x)}{30\,a\,b^2+40\,a^2\,b+30\,a^3+10\,b^3+\frac{12\,a^4}{b}+\frac{2\,a^5}{b^2}}+\frac{12\,a^4\tan(e+f\,x)}{30\,a\,b^3+30\,a^3\,b+12\,a^4+10\,b^4+40\,a^2\,b^2+\frac{2}{}}\right.}{}$$

$$-\frac{\tan(e+f\,x)\,(a+2\,b)}{b^2\,f}$$

$$-\frac{\operatorname{atan}\left(\frac{\sqrt{-b^5\,(a+b)^5}\left(\frac{2\tan(e+f\,x)\left(a^6+6\,a^5\,b+15\,a^4\,b^2+20\,a^3\,b^3+15\,a^2\,b^4+6\,a\,b^5+2\,b^6\right)}{b^3}+\frac{\sqrt{-b^5\,(a+b)^5}\left(\frac{4\,a^4\,b^3+12\,a^3\,b^4+8\,a^2}{b^3}\right.\right.}{2\,a\,b^5}\right.}{\frac{2\left(a^5+6\,a^4\,b+15\,a^3\,b^2+19\,a^2\,b^3+12\,a\,b^4+3\,b^5\right)}{b^3}-\frac{\sqrt{-b^5\,(a+b)^5}\left(\frac{2\tan(e+f\,x)\left(a^6+6\,a^5\,b+15\,a^4\,b^2+20\,a^3\,b^3+15\,a^2\,b^4+6\,a\,b^5+2\,b^6\right)}{b^3}+\frac{\sqrt{-b^5\,(a+b}\right.}{2\,a\,b^5}}\right.}$$

input
```
int(tan(e + f*x)^6/(a + b/cos(e + f*x)^2),x)
```

output

```
tan(e + f*x)^3/(3*b*f) - atan((40*a^2*tan(e + f*x))/(30*a*b + 40*a^2 + 10*
b^2 + (30*a^3)/b + (12*a^4)/b^2 + (2*a^5)/b^3) + (30*a^3*tan(e + f*x))/(30
*a*b^2 + 40*a^2*b + 30*a^3 + 10*b^3 + (12*a^4)/b + (2*a^5)/b^2) + (12*a^4*
tan(e + f*x))/(30*a*b^3 + 30*a^3*b + 12*a^4 + 10*b^4 + 40*a^2*b^2 + (2*a^5
)/b) + (2*a^5*tan(e + f*x))/(30*a*b^4 + 12*a^4*b + 2*a^5 + 10*b^5 + 40*a^2
*b^3 + 30*a^3*b^2) + (10*b^2*tan(e + f*x))/(30*a*b + 40*a^2 + 10*b^2 + (30
*a^3)/b + (12*a^4)/b^2 + (2*a^5)/b^3) + (30*a*b*tan(e + f*x))/(30*a*b + 40
*a^2 + 10*b^2 + (30*a^3)/b + (12*a^4)/b^2 + (2*a^5)/b^3))/(a*f) - (tan(e +
 f*x)*(a + 2*b))/(b^2*f) - (atan((((-b^5*(a + b)^5)^(1/2)*((2*tan(e + f*x)
*(6*a*b^5 + 6*a^5*b + a^6 + 2*b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2))
/b^3 + ((-b^5*(a + b)^5)^(1/2)*((8*a^2*b^5 + 12*a^3*b^4 + 4*a^4*b^3)/b^3 +
 (tan(e + f*x)*(8*a^2*b^6 + 4*a^3*b^5)*(-b^5*(a + b)^5)^(1/2))/(a*b^8)))/(
2*a*b^5))*1i)/(2*a*b^5) + ((-b^5*(a + b)^5)^(1/2)*((2*tan(e + f*x)*(6*a*b^
5 + 6*a^5*b + a^6 + 2*b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2))/b^3 - (
(-b^5*(a + b)^5)^(1/2)*((8*a^2*b^5 + 12*a^3*b^4 + 4*a^4*b^3)/b^3 - (tan(e
+ f*x)*(8*a^2*b^6 + 4*a^3*b^5)*(-b^5*(a + b)^5)^(1/2))/(a*b^8)))/(2*a*b^5)
)*1i)/(2*a*b^5))/((2*(12*a*b^4 + 6*a^4*b + a^5 + 3*b^5 + 19*a^2*b^3 + 15*a
^3*b^2))/b^3 - ((-b^5*(a + b)^5)^(1/2)*((2*tan(e + f*x)*(6*a*b^5 + 6*a^5*b
 + a^6 + 2*b^6 + 15*a^2*b^4 + 20*a^3*b^3 + 15*a^4*b^2))/b^3 + ((-b^5*(a +
b)^5)^(1/2)*((8*a^2*b^5 + 12*a^3*b^4 + 4*a^4*b^3)/b^3 + (tan(e + f*x)*(...
```

3.344 $\int \frac{\tan^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$

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3.344.2 Mathematica [C] (verified) . . . 2499
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3.344.1 Optimal result

Integrand size = 23, antiderivative size = 59

$$\int \frac{\tan^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{x}{a} - \frac{(a+b)^{3/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{ab^{3/2}f} + \frac{\tan(e+fx)}{bf}$$

output
```
x/a-(a+b)^(3/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a/b^(3/2)/f+tan(f*x
+e)/b/f
```

3.344.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.12 (sec) , antiderivative size = 206, normalized size of antiderivative = 3.49

$$\int \frac{\tan^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left((a+b)^2\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right.\right.}{2ab\sqrt{a+b}f\,(a+b\sec^2(e+fx))\sqrt{}}$$

input
```
Integrate[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*((a + b)^2*ArcTan[(Sec[f*x]
*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sq
rt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]) + Sqrt[a
 + b]*Sqrt[b*(I*Cos[e] + Sin[e])^4]*(b*f*x + a*Sec[e]*Sec[e + f*x]*Sin[f*x
]))))/(2*a*b*Sqrt[a + b]*f*(a + b*Sec[e + f*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e]
)^4])
```

3.344.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4629, 2075, 381, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\tan(e+fx)^4}{a+b\sec(e+fx)^2}\,dx$$

$$\downarrow 4629$$

$$\frac{\int \frac{\tan^4(e+fx)}{(\tan^2(e+fx)+1)(a+b(\tan^2(e+fx)+1))}\,d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\tan^4(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)}\,d\tan(e+fx)}{f}$$

$$\downarrow 381$$

$$\frac{\frac{\tan(e+fx)}{b} - \frac{\int \frac{(a+2b)\tan^2(e+fx)+a+b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}\,d\tan(e+fx)}{b}}{f}$$

$$\downarrow 397$$

$$\frac{\frac{\tan(e+fx)}{b} - \frac{\frac{(a+b)^2\int \frac{1}{b\tan^2(e+fx)+a+b}\,d\tan(e+fx)}{a} - b\int \frac{1}{\tan^2(e+fx)+1}\,d\tan(e+fx)}{a}}{b}}{f}$$

$$\downarrow 216$$

$$\frac{\frac{\tan(e+fx)}{b} - \frac{\frac{(a+b)^2 \int \frac{1}{b\tan^2(e+fx)+a+b}\, d\tan(e+fx)}{a} - \frac{b\arctan(\tan(e+fx))}{a}}{b}}{f}$$

$$\downarrow 218$$

$$\frac{\frac{\tan(e+fx)}{b} - \frac{\frac{(a+b)^{3/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{b}} - \frac{b\arctan(\tan(e+fx))}{a}}{b}}{f}$$

input
```
Int[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

output
```
(-((-((b*ArcTan[Tan[e + f*x]])/a) + ((a + b)^(3/2)*ArcTan[(Sqrt[b]*Tan[e +
 f*x])/Sqrt[a + b]])/(a*Sqrt[b]))/b) + Tan[e + f*x]/b)/f
```

3.344.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 381
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(b*d*(m + 2*(p + q) + 1))), x] - Simp[e^4/(b*d*(m + 2*(p + q) + 1))
Int[(e*x)^(m - 4)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*c*(m - 3) + (a*d*(m +
2*q - 1) + b*c*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 3] && IntBinomialQ[a, b, c, d, e, m, 2
, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.344.4 Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.22

method	result
derivativedivides	$\frac{\frac{\tan(fx+e)}{b}+\frac{\arctan(\tan(fx+e))}{a}+\frac{\left(-a^2-2ab-b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{ab\sqrt{(a+b)b}}}{f}$
default	$\frac{\frac{\tan(fx+e)}{b}+\frac{\arctan(\tan(fx+e))}{a}+\frac{\left(-a^2-2ab-b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{ab\sqrt{(a+b)b}}}{f}$
risch	$\frac{x}{a}+\frac{2i}{fb\left(e^{2i(fx+e)}+1\right)}+\frac{\sqrt{-(a+b)b}\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-(a+b)b}+a+2b}{a}\right)}{2b^2f}+\frac{\sqrt{-(a+b)b}\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-(a+b)b}+}{a}\right.}{2bfa}$

input
```
int(tan(f*x+e)^4/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(tan(f*x+e)/b+1/a*arctan(tan(f*x+e))+(-a^2-2*a*b-b^2)/a/b/((a+b)*b)^(1
/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))
```

3.344.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(51) = 102$.

Time = 0.28 (sec) , antiderivative size = 297, normalized size of antiderivative = 5.03

$$\int \frac{\tan^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$= \left[\frac{4\,bfx\cos(fx+e) + (a+b)\sqrt{-\frac{a+b}{b}}\cos(fx+e)\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4 - 2\,(3\,ab+4\,b^2)\cos(fx+e)^2 + 4\left((ab+2\,b^2\right.\right.}{a^2\cos(fx+e)^4 + 2\,ab\cos(}}{4\,abf\cos(fx+e)}\right.$$

input
```
integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/4*(4*b*f*x*cos(f*x + e) + (a + b)*sqrt(-(a + b)/b)*cos(f*x + e)*log(((a
^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*
((a*b + 2*b^2)*cos(f*x + e)^3 - b^2*cos(f*x + e))*sqrt(-(a + b)/b)*sin(f*x
 + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) + 4*a*sin(
f*x + e))/(a*b*f*cos(f*x + e)), 1/2*(2*b*f*x*cos(f*x + e) + (a + b)*sqrt((
a + b)/b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt((a + b)/b)/((a +
b)*cos(f*x + e)*sin(f*x + e)))*cos(f*x + e) + 2*a*sin(f*x + e))/(a*b*f*cos
(f*x + e))]
```

3.344.6 Sympy [F]

$$\int \frac{\tan^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\tan^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

input
```
integrate(tan(f*x+e)**4/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(tan(e + f*x)**4/(a + b*sec(e + f*x)**2), x)
```

3.344.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.12

$$\int \frac{\tan^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{fx+e}{a} + \frac{\tan(fx+e)}{b} - \frac{(a^2+2ab+b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}ab}}{f}$$

input
```
integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
((f*x + e)/a + tan(f*x + e)/b - (a^2 + 2*a*b + b^2)*arctan(b*tan(f*x + e)/
sqrt((a + b)*b))/(sqrt((a + b)*b)*a*b))/f
```

3.344.8 Giac [A] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.47

$$\int \frac{\tan^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{fx+e}{a} + \frac{\tan(fx+e)}{b} - \frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(a^2+2ab+b^2)}{\sqrt{ab+b^2}ab}}{f}$$

input
```
integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
((f*x + e)/a + tan(f*x + e)/b - (pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arc
tan(b*tan(f*x + e)/sqrt(a*b + b^2)))*(a^2 + 2*a*b + b^2)/(sqrt(a*b + b^2)*
a*b))/f
```

3.344.9 Mupad [B] (verification not implemented)

Time = 19.16 (sec) , antiderivative size = 410, normalized size of antiderivative = 6.95

$$\int \frac{\tan^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{\operatorname{atan}\left(\frac{8a^2\tan(e+fx)}{12ab+8a^2+6b^2+\frac{2a^3}{b}} + \frac{2a^3\tan(e+fx)}{2a^3+8a^2b+12ab^2+6b^3} + \frac{6b^2\tan(e+fx)}{12ab+8a^2+6b^2+\frac{2a^3}{b}} + \frac{12ab\tan(e+fx)}{12ab+8a^2+6b^2+\frac{2a^3}{b}}\right)}{af}$$
$$+ \frac{\tan(e+fx)}{bf}$$
$$+ \frac{\operatorname{atanh}\left(\frac{6\tan(e+fx)\sqrt{-a^3b^3-3a^2b^4-3ab^5-b^6}}{18ab^2+20a^2b+10a^3+6b^3+\frac{2a^4}{b}} + \frac{6a\tan(e+fx)\sqrt{-a^3b^3-3a^2b^4-3ab^5-b^6}}{2a^4+10a^3b+20a^2b^2+18ab^3+6b^4} + \frac{2a^2\tan(e+fx)\sqrt{-a^3b^3-3a^2b^4-3}}{2a^4b+10a^3b^2+20a^2b^3+18ab^4+}\right.}{ab^3f}$$

input
```
int(tan(e + f*x)^4/(a + b/cos(e + f*x)^2),x)
```

output
```
atan((8*a^2*tan(e + f*x))/(12*a*b + 8*a^2 + 6*b^2 + (2*a^3)/b) + (2*a^3*ta
n(e + f*x))/(12*a*b^2 + 8*a^2*b + 2*a^3 + 6*b^3) + (6*b^2*tan(e + f*x))/(1
2*a*b + 8*a^2 + 6*b^2 + (2*a^3)/b) + (12*a*b*tan(e + f*x))/(12*a*b + 8*a^2
 + 6*b^2 + (2*a^3)/b))/(a*f) + tan(e + f*x)/(b*f) + (atanh((6*tan(e + f*x)
*(- 3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*b^3)^(1/2))/(18*a*b^2 + 20*a^2*b + 10*
a^3 + 6*b^3 + (2*a^4)/b) + (6*a*tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4
- a^3*b^3)^(1/2))/(18*a*b^3 + 10*a^3*b + 2*a^4 + 6*b^4 + 20*a^2*b^2) + (2*
a^2*tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*b^3)^(1/2))/(18*a*b^4
+ 2*a^4*b + 6*b^5 + 20*a^2*b^3 + 10*a^3*b^2))*(-b^3*(a + b)^3)^(1/2))/(a*b
^3*f)
```

3.345 $\int \frac{\tan^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$

3.345.1 Optimal result . . . 2506
3.345.2 Mathematica [C] (verified) . . . 2506
3.345.3 Rubi [A] (verified) . . . 2507
3.345.4 Maple [A] (verified) . . . 2509
3.345.5 Fricas [A] (verification not implemented) . . . 2510
3.345.6 Sympy [F] . . . 2510
3.345.7 Maxima [A] (verification not implemented) . . . 2511
3.345.8 Giac [A] (verification not implemented) . . . 2511
3.345.9 Mupad [B] (verification not implemented) . . . 2511

3.345.1 Optimal result

Integrand size = 23, antiderivative size = 46

$$\int \frac{\tan^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{x}{a} + \frac{\sqrt{a+b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{b}f}$$

output
```
-x/a+arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*(a+b)^(1/2)/a/f/b^(1/2)
```

3.345.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.76 (sec) , antiderivative size = 184, normalized size of antiderivative = 4.00

$$\int \frac{\tan^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(\sqrt{a+b}fx\sqrt{b(\cos(e)-i\sin(e))^4}+(a+b)\arctan\left(\frac{\sec(fx)(\cos($$
$$2a\sqrt{a+b}f\,(a+b\sec^2(e+fx))\sqrt{b(\cos(e)-i\sin$$

input
```
Integrate[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

output
```
-1/2*((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(Sqrt[a + b]*f*x*Sqrt[
b*(Cos[e] - I*Sin[e])^4] + (a + b)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e]
)*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e
] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e])))/(a*Sqrt[a + b]*f*(a + b*Sec[e
 + f*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

3.345.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4629, 2075, 383, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\tan(e+fx)^2}{a+b\sec(e+fx)^2}\,dx$$

$$\downarrow 4629$$

$$\frac{\int \frac{\tan^2(e+fx)}{(\tan^2(e+fx)+1)(a+b(\tan^2(e+fx)+1))}\,d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\tan^2(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)}\,d\tan(e+fx)}{f}$$

$$\downarrow 383$$

$$\frac{\frac{(a+b)\int \frac{1}{b\tan^2(e+fx)+a+b}\,d\tan(e+fx)}{a} - \frac{\int \frac{1}{\tan^2(e+fx)+1}\,d\tan(e+fx)}{a}}{f}$$

$$\downarrow 216$$

$$\frac{\frac{(a+b)\int \frac{1}{b\tan^2(e+fx)+a+b}\,d\tan(e+fx)}{a} - \frac{\arctan(\tan(e+fx))}{a}}{f}$$

$$\downarrow 218$$

$$\frac{\frac{\sqrt{a+b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{b}} - \frac{\arctan(\tan(e+fx))}{a}}{f}$$

input
```
Int[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

output
```
(-(ArcTan[Tan[e + f*x]]/a) + (Sqrt[a + b]*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sq
rt[a + b]])/(a*Sqrt[b]))/f
```

3.345.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 383
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Sym
bol] :> Simp[(-a)*(e^2/(b*c - a*d))   Int[(e*x)^(m - 2)/(a + b*x^2), x], x]
 + Simp[c*(e^2/(b*c - a*d))   Int[(e*x)^(m - 2)/(c + d*x^2), x], x] /; Free
Q[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LeQ[2, m, 3]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.345.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{-\frac{\arctan(\tan(fx+e))}{a}+\frac{(a+b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a\sqrt{(a+b)b}}}{f}$	48
default	$\frac{-\frac{\arctan(\tan(fx+e))}{a}+\frac{(a+b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a\sqrt{(a+b)b}}}{f}$	48
risch	$-\frac{x}{a}-\frac{\sqrt{-(a+b)b}\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-(a+b)b}+a+2b}{a}\right)}{2bfa}+\frac{\sqrt{-(a+b)b}\ln\left(e^{2i(fx+e)}-\frac{2i\sqrt{-(a+b)b}-a-2b}{a}\right)}{2bfa}$	111

input
```
int(tan(f*x+e)^2/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/a*arctan(tan(f*x+e))+(a+b)/a/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/(
(a+b)*b)^(1/2)))
```

3.345.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 226, normalized size of antiderivative = 4.91

$$\int \frac{\tan^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$= \left[-\frac{4\,fx - \sqrt{-\frac{a+b}{b}}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4 - 2\,(3\,ab+4\,b^2)\cos(fx+e)^2 - 4\left((ab+2\,b^2)\cos(fx+e)^3 - b^2\cos(fx+e)\right)\sqrt{-\frac{a+b}{b}}\sin(}{a^2\cos(fx+e)^4 + 2\,ab\cos(fx+e)^2 + b^2}\right.}{4\,af}\right.$$

$$\left. -\frac{2\,fx + \sqrt{\frac{a+b}{b}}\arctan\left(\frac{\left((a+2\,b)\cos(fx+e)^2 - b\right)\sqrt{\frac{a+b}{b}}}{2\,(a+b)\cos(fx+e)\sin(fx+e)}\right)}{2\,af}\right]$$

input
```
integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[-1/4*(4*f*x - sqrt(-(a + b)/b)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4
- 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a*b + 2*b^2)*cos(f*x + e)^3 - b^2
*cos(f*x + e))*sqrt(-(a + b)/b)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 +
2*a*b*cos(f*x + e)^2 + b^2)))/(a*f), -1/2*(2*f*x + sqrt((a + b)/b)*arctan(
1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt((a + b)/b)/((a + b)*cos(f*x + e)*s
in(f*x + e))))/(a*f)]
```

3.345.6 Sympy [F]

$$\int \frac{\tan^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\tan^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

input
```
integrate(tan(f*x+e)**2/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(tan(e + f*x)**2/(a + b*sec(e + f*x)**2), x)
```

3.345.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \frac{\tan^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{(a+b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}a} - \frac{fx+e}{a}}{f}$$

input
```
integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
((a + b)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a) - (f*x
 + e)/a)/f
```

3.345.8 Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.43

$$\int \frac{\tan^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(a+b)}{\sqrt{ab+b^2}a} - \frac{fx+e}{a}}{f}$$

input
```
integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b
^2)))*(a + b)/(sqrt(a*b + b^2)*a) - (f*x + e)/a)/f
```

3.345.9 Mupad [B] (verification not implemented)

Time = 19.66 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.74

$$\int \frac{\tan^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\operatorname{atan}\left(\frac{2\,a\,b^2\tan(e+f\,x)}{2\,a^2\,b+2\,a\,b^2}+\frac{2\,a^2\,b\tan(e+f\,x)}{2\,a^2\,b+2\,a\,b^2}\right)}{a\,f} - \frac{\operatorname{atanh}\left(\frac{2\,a\,b^2\tan(e+f\,x)\sqrt{-b^2-a\,b}}{2\,a^2\,b^2+2\,a\,b^3}\right)\sqrt{-b\,(a+b)}}{a\,b\,f}$$

input
```
int(tan(e + f*x)^2/(a + b/cos(e + f*x)^2),x)
```

output
```
- atan((2*a*b^2*tan(e + f*x))/(2*a*b^2 + 2*a^2*b) + (2*a^2*b*tan(e + f*x))
/(2*a*b^2 + 2*a^2*b))/(a*f) - (atanh((2*a*b^2*tan(e + f*x)*(- a*b - b^2)^(
1/2))/(2*a*b^3 + 2*a^2*b^2))*(-b*(a + b))^(1/2))/(a*b*f)
```

3.346 $\int \frac{1}{a+b\sec^2(e+fx)}\,dx$

3.346.1 Optimal result . . . 2513
3.346.2 Mathematica [C] (verified) . . . 2513
3.346.3 Rubi [A] (verified) . . . 2514
3.346.4 Maple [A] (verified) . . . 2515
3.346.5 Fricas [A] (verification not implemented) . . . 2516
3.346.6 Sympy [F] . . . 2516
3.346.7 Maxima [A] (verification not implemented) . . . 2517
3.346.8 Giac [A] (verification not implemented) . . . 2517
3.346.9 Mupad [B] (verification not implemented) . . . 2517

3.346.1 Optimal result

Integrand size = 14, antiderivative size = 45

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx = \frac{x}{a} + \frac{\sqrt{b}\arctan\left(\frac{\sqrt{a+b}\cot(e+fx)}{\sqrt{b}}\right)}{a\sqrt{a+b}f}$$

output
```
x/a+arctan(cot(f*x+e)*(a+b)^(1/2)/b^(1/2))*b^(1/2)/a/f/(a+b)^(1/2)
```

3.346.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 182, normalized size of antiderivative = 4.04

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(\sqrt{a+b}fx\sqrt{b(\cos(e)-i\sin(e))^4}+b\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e}{2\sqrt{a+b}}\right.\right.}{2a\sqrt{a+b}f\,(a+b\sec^2(e+fx))\sqrt{b(\cos(e)-i\sin(e))^4}}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(-1),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(Sqrt[a + b]*f*x*Sqrt[b*(Co
s[e] - I*Sin[e])^4] + b*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2
*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e
])^4])]*(Cos[2*e] - I*Sin[2*e])))/(2*a*Sqrt[a + b]*f*(a + b*Sec[e + f*x]^2
)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

3.346.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 4615, 3042, 3660, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{a+b\sec(e+fx)^2}\,dx$$
$$\downarrow 4615$$
$$\frac{x}{a} - \frac{b\int \frac{1}{a\cos^2(e+fx)+b}\,dx}{a}$$
$$\downarrow 3042$$
$$\frac{x}{a} - \frac{b\int \frac{1}{a\sin\left(e+fx+\frac{\pi}{2}\right)^2+b}\,dx}{a}$$
$$\downarrow 3660$$
$$\frac{b\int \frac{1}{(a+b)\cot^2(e+fx)+b}\,d\cot(e+fx)}{af} + \frac{x}{a}$$
$$\downarrow 218$$
$$\frac{\sqrt{b}\arctan\left(\frac{\sqrt{a+b}\cot(e+fx)}{\sqrt{b}}\right)}{af\sqrt{a+b}} + \frac{x}{a}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^(-1),x]
```

output
```
x/a + (Sqrt[b]*ArcTan[(Sqrt[a + b]*Cot[e + f*x])/Sqrt[b]])/(a*Sqrt[a + b]*
f)
```

3.346.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f  Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

rule 4615
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> Simp[x/a, x
] - Simp[b/a  Int[1/(b + a*Cos[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f},
x] && NeQ[a + b, 0]
```

3.346.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

method	result	size
derivativedivides	$\frac{-\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a\sqrt{(a+b)b}}+\frac{\arctan(\tan(fx+e))}{a}}{f}$	46
default	$\frac{-\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{a\sqrt{(a+b)b}}+\frac{\arctan(\tan(fx+e))}{a}}{f}$	46
risch	$\frac{x}{a}+\frac{\sqrt{-(a+b)b}\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-(a+b)b}+a+2b}{a}\right)}{2(a+b)fa}-\frac{\sqrt{-(a+b)b}\ln\left(e^{2i(fx+e)}-\frac{2i\sqrt{-(a+b)b}-a-2b}{a}\right)}{2(a+b)fa}$	114

input
```
int(1/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-b/a/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))+1/a*arctan(
tan(f*x+e)))
```

3.346.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 231, normalized size of antiderivative = 5.13

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx$$
$$= \left[\frac{4\,fx+\sqrt{-\frac{b}{a+b}}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4-2\,(3\,ab+4\,b^2)\cos(fx+e)^2+4\left((a^2+3\,ab+2\,b^2)\cos(fx+e)^3-(ab+b^2)\cos(fx+e)\right)\sqrt{-}}{a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2}\right.}{4\,af}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/4*(4*f*x + sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 -
 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^
3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*co
s(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)))/(a*f), 1/2*(2*f*x + sqrt(b/(a
 + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*
x + e)*sin(f*x + e))))/(a*f)]
```

3.346.6 Sympy [F]

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx = \int \frac{1}{a+b\sec^2(e+fx)}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(1/(a + b*sec(e + f*x)**2), x)
```

3.346.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{b\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}a} - \frac{fx+e}{a}}{f}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
-(b*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a) - (f*x + e)
/a)/f
```

3.346.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.44

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b}{\sqrt{ab+b^2}a} - \frac{fx+e}{a}}{f}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
-((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b +
b^2)))*b/(sqrt(a*b + b^2)*a) - (f*x + e)/a)/f
```

3.346.9 Mupad [B] (verification not implemented)

Time = 19.37 (sec) , antiderivative size = 460, normalized size of antiderivative = 10.22

$$\int \frac{1}{a+b\sec^2(e+fx)}\,dx = \frac{x}{a} - \frac{\mathrm{atan}\left(\frac{\frac{\left(2\,b^3\tan(e+f\,x)-\frac{\left(2\,a^2\,b^2-\frac{\tan(e+f\,x)\left(8\,a^3\,b^2+16\,a^2\,b^3\right)\sqrt{-b\,(a+b)}}{4\left(a^2+b\,a\right)}\right)\sqrt{-b\,(a+b)}}{2\left(a^2+b\,a\right)}\right)\sqrt{-b\,(a+b)}\,1i}{a^2+b\,a}+\frac{\left(2\,b^3\tan(e+f\,x)+\frac{\left(2\,a^2\,b^2+\frac{\tan(e+f\,x)\left(8 \cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}}{\frac{\left(2\,b^3\tan(e+f\,x)-\frac{\left(2\,a^2\,b^2-\frac{\tan(e+f\,x)\left(8\,a^3\,b^2+16\,a^2\,b^3\right)\sqrt{-b\,(a+b)}}{4\left(a^2+b\,a\right)}\right)\sqrt{-b\,(a+b)}}{2\left(a^2+b\,a\right)}\right)\sqrt{-b\,(a+b)}}{a^2+b\,a}-\frac{\left(2\,b^3\tan(e+f\,x)+\frac{\left(2\,a^2\,b^2+\frac{\tan(e+f\,x)\left(8 \cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}}\right)\cdots}{f\,\left(a^2+b\,a\right)}$$

input
```
int(1/(a + b/cos(e + f*x)^2),x)
```

output
```
x/a - (atan((((2*b^3*tan(e + f*x) - ((2*a^2*b^2 - (tan(e + f*x)*(16*a^2*b^
3 + 8*a^3*b^2)*(-b*(a + b))^(1/2))/(4*(a*b + a^2)))*(-b*(a + b))^(1/2))/(2
*(a*b + a^2)))*(-b*(a + b))^(1/2)*1i)/(a*b + a^2) + ((2*b^3*tan(e + f*x) +
 ((2*a^2*b^2 + (tan(e + f*x)*(16*a^2*b^3 + 8*a^3*b^2)*(-b*(a + b))^(1/2))/
(4*(a*b + a^2)))*(-b*(a + b))^(1/2))/(2*(a*b + a^2)))*(-b*(a + b))^(1/2)*1
i)/(a*b + a^2))/(((2*b^3*tan(e + f*x) - ((2*a^2*b^2 - (tan(e + f*x)*(16*a^
2*b^3 + 8*a^3*b^2)*(-b*(a + b))^(1/2))/(4*(a*b + a^2)))*(-b*(a + b))^(1/2)
)/(2*(a*b + a^2)))*(-b*(a + b))^(1/2))/(a*b + a^2) - ((2*b^3*tan(e + f*x)
+ ((2*a^2*b^2 + (tan(e + f*x)*(16*a^2*b^3 + 8*a^3*b^2)*(-b*(a + b))^(1/2))
/(4*(a*b + a^2)))*(-b*(a + b))^(1/2))/(2*(a*b + a^2)))*(-b*(a + b))^(1/2))
/(a*b + a^2)))*(-b*(a + b))^(1/2)*1i)/(f*(a*b + a^2))
```

3.347 $\int \frac{\cot^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$

3.347.1 Optimal result . . . 2519
3.347.2 Mathematica [C] (verified) . . . 2519
3.347.3 Rubi [A] (verified) . . . 2520
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3.347.7 Maxima [A] (verification not implemented) . . . 2524
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3.347.1 Optimal result

Integrand size = 23, antiderivative size = 62

$$\int \frac{\cot^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{x}{a} + \frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a(a+b)^{3/2}f} - \frac{\cot(e+fx)}{(a+b)f}$$

output
```
-x/a+b^(3/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a/(a+b)^(3/2)/f-cot(f*
x+e)/(a+b)/f
```

3.347.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.25 (sec) , antiderivative size = 204, normalized size of antiderivative = 3.29

$$\int \frac{\cot^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(b^2\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(c}{2a(a+b)^{3/2}f\,(a+b\sec^2(e+fx)}$$

input
```
Integrate[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

output
```
-1/2*((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(b^2*ArcTan[(Sec[f*x]*
(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqr
t[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]) + Sqrt[a
+ b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]*((a + b)*f*x - a*Csc[e]*Csc[e + f*x]*Si
n[f*x])))/(a*(a + b)^(3/2)*f*(a + b*Sec[e + f*x]^2)*Sqrt[b*(Cos[e] - I*Sin
[e])^4])
```

3.347.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.24, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4629, 2075, 382, 25, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\tan(e+fx)^2\left(a+b\sec(e+fx)^2\right)}dx$$

$$\downarrow 4629$$

$$\frac{\int \frac{\cot^2(e+fx)}{(\tan^2(e+fx)+1)(a+b(\tan^2(e+fx)+1))}d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\cot^2(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)}d\tan(e+fx)}{f}$$

$$\downarrow 382$$

$$\frac{\frac{\int -\frac{b\tan^2(e+fx)+a+2b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{a+b} - \frac{\cot(e+fx)}{a+b}}{f}$$

$$\downarrow 25$$

$$\frac{-\frac{\int \frac{b\tan^2(e+fx)+a+2b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{a+b} - \frac{\cot(e+fx)}{a+b}}{f}$$

$$\downarrow 397$$

$$\frac{-\frac{(a+b)\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}-\frac{b^2\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{a+b}-\frac{\cot(e+fx)}{a+b}}{f}$$

$$\downarrow 216$$

$$\frac{-\frac{(a+b)\arctan(\tan(e+fx))}{a}-\frac{b^2\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{a+b}-\frac{\cot(e+fx)}{a+b}}{f}$$

$$\downarrow 218$$

$$\frac{-\frac{(a+b)\arctan(\tan(e+fx))}{a}-\frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{a+b}-\frac{\cot(e+fx)}{a+b}}{f}$$

input
```
Int[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2),x]
```

output
```
(-((((a + b)*ArcTan[Tan[e + f*x]])/a - (b^(3/2)*ArcTan[(Sqrt[b]*Tan[e + f*
x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(a + b)) - Cot[e + f*x]/(a + b))/f
```

3.347.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.347.4 Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{\frac{b^2 \arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a+b)a\sqrt{(a+b)b}} - \frac{1}{(a+b)\tan(fx+e)} - \frac{\arctan(\tan(fx+e))}{a}}{f}$
default	$\frac{\frac{b^2 \arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a+b)a\sqrt{(a+b)b}} - \frac{1}{(a+b)\tan(fx+e)} - \frac{\arctan(\tan(fx+e))}{a}}{f}$
risch	$-\frac{x}{a} - \frac{2i}{f(a+b)\left(e^{2i(fx+e)}-1\right)} - \frac{\sqrt{-(a+b)b}\,b\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-(a+b)b}+a+2b}{a}\right)}{2(a+b)^2 fa} + \frac{\sqrt{-(a+b)b}\,b\ln\left(e^{2i(fx+e)}-\frac{2i\sqrt{}\right.}{2(a+b)^2 fa}$

input
```
int(cot(f*x+e)^2/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/(a+b)*b^2/a/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))-1/
(a+b)/tan(f*x+e)-1/a*arctan(tan(f*x+e)))
```

3.347.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(54) = 108$.

Time = 0.28 (sec) , antiderivative size = 310, normalized size of antiderivative = 5.00

$$\int \frac{\cot^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$= \left[-\frac{4\,(a+b)fx\sin(fx+e) - b\sqrt{-\frac{b}{a+b}}\log\left(\frac{(a^2+8\,ab+8\,b^2)\cos(fx+e)^4 - 2\,(3\,ab+4\,b^2)\cos(fx+e)^2 - 4\left((a^2+3\,ab+2\,b^2)\cos(fx+e)^2+b\right.\right.}{a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b}}{4\,(a^2+ab)f\sin(fx+e)} \right.$$

$$\left. -\frac{2\,(a+b)fx\sin(fx+e) + b\sqrt{\frac{b}{a+b}}\arctan\left(\frac{\left((a+2\,b)\cos(fx+e)^2-b\right)\sqrt{\frac{b}{a+b}}}{2\,b\cos(fx+e)\sin(fx+e)}\right)\sin(fx+e) + 2\,a\cos(fx+e)}{2\,(a^2+ab)f\sin(fx+e)} \right]$$

input
```
integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[-1/4*(4*(a + b)*f*x*sin(f*x + e) - b*sqrt(-b/(a + b))*log(((a^2 + 8*a*b +
 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*
b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin
(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*
x + e) + 4*a*cos(f*x + e))/((a^2 + a*b)*f*sin(f*x + e)), -1/2*(2*(a + b)*f
*x*sin(f*x + e) + b*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 -
 b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) + 2*a*cos(
f*x + e))/((a^2 + a*b)*f*sin(f*x + e))]
```

3.347.6 Sympy [F]

$$\int \frac{\cot^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\cot^2(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

input
```
integrate(cot(f*x+e)**2/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(cot(e + f*x)**2/(a + b*sec(e + f*x)**2), x)
```

3.347.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.06

$$\int \frac{\cot^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{b^2 \arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^2+ab)\sqrt{(a+b)b}} - \frac{fx+e}{a} - \frac{1}{(a+b)\tan(fx+e)}}{f}$$

input
```
integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
(b^2*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^2 + a*b)*sqrt((a + b)*b))
- (f*x + e)/a - 1/((a + b)*tan(f*x + e)))/f
```

3.347.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.40

$$\int \frac{\cot^2(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b^2}{(a^2+ab)\sqrt{ab+b^2}} - \frac{fx+e}{a} - \frac{1}{(a+b)\tan(fx+e)}}{f}$$

input
```
integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b
^2)))*b^2/((a^2 + a*b)*sqrt(a*b + b^2)) - (f*x + e)/a - 1/((a + b)*tan(f*x
 + e)))/f
```

3.347.9 Mupad [B] (verification not implemented)

Time = 21.80 (sec) , antiderivative size = 637, normalized size of antiderivative = 10.27

$$\int \frac{\cot^2(e+fx)}{a+b\sec^2(e+fx)}\,dx =$$

$$-\frac{a\,b^2+2\,a^2\,b+a^3+a^3\tan(e+f\,x)\,\operatorname{atan}(\tan(e+f\,x))+b^3\tan(e+f\,x)\,\operatorname{atan}(\tan(e+f\,x))+3\,a\,b^2\,\tan$$

input
```
int(cot(e + f*x)^2/(a + b/cos(e + f*x)^2),x)
```

output
```
-(a*b^2 + 2*a^2*b + a^3 - atan((a*tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^
4 - a^3*b^3)^(3/2)*1i + b*tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*
b^3)^(3/2)*2i + b^7*tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*b^3)^(
1/2)*2i + a*b^6*tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*b^3)^(1/2)
*10i + a^6*b*tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*b^3)^(1/2)*1i
 + a^2*b^5*tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*b^3)^(1/2)*21i
+ a^3*b^4*tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*b^3)^(1/2)*24i +
 a^4*b^3*tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*b^3)^(1/2)*16i +
a^5*b^2*tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*b^3)^(1/2)*6i)/(3*
a*b^9 + 18*a^2*b^8 + 46*a^3*b^7 + 65*a^4*b^6 + 55*a^5*b^5 + 28*a^6*b^4 + 8
*a^7*b^3 + a^8*b^2))*tan(e + f*x)*(- 3*a*b^5 - b^6 - 3*a^2*b^4 - a^3*b^3)^
(1/2)*1i + a^3*tan(e + f*x)*atan(tan(e + f*x)) + b^3*tan(e + f*x)*atan(tan
(e + f*x)) + 3*a*b^2*tan(e + f*x)*atan(tan(e + f*x)) + 3*a^2*b*tan(e + f*x
)*atan(tan(e + f*x)))/(a^4*f*tan(e + f*x) + a*b^3*f*tan(e + f*x) + 3*a^3*b
*f*tan(e + f*x) + 3*a^2*b^2*f*tan(e + f*x))
```

3.348 $\int \frac{\cot^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$

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3.348.2 Mathematica [C] (warning: unable to verify) . . . 2526
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3.348.1 Optimal result

Integrand size = 23, antiderivative size = 86

$$\int \frac{\cot^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{x}{a} - \frac{b^{5/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a(a+b)^{5/2}f} + \frac{(a+2b)\cot(e+fx)}{(a+b)^2 f} - \frac{\cot^3(e+fx)}{3(a+b)f}$$

output
```
x/a-b^(5/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a/(a+b)^(5/2)/f+(a+2*b)
*cot(f*x+e)/(a+b)^2/f-1/3*cot(f*x+e)^3/(a+b)/f
```

3.348.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.69 (sec) , antiderivative size = 390, normalized size of antiderivative = 4.53

$$\int \frac{\cot^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(3b^3\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(\cos$$

input
```
Integrate[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(3*b^3*ArcTan[(Sec[f*x]*(Co
s[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a
 + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]) + (Sqrt[a +
b]*Csc[e]*Csc[e + f*x]^3*Sqrt[b*(Cos[e] - I*Sin[e])^4]*(9*(a + b)^2*f*x*Co
s[f*x] - 9*(a + b)^2*f*x*Cos[2*e + f*x] - 3*a^2*f*x*Cos[2*e + 3*f*x] - 6*a
*b*f*x*Cos[2*e + 3*f*x] - 3*b^2*f*x*Cos[2*e + 3*f*x] + 3*a^2*f*x*Cos[4*e +
 3*f*x] + 6*a*b*f*x*Cos[4*e + 3*f*x] + 3*b^2*f*x*Cos[4*e + 3*f*x] - 12*a^2
*Sin[f*x] - 24*a*b*Sin[f*x] - 12*a^2*Sin[2*e + f*x] - 18*a*b*Sin[2*e + f*x
] + 8*a^2*Sin[2*e + 3*f*x] + 14*a*b*Sin[2*e + 3*f*x]))/8))/(6*a*(a + b)^(5
/2)*f*(a + b*Sec[e + f*x]^2)*Sqrt[b*(Cos[e] - I*Sin[e])^4])
```

3.348.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.27, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4629, 2075, 382, 27, 445, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\tan(e+fx)^4\left(a+b\sec(e+fx)^2\right)}dx$$

$$\downarrow 4629$$

$$\frac{\int \frac{\cot^4(e+fx)}{(\tan^2(e+fx)+1)(a+b(\tan^2(e+fx)+1))}d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\cot^4(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)}d\tan(e+fx)}{f}$$

$$\downarrow 382$$

$$\frac{\int -\frac{3\cot^2(e+fx)\left(b\tan^2(e+fx)+a+2b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{3(a+b)} - \frac{\cot^3(e+fx)}{3(a+b)}}{f}$$

$$\downarrow 27$$

$$\frac{-\frac{\int \frac{\cot^2(e+fx)\left(b\tan^2(e+fx)+a+2b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{a+b}-\frac{\cot^3(e+fx)}{3(a+b)}}{f}$$

$$\downarrow 445$$

$$\frac{-\frac{\int \frac{a^2+3ba+3b^2+b(a+2b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{a+b}-\frac{(a+2b)\cot(e+fx)}{a+b}}{a+b}-\frac{\cot^3(e+fx)}{3(a+b)}}{f}$$

$$\downarrow 397$$

$$\frac{-\frac{\frac{(a+b)^2\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}-\frac{b^3\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{a+b}-\frac{(a+2b)\cot(e+fx)}{a+b}}{a+b}-\frac{\cot^3(e+fx)}{3(a+b)}}{f}$$

$$\downarrow 216$$

$$\frac{-\frac{\frac{(a+b)^2\arctan(\tan(e+fx))}{a}-\frac{b^3\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{a+b}-\frac{(a+2b)\cot(e+fx)}{a+b}}{a+b}-\frac{\cot^3(e+fx)}{3(a+b)}}{f}$$

$$\downarrow 218$$

$$\frac{-\frac{\frac{(a+b)^2\arctan(\tan(e+fx))}{a}-\frac{b^{5/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{a+b}-\frac{(a+2b)\cot(e+fx)}{a+b}}{a+b}-\frac{\cot^3(e+fx)}{3(a+b)}}{f}$$

input
```
Int[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2),x]
```

output
```
(-1/3*Cot[e + f*x]^3/(a + b) - (-((((a + b)^2*ArcTan[Tan[e + f*x]])/a - (b
^(5/2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(a + b
)) - ((a + 2*b)*Cot[e + f*x])/(a + b))/(a + b))/f
```

3.348.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f  Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.348.4 Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{\frac{\arctan(\tan(fx+e))}{a}-\frac{1}{3(a+b)\tan(fx+e)^3}-\frac{-a-2b}{(a+b)^2\tan(fx+e)}-\frac{b^3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a+b)^2a\sqrt{(a+b)b}}}{f}$
default	$\frac{\frac{\arctan(\tan(fx+e))}{a}-\frac{1}{3(a+b)\tan(fx+e)^3}-\frac{-a-2b}{(a+b)^2\tan(fx+e)}-\frac{b^3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a+b)^2a\sqrt{(a+b)b}}}{f}$
risch	$\frac{x}{a}+\frac{2i\left(6a\,e^{4i(fx+e)}+9b\,e^{4i(fx+e)}-6a\,e^{2i(fx+e)}-12b\,e^{2i(fx+e)}+4a+7b\right)}{3f(a+b)^2\left(e^{2i(fx+e)}-1\right)^3}+\frac{\sqrt{-(a+b)b}\,b^2\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-(a+b)b}}{a}\right.}{2(a+b)^3fa}$

input
```
int(cot(f*x+e)^4/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/a*arctan(tan(f*x+e))-1/3/(a+b)/tan(f*x+e)^3-(-a-2*b)/(a+b)^2/tan(f*
x+e)-1/(a+b)^2*b^3/a/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))
```

3.348.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. 2(76) = 152.

Time = 0.31 (sec) , antiderivative size = 533, normalized size of antiderivative = 6.20

$$\int \frac{\cot^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$=\left[\frac{4\left(4\,a^2+7\,ab\right)\cos\left(fx+e\right)^3+3\left(b^2\cos\left(fx+e\right)^2-b^2\right)\sqrt{-\frac{b}{a+b}}\log\left(\frac{\left(a^2+8\,ab+8\,b^2\right)\cos(fx+e)^4-2\left(3\,ab+4\,b^2\right)\cos(f\right.}{}\right.$$

input
```
integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[1/12*(4*(4*a^2 + 7*a*b)*cos(f*x + e)^3 + 3*(b^2*cos(f*x + e)^2 - b^2)*sqr
t(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2
)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*c
os(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*
a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) - 12*(a^2 + 2*a*b)*cos(f*x + e) +
12*((a^2 + 2*a*b + b^2)*f*x*cos(f*x + e)^2 - (a^2 + 2*a*b + b^2)*f*x)*sin(
f*x + e))/(((a^3 + 2*a^2*b + a*b^2)*f*cos(f*x + e)^2 - (a^3 + 2*a^2*b + a*
b^2)*f)*sin(f*x + e)), 1/6*(2*(4*a^2 + 7*a*b)*cos(f*x + e)^3 + 3*(b^2*cos(
f*x + e)^2 - b^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b
)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) - 6*(a^2 + 2
*a*b)*cos(f*x + e) + 6*((a^2 + 2*a*b + b^2)*f*x*cos(f*x + e)^2 - (a^2 + 2*
a*b + b^2)*f*x)*sin(f*x + e))/(((a^3 + 2*a^2*b + a*b^2)*f*cos(f*x + e)^2 -
 (a^3 + 2*a^2*b + a*b^2)*f)*sin(f*x + e))]
```

3.348.6 Sympy [F]

$$\int \frac{\cot^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\cot^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

input
```
integrate(cot(f*x+e)**4/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(cot(e + f*x)**4/(a + b*sec(e + f*x)**2), x)
```

3.348.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.23

$$\int \frac{\cot^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{\frac{3\,b^3\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^3+2\,a^2b+ab^2)\sqrt{(a+b)b}} - \frac{3\,(fx+e)}{a} - \frac{3\,(a+2\,b)\tan(fx+e)^2-a-b}{(a^2+2\,ab+b^2)\tan(fx+e)^3}}{3\,f}$$

input
```
integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
-1/3*(3*b^3*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^3 + 2*a^2*b + a*b^2
)*sqrt((a + b)*b)) - 3*(f*x + e)/a - (3*(a + 2*b)*tan(f*x + e)^2 - a - b)/
((a^2 + 2*a*b + b^2)*tan(f*x + e)^3))/f
```

3.348.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.56

$$\int \frac{\cot^4(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= -\frac{\frac{3\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b^3}{(a^3+2a^2b+ab^2)\sqrt{ab+b^2}} - \frac{3(fx+e)}{a} - \frac{3a\tan(fx+e)^2+6b\tan(fx+e)^2-a-b}{(a^2+2ab+b^2)\tan(fx+e)^3}}{3f}$$

input
```
integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
-1/3*(3*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(
a*b + b^2)))*b^3/((a^3 + 2*a^2*b + a*b^2)*sqrt(a*b + b^2)) - 3*(f*x + e)/a
 - (3*a*tan(f*x + e)^2 + 6*b*tan(f*x + e)^2 - a - b)/((a^2 + 2*a*b + b^2)*
tan(f*x + e)^3))/f
```

3.348.9 Mupad [B] (verification not implemented)

Time = 24.73 (sec) , antiderivative size = 2644, normalized size of antiderivative = 30.74

$$\int \frac{\cot^4(e+fx)}{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
int(cot(e + f*x)^4/(a + b/cos(e + f*x)^2),x)
```

output

```
atan((10*b^12*tan(e + f*x))/(80*a*b^11 + 10*b^12 + 290*a^2*b^10 + 630*a^3*
b^9 + 912*a^4*b^8 + 922*a^5*b^7 + 660*a^6*b^6 + 330*a^7*b^5 + 110*a^8*b^4
+ 22*a^9*b^3 + 2*a^10*b^2) + (80*a*b^11*tan(e + f*x))/(80*a*b^11 + 10*b^12
 + 290*a^2*b^10 + 630*a^3*b^9 + 912*a^4*b^8 + 922*a^5*b^7 + 660*a^6*b^6 +
330*a^7*b^5 + 110*a^8*b^4 + 22*a^9*b^3 + 2*a^10*b^2) + (290*a^2*b^10*tan(e
 + f*x))/(80*a*b^11 + 10*b^12 + 290*a^2*b^10 + 630*a^3*b^9 + 912*a^4*b^8 +
 922*a^5*b^7 + 660*a^6*b^6 + 330*a^7*b^5 + 110*a^8*b^4 + 22*a^9*b^3 + 2*a^
10*b^2) + (630*a^3*b^9*tan(e + f*x))/(80*a*b^11 + 10*b^12 + 290*a^2*b^10 +
 630*a^3*b^9 + 912*a^4*b^8 + 922*a^5*b^7 + 660*a^6*b^6 + 330*a^7*b^5 + 110
*a^8*b^4 + 22*a^9*b^3 + 2*a^10*b^2) + (912*a^4*b^8*tan(e + f*x))/(80*a*b^1
1 + 10*b^12 + 290*a^2*b^10 + 630*a^3*b^9 + 912*a^4*b^8 + 922*a^5*b^7 + 660
*a^6*b^6 + 330*a^7*b^5 + 110*a^8*b^4 + 22*a^9*b^3 + 2*a^10*b^2) + (922*a^5
*b^7*tan(e + f*x))/(80*a*b^11 + 10*b^12 + 290*a^2*b^10 + 630*a^3*b^9 + 912
*a^4*b^8 + 922*a^5*b^7 + 660*a^6*b^6 + 330*a^7*b^5 + 110*a^8*b^4 + 22*a^9*
b^3 + 2*a^10*b^2) + (660*a^6*b^6*tan(e + f*x))/(80*a*b^11 + 10*b^12 + 290*
a^2*b^10 + 630*a^3*b^9 + 912*a^4*b^8 + 922*a^5*b^7 + 660*a^6*b^6 + 330*a^7
*b^5 + 110*a^8*b^4 + 22*a^9*b^3 + 2*a^10*b^2) + (330*a^7*b^5*tan(e + f*x))
/(80*a*b^11 + 10*b^12 + 290*a^2*b^10 + 630*a^3*b^9 + 912*a^4*b^8 + 922*a^5
*b^7 + 660*a^6*b^6 + 330*a^7*b^5 + 110*a^8*b^4 + 22*a^9*b^3 + 2*a^10*b^2)
+ (110*a^8*b^4*tan(e + f*x))/(80*a*b^11 + 10*b^12 + 290*a^2*b^10 + 630*...
```

3.349 $\int \frac{\cot^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$

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3.349.1 Optimal result

Integrand size = 23, antiderivative size = 120

$$\int \frac{\cot^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = -\frac{x}{a} + \frac{b^{7/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a(a+b)^{7/2}f} - \frac{(a^2+3ab+3b^2)\cot(e+fx)}{(a+b)^3 f} + \frac{(a+2b)\cot^3(e+fx)}{3(a+b)^2 f} - \frac{\cot^5(e+fx)}{5(a+b)f}$$

output

```
-x/a+b^(7/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a/(a+b)^(7/2)/f-(a^2+3
*a*b+3*b^2)*cot(f*x+e)/(a+b)^3/f+1/3*(a+2*b)*cot(f*x+e)^3/(a+b)^2/f-1/5*co
t(f*x+e)^5/(a+b)/f
```

3.349.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.36 (sec) , antiderivative size = 671, normalized size of antiderivative = 5.59

$$\int \frac{\cot^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\left(-\frac{480b^4\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(\cos(2e)-i}{\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right.$$

input
```
Integrate[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^2*((-480*b^4*ArcTan[(Sec[f*x]
*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sq
rt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4])]*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a
 + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + Csc[e]*Csc[e + f*x]^5*(-150*(a + b)
^3*f*x*Cos[f*x] + 150*(a + b)^3*f*x*Cos[2*e + f*x] + 75*a^3*f*x*Cos[2*e +
3*f*x] + 225*a^2*b*f*x*Cos[2*e + 3*f*x] + 225*a*b^2*f*x*Cos[2*e + 3*f*x] +
 75*b^3*f*x*Cos[2*e + 3*f*x] - 75*a^3*f*x*Cos[4*e + 3*f*x] - 225*a^2*b*f*x
*Cos[4*e + 3*f*x] - 225*a*b^2*f*x*Cos[4*e + 3*f*x] - 75*b^3*f*x*Cos[4*e +
3*f*x] - 15*a^3*f*x*Cos[4*e + 5*f*x] - 45*a^2*b*f*x*Cos[4*e + 5*f*x] - 45*
a*b^2*f*x*Cos[4*e + 5*f*x] - 15*b^3*f*x*Cos[4*e + 5*f*x] + 15*a^3*f*x*Cos[
6*e + 5*f*x] + 45*a^2*b*f*x*Cos[6*e + 5*f*x] + 45*a*b^2*f*x*Cos[6*e + 5*f*
x] + 15*b^3*f*x*Cos[6*e + 5*f*x] + 280*a^3*Sin[f*x] + 780*a^2*b*Sin[f*x] +
 680*a*b^2*Sin[f*x] + 180*a^3*Sin[2*e + f*x] + 540*a^2*b*Sin[2*e + f*x] +
480*a*b^2*Sin[2*e + f*x] - 140*a^3*Sin[2*e + 3*f*x] - 420*a^2*b*Sin[2*e +
3*f*x] - 400*a*b^2*Sin[2*e + 3*f*x] - 90*a^3*Sin[4*e + 3*f*x] - 240*a^2*b*
Sin[4*e + 3*f*x] - 180*a*b^2*Sin[4*e + 3*f*x] + 46*a^3*Sin[4*e + 5*f*x] +
132*a^2*b*Sin[4*e + 5*f*x] + 116*a*b^2*Sin[4*e + 5*f*x])))/(960*a*(a + b)^
3*f*(a + b*Sec[e + f*x]^2))
```

3.349.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.22, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 4629, 2075, 382, 27, 445, 27, 445, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\tan(e+fx)^6\,(a+b\sec(e+fx)^2)}dx$$

$$\downarrow 4629$$

$$\frac{\int \frac{\cot^6(e+fx)}{(\tan^2(e+fx)+1)(a+b(\tan^2(e+fx)+1))}d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\cot^6(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)}\,d\tan(e+fx)}{f}$$

$$\downarrow 382$$

$$\frac{\int -\frac{5\cot^4(e+fx)\left(b\tan^2(e+fx)+a+2b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}\,d\tan(e+fx)}{5(a+b)} - \frac{\cot^5(e+fx)}{5(a+b)}}{f}$$

$$\downarrow 27$$

$$\frac{-\frac{\int \frac{\cot^4(e+fx)\left(b\tan^2(e+fx)+a+2b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}\,d\tan(e+fx)}{a+b} - \frac{\cot^5(e+fx)}{5(a+b)}}{f}$$

$$\downarrow 445$$

$$\frac{-\frac{\int \frac{3\cot^2(e+fx)\left(a^2+3ba+3b^2+b(a+2b)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}\,d\tan(e+fx)}{3(a+b)} - \frac{(a+2b)\cot^3(e+fx)}{3(a+b)}}{a+b} - \frac{\cot^5(e+fx)}{5(a+b)}}{f}$$

$$\downarrow 27$$

$$\frac{-\frac{\int \frac{\cot^2(e+fx)\left(a^2+3ba+3b^2+b(a+2b)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}\,d\tan(e+fx)}{a+b} - \frac{(a+2b)\cot^3(e+fx)}{3(a+b)}}{a+b} - \frac{\cot^5(e+fx)}{5(a+b)}}{f}$$

$$\downarrow 445$$

$$\frac{-\frac{-\frac{\int \frac{b\left(a^2+3ba+3b^2\right)\tan^2(e+fx)+(a+2b)\left(a^2+2ba+2b^2\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}\,d\tan(e+fx)}{a+b} - \frac{\left(a^2+3ab+3b^2\right)\cot(e+fx)}{a+b}}{a+b} - \frac{(a+2b)\cot^3(e+fx)}{3(a+b)}}{a+b} - \frac{\cot^5(e+fx)}{5(a+b)}}{f}$$

$$\downarrow 397$$

$$\frac{-\frac{-\frac{(a+b)^3\int \frac{1}{\tan^2(e+fx)+1}\,d\tan(e+fx)}{a} - b^4\int \frac{1}{b\tan^2(e+fx)+a+b}\,d\tan(e+fx)}{a+b} - \frac{\left(a^2+3ab+3b^2\right)\cot(e+fx)}{a+b}}{a+b} - \frac{(a+2b)\cot^3(e+fx)}{3(a+b)}}{a+b} - \frac{\cot^5(e+fx)}{5(a+b)}}{f}$$

$$\downarrow 216$$

$$\frac{-\frac{-\frac{(a+b)^3\arctan(\tan(e+fx))}{a}-\frac{b^4\int\frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{a+b}-\frac{\left(a^2+3ab+3b^2\right)\cot(e+fx)}{a+b}}{a+b}-\frac{(a+2b)\cot^3(e+fx)}{3(a+b)}-\frac{\cot^5(e+fx)}{5(a+b)}}{f}$$

$$\downarrow 218$$

$$\frac{-\frac{-\frac{\left(a^2+3ab+3b^2\right)\cot(e+fx)}{a+b}-\frac{(a+b)^3\arctan(\tan(e+fx))}{a}-\frac{b^{7/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{a+b}}{a+b}-\frac{(a+2b)\cot^3(e+fx)}{3(a+b)}-\frac{\cot^5(e+fx)}{5(a+b)}}{f}$$

input
```
Int[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2),x]
```

output
```
(-1/5*Cot[e + f*x]^5/(a + b) - (-1/3*((a + 2*b)*Cot[e + f*x]^3)/(a + b) -
(-((((a + b)^3*ArcTan[Tan[e + f*x]])/a - (b^(7/2)*ArcTan[(Sqrt[b]*Tan[e +
f*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(a + b)) - ((a^2 + 3*a*b + 3*b^2)*Cot
[e + f*x])/(a + b))/(a + b))/(a + b))/f
```

3.349.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.349.4 Maple [A] (verified)

Time = 3.66 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{-\frac{\arctan(\tan(fx+e))}{a}+\frac{b^4\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a+b)^3a\sqrt{(a+b)b}}-\frac{1}{5(a+b)\tan(fx+e)^5}-\frac{-a-2b}{3(a+b)^2\tan(fx+e)^3}-\frac{a^2+3ab+3b^2}{(a+b)^3\tan(fx+e)}}{f}$
default	$\frac{-\frac{\arctan(\tan(fx+e))}{a}+\frac{b^4\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a+b)^3a\sqrt{(a+b)b}}-\frac{1}{5(a+b)\tan(fx+e)^5}-\frac{-a-2b}{3(a+b)^2\tan(fx+e)^3}-\frac{a^2+3ab+3b^2}{(a+b)^3\tan(fx+e)}}{f}$
risch	$-\frac{x}{a}-\frac{2i\left(45a^2e^{8i(fx+e)}+120abe^{8i(fx+e)}+90b^2e^{8i(fx+e)}-90a^2e^{6i(fx+e)}-270abe^{6i(fx+e)}-240b^2e^{6i(fx+e)}+140a^2e^{4}\right.}{15f(a+b)^3(}$

input
```
int(cot(f*x+e)^6/(a+b*sec(f*x+e)^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/a*arctan(tan(f*x+e))+1/(a+b)^3*b^4/a/((a+b)*b)^(1/2)*arctan(b*tan(
f*x+e)/((a+b)*b)^(1/2))-1/5/(a+b)/tan(f*x+e)^5-1/3*(-a-2*b)/(a+b)^2/tan(f*
x+e)^3-(a^2+3*a*b+3*b^2)/(a+b)^3/tan(f*x+e))
```

3.349.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 373 vs. 2(108) = 216.

Time = 0.32 (sec) , antiderivative size = 833, normalized size of antiderivative = 6.94

$$\int \frac{\cot^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

$$= \left[-\frac{4\left(23\,a^3+66\,a^2b+58\,ab^2\right)\cos\left(fx+e\right)^5-20\left(7\,a^3+21\,a^2b+20\,ab^2\right)\cos\left(fx+e\right)^3-15\left(b^3\cos\left(fx+\right.\right.}{} \right.$$

$$\left. -\frac{2\left(23\,a^3+66\,a^2b+58\,ab^2\right)\cos\left(fx+e\right)^5-10\left(7\,a^3+21\,a^2b+20\,ab^2\right)\cos\left(fx+e\right)^3+15\left(b^3\cos\left(fx+e\right.\right.}{} \right]$$

input
```
integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="fricas")
```

output
```
[-1/60*(4*(23*a^3 + 66*a^2*b + 58*a*b^2)*cos(f*x + e)^5 - 20*(7*a^3 + 21*a
^2*b + 20*a*b^2)*cos(f*x + e)^3 - 15*(b^3*cos(f*x + e)^4 - 2*b^3*cos(f*x +
 e)^2 + b^3)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 -
2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3
 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos
(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) + 60*(a^3 + 3*a^2*
b + 3*a*b^2)*cos(f*x + e) + 60*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*x*cos(f*
x + e)^4 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*x*cos(f*x + e)^2 + (a^3 + 3
*a^2*b + 3*a*b^2 + b^3)*f*x)*sin(f*x + e))/(((a^4 + 3*a^3*b + 3*a^2*b^2 +
a*b^3)*f*cos(f*x + e)^4 - 2*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x
+ e)^2 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f)*sin(f*x + e)), -1/30*(2*(2
3*a^3 + 66*a^2*b + 58*a*b^2)*cos(f*x + e)^5 - 10*(7*a^3 + 21*a^2*b + 20*a*
b^2)*cos(f*x + e)^3 + 15*(b^3*cos(f*x + e)^4 - 2*b^3*cos(f*x + e)^2 + b^3)
*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))
/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) + 30*(a^3 + 3*a^2*b + 3*a*b^2
)*cos(f*x + e) + 30*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*x*cos(f*x + e)^4 -
2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*x*cos(f*x + e)^2 + (a^3 + 3*a^2*b + 3*
a*b^2 + b^3)*f*x)*sin(f*x + e))/(((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*co
s(f*x + e)^4 - 2*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^2 + (a
^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f)*sin(f*x + e))]
```

3.349.6 Sympy [F]

$$\int \frac{\cot^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \int \frac{\cot^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$

input
```
integrate(cot(f*x+e)**6/(a+b*sec(f*x+e)**2),x)
```

output
```
Integral(cot(e + f*x)**6/(a + b*sec(e + f*x)**2), x)
```

3.349.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.34

$$\int \frac{\cot^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{\frac{15\,b^4\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^4+3\,a^3b+3\,a^2b^2+ab^3)\sqrt{(a+b)b}} - \frac{15\,(fx+e)}{a} - \frac{15\,(a^2+3\,ab+3\,b^2)\tan(fx+e)^4 - 5\,(a^2+3\,ab+2\,b^2)\tan(fx+e)^2 + 3\,a^2+6\,ab+3\,b^2}{(a^3+3\,a^2b+3\,ab^2+b^3)\tan(fx+e)^5}}{15\,f}$$

input
```
integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="maxima")
```

output
```
1/15*(15*b^4*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^4 + 3*a^3*b + 3*a^
2*b^2 + a*b^3)*sqrt((a + b)*b)) - 15*(f*x + e)/a - (15*(a^2 + 3*a*b + 3*b^
2)*tan(f*x + e)^4 - 5*(a^2 + 3*a*b + 2*b^2)*tan(f*x + e)^2 + 3*a^2 + 6*a*b
 + 3*b^2)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*tan(f*x + e)^5))/f
```

3.349.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.77

$$\int \frac{\cot^6(e+fx)}{a+b\sec^2(e+fx)}\,dx$$
$$= \frac{\frac{15\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)b^4}{(a^4+3\,a^3b+3\,a^2b^2+ab^3)\sqrt{ab+b^2}} - \frac{15\,(fx+e)}{a} - \frac{15\,a^2\tan(fx+e)^4+45\,ab\tan(fx+e)^4+45\,b^2\tan(fx+e)^4-5\,a^2\tan(fx+e)^2}{(a^3+3\,a^2b+3\,ab^2+b^3)\tan(}}{15\,f}$$

input
```
integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

output
```
1/15*(15*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt
(a*b + b^2)))*b^4/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sqrt(a*b + b^2)) -
15*(f*x + e)/a - (15*a^2*tan(f*x + e)^4 + 45*a*b*tan(f*x + e)^4 + 45*b^2*t
an(f*x + e)^4 - 5*a^2*tan(f*x + e)^2 - 15*a*b*tan(f*x + e)^2 - 10*b^2*tan(
f*x + e)^2 + 3*a^2 + 6*a*b + 3*b^2)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*tan(f
*x + e)^5))/f
```

3.349.9 Mupad [B] (verification not implemented)

Time = 25.76 (sec) , antiderivative size = 4324, normalized size of antiderivative = 36.03

$$\int \frac{\cot^6(e+fx)}{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
int(cot(e + f*x)^6/(a + b/cos(e + f*x)^2),x)
```

output
```
(atan(((((tan(e + f*x)*(48*a*b^17 + 4*b^18 + 282*a^2*b^16 + 1078*a^3*b^15
+ 2982*a^4*b^14 + 6258*a^5*b^13 + 10178*a^6*b^12 + 12942*a^7*b^11 + 12888*
a^8*b^10 + 10012*a^9*b^9 + 6006*a^10*b^8 + 2730*a^11*b^7 + 910*a^12*b^6 +
210*a^13*b^5 + 30*a^14*b^4 + 2*a^15*b^3))/2 - ((-b^7*(a + b)^7)^(1/2)*(8*a
^2*b^17 + 108*a^3*b^16 + 680*a^4*b^15 + 2650*a^5*b^14 + 7152*a^6*b^13 + 14
168*a^7*b^12 + 21296*a^8*b^11 + 24750*a^9*b^10 + 22440*a^10*b^9 + 15884*a^
11*b^8 + 8712*a^12*b^7 + 3638*a^13*b^6 + 1120*a^14*b^5 + 240*a^15*b^4 + 32
*a^16*b^3 + 2*a^17*b^2 - (tan(e + f*x)*(-b^7*(a + b)^7)^(1/2)*(16*a^2*b^18
 + 248*a^3*b^17 + 1800*a^4*b^16 + 8120*a^5*b^15 + 25480*a^6*b^14 + 58968*a
^7*b^13 + 104104*a^8*b^12 + 143000*a^9*b^11 + 154440*a^10*b^10 + 131560*a^
11*b^9 + 88088*a^12*b^8 + 45864*a^13*b^7 + 18200*a^14*b^6 + 5320*a^15*b^5
+ 1080*a^16*b^4 + 136*a^17*b^3 + 8*a^18*b^2))/(4*a*(a + b)^7)))/(2*a*(a +
b)^7))*(-b^7*(a + b)^7)^(1/2)*1i)/(a*(a + b)^7) + (((tan(e + f*x)*(48*a*b^
17 + 4*b^18 + 282*a^2*b^16 + 1078*a^3*b^15 + 2982*a^4*b^14 + 6258*a^5*b^13
 + 10178*a^6*b^12 + 12942*a^7*b^11 + 12888*a^8*b^10 + 10012*a^9*b^9 + 6006
*a^10*b^8 + 2730*a^11*b^7 + 910*a^12*b^6 + 210*a^13*b^5 + 30*a^14*b^4 + 2*
a^15*b^3))/2 + ((-b^7*(a + b)^7)^(1/2)*(8*a^2*b^17 + 108*a^3*b^16 + 680*a^
4*b^15 + 2650*a^5*b^14 + 7152*a^6*b^13 + 14168*a^7*b^12 + 21296*a^8*b^11 +
 24750*a^9*b^10 + 22440*a^10*b^9 + 15884*a^11*b^8 + 8712*a^12*b^7 + 3638*a
^13*b^6 + 1120*a^14*b^5 + 240*a^15*b^4 + 32*a^16*b^3 + 2*a^17*b^2 + (ta...
```

3.350 $\int \frac{\tan^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

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3.350.1 Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \frac{\tan^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{(a+b)^2}{2a^2bf\left(b+a\cos^2(e+fx)\right)} - \frac{\log(\cos(e+fx))}{b^2f} - \frac{\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\log\left(b+a\cos^2(e+fx)\right)}{2f}$$

output
```
-1/2*(a+b)^2/a^2/b/f/(b+a*cos(f*x+e)^2)-ln(cos(f*x+e))/b^2/f-1/2*(1/a^2-1/
b^2)*ln(b+a*cos(f*x+e)^2)/f
```

3.350.2 Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.42

$$\int \frac{\tan^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{(a+2b+a\cos(2e+2fx))^2\left(\frac{(a+b)^2}{a^2b(b+a\cos^2(e+fx))}+\frac{2\log(\cos(e+fx))}{b^2}+\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\log\left(b+a\cos^2(e+fx)\right)\right)\sec}{8f\left(a+b\sec^2(e+fx)\right)^2}$$

input
```
Integrate[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
-1/8*((a + 2*b + a*Cos[2*e + 2*f*x])^2*((a + b)^2/(a^2*b*(b + a*Cos[e + f*
x]^2)) + (2*Log[Cos[e + f*x]])/b^2 + (a^(-2) - b^(-2))*Log[b + a*Cos[e + f
*x]^2])*Sec[e + f*x]^4)/(f*(a + b*Sec[e + f*x]^2)^2)
```

3.350.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4626, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\tan(e+fx)^5}{\left(a+b\sec(e+fx)^2\right)^2}\,dx$$

$$\downarrow 4626$$

$$-\frac{\int \frac{\left(1-\cos^2(e+fx)\right)^2\sec(e+fx)}{\left(a\cos^2(e+fx)+b\right)^2}\,d\cos(e+fx)}{f}$$

$$\downarrow 354$$

$$-\frac{\int \frac{\left(1-\cos^2(e+fx)\right)^2\sec(e+fx)}{\left(a\cos^2(e+fx)+b\right)^2}\,d\cos^2(e+fx)}{2f}$$

$$\downarrow 99$$

$$-\frac{\int \left(-\frac{(a+b)^2}{ab\left(a\cos^2(e+fx)+b\right)^2}+\frac{\sec(e+fx)}{b^2}+\frac{b^2-a^2}{ab^2\left(a\cos^2(e+fx)+b\right)}\right)d\cos^2(e+fx)}{2f}$$

$$\downarrow 2009$$

$$-\frac{\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\log\left(a\cos^2(e+fx)+b\right)+\frac{(a+b)^2}{a^2b\left(a\cos^2(e+fx)+b\right)}+\frac{\log\left(\cos^2(e+fx)\right)}{b^2}}{2f}$$

input
```
Int[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
-1/2*((a + b)^2/(a^2*b*(b + a*Cos[e + f*x]^2)) + Log[Cos[e + f*x]^2]/b^2 +
 (a^(-2) - b^(-2))*Log[b + a*Cos[e + f*x]^2])/f
```

3.350.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.350.4 Maple [A] (verified)

Time = 6.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{\frac{(a+b)\left(\frac{(a-b)\ln\left(b+a\cos(fx+e)^2\right)}{a^2}-\frac{(a+b)b}{a^2\left(b+a\cos(fx+e)^2\right)}\right)}{2b^2}-\frac{\ln(\cos(fx+e))}{b^2}}{f}$
default	$\frac{\frac{(a+b)\left(\frac{(a-b)\ln\left(b+a\cos(fx+e)^2\right)}{a^2}-\frac{(a+b)b}{a^2\left(b+a\cos(fx+e)^2\right)}\right)}{2b^2}-\frac{\ln(\cos(fx+e))}{b^2}}{f}$
risch	$\frac{ix}{a^2}+\frac{2ie}{a^2f}-\frac{2\left(a^2+2ab+b^2\right)e^{2i(fx+e)}}{a^2bf\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)}-\frac{\ln\left(e^{2i(fx+e)}+1\right)}{b^2f}+\frac{\ln\left(e^{4i(fx+e)}+\frac{2(a+2b)e^{2i(fx}{a}\right.}{2b^2f}$

input
```
int(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/2/b^2*(a+b)*((a-b)/a^2*ln(b+a*cos(f*x+e)^2)-(a+b)*b/a^2/(b+a*cos(f*
x+e)^2))-1/b^2*ln(cos(f*x+e)))
```

3.350.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.53

$$\int \frac{\tan^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{a^2b+2\,ab^2+b^3-\left(a^2b-b^3+\left(a^3-ab^2\right)\cos\left(fx+e\right)^2\right)\log\left(a\cos\left(fx+e\right)^2+b\right)+2\left(a^3\cos\left(fx+e\right)^2+\right.}{2\left(a^3b^2f\cos\left(fx+e\right)^2+a^2b^3f\right)}$$

input
```
integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
-1/2*(a^2*b + 2*a*b^2 + b^3 - (a^2*b - b^3 + (a^3 - a*b^2)*cos(f*x + e)^2)
*log(a*cos(f*x + e)^2 + b) + 2*(a^3*cos(f*x + e)^2 + a^2*b)*log(-cos(f*x +
 e)))/(a^3*b^2*f*cos(f*x + e)^2 + a^2*b^3*f)
```

3.350.6 Sympy [F]

$$\int \frac{\tan^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \int \frac{\tan^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

input
```
integrate(tan(f*x+e)**5/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral(tan(e + f*x)**5/(a + b*sec(e + f*x)**2)**2, x)
```

3.350.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.27

$$\int \frac{\tan^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\frac{a^2+2ab+b^2}{a^3b\sin(fx+e)^2-a^3b-a^2b^2} - \frac{\log\left(\sin(fx+e)^2-1\right)}{b^2} + \frac{\left(a^2-b^2\right)\log\left(a\sin(fx+e)^2-a-b\right)}{a^2b^2}}{2f}$$

input
```
integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/2*((a^2 + 2*a*b + b^2)/(a^3*b*sin(f*x + e)^2 - a^3*b - a^2*b^2) - log(si
n(f*x + e)^2 - 1)/b^2 + (a^2 - b^2)*log(a*sin(f*x + e)^2 - a - b)/(a^2*b^2
))/f
```

3.350.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 532 vs. 2(73) = 146.

Time = 1.65 (sec) , antiderivative size = 532, normalized size of antiderivative = 6.91

$$\int \frac{\tan^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\left(a^3+a^2b-ab^2-b^3\right)\log\left(\left|-a\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)-b\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1}+\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)-2a+2b\right|\right)}{a^3b^2+a^2b^3} + \frac{\log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+2\right|\right)}{a^2}$$

input
```
integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/2*((a^3 + a^2*b - a*b^2 - b^3)*log(abs(-a*((cos(f*x + e) + 1)/(cos(f*x +
 e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) - b*((cos(f*x + e) + 1)/
(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) - 2*a + 2*b))/
(a^3*b^2 + a^2*b^3) + log(abs(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - (co
s(f*x + e) - 1)/(cos(f*x + e) + 1) + 2))/a^2 - log(abs(-(cos(f*x + e) + 1)
/(cos(f*x + e) - 1) - (cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2))/b^2 - (a
^3*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1)/(cos(f*x +
e) + 1)) + a^2*b*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) -
1)/(cos(f*x + e) + 1)) - a*b^2*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (c
os(f*x + e) - 1)/(cos(f*x + e) + 1)) - b^3*((cos(f*x + e) + 1)/(cos(f*x +
e) - 1) + (cos(f*x + e) - 1)/(cos(f*x + e) + 1)) + 2*a^3 - 6*a^2*b - 6*a*b
^2 + 2*b^3)/((a*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*x + e) - 1
)/(cos(f*x + e) + 1)) + b*((cos(f*x + e) + 1)/(cos(f*x + e) - 1) + (cos(f*
x + e) - 1)/(cos(f*x + e) + 1)) + 2*a - 2*b)*a^2*b^2))/f
```

3.350.9 Mupad [B] (verification not implemented)

Time = 20.34 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.21

$$\int \frac{\tan^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\ln\left(b\tan(e+f\,x)^2+a+b\right)}{2\,b^2\,f} - \frac{\ln\left(b\tan(e+f\,x)^2+a+b\right)}{2\,a^2\,f} + \frac{a^2}{2\,f\,\left(a^2\,b^2+a\,b^3\tan\left(e+f\,x\right)^2+a\,b^3\right)} + \frac{b^2}{2\,f\,\left(a^2\,b^2+a\,b^3\tan\left(e+f\,x\right)^2+a\,b^3\right)} + \frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,a^2\,f} + \frac{a\,b}{f\,\left(a^2\,b^2+a\,b^3\tan\left(e+f\,x\right)^2+a\,b^3\right)}$$

input
```
int(tan(e + f*x)^5/(a + b/cos(e + f*x)^2)^2,x)
```

output
```
log(a + b + b*tan(e + f*x)^2)/(2*b^2*f) - log(a + b + b*tan(e + f*x)^2)/(2
*a^2*f) + a^2/(2*f*(a*b^3 + a^2*b^2 + a*b^3*tan(e + f*x)^2)) + b^2/(2*f*(a
*b^3 + a^2*b^2 + a*b^3*tan(e + f*x)^2)) + log(tan(e + f*x)^2 + 1)/(2*a^2*f
) + (a*b)/(f*(a*b^3 + a^2*b^2 + a*b^3*tan(e + f*x)^2))
```

3.351 $\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

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3.351.1 Optimal result

Integrand size = 23, antiderivative size = 51

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{a+b}{2a^2f\,(b+a\cos^2(e+fx))} + \frac{\log\left(b+a\cos^2(e+fx)\right)}{2a^2f}$$

output
```
1/2*(a+b)/a^2/f/(b+a*cos(f*x+e)^2)+1/2*ln(b+a*cos(f*x+e)^2)/a^2/f
```

3.351.2 Mathematica [A] (verified)

Time = 2.34 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.59

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{2(a+b)+(a+2b)\log(a+2b+a\cos(2(e+fx)))+a\cos(2(e+fx))\log(a+2b+a\cos(2(e+fx)))}{2a^2f(a+2b+a\cos(2(e+fx)))}$$

input
```
Integrate[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(2*(a + b) + (a + 2*b)*Log[a + 2*b + a*Cos[2*(e + f*x)]] + a*Cos[2*(e + f*
x)]*Log[a + 2*b + a*Cos[2*(e + f*x)]])/(2*a^2*f*(a + 2*b + a*Cos[2*(e + f*
x)]))
```

3.351.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4626, 353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\tan(e+fx)^3}{(a+b\sec(e+fx)^2)^2}\,dx$$

$$\downarrow 4626$$

$$-\frac{\int \frac{\cos(e+fx)(1-\cos^2(e+fx))}{(a\cos^2(e+fx)+b)^2}\,d\cos(e+fx)}{f}$$

$$\downarrow 353$$

$$-\frac{\int \frac{1-\cos^2(e+fx)}{(a\cos^2(e+fx)+b)^2}\,d\cos^2(e+fx)}{2f}$$

$$\downarrow 49$$

$$-\frac{\int \left(\frac{a+b}{a(a\cos^2(e+fx)+b)^2}-\frac{1}{a(a\cos^2(e+fx)+b)}\right)\,d\cos^2(e+fx)}{2f}$$

$$\downarrow 2009$$

$$-\frac{-\frac{a+b}{a^2(a\cos^2(e+fx)+b)}-\frac{\log(a\cos^2(e+fx)+b)}{a^2}}{2f}$$

input
```
Int[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
-1/2*(-((a + b)/(a^2*(b + a*Cos[e + f*x]^2))) - Log[b + a*Cos[e + f*x]^2]/
a^2)/f
```

3.351.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.351.4 Maple [A] (verified)

Time = 3.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.98

method	result	size
derivativedivides	$\frac{\frac{\ln\left(b+a\cos(fx+e)^2\right)}{2a^2}-\frac{-a-b}{2a^2\left(b+a\cos(fx+e)^2\right)}}{f}$	50
default	$\frac{\frac{\ln\left(b+a\cos(fx+e)^2\right)}{2a^2}-\frac{-a-b}{2a^2\left(b+a\cos(fx+e)^2\right)}}{f}$	50
risch	$-\frac{ix}{a^2}-\frac{2ie}{a^2f}+\frac{2(a+b)e^{2i(fx+e)}}{a^2f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)}+\frac{\ln\left(e^{4i(fx+e)}+\frac{2(a+2b)e^{2i(fx+e)}}{a}+1\right)}{2a^2f}$	117

input
```
int(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/2/a^2*ln(b+a*cos(f*x+e)^2)-1/2*(-a-b)/a^2/(b+a*cos(f*x+e)^2))
```

3.351.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

$$\int \frac{\tan^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\left(a\cos\left(fx+e\right)^2+b\right)\log\left(a\cos\left(fx+e\right)^2+b\right)+a+b}{2\left(a^3f\cos\left(fx+e\right)^2+a^2bf\right)}$$

input
```
integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/2*((a*cos(f*x + e)^2 + b)*log(a*cos(f*x + e)^2 + b) + a + b)/(a^3*f*cos(
f*x + e)^2 + a^2*b*f)
```

3.351.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\tan^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(tan(f*x+e)**3/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Timed out
```

3.351.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.16

$$\int \frac{\tan^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{\frac{a+b}{a^3\sin(fx+e)^2-a^3-a^2b}-\frac{\log\left(a\sin(fx+e)^2-a-b\right)}{a^2}}{2f}$$

input
```
integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/2*((a + b)/(a^3*sin(f*x + e)^2 - a^3 - a^2*b) - log(a*sin(f*x + e)^2 -
a - b)/a^2)/f
```

3.351.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(47) = 94$.

Time $= 0.72$ (sec) , antiderivative size $= 314$, normalized size of antiderivative $= 6.16$

$$\int \frac{\tan^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$= \frac{\frac{\log\left(a+b+\frac{2\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{a^2} - \frac{2\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)}{a^2} - \frac{a+b+\frac{6\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}}{\left(a+b+\frac{2\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-}$$

$$2\,f$$

input
```
integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/2*(log(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(cos(f*x
+ e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2
 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/a^2 - 2*log(abs(-(cos(f*x
+ e) - 1)/(cos(f*x + e) + 1) + 1))/a^2 - (a + b + 6*a*(cos(f*x + e) - 1)/(
cos(f*x + e) + 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x
 + e) - 1)^2/(cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) +
 1)^2)/((a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(cos(f*x
+ e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2
 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)*a^2))/f
```

3.351.9 Mupad [B] (verification not implemented)

Time $= 20.32$ (sec) , antiderivative size $= 97$, normalized size of antiderivative $= 1.90$

$$\int \frac{\tan^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{\operatorname{atanh}\left(\frac{4\,b^2\tan(e+f\,x)^2}{8\,b^2+\frac{8\,b^3}{a}+4\,b^2\tan(e+f\,x)^2+\frac{8\,b^3\tan(e+f\,x)^2}{a}}\right)}{a^2\,f} - \frac{a+b}{2\,a\,b\,f\,\left(b\tan\left(e+f\,x\right)^2+a+b\right)}$$

input
```
int(tan(e + f*x)^3/(a + b/cos(e + f*x)^2)^2,x)
```

output
```
- atanh((4*b^2*tan(e + f*x)^2)/(8*b^2 + (8*b^3)/a + 4*b^2*tan(e + f*x)^2 +
 (8*b^3*tan(e + f*x)^2)/a))/(a^2*f) - (a + b)/(2*a*b*f*(a + b + b*tan(e +
f*x)^2))
```

3.352 $\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

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3.352.1 Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{b}{2a^2f\left(b+a\cos^2(e+fx)\right)} - \frac{\log\left(b+a\cos^2(e+fx)\right)}{2a^2f}$$

output
```
-1/2*b/a^2/f/(b+a*cos(f*x+e)^2)-1/2*ln(b+a*cos(f*x+e)^2)/a^2/f
```

3.352.2 Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.61

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{2b+(a+2b)\log(a+2b+a\cos(2(e+fx)))+a\cos(2(e+fx))\log(a+2b+a\cos(2(e+fx)))}{2a^2f(a+2b+a\cos(2(e+fx)))}$$

input
```
Integrate[Tan[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
-1/2*(2*b + (a + 2*b)*Log[a + 2*b + a*Cos[2*(e + f*x)]] + a*Cos[2*(e + f*x
)]*Log[a + 2*b + a*Cos[2*(e + f*x)]])/(a^2*f*(a + 2*b + a*Cos[2*(e + f*x)]
))
```

3.352.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4626, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\tan(e+fx)}{(a+b\sec(e+fx)^2)^2}dx$$

$$\downarrow 4626$$

$$-\frac{\int \frac{\cos^3(e+fx)}{(a\cos^2(e+fx)+b)^2}d\cos(e+fx)}{f}$$

$$\downarrow 243$$

$$-\frac{\int \frac{\cos^2(e+fx)}{(a\cos^2(e+fx)+b)^2}d\cos^2(e+fx)}{2f}$$

$$\downarrow 49$$

$$-\frac{\int \left(\frac{1}{a(a\cos^2(e+fx)+b)}-\frac{b}{a(a\cos^2(e+fx)+b)^2}\right)d\cos^2(e+fx)}{2f}$$

$$\downarrow 2009$$

$$-\frac{\frac{b}{a^2(a\cos^2(e+fx)+b)}+\frac{\log(a\cos^2(e+fx)+b)}{a^2}}{2f}$$

input
```
Int[Tan[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
-1/2*(b/(a^2*(b + a*Cos[e + f*x]^2)) + Log[b + a*Cos[e + f*x]^2]/a^2)/f
```

3.352.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.352.4 Maple [A] (verified)

Time = 2.32 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.20

method	result	size
derivativedivides	$-\frac{\ln\left(a+b\sec(fx+e)^2\right)}{2fa^2}+\frac{1}{2fa\left(a+b\sec(fx+e)^2\right)}+\frac{\ln(\sec(fx+e))}{fa^2}$	59
default	$-\frac{\ln\left(a+b\sec(fx+e)^2\right)}{2fa^2}+\frac{1}{2fa\left(a+b\sec(fx+e)^2\right)}+\frac{\ln(\sec(fx+e))}{fa^2}$	59
risch	$\frac{ix}{a^2}+\frac{2ie}{a^2f}-\frac{2b\,e^{2i(fx+e)}}{a^2f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)}-\frac{\ln\left(e^{4i(fx+e)}+\frac{2(a+2b)e^{2i(fx+e)}}{a}+1\right)}{2a^2f}$	115

input
```
int(tan(f*x+e)/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/f/a^2*ln(a+b*sec(f*x+e)^2)+1/2/f/a/(a+b*sec(f*x+e)^2)+1/f/a^2*ln(sec(
f*x+e))
```

3.352.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06

$$\int \frac{\tan(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{\left(a\cos\left(fx+e\right)^2+b\right)\log\left(a\cos\left(fx+e\right)^2+b\right)+b}{2\left(a^3 f\cos\left(fx+e\right)^2+a^2bf\right)}$$

input
```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
-1/2*((a*cos(f*x + e)^2 + b)*log(a*cos(f*x + e)^2 + b) + b)/(a^3*f*cos(f*x
 + e)^2 + a^2*b*f)
```

3.352.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\tan(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Timed out
```

3.352.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.16

$$\int \frac{\tan(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\frac{b}{a^3\sin(fx+e)^2-a^3-a^2b}-\frac{\log\left(a\sin(fx+e)^2-a-b\right)}{a^2}}{2f}$$

input
```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/2*(b/(a^3*sin(f*x + e)^2 - a^3 - a^2*b) - log(a*sin(f*x + e)^2 - a - b)/
a^2)/f
```

3.352.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(45) = 90$.

Time = 0.43 (sec) , antiderivative size = 384, normalized size of antiderivative = 7.84

$$\int \frac{\tan(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$= \frac{\frac{a^2+2ab+b^2+\frac{2a^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{4ab(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2b^2(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{2ab(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{b^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}}{(a^3+a^2b)\left(a+b+\frac{2a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)} - \frac{\log\left(a+b+}{2f}$$

input
```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/2*((a^2 + 2*a*b + b^2 + 2*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 4*
a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b^2*(cos(f*x + e) - 1)/(cos(
f*x + e) + 1) + a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 2*a*b*(cos
(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + b^2*(cos(f*x + e) - 1)^2/(cos(f*x
+ e) + 1)^2)/((a^3 + a^2*b)*(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e)
+ 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/
(cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)) - log
(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(cos(f*x + e) - 1
)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + b*(co
s(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/a^2 + 2*log(abs(-(cos(f*x + e) - 1
)/(cos(f*x + e) + 1) + 1))/a^2)/f
```

3.352.9 Mupad [B] (verification not implemented)

Time = 19.57 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.84

$$\int \frac{\tan(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\operatorname{atanh}\left(\frac{4b^2\tan(e+fx)^2}{8b^2+\frac{8b^3}{a}+4b^2\tan(e+fx)^2+\frac{8b^3\tan(e+fx)^2}{a}}\right)}{a^2 f} + \frac{1}{2af\left(b\tan(e+fx)^2+a+b\right)}$$

input
```
int(tan(e + f*x)/(a + b/cos(e + f*x)^2)^2,x)
```

output
```
atanh((4*b^2*tan(e + f*x)^2)/(8*b^2 + (8*b^3)/a + 4*b^2*tan(e + f*x)^2 + (
8*b^3*tan(e + f*x)^2)/a))/(a^2*f) + 1/(2*a*f*(a + b + b*tan(e + f*x)^2))
```

3.353 $\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

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3.353.8 Giac [B] (verification not implemented) . . . 2563
3.353.9 Mupad [B] (verification not implemented) . . . 2564

3.353.1 Optimal result

Integrand size = 21, antiderivative size = 83

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{b^2}{2a^2(a+b)f\left(b+a\cos^2(e+fx)\right)} + \frac{b(2a+b)\log\left(b+a\cos^2(e+fx)\right)}{2a^2(a+b)^2f} + \frac{\log(\sin(e+fx))}{(a+b)^2f}$$

output
```
1/2*b^2/a^2/(a+b)/f/(b+a*cos(f*x+e)^2)+1/2*b*(2*a+b)*ln(b+a*cos(f*x+e)^2)/
a^2/(a+b)^2/f+ln(sin(f*x+e))/(a+b)^2/f
```

3.353.2 Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.35

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{(a+2b+a\cos(2(e+fx)))^2\sec^4(e+fx)\left(2\log(\sin(e+fx)) + \frac{b(2a+b)\log(a+b-a\sin^2(e+fx))}{a^2} + \frac{b^2(a+b)}{a^2(a+b-a\sin^2(e+}\right.}{8(a+b)^2f\left(a+b\sec^2(e+fx)\right)^2}$$

input
```
Integrate[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])^2*Sec[e + f*x]^4*(2*Log[Sin[e + f*x]] + (b
*(2*a + b)*Log[a + b - a*Sin[e + f*x]^2])/a^2 + (b^2*(a + b))/(a^2*(a + b
- a*Sin[e + f*x]^2))))/(8*(a + b)^2*f*(a + b*Sec[e + f*x]^2)^2)
```

3.353.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4626, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\tan(e+fx)\left(a+b\sec(e+fx)^2\right)^2}dx$$

$$\downarrow 4626$$

$$-\frac{\int \frac{\cos^5(e+fx)}{(1-\cos^2(e+fx))(a\cos^2(e+fx)+b)^2}d\cos(e+fx)}{f}$$

$$\downarrow 354$$

$$-\frac{\int \frac{\cos^4(e+fx)}{(1-\cos^2(e+fx))(a\cos^2(e+fx)+b)^2}d\cos^2(e+fx)}{2f}$$

$$\downarrow 99$$

$$-\frac{\int \left(\frac{b^2}{a(a+b)(a\cos^2(e+fx)+b)^2}-\frac{(2a+b)b}{a(a+b)^2(a\cos^2(e+fx)+b)}-\frac{1}{(a+b)^2(\cos^2(e+fx)-1)}\right)d\cos^2(e+fx)}{2f}$$

$$\downarrow 2009$$

$$-\frac{-\frac{b^2}{a^2(a+b)(a\cos^2(e+fx)+b)}-\frac{b(2a+b)\log(a\cos^2(e+fx)+b)}{a^2(a+b)^2}-\frac{\log(1-\cos^2(e+fx))}{(a+b)^2}}{2f}$$

input
```
Int[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
-1/2*(-(b^2/(a^2*(a + b)*(b + a*Cos[e + f*x]^2))) - Log[1 - Cos[e + f*x]^2
]/(a + b)^2 - (b*(2*a + b)*Log[b + a*Cos[e + f*x]^2])/(a^2*(a + b)^2))/f
```

3.353.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.353.4 Maple [A] (verified)

Time = 3.66 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{\frac{\ln(-1+\cos(fx+e))}{2(a+b)^2}+\frac{b\left(\frac{(a+b)b}{a^2\left(b+a\cos(fx+e)^2\right)}+\frac{(2a+b)\ln\left(b+a\cos(fx+e)^2\right)}{a^2}\right)}{2(a+b)^2}+\frac{\ln(1+\cos(fx+e))}{2(a+b)^2}}{f}$
default	$\frac{\frac{\ln(-1+\cos(fx+e))}{2(a+b)^2}+\frac{b\left(\frac{(a+b)b}{a^2\left(b+a\cos(fx+e)^2\right)}+\frac{(2a+b)\ln\left(b+a\cos(fx+e)^2\right)}{a^2}\right)}{2(a+b)^2}+\frac{\ln(1+\cos(fx+e))}{2(a+b)^2}}{f}$
risch	$\frac{ix}{a^2}-\frac{2ix}{a^2+2ab+b^2}-\frac{2ie}{f(a^2+2ab+b^2)}-\frac{4ibx}{a(a^2+2ab+b^2)}-\frac{4ibe}{af(a^2+2ab+b^2)}-\frac{2ib^2x}{a^2(a^2+2ab+b^2)}-\frac{2ib^2e}{a^2f(a^2+2ab+b$

input
```
int(cot(f*x+e)/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/2/(a+b)^2*ln(-1+cos(f*x+e))+1/2*b/(a+b)^2*((a+b)*b/a^2/(b+a*cos(f*x
+e)^2)+(2*a+b)/a^2*ln(b+a*cos(f*x+e)^2))+1/2/(a+b)^2*ln(1+cos(f*x+e)))
```

3.353.5 Fricas [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.66

$$\int \frac{\cot(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$
$$= \frac{ab^2+b^3+\left(2\,ab^2+b^3+\left(2\,a^2b+ab^2\right)\cos\left(fx+e\right)^2\right)\log\left(a\cos\left(fx+e\right)^2+b\right)+2\left(a^3\cos\left(fx+e\right)^2+a^2b\right)}{2\left(\left(a^5+2\,a^4b+a^3b^2\right)f\cos\left(fx+e\right)^2+\left(a^4b+2\,a^3b^2+a^2b^3\right)f\right)}$$

input
```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/2*(a*b^2 + b^3 + (2*a*b^2 + b^3 + (2*a^2*b + a*b^2)*cos(f*x + e)^2)*log(
a*cos(f*x + e)^2 + b) + 2*(a^3*cos(f*x + e)^2 + a^2*b)*log(1/2*sin(f*x + e
)))/((a^5 + 2*a^4*b + a^3*b^2)*f*cos(f*x + e)^2 + (a^4*b + 2*a^3*b^2 + a^2
*b^3)*f)
```

3.353.6 Sympy [F]

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

input
```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral(cot(e + f*x)/(a + b*sec(e + f*x)**2)**2, x)
```

3.353.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.41

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{b^2}{a^4+2a^3b+a^2b^2-(a^4+a^3b)\sin(fx+e)^2} + \frac{(2ab+b^2)\log\left(a\sin(fx+e)^2-a-b\right)}{a^4+2a^3b+a^2b^2} + \frac{\log\left(\sin(fx+e)^2\right)}{a^2+2ab+b^2}}{2f}$$

input
```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/2*(b^2/(a^4 + 2*a^3*b + a^2*b^2 - (a^4 + a^3*b)*sin(f*x + e)^2) + (2*a*b
 + b^2)*log(a*sin(f*x + e)^2 - a - b)/(a^4 + 2*a^3*b + a^2*b^2) + log(sin(
f*x + e)^2)/(a^2 + 2*a*b + b^2))/f
```

3.353.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. 2(79) = 158.

Time = 0.34 (sec) , antiderivative size = 396, normalized size of antiderivative = 4.77

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{(2ab+b^2)\log\left(a+b+\frac{2a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{a^4+2a^3b+a^2b^2} + \frac{\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a^2+2ab+b^2} - \frac{2ab+b^2+\frac{4ab(\cos(}{\cos(fx}}{(a^3+a^2b)\left(a+b+\frac{2a}{c}\right.}$$

input
```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/2*((2*a*b + b^2)*log(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) -
 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f
*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/(a^4 + 2*a^3
*b + a^2*b^2) + log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1))/(a^2 + 2
*a*b + b^2) - (2*a*b + b^2 + 4*a*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) -
 2*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 2*a*b*(cos(f*x + e) - 1)^2/
(cos(f*x + e) + 1)^2 + b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/((a^
3 + a^2*b)*(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(cos(f
*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1
)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)) - 2*log(abs(-(cos(f*x
+ e) - 1)/(cos(f*x + e) + 1) + 1))/a^2)/f
```

3.353.9 Mupad [B] (verification not implemented)

Time = 19.76 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.28

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\ln\left(\tan(e+f\,x)\right)}{f\,\left(a^2+2\,a\,b+b^2\right)} - \frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,a^2\,f} - \frac{b}{2\,a\,f\,\left(a+b\right)\,\left(b\tan\left(e+f\,x\right)^2+a+b\right)} + \frac{b\,\ln\left(b\tan(e+f\,x)^2+a+b\right)\,\left(2\,a+b\right)}{2\,a^2\,f\,\left(a+b\right)^2}$$

input
```
int(cot(e + f*x)/(a + b/cos(e + f*x)^2)^2,x)
```

output
```
log(tan(e + f*x))/(f*(2*a*b + a^2 + b^2)) - log(tan(e + f*x)^2 + 1)/(2*a^2
*f) - b/(2*a*f*(a + b)*(a + b + b*tan(e + f*x)^2)) + (b*log(a + b + b*tan(
e + f*x)^2)*(2*a + b))/(2*a^2*f*(a + b)^2)
```

3.354 $\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

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3.354.1 Optimal result

Integrand size = 23, antiderivative size = 111

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{b^3}{2a^2(a+b)^2 f\,(b+a\cos^2(e+fx))} - \frac{\csc^2(e+fx)}{2(a+b)^2 f} - \frac{b^2(3a+b)\log\left(b+a\cos^2(e+fx)\right)}{2a^2(a+b)^3 f} - \frac{(a+3b)\log(\sin(e+fx))}{(a+b)^3 f}$$

output

```
-1/2*b^3/a^2/(a+b)^2/f/(b+a*cos(f*x+e)^2)-1/2*csc(f*x+e)^2/(a+b)^2/f-1/2*b
^2*(3*a+b)*ln(b+a*cos(f*x+e)^2)/a^2/(a+b)^3/f-(a+3*b)*ln(sin(f*x+e))/(a+b)
^3/f
```

3.354.2 Mathematica [A] (verified)

Time = 1.78 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.17

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))^2\left((a+b)\csc^2(e+fx)+2(a+3b)\log(\sin(e+fx))+\frac{b^2\left(\frac{2b(a+b)}{a+2b+a\cos(2(e+fx))}+(3\right.}{8(a+b)^3 f\,(a+b\sec^2(e+fx))^2}$$

input
```
Integrate[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
-1/8*((a + 2*b + a*Cos[2*(e + f*x)])^2*((a + b)*Csc[e + f*x]^2 + 2*(a + 3*
b)*Log[Sin[e + f*x]] + (b^2*((2*b*(a + b))/(a + 2*b + a*Cos[2*(e + f*x)])
+ (3*a + b)*Log[a + b - a*Sin[e + f*x]^2]))/a^2)*Sec[e + f*x]^4)/((a + b)^
3*f*(a + b*Sec[e + f*x]^2)^2)
```

3.354.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4626, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\tan(e+fx)^3\left(a+b\sec(e+fx)^2\right)^2}dx$$

$$\downarrow 4626$$

$$-\frac{\int \frac{\cos^7(e+fx)}{\left(1-\cos^2(e+fx)\right)^2\left(a\cos^2(e+fx)+b\right)^2}d\cos(e+fx)}{f}$$

$$\downarrow 354$$

$$-\frac{\int \frac{\cos^6(e+fx)}{\left(1-\cos^2(e+fx)\right)^2\left(a\cos^2(e+fx)+b\right)^2}d\cos^2(e+fx)}{2f}$$

$$\downarrow 99$$

$$-\frac{\int \left(-\frac{b^3}{a(a+b)^2\left(a\cos^2(e+fx)+b\right)^2}+\frac{(3a+b)b^2}{a(a+b)^3\left(a\cos^2(e+fx)+b\right)}+\frac{a+3b}{(a+b)^3\left(\cos^2(e+fx)-1\right)}+\frac{1}{(a+b)^2\left(\cos^2(e+fx)-1\right)^2}\right)d\cos^2(e+fx)}{2f}$$

$$\downarrow 2009$$

$$-\frac{\frac{b^3}{a^2(a+b)^2\left(a\cos^2(e+fx)+b\right)}+\frac{b^2(3a+b)\log\left(a\cos^2(e+fx)+b\right)}{a^2(a+b)^3}+\frac{1}{(a+b)^2\left(1-\cos^2(e+fx)\right)}+\frac{(a+3b)\log\left(1-\cos^2(e+fx)\right)}{(a+b)^3}}{2f}$$

input
```
Int[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
-1/2*(1/((a + b)^2*(1 - Cos[e + f*x]^2)) + b^3/(a^2*(a + b)^2*(b + a*Cos[e
 + f*x]^2)) + ((a + 3*b)*Log[1 - Cos[e + f*x]^2])/(a + b)^3 + (b^2*(3*a +
b)*Log[b + a*Cos[e + f*x]^2])/(a^2*(a + b)^3))/f
```

3.354.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.354.4 Maple [A] (verified)

Time = 7.28 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.27

method	result
derivativedivides	$\frac{-\frac{b^2\left(\frac{(a+b)b}{a^2\left(b+a\cos(fx+e)^2\right)}+\frac{(3a+b)\ln\left(b+a\cos(fx+e)^2\right)}{a^2}\right)}{2(a+b)^3}-\frac{1}{4(a+b)^2(1+\cos(fx+e))}+\frac{(-a-3b)\ln(1+\cos(fx+e))}{2(a+b)^3}+\frac{1}{4(a+b)^2(-1+c}}{f}$
default	$\frac{-\frac{b^2\left(\frac{(a+b)b}{a^2\left(b+a\cos(fx+e)^2\right)}+\frac{(3a+b)\ln\left(b+a\cos(fx+e)^2\right)}{a^2}\right)}{2(a+b)^3}-\frac{1}{4(a+b)^2(1+\cos(fx+e))}+\frac{(-a-3b)\ln(1+\cos(fx+e))}{2(a+b)^3}+\frac{1}{4(a+b)^2(-1+c}}{f}$
risch	$-\frac{ix}{a^2}+\frac{2iax}{a^3+3a^2b+3ab^2+b^3}+\frac{2iae}{f(a^3+3a^2b+3ab^2+b^3)}+\frac{6ibx}{a^3+3a^2b+3ab^2+b^3}+\frac{6ibe}{f(a^3+3a^2b+3ab^2+b^3)}+\frac{}{a(a^3+}$

input
```
int(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/2*b^2/(a+b)^3*((a+b)*b/a^2/(b+a*cos(f*x+e)^2)+(3*a+b)/a^2*ln(b+a*c
os(f*x+e)^2))-1/4/(a+b)^2/(1+cos(f*x+e))+1/2*(-a-3*b)/(a+b)^3*ln(1+cos(f*x
+e))+1/4/(a+b)^2/(-1+cos(f*x+e))+1/2*(-a-3*b)/(a+b)^3*ln(-1+cos(f*x+e)))
```

3.354.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 312 vs. 2(105) = 210.

Time = 0.50 (sec) , antiderivative size = 312, normalized size of antiderivative = 2.81

$$\int \frac{\cot^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$
$$= \frac{a^3b+a^2b^2+ab^3+b^4+\left(a^4+a^3b-ab^3-b^4\right)\cos(fx+e)^2-\left(\left(3\,a^2b^2+ab^3\right)\cos(fx+e)^4-3\,ab^3-b^4-\right.}{2\left(\left(a^6+3\,a^5b+3\,a^4b^2+a^3b^3\right)f\,c\right.}$$

input
```
integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/2*(a^3*b + a^2*b^2 + a*b^3 + b^4 + (a^4 + a^3*b - a*b^3 - b^4)*cos(f*x +
 e)^2 - ((3*a^2*b^2 + a*b^3)*cos(f*x + e)^4 - 3*a*b^3 - b^4 - (3*a^2*b^2 -
 2*a*b^3 - b^4)*cos(f*x + e)^2)*log(a*cos(f*x + e)^2 + b) - 2*((a^4 + 3*a^
3*b)*cos(f*x + e)^4 - a^3*b - 3*a^2*b^2 - (a^4 + 2*a^3*b - 3*a^2*b^2)*cos(
f*x + e)^2)*log(1/2*sin(f*x + e)))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*
f*cos(f*x + e)^4 - (a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*f*cos(f*x + e)^2
 - (a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*f)
```

3.354.6 Sympy [F]

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

input
```
integrate(cot(f*x+e)**3/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral(cot(e + f*x)**3/(a + b*sec(e + f*x)**2)**2, x)
```

3.354.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.73

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{\frac{(3ab^2+b^3)\log\left(a\sin(fx+e)^2-a-b\right)}{a^5+3a^4b+3a^3b^2+a^2b^3} + \frac{(a+3b)\log\left(\sin(fx+e)^2\right)}{a^3+3a^2b+3ab^2+b^3} - \frac{a^3+a^2b-(a^3-b^3)\sin(fx+e)^2}{(a^5+2a^4b+a^3b^2)\sin(fx+e)^4-(a^5+3a^4b+3a^3b^2+a^2b^3)\sin(fx+e)^2}}{2f}$$

input
```
integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/2*((3*a*b^2 + b^3)*log(a*sin(f*x + e)^2 - a - b)/(a^5 + 3*a^4*b + 3*a^3
*b^2 + a^2*b^3) + (a + 3*b)*log(sin(f*x + e)^2)/(a^3 + 3*a^2*b + 3*a*b^2 +
 b^3) - (a^3 + a^2*b - (a^3 - b^3)*sin(f*x + e)^2)/((a^5 + 2*a^4*b + a^3*b
^2)*sin(f*x + e)^4 - (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sin(f*x + e)^2)
)/f
```

3.354.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 811 vs. 2(105) = 210.

Time = 0.42 (sec) , antiderivative size = 811, normalized size of antiderivative = 7.31

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-1/24*(12*(3*a*b^2 + b^3)*log(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e)
 + 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2
/(cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/(a^5
+ 3*a^4*b + 3*a^3*b^2 + a^2*b^3) + 12*(a + 3*b)*log(abs(-cos(f*x + e) + 1)
/abs(cos(f*x + e) + 1))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (3*a^4 + 6*a^3*b
 + 3*a^2*b^2 + 10*a^4*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 16*a^3*b*(co
s(f*x + e) - 1)/(cos(f*x + e) + 1) + 30*a^2*b^2*(cos(f*x + e) - 1)/(cos(f*
x + e) + 1) + 32*a*b^3*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 8*b^4*(cos(
f*x + e) - 1)/(cos(f*x + e) + 1) + 11*a^4*(cos(f*x + e) - 1)^2/(cos(f*x +
e) + 1)^2 + 22*a^3*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 27*a^2*b^
2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 16*a*b^3*(cos(f*x + e) - 1)^
2/(cos(f*x + e) + 1)^2 - 16*b^4*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2
+ 4*a^4*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 16*a^3*b*(cos(f*x + e)
 - 1)^3/(cos(f*x + e) + 1)^3 + 36*a^2*b^2*(cos(f*x + e) - 1)^3/(cos(f*x +
e) + 1)^3 + 32*a*b^3*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 8*b^4*(co
s(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3)/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*
b^3)*(a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + b*(cos(f*x + e) - 1)/(cos(
f*x + e) + 1) + 2*a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 2*b*(cos(f
*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + a*(cos(f*x + e) - 1)^3/(cos(f*x + e)
 + 1)^3 + b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3)) - 3*(cos(f*x + ...
```

3.354.9 Mupad [B] (verification not implemented)

Time = 20.51 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.44

$$\int \frac{\cot^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,a^2\,f} - \frac{\frac{1}{2\,(a+b)}+\frac{\tan(e+f\,x)^2\,(a\,b-b^2)}{2\,a\,(a+b)^2}}{f\,\left(b\tan\left(e+f\,x\right)^4+(a+b)\,\tan\left(e+f\,x\right)^2\right)} - \frac{\ln\left(\tan(e+f\,x)\right)\,(a+3\,b)}{f\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)} - \frac{b^2\,\ln\left(b\tan(e+f\,x)^2+a+b\right)\,(3\,a+b)}{2\,a^2\,f\,(a+b)^3}$$

input
```
int(cot(e + f*x)^3/(a + b/cos(e + f*x)^2)^2,x)
```

output
```
log(tan(e + f*x)^2 + 1)/(2*a^2*f) - (1/(2*(a + b)) + (tan(e + f*x)^2*(a*b
- b^2))/(2*a*(a + b)^2))/(f*(tan(e + f*x)^2*(a + b) + b*tan(e + f*x)^4)) -
 (log(tan(e + f*x))*(a + 3*b))/(f*(3*a*b^2 + 3*a^2*b + a^3 + b^3)) - (b^2*
log(a + b + b*tan(e + f*x)^2)*(3*a + b))/(2*a^2*f*(a + b)^3)
```

3.355 $\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

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3.355.1 Optimal result

Integrand size = 23, antiderivative size = 140

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{b^4}{2a^2(a+b)^3 f\,(b+a\cos^2(e+fx))} + \frac{(a+2b)\csc^2(e+fx)}{(a+b)^3 f} - \frac{\csc^4(e+fx)}{4(a+b)^2 f} + \frac{b^3(4a+b)\log\left(b+a\cos^2(e+fx)\right)}{2a^2(a+b)^4 f} + \frac{(a^2+4ab+6b^2)\log(\sin(e+fx))}{(a+b)^4 f}$$

output
```
1/2*b^4/a^2/(a+b)^3/f/(b+a*cos(f*x+e)^2)+(a+2*b)*csc(f*x+e)^2/(a+b)^3/f-1/
4*csc(f*x+e)^4/(a+b)^2/f+1/2*b^3*(4*a+b)*ln(b+a*cos(f*x+e)^2)/a^2/(a+b)^4/
f+(a^2+4*a*b+6*b^2)*ln(sin(f*x+e))/(a+b)^4/f
```

3.355.2 Mathematica [A] (verified)

Time = 2.96 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.16

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{(a+2b+a\cos(2(e+fx)))^2\sec^4(e+fx)\left(4(a+b)(a+2b)\csc^2(e+fx) - (a+b)^2\csc^4(e+fx) + 4(a^2 -}{16(a+b)^4 f\,(a+b\sec^2(e+f}$$

input
```
Integrate[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])^2*Sec[e + f*x]^4*(4*(a + b)*(a + 2*b)*Csc[
e + f*x]^2 - (a + b)^2*Csc[e + f*x]^4 + 4*(a^2 + 4*a*b + 6*b^2)*Log[Sin[e
+ f*x]] + (2*b^3*(4*a + b)*Log[a + b - a*Sin[e + f*x]^2])/a^2 + (2*b^4*(a
+ b))/(a^2*(a + b - a*Sin[e + f*x]^2))))/(16*(a + b)^4*f*(a + b*Sec[e + f*
x]^2)^2)
```

3.355.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4626, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\tan(e+fx)^5\left(a+b\sec(e+fx)^2\right)^2}dx$$

$$\downarrow 4626$$

$$-\frac{\int \frac{\cos^9(e+fx)}{(1-\cos^2(e+fx))^3(a\cos^2(e+fx)+b)^2}d\cos(e+fx)}{f}$$

$$\downarrow 354$$

$$-\frac{\int \frac{\cos^8(e+fx)}{(1-\cos^2(e+fx))^3(a\cos^2(e+fx)+b)^2}d\cos^2(e+fx)}{2f}$$

$$\downarrow 99$$

$$-\frac{\int \left(\frac{b^4}{a(a+b)^3(a\cos^2(e+fx)+b)^2}-\frac{(4a+b)b^3}{a(a+b)^4(a\cos^2(e+fx)+b)}+\frac{-a^2-4ba-6b^2}{(a+b)^4(\cos^2(e+fx)-1)}-\frac{2(a+2b)}{(a+b)^3(\cos^2(e+fx)-1)^2}-\frac{1}{(a+b)^2(\cos^2(e+fx)}\right.}{2f}$$

$$\downarrow 2009$$

$$-\frac{-\frac{b^4}{a^2(a+b)^3(a\cos^2(e+fx)+b)}-\frac{b^3(4a+b)\log(a\cos^2(e+fx)+b)}{a^2(a+b)^4}-\frac{(a^2+4ab+6b^2)\log(1-\cos^2(e+fx))}{(a+b)^4}-\frac{2(a+2b)}{(a+b)^3(1-\cos^2(e+fx))}+\frac{2(a+b}{}}{2f}$$

input
```
Int[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
-1/2*(1/(2*(a + b)^2*(1 - Cos[e + f*x]^2)^2) - (2*(a + 2*b))/((a + b)^3*(1
 - Cos[e + f*x]^2)) - b^4/(a^2*(a + b)^3*(b + a*Cos[e + f*x]^2)) - ((a^2 +
 4*a*b + 6*b^2)*Log[1 - Cos[e + f*x]^2])/(a + b)^4 - (b^3*(4*a + b)*Log[b
+ a*Cos[e + f*x]^2])/(a^2*(a + b)^4))/f
```

3.355.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.355.4 Maple [A] (verified)

Time = 13.27 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.44

method	result
derivativedivides	$\frac{b^3\left(\frac{(a+b)b}{a^2\left(b+a\cos(fx+e)^2\right)}+\frac{(4a+b)\ln\left(b+a\cos(fx+e)^2\right)}{a^2}\right)}{2(a+b)^4}-\frac{1}{16(a+b)^2(1+\cos(fx+e))^2}-\frac{-7a-15b}{16(a+b)^3(1+\cos(fx+e))}+\frac{\left(a^2+4ab+6b^2\right)}{2(a-}$ / f
default	$\frac{b^3\left(\frac{(a+b)b}{a^2\left(b+a\cos(fx+e)^2\right)}+\frac{(4a+b)\ln\left(b+a\cos(fx+e)^2\right)}{a^2}\right)}{2(a+b)^4}-\frac{1}{16(a+b)^2(1+\cos(fx+e))^2}-\frac{-7a-15b}{16(a+b)^3(1+\cos(fx+e))}+\frac{\left(a^2+4ab+6b^2\right)}{2(a-}$ / f
risch	$-\frac{2ia^2x}{a^4+4a^3b+6a^2b^2+4ab^3+b^4}-\frac{8iabx}{a^4+4a^3b+6a^2b^2+4ab^3+b^4}-\frac{8iabe}{f(a^4+4a^3b+6a^2b^2+4ab^3+b^4)}-\frac{2ib^4}{a^2f(a^4+4a^3b+6a$

input
```
int(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/2*b^3/(a+b)^4*((a+b)*b/a^2/(b+a*cos(f*x+e)^2)+(4*a+b)/a^2*ln(b+a*co
s(f*x+e)^2))-1/16/(a+b)^2/(1+cos(f*x+e))^2-1/16*(-7*a-15*b)/(a+b)^3/(1+cos
(f*x+e))+1/2*(a^2+4*a*b+6*b^2)/(a+b)^4*ln(1+cos(f*x+e))-1/16/(a+b)^2/(-1+c
os(f*x+e))^2-1/16*(7*a+15*b)/(a+b)^3/(-1+cos(f*x+e))+1/2*(a^2+4*a*b+6*b^2)
/(a+b)^4*ln(-1+cos(f*x+e)))
```

3.355.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 557 vs. $2(134) = 268$.

Time = 0.80 (sec) , antiderivative size = 557, normalized size of antiderivative = 3.98

$$\int \frac{\cot^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$
$$= \frac{3\,a^4b+10\,a^3b^2+7\,a^2b^3+2\,ab^4+2\,b^5-2\left(2\,a^5+6\,a^4b+4\,a^3b^2-ab^4-b^5\right)\cos\left(fx+e\right)^4+\left(3\,a^5+6\,a^4b+\right.}$$

input
```
integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
1/4*(3*a^4*b + 10*a^3*b^2 + 7*a^2*b^3 + 2*a*b^4 + 2*b^5 - 2*(2*a^5 + 6*a^4
*b + 4*a^3*b^2 - a*b^4 - b^5)*cos(f*x + e)^4 + (3*a^5 + 6*a^4*b - 5*a^3*b^
2 - 8*a^2*b^3 - 4*a*b^4 - 4*b^5)*cos(f*x + e)^2 + 2*((4*a^2*b^3 + a*b^4)*c
os(f*x + e)^6 + 4*a*b^4 + b^5 - (8*a^2*b^3 - 2*a*b^4 - b^5)*cos(f*x + e)^4
 + (4*a^2*b^3 - 7*a*b^4 - 2*b^5)*cos(f*x + e)^2)*log(a*cos(f*x + e)^2 + b)
 + 4*((a^5 + 4*a^4*b + 6*a^3*b^2)*cos(f*x + e)^6 + a^4*b + 4*a^3*b^2 + 6*a
^2*b^3 - (2*a^5 + 7*a^4*b + 8*a^3*b^2 - 6*a^2*b^3)*cos(f*x + e)^4 + (a^5 +
 2*a^4*b - 2*a^3*b^2 - 12*a^2*b^3)*cos(f*x + e)^2)*log(1/2*sin(f*x + e)))/
((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^6 - (2*a
^7 + 7*a^6*b + 8*a^5*b^2 + 2*a^4*b^3 - 2*a^3*b^4 - a^2*b^5)*f*cos(f*x + e)
^4 + (a^7 + 2*a^6*b - 2*a^5*b^2 - 8*a^4*b^3 - 7*a^3*b^4 - 2*a^2*b^5)*f*cos
(f*x + e)^2 + (a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*f)
```

3.355.6 Sympy [F]

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

input
```
integrate(cot(f*x+e)**5/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral(cot(e + f*x)**5/(a + b*sec(e + f*x)**2)**2, x)
```

3.355.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(134) = 268$.

Time $= 0.20$ (sec) , antiderivative size $= 279$, normalized size of antiderivative $= 1.99$

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$
$$= \frac{\frac{2\,(4\,ab^3+b^4)\log\left(a\sin(fx+e)^2-a-b\right)}{a^6+4\,a^5b+6\,a^4b^2+4\,a^3b^3+a^2b^4} + \frac{2\,(a^2+4\,ab+6\,b^2)\log\left(\sin(fx+e)^2\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4} + \frac{2\,(2\,a^4+4\,a^3b-b^4)\sin(fx+e)^4+a^4+2\,a^3b+a^2b^2-(5\,a^4+13\,a^3}{(a^6+3\,a^5b+3\,a^4b^2+a^3b^3)\sin(fx+e)^6-(a^6+4\,a^5b+6\,a^4b^2+4\,a}}{4\,f}$$

input
```
integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/4*(2*(4*a*b^3 + b^4)*log(a*sin(f*x + e)^2 - a - b)/(a^6 + 4*a^5*b + 6*a^
4*b^2 + 4*a^3*b^3 + a^2*b^4) + 2*(a^2 + 4*a*b + 6*b^2)*log(sin(f*x + e)^2)
/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + (2*(2*a^4 + 4*a^3*b - b^4)*
sin(f*x + e)^4 + a^4 + 2*a^3*b + a^2*b^2 - (5*a^4 + 13*a^3*b + 8*a^2*b^2)*
sin(f*x + e)^2)/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*sin(f*x + e)^6 - (a
^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*sin(f*x + e)^4))/f
```

3.355.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 913 vs. $2(134) = 268$.

Time = 0.50 (sec) , antiderivative size = 913, normalized size of antiderivative = 6.52

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/64*(32*(4*a*b^3 + b^4)*log(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e)
+ 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/
(cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/(a^6 +
 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4) + 32*(a^2 + 4*a*b + 6*b^2)*log
(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1))/(a^4 + 4*a^3*b + 6*a^2*b^2
+ 4*a*b^3 + b^4) - (12*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 40*a*b*
(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 28*b^2*(cos(f*x + e) - 1)/(cos(f*x
 + e) + 1) + a^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 2*a*b*(cos(f*
x + e) - 1)^2/(cos(f*x + e) + 1)^2 + b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e
) + 1)^2)/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - (a^2 + 2*a*b + b^2
 + 12*a^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 40*a*b*(cos(f*x + e) - 1
)/(cos(f*x + e) + 1) + 28*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 48*a
^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 192*a*b*(cos(f*x + e) - 1)^
2/(cos(f*x + e) + 1)^2 + 288*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2
)*(cos(f*x + e) + 1)^2/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(cos(f
*x + e) - 1)^2) - 32*(4*a^2*b^3 + 5*a*b^4 + b^5 + 8*a^2*b^3*(cos(f*x + e)
- 1)/(cos(f*x + e) + 1) - 2*a*b^4*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) -
2*b^5*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 4*a^2*b^3*(cos(f*x + e) - 1)
^2/(cos(f*x + e) + 1)^2 + 5*a*b^4*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^
2 + b^5*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/((a^6 + 4*a^5*b + 6*...
```

3.355.9 Mupad [B] (verification not implemented)

Time = 21.17 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.47

$$\int \frac{\cot^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\frac{\tan(e+f\,x)^2\,(2\,a+5\,b)}{4\,(a+b)^2} - \frac{1}{4\,(a+b)} + \frac{\tan(e+f\,x)^4\,(a^2\,b+3\,a\,b^2-b^3)}{2\,a\,(a+b)^3}}{f\,\left(b\tan\left(e+f\,x\right)^6+(a+b)\,\tan\left(e+f\,x\right)^4\right)} - \frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,a^2\,f} + \frac{\ln\left(\tan(e+f\,x)\right)\,(a^2+4\,a\,b+6\,b^2)}{f\,(a^4+4\,a^3\,b+6\,a^2\,b^2+4\,a\,b^3+b^4)} + \frac{b^3\,\ln\left(b\tan(e+f\,x)^2+a+b\right)\,(4\,a+b)}{2\,a^2\,f\,(a+b)^4}$$

input
```
int(cot(e + f*x)^5/(a + b/cos(e + f*x)^2)^2,x)
```

output
```
((tan(e + f*x)^2*(2*a + 5*b))/(4*(a + b)^2) - 1/(4*(a + b)) + (tan(e + f*x
)^4*(3*a*b^2 + a^2*b - b^3))/(2*a*(a + b)^3))/(f*(tan(e + f*x)^4*(a + b) +
 b*tan(e + f*x)^6)) - log(tan(e + f*x)^2 + 1)/(2*a^2*f) + (log(tan(e + f*x
))*(4*a*b + a^2 + 6*b^2))/(f*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2))
+ (b^3*log(a + b + b*tan(e + f*x)^2)*(4*a + b))/(2*a^2*f*(a + b)^4)
```

3.356 $\int \frac{\tan^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

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3.356.1 Optimal result

Integrand size = 23, antiderivative size = 119

$$\int \frac{\tan^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{x}{a^2} - \frac{(3a-2b)(a+b)^{3/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2b^{5/2}f} + \frac{(3a+b)\tan(e+fx)}{2ab^2f} - \frac{(a+b)\tan^3(e+fx)}{2abf\left(a+b+b\tan^2(e+fx)\right)}$$

output

```
-x/a^2-1/2*(3*a-2*b)*(a+b)^(3/2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^
2/b^(5/2)/f+1/2*(3*a+b)*tan(f*x+e)/a/b^2/f-1/2*(a+b)*tan(f*x+e)^3/a/b/f/(a
+b+b*tan(f*x+e)^2)
```

3.356.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.80 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.40

$$\int \frac{\tan^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{(a+2b+a\cos(2(e+fx)))\sec^4(e+fx)\left(-\frac{2x(a+2b+a\cos(2(e+fx)))}{a^2} + \frac{(3a-2b)(a+b)^{3/2}\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(}{2\sqrt{a+b}\sqrt{b(}}\right.\right.}{a^2}$$

input
```
Integrate[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*((-2*x*(a + 2*b + a*Cos[2*(
e + f*x)]))/a^2 + ((3*a - 2*b)*(a + b)^(3/2)*ArcTan[(Sec[f*x]*(Cos[2*e] -
I*Sin[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqr
t[b*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[2*e] - I*
Sin[2*e]))/(a^2*b^2*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (2*(a + 2*b + a*Cos
[2*(e + f*x)])*Sec[e]*Sec[e + f*x]*Sin[f*x])/(b^2*f) - ((a + b)^2*((a + 2*
b)*Sin[2*e] - a*Sin[2*f*x]))/(a^2*b^2*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]
))))/(8*(a + b*Sec[e + f*x]^2)^2)
```

3.356.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4629, 2075, 372, 444, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\tan(e+fx)^6}{(a+b\sec(e+fx)^2)^2}\,dx$$

$$\downarrow 4629$$

$$\frac{\int \frac{\tan^6(e+fx)}{(\tan^2(e+fx)+1)(a+b(\tan^2(e+fx)+1))^2}\,d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\tan^6(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^2}\,d\tan(e+fx)}{f}$$

$$\downarrow 372$$

$$\frac{\int \frac{\tan^2(e+fx)\left((3a+b)\tan^2(e+fx)+3(a+b)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}\,d\tan(e+fx)}{2ab} - \frac{(a+b)\tan^3(e+fx)}{2ab(a+b\tan^2(e+fx)+b)}}{f}$$

$$\downarrow 444$$

$$\frac{\frac{(3a+b)\tan(e+fx)}{b}-\frac{\int\frac{\left(3a^2+4ba-b^2\right)\tan^2(e+fx)+(a+b)(3a+b)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{b}}{2ab}-\frac{(a+b)\tan^3(e+fx)}{2ab(a+b\tan^2(e+fx)+b)}}{f}$$

$$\downarrow 397$$

$$\frac{\frac{(3a+b)\tan(e+fx)}{b}-\frac{2b^2\int\frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}+\frac{(3a-2b)(a+b)^2\int\frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{b}}{2ab}-\frac{(a+b)\tan^3(e+fx)}{2ab(a+b\tan^2(e+fx)+b)}}{f}$$

$$\downarrow 216$$

$$\frac{\frac{(3a+b)\tan(e+fx)}{b}-\frac{(3a-2b)(a+b)^2\int\frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}+\frac{2b^2\arctan(\tan(e+fx))}{a}}{b}}{2ab}-\frac{(a+b)\tan^3(e+fx)}{2ab(a+b\tan^2(e+fx)+b)}}{f}$$

$$\downarrow 218$$

$$\frac{\frac{(3a+b)\tan(e+fx)}{b}-\frac{2b^2\arctan(\tan(e+fx))}{a}+\frac{(3a-2b)(a+b)^{3/2}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{b}}}{b}}{2ab}-\frac{(a+b)\tan^3(e+fx)}{2ab(a+b\tan^2(e+fx)+b)}}{f}$$

input
```
Int[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((-(((2*b^2*ArcTan[Tan[e + f*x]])/a + ((3*a - 2*b)*(a + b)^(3/2)*ArcTan[(S
qrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*Sqrt[b]))/b) + ((3*a + b)*Tan[e + f*
x])/b)/(2*a*b) - ((a + b)*Tan[e + f*x]^3)/(2*a*b*(a + b + b*Tan[e + f*x]^2
)))/f
```

3.356.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.356.4 Maple [A] (verified)

Time = 8.53 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{\frac{\tan(fx+e)}{b^2}-\frac{\left(-\frac{1}{2}a^3-a^2b-\frac{1}{2}ab^2\right)\tan(fx+e)}{a+b+b\tan(fx+e)^2}+\frac{\left(3a^3+4a^2b-ab^2-2b^3\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}}{a^2b^2}-\frac{\arctan(\tan(fx+e))}{a^2}}{f}$
default	$\frac{\frac{\tan(fx+e)}{b^2}-\frac{\left(-\frac{1}{2}a^3-a^2b-\frac{1}{2}ab^2\right)\tan(fx+e)}{a+b+b\tan(fx+e)^2}+\frac{\left(3a^3+4a^2b-ab^2-2b^3\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}}{a^2b^2}-\frac{\arctan(\tan(fx+e))}{a^2}}{f}$
risch	$-\frac{x}{a^2}+\frac{i\left(3a^3e^{4i(fx+e)}+4a^2be^{4i(fx+e)}+5ab^2e^{4i(fx+e)}+2b^3e^{4i(fx+e)}+6a^3e^{2i(fx+e)}+14a^2be^{2i(fx+e)}+6ab^2e^{2i(fx+e}\right.}{a^2b^2f\left(ae^{4i(fx+e)}+2ae^{2i(fx+e)}+4be^{2i(fx+e)}+a\right)\left(e^{2i(fx+e)}+1\right)}$

input
```
int(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(tan(f*x+e)/b^2-1/a^2/b^2*((-1/2*a^3-a^2*b-1/2*a*b^2)*tan(f*x+e)/(a+b+
b*tan(f*x+e)^2)+1/2*(3*a^3+4*a^2*b-a*b^2-2*b^3)/((a+b)*b)^(1/2)*arctan(b*t
an(f*x+e)/((a+b)*b)^(1/2)))-1/a^2*arctan(tan(f*x+e)))
```

3.356.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(105) = 210.

Time = 0.30 (sec) , antiderivative size = 514, normalized size of antiderivative = 4.32

$$\int \frac{\tan^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$= \left[-\frac{8\,ab^2 fx\cos(fx+e)^3+8\,b^3 fx\cos(fx+e)+\left(3\,a^3+a^2b-2\,ab^2\right)\cos(fx+e)^3+\left(3\,a^2b+ab^2-2\,b^3\right)}{} \right.$$

$$\left. -\frac{4\,ab^2 fx\cos(fx+e)^3+4\,b^3 fx\cos(fx+e)-\left(3\,a^3+a^2b-2\,ab^2\right)\cos(fx+e)^3+\left(3\,a^2b+ab^2-2\,b^3\right)}{4\left(a^3b^2 f\cos(fx\right.} \right]$$

input
```
integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[-1/8*(8*a*b^2*f*x*cos(f*x + e)^3 + 8*b^3*f*x*cos(f*x + e) + ((3*a^3 + a^2
*b - 2*a*b^2)*cos(f*x + e)^3 + (3*a^2*b + a*b^2 - 2*b^3)*cos(f*x + e))*sqr
t(-(a + b)/b)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2
)*cos(f*x + e)^2 - 4*((a*b + 2*b^2)*cos(f*x + e)^3 - b^2*cos(f*x + e))*sqr
t(-(a + b)/b)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)
^2 + b^2)) - 4*(2*a^2*b + (3*a^3 + 2*a^2*b + a*b^2)*cos(f*x + e)^2)*sin(f*
x + e))/(a^3*b^2*f*cos(f*x + e)^3 + a^2*b^3*f*cos(f*x + e)), -1/4*(4*a*b^2
*f*x*cos(f*x + e)^3 + 4*b^3*f*x*cos(f*x + e) - ((3*a^3 + a^2*b - 2*a*b^2)*
cos(f*x + e)^3 + (3*a^2*b + a*b^2 - 2*b^3)*cos(f*x + e))*sqrt((a + b)/b)*a
rctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt((a + b)/b)/((a + b)*cos(f*x
+ e)*sin(f*x + e))) - 2*(2*a^2*b + (3*a^3 + 2*a^2*b + a*b^2)*cos(f*x + e)^
2)*sin(f*x + e))/(a^3*b^2*f*cos(f*x + e)^3 + a^2*b^3*f*cos(f*x + e))]
```

3.356.6 Sympy [F]

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

input
```
integrate(tan(f*x+e)**6/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral(tan(e + f*x)**6/(a + b*sec(e + f*x)**2)**2, x)
```

3.356.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.07

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$
$$= \frac{\frac{(a^2+2ab+b^2)\tan(fx+e)}{ab^3\tan(fx+e)^2+a^2b^2+ab^3} - \frac{2(fx+e)}{a^2} + \frac{2\tan(fx+e)}{b^2} - \frac{(3a^3+4a^2b-ab^2-2b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}a^2b^2}}{2f}$$

input
```
integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/2*((a^2 + 2*a*b + b^2)*tan(f*x + e)/(a*b^3*tan(f*x + e)^2 + a^2*b^2 + a*
b^3) - 2*(f*x + e)/a^2 + 2*tan(f*x + e)/b^2 - (3*a^3 + 4*a^2*b - a*b^2 - 2
*b^3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^2*b^2))/f
```

3.356.8 Giac [A] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.30

$$\int \frac{\tan^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx =$$

$$-\frac{\frac{2\,(fx+e)}{a^2}-\frac{2\tan(fx+e)}{b^2}+\frac{\left(3a^3+4a^2b-ab^2-2b^3\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{\sqrt{ab+b^2}a^2b^2}-\frac{a^2\tan(fx+e)+2ab\tan(fx+e)+b^2t}{\left(b\tan(fx+e)^2+a+b\right)ab^2}}{2f}$$

input
```
integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-1/2*(2*(f*x + e)/a^2 - 2*tan(f*x + e)/b^2 + (3*a^3 + 4*a^2*b - a*b^2 - 2*
b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
 + b^2)))/(sqrt(a*b + b^2)*a^2*b^2) - (a^2*tan(f*x + e) + 2*a*b*tan(f*x +
e) + b^2*tan(f*x + e))/((b*tan(f*x + e)^2 + a + b)*a*b^2))/f
```

3.356.9 Mupad [B] (verification not implemented)

Time = 20.05 (sec) , antiderivative size = 765, normalized size of antiderivative = 6.43

$$\int \frac{\tan^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\tan(e+f\,x)}{b^2\,f}$$

$$-\frac{\operatorname{atan}\left(\frac{5\tan(e+f\,x)}{\frac{12\,a}{b}-\frac{10\,b}{a}-\frac{15\,b^2}{2\,a^2}+\frac{9\,a^2}{2\,b^2}+5}-\frac{10\tan(e+f\,x)}{\frac{5\,a}{b}-\frac{15\,b}{2\,a}+\frac{12\,a^2}{b^2}+\frac{9\,a^3}{2\,b^3}-10}+\frac{12\,a\tan(e+f\,x)}{12\,a+5\,b-\frac{10\,b^2}{a}+\frac{9\,a^2}{2\,b}-\frac{15\,b^3}{2\,a^2}}-\frac{15\,b\tan(e+f\,x)}{2\left(\frac{5\,a^2}{b}-\frac{15\,b}{2}-10\,a+\frac{12\,a^3}{b^2}+\frac{9\,a^4}{2\,b^3}\right)}+\right.}{a^2\,f}$$

$$+\frac{\tan(e+f\,x)\,\left(a^2+2\,a\,b+b^2\right)}{2\,a\,f\,\left(b^3\tan\left(e+f\,x\right)^2+b^3+a\,b^2\right)}$$

$$-\frac{\operatorname{atan}\left(-\frac{\tan(e+f\,x)\,\sqrt{-a^3\,b^5-3\,a^2\,b^6-3\,a\,b^7-b^8}\,35\mathrm{i}}{4\left(9\,a^3\,b-\frac{85\,a\,b^3}{4}+\frac{81\,a^4}{4}+\frac{25\,b^4}{4}-\frac{49\,a^2\,b^2}{2}+\frac{15\,b^5}{2\,a}+\frac{27\,a^5}{4\,b}\right)}+\frac{\tan(e+f\,x)\,\sqrt{-a^3\,b^5-3\,a^2\,b^6-3\,a\,b^7-b^8}\,15\mathrm{i}}{2\left(\frac{25\,a\,b^3}{4}-\frac{49\,a^3\,b}{2}+9\,a^4+\frac{15\,b^4}{2}-\frac{85\,a^2\,b^2}{4}+\frac{81\,a^5}{4\,b}+\frac{27\,a^6}{4\,b^2}\right)}-\frac{ta}{4\left(\frac{81\,a^3}{4}\right.}\right.}$$

input
```
int(tan(e + f*x)^6/(a + b/cos(e + f*x)^2)^2,x)
```

output

```
tan(e + f*x)/(b^2*f) - atan((5*tan(e + f*x))/((12*a)/b - (10*b)/a - (15*b^
2)/(2*a^2) + (9*a^2)/(2*b^2) + 5) - (10*tan(e + f*x))/((5*a)/b - (15*b)/(2
*a) + (12*a^2)/b^2 + (9*a^3)/(2*b^3) - 10) + (12*a*tan(e + f*x))/(12*a + 5
*b - (10*b^2)/a + (9*a^2)/(2*b) - (15*b^3)/(2*a^2)) - (15*b*tan(e + f*x))/
(2*((5*a^2)/b - (15*b)/2 - 10*a + (12*a^3)/b^2 + (9*a^4)/(2*b^3))) + (9*a^
2*tan(e + f*x))/(2*(12*a*b + (9*a^2)/2 + 5*b^2 - (10*b^3)/a - (15*b^4)/(2*
a^2))))/(a^2*f) + (tan(e + f*x)*(2*a*b + a^2 + b^2))/(2*a*f*(a*b^2 + b^3 +
 b^3*tan(e + f*x)^2)) - (atan((tan(e + f*x)*(- 3*a*b^7 - b^8 - 3*a^2*b^6 -
 a^3*b^5)^(1/2)*15i)/(2*((25*a*b^3)/4 - (49*a^3*b)/2 + 9*a^4 + (15*b^4)/2
- (85*a^2*b^2)/4 + (81*a^5)/(4*b) + (27*a^6)/(4*b^2))) - (tan(e + f*x)*(-
3*a*b^7 - b^8 - 3*a^2*b^6 - a^3*b^5)^(1/2)*35i)/(4*(9*a^3*b - (85*a*b^3)/4
 + (81*a^4)/4 + (25*b^4)/4 - (49*a^2*b^2)/2 + (15*b^5)/(2*a) + (27*a^5)/(4
*b))) - (tan(e + f*x)*(- 3*a*b^7 - b^8 - 3*a^2*b^6 - a^3*b^5)^(1/2)*45i)/(
4*((81*a^3*b)/4 - (49*a*b^3)/2 + (27*a^4)/4 - (85*b^4)/4 + 9*a^2*b^2 + (25
*b^5)/(4*a) + (15*b^6)/(2*a^2))) + (a^2*tan(e + f*x)*(- 3*a*b^7 - b^8 - 3*
a^2*b^6 - a^3*b^5)^(1/2)*27i)/(4*(9*a^2*b^4 - (85*b^6)/4 - (49*a*b^5)/2 +
(81*a^3*b^3)/4 + (27*a^4*b^2)/4 + (25*b^7)/(4*a) + (15*b^8)/(2*a^2))) + (a
*tan(e + f*x)*(- 3*a*b^7 - b^8 - 3*a^2*b^6 - a^3*b^5)^(1/2)*27i)/(4*((27*a
^4*b)/4 - (49*a*b^4)/2 - (85*b^5)/4 + 9*a^2*b^3 + (81*a^3*b^2)/4 + (25*b^6
)/(4*a) + (15*b^7)/(2*a^2))))*(-b^5*(a + b)^3)^(1/2)*(3*a - 2*b)*1i)/(2...
```

3.357 $\int \frac{\tan^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

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3.357.1 Optimal result

Integrand size = 23, antiderivative size = 90

$$\int \frac{\tan^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{x}{a^2} + \frac{(a-2b)\sqrt{a+b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2b^{3/2}f} - \frac{(a+b)\tan(e+fx)}{2abf\left(a+b+b\tan^2(e+fx)\right)}$$

output

```
x/a^2+1/2*(a-2*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*(a+b)^(1/2)/a^2/b
^(3/2)/f-1/2*(a+b)*tan(f*x+e)/a/b/f/(a+b+b*tan(f*x+e)^2)
```

3.357.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.73 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.77

$$\int \frac{\tan^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^4(e+fx)\left(2x(a+2b+a\cos(2(e+fx))) + \frac{(-a^2+ab+2b^2)\arctan\left(\frac{\sec(fx)(\cos(2e)-}{}\right.\right.}{8a^2\left(a+b\sec^2(e+\right.}$$

input
```
Integrate[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*(2*x*(a + 2*b + a*Cos[2*(e
+ f*x)]) + ((-a^2 + a*b + 2*b^2)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*
(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e]
- I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[2*e] - I*Sin[2*e]))/(
b*Sqrt[a + b]*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + ((a + b)*((a + 2*b)*Sin[2
*e] - a*Sin[2*f*x]))/(b*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))))/(8*a^2*(a
 + b*Sec[e + f*x]^2)^2)
```

3.357.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4629, 2075, 372, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\tan(e+fx)^4}{(a+b\sec(e+fx)^2)^2}\,dx$$

$$\downarrow 4629$$

$$\frac{\int \frac{\tan^4(e+fx)}{(\tan^2(e+fx)+1)(a+b(\tan^2(e+fx)+1))^2}\,d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\tan^4(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^2}\,d\tan(e+fx)}{f}$$

$$\downarrow 372$$

$$\frac{\int \frac{(a-b)\tan^2(e+fx)+a+b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}\,d\tan(e+fx)}{2ab} - \frac{(a+b)\tan(e+fx)}{2ab(a+b\tan^2(e+fx)+b)}}{f}$$

$$\downarrow 397$$

$$\frac{\frac{2b\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}+\frac{(a-2b)(a+b)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{2ab}-\frac{(a+b)\tan(e+fx)}{2ab(a+b\tan^2(e+fx)+b)}}{f}$$

$$\downarrow 216$$

$$\frac{\frac{(a-2b)(a+b)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}+\frac{2b\arctan(\tan(e+fx))}{a}}{2ab}-\frac{(a+b)\tan(e+fx)}{2ab(a+b\tan^2(e+fx)+b)}}{f}$$

$$\downarrow 218$$

$$\frac{\frac{2b\arctan(\tan(e+fx))}{a}+\frac{(a-2b)\sqrt{a+b}\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{b}}}{2ab}-\frac{(a+b)\tan(e+fx)}{2ab(a+b\tan^2(e+fx)+b)}}{f}$$

input
```
Int[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(((2*b*ArcTan[Tan[e + f*x]])/a + ((a - 2*b)*Sqrt[a + b]*ArcTan[(Sqrt[b]*Ta
n[e + f*x])/Sqrt[a + b]])/(a*Sqrt[b]))/(2*a*b) - ((a + b)*Tan[e + f*x])/(2
*a*b*(a + b + b*Tan[e + f*x]^2)))/f
```

3.357.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.357.4 Maple [A] (verified)

Time = 4.55 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{\frac{\arctan(\tan(fx+e))}{a^2}+\frac{(a+b)\left(-\frac{a\tan(fx+e)}{2b\left(a+b+b\tan(fx+e)^2\right)}+\frac{(a-2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2b\sqrt{(a+b)b}}\right)}{a^2}}{f}$
default	$\frac{\frac{\arctan(\tan(fx+e))}{a^2}+\frac{(a+b)\left(-\frac{a\tan(fx+e)}{2b\left(a+b+b\tan(fx+e)^2\right)}+\frac{(a-2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2b\sqrt{(a+b)b}}\right)}{a^2}}{f}$
risch	$\frac{x}{a^2}-\frac{i\left(a^2e^{2i(fx+e)}+3ab\,e^{2i(fx+e)}+2b^2e^{2i(fx+e)}+a^2+ab\right)}{a^2bf\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)}+\frac{\sqrt{-(a+b)b}\,\ln\left(e^{2i(fx+e)}-\frac{2i\sqrt{-(a+b)b}-a-2b}{a}\right)}{4b^2fa}-\frac{\sqrt{}$

input
```
int(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/a^2*arctan(tan(f*x+e))+(a+b)/a^2*(-1/2*a/b*tan(f*x+e)/(a+b+b*tan(f*
x+e)^2)+1/2*(a-2*b)/b/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))
))
```

3.357.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(78) = 156$.

Time $= 0.29$ (sec) , antiderivative size $= 393$, normalized size of antiderivative $= 4.37$

$$\int \frac{\tan^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$= \left[\frac{8\,abfx\cos\left(fx+e\right)^2+8\,b^2fx-4\left(a^2+ab\right)\cos\left(fx+e\right)\sin\left(fx+e\right)-\left(\left(a^2-2\,ab\right)\cos\left(fx+e\right)^2+ab-\right.}{8\left(a^3bf\cos\left(\right.\right.}\right.$$

input
```
integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[1/8*(8*a*b*f*x*cos(f*x + e)^2 + 8*b^2*f*x - 4*(a^2 + a*b)*cos(f*x + e)*si
n(f*x + e) - ((a^2 - 2*a*b)*cos(f*x + e)^2 + a*b - 2*b^2)*sqrt(-(a + b)/b)
*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e
)^2 + 4*((a*b + 2*b^2)*cos(f*x + e)^3 - b^2*cos(f*x + e))*sqrt(-(a + b)/b)
*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)))/(
a^3*b*f*cos(f*x + e)^2 + a^2*b^2*f), 1/4*(4*a*b*f*x*cos(f*x + e)^2 + 4*b^2
*f*x - 2*(a^2 + a*b)*cos(f*x + e)*sin(f*x + e) - ((a^2 - 2*a*b)*cos(f*x +
e)^2 + a*b - 2*b^2)*sqrt((a + b)/b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 -
 b)*sqrt((a + b)/b)/((a + b)*cos(f*x + e)*sin(f*x + e))))/(a^3*b*f*cos(f*x
 + e)^2 + a^2*b^2*f)]
```

3.357.6 Sympy [F]

$$\int \frac{\tan^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \int \frac{\tan^4\left(e+fx\right)}{\left(a+b\sec^2\left(e+fx\right)\right)^2}\,dx$$

input
```
integrate(tan(f*x+e)**4/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral(tan(e + f*x)**4/(a + b*sec(e + f*x)**2)**2, x)
```

3.357.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.07

$$\int \frac{\tan^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{\frac{(a+b)\tan(fx+e)}{ab^2\tan(fx+e)^2+a^2b+ab^2} - \frac{2(fx+e)}{a^2} - \frac{\left(a^2-ab-2b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}a^2b}}{2f}$$

input
```
integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/2*((a + b)*tan(f*x + e)/(a*b^2*tan(f*x + e)^2 + a^2*b + a*b^2) - 2*(f*x
 + e)/a^2 - (a^2 - a*b - 2*b^2)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sq
rt((a + b)*b)*a^2*b))/f
```

3.357.8 Giac [A] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.33

$$\int \frac{\tan^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$
$$= \frac{\frac{2(fx+e)}{a^2} + \frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)\left(a^2-ab-2b^2\right)}{\sqrt{ab+b^2}a^2b} - \frac{a\tan(fx+e)+b\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)ab}}{2f}$$

input
```
integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/2*(2*(f*x + e)/a^2 + (pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan
(f*x + e)/sqrt(a*b + b^2)))*(a^2 - a*b - 2*b^2)/(sqrt(a*b + b^2)*a^2*b) -
(a*tan(f*x + e) + b*tan(f*x + e))/((b*tan(f*x + e)^2 + a + b)*a*b))/f
```

3.357.9 Mupad [B] (verification not implemented)

Time = 19.79 (sec) , antiderivative size = 285, normalized size of antiderivative = 3.17

$$\int \frac{\tan^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$= \frac{\operatorname{atan}\left(\frac{\tan(e+f\,x)}{\frac{3\,b}{2\,a}-\frac{a}{2\,b}+1}-\frac{\tan(e+f\,x)}{2\left(\frac{b}{a}+\frac{3\,b^2}{2\,a^2}-\frac{1}{2}\right)}+\frac{3\,b\tan(e+f\,x)}{2\left(a+\frac{3\,b}{2}-\frac{a^2}{2\,b}\right)}\right)}{a^2\,f}-\frac{\tan(e+f\,x)\,(a+b)}{2\,a\,b\,f\,\left(b\tan\left(e+f\,x\right)^2+a+b\right)}$$
$$-\frac{\operatorname{atanh}\left(\frac{3\tan(e+f\,x)\,\sqrt{-b^4-a\,b^3}}{2\left(\frac{a\,b}{4}-a^2+\frac{3\,b^2}{2}+\frac{a^3}{4\,b}\right)}-\frac{5\tan(e+f\,x)\,\sqrt{-b^4-a\,b^3}}{4\left(\frac{a^2}{4}-a\,b+\frac{b^2}{4}+\frac{3\,b^3}{2\,a}\right)}+\frac{\tan(e+f\,x)\,\sqrt{-b^4-a\,b^3}}{4\left(\frac{a\,b}{4}-b^2+\frac{b^3}{4\,a}+\frac{3\,b^4}{2\,a^2}\right)}\right)\,\sqrt{-b^3\,\left(a+b\right)}\,(a-2\,b)}{2\,a^2\,b^3\,f}$$

input
```
int(tan(e + f*x)^4/(a + b/cos(e + f*x)^2)^2,x)
```

output
```
atan(tan(e + f*x)/((3*b)/(2*a) - a/(2*b) + 1) - tan(e + f*x)/(2*(b/a + (3*
b^2)/(2*a^2) - 1/2)) + (3*b*tan(e + f*x))/(2*(a + (3*b)/2 - a^2/(2*b))))/(
a^2*f) - (tan(e + f*x)*(a + b))/(2*a*b*f*(a + b + b*tan(e + f*x)^2)) - (at
anh((3*tan(e + f*x)*(- a*b^3 - b^4)^(1/2))/(2*((a*b)/4 - a^2 + (3*b^2)/2 +
 a^3/(4*b))) - (5*tan(e + f*x)*(- a*b^3 - b^4)^(1/2))/(4*(a^2/4 - a*b + b^
2/4 + (3*b^3)/(2*a))) + (tan(e + f*x)*(- a*b^3 - b^4)^(1/2))/(4*((a*b)/4 -
 b^2 + b^3/(4*a) + (3*b^4)/(2*a^2))))*(-b^3*(a + b))^(1/2)*(a - 2*b))/(2*a
^2*b^3*f)
```

3.358 $\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

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3.358.1 Optimal result

Integrand size = 23, antiderivative size = 85

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = -\frac{x}{a^2} + \frac{(a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2\sqrt{b}\sqrt{a+b}f} + \frac{\tan(e+fx)}{2af\left(a+b+b\tan^2(e+fx)\right)}$$

output
```
-x/a^2+1/2*(a+2*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^2/f/b^(1/2)/(a
+b)^(1/2)+1/2*tan(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)
```

3.358.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 8.55 (sec) , antiderivative size = 346, normalized size of antiderivative = 4.07

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$= (a+2b+a\cos(2(e+fx)))^2\sec^4(e+fx)\left(-\frac{16x+\frac{\left(-a^3+6a^2b+24ab^2+16b^3\right)\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right.}{b(a+b)^{3/2}f\sqrt{b(\cos(e)-i\sin(e))^4}}\right.$$

$$64\,(a+b$$

input
```
Integrate[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])^2*Sec[e + f*x]^4*(-((16*x + ((-a^3 + 6*a^2
*b + 24*a*b^2 + 16*b^3)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2
*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e
])^4])]*(Cos[2*e] - I*Sin[2*e]))/(b*(a + b)^(3/2)*f*Sqrt[b*(Cos[e] - I*Sin
[e])^4]) + ((a^2 + 8*a*b + 8*b^2)*((a + 2*b)*Sin[2*e] - a*Sin[2*f*x]))/(b*
(a + b)*f*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[e] - Sin[e])*(Cos[e] + Sin[e
]))))/a^2) + (((a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b
)^(3/2) - (a*Sqrt[b]*Sin[2*(e + f*x)])/((a + b)*(a + 2*b + a*Cos[2*(e + f*
x)])))/(b^(3/2)*f)))/(64*(a + b*Sec[e + f*x]^2)^2)
```

3.358.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4629, 2075, 373, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\tan(e+fx)^2}{\left(a+b\sec(e+fx)^2\right)^2}\,dx$$

$$\downarrow 4629$$

$$\frac{\int \frac{\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(a+b\left(\tan^2(e+fx)+1\right)\right)^2}\,d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}\,d\tan(e+fx)}{f}$$

$$\downarrow 373$$

$$\frac{\frac{\tan(e+fx)}{2a(a+b\tan^2(e+fx)+b)} - \frac{\int \frac{1-\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}\,d\tan(e+fx)}{2a}}{f}$$

$$\downarrow 397$$

$$\frac{\tan(e+fx)}{2a(a+b\tan^2(e+fx)+b)} - \frac{\frac{2\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a} - \frac{(a+2b)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{2a}}{f}$$

$$\downarrow 216$$

$$\frac{\tan(e+fx)}{2a(a+b\tan^2(e+fx)+b)} - \frac{\frac{2\arctan(\tan(e+fx))}{a} - \frac{(a+2b)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{2a}}{f}$$

$$\downarrow 218$$

$$\frac{\tan(e+fx)}{2a(a+b\tan^2(e+fx)+b)} - \frac{\frac{2\arctan(\tan(e+fx))}{a} - \frac{(a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{b}\sqrt{a+b}}}{2a}}{f}$$

input
```
Int[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
(-1/2*((2*ArcTan[Tan[e + f*x]])/a - ((a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x
])/Sqrt[a + b]])/(a*Sqrt[b]*Sqrt[a + b]))/a + Tan[e + f*x]/(2*a*(a + b + b
*Tan[e + f*x]^2)))/f
```

3.358.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.358.4 Maple [A] (verified)

Time = 2.40 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{\frac{a\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{(a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}}{a^2}-\frac{\arctan(\tan(fx+e))}{a^2}}{f}$
default	$\frac{\frac{\frac{a\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{(a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}}{a^2}-\frac{\arctan(\tan(fx+e))}{a^2}}{f}$
risch	$-\frac{x}{a^2}+\frac{i\left(a\,e^{2i(fx+e)}+2b\,e^{2i(fx+e)}+a\right)}{a^2f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)}+\frac{\ln\left(e^{2i(fx+e)}+\frac{-2iba-2ib^2+a\sqrt{-ab-b^2}+2b\sqrt{-ab-b^2}}{a\sqrt{-ab-b^2}}\right)}{4\sqrt{-ab-b^2}\,fa}+$

input
```
int(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/a^2*(1/2*a*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(a+2*b)/((a+b)*b)^(1
/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))-1/a^2*arctan(tan(f*x+e)))
```

3.358.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(73) = 146.

Time = 0.30 (sec) , antiderivative size = 458, normalized size of antiderivative = 5.39

$$\int \frac{\tan^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$= \left[-\frac{8\left(a^2b+ab^2\right)fx\cos\left(fx+e\right)^2 + 8\left(ab^2+b^3\right)fx - 4\left(a^2b+ab^2\right)\cos\left(fx+e\right)\sin\left(fx+e\right) + \left(\left(a^2+2\,ab\right)\right.}{8\left(\left(a^4b+\right.\right.}\right.$$

$$\left. -\frac{4\left(a^2b+ab^2\right)fx\cos\left(fx+e\right)^2 + 4\left(ab^2+b^3\right)fx - 2\left(a^2b+ab^2\right)\cos\left(fx+e\right)\sin\left(fx+e\right) + \left(\left(a^2+2\,ab\right)\right.}{4\left(\left(a^4b+a^3b^2\right)f\cos\left(fx+e\right)^2 + \left(a^3b^2+\right.\right.}\right]$$

input
```
integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[-1/8*(8*(a^2*b + a*b^2)*f*x*cos(f*x + e)^2 + 8*(a*b^2 + b^3)*f*x - 4*(a^2
*b + a*b^2)*cos(f*x + e)*sin(f*x + e) + ((a^2 + 2*a*b)*cos(f*x + e)^2 + a*
b + 2*b^2)*sqrt(-a*b - b^2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*
(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x +
 e))*sqrt(-a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(
f*x + e)^2 + b^2)))/((a^4*b + a^3*b^2)*f*cos(f*x + e)^2 + (a^3*b^2 + a^2*b
^3)*f), -1/4*(4*(a^2*b + a*b^2)*f*x*cos(f*x + e)^2 + 4*(a*b^2 + b^3)*f*x -
 2*(a^2*b + a*b^2)*cos(f*x + e)*sin(f*x + e) + ((a^2 + 2*a*b)*cos(f*x + e)
^2 + a*b + 2*b^2)*sqrt(a*b + b^2)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b
)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e))))/((a^4*b + a^3*b^2)*f*cos(f
*x + e)^2 + (a^3*b^2 + a^2*b^3)*f)]
```

3.358.6 Sympy [F]

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

input
```
integrate(tan(f*x+e)**2/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral(tan(e + f*x)**2/(a + b*sec(e + f*x)**2)**2, x)
```

3.358.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.88

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{\tan(fx+e)}{ab\tan(fx+e)^2+a^2+ab} + \frac{(a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}a^2} - \frac{2(fx+e)}{a^2}}{2f}$$

input
```
integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/2*(tan(f*x + e)/(a*b*tan(f*x + e)^2 + a^2 + a*b) + (a + 2*b)*arctan(b*ta
n(f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^2) - 2*(f*x + e)/a^2)/f
```

3.358.8 Giac [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.11

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$
$$= \frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(a+2b)}{\sqrt{ab+b^2}a^2} - \frac{2(fx+e)}{a^2} + \frac{\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)a}}{2f}$$

input
```
integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/2*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
 + b^2)))*(a + 2*b)/(sqrt(a*b + b^2)*a^2) - 2*(f*x + e)/a^2 + tan(f*x + e)
/((b*tan(f*x + e)^2 + a + b)*a))/f
```

3.358.9 Mupad [B] (verification not implemented)

Time = 20.67 (sec) , antiderivative size = 711, normalized size of antiderivative = 8.36

$$\int \frac{\tan^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{\tan(e+f\,x)}{2\,a\,f\,\left(b\tan\left(e+f\,x\right)^2+a+b\right)} - \frac{x}{a^2}$$

$$\operatorname{atan}\left(\frac{\left(\frac{\tan(e+f\,x)\left(a^2\,b+4\,a\,b^2+8\,b^3\right)}{2\,a^2}-\frac{\left(2\,a\,b^2-\frac{\tan(e+f\,x)\left(16\,a^5\,b^2+32\,a^4\,b^3\right)(a+2\,b)\sqrt{-b\,(a+b)}}{8\,a^2\left(a^3\,b+a^2\,b^2\right)}\right)(a+2\,b)\sqrt{-b\,(a+b)}}{4\left(a^3\,b+a^2\,b^2\right)}\right)(a+2\,b)\sqrt{-b\,(a+b)}\,1\mathrm{i}}{4\left(a^3\,b+a^2\,b^2\right)}+\cdots}{\frac{b^2+\frac{a\,b}{2}}{a^3}-\frac{\left(\frac{\tan(e+f\,x)\left(a^2\,b+4\,a\,b^2+8\,b^3\right)}{2\,a^2}-\frac{\left(2\,a\,b^2-\frac{\tan(e+f\,x)\left(16\,a^5\,b^2+32\,a^4\,b^3\right)(a+2\,b)\sqrt{-b\,(a+b)}}{8\,a^2\left(a^3\,b+a^2\,b^2\right)}\right)(a+2\,b)\sqrt{-b\,(a+b)}}{4\left(a^3\,b+a^2\,b^2\right)}\right)(a+2\,b)\sqrt{-b\,(a+}}{4\left(a^3\,b+a^2\,b^2\right)}}\right)$$

input
```
int(tan(e + f*x)^2/(a + b/cos(e + f*x)^2)^2,x)
```

output
```
tan(e + f*x)/(2*a*f*(a + b + b*tan(e + f*x)^2)) - x/a^2 - (atan(((((tan(e
+ f*x)*(4*a*b^2 + a^2*b + 8*b^3))/(2*a^2) - ((2*a*b^2 - (tan(e + f*x)*(32*
a^4*b^3 + 16*a^5*b^2)*(a + 2*b)*(-b*(a + b))^(1/2))/(8*a^2*(a^3*b + a^2*b^
2)))*(a + 2*b)*(-b*(a + b))^(1/2))/(4*(a^3*b + a^2*b^2)))*(a + 2*b)*(-b*(a
 + b))^(1/2)*1i)/(4*(a^3*b + a^2*b^2)) + (((tan(e + f*x)*(4*a*b^2 + a^2*b
+ 8*b^3))/(2*a^2) + ((2*a*b^2 + (tan(e + f*x)*(32*a^4*b^3 + 16*a^5*b^2)*(a
 + 2*b)*(-b*(a + b))^(1/2))/(8*a^2*(a^3*b + a^2*b^2)))*(a + 2*b)*(-b*(a +
b))^(1/2))/(4*(a^3*b + a^2*b^2)))*(a + 2*b)*(-b*(a + b))^(1/2)*1i)/(4*(a^3
*b + a^2*b^2)))/(((a*b)/2 + b^2)/a^3 - (((tan(e + f*x)*(4*a*b^2 + a^2*b +
8*b^3))/(2*a^2) - ((2*a*b^2 - (tan(e + f*x)*(32*a^4*b^3 + 16*a^5*b^2)*(a +
 2*b)*(-b*(a + b))^(1/2))/(8*a^2*(a^3*b + a^2*b^2)))*(a + 2*b)*(-b*(a + b)
)^(1/2))/(4*(a^3*b + a^2*b^2)))*(a + 2*b)*(-b*(a + b))^(1/2))/(4*(a^3*b +
a^2*b^2)) + (((tan(e + f*x)*(4*a*b^2 + a^2*b + 8*b^3))/(2*a^2) + ((2*a*b^2
 + (tan(e + f*x)*(32*a^4*b^3 + 16*a^5*b^2)*(a + 2*b)*(-b*(a + b))^(1/2))/(
8*a^2*(a^3*b + a^2*b^2)))*(a + 2*b)*(-b*(a + b))^(1/2))/(4*(a^3*b + a^2*b^
2)))*(a + 2*b)*(-b*(a + b))^(1/2))/(4*(a^3*b + a^2*b^2))))*(a + 2*b)*(-b*(
a + b))^(1/2)*1i)/(2*f*(a^3*b + a^2*b^2))
```

3.359 $\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

3.359.1 Optimal result 2601
3.359.2 Mathematica [C] (warning: unable to verify) 2601
3.359.3 Rubi [A] (verified) 2602
3.359.4 Maple [A] (verified) 2604
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3.359.1 Optimal result

Integrand size = 14, antiderivative size = 92

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{x}{a^2} - \frac{\sqrt{b}(3a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{3/2}f} - \frac{b\tan(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)}$$

output
```
x/a^2-1/2*(3*a+2*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(1/2)/a^2/(a+
b)^(3/2)/f-1/2*b*tan(f*x+e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

3.359.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.20 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.61

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$= \frac{(a+2b+a\cos(2(e+fx)))\sec^4(e+fx)\left(2x(a+2b+a\cos(2(e+fx))) + \frac{b(3a+2b)\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e}{2\sqrt{a+b}}\right.\right.}{8a^2\left(a+b\sec^2(e+\right.}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(-2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*(2*x*(a + 2*b + a*Cos[2*(e
+ f*x)]) + (b*(3*a + 2*b)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a +
 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin
[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[2*e] - I*Sin[2*e]))/((a + b)
^(3/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (b*((a + 2*b)*Sin[2*e] - a*Sin[2
*f*x]))/((a + b)*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e]))))/(8*a^2*(a + b*Se
c[e + f*x]^2)^2)
```

3.359.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.21, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4616, 316, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\left(a+b\sec(e+fx)^2\right)^2}dx$$

$$\downarrow 4616$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^2}d\tan(e+fx)}{f}$$

$$\downarrow 316$$

$$\frac{\int \frac{-b\tan^2(e+fx)+2a+b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2a(a+b)} - \frac{b\tan(e+fx)}{2a(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

$$\downarrow 397$$

$$\frac{\frac{2(a+b)\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a} - \frac{b(3a+2b)\int \frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{2a(a+b)} - \frac{b\tan(e+fx)}{2a(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

$$\downarrow 216$$

$$\frac{\frac{2(a+b)\arctan(\tan(e+fx))}{a}-\frac{b(3a+2b)\int\frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{2a(a+b)}-\frac{b\tan(e+fx)}{2a(a+b)(a+b\tan^2(e+fx)+b)}$$
$$f$$
$$\downarrow 218$$
$$\frac{\frac{2(a+b)\arctan(\tan(e+fx))}{a}-\frac{\sqrt{b}(3a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{2a(a+b)}-\frac{b\tan(e+fx)}{2a(a+b)(a+b\tan^2(e+fx)+b)}$$
$$f$$

input
```
Int[(a + b*Sec[e + f*x]^2)^(-2),x]
```

output
```
(((2*(a + b)*ArcTan[Tan[e + f*x]])/a - (Sqrt[b]*(3*a + 2*b)*ArcTan[(Sqrt[b
]*Tan[e + f*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(2*a*(a + b)) - (b*Tan[e +
f*x])/(2*a*(a + b)*(a + b + b*Tan[e + f*x]^2)))/f
```

3.359.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f  Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

3.359.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{a\tan(fx+e)}{2(a+b)\left(a+b+b\tan(fx+e)^2\right)}+\frac{(3a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}\right)}{a^2}+\frac{\arctan(\tan(fx+e))}{a^2}}{f}$
default	$\frac{-\frac{b\left(\frac{a\tan(fx+e)}{2(a+b)\left(a+b+b\tan(fx+e)^2\right)}+\frac{(3a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}\right)}{a^2}+\frac{\arctan(\tan(fx+e))}{a^2}}{f}$
risch	$\frac{x}{a^2}-\frac{ib\left(a\,e^{2i(fx+e)}+2b\,e^{2i(fx+e)}+a\right)}{a^2(a+b)f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)}+\frac{3\sqrt{-(a+b)b}\,\ln\left(e^{2i(fx+e)}+\frac{2i\sqrt{-(a+b)b}+a+2b}{a}\right)}{4(a+b)^2fa}+$

input
```
int(1/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-b/a^2*(1/2*a/(a+b)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(3*a+2*b)/(a+
b)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))+1/a^2*arctan(tan(
f*x+e)))
```

3.359.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(80) = 160$.

Time $= 0.29$ (sec) , antiderivative size $= 435$, normalized size of antiderivative $= 4.73$

$$\int \frac{1}{\left(a + b\sec^2(e+fx)\right)^2}\,dx$$

$$= \left[\frac{8\left(a^2+ab\right)fx\cos\left(fx+e\right)^2 - 4\,ab\cos\left(fx+e\right)\sin\left(fx+e\right) + 8\left(ab+b^2\right)fx + \left(\left(3\,a^2+2\,ab\right)\cos\left(fx+e\right)\right.}{8\left(\left(a^4+a^\right.\right.}\right.$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[1/8*(8*(a^2 + a*b)*f*x*cos(f*x + e)^2 - 4*a*b*cos(f*x + e)*sin(f*x + e) +
 8*(a*b + b^2)*f*x + ((3*a^2 + 2*a*b)*cos(f*x + e)^2 + 3*a*b + 2*b^2)*sqrt
(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)
*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*co
s(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a
*b*cos(f*x + e)^2 + b^2)))/((a^4 + a^3*b)*f*cos(f*x + e)^2 + (a^3*b + a^2*
b^2)*f), 1/4*(4*(a^2 + a*b)*f*x*cos(f*x + e)^2 - 2*a*b*cos(f*x + e)*sin(f*
x + e) + 4*(a*b + b^2)*f*x + ((3*a^2 + 2*a*b)*cos(f*x + e)^2 + 3*a*b + 2*b
^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a +
b))/(b*cos(f*x + e)*sin(f*x + e))))/((a^4 + a^3*b)*f*cos(f*x + e)^2 + (a^3
*b + a^2*b^2)*f)]
```

3.359.6 Sympy [F]

$$\int \frac{1}{\left(a + b\sec^2(e+fx)\right)^2}\,dx = \int \frac{1}{\left(a + b\sec^2\left(e+fx\right)\right)^2}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(-2), x)
```

3.359.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.15

$$\int \frac{1}{(a+b\sec^2(e+fx))^2}\,dx$$
$$= -\frac{\frac{b\tan(fx+e)}{a^3+2a^2b+ab^2+(a^2b+ab^2)\tan(fx+e)^2} + \frac{(3ab+2b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^3+a^2b)\sqrt{(a+b)b}} - \frac{2(fx+e)}{a^2}}{2f}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/2*(b*tan(f*x + e)/(a^3 + 2*a^2*b + a*b^2 + (a^2*b + a*b^2)*tan(f*x + e)
^2) + (3*a*b + 2*b^2)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^3 + a^2*b
)*sqrt((a + b)*b)) - 2*(f*x + e)/a^2)/f
```

3.359.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.24

$$\int \frac{1}{(a+b\sec^2(e+fx))^2}\,dx$$
$$= -\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(3ab+2b^2)}{(a^3+a^2b)\sqrt{ab+b^2}} + \frac{b\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)(a^2+ab)} - \frac{2(fx+e)}{a^2}}{2f}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-1/2*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*
b + b^2)))*(3*a*b + 2*b^2)/((a^3 + a^2*b)*sqrt(a*b + b^2)) + b*tan(f*x + e
)/((b*tan(f*x + e)^2 + a + b)*(a^2 + a*b)) - 2*(f*x + e)/a^2)/f
```

3.359.9 Mupad [B] (verification not implemented)

Time = 21.73 (sec) , antiderivative size = 2056, normalized size of antiderivative = 22.35

$$\int \frac{1}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(1/(a + b/cos(e + f*x)^2)^2,x)
```

output
```
atan((((((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)*1i)/(2*(2*a^4*b + a^5 + a^3*b
^2)) - (tan(e + f*x)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/
(8*a^2*(2*a^3*b + a^4 + a^2*b^2)))/(2*a^2) + (tan(e + f*x)*(20*a*b^4 + 8*b
^5 + 13*a^2*b^3))/(4*(2*a^3*b + a^4 + a^2*b^2)))/a^2 - ((((2*a^4*b^4 + 6*a
^5*b^3 + 4*a^6*b^2)*1i)/(2*(2*a^4*b + a^5 + a^3*b^2)) + (tan(e + f*x)*(32*
a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*a^2*(2*a^3*b + a^4 + a
^2*b^2)))/(2*a^2) - (tan(e + f*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3))/(4*(2*a
^3*b + a^4 + a^2*b^2)))/a^2)/((((((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)*1i)/
(2*(2*a^4*b + a^5 + a^3*b^2)) - (tan(e + f*x)*(32*a^4*b^5 + 80*a^5*b^4 + 6
4*a^6*b^3 + 16*a^7*b^2))/(8*a^2*(2*a^3*b + a^4 + a^2*b^2)))*1i)/(2*a^2) +
(tan(e + f*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3)*1i)/(4*(2*a^3*b + a^4 + a^2*
b^2)))/a^2 + (((((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)*1i)/(2*(2*a^4*b + a^5
 + a^3*b^2)) + (tan(e + f*x)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^
7*b^2))/(8*a^2*(2*a^3*b + a^4 + a^2*b^2)))*1i)/(2*a^2) - (tan(e + f*x)*(20
*a*b^4 + 8*b^5 + 13*a^2*b^3)*1i)/(4*(2*a^3*b + a^4 + a^2*b^2)))/a^2 + ((3*
a*b^3)/2 + b^4)/(2*a^4*b + a^5 + a^3*b^2)))/(a^2*f) + (atan((((tan(e + f*
x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3))/(2*(2*a^3*b + a^4 + a^2*b^2)) - ((-b*(
a + b)^3)^(1/2)*((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)/(2*a^4*b + a^5 + a^3*
b^2) - (tan(e + f*x)*(-b*(a + b)^3)^(1/2)*(3*a + 2*b)*(32*a^4*b^5 + 80*a^5
*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*(2*a^3*b + a^4 + a^2*b^2)*(3*a^4*b ...
```

3.360 $\int \frac{\cot^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

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3.360.1 Optimal result

Integrand size = 23, antiderivative size = 121

$$\int \frac{\cot^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{x}{a^2} + \frac{b^{3/2}(5a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{5/2}f} - \frac{(2a-b)\cot(e+fx)}{2a(a+b)^2 f} - \frac{b\cot(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)}$$

output
```
-x/a^2+1/2*b^(3/2)*(5*a+2*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^2/(a
+b)^(5/2)/f-1/2*(2*a-b)*cot(f*x+e)/a/(a+b)^2/f-1/2*b*cot(f*x+e)/a/(a+b)/f/
(a+b+b*tan(f*x+e)^2)
```

3.360.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.34 (sec) , antiderivative size = 288, normalized size of antiderivative = 2.38

$$\int \frac{\cot^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{(a+2b+a\cos(2(e+fx)))\sec^4(e+fx)\left(-\frac{2x(a+2b+a\cos(2(e+fx)))}{a^2} - \frac{b^2(5a+2b)\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b}{2\sqrt{a+b}\sqrt{b(\cos(e)-}}\right.}{a^2(a+b)^{}}$$

8

input
```
Integrate[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^4*((-2*x*(a + 2*b + a*Cos[2*(
e + f*x)]))/a^2 - (b^2*(5*a + 2*b)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e]
)*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e
] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])*(Cos[2*e] - I*Sin[2*e]))
/(a^2*(a + b)^(5/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) + (2*(a + 2*b + a*Cos
[2*(e + f*x)])*Csc[e]*Csc[e + f*x]*Sin[f*x])/((a + b)^2*f) + (b^2*(-((a +
2*b)*Sin[2*e]) + a*Sin[2*f*x]))/(a^2*(a + b)^2*f*(Cos[e] - Sin[e])*(Cos[e]
 + Sin[e])))/(8*(a + b*Sec[e + f*x]^2)^2)
```

3.360.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.17, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4629, 2075, 374, 445, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\tan(e+fx)^2\,(a+b\sec(e+fx)^2)^2}\,dx$$

$$\downarrow 4629$$

$$\frac{\int \frac{\cot^2(e+fx)}{(\tan^2(e+fx)+1)(a+b(\tan^2(e+fx)+1))^2}\,d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\cot^2(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^2}\,d\tan(e+fx)}{f}$$

$$\downarrow 374$$

$$\frac{\int \frac{\cot^2(e+fx)\left(-3b\tan^2(e+fx)+2a-b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}\,d\tan(e+fx)}{2a(a+b)} - \frac{b\cot(e+fx)}{2a(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

$$\downarrow 445$$

$$\frac{-\frac{\int \frac{2a^2+6ba+b^2+(2a-b)b\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{a+b}-\frac{(2a-b)\cot(e+fx)}{a+b}}{2a(a+b)}-\frac{b\cot(e+fx)}{2a(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

$$\downarrow 397$$

$$\frac{-\frac{\frac{2(a+b)^2\int\frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}-\frac{b^2(5a+2b)\int\frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{a+b}-\frac{(2a-b)\cot(e+fx)}{a+b}}{2a(a+b)}-\frac{b\cot(e+fx)}{2a(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

$$\downarrow 216$$

$$\frac{-\frac{\frac{2(a+b)^2\arctan(\tan(e+fx))}{a}-\frac{b^2(5a+2b)\int\frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{a+b}-\frac{(2a-b)\cot(e+fx)}{a+b}}{2a(a+b)}-\frac{b\cot(e+fx)}{2a(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

$$\downarrow 218$$

$$\frac{-\frac{\frac{2(a+b)^2\arctan(\tan(e+fx))}{a}-\frac{b^{3/2}(5a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{a+b}-\frac{(2a-b)\cot(e+fx)}{a+b}}{2a(a+b)}-\frac{b\cot(e+fx)}{2a(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

input
```
Int[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((-(((2*(a + b)^2*ArcTan[Tan[e + f*x]])/a - (b^(3/2)*(5*a + 2*b)*ArcTan[(S
qrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(a + b)) - ((2*a - b)*
Cot[e + f*x])/(a + b))/(2*a*(a + b)) - (b*Cot[e + f*x])/(2*a*(a + b)*(a +
b + b*Tan[e + f*x]^2)))/f
```

3.360.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.360.4 Maple [A] (verified)

Time = 5.16 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{-\frac{\arctan(\tan(fx+e))}{a^2}-\frac{1}{(a+b)^2\tan(fx+e)}+\frac{b^2\left(\frac{a\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{(5a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{a^2(a+b)^2}}{f}$
default	$\frac{-\frac{\arctan(\tan(fx+e))}{a^2}-\frac{1}{(a+b)^2\tan(fx+e)}+\frac{b^2\left(\frac{a\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{(5a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{a^2(a+b)^2}}{f}$
risch	$-\frac{x}{a^2}+\frac{i\left(-2a^3e^{4i(fx+e)}+ab^2e^{4i(fx+e)}+2b^3e^{4i(fx+e)}-4a^3e^{2i(fx+e)}-8a^2be^{2i(fx+e)}-2b^3e^{2i(fx+e)}-2a^3-ab^2\right)}{a^2(a+b)^2f\left(ae^{4i(fx+e)}+2ae^{2i(fx+e)}+4be^{2i(fx+e)}+a\right)\left(e^{2i(fx+e)}-1\right)}-$

input
```
int(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/a^2*arctan(tan(f*x+e))-1/(a+b)^2/tan(f*x+e)+b^2/a^2/(a+b)^2*(1/2*a
*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(5*a+2*b)/((a+b)*b)^(1/2)*arctan(b*ta
n(f*x+e)/((a+b)*b)^(1/2))))
```

3.360.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. 2(107) = 214.

Time = 0.30 (sec) , antiderivative size = 604, normalized size of antiderivative = 4.99

$$\int \frac{\cot^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$= \left[-\frac{4\left(2a^3+ab^2\right)\cos(fx+e)^3-\left(5ab^2+2b^3+\left(5a^2b+2ab^2\right)\cos(fx+e)^2\right)\sqrt{-\frac{b}{a+b}}\log\left(\frac{(a^2+8ab+8b^2)\cos}{}\right.}{} \right.$$

$$\left. -\frac{2\left(2a^3+ab^2\right)\cos(fx+e)^3+\left(5ab^2+2b^3+\left(5a^2b+2ab^2\right)\cos(fx+e)^2\right)\sqrt{\frac{b}{a+b}}\arctan\left(\frac{\left((a+2b)\cos(fx+\right.}{2b\cos(fx+e)s}\right.}{4\left((a^5+2a^4b+a^3b^2)f\right.} \right]$$

input
```
integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[-1/8*(4*(2*a^3 + a*b^2)*cos(f*x + e)^3 - (5*a*b^2 + 2*b^3 + (5*a^2*b + 2*
a*b^2)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x
 + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*b + 2*b^2)*cos(
f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2
)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) + 4*(2*a
^2*b - a*b^2)*cos(f*x + e) + 8*((a^3 + 2*a^2*b + a*b^2)*f*x*cos(f*x + e)^2
 + (a^2*b + 2*a*b^2 + b^3)*f*x)*sin(f*x + e))/(((a^5 + 2*a^4*b + a^3*b^2)*
f*cos(f*x + e)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*f)*sin(f*x + e)), -1/4*(2
*(2*a^3 + a*b^2)*cos(f*x + e)^3 + (5*a*b^2 + 2*b^3 + (5*a^2*b + 2*a*b^2)*c
os(f*x + e)^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*s
qrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e) + 2*(2*a^2*b -
a*b^2)*cos(f*x + e) + 4*((a^3 + 2*a^2*b + a*b^2)*f*x*cos(f*x + e)^2 + (a^2
*b + 2*a*b^2 + b^3)*f*x)*sin(f*x + e))/(((a^5 + 2*a^4*b + a^3*b^2)*f*cos(f
*x + e)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*f)*sin(f*x + e))]
```

3.360.6 Sympy [F]

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

input
```
integrate(cot(f*x+e)**2/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral(cot(e + f*x)**2/(a + b*sec(e + f*x)**2)**2, x)
```

3.360.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.35

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \frac{\frac{(5ab^2+2b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^4+2a^3b+a^2b^2)\sqrt{(a+b)b}} - \frac{(2ab-b^2)\tan(fx+e)^2+2a^2+2ab}{(a^3b+2a^2b^2+ab^3)\tan(fx+e)^3+(a^4+3a^3b+3a^2b^2+ab^3)\tan(fx+e)} - \frac{2(fx+e)}{a^2}}{2f}$$

input
```
integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/2*((5*a*b^2 + 2*b^3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^4 + 2*a^
3*b + a^2*b^2)*sqrt((a + b)*b)) - ((2*a*b - b^2)*tan(f*x + e)^2 + 2*a^2 +
2*a*b)/((a^3*b + 2*a^2*b^2 + a*b^3)*tan(f*x + e)^3 + (a^4 + 3*a^3*b + 3*a^
2*b^2 + a*b^3)*tan(f*x + e)) - 2*(f*x + e)/a^2)/f
```

3.360.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.45

$$\int \frac{\cot^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$
$$= \frac{\frac{(5\,ab^2+2\,b^3)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^4+2\,a^3b+a^2b^2)\sqrt{ab+b^2}} - \frac{2\,ab\tan(fx+e)^2-b^2\tan(fx+e)^2+2\,a^2+2\,ab}{\left(b\tan(fx+e)^3+a\tan(fx+e)+b\tan(fx+e)\right)(a^3+2\,a^2b+ab^2)} - \frac{2\,(fx+e)}{a^2}}{2\,f}$$

input
```
integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/2*((5*a*b^2 + 2*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan
(f*x + e)/sqrt(a*b + b^2)))/((a^4 + 2*a^3*b + a^2*b^2)*sqrt(a*b + b^2)) -
(2*a*b*tan(f*x + e)^2 - b^2*tan(f*x + e)^2 + 2*a^2 + 2*a*b)/((b*tan(f*x +
e)^3 + a*tan(f*x + e) + b*tan(f*x + e))*(a^3 + 2*a^2*b + a*b^2)) - 2*(f*x
+ e)/a^2)/f
```

3.360.9 Mupad [B] (verification not implemented)

Time = 24.79 (sec) , antiderivative size = 3146, normalized size of antiderivative = 26.00

$$\int \frac{\cot^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(cot(e + f*x)^2/(a + b/cos(e + f*x)^2)^2,x)
```

output

```
(atan((((tan(e + f*x)*(128*a^3*b^13 + 1344*a^4*b^12 + 6160*a^5*b^11 + 1616
0*a^6*b^10 + 26800*a^7*b^9 + 29312*a^8*b^8 + 21424*a^9*b^7 + 10400*a^10*b^
6 + 3280*a^11*b^5 + 640*a^12*b^4 + 64*a^13*b^3) - ((-b^3*(a + b)^5)^(1/2)*
(5*a + 2*b)*(64*a^6*b^12 + 896*a^7*b^11 + 4992*a^8*b^10 + 15360*a^9*b^9 +
29568*a^10*b^8 + 37632*a^11*b^7 + 32256*a^12*b^6 + 18432*a^13*b^5 + 6720*a
^14*b^4 + 1408*a^15*b^3 + 128*a^16*b^2 - (tan(e + f*x)*(-b^3*(a + b)^5)^(1
/2)*(5*a + 2*b)*(512*a^7*b^13 + 5376*a^8*b^12 + 25600*a^9*b^11 + 72960*a^1
0*b^10 + 138240*a^11*b^9 + 182784*a^12*b^8 + 172032*a^13*b^7 + 115200*a^14
*b^6 + 53760*a^15*b^5 + 16640*a^16*b^4 + 3072*a^17*b^3 + 256*a^18*b^2))/(4
*(5*a^6*b + a^7 + a^2*b^5 + 5*a^3*b^4 + 10*a^4*b^3 + 10*a^5*b^2))))/(4*(5*
a^6*b + a^7 + a^2*b^5 + 5*a^3*b^4 + 10*a^4*b^3 + 10*a^5*b^2)))*(-b^3*(a +
b)^5)^(1/2)*(5*a + 2*b)*1i)/(4*(5*a^6*b + a^7 + a^2*b^5 + 5*a^3*b^4 + 10*a
^4*b^3 + 10*a^5*b^2)) + ((tan(e + f*x)*(128*a^3*b^13 + 1344*a^4*b^12 + 616
0*a^5*b^11 + 16160*a^6*b^10 + 26800*a^7*b^9 + 29312*a^8*b^8 + 21424*a^9*b^
7 + 10400*a^10*b^6 + 3280*a^11*b^5 + 640*a^12*b^4 + 64*a^13*b^3) + ((-b^3*
(a + b)^5)^(1/2)*(5*a + 2*b)*(64*a^6*b^12 + 896*a^7*b^11 + 4992*a^8*b^10 +
 15360*a^9*b^9 + 29568*a^10*b^8 + 37632*a^11*b^7 + 32256*a^12*b^6 + 18432*
a^13*b^5 + 6720*a^14*b^4 + 1408*a^15*b^3 + 128*a^16*b^2 + (tan(e + f*x)*(-
b^3*(a + b)^5)^(1/2)*(5*a + 2*b)*(512*a^7*b^13 + 5376*a^8*b^12 + 25600*a^9
*b^11 + 72960*a^10*b^10 + 138240*a^11*b^9 + 182784*a^12*b^8 + 172032*a^...
```

3.361 $\int \frac{\cot^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

3.361.1 Optimal result 2616
3.361.2 Mathematica [C] (warning: unable to verify) 2616
3.361.3 Rubi [A] (verified) 2617
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3.361.9 Mupad [B] (verification not implemented) 2624

3.361.1 Optimal result

Integrand size = 23, antiderivative size = 160

$$\int \frac{\cot^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \frac{x}{a^2} - \frac{b^{5/2}(7a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{7/2}f} + \frac{(2a^2+6ab-b^2)\cot(e+fx)}{2a(a+b)^3 f} - \frac{(2a-3b)\cot^3(e+fx)}{6a(a+b)^2 f} - \frac{b\cot^3(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)}$$

output
```
x/a^2-1/2*b^(5/2)*(7*a+2*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^2/(a+
b)^(7/2)/f+1/2*(2*a^2+6*a*b-b^2)*cot(f*x+e)/a/(a+b)^3/f-1/6*(2*a-3*b)*cot(
f*x+e)^3/a/(a+b)^2/f-1/2*b*cot(f*x+e)^3/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

3.361.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 8.33 (sec) , antiderivative size = 1896, normalized size of antiderivative = 11.85

$$\int \frac{\cot^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((7*a + 2*b)*(a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[e + f*x]^4*((b^3*ArcTan[
Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2
)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x])
- 2*b*Sin[f*x] + a*Sin[2*e + f*x])]*Cos[2*e])/(8*a^2*Sqrt[a + b]*f*Sqrt[b*
Cos[4*e] - I*b*Sin[4*e]]) - ((I/8)*b^3*ArcTan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a
 + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[2*e])/(Sqrt[a + b]*Sqr
t[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*Sin[f*x] + a*Sin[2*e +
 f*x])]*Sin[2*e])/(a^2*Sqrt[a + b]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]])))/((
a + b)^3*(a + b*Sec[e + f*x]^2)^2) + ((a + 2*b + a*Cos[2*e + 2*f*x])*Csc[e
]*Csc[e + f*x]^3*Sec[2*e]*Sec[e + f*x]^4*(-6*a^4*f*x*Cos[f*x] - 54*a^3*b*f
*x*Cos[f*x] - 126*a^2*b^2*f*x*Cos[f*x] - 114*a*b^3*f*x*Cos[f*x] - 36*b^4*f
*x*Cos[f*x] + 3*a^4*f*x*Cos[3*f*x] - 3*a^3*b*f*x*Cos[3*f*x] - 27*a^2*b^2*f
*x*Cos[3*f*x] - 33*a*b^3*f*x*Cos[3*f*x] - 12*b^4*f*x*Cos[3*f*x] + 6*a^4*f*
x*Cos[2*e - f*x] + 54*a^3*b*f*x*Cos[2*e - f*x] + 126*a^2*b^2*f*x*Cos[2*e -
 f*x] + 114*a*b^3*f*x*Cos[2*e - f*x] + 36*b^4*f*x*Cos[2*e - f*x] + 6*a^4*f
*x*Cos[2*e + f*x] + 54*a^3*b*f*x*Cos[2*e + f*x] + 126*a^2*b^2*f*x*Cos[2*e
+ f*x] + 114*a*b^3*f*x*Cos[2*e + f*x] + 36*b^4*f*x*Cos[2*e + f*x] - 6*a^4*
f*x*Cos[4*e + f*x] - 54*a^3*b*f*x*Cos[4*e + f*x] - 126*a^2*b^2*f*x*Cos[4*e
 + f*x] - 114*a*b^3*f*x*Cos[4*e + f*x] - 36*b^4*f*x*Cos[4*e + f*x] - 3*a^4
*f*x*Cos[2*e + 3*f*x] + 3*a^3*b*f*x*Cos[2*e + 3*f*x] + 27*a^2*b^2*f*x*C...
```

3.361.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 4629, 2075, 374, 445, 27, 445, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\tan(e+fx)^4\left(a+b\sec(e+fx)^2\right)^2}dx$$

$$\downarrow 4629$$

$$\frac{\int \frac{\cot^4(e+fx)}{(\tan^2(e+fx)+1)(a+b(\tan^2(e+fx)+1))^2}\,d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\cot^4(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^2}\,d\tan(e+fx)}{f}$$

$$\downarrow 374$$

$$\frac{\frac{\int \frac{\cot^4(e+fx)\left(-5b\tan^2(e+fx)+2a-3b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}\,d\tan(e+fx)}{2a(a+b)} - \frac{b\cot^3(e+fx)}{2a(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

$$\downarrow 445$$

$$\frac{-\frac{\int \frac{3\cot^2(e+fx)\left(2a^2+6ba-b^2+(2a-3b)b\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}\,d\tan(e+fx)}{3(a+b)} - \frac{(2a-3b)\cot^3(e+fx)}{3(a+b)}}{2a(a+b)} - \frac{b\cot^3(e+fx)}{2a(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

$$\downarrow 27$$

$$\frac{-\frac{\int \frac{\cot^2(e+fx)\left(2a^2+6ba-b^2+(2a-3b)b\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}\,d\tan(e+fx)}{a+b} - \frac{(2a-3b)\cot^3(e+fx)}{3(a+b)}}{2a(a+b)} - \frac{b\cot^3(e+fx)}{2a(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

$$\downarrow 445$$

$$\frac{-\frac{-\frac{\int \frac{2a^3+8ba^2+12b^2a+b^3+b\left(2a^2+6ba-b^2\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}\,d\tan(e+fx)}{a+b} - \frac{\left(2a^2+6ab-b^2\right)\cot(e+fx)}{a+b}}{a+b} - \frac{(2a-3b)\cot^3(e+fx)}{3(a+b)}}{2a(a+b)} - \frac{b\cot^3(e+fx)}{2a(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

$$\downarrow 397$$

$$\frac{-\frac{-\frac{\frac{2(a+b)^3\int \frac{1}{\tan^2(e+fx)+1}\,d\tan(e+fx)}{a} - \frac{b^3(7a+2b)\int \frac{1}{b\tan^2(e+fx)+a+b}\,d\tan(e+fx)}{a}}{a+b} - \frac{\left(2a^2+6ab-b^2\right)\cot(e+fx)}{a+b}}{a+b} - \frac{(2a-3b)\cot^3(e+fx)}{3(a+b)}}{2a(a+b)} - \frac{b\cot^3}{2a(a+b)(a+b}}{f}$$

$$\downarrow 216$$

$$\frac{-\frac{-\frac{\frac{2(a+b)^3\arctan(\tan(e+fx))}{a} - \frac{b^3(7a+2b)\int \frac{1}{b\tan^2(e+fx)+a+b}\,d\tan(e+fx)}{a}}{a+b} - \frac{\left(2a^2+6ab-b^2\right)\cot(e+fx)}{a+b}}{a+b} - \frac{(2a-3b)\cot^3(e+fx)}{3(a+b)}}{2a(a+b)} - \frac{b\cot^3(e+fx)}{2a(a+b)(a+b\tan^2(e+f}}{f}$$

3.361. $\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$

$$\downarrow 218$$

$$\frac{-\frac{\left(2a^2+6ab-b^2\right)\cot(e+fx)}{a+b}-\frac{2(a+b)^3\arctan(\tan(e+fx))}{a}-\frac{b^{5/2}(7a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{2a(a+b)}-\frac{(2a-3b)\cot^3(e+fx)}{3(a+b)}}{a+b}-\frac{b\cot^3(e+fx)}{2a(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

input
```
Int[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((-1/3*((2*a - 3*b)*Cot[e + f*x]^3)/(a + b) - (-(((2*(a + b)^3*ArcTan[Tan[
e + f*x]])/a - (b^(5/2)*(7*a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a +
 b]])/(a*Sqrt[a + b]))/(a + b)) - ((2*a^2 + 6*a*b - b^2)*Cot[e + f*x])/(a
+ b))/(a + b))/(2*a*(a + b)) - (b*Cot[e + f*x]^3)/(2*a*(a + b)*(a + b + b*
Tan[e + f*x]^2)))/f
```

3.361.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.361.4 Maple [A] (verified)

Time = 9.73 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{\frac{\arctan(\tan(fx+e))}{a^2}-\frac{1}{3(a+b)^2\tan(fx+e)^3}-\frac{-a-3b}{(a+b)^3\tan(fx+e)}-\frac{b^3\left(\frac{a\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{(7a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{a^2(a+b)^3}}{f}$
default	$\frac{\frac{\arctan(\tan(fx+e))}{a^2}-\frac{1}{3(a+b)^2\tan(fx+e)^3}-\frac{-a-3b}{(a+b)^3\tan(fx+e)}-\frac{b^3\left(\frac{a\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{(7a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{a^2(a+b)^3}}{f}$
risch	$\frac{x}{a^2}-\frac{i\left(-12a^4e^{8i(fx+e)}-24a^3b\,e^{8i(fx+e)}+3a\,b^3e^{8i(fx+e)}+6b^4e^{8i(fx+e)}-12a^4e^{6i(fx+e)}-60a^3b\,e^{6i(fx+e)}-96a^2b^2e^{6i(}\right.$

input
```
int(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/a^2*arctan(tan(f*x+e))-1/3/(a+b)^2/tan(f*x+e)^3-(-a-3*b)/(a+b)^3/ta
n(f*x+e)-b^3/a^2/(a+b)^3*(1/2*a*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)+1/2*(7*a+2
*b)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))))
```

3.361.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 446 vs. $2(144) = 288$.

Time = 0.32 (sec) , antiderivative size = 979, normalized size of antiderivative = 6.12

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[1/24*(4*(8*a^4 + 20*a^3*b + 3*a*b^3)*cos(f*x + e)^5 - 8*(3*a^4 + 5*a^3*b
- 10*a^2*b^2 + 3*a*b^3)*cos(f*x + e)^3 + 3*((7*a^2*b^2 + 2*a*b^3)*cos(f*x
+ e)^4 - 7*a*b^3 - 2*b^4 - (7*a^2*b^2 - 5*a*b^3 - 2*b^4)*cos(f*x + e)^2)*s
qrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b
^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)
*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 +
2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) - 12*(2*a^3*b + 6*a^2*b^2 - a*b^
3)*cos(f*x + e) + 24*((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*x*cos(f*x + e)
^4 - (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*f*x*cos(f*x + e)^2 - (a^3*b + 3*a^2*b
^2 + 3*a*b^3 + b^4)*f*x)*sin(f*x + e))/(((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*
b^3)*f*cos(f*x + e)^4 - (a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*f*cos(f*x +
e)^2 - (a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*f)*sin(f*x + e)), 1/12*(2
*(8*a^4 + 20*a^3*b + 3*a*b^3)*cos(f*x + e)^5 - 4*(3*a^4 + 5*a^3*b - 10*a^2
*b^2 + 3*a*b^3)*cos(f*x + e)^3 + 3*((7*a^2*b^2 + 2*a*b^3)*cos(f*x + e)^4 -
 7*a*b^3 - 2*b^4 - (7*a^2*b^2 - 5*a*b^3 - 2*b^4)*cos(f*x + e)^2)*sqrt(b/(a
 + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*
x + e)*sin(f*x + e)))*sin(f*x + e) - 6*(2*a^3*b + 6*a^2*b^2 - a*b^3)*cos(f
*x + e) + 12*((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*x*cos(f*x + e)^4 - (a^
4 + 2*a^3*b - 2*a*b^3 - b^4)*f*x*cos(f*x + e)^2 - (a^3*b + 3*a^2*b^2 + 3*a
*b^3 + b^4)*f*x)*sin(f*x + e))/(((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*...
```

3.361.6 Sympy [F]

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx = \int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

input
```
integrate(cot(f*x+e)**4/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Integral(cot(e + f*x)**4/(a + b*sec(e + f*x)**2)**2, x)
```

3.361.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.47

$$\int \frac{\cot^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{\frac{3\left(7ab^3+2b^4\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\left(a^5+3a^4b+3a^3b^2+a^2b^3\right)\sqrt{(a+b)b}} - \frac{3\left(2a^2b+6ab^2-b^3\right)\tan(fx+e)^4-2a^3-4a^2b-2ab^2+2\left(3a^3+11a^2b+8ab^2\right)\tan(fx+e)^2}{\left(a^4b+3a^3b^2+3a^2b^3+ab^4\right)\tan(fx+e)^5+\left(a^5+4a^4b+6a^3b^2+4a^2b^3+ab^4\right)\tan(fx+e)^3} - \frac{6(fx+e}{a^2}}{6f}$$

input
```
integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
-1/6*(3*(7*a*b^3 + 2*b^4)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^5 + 3
*a^4*b + 3*a^3*b^2 + a^2*b^3)*sqrt((a + b)*b)) - (3*(2*a^2*b + 6*a*b^2 - b
^3)*tan(f*x + e)^4 - 2*a^3 - 4*a^2*b - 2*a*b^2 + 2*(3*a^3 + 11*a^2*b + 8*a
*b^2)*tan(f*x + e)^2)/((a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*tan(f*x + e
)^5 + (a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*tan(f*x + e)^3) - 6*
(f*x + e)/a^2)/f
```

3.361.8 Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.32

$$\int \frac{\cot^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{\frac{3b^3\tan(fx+e)}{\left(a^4+3a^3b+3a^2b^2+ab^3\right)\left(b\tan(fx+e)^2+a+b\right)} + \frac{3\left(7ab^3+2b^4\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{\left(a^5+3a^4b+3a^3b^2+a^2b^3\right)\sqrt{ab+b^2}} - \frac{6(fx+e)}{a^2} - \frac{2\left(3a\tan(}{(a^3+3a}}{6f}$$

input
```
integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
-1/6*(3*b^3*tan(f*x + e)/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*(b*tan(f*x +
 e)^2 + a + b)) + 3*(7*a*b^3 + 2*b^4)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b)
 + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a
^2*b^3)*sqrt(a*b + b^2)) - 6*(f*x + e)/a^2 - 2*(3*a*tan(f*x + e)^2 + 9*b*t
an(f*x + e)^2 - a - b)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*tan(f*x + e)^3))/f
```

3.361.9 Mupad [B] (verification not implemented)

Time = 25.73 (sec) , antiderivative size = 4987, normalized size of antiderivative = 31.17

$$\int \frac{\cot^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(cot(e + f*x)^4/(a + b/cos(e + f*x)^2)^2,x)
```

output
```
((tan(e + f*x)^2*(3*a + 8*b))/(3*(a + b)^2) - 1/(3*(a + b)) + (tan(e + f*x
)^4*(6*a*b^2 + 2*a^2*b - b^3))/(2*a*(a + b)^3))/(f*(tan(e + f*x)^3*(a + b)
 + b*tan(e + f*x)^5)) + atan((560*a^3*b^16*tan(e + f*x))/(560*a^3*b^16 + 7
280*a^4*b^15 + 42560*a^5*b^14 + 149184*a^6*b^13 + 351904*a^7*b^12 + 593440
*a^8*b^11 + 741120*a^9*b^10 + 699840*a^10*b^9 + 505008*a^11*b^8 + 278768*a
^12*b^7 + 116480*a^13*b^6 + 35840*a^14*b^5 + 7680*a^15*b^4 + 1024*a^16*b^3
 + 64*a^17*b^2) + (7280*a^4*b^15*tan(e + f*x))/(560*a^3*b^16 + 7280*a^4*b^
15 + 42560*a^5*b^14 + 149184*a^6*b^13 + 351904*a^7*b^12 + 593440*a^8*b^11
+ 741120*a^9*b^10 + 699840*a^10*b^9 + 505008*a^11*b^8 + 278768*a^12*b^7 +
116480*a^13*b^6 + 35840*a^14*b^5 + 7680*a^15*b^4 + 1024*a^16*b^3 + 64*a^17
*b^2) + (42560*a^5*b^14*tan(e + f*x))/(560*a^3*b^16 + 7280*a^4*b^15 + 4256
0*a^5*b^14 + 149184*a^6*b^13 + 351904*a^7*b^12 + 593440*a^8*b^11 + 741120*
a^9*b^10 + 699840*a^10*b^9 + 505008*a^11*b^8 + 278768*a^12*b^7 + 116480*a^
13*b^6 + 35840*a^14*b^5 + 7680*a^15*b^4 + 1024*a^16*b^3 + 64*a^17*b^2) + (
149184*a^6*b^13*tan(e + f*x))/(560*a^3*b^16 + 7280*a^4*b^15 + 42560*a^5*b^
14 + 149184*a^6*b^13 + 351904*a^7*b^12 + 593440*a^8*b^11 + 741120*a^9*b^10
 + 699840*a^10*b^9 + 505008*a^11*b^8 + 278768*a^12*b^7 + 116480*a^13*b^6 +
 35840*a^14*b^5 + 7680*a^15*b^4 + 1024*a^16*b^3 + 64*a^17*b^2) + (351904*a
^7*b^12*tan(e + f*x))/(560*a^3*b^16 + 7280*a^4*b^15 + 42560*a^5*b^14 + 149
184*a^6*b^13 + 351904*a^7*b^12 + 593440*a^8*b^11 + 741120*a^9*b^10 + 69...
```

3.362 $\int \frac{\cot^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$

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3.362.1 Optimal result

Integrand size = 23, antiderivative size = 207

$$\int \frac{\cot^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = -\frac{x}{a^2} + \frac{b^{7/2}(9a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{9/2}f} - \frac{(2a^3+8a^2b+12ab^2-b^3)\cot(e+fx)}{2a(a+b)^4 f} + \frac{(2a^2+6ab-3b^2)\cot^3(e+fx)}{6a(a+b)^3 f} - \frac{(2a-5b)\cot^5(e+fx)}{10a(a+b)^2 f} - \frac{b\cot^5(e+fx)}{2a(a+b)f\left(a+b+b\tan^2(e+fx)\right)}$$

output

```
-x/a^2+1/2*b^(7/2)*(9*a+2*b)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^2/(a
+b)^(9/2)/f-1/2*(2*a^3+8*a^2*b+12*a*b^2-b^3)*cot(f*x+e)/a/(a+b)^4/f+1/6*(2
*a^2+6*a*b-3*b^2)*cot(f*x+e)^3/a/(a+b)^3/f-1/10*(2*a-5*b)*cot(f*x+e)^5/a/(
a+b)^2/f-1/2*b*cot(f*x+e)^5/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

3.362.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.49 (sec) , antiderivative size = 3028, normalized size of antiderivative = 14.63

$$\int \frac{\cot^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]
```

output
```
((9*a + 2*b)*(a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[e + f*x]^4*(-1/8*(b^4*Ar
cTan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) -
((I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f
*x]) - 2*b*Sin[f*x] + a*Sin[2*e + f*x])]*Cos[2*e])/(a^2*Sqrt[a + b]*f*Sqrt
[b*Cos[4*e] - I*b*Sin[4*e]]) + ((I/8)*b^4*ArcTan[Sec[f*x]*(Cos[2*e]/(2*Sqr
t[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[2*e])/(Sqrt[a + b]*
Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*Sin[f*x] + a*Sin[2*
e + f*x])]*Sin[2*e])/(a^2*Sqrt[a + b]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]])))
/((a + b)^4*(a + b*Sec[e + f*x]^2)^2) + ((a + 2*b + a*Cos[2*e + 2*f*x])*Cs
c[e]*Csc[e + f*x]^5*Sec[2*e]*Sec[e + f*x]^4*(75*a^5*f*x*Cos[f*x] + 900*a^4
*b*f*x*Cos[f*x] + 2850*a^3*b^2*f*x*Cos[f*x] + 3900*a^2*b^3*f*x*Cos[f*x] +
2475*a*b^4*f*x*Cos[f*x] + 600*b^5*f*x*Cos[f*x] - 15*a^5*f*x*Cos[3*f*x] + 2
40*a^4*b*f*x*Cos[3*f*x] + 1110*a^3*b^2*f*x*Cos[3*f*x] + 1740*a^2*b^3*f*x*C
os[3*f*x] + 1185*a*b^4*f*x*Cos[3*f*x] + 300*b^5*f*x*Cos[3*f*x] - 75*a^5*f*
x*Cos[2*e - f*x] - 900*a^4*b*f*x*Cos[2*e - f*x] - 2850*a^3*b^2*f*x*Cos[2*e
 - f*x] - 3900*a^2*b^3*f*x*Cos[2*e - f*x] - 2475*a*b^4*f*x*Cos[2*e - f*x]
- 600*b^5*f*x*Cos[2*e - f*x] - 75*a^5*f*x*Cos[2*e + f*x] - 900*a^4*b*f*x*C
os[2*e + f*x] - 2850*a^3*b^2*f*x*Cos[2*e + f*x] - 3900*a^2*b^3*f*x*Cos[2*e
 + f*x] - 2475*a*b^4*f*x*Cos[2*e + f*x] - 600*b^5*f*x*Cos[2*e + f*x] + 75*
a^5*f*x*Cos[4*e + f*x] + 900*a^4*b*f*x*Cos[4*e + f*x] + 2850*a^3*b^2*f*...
```

3.362.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.12, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 4629, 2075, 374, 445, 27, 445, 27, 445, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\tan(e+fx)^6\,(a+b\sec(e+fx)^2)^2}\,dx$$

$$\downarrow 4629$$

$$\frac{\int \frac{\cot^6(e+fx)}{(\tan^2(e+fx)+1)(a+b(\tan^2(e+fx)+1))^2}\,d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\cot^6(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^2}\,d\tan(e+fx)}{f}$$

$$\downarrow 374$$

$$\frac{\frac{\int \frac{\cot^6(e+fx)\left(-7b\tan^2(e+fx)+2a-5b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}\,d\tan(e+fx)}{2a(a+b)} - \frac{b\cot^5(e+fx)}{2a(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

$$\downarrow 445$$

$$\frac{-\frac{\int \frac{5\cot^4(e+fx)\left(2a^2+6ba-3b^2+(2a-5b)b\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}\,d\tan(e+fx)}{5(a+b)} - \frac{(2a-5b)\cot^5(e+fx)}{5(a+b)}}{2a(a+b)} - \frac{b\cot^5(e+fx)}{2a(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

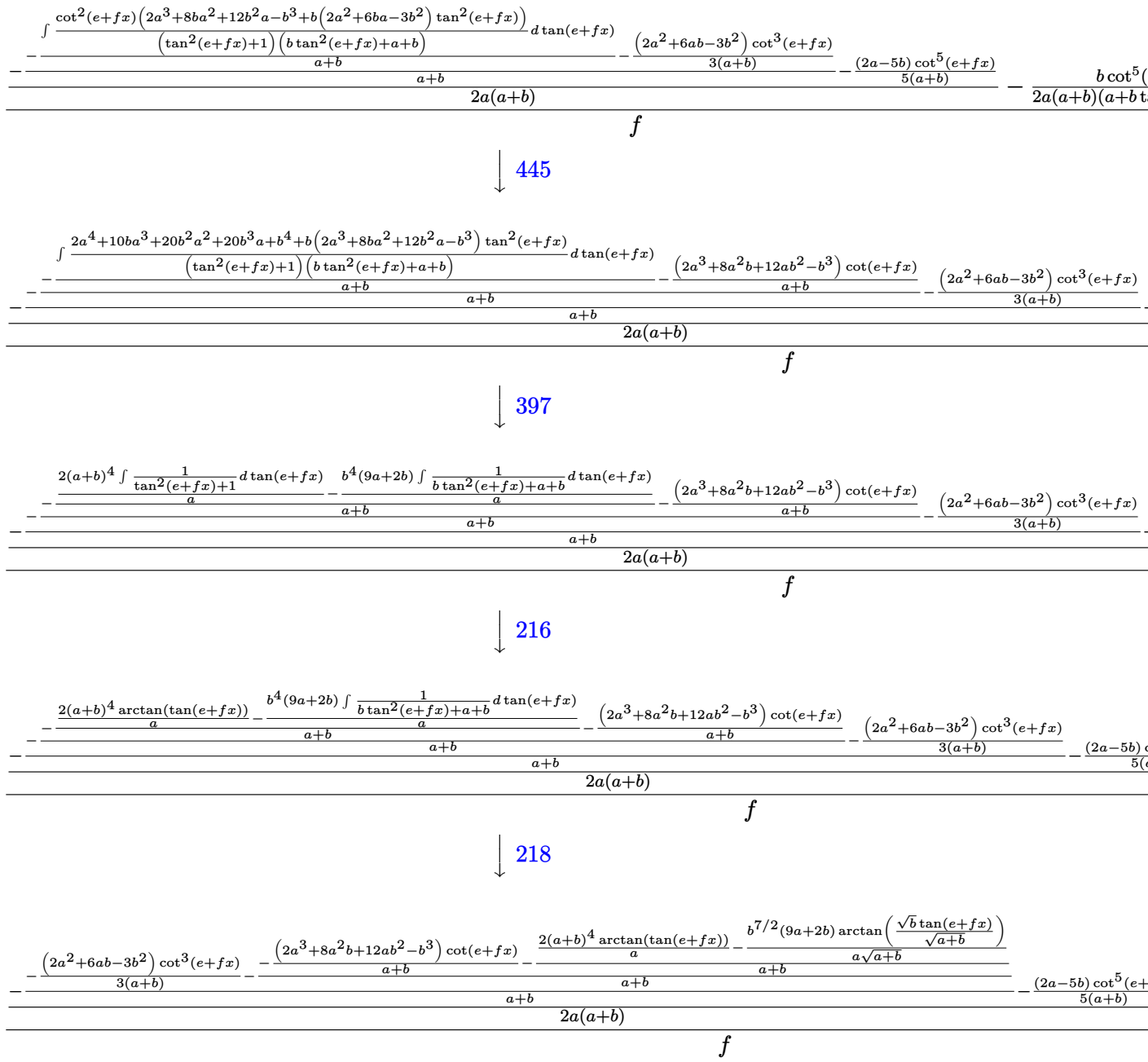
$$\downarrow 27$$

$$\frac{-\frac{\int \frac{\cot^4(e+fx)\left(2a^2+6ba-3b^2+(2a-5b)b\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}\,d\tan(e+fx)}{a+b} - \frac{(2a-5b)\cot^5(e+fx)}{5(a+b)}}{2a(a+b)} - \frac{b\cot^5(e+fx)}{2a(a+b)(a+b\tan^2(e+fx)+b)}}{f}$$

$$\downarrow 445$$

$$\frac{-\frac{-\frac{\int \frac{3\cot^2(e+fx)\left(2a^3+8ba^2+12b^2a-b^3+b\left(2a^2+6ba-3b^2\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}\,d\tan(e+fx)}{3(a+b)} - \frac{\left(2a^2+6ab-3b^2\right)\cot^3(e+fx)}{3(a+b)}}{a+b} - \frac{(2a-5b)\cot^5(e+fx)}{5(a+b)}}{2a(a+b)} - \frac{b\cot^5}{2a(a+b)(a+b}}{f}$$

$$\downarrow 27$$



input `Int[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2)^2,x]`

output
```
((-1/5*((2*a - 5*b)*Cot[e + f*x]^5)/(a + b) - (-1/3*((2*a^2 + 6*a*b - 3*b^
2)*Cot[e + f*x]^3)/(a + b) - (-(((2*(a + b)^4*ArcTan[Tan[e + f*x]])/a - (b
^(7/2)*(9*a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*Sqrt[a +
 b]))/(a + b)) - ((2*a^3 + 8*a^2*b + 12*a*b^2 - b^3)*Cot[e + f*x])/(a + b)
)/(a + b))/(a + b))/(2*a*(a + b)) - (b*Cot[e + f*x]^5)/(2*a*(a + b)*(a + b
 + b*Tan[e + f*x]^2)))/f
```

3.362.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.362.4 Maple [A] (verified)

Time = 18.38 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.73

method	result
derivativedivides	$\frac{-\frac{\arctan(\tan(fx+e))}{a^2}+\frac{b^4\left(\frac{a\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{(9a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{(a+b)^4a^2}-\frac{1}{5(a+b)^2\tan(fx+e)^5}-\frac{-a-3b}{3(a+b)^3\tan(fx+e)^3}}{f}$
default	$\frac{-\frac{\arctan(\tan(fx+e))}{a^2}+\frac{b^4\left(\frac{a\tan(fx+e)}{2a+2b+2b\tan(fx+e)^2}+\frac{(9a+2b)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}\right)}{(a+b)^4a^2}-\frac{1}{5(a+b)^2\tan(fx+e)^5}-\frac{-a-3b}{3(a+b)^3\tan(fx+e)^3}}{f}$
risch	$-\frac{x}{a^2}-\frac{i\left(172a^4b+216a^3b^2+15ab^4+46a^5+300a^4b\,e^{12i(fx+e)}+900a^3b^2e^{10i(fx+e)}+1200a^2b^3e^{10i(fx+e)}+60ab^4e^{10i(f}$

input
```
int(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/a^2*arctan(tan(f*x+e))+b^4/(a+b)^4/a^2*(1/2*a*tan(f*x+e)/(a+b+b*ta
n(f*x+e)^2)+1/2*(9*a+2*b)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1
/2)))-1/5/(a+b)^2/tan(f*x+e)^5-1/3*(-a-3*b)/(a+b)^3/tan(f*x+e)^3-(a^2+4*a*
b+6*b^2)/(a+b)^4/tan(f*x+e))
```

3.362.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 709 vs. $2(189) = 378$.

Time $= 0.36$ (sec) , antiderivative size $= 1505$, normalized size of antiderivative $= 7.27$

$$\int \frac{\cot^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="fricas")
```

output
```
[-1/120*(4*(46*a^5 + 172*a^4*b + 216*a^3*b^2 + 15*a*b^4)*cos(f*x + e)^7 -
4*(70*a^5 + 234*a^4*b + 218*a^3*b^2 - 216*a^2*b^3 + 45*a*b^4)*cos(f*x + e)
^5 + 20*(6*a^5 + 10*a^4*b - 20*a^3*b^2 - 78*a^2*b^3 + 9*a*b^4)*cos(f*x + e
)^3 - 15*((9*a^2*b^3 + 2*a*b^4)*cos(f*x + e)^6 + 9*a*b^4 + 2*b^5 - (18*a^2
*b^3 - 5*a*b^4 - 2*b^5)*cos(f*x + e)^4 + (9*a^2*b^3 - 16*a*b^4 - 4*b^5)*co
s(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 -
 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^
3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*co
s(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) + 60*(2*a^4*b + 8
*a^3*b^2 + 12*a^2*b^3 - a*b^4)*cos(f*x + e) + 120*((a^5 + 4*a^4*b + 6*a^3*
b^2 + 4*a^2*b^3 + a*b^4)*f*x*cos(f*x + e)^6 - (2*a^5 + 7*a^4*b + 8*a^3*b^2
 + 2*a^2*b^3 - 2*a*b^4 - b^5)*f*x*cos(f*x + e)^4 + (a^5 + 2*a^4*b - 2*a^3*
b^2 - 8*a^2*b^3 - 7*a*b^4 - 2*b^5)*f*x*cos(f*x + e)^2 + (a^4*b + 4*a^3*b^2
 + 6*a^2*b^3 + 4*a*b^4 + b^5)*f*x)*sin(f*x + e))/(((a^7 + 4*a^6*b + 6*a^5*
b^2 + 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^6 - (2*a^7 + 7*a^6*b + 8*a^5*b^2
 + 2*a^4*b^3 - 2*a^3*b^4 - a^2*b^5)*f*cos(f*x + e)^4 + (a^7 + 2*a^6*b - 2*
a^5*b^2 - 8*a^4*b^3 - 7*a^3*b^4 - 2*a^2*b^5)*f*cos(f*x + e)^2 + (a^6*b + 4
*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*f)*sin(f*x + e)), -1/60*(2*(46
*a^5 + 172*a^4*b + 216*a^3*b^2 + 15*a*b^4)*cos(f*x + e)^7 - 2*(70*a^5 + 23
4*a^4*b + 218*a^3*b^2 - 216*a^2*b^3 + 45*a*b^4)*cos(f*x + e)^5 + 10*(6*...
```

3.362.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)**6/(a+b*sec(f*x+e)**2)**2,x)
```

output
```
Timed out
```

3.362.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.54

$$\int \frac{\cot^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$
$$= \frac{15\left(9ab^4+2b^5\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\left(a^6+4a^5b+6a^4b^2+4a^3b^3+a^2b^4\right)\sqrt{(a+b)b}} - \frac{15\left(2a^3b+8a^2b^2+12ab^3-b^4\right)\tan(fx+e)^6+10\left(3a^4+14a^3b+26a^2b^2+15ab^3\right)\tan(fx+e)^4+6a^4-}{\left(a^5b+4a^4b^2+6a^3b^3+4a^2b^4+ab^5\right)\tan(fx+e)^7+\left(a^6+5a^5b+10a^4-\right.}$$
$$\frac{}{30\,f}$$

input
```
integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="maxima")
```

output
```
1/30*(15*(9*a*b^4 + 2*b^5)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^6 +
4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*sqrt((a + b)*b)) - (15*(2*a^3*b
 + 8*a^2*b^2 + 12*a*b^3 - b^4)*tan(f*x + e)^6 + 10*(3*a^4 + 14*a^3*b + 26*
a^2*b^2 + 15*a*b^3)*tan(f*x + e)^4 + 6*a^4 + 18*a^3*b + 18*a^2*b^2 + 6*a*b
^3 - 2*(5*a^4 + 22*a^3*b + 29*a^2*b^2 + 12*a*b^3)*tan(f*x + e)^2)/((a^5*b
+ 4*a^4*b^2 + 6*a^3*b^3 + 4*a^2*b^4 + a*b^5)*tan(f*x + e)^7 + (a^6 + 5*a^5
*b + 10*a^4*b^2 + 10*a^3*b^3 + 5*a^2*b^4 + a*b^5)*tan(f*x + e)^5) - 30*(f*
x + e)/a^2)/f
```

3.362.8 Giac [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.44

$$\int \frac{\cot^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx$$

$$= \frac{\frac{15\,b^4\tan(fx+e)}{(a^5+4\,a^4b+6\,a^3b^2+4\,a^2b^3+ab^4)\left(b\tan(fx+e)^2+a+b\right)} + \frac{15\,(9\,ab^4+2\,b^5)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{(a^6+4\,a^5b+6\,a^4b^2+4\,a^3b^3+a^2b^4)\sqrt{ab+b^2}} - \frac{30\,(fx+e)}{a^2} - \frac{2\left(15}{}}{30\,f}$$

input
```
integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

output
```
1/30*(15*b^4*tan(f*x + e)/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)
*(b*tan(f*x + e)^2 + a + b)) + 15*(9*a*b^4 + 2*b^5)*(pi*floor((f*x + e)/pi
 + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^6 + 4*a^5*b +
 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*sqrt(a*b + b^2)) - 30*(f*x + e)/a^2 - 2*
(15*a^2*tan(f*x + e)^4 + 60*a*b*tan(f*x + e)^4 + 90*b^2*tan(f*x + e)^4 - 5
*a^2*tan(f*x + e)^2 - 20*a*b*tan(f*x + e)^2 - 15*b^2*tan(f*x + e)^2 + 3*a^
2 + 6*a*b + 3*b^2)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*tan(f*x +
e)^5))/f
```

3.362.9 Mupad [B] (verification not implemented)

Time = 26.62 (sec) , antiderivative size = 6017, normalized size of antiderivative = 29.07

$$\int \frac{\cot^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(cot(e + f*x)^6/(a + b/cos(e + f*x)^2)^2,x)
```

output

```
atan((((64*a^6*b^22 + 2304*a^7*b^21 + 29440*a^8*b^20 + 210560*a^9*b^19 +
997248*a^10*b^18 + 3404800*a^11*b^17 + 8806912*a^12*b^16 + 17809920*a^13*b
^15 + 28745600*a^14*b^14 + 37533184*a^15*b^13 + 39975936*a^16*b^12 + 34874
112*a^17*b^11 + 24926720*a^18*b^10 + 14545920*a^19*b^9 + 6874624*a^20*b^8
+ 2595328*a^21*b^7 + 765504*a^22*b^6 + 170240*a^23*b^5 + 26880*a^24*b^4 +
2688*a^25*b^3 + 128*a^26*b^2 + (tan(e + f*x)*(512*a^7*b^23 + 10496*a^8*b^2
2 + 102400*a^9*b^21 + 632320*a^10*b^20 + 2772480*a^11*b^19 + 9178368*a^12*
b^18 + 23814144*a^13*b^17 + 49612800*a^14*b^16 + 84341760*a^15*b^15 + 1182
43840*a^16*b^14 + 137592832*a^17*b^13 + 133293056*a^18*b^12 + 107494400*a^
19*b^11 + 71938560*a^20*b^10 + 39690240*a^21*b^9 + 17860608*a^22*b^8 + 644
9664*a^23*b^7 + 1824000*a^24*b^6 + 389120*a^25*b^5 + 58880*a^26*b^4 + 5632
*a^27*b^3 + 256*a^28*b^2)*1i)/(2*a^2))*1i)/(2*a^2) + tan(e + f*x)*(128*a^3
*b^23 + 2624*a^4*b^22 + 24592*a^5*b^21 + 140608*a^6*b^20 + 554016*a^7*b^19
 + 1613184*a^8*b^18 + 3637488*a^9*b^17 + 6570624*a^10*b^16 + 9747456*a^11*
b^15 + 12075072*a^12*b^14 + 12596848*a^13*b^13 + 11073344*a^14*b^12 + 8154
592*a^15*b^11 + 4977408*a^16*b^10 + 2481936*a^17*b^9 + 992256*a^18*b^8 + 3
10080*a^19*b^7 + 72960*a^20*b^6 + 12160*a^21*b^5 + 1280*a^22*b^4 + 64*a^23
*b^3))/(2*a^2) - (((64*a^6*b^22 + 2304*a^7*b^21 + 29440*a^8*b^20 + 210560*
a^9*b^19 + 997248*a^10*b^18 + 3404800*a^11*b^17 + 8806912*a^12*b^16 + 1780
9920*a^13*b^15 + 28745600*a^14*b^14 + 37533184*a^15*b^13 + 39975936*a^1...
```

3.363 $\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

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3.363.1 Optimal result

Integrand size = 23, antiderivative size = 78

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{(a+b)^2}{4a^3f\left(b+a\cos^2(e+fx)\right)^2} - \frac{a+b}{a^3f\left(b+a\cos^2(e+fx)\right)} - \frac{\log\left(b+a\cos^2(e+fx)\right)}{2a^3f}$$

output
```
1/4*(a+b)^2/a^3/f/(b+a*cos(f*x+e)^2)^2+(-a-b)/a^3/f/(b+a*cos(f*x+e)^2)-1/2
*ln(b+a*cos(f*x+e)^2)/a^3/f
```

3.363.2 Mathematica [A] (verified)

Time = 3.26 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.74

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{2(a^2+4ab+3b^2)+(a+2b)^2\log(a+2b+a\cos(2(e+fx)))+a^2\cos^2(2(e+fx))\log(a+2b+a\cos(2}{2a^3f(a+2b+a\cos(2(e+}$$

input
```
Integrate[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
-1/2*(2*(a^2 + 4*a*b + 3*b^2) + (a + 2*b)^2*Log[a + 2*b + a*Cos[2*(e + f*x
)]] + a^2*Cos[2*(e + f*x)]^2*Log[a + 2*b + a*Cos[2*(e + f*x)]] + 2*a*Cos[2
*(e + f*x)]*(2*(a + b) + (a + 2*b)*Log[a + 2*b + a*Cos[2*(e + f*x)]]))/(a^
3*f*(a + 2*b + a*Cos[2*(e + f*x)])^2)
```

3.363.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4626, 353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\tan(e+fx)^5}{(a+b\sec(e+fx)^2)^3}\,dx$$

$$\downarrow 4626$$

$$-\frac{\int \frac{\cos(e+fx)\left(1-\cos^2(e+fx)\right)^2}{\left(a\cos^2(e+fx)+b\right)^3}\,d\cos(e+fx)}{f}$$

$$\downarrow 353$$

$$-\frac{\int \frac{\left(1-\cos^2(e+fx)\right)^2}{\left(a\cos^2(e+fx)+b\right)^3}\,d\cos^2(e+fx)}{2f}$$

$$\downarrow 49$$

$$-\frac{\int \left(\frac{(a+b)^2}{a^2\left(a\cos^2(e+fx)+b\right)^3}-\frac{2(a+b)}{a^2\left(a\cos^2(e+fx)+b\right)^2}+\frac{1}{a^2\left(a\cos^2(e+fx)+b\right)}\right)d\cos^2(e+fx)}{2f}$$

$$\downarrow 2009$$

$$-\frac{-\frac{(a+b)^2}{2a^3\left(a\cos^2(e+fx)+b\right)^2}+\frac{2(a+b)}{a^3\left(a\cos^2(e+fx)+b\right)}+\frac{\log\left(a\cos^2(e+fx)+b\right)}{a^3}}{2f}$$

input
```
Int[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
-1/2*(-1/2*(a + b)^2/(a^3*(b + a*Cos[e + f*x]^2)^2) + (2*(a + b))/(a^3*(b
+ a*Cos[e + f*x]^2)) + Log[b + a*Cos[e + f*x]^2]/a^3)/f
```

3.363.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.363.4 Maple [A] (verified)

Time = 20.49 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.03

method	result
derivativedivides	$\frac{\frac{-2a-2b}{2a^3\left(b+a\cos(fx+e)^2\right)}+\frac{a^2+2ab+b^2}{4a^3\left(b+a\cos(fx+e)^2\right)^2}-\frac{\ln\left(b+a\cos(fx+e)^2\right)}{2a^3}}{f}$
default	$\frac{\frac{-2a-2b}{2a^3\left(b+a\cos(fx+e)^2\right)}+\frac{a^2+2ab+b^2}{4a^3\left(b+a\cos(fx+e)^2\right)^2}-\frac{\ln\left(b+a\cos(fx+e)^2\right)}{2a^3}}{f}$
risch	$\frac{ix}{a^3}+\frac{2ie}{a^3f}-\frac{4\left(a^2e^{6i(fx+e)}+ab\,e^{6i(fx+e)}+a^2e^{4i(fx+e)}+4ab\,e^{4i(fx+e)}+3b^2e^{4i(fx+e)}+a^2e^{2i(fx+e)}+ab\,e^{2i(fx+e)}\right)}{a^3f\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)^2}-$

input
```
int(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/2*(-2*a-2*b)/a^3/(b+a*cos(f*x+e)^2)+1/4*(a^2+2*a*b+b^2)/a^3/(b+a*co
s(f*x+e)^2)^2-1/2/a^3*ln(b+a*cos(f*x+e)^2))
```

3.363.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.49

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{4\left(a^2+ab\right)\cos\left(fx+e\right)^2-a^2+2\,ab+3\,b^2+2\left(a^2\cos\left(fx+e\right)^4+2\,ab\cos\left(fx+e\right)^2+b^2\right)\log\left(a\cos\left(f\right.\right.}{4\left(a^5f\cos\left(fx+e\right)^4+2\,a^4bf\cos\left(fx+e\right)^2+a^3b^2f\right)}$$

input
```
integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
-1/4*(4*(a^2 + a*b)*cos(f*x + e)^2 - a^2 + 2*a*b + 3*b^2 + 2*(a^2*cos(f*x
+ e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*log(a*cos(f*x + e)^2 + b))/(a^5*f*cos
(f*x + e)^4 + 2*a^4*b*f*cos(f*x + e)^2 + a^3*b^2*f)
```

3.363.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(tan(f*x+e)**5/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Timed out
```

3.363.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.44

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{4\,(a^2+ab)\sin(fx+e)^2-3\,a^2-6\,ab-3\,b^2}{a^5\sin(fx+e)^4+a^5+2\,a^4b+a^3b^2-2\,(a^5+a^4b)\sin(fx+e)^2}-\frac{2\log\left(a\sin(fx+e)^2-a-b\right)}{a^3}}{4\,f}$$

input
```
integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
1/4*((4*(a^2 + a*b)*sin(f*x + e)^2 - 3*a^2 - 6*a*b - 3*b^2)/(a^5*sin(f*x +
 e)^4 + a^5 + 2*a^4*b + a^3*b^2 - 2*(a^5 + a^4*b)*sin(f*x + e)^2) - 2*log(
a*sin(f*x + e)^2 - a - b)/a^3)/f
```

3.363.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 531 vs. 2(74) = 148.

Time = 1.93 (sec) , antiderivative size = 531, normalized size of antiderivative = 6.81

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{2\log\left(a+b+\frac{2\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{a^3}-\frac{4\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)}{a^3}-\frac{3\,a^2+6\,ab+3\,b^2+\frac{20}{}}{}$$

input
```
integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
-1/4*(2*log(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(cos(f
*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1
)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/a^3 - 4*log(abs(-(cos(f
*x + e) - 1)/(cos(f*x + e) + 1) + 1))/a^3 - (3*a^2 + 6*a*b + 3*b^2 + 20*a^
2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 8*a*b*(cos(f*x + e) - 1)/(cos(f*
x + e) + 1) - 12*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 50*a^2*(cos(f
*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 28*a*b*(cos(f*x + e) - 1)^2/(cos(f*x
 + e) + 1)^2 + 18*b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 20*a^2*(
cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 8*a*b*(cos(f*x + e) - 1)^3/(cos
(f*x + e) + 1)^3 - 12*b^2*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 3*a^
2*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 6*a*b*(cos(f*x + e) - 1)^4/(
cos(f*x + e) + 1)^4 + 3*b^2*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4)/((a
 + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(cos(f*x + e) - 1)/
(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + b*(cos(
f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)^2*a^3))/f
```

3.363.9 Mupad [B] (verification not implemented)

Time = 19.93 (sec) , antiderivative size = 166, normalized size of antiderivative = 2.13

$$\int \frac{\tan^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$
$$= \frac{\operatorname{atanh}\left(\frac{4b^2\tan(e+fx)^2}{8b^2+\frac{8b^3}{a}+4b^2\tan(e+fx)^2+\frac{8b^3\tan(e+fx)^2}{a}}\right)}{a^3 f}$$
$$+ \frac{\frac{-a^3+3ab^2+2b^3}{4a^2b^2} - \frac{\tan(e+fx)^2\,(a^2-b^2)}{2a^2b}}{f\,\left(2ab+a^2+b^2+\tan(e+fx)^2\,(2b^2+2ab)+b^2\tan(e+fx)^4\right)}$$

input
```
int(tan(e + f*x)^5/(a + b/cos(e + f*x)^2)^3,x)
```

output
```
atanh((4*b^2*tan(e + f*x)^2)/(8*b^2 + (8*b^3)/a + 4*b^2*tan(e + f*x)^2 + (
8*b^3*tan(e + f*x)^2)/a))/(a^3*f) + ((3*a*b^2 - a^3 + 2*b^3)/(4*a^2*b^2) -
 (tan(e + f*x)^2*(a^2 - b^2))/(2*a^2*b))/(f*(2*a*b + a^2 + b^2 + tan(e + f
*x)^2*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^4))
```

3.364 $\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

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3.364.2 Mathematica [A] (verified) 2641
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3.364.1 Optimal result

Integrand size = 23, antiderivative size = 81

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{b(a+b)}{4a^3f\left(b+a\cos^2(e+fx)\right)^2} + \frac{a+2b}{2a^3f\left(b+a\cos^2(e+fx)\right)} + \frac{\log\left(b+a\cos^2(e+fx)\right)}{2a^3f}$$

output
```
-1/4*b*(a+b)/a^3/f/(b+a*cos(f*x+e)^2)^2+1/2*(a+2*b)/a^3/f/(b+a*cos(f*x+e)^
2)+1/2*ln(b+a*cos(f*x+e)^2)/a^3/f
```

3.364.2 Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.62

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{2(a^2+3ab+3b^2)+(a+2b)^2\log(a+2b+a\cos(2(e+fx)))+a^2\cos^2(2(e+fx))\log(a+2b+a\cos(2(e+fx)))}{2a^3f(a+2b+a\cos(2(e+fx)))^2}$$

input
```
Integrate[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
(2*(a^2 + 3*a*b + 3*b^2) + (a + 2*b)^2*Log[a + 2*b + a*Cos[2*(e + f*x)]] +
 a^2*Cos[2*(e + f*x)]^2*Log[a + 2*b + a*Cos[2*(e + f*x)]] + 2*a*(a + 2*b)*
Cos[2*(e + f*x)]*(1 + Log[a + 2*b + a*Cos[2*(e + f*x)]]))/(2*a^3*f*(a + 2*
b + a*Cos[2*(e + f*x)])^2)
```

3.364.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4626, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\tan(e+fx)^3}{(a+b\sec(e+fx)^2)^3}\,dx$$

$$\downarrow 4626$$

$$-\frac{\int \frac{\cos^3(e+fx)(1-\cos^2(e+fx))}{(a\cos^2(e+fx)+b)^3}\,d\cos(e+fx)}{f}$$

$$\downarrow 354$$

$$-\frac{\int \frac{\cos^2(e+fx)(1-\cos^2(e+fx))}{(a\cos^2(e+fx)+b)^3}\,d\cos^2(e+fx)}{2f}$$

$$\downarrow 86$$

$$-\frac{\int \left(-\frac{b(a+b)}{a^2(a\cos^2(e+fx)+b)^3}-\frac{1}{a^2(a\cos^2(e+fx)+b)}+\frac{a+2b}{a^2(a\cos^2(e+fx)+b)^2}\right)d\cos^2(e+fx)}{2f}$$

$$\downarrow 2009$$

$$-\frac{\frac{b(a+b)}{2a^3(a\cos^2(e+fx)+b)^2}-\frac{a+2b}{a^3(a\cos^2(e+fx)+b)}-\frac{\log(a\cos^2(e+fx)+b)}{a^3}}{2f}$$

input
```
Int[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
-1/2*((b*(a + b))/(2*a^3*(b + a*Cos[e + f*x]^2)^2) - (a + 2*b)/(a^3*(b + a
*Cos[e + f*x]^2)) - Log[b + a*Cos[e + f*x]^2]/a^3)/f
```

3.364.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.364.4 Maple [A] (verified)

Time = 11.32 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{-\frac{(a+b)b}{4a^3\left(b+a\cos(fx+e)^2\right)^2}-\frac{-a-2b}{2a^3\left(b+a\cos(fx+e)^2\right)}+\frac{\ln\left(b+a\cos(fx+e)^2\right)}{2a^3}}{f}$
default	$\frac{-\frac{(a+b)b}{4a^3\left(b+a\cos(fx+e)^2\right)^2}-\frac{-a-2b}{2a^3\left(b+a\cos(fx+e)^2\right)}+\frac{\ln\left(b+a\cos(fx+e)^2\right)}{2a^3}}{f}$
risch	$-\frac{ix}{a^3}-\frac{2ie}{a^3f}+\frac{2a^2e^{6i(fx+e)}+4abe^{6i(fx+e)}+4a^2e^{4i(fx+e)}+12abe^{4i(fx+e)}+12b^2e^{4i(fx+e)}+2a^2e^{2i(fx+e)}+4abe^{2i(f}}{a^3f\left(ae^{4i(fx+e)}+2ae^{2i(fx+e)}+4be^{2i(fx+e)}+a\right)^2}$

input
```
int(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/4*(a+b)*b/a^3/(b+a*cos(f*x+e)^2)^2-1/2*(-a-2*b)/a^3/(b+a*cos(f*x+e
)^2)+1/2/a^3*ln(b+a*cos(f*x+e)^2))
```

3.364.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.37

$$\int \frac{\tan^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$
$$= \frac{2\left(a^2+2ab\right)\cos\left(fx+e\right)^2+ab+3b^2+2\left(a^2\cos\left(fx+e\right)^4+2ab\cos\left(fx+e\right)^2+b^2\right)\log\left(a\cos\left(fx+e\right)^2\right.}{4\left(a^5f\cos\left(fx+e\right)^4+2a^4bf\cos\left(fx+e\right)^2+a^3b^2f\right)}$$

input
```
integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
1/4*(2*(a^2 + 2*a*b)*cos(f*x + e)^2 + a*b + 3*b^2 + 2*(a^2*cos(f*x + e)^4
+ 2*a*b*cos(f*x + e)^2 + b^2)*log(a*cos(f*x + e)^2 + b))/(a^5*f*cos(f*x +
e)^4 + 2*a^4*b*f*cos(f*x + e)^2 + a^3*b^2*f)
```

3.364.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(tan(f*x+e)**3/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Timed out
```

3.364.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.40

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{2\,(a^2+2\,ab)\sin(fx+e)^2-2\,a^2-5\,ab-3\,b^2}{a^5\sin(fx+e)^4+a^5+2\,a^4b+a^3b^2-2\,(a^5+a^4b)\sin(fx+e)^2}-\frac{2\log\left(a\sin(fx+e)^2-a-b\right)}{a^3}}{4\,f}$$

input
```
integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
-1/4*((2*(a^2 + 2*a*b)*sin(f*x + e)^2 - 2*a^2 - 5*a*b - 3*b^2)/(a^5*sin(f*
x + e)^4 + a^5 + 2*a^4*b + a^3*b^2 - 2*(a^5 + a^4*b)*sin(f*x + e)^2) - 2*l
og(a*sin(f*x + e)^2 - a - b)/a^3)/f
```

3.364.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 656 vs. 2(75) = 150.

Time = 0.96 (sec) , antiderivative size = 656, normalized size of antiderivative = 8.10

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx =$$

$$\frac{3\,a^3+9\,a^2b+9\,ab^2+3\,b^3+\frac{20\,a^3(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{28\,a^2b(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{4\,ab^2(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{12\,b^3(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{34\,a^3(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{22\,a^2b(\cos(f}{(a^4}$$

$$-$$

input
```
integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
-1/4*((3*a^3 + 9*a^2*b + 9*a*b^2 + 3*b^3 + 20*a^3*(cos(f*x + e) - 1)/(cos(
f*x + e) + 1) + 28*a^2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 4*a*b^2*(
cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 12*b^3*(cos(f*x + e) - 1)/(cos(f*x
+ e) + 1) + 34*a^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 22*a^2*b*(c
os(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 10*a*b^2*(cos(f*x + e) - 1)^2/(c
os(f*x + e) + 1)^2 + 18*b^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 20
*a^3*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 28*a^2*b*(cos(f*x + e) -
1)^3/(cos(f*x + e) + 1)^3 - 4*a*b^2*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1
)^3 - 12*b^3*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 3*a^3*(cos(f*x +
e) - 1)^4/(cos(f*x + e) + 1)^4 + 9*a^2*b*(cos(f*x + e) - 1)^4/(cos(f*x + e
) + 1)^4 + 9*a*b^2*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 3*b^3*(cos(
f*x + e) - 1)^4/(cos(f*x + e) + 1)^4)/((a^4 + a^3*b)*(a + b + 2*a*(cos(f*x
 + e) - 1)/(cos(f*x + e) + 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1)
+ a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(co
s(f*x + e) + 1)^2)^2) - 2*log(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e)
 + 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2
/(cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)/a^3 +
 4*log(abs(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1))/a^3)/f
```

3.364.9 Mupad [B] (verification not implemented)

Time = 19.47 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.89

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= -\frac{\frac{a^2+3ab+2b^2}{4a^2b} + \frac{b\tan(e+fx)^2}{2a^2}}{f\left(2ab+a^2+b^2+\tan(e+fx)^2\left(2b^2+2ab\right)+b^2\tan(e+fx)^4\right)} - \frac{\operatorname{atanh}\left(\frac{4b^2\tan(e+fx)^2}{8b^2+\frac{8b^3}{a}+4b^2\tan(e+fx)^2+\frac{8b^3\tan(e+fx)^2}{a}}\right)}{a^3 f}$$

input
```
int(tan(e + f*x)^3/(a + b/cos(e + f*x)^2)^3,x)
```

output
```
- ((3*a*b + a^2 + 2*b^2)/(4*a^2*b) + (b*tan(e + f*x)^2)/(2*a^2))/(f*(2*a*b
 + a^2 + b^2 + tan(e + f*x)^2*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^4)) - ata
nh((4*b^2*tan(e + f*x)^2)/(8*b^2 + (8*b^3)/a + 4*b^2*tan(e + f*x)^2 + (8*b
^3*tan(e + f*x)^2)/a))/(a^3*f)
```

3.365 $\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

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3.365.2 Mathematica [A] (verified) . . . 2647
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3.365.1 Optimal result

Integrand size = 21, antiderivative size = 74

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{b^2}{4a^3f\left(b+a\cos^2(e+fx)\right)^2} - \frac{b}{a^3f\left(b+a\cos^2(e+fx)\right)} - \frac{\log\left(b+a\cos^2(e+fx)\right)}{2a^3f}$$

output

```
1/4*b^2/a^3/f/(b+a*cos(f*x+e)^2)^2-b/a^3/f/(b+a*cos(f*x+e)^2)-1/2*ln(b+a*c
os(f*x+e)^2)/a^3/f
```

3.365.2 Mathematica [A] (verified)

Time = 1.91 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.74

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{2b(2a+3b)+(a+2b)^2\log(a+2b+a\cos(2(e+fx)))+a^2\cos^2(2(e+fx))\log(a+2b+a\cos(2(e+f}{2a^3f(a+2b+a\cos(2(e+fx)))^2}$$

input

```
Integrate[Tan[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
-1/2*(2*b*(2*a + 3*b) + (a + 2*b)^2*Log[a + 2*b + a*Cos[2*(e + f*x)]] + a^
2*Cos[2*(e + f*x)]^2*Log[a + 2*b + a*Cos[2*(e + f*x)]] + 2*a*Cos[2*(e + f*
x)]*(2*b + (a + 2*b)*Log[a + 2*b + a*Cos[2*(e + f*x)]]))/(a^3*f*(a + 2*b +
 a*Cos[2*(e + f*x)])^2)
```

3.365.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4626, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\tan(e+fx)}{(a+b\sec(e+fx)^2)^3}\,dx$$

$$\downarrow 4626$$

$$-\frac{\int \frac{\cos^5(e+fx)}{(a\cos^2(e+fx)+b)^3}\,d\cos(e+fx)}{f}$$

$$\downarrow 243$$

$$-\frac{\int \frac{\cos^4(e+fx)}{(a\cos^2(e+fx)+b)^3}\,d\cos^2(e+fx)}{2f}$$

$$\downarrow 49$$

$$-\frac{\int \left(\frac{b^2}{a^2(a\cos^2(e+fx)+b)^3}-\frac{2b}{a^2(a\cos^2(e+fx)+b)^2}+\frac{1}{a^2(a\cos^2(e+fx)+b)}\right)d\cos^2(e+fx)}{2f}$$

$$\downarrow 2009$$

$$-\frac{-\frac{b^2}{2a^3(a\cos^2(e+fx)+b)^2}+\frac{2b}{a^3(a\cos^2(e+fx)+b)}+\frac{\log(a\cos^2(e+fx)+b)}{a^3}}{2f}$$

input
```
Int[Tan[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
-1/2*(-1/2*b^2/(a^3*(b + a*Cos[e + f*x]^2)^2) + (2*b)/(a^3*(b + a*Cos[e +
f*x]^2)) + Log[b + a*Cos[e + f*x]^2]/a^3)/f
```

3.365.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.365.4 Maple [A] (verified)

Time = 11.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.09

method	result	si
derivativedivides	$\frac{1}{4fa\left(a+b\sec(fx+e)^2\right)^2}+\frac{1}{2fa^2\left(a+b\sec(fx+e)^2\right)}-\frac{\ln\left(a+b\sec(fx+e)^2\right)}{2fa^3}+\frac{\ln(\sec(fx+e))}{fa^3}$	8
default	$\frac{1}{4fa\left(a+b\sec(fx+e)^2\right)^2}+\frac{1}{2fa^2\left(a+b\sec(fx+e)^2\right)}-\frac{\ln\left(a+b\sec(fx+e)^2\right)}{2fa^3}+\frac{\ln(\sec(fx+e))}{fa^3}$	8
risch	$\frac{ix}{a^3}+\frac{2ie}{a^3f}-\frac{4b\left(a\,e^{6i(fx+e)}+2a\,e^{4i(fx+e)}+3b\,e^{4i(fx+e)}+a\,e^{2i(fx+e)}\right)}{a^3\left(a\,e^{4i(fx+e)}+2a\,e^{2i(fx+e)}+4b\,e^{2i(fx+e)}+a\right)^2f}-\frac{\ln\left(e^{4i(fx+e)}+\frac{2(a+2b)e^{2i(fx+e)}}{a}+1\right)}{2a^3f}$	1

input
```
int(tan(f*x+e)/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4/f/a/(a+b*sec(f*x+e)^2)^2+1/2/f/a^2/(a+b*sec(f*x+e)^2)-1/2/f/a^3*ln(a+b
*sec(f*x+e)^2)+1/f/a^3*ln(sec(f*x+e))
```

3.365.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.38

$$\int \frac{\tan(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{4ab\cos(fx+e)^2+3b^2+2\left(a^2\cos(fx+e)^4+2ab\cos(fx+e)^2+b^2\right)\log\left(a\cos(fx+e)^2+b\right)}{4\left(a^5f\cos(fx+e)^4+2a^4bf\cos(fx+e)^2+a^3b^2f\right)}$$

input
```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
-1/4*(4*a*b*cos(f*x + e)^2 + 3*b^2 + 2*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x
 + e)^2 + b^2)*log(a*cos(f*x + e)^2 + b))/(a^5*f*cos(f*x + e)^4 + 2*a^4*b*
f*cos(f*x + e)^2 + a^3*b^2*f)
```

3.365.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Timed out
```

3.365.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.38

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{4ab\sin(fx+e)^2-4ab-3b^2}{a^5\sin(fx+e)^4+a^5+2a^4b+a^3b^2-2(a^5+a^4b)\sin(fx+e)^2}-\frac{2\log\left(a\sin(fx+e)^2-a-b\right)}{a^3}}{4f}$$

input
```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
1/4*((4*a*b*sin(f*x + e)^2 - 4*a*b - 3*b^2)/(a^5*sin(f*x + e)^4 + a^5 + 2*
a^4*b + a^3*b^2 - 2*(a^5 + a^4*b)*sin(f*x + e)^2) - 2*log(a*sin(f*x + e)^2
 - a - b)/a^3)/f
```

3.365.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 782 vs. 2(70) = 140.

Time = 0.58 (sec) , antiderivative size = 782, normalized size of antiderivative = 10.57

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
1/4*((3*a^4 + 12*a^3*b + 18*a^2*b^2 + 12*a*b^3 + 3*b^4 + 12*a^4*(cos(f*x +
 e) - 1)/(cos(f*x + e) + 1) + 40*a^3*b*(cos(f*x + e) - 1)/(cos(f*x + e) +
1) + 24*a^2*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 16*a*b^3*(cos(f*x
+ e) - 1)/(cos(f*x + e) + 1) - 12*b^4*(cos(f*x + e) - 1)/(cos(f*x + e) + 1
) + 18*a^4*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 56*a^3*b*(cos(f*x +
 e) - 1)^2/(cos(f*x + e) + 1)^2 + 12*a^2*b^2*(cos(f*x + e) - 1)^2/(cos(f*x
 + e) + 1)^2 + 8*a*b^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 18*b^4*
(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 12*a^4*(cos(f*x + e) - 1)^3/(c
os(f*x + e) + 1)^3 + 40*a^3*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 +
24*a^2*b^2*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 16*a*b^3*(cos(f*x +
 e) - 1)^3/(cos(f*x + e) + 1)^3 - 12*b^4*(cos(f*x + e) - 1)^3/(cos(f*x + e
) + 1)^3 + 3*a^4*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 12*a^3*b*(cos
(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 18*a^2*b^2*(cos(f*x + e) - 1)^4/(c
os(f*x + e) + 1)^4 + 12*a*b^3*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 +
3*b^4*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4)/((a^5 + 2*a^4*b + a^3*b^2
)*(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(cos(f*x + e) -
 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + b*(
cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2)^2) - 2*log(a + b + 2*a*(cos(f*x
+ e) - 1)/(cos(f*x + e) + 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) +
 a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(...
```

3.365.9 Mupad [B] (verification not implemented)

Time = 19.33 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.92

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \frac{\frac{3a+2b}{4a^2} + \frac{b\tan(e+fx)^2}{2a^2}}{f\left(2ab+a^2+b^2+\tan(e+fx)^2\left(2b^2+2ab\right)+b^2\tan(e+fx)^4\right)} + \frac{\operatorname{atanh}\left(\frac{4b^2\tan(e+fx)^2}{8b^2+\frac{8b^3}{a}+4b^2\tan(e+fx)^2+\frac{8b^3\tan(e+fx)^2}{a}}\right)}{a^3 f}$$

input
```
int(tan(e + f*x)/(a + b/cos(e + f*x)^2)^3,x)
```

output
```
((3*a + 2*b)/(4*a^2) + (b*tan(e + f*x)^2)/(2*a^2))/(f*(2*a*b + a^2 + b^2 +
 tan(e + f*x)^2*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^4)) + atanh((4*b^2*tan(
e + f*x)^2)/(8*b^2 + (8*b^3)/a + 4*b^2*tan(e + f*x)^2 + (8*b^3*tan(e + f*x
)^2)/a))/(a^3*f)
```

3.366 $\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

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3.366.1 Optimal result

Integrand size = 21, antiderivative size = 130

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{b^3}{4a^3(a+b)f\left(b+a\cos^2(e+fx)\right)^2} + \frac{b^2(3a+2b)}{2a^3(a+b)^2 f\left(b+a\cos^2(e+fx)\right)} + \frac{b(3a^2+3ab+b^2)\log\left(b+a\cos^2(e+fx)\right)}{2a^3(a+b)^3 f} + \frac{\log(\sin(e+fx))}{(a+b)^3 f}$$

output

```
-1/4*b^3/a^3/(a+b)/f/(b+a*cos(f*x+e)^2)^2+1/2*b^2*(3*a+2*b)/a^3/(a+b)^2/f/
(b+a*cos(f*x+e)^2)+1/2*b*(3*a^2+3*a*b+b^2)*ln(b+a*cos(f*x+e)^2)/a^3/(a+b)^
3/f+ln(sin(f*x+e))/(a+b)^3/f
```

3.366.2 Mathematica [A] (verified)

Time = 1.42 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.22

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{(a+2b+a\cos(2(e+fx)))^3\sec^6(e+fx)\left(4\log(\sin(e+fx)) + \frac{2b(3a^2+3ab+b^2)\log(a+b-a\sin^2(e+fx))}{a^3} - \frac{b^3}{a^3(a+b-}\right.}{32(a+b)^3 f\left(a+b\sec^2(e+fx)\right)^3}$$

input
```
Integrate[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])^3*Sec[e + f*x]^6*(4*Log[Sin[e + f*x]] + (2
*b*(3*a^2 + 3*a*b + b^2)*Log[a + b - a*Sin[e + f*x]^2])/a^3 - (b^3*(a + b)
^2)/(a^3*(a + b - a*Sin[e + f*x]^2)^2) + (2*b^2*(a + b)*(3*a + 2*b))/(a^3*
(a + b - a*Sin[e + f*x]^2))))/(32*(a + b)^3*f*(a + b*Sec[e + f*x]^2)^3)
```

3.366.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4626, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\tan(e+fx)\,(a+b\sec(e+fx)^2)^3}\,dx$$

$$\downarrow 4626$$

$$-\frac{\int \frac{\cos^7(e+fx)}{(1-\cos^2(e+fx))(a\cos^2(e+fx)+b)^3}\,d\cos(e+fx)}{f}$$

$$\downarrow 354$$

$$-\frac{\int \frac{\cos^6(e+fx)}{(1-\cos^2(e+fx))(a\cos^2(e+fx)+b)^3}\,d\cos^2(e+fx)}{2f}$$

$$\downarrow 99$$

$$-\frac{\int \left(-\frac{b^3}{a^2(a+b)(a\cos^2(e+fx)+b)^3}+\frac{(3a+2b)b^2}{a^2(a+b)^2(a\cos^2(e+fx)+b)^2}-\frac{(3a^2+3ba+b^2)b}{a^2(a+b)^3(a\cos^2(e+fx)+b)}-\frac{1}{(a+b)^3(\cos^2(e+fx)-1)}\right)d\cos^2(e+}{2f}$$

$$\downarrow 2009$$

$$-\frac{\frac{b^3}{2a^3(a+b)(a\cos^2(e+fx)+b)^2}-\frac{b^2(3a+2b)}{a^3(a+b)^2(a\cos^2(e+fx)+b)}-\frac{b(3a^2+3ab+b^2)\log(a\cos^2(e+fx)+b)}{a^3(a+b)^3}-\frac{\log(1-\cos^2(e+fx))}{(a+b)^3}}{2f}$$

input
```
Int[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
-1/2*(b^3/(2*a^3*(a + b)*(b + a*Cos[e + f*x]^2)^2) - (b^2*(3*a + 2*b))/(a^
3*(a + b)^2*(b + a*Cos[e + f*x]^2)) - Log[1 - Cos[e + f*x]^2]/(a + b)^3 -
(b*(3*a^2 + 3*a*b + b^2)*Log[b + a*Cos[e + f*x]^2])/(a^3*(a + b)^3))/f
```

3.366.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.366.4 Maple [A] (verified)

Time = 17.52 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.11

method	result
derivativedivides	$\frac{b\left(\frac{b\left(3a^2+5ab+2b^2\right)}{a^3\left(b+a\cos(fx+e)^2\right)}+\frac{\left(3a^2+3ab+b^2\right)\ln\left(b+a\cos(fx+e)^2\right)}{a^3}-\frac{b^2\left(a^2+2ab+b^2\right)}{2a^3\left(b+a\cos(fx+e)^2\right)^2}\right)}{2(a+b)^3}+\frac{\ln(1+\cos(fx+e))}{2(a+b)^3}+\frac{\ln(-1+\cos(fx+e))}{2(a+b)^3}}{f}$
default	$\frac{b\left(\frac{b\left(3a^2+5ab+2b^2\right)}{a^3\left(b+a\cos(fx+e)^2\right)}+\frac{\left(3a^2+3ab+b^2\right)\ln\left(b+a\cos(fx+e)^2\right)}{a^3}-\frac{b^2\left(a^2+2ab+b^2\right)}{2a^3\left(b+a\cos(fx+e)^2\right)^2}\right)}{2(a+b)^3}+\frac{\ln(1+\cos(fx+e))}{2(a+b)^3}+\frac{\ln(-1+\cos(fx+e))}{2(a+b)^3}}{f}$
risch	$\frac{ix}{a^3}-\frac{2ix}{a^3+3a^2b+3ab^2+b^3}-\frac{2ie}{f(a^3+3a^2b+3ab^2+b^3)}-\frac{6ibx}{a(a^3+3a^2b+3ab^2+b^3)}-\frac{6ibe}{af(a^3+3a^2b+3ab^2+b^3)}-\frac{}{a^2(}$

input
```
int(cot(f*x+e)/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/2*b/(a+b)^3*(1/a^3*b*(3*a^2+5*a*b+2*b^2)/(b+a*cos(f*x+e)^2)+(3*a^2+
3*a*b+b^2)/a^3*ln(b+a*cos(f*x+e)^2)-1/2*b^2*(a^2+2*a*b+b^2)/a^3/(b+a*cos(f
*x+e)^2)^2)+1/2/(a+b)^3*ln(1+cos(f*x+e))+1/2/(a+b)^3*ln(-1+cos(f*x+e)))
```

3.366.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 307 vs. 2(124) = 248.

Time = 0.61 (sec) , antiderivative size = 307, normalized size of antiderivative = 2.36

$$\int \frac{\cot(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$
$$= \frac{5\,a^2b^3+8\,ab^4+3\,b^5+2\left(3\,a^3b^2+5\,a^2b^3+2\,ab^4\right)\cos\left(fx+e\right)^2+2\left(3\,a^2b^3+3\,ab^4+b^5+\left(3\,a^4b+3\,a^3b^2+\right.\right.}{4\left(\left(a^8+3\,a^7b+3\,a^6b^2+a^5b^3\right)f\cos\left(fx\right.\right.}$$

input
```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
1/4*(5*a^2*b^3 + 8*a*b^4 + 3*b^5 + 2*(3*a^3*b^2 + 5*a^2*b^3 + 2*a*b^4)*cos
(f*x + e)^2 + 2*(3*a^2*b^3 + 3*a*b^4 + b^5 + (3*a^4*b + 3*a^3*b^2 + a^2*b^
3)*cos(f*x + e)^4 + 2*(3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*cos(f*x + e)^2)*log(
a*cos(f*x + e)^2 + b) + 4*(a^5*cos(f*x + e)^4 + 2*a^4*b*cos(f*x + e)^2 + a
^3*b^2)*log(1/2*sin(f*x + e)))/((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*f*co
s(f*x + e)^4 + 2*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*f*cos(f*x + e)^
2 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*f)
```

3.366.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Timed out
```

3.366.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.87

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \frac{\frac{2\left(3a^2b+3ab^2+b^3\right)\log\left(a\sin(fx+e)^2-a-b\right)}{a^6+3a^5b+3a^4b^2+a^3b^3} + \frac{6a^2b^2+9ab^3+3b^4-2\left(3a^2b^2+2ab^3\right)\sin(fx+e)^2}{a^7+4a^6b+6a^5b^2+4a^4b^3+a^3b^4+\left(a^7+2a^6b+a^5b^2\right)\sin(fx+e)^4-2\left(a^7+3a^6b+3a^5b^2+a^4b^3\right)\sin}}{4f}$$

input
```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
1/4*(2*(3*a^2*b + 3*a*b^2 + b^3)*log(a*sin(f*x + e)^2 - a - b)/(a^6 + 3*a^
5*b + 3*a^4*b^2 + a^3*b^3) + (6*a^2*b^2 + 9*a*b^3 + 3*b^4 - 2*(3*a^2*b^2 +
 2*a*b^3)*sin(f*x + e)^2)/(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4
 + (a^7 + 2*a^6*b + a^5*b^2)*sin(f*x + e)^4 - 2*(a^7 + 3*a^6*b + 3*a^5*b^2
 + a^4*b^3)*sin(f*x + e)^2) + 2*log(sin(f*x + e)^2)/(a^3 + 3*a^2*b + 3*a*b
^2 + b^3))/f
```

3.366.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 767 vs. $2(124) = 248$.

Time = 0.42 (sec) , antiderivative size = 767, normalized size of antiderivative = 5.90

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
1/4*(2*(3*a^2*b + 3*a*b^2 + b^3)*log(a + b + 2*a*(cos(f*x + e) - 1)/(cos(f
*x + e) + 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x + e)
 - 1)^2/(cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2
)/(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3) + 2*log(abs(-cos(f*x + e) + 1)/abs
(cos(f*x + e) + 1))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (9*a^3*b + 18*a^2*b^
2 + 12*a*b^3 + 3*b^4 + 36*a^3*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 24
*a^2*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 16*a*b^3*(cos(f*x + e) -
1)/(cos(f*x + e) + 1) - 12*b^4*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 54*
a^3*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 12*a^2*b^2*(cos(f*x + e)
 - 1)^2/(cos(f*x + e) + 1)^2 + 8*a*b^3*(cos(f*x + e) - 1)^2/(cos(f*x + e)
+ 1)^2 + 18*b^4*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 36*a^3*b*(cos(
f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 24*a^2*b^2*(cos(f*x + e) - 1)^3/(co
s(f*x + e) + 1)^3 - 16*a*b^3*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 1
2*b^4*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 9*a^3*b*(cos(f*x + e) -
1)^4/(cos(f*x + e) + 1)^4 + 18*a^2*b^2*(cos(f*x + e) - 1)^4/(cos(f*x + e)
+ 1)^4 + 12*a*b^3*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 3*b^4*(cos(f
*x + e) - 1)^4/(cos(f*x + e) + 1)^4)/((a^5 + 2*a^4*b + a^3*b^2)*(a + b + 2
*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x
 + e) + 1) + a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + b*(cos(f*x + e)
 - 1)^2/(cos(f*x + e) + 1)^2)^2) - 4*log(abs(-(cos(f*x + e) - 1)/(cos(f...
```

3.366.9 Mupad [B] (verification not implemented)

Time = 20.77 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.46

$$\begin{aligned}
&\int \frac{\cot(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx \\
&= \frac{\ln\left(\tan(e+f\,x)\right)}{f\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)} \\
&\quad - \frac{\frac{2\,b^2+5\,a\,b}{4\,a^2\,(a+b)} + \frac{b\tan(e+f\,x)^2\,\left(b^2+2\,a\,b\right)}{2\,a^2\,(a+b)^2}}{f\,\left(2\,a\,b+a^2+b^2+\tan\left(e+f\,x\right)^2\,\left(2\,b^2+2\,a\,b\right)+b^2\tan\left(e+f\,x\right)^4\right)} \\
&\quad - \frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,a^3\,f} + \frac{b\,\ln\left(b\tan(e+f\,x)^2+a+b\right)\,\left(3\,a^2+3\,a\,b+b^2\right)}{2\,a^3\,f\,(a+b)^3}
\end{aligned}$$

input
```
int(cot(e + f*x)/(a + b/cos(e + f*x)^2)^3,x)
```

output
```
log(tan(e + f*x))/(f*(3*a*b^2 + 3*a^2*b + a^3 + b^3)) - ((5*a*b + 2*b^2)/(
4*a^2*(a + b)) + (b*tan(e + f*x)^2*(2*a*b + b^2))/(2*a^2*(a + b)^2))/(f*(2
*a*b + a^2 + b^2 + tan(e + f*x)^2*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^4)) -
 log(tan(e + f*x)^2 + 1)/(2*a^3*f) + (b*log(a + b + b*tan(e + f*x)^2)*(3*a
*b + 3*a^2 + b^2))/(2*a^3*f*(a + b)^3)
```

3.367 $\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

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3.367.2 Mathematica [A] (verified) . . . 2660
3.367.3 Rubi [A] (verified) . . . 2661
3.367.4 Maple [A] (verified) . . . 2663
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3.367.1 Optimal result

Integrand size = 23, antiderivative size = 154

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{b^4}{4a^3(a+b)^2 f\left(b+a\cos^2(e+fx)\right)^2} - \frac{b^3(2a+b)}{a^3(a+b)^3 f\left(b+a\cos^2(e+fx)\right)} - \frac{\csc^2(e+fx)}{2(a+b)^3 f} - \frac{b^2(6a^2+4ab+b^2)\log\left(b+a\cos^2(e+fx)\right)}{2a^3(a+b)^4 f} - \frac{(a+4b)\log(\sin(e+fx))}{(a+b)^4 f}$$

output

```
1/4*b^4/a^3/(a+b)^2/f/(b+a*cos(f*x+e)^2)^2-b^3*(2*a+b)/a^3/(a+b)^3/f/(b+a*
cos(f*x+e)^2)-1/2*csc(f*x+e)^2/(a+b)^3/f-1/2*b^2*(6*a^2+4*a*b+b^2)*ln(b+a*
cos(f*x+e)^2)/a^3/(a+b)^4/f-(a+4*b)*ln(sin(f*x+e))/(a+b)^4/f
```

3.367.2 Mathematica [A] (verified)

Time = 1.98 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.14

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{(a+2b+a\cos(2(e+fx)))^3\sec^6(e+fx)\left(2(a+b)\csc^2(e+fx)+4(a+4b)\log(\sin(e+fx))+\frac{2b^2(6a^2}{32(a+b)^4 f\left(a+b\sec^2(e+fx)\right)^3}$$

input
```
Integrate[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
-1/32*((a + 2*b + a*Cos[2*(e + f*x)])^3*Sec[e + f*x]^6*(2*(a + b)*Csc[e +
f*x]^2 + 4*(a + 4*b)*Log[Sin[e + f*x]] + (2*b^2*(6*a^2 + 4*a*b + b^2)*Log[
a + b - a*Sin[e + f*x]^2])/a^3 - (b^4*(a + b)^2)/(a^3*(a + b - a*Sin[e + f
*x]^2)^2) + (4*b^3*(a + b)*(2*a + b))/(a^3*(a + b - a*Sin[e + f*x]^2))))/(
(a + b)^4*f*(a + b*Sec[e + f*x]^2)^3)
```

3.367.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4626, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\tan(e+fx)^3\,(a+b\sec(e+fx)^2)^3}\,dx$$

$$\downarrow 4626$$

$$-\frac{\int \frac{\cos^9(e+fx)}{(1-\cos^2(e+fx))^2(a\cos^2(e+fx)+b)^3}\,d\cos(e+fx)}{f}$$

$$\downarrow 354$$

$$-\frac{\int \frac{\cos^8(e+fx)}{(1-\cos^2(e+fx))^2(a\cos^2(e+fx)+b)^3}\,d\cos^2(e+fx)}{2f}$$

$$\downarrow 99$$

$$-\frac{\int \left(\frac{b^4}{a^2(a+b)^2(a\cos^2(e+fx)+b)^3}-\frac{2(2a+b)b^3}{a^2(a+b)^3(a\cos^2(e+fx)+b)^2}+\frac{(6a^2+4ba+b^2)b^2}{a^2(a+b)^4(a\cos^2(e+fx)+b)}+\frac{a+4b}{(a+b)^4(\cos^2(e+fx)-1)}+\frac{1}{(a+b)^3(\cos^2(}\right.}{2f}$$

$$\downarrow 2009$$

$$-\frac{-\frac{b^4}{2a^3(a+b)^2(a\cos^2(e+fx)+b)^2}+\frac{2b^3(2a+b)}{a^3(a+b)^3(a\cos^2(e+fx)+b)}+\frac{b^2\left(6a^2+4ab+b^2\right)\log\left(a\cos^2(e+fx)+b\right)}{a^3(a+b)^4}+\frac{1}{(a+b)^3(1-\cos^2(e+fx))}+\frac{(a+}{2f}$$

input
```
Int[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
-1/2*(1/((a + b)^3*(1 - Cos[e + f*x]^2)) - b^4/(2*a^3*(a + b)^2*(b + a*Cos
[e + f*x]^2)^2) + (2*b^3*(2*a + b))/(a^3*(a + b)^3*(b + a*Cos[e + f*x]^2))
 + ((a + 4*b)*Log[1 - Cos[e + f*x]^2])/(a + b)^4 + (b^2*(6*a^2 + 4*a*b + b
^2)*Log[b + a*Cos[e + f*x]^2])/(a^3*(a + b)^4))/f
```

3.367.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.367.4 Maple [A] (verified)

Time = 30.71 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.25

method	result
derivativedivides	$-\frac{1}{4(a+b)^3(1+\cos(fx+e))}+\frac{(-a-4b)\ln(1+\cos(fx+e))}{2(a+b)^4}-\frac{b^2\left(\frac{2b\left(2a^2+3ab+b^2\right)}{a^3\left(b+a\cos(fx+e)^2\right)}+\frac{\left(6a^2+4ab+b^2\right)\ln\left(b+a\cos(fx+e)^2\right)}{a^3}-\frac{b^2\left(a^2\right.}{2a^3\left(b+a\right.}\right.}{2(a+b)^4}}{f}$
default	$-\frac{1}{4(a+b)^3(1+\cos(fx+e))}+\frac{(-a-4b)\ln(1+\cos(fx+e))}{2(a+b)^4}-\frac{b^2\left(\frac{2b\left(2a^2+3ab+b^2\right)}{a^3\left(b+a\cos(fx+e)^2\right)}+\frac{\left(6a^2+4ab+b^2\right)\ln\left(b+a\cos(fx+e)^2\right)}{a^3}-\frac{b^2\left(a^2\right.}{2a^3\left(b+a\right.}\right.}{2(a+b)^4}}{f}$
risch	Expression too large to display

input
```
int(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/4/(a+b)^3/(1+cos(f*x+e))+1/2*(-a-4*b)/(a+b)^4*ln(1+cos(f*x+e))-1/2
*b^2/(a+b)^4*(2/a^3*b*(2*a^2+3*a*b+b^2)/(b+a*cos(f*x+e)^2)+(6*a^2+4*a*b+b^
2)/a^3*ln(b+a*cos(f*x+e)^2)-1/2*b^2*(a^2+2*a*b+b^2)/a^3/(b+a*cos(f*x+e)^2)
^2)+1/4/(a+b)^3/(-1+cos(f*x+e))+1/2*(-a-4*b)/(a+b)^4*ln(-1+cos(f*x+e)))
```

3.367.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 584 vs. $2(148) = 296$.

Time = 0.93 (sec) , antiderivative size = 584, normalized size of antiderivative = 3.79

$$\int \frac{\cot^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$
$$= \frac{2\,a^4b^2+2\,a^3b^3+7\,a^2b^4+10\,ab^5+3\,b^6+2\left(a^6+a^5b-4\,a^3b^3-6\,a^2b^4-2\,ab^5\right)\cos\left(fx+e\right)^4+\left(4\,a^5b+4\,a\right.}{}$$

input
```
integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
1/4*(2*a^4*b^2 + 2*a^3*b^3 + 7*a^2*b^4 + 10*a*b^5 + 3*b^6 + 2*(a^6 + a^5*b
 - 4*a^3*b^3 - 6*a^2*b^4 - 2*a*b^5)*cos(f*x + e)^4 + (4*a^5*b + 4*a^4*b^2
+ 8*a^3*b^3 + 5*a^2*b^4 - 6*a*b^5 - 3*b^6)*cos(f*x + e)^2 - 2*((6*a^4*b^2
+ 4*a^3*b^3 + a^2*b^4)*cos(f*x + e)^6 - 6*a^2*b^4 - 4*a*b^5 - b^6 - (6*a^4
*b^2 - 8*a^3*b^3 - 7*a^2*b^4 - 2*a*b^5)*cos(f*x + e)^4 - (12*a^3*b^3 + 2*a
^2*b^4 - 2*a*b^5 - b^6)*cos(f*x + e)^2)*log(a*cos(f*x + e)^2 + b) - 4*((a^
6 + 4*a^5*b)*cos(f*x + e)^6 - a^4*b^2 - 4*a^3*b^3 - (a^6 + 2*a^5*b - 8*a^4
*b^2)*cos(f*x + e)^4 - (2*a^5*b + 7*a^4*b^2 - 4*a^3*b^3)*cos(f*x + e)^2)*l
og(1/2*sin(f*x + e)))/((a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5*b^4)*f
*cos(f*x + e)^6 - (a^9 + 2*a^8*b - 2*a^7*b^2 - 8*a^6*b^3 - 7*a^5*b^4 - 2*a
^4*b^5)*f*cos(f*x + e)^4 - (2*a^8*b + 7*a^7*b^2 + 8*a^6*b^3 + 2*a^5*b^4 -
2*a^4*b^5 - a^3*b^6)*f*cos(f*x + e)^2 - (a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 +
 4*a^4*b^5 + a^3*b^6)*f)
```

3.367.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)**3/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Timed out
```

3.367.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 344 vs. $2(148) = 296$.

Time $= 0.21$ (sec) , antiderivative size $= 344$, normalized size of antiderivative $= 2.23$

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{\frac{2\,(6\,a^2b^2+4\,ab^3+b^4)\log\left(a\sin(fx+e)^2-a-b\right)}{a^7+4\,a^6b+6\,a^5b^2+4\,a^4b^3+a^3b^4}+\frac{2\,(a+4\,b)\log\left(\sin(fx+e)^2\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}+\frac{2\,a^5+4\,a^4b+2\,a^3b^2+2\,(a^5-4\,a^2b^3-2}{(a^8+3\,a^7b+3\,a^6b^2+a^5b^3)\sin(fx+e)^6-2\,(a^8+4\,a^7b+6\,a^6b}}{4\,f}$$

input
```
integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
-1/4*(2*(6*a^2*b^2 + 4*a*b^3 + b^4)*log(a*sin(f*x + e)^2 - a - b)/(a^7 + 4
*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4) + 2*(a + 4*b)*log(sin(f*x + e)^2
)/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + (2*a^5 + 4*a^4*b + 2*a^3*b
^2 + 2*(a^5 - 4*a^2*b^3 - 2*a*b^4)*sin(f*x + e)^4 - (4*a^5 + 4*a^4*b - 8*a
^2*b^3 - 11*a*b^4 - 3*b^5)*sin(f*x + e)^2)/((a^8 + 3*a^7*b + 3*a^6*b^2 + a
^5*b^3)*sin(f*x + e)^6 - 2*(a^8 + 4*a^7*b + 6*a^6*b^2 + 4*a^5*b^3 + a^4*b^
4)*sin(f*x + e)^4 + (a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 +
 a^3*b^5)*sin(f*x + e)^2))/f
```

3.367.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1069 vs. $2(148) = 296$.

Time $= 0.56$ (sec) , antiderivative size $= 1069$, normalized size of antiderivative $= 6.94$

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
-1/8*(4*(6*a^2*b^2 + 4*a*b^3 + b^4)*log(a + b + 2*a*(cos(f*x + e) - 1)/(co
s(f*x + e) + 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x +
 e) - 1)^2/(cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1
)^2)/(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4) + 4*(a + 4*b)*log(a
bs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 +
4*a*b^3 + b^4) - (a + b + 4*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 16*b
*(cos(f*x + e) - 1)/(cos(f*x + e) + 1))*(cos(f*x + e) + 1)/((a^4 + 4*a^3*b
 + 6*a^2*b^2 + 4*a*b^3 + b^4)*(cos(f*x + e) - 1)) - (cos(f*x + e) - 1)/((a
^3 + 3*a^2*b + 3*a*b^2 + b^3)*(cos(f*x + e) + 1)) - 2*(18*a^4*b^2 + 48*a^3
*b^3 + 45*a^2*b^4 + 18*a*b^5 + 3*b^6 + 72*a^4*b^2*(cos(f*x + e) - 1)/(cos(
f*x + e) + 1) + 80*a^3*b^3*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 20*a^2*
b^4*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 40*a*b^5*(cos(f*x + e) - 1)/(c
os(f*x + e) + 1) - 12*b^6*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 108*a^4*
b^2*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 64*a^3*b^3*(cos(f*x + e) -
 1)^2/(cos(f*x + e) + 1)^2 + 46*a^2*b^4*(cos(f*x + e) - 1)^2/(cos(f*x + e)
 + 1)^2 + 44*a*b^5*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 18*b^6*(cos
(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 72*a^4*b^2*(cos(f*x + e) - 1)^3/(c
os(f*x + e) + 1)^3 + 80*a^3*b^3*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3
- 20*a^2*b^4*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 - 40*a*b^5*(cos(f*x
 + e) - 1)^3/(cos(f*x + e) + 1)^3 - 12*b^6*(cos(f*x + e) - 1)^3/(cos(f*...
```

3.367.9 Mupad [B] (verification not implemented)

Time = 21.19 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.77

$$\int \frac{\cot^3(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$

$$= \frac{\frac{\tan(e+fx)^2\left(-4a^2b+7ab^2+2b^3\right)}{4a^2\left(a^2+2ab+b^2\right)} - \frac{1}{2(a+b)} + \frac{\tan(e+fx)^4\left(-a^2b^2+3ab^3+b^4\right)}{2a^2(a+b)\left(a^2+2ab+b^2\right)}}{f\left(\tan(e+fx)^2\left(a^2+2ab+b^2\right)+\tan(e+fx)^4\left(2b^2+2ab\right)+b^2\tan(e+fx)^6\right)}$$
$$+ \frac{\ln\left(\tan(e+fx)^2+1\right)}{2a^3f} - \frac{\ln\left(\tan(e+fx)\right)(a+4b)}{f\left(a^4+4a^3b+6a^2b^2+4ab^3+b^4\right)}$$
$$- \frac{b^2\ln\left(b\tan(e+fx)^2+a+b\right)\left(6a^2+4ab+b^2\right)}{2a^3f(a+b)^4}$$

input
```
int(cot(e + f*x)^3/(a + b/cos(e + f*x)^2)^3,x)
```

output
```
((tan(e + f*x)^2*(7*a*b^2 - 4*a^2*b + 2*b^3))/(4*a^2*(2*a*b + a^2 + b^2))
- 1/(2*(a + b)) + (tan(e + f*x)^4*(3*a*b^3 + b^4 - a^2*b^2))/(2*a^2*(a + b
)*(2*a*b + a^2 + b^2)))/(f*(tan(e + f*x)^2*(2*a*b + a^2 + b^2) + tan(e + f
*x)^4*(2*a*b + 2*b^2) + b^2*tan(e + f*x)^6)) + log(tan(e + f*x)^2 + 1)/(2*
a^3*f) - (log(tan(e + f*x))*(a + 4*b))/(f*(4*a*b^3 + 4*a^3*b + a^4 + b^4 +
 6*a^2*b^2)) - (b^2*log(a + b + b*tan(e + f*x)^2)*(4*a*b + 6*a^2 + b^2))/(
2*a^3*f*(a + b)^4)
```

3.368 $\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

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3.368.1 Optimal result

Integrand size = 23, antiderivative size = 192

$$\begin{aligned}\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx &= -\frac{b^5}{4a^3(a+b)^3 f\left(b+a\cos^2(e+fx)\right)^2}\\ &+\frac{b^4(5a+2b)}{2a^3(a+b)^4 f\left(b+a\cos^2(e+fx)\right)}+\frac{(2a+5b)\csc^2(e+fx)}{2(a+b)^4 f}\\ &-\frac{\csc^4(e+fx)}{4(a+b)^3 f}+\frac{b^3(10a^2+5ab+b^2)\log\left(b+a\cos^2(e+fx)\right)}{2a^3(a+b)^5 f}\\ &+\frac{(a^2+5ab+10b^2)\log(\sin(e+fx))}{(a+b)^5 f}\end{aligned}$$

output

```
-1/4*b^5/a^3/(a+b)^3/f/(b+a*cos(f*x+e)^2)^2+1/2*b^4*(5*a+2*b)/a^3/(a+b)^4/
f/(b+a*cos(f*x+e)^2)+1/2*(2*a+5*b)*csc(f*x+e)^2/(a+b)^4/f-1/4*csc(f*x+e)^4
/(a+b)^3/f+1/2*b^3*(10*a^2+5*a*b+b^2)*ln(b+a*cos(f*x+e)^2)/a^3/(a+b)^5/f+(
a^2+5*a*b+10*b^2)*ln(sin(f*x+e))/(a+b)^5/f
```

3.368.2 Mathematica [A] (verified)

Time = 5.53 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.08

$$\int \frac{\cot^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$
$$= \frac{(a+2b+a\cos(2(e+fx)))^3\sec^6(e+fx)\left(2(a+b)(2a+5b)\csc^2(e+fx)-(a+b)^2\csc^4(e+fx)+4(a^2\right.}{32(a+b)^5 f\,(a}$$

input
```
Integrate[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])^3*Sec[e + f*x]^6*(2*(a + b)*(2*a + 5*b)*Cs
c[e + f*x]^2 - (a + b)^2*Csc[e + f*x]^4 + 4*(a^2 + 5*a*b + 10*b^2)*Log[Sin
[e + f*x]] + (2*b^3*(10*a^2 + 5*a*b + b^2)*Log[a + b - a*Sin[e + f*x]^2])/
a^3 - (b^5*(a + b)^2)/(a^3*(a + b - a*Sin[e + f*x]^2)^2) + (2*b^4*(a + b)*
(5*a + 2*b))/(a^3*(a + b - a*Sin[e + f*x]^2))))/(32*(a + b)^5*f*(a + b*Sec
[e + f*x]^2)^3)
```

3.368.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4626, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{\tan(e+fx)^5\left(a+b\sec(e+fx)^2\right)^3}dx$$
$$\downarrow 4626$$
$$-\frac{\int \frac{\cos^{11}(e+fx)}{(1-\cos^2(e+fx))^3(a\cos^2(e+fx)+b)^3}d\cos(e+fx)}{f}$$
$$\downarrow 354$$

$$-\frac{\int \frac{\cos^{10}(e+fx)}{(1-\cos^2(e+fx))^3(a\cos^2(e+fx)+b)^3}d\cos^2(e+fx)}{2f}$$

$$\downarrow 99$$

$$-\frac{\int\left(-\frac{b^5}{a^2(a+b)^3(a\cos^2(e+fx)+b)^3}+\frac{(5a+2b)b^4}{a^2(a+b)^4(a\cos^2(e+fx)+b)^2}-\frac{(10a^2+5ba+b^2)b^3}{a^2(a+b)^5(a\cos^2(e+fx)+b)}+\frac{-a^2-5ba-10b^2}{(a+b)^5(\cos^2(e+fx)-1)}+\frac{-2}{(a+b)^4(\cos}\right.}{2f}$$

$$\downarrow 2009$$

$$-\frac{\frac{b^5}{2a^3(a+b)^3(a\cos^2(e+fx)+b)^2}-\frac{b^4(5a+2b)}{a^3(a+b)^4(a\cos^2(e+fx)+b)}-\frac{(a^2+5ab+10b^2)\log(1-\cos^2(e+fx))}{(a+b)^5}-\frac{b^3(10a^2+5ab+b^2)\log(a\cos^2(e+fx)}{a^3(a+b)^5}}{2f}$$

input
```
Int[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
-1/2*(1/(2*(a + b)^3*(1 - Cos[e + f*x]^2)^2) - (2*a + 5*b)/((a + b)^4*(1 -
 Cos[e + f*x]^2)) + b^5/(2*a^3*(a + b)^3*(b + a*Cos[e + f*x]^2)^2) - (b^4*
(5*a + 2*b))/(a^3*(a + b)^4*(b + a*Cos[e + f*x]^2)) - ((a^2 + 5*a*b + 10*b
^2)*Log[1 - Cos[e + f*x]^2])/(a + b)^5 - (b^3*(10*a^2 + 5*a*b + b^2)*Log[b
 + a*Cos[e + f*x]^2])/(a^3*(a + b)^5))/f
```

3.368.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.368.4 Maple [A] (verified)

Time = 53.50 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.32

method	result
derivativedivides	$-\frac{1}{16(a+b)^3(-1+\cos(fx+e))^2}-\frac{7a+19b}{16(a+b)^4(-1+\cos(fx+e))}+\frac{\left(a^2+5ab+10b^2\right)\ln(-1+\cos(fx+e))}{2(a+b)^5}-\frac{1}{16(a+b)^3(1+\cos(fx+e))^2}-\frac{}{16(a}$
default	$-\frac{1}{16(a+b)^3(-1+\cos(fx+e))^2}-\frac{7a+19b}{16(a+b)^4(-1+\cos(fx+e))}+\frac{\left(a^2+5ab+10b^2\right)\ln(-1+\cos(fx+e))}{2(a+b)^5}-\frac{1}{16(a+b)^3(1+\cos(fx+e))^2}-\frac{}{16(a}$
risch	Expression too large to display

input
```
int(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/16/(a+b)^3/(-1+cos(f*x+e))^2-1/16*(7*a+19*b)/(a+b)^4/(-1+cos(f*x+e
))+1/2*(a^2+5*a*b+10*b^2)/(a+b)^5*ln(-1+cos(f*x+e))-1/16/(a+b)^3/(1+cos(f*
x+e))^2-1/16*(-7*a-19*b)/(a+b)^4/(1+cos(f*x+e))+1/2*(a^2+5*a*b+10*b^2)/(a+
b)^5*ln(1+cos(f*x+e))+1/2*b^3/(a+b)^5*((10*a^2+5*a*b+b^2)/a^3*ln(b+a*cos(f
*x+e)^2)-1/2*b^2*(a^2+2*a*b+b^2)/a^3/(b+a*cos(f*x+e)^2)^2+1/a^3*b*(5*a^2+7
*a*b+2*b^2)/(b+a*cos(f*x+e)^2)))
```

3.368.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 859 vs. $2(182) = 364$.

Time = 1.68 (sec) , antiderivative size = 859, normalized size of antiderivative = 4.47

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \frac{3\,a^5b^2 + 12\,a^4b^3 + 9\,a^3b^4 + 9\,a^2b^5 + 12\,ab^6 + 3\,b^7 - 2\,(2\,a^7 + 7\,a^6b + 5\,a^5b^2 - 5\,a^3b^4 - 7\,a^2b^5 - 2\,ab^6)\cos(}{}$$

input
```
integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
1/4*(3*a^5*b^2 + 12*a^4*b^3 + 9*a^3*b^4 + 9*a^2*b^5 + 12*a*b^6 + 3*b^7 - 2
*(2*a^7 + 7*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 7*a^2*b^5 - 2*a*b^6)*cos(f*x +
 e)^6 + (3*a^7 + 4*a^6*b - 19*a^5*b^2 - 20*a^4*b^3 - 20*a^3*b^4 - 19*a^2*b
^5 + 4*a*b^6 + 3*b^7)*cos(f*x + e)^4 + 2*(3*a^6*b + 10*a^5*b^2 + 2*a^4*b^3
 - 2*a^2*b^5 - 10*a*b^6 - 3*b^7)*cos(f*x + e)^2 + 2*((10*a^4*b^3 + 5*a^3*b
^4 + a^2*b^5)*cos(f*x + e)^8 + 10*a^2*b^5 + 5*a*b^6 + b^7 - 2*(10*a^4*b^3
 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*cos(f*x + e)^6 + (10*a^4*b^3 - 35*a^3*b^4
 - 9*a^2*b^5 + a*b^6 + b^7)*cos(f*x + e)^4 + 2*(10*a^3*b^4 - 5*a^2*b^5 - 4
*a*b^6 - b^7)*cos(f*x + e)^2)*log(a*cos(f*x + e)^2 + b) + 4*((a^7 + 5*a^6*
b + 10*a^5*b^2)*cos(f*x + e)^8 + a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 - 2*(a^7
 + 4*a^6*b + 5*a^5*b^2 - 10*a^4*b^3)*cos(f*x + e)^6 + (a^7 + a^6*b - 9*a^5
*b^2 - 35*a^4*b^3 + 10*a^3*b^4)*cos(f*x + e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5*
a^4*b^3 - 10*a^3*b^4)*cos(f*x + e)^2)*log(1/2*sin(f*x + e)))/((a^10 + 5*a^
9*b + 10*a^8*b^2 + 10*a^7*b^3 + 5*a^6*b^4 + a^5*b^5)*f*cos(f*x + e)^8 - 2*
(a^10 + 4*a^9*b + 5*a^8*b^2 - 5*a^6*b^4 - 4*a^5*b^5 - a^4*b^6)*f*cos(f*x +
 e)^6 + (a^10 + a^9*b - 9*a^8*b^2 - 25*a^7*b^3 - 25*a^6*b^4 - 9*a^5*b^5 +
a^4*b^6 + a^3*b^7)*f*cos(f*x + e)^4 + 2*(a^9*b + 4*a^8*b^2 + 5*a^7*b^3 - 5
*a^5*b^5 - 4*a^4*b^6 - a^3*b^7)*f*cos(f*x + e)^2 + (a^8*b^2 + 5*a^7*b^3 +
10*a^6*b^4 + 10*a^5*b^5 + 5*a^4*b^6 + a^3*b^7)*f)
```

3.368.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)**5/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Timed out
```

3.368.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 454 vs. 2(182) = 364.

Time = 0.23 (sec) , antiderivative size = 454, normalized size of antiderivative = 2.36

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \frac{2\,(10\,a^2b^3+5\,ab^4+b^5)\log\left(a\sin(fx+e)^2-a-b\right)}{a^8+5\,a^7b+10\,a^6b^2+10\,a^5b^3+5\,a^4b^4+a^3b^5} + \frac{2\,(a^2+5\,ab+10\,b^2)\log\left(\sin(fx+e)^2\right)}{a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3+5\,ab^4+b^5} + \frac{2\,(2\,a^6+5\,a^5b-5\,a^2b^4-2\,ab^5)\sin(fx+e)^6-a^6-3\,a}{(a^9+4\,a^8b+6\,a^7b^2+4\,a^6b^3+a^5b^4)\sin(fx+e)^8-2}$$
$$4\,f$$

input
```
integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
1/4*(2*(10*a^2*b^3 + 5*a*b^4 + b^5)*log(a*sin(f*x + e)^2 - a - b)/(a^8 + 5
*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5) + 2*(a^2 + 5*a*b +
 10*b^2)*log(sin(f*x + e)^2)/(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*
a*b^4 + b^5) + (2*(2*a^6 + 5*a^5*b - 5*a^2*b^4 - 2*a*b^5)*sin(f*x + e)^6 -
 a^6 - 3*a^5*b - 3*a^4*b^2 - a^3*b^3 - (9*a^6 + 29*a^5*b + 20*a^4*b^2 - 10
*a^2*b^4 - 13*a*b^5 - 3*b^6)*sin(f*x + e)^4 + 2*(3*a^6 + 11*a^5*b + 13*a^4
*b^2 + 5*a^3*b^3)*sin(f*x + e)^2)/((a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3
+ a^5*b^4)*sin(f*x + e)^8 - 2*(a^9 + 5*a^8*b + 10*a^7*b^2 + 10*a^6*b^3 + 5
*a^5*b^4 + a^4*b^5)*sin(f*x + e)^6 + (a^9 + 6*a^8*b + 15*a^7*b^2 + 20*a^6*
b^3 + 15*a^5*b^4 + 6*a^4*b^5 + a^3*b^6)*sin(f*x + e)^4))/f
```

3.368.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1912 vs. 2(182) = 364.

Time = 0.53 (sec) , antiderivative size = 1912, normalized size of antiderivative = 9.96

$$\int \frac{\cot^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
1/64*(32*(10*a^2*b^3 + 5*a*b^4 + b^5)*log(a + b + 2*a*(cos(f*x + e) - 1)/(
cos(f*x + e) + 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + a*(cos(f*x
 + e) - 1)^2/(cos(f*x + e) + 1)^2 + b*(cos(f*x + e) - 1)^2/(cos(f*x + e) +
 1)^2)/(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5) + 3
2*(a^2 + 5*a*b + 10*b^2)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1))
/(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5) - (12*a^3*(cos(
f*x + e) - 1)/(cos(f*x + e) + 1) + 60*a^2*b*(cos(f*x + e) - 1)/(cos(f*x +
e) + 1) + 84*a*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 36*b^3*(cos(f*x
 + e) - 1)/(cos(f*x + e) + 1) + a^3*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1
)^2 + 3*a^2*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 3*a*b^2*(cos(f*x
 + e) - 1)^2/(cos(f*x + e) + 1)^2 + b^3*(cos(f*x + e) - 1)^2/(cos(f*x + e)
 + 1)^2)/(a^6 + 6*a^5*b + 15*a^4*b^2 + 20*a^3*b^3 + 15*a^2*b^4 + 6*a*b^5 +
 b^6) - (a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4 + 16*a^7*(cos(f*x
 + e) - 1)/(cos(f*x + e) + 1) + 80*a^6*b*(cos(f*x + e) - 1)/(cos(f*x + e)
+ 1) + 144*a^5*b^2*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 112*a^4*b^3*(co
s(f*x + e) - 1)/(cos(f*x + e) + 1) + 32*a^3*b^4*(cos(f*x + e) - 1)/(cos(f*
x + e) + 1) + 70*a^7*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 312*a^6*b
*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 436*a^5*b^2*(cos(f*x + e) - 1
)^2/(cos(f*x + e) + 1)^2 + 536*a^4*b^3*(cos(f*x + e) - 1)^2/(cos(f*x + e)
+ 1)^2 + 822*a^3*b^4*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 672*a^...
```

3.368.9 Mupad [B] (verification not implemented)

Time = 22.49 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.70

$$\int \frac{\cot^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \frac{\ln\left(\tan(e+f\,x)\right)\,\left(a^2+5\,a\,b+10\,b^2\right)}{f\,\left(a^5+5\,a^4\,b+10\,a^3\,b^2+10\,a^2\,b^3+5\,a\,b^4+b^5\right)}$$
$$-\frac{\frac{1}{4\,(a+b)}-\frac{\tan(e+f\,x)^2\,(a+3\,b)}{2\,(a+b)^2}+\frac{\tan(e+f\,x)^4\,\left(-4\,a^3\,b-15\,a^2\,b^2+9\,a\,b^3+2\,b^4\right)}{4\,a^2\,(a+b)\,\left(a^2+2\,a\,b+b^2\right)}+\frac{\tan(e+f\,x)^6\,\left(-a^3\,b^2-4\,a^2\,b^3+4\,a\,b^4+b^5\right)}{2\,a^2\,(a+b)^2\,\left(a^2+2\,a\,b+b^2\right)}}{f\,\left(\tan\left(e+f\,x\right)^4\,\left(a^2+2\,a\,b+b^2\right)+\tan\left(e+f\,x\right)^6\,\left(2\,b^2+2\,a\,b\right)+b^2\,\tan\left(e+f\,x\right)^8\right)}$$
$$-\frac{\ln\left(\tan(e+f\,x)^2+1\right)}{2\,a^3\,f}+\frac{b^3\,\ln\left(b\,\tan(e+f\,x)^2+a+b\right)\,\left(10\,a^2+5\,a\,b+b^2\right)}{2\,a^3\,f\,(a+b)^5}$$

input
```
int(cot(e + f*x)^5/(a + b/cos(e + f*x)^2)^3,x)
```

output
```
(log(tan(e + f*x))*(5*a*b + a^2 + 10*b^2))/(f*(5*a*b^4 + 5*a^4*b + a^5 + b
^5 + 10*a^2*b^3 + 10*a^3*b^2)) - (1/(4*(a + b)) - (tan(e + f*x)^2*(a + 3*b
))/(2*(a + b)^2) + (tan(e + f*x)^4*(9*a*b^3 - 4*a^3*b + 2*b^4 - 15*a^2*b^2
))/(4*a^2*(a + b)*(2*a*b + a^2 + b^2)) + (tan(e + f*x)^6*(4*a*b^4 + b^5 -
4*a^2*b^3 - a^3*b^2))/(2*a^2*(a + b)^2*(2*a*b + a^2 + b^2)))/(f*(tan(e + f
*x)^4*(2*a*b + a^2 + b^2) + tan(e + f*x)^6*(2*a*b + 2*b^2) + b^2*tan(e + f
*x)^8)) - log(tan(e + f*x)^2 + 1)/(2*a^3*f) + (b^3*log(a + b + b*tan(e + f
*x)^2)*(5*a*b + 10*a^2 + b^2))/(2*a^3*f*(a + b)^5)
```

3.369 $\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

3.369.1 Optimal result 2675
3.369.2 Mathematica [C] (warning: unable to verify) 2675
3.369.3 Rubi [A] (verified) 2677
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3.369.8 Giac [A] (verification not implemented) 2682
3.369.9 Mupad [B] (verification not implemented) 2683

3.369.1 Optimal result

Integrand size = 23, antiderivative size = 147

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{x}{a^3} + \frac{\sqrt{a+b}(3a^2-4ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3b^{5/2}f} - \frac{(a+b)\tan^3(e+fx)}{4abf\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{(3a-4b)(a+b)\tan(e+fx)}{8a^2b^2f\left(a+b+b\tan^2(e+fx)\right)}$$

output

```
-x/a^3+1/8*(3*a^2-4*a*b+8*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*(a+b
)^(1/2)/a^3/b^(5/2)/f-1/4*(a+b)*tan(f*x+e)^3/a/b/f/(a+b+b*tan(f*x+e)^2)^2-
1/8*(3*a-4*b)*(a+b)*tan(f*x+e)/a^2/b^2/f/(a+b+b*tan(f*x+e)^2)
```

3.369.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.61 (sec) , antiderivative size = 760, normalized size of antiderivative = 5.17

$$\int \frac{\tan^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$

$$= \frac{(-3a^3+a^2b-4ab^2-8b^3)\,(a+2b+a\cos(2e+2fx))^3\sec^6(e+fx)\left(\frac{\arctan\left(\sec(fx)\left(\frac{\cos(2e)}{2\sqrt{a+b}\sqrt{b\cos(4e)-ib\sin(4e)}}-\frac{}{2\sqrt{}}\right.\right.}{64a^3}$$

$$+\frac{(a+2b+a\cos(2e+2fx))\sec(2e)\sec^6(e+fx)\,(-24a^2b^2fx\cos(2e)-64ab^3fx\cos(2e)-64b^4fx\cos(2e}{}$$

input
```
Integrate[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
((-3*a^3 + a^2*b - 4*a*b^2 - 8*b^3)*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e
 + f*x]^6*((ArcTan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b
*Sin[4*e]]) - ((I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]
]))*(-(a*Sin[f*x]) - 2*b*Sin[f*x] + a*Sin[2*e + f*x])]*Cos[2*e])/(64*a^3*b
^2*Sqrt[a + b]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/64)*ArcTan[Sec[f*x
]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[2
*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*S
in[f*x] + a*Sin[2*e + f*x])]*Sin[2*e])/(a^3*b^2*Sqrt[a + b]*f*Sqrt[b*Cos[4
*e] - I*b*Sin[4*e]])))/(a + b*Sec[e + f*x]^2)^3 + ((a + 2*b + a*Cos[2*e +
2*f*x])*Sec[2*e]*Sec[e + f*x]^6*(-24*a^2*b^2*f*x*Cos[2*e] - 64*a*b^3*f*x*C
os[2*e] - 64*b^4*f*x*Cos[2*e] - 16*a^2*b^2*f*x*Cos[2*f*x] - 32*a*b^3*f*x*C
os[2*f*x] - 16*a^2*b^2*f*x*Cos[4*e + 2*f*x] - 32*a*b^3*f*x*Cos[4*e + 2*f*x
] - 4*a^2*b^2*f*x*Cos[2*e + 4*f*x] - 4*a^2*b^2*f*x*Cos[6*e + 4*f*x] + 9*a^
4*Sin[2*e] + 15*a^3*b*Sin[2*e] - 18*a^2*b^2*Sin[2*e] - 72*a*b^3*Sin[2*e] -
 48*b^4*Sin[2*e] - 9*a^4*Sin[2*f*x] - 13*a^3*b*Sin[2*f*x] + 28*a^2*b^2*Sin
[2*f*x] + 32*a*b^3*Sin[2*f*x] + 3*a^4*Sin[4*e + 2*f*x] - a^3*b*Sin[4*e + 2
*f*x] - 20*a^2*b^2*Sin[4*e + 2*f*x] - 16*a*b^3*Sin[4*e + 2*f*x] - 3*a^4*Si
n[2*e + 4*f*x] + 3*a^3*b*Sin[2*e + 4*f*x] + 6*a^2*b^2*Sin[2*e + 4*f*x]))/(
128*a^3*b^2*f*(a + b*Sec[e + f*x]^2)^3)
```

3.369.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.16, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4629, 2075, 372, 440, 25, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\tan(e+fx)^6}{(a+b\sec(e+fx)^2)^3}\,dx$$

$$\downarrow 4629$$

$$\frac{\int \frac{\tan^6(e+fx)}{(\tan^2(e+fx)+1)(a+b(\tan^2(e+fx)+1))^3}\,d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\tan^6(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^3}\,d\tan(e+fx)}{f}$$

$$\downarrow 372$$

$$\frac{\frac{\int \frac{\tan^2(e+fx)\left((3a-b)\tan^2(e+fx)+3(a+b)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}\,d\tan(e+fx)}{4ab} - \frac{(a+b)\tan^3(e+fx)}{4ab(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\downarrow 440$$

$$\frac{-\frac{\int -\frac{\left(3a^2-ba+4b^2\right)\tan^2(e+fx)+(3a-4b)(a+b)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}\,d\tan(e+fx)}{2ab} - \frac{(3a-4b)(a+b)\tan(e+fx)}{2ab\left(a+b\tan^2(e+fx)+b\right)}}{4ab} - \frac{(a+b)\tan^3(e+fx)}{4ab(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\downarrow 25$$

$$\frac{\frac{\int \frac{\left(3a^2-ba+4b^2\right)\tan^2(e+fx)+(3a-4b)(a+b)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}\,d\tan(e+fx)}{2ab} - \frac{(3a-4b)(a+b)\tan(e+fx)}{2ab\left(a+b\tan^2(e+fx)+b\right)}}{4ab} - \frac{(a+b)\tan^3(e+fx)}{4ab(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\downarrow 397$$

$$\frac{\frac{(a+b)\left(3a^2-4ab+8b^2\right)\int\frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{2ab}-\frac{8b^2\int\frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}}{4ab}-\frac{(3a-4b)(a+b)\tan(e+fx)}{2ab\left(a+b\tan^2(e+fx)+b\right)}-\frac{(a+b)\tan^3(e+fx)}{4ab(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\downarrow 216$$

$$\frac{\frac{\frac{(a+b)\left(3a^2-4ab+8b^2\right)\int\frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{2ab}-\frac{8b^2\arctan(\tan(e+fx))}{a}}{4ab}-\frac{(3a-4b)(a+b)\tan(e+fx)}{2ab\left(a+b\tan^2(e+fx)+b\right)}-\frac{(a+b)\tan^3(e+fx)}{4ab(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\downarrow 218$$

$$\frac{\frac{\frac{\sqrt{a+b}\left(3a^2-4ab+8b^2\right)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{b}}-\frac{8b^2\arctan(\tan(e+fx))}{a}}{2ab}-\frac{(3a-4b)(a+b)\tan(e+fx)}{2ab\left(a+b\tan^2(e+fx)+b\right)}}{4ab}-\frac{(a+b)\tan^3(e+fx)}{4ab(a+b\tan^2(e+fx)+b)^2}}{f}$$

input
```
Int[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
(-1/4*((a + b)*Tan[e + f*x]^3)/(a*b*(a + b + b*Tan[e + f*x]^2)^2) + (((-8*
b^2*ArcTan[Tan[e + f*x]])/a + (Sqrt[a + b]*(3*a^2 - 4*a*b + 8*b^2)*ArcTan[
(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*Sqrt[b]))/(2*a*b) - ((3*a - 4*b)*(
a + b)*Tan[e + f*x])/(2*a*b*(a + b + b*Tan[e + f*x]^2)))/(4*a*b))/f
```

3.369.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.369.4 Maple [A] (verified)

Time = 27.17 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{(a+b)\left(\frac{-\frac{a(5a-4b)\tan(fx+e)^3}{8b}-\frac{a\left(3a^2-ab-4b^2\right)\tan(fx+e)}{8b^2}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(3a^2-4ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8b^2\sqrt{(a+b)b}}\right)}{a^3}-\frac{\arctan(\tan(fx+e))}{a^3}}{f}$
default	$\frac{(a+b)\left(\frac{-\frac{a(5a-4b)\tan(fx+e)^3}{8b}-\frac{a\left(3a^2-ab-4b^2\right)\tan(fx+e)}{8b^2}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(3a^2-4ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8b^2\sqrt{(a+b)b}}\right)}{a^3}-\frac{\arctan(\tan(fx+e))}{a^3}}{f}$
risch	$-\frac{x}{a^3}+\frac{i\left(-3a^4e^{6i(fx+e)}+a^3be^{6i(fx+e)}+20a^2b^2e^{6i(fx+e)}+16ab^3e^{6i(fx+e)}-9a^4e^{4i(fx+e)}-15a^3be^{4i(fx+e)}+18a^2b^2e\right.}{4a^3b^2f\left(ae^{4i(fx}$

input
```
int(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*((a+b)/a^3*((-1/8*a*(5*a-4*b)/b*tan(f*x+e)^3-1/8*a*(3*a^2-a*b-4*b^2)/b
^2*tan(f*x+e))/(a+b+b*tan(f*x+e)^2)^2+1/8*(3*a^2-4*a*b+8*b^2)/b^2/((a+b)*b
)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))-1/a^3*arctan(tan(f*x+e)))
```

3.369.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. 2(133) = 266.

Time = 0.30 (sec) , antiderivative size = 664, normalized size of antiderivative = 4.52

$$\int \frac{\tan^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$

$$= \left[-\frac{32\,a^2b^2fx\cos(fx+e)^4+64\,ab^3fx\cos(fx+e)^2+32\,b^4fx-\left((3\,a^4-4\,a^3b+8\,a^2b^2\right)\cos(fx+e)^4+3}{} \right.$$

$$\left. -\frac{16\,a^2b^2fx\cos(fx+e)^4+32\,ab^3fx\cos(fx+e)^2+16\,b^4fx+\left((3\,a^4-4\,a^3b+8\,a^2b^2\right)\cos(fx+e)^4+3}{} \right.$$

input
```
integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[-1/32*(32*a^2*b^2*f*x*cos(f*x + e)^4 + 64*a*b^3*f*x*cos(f*x + e)^2 + 32*b
^4*f*x - ((3*a^4 - 4*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 3*a^2*b^2 - 4*a*b
^3 + 8*b^4 + 2*(3*a^3*b - 4*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*sqrt(-(a +
b)/b)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*
x + e)^2 - 4*((a*b + 2*b^2)*cos(f*x + e)^3 - b^2*cos(f*x + e))*sqrt(-(a +
b)/b)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2
)) + 4*(3*(a^4 - a^3*b - 2*a^2*b^2)*cos(f*x + e)^3 + (5*a^3*b + a^2*b^2 -
4*a*b^3)*cos(f*x + e))*sin(f*x + e))/(a^5*b^2*f*cos(f*x + e)^4 + 2*a^4*b^3
*f*cos(f*x + e)^2 + a^3*b^4*f), -1/16*(16*a^2*b^2*f*x*cos(f*x + e)^4 + 32*
a*b^3*f*x*cos(f*x + e)^2 + 16*b^4*f*x + ((3*a^4 - 4*a^3*b + 8*a^2*b^2)*cos
(f*x + e)^4 + 3*a^2*b^2 - 4*a*b^3 + 8*b^4 + 2*(3*a^3*b - 4*a^2*b^2 + 8*a*b
^3)*cos(f*x + e)^2)*sqrt((a + b)/b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 -
 b)*sqrt((a + b)/b)/((a + b)*cos(f*x + e)*sin(f*x + e))) + 2*(3*(a^4 - a^3
*b - 2*a^2*b^2)*cos(f*x + e)^3 + (5*a^3*b + a^2*b^2 - 4*a*b^3)*cos(f*x + e
))*sin(f*x + e))/(a^5*b^2*f*cos(f*x + e)^4 + 2*a^4*b^3*f*cos(f*x + e)^2 +
a^3*b^4*f)]
```

3.369.6 Sympy [F]

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

input
```
integrate(tan(f*x+e)**6/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Integral(tan(e + f*x)**6/(a + b*sec(e + f*x)**2)**3, x)
```

3.369.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.31

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx =$$
$$-\frac{\frac{\left(5a^2b+ab^2-4b^3\right)\tan(fx+e)^3+\left(3a^3+2a^2b-5ab^2-4b^3\right)\tan(fx+e)}{a^2b^4\tan(fx+e)^4+a^4b^2+2a^3b^3+a^2b^4+2\left(a^3b^3+a^2b^4\right)\tan(fx+e)^2}+\frac{8(fx+e)}{a^3}-\frac{\left(3a^3-a^2b+4ab^2+8b^3\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}a^3b^2}}{8f}$$

input
```
integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
-1/8*(((5*a^2*b + a*b^2 - 4*b^3)*tan(f*x + e)^3 + (3*a^3 + 2*a^2*b - 5*a*b
^2 - 4*b^3)*tan(f*x + e))/(a^2*b^4*tan(f*x + e)^4 + a^4*b^2 + 2*a^3*b^3 +
a^2*b^4 + 2*(a^3*b^3 + a^2*b^4)*tan(f*x + e)^2) + 8*(f*x + e)/a^3 - (3*a^3
 - a^2*b + 4*a*b^2 + 8*b^3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/(sqrt((
a + b)*b)*a^3*b^2))/f
```

3.369.8 Giac [A] (verification not implemented)

Time = 2.54 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.36

$$\int \frac{\tan^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx =$$

$$-\frac{\frac{8\,(fx+e)}{a^3}-\frac{\left(3\,a^3-a^2b+4\,ab^2+8\,b^3\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{\sqrt{ab+b^2}a^3b^2}+\frac{5\,a^2b\tan(fx+e)^3+ab^2\tan(fx+e)^3-4\,b^3\tan(fx+e)^3}{\left(b\tan\right.}}{8\,f}$$

input
```
integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
-1/8*(8*(f*x + e)/a^3 - (3*a^3 - a^2*b + 4*a*b^2 + 8*b^3)*(pi*floor((f*x +
 e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/(sqrt(a*b +
 b^2)*a^3*b^2) + (5*a^2*b*tan(f*x + e)^3 + a*b^2*tan(f*x + e)^3 - 4*b^3*ta
n(f*x + e)^3 + 3*a^3*tan(f*x + e) + 2*a^2*b*tan(f*x + e) - 5*a*b^2*tan(f*x
 + e) - 4*b^3*tan(f*x + e))/((b*tan(f*x + e)^2 + a + b)^2*a^2*b^2))/f
```

3.369.9 Mupad [B] (verification not implemented)

Time = 20.95 (sec) , antiderivative size = 615, normalized size of antiderivative = 4.18

$$\int \frac{\tan^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx =$$

$$-\frac{\operatorname{atan}\left(\frac{25\tan(e+fx)}{32\left(\frac{5b}{4a}-\frac{3a}{16b}+\frac{9a^2}{32b^2}+\frac{25}{32}\right)}-\frac{3\tan(e+fx)}{16\left(\frac{9a}{32b}+\frac{25b}{32a}+\frac{5b^2}{4a^2}-\frac{3}{16}\right)}+\frac{9\tan(e+fx)}{32\left(\frac{25b^2}{32a^2}-\frac{3b}{16a}+\frac{5b^3}{4a^3}+\frac{9}{32}\right)}+\frac{5\tan(e+fx)}{4\left(\frac{25a}{32b}-\frac{3a^2}{16b^2}+\frac{9a^3}{32b^3}+\frac{5}{4}\right)}\right)}{a^3 f}$$

$$-\frac{\frac{\tan(e+fx)^3\left(5a^2+ab-4b^2\right)}{8a^2b}-\frac{\tan(e+fx)\,(a+b)\left(-3a^2+ab+4b^2\right)}{8a^2b^2}}{f\left(2ab+a^2+b^2+\tan(e+fx)^2\left(2b^2+2ab\right)+b^2\tan(e+fx)^4\right)}$$

$$-\frac{\operatorname{atanh}\left(\frac{27\tan(e+fx)\sqrt{-b^6-ab^5}}{256\left(\frac{27ab^2}{256}-\frac{27b^3}{128}+\frac{171b^4}{256a}-\frac{7b^5}{64a^2}+\frac{5b^6}{32a^3}+\frac{5b^7}{4a^4}\right)}-\frac{81\tan(e+fx)\sqrt{-b^6-ab^5}}{256\left(\frac{27a^2b}{256}-\frac{27ab^2}{128}+\frac{171b^3}{256}-\frac{7b^4}{64a}+\frac{5b^5}{32a^2}+\frac{5b^6}{4a^3}\right)}-\frac{35\tan(e+fx}{32\left(\frac{171a^2b}{256}-\frac{7ab^2}{64}-\frac{27}{1}\right.}\right.}$$

input
```
int(tan(e + f*x)^6/(a + b/cos(e + f*x)^2)^3,x)
```

output
```
- atan((25*tan(e + f*x))/(32*((5*b)/(4*a) - (3*a)/(16*b) + (9*a^2)/(32*b^2
) + 25/32)) - (3*tan(e + f*x))/(16*((9*a)/(32*b) + (25*b)/(32*a) + (5*b^2)
/(4*a^2) - 3/16)) + (9*tan(e + f*x))/(32*((25*b^2)/(32*a^2) - (3*b)/(16*a)
 + (5*b^3)/(4*a^3) + 9/32)) + (5*tan(e + f*x))/(4*((25*a)/(32*b) - (3*a^2)
/(16*b^2) + (9*a^3)/(32*b^3) + 5/4)))/(a^3*f) - ((tan(e + f*x)^3*(a*b + 5*
a^2 - 4*b^2))/(8*a^2*b) - (tan(e + f*x)*(a + b)*(a*b - 3*a^2 + 4*b^2))/(8*
a^2*b^2))/(f*(2*a*b + a^2 + b^2 + tan(e + f*x)^2*(2*a*b + 2*b^2) + b^2*tan
(e + f*x)^4)) - (atanh((27*tan(e + f*x)*(- a*b^5 - b^6)^(1/2))/(256*((27*a
*b^2)/256 - (27*b^3)/128 + (171*b^4)/(256*a) - (7*b^5)/(64*a^2) + (5*b^6)/
(32*a^3) + (5*b^7)/(4*a^4))) - (81*tan(e + f*x)*(- a*b^5 - b^6)^(1/2))/(25
6*((27*a^2*b)/256 - (27*a*b^2)/128 + (171*b^3)/256 - (7*b^4)/(64*a) + (5*b
^5)/(32*a^2) + (5*b^6)/(4*a^3))) - (35*tan(e + f*x)*(- a*b^5 - b^6)^(1/2))
/(32*((171*a^2*b)/256 - (7*a*b^2)/64 - (27*a^3)/128 + (5*b^3)/32 + (5*b^4)
/(4*a) + (27*a^4)/(256*b))) + (5*tan(e + f*x)*(- a*b^5 - b^6)^(1/2))/(4*((
5*a*b^2)/32 - (7*a^2*b)/64 + (171*a^3)/256 + (5*b^3)/4 - (27*a^4)/(128*b)
+ (27*a^5)/(256*b^2))) + (63*tan(e + f*x)*(- a*b^5 - b^6)^(1/2))/(64*((171
*a*b^2)/256 - (27*a^2*b)/128 + (27*a^3)/256 - (7*b^3)/64 + (5*b^4)/(32*a)
+ (5*b^5)/(4*a^2))))*(-b^5*(a + b))^(1/2)*(3*a^2 - 4*a*b + 8*b^2))/(8*a^3*
b^5*f)
```

3.370 $\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

3.370.1 Optimal result 2684
3.370.2 Mathematica [C] (warning: unable to verify) 2684
3.370.3 Rubi [A] (verified) 2686
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3.370.5 Fricas [B] (verification not implemented) 2690
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3.370.7 Maxima [A] (verification not implemented) 2691
3.370.8 Giac [A] (verification not implemented) 2691
3.370.9 Mupad [B] (verification not implemented) 2692

3.370.1 Optimal result

Integrand size = 23, antiderivative size = 137

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{x}{a^3} + \frac{(a^2-4ab-8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3b^{3/2}\sqrt{a+b}f} - \frac{(a+b)\tan(e+fx)}{4abf\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{(a-4b)\tan(e+fx)}{8a^2bf\left(a+b+b\tan^2(e+fx)\right)}$$

output
```
x/a^3+1/8*(a^2-4*a*b-8*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^3/b^(
3/2)/f/(a+b)^(1/2)-1/4*(a+b)*tan(f*x+e)/a/b/f/(a+b+b*tan(f*x+e)^2)^2+1/8*(
a-4*b)*tan(f*x+e)/a^2/b/f/(a+b+b*tan(f*x+e)^2)
```

3.370.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 16.89 (sec) , antiderivative size = 1473, normalized size of antiderivative = 10.75

$$\int \frac{\tan^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$

$$= \frac{(a+2b+a\cos(2e+2fx))^3\sec^6(e+fx)\left(\frac{(3a^2+8ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} - \frac{a\sqrt{b}(3a^2+16ab+16b^2+3a(a+2b)\cos(2(e+}{(a+b)^2(a+2b+a\cos(2(e+f}\right.}{1024b^{5/2}f\left(a+b\sec^2(e+fx)\right)^3}$$

$$- \frac{(a+2b+a\cos(2e+2fx))^3\sec^6(e+fx)\left(-\frac{3a(a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} + \frac{\sqrt{b}(3a^3+14a^2b+24ab^2+16b^3+a(3a^2+4ab}{(a+b)^2(a+2b+a\cos}\right.}{2048b^{5/2}f\left(a+b\sec^2(e+fx)\right)^3}$$

$$+ \frac{(a+2b+a\cos(2e+2fx))^3\sec^6(e+fx)\left(\frac{2(3a^5-10a^4b+80a^3b^2+480a^2b^3+640ab^4+256b^5)\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))}{2\sqrt{a+b}\sqrt{}}\right.\right.}{\sqrt{a+b}\sqrt{b}(\cos(e)-i\sin(e))^4}$$

$$- \frac{(a+2b+a\cos(2e+2fx))^3\sec^6(e+fx)\left(-\frac{6a^2\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(\cos(2e)-\right.\right.}{\sqrt{a+b}\sqrt{b}(\cos(e)-i\sin(e))^4}$$

input
```
Integrate[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

output

```
((a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(((3*a^2 + 8*a*b + 8*b^2)
*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) - (a*Sqrt[b]*(3
*a^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])
/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2)))/(1024*b^(5/2)*f*(a + b*Sec
[e + f*x]^2)^3) - ((a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*((-3*a*
(a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) + (Sqr
t[b]*(3*a^3 + 14*a^2*b + 24*a*b^2 + 16*b^3 + a*(3*a^2 + 4*a*b + 4*b^2)*Cos
[2*(e + f*x)])*Sin[2*(e + f*x)])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])
^2)))/(2048*b^(5/2)*f*(a + b*Sec[e + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e +
2*f*x])^3*Sec[e + f*x]^6*((2*(3*a^5 - 10*a^4*b + 80*a^3*b^2 + 480*a^2*b^3
+ 640*a*b^4 + 256*b^5)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*
b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e]
)^4])]*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]
) + (Sec[2*e]*(256*b^2*(a + b)^2*(3*a^2 + 8*a*b + 8*b^2)*f*x*Cos[2*e] + 51
2*a*b^2*(a + b)^2*(a + 2*b)*f*x*Cos[2*f*x] + 128*a^4*b^2*f*x*Cos[2*(e + 2*
f*x)] + 256*a^3*b^3*f*x*Cos[2*(e + 2*f*x)] + 128*a^2*b^4*f*x*Cos[2*(e + 2*
f*x)] + 512*a^4*b^2*f*x*Cos[4*e + 2*f*x] + 2048*a^3*b^3*f*x*Cos[4*e + 2*f*
x] + 2560*a^2*b^4*f*x*Cos[4*e + 2*f*x] + 1024*a*b^5*f*x*Cos[4*e + 2*f*x] +
 128*a^4*b^2*f*x*Cos[6*e + 4*f*x] + 256*a^3*b^3*f*x*Cos[6*e + 4*f*x] + 128
*a^2*b^4*f*x*Cos[6*e + 4*f*x] - 9*a^6*Sin[2*e] + 12*a^5*b*Sin[2*e] + 68...
```

3.370.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4629, 2075, 372, 402, 27, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\tan(e+fx)^4}{\left(a+b\sec(e+fx)^2\right)^3}\,dx$$

$$\downarrow 4629$$

$$\frac{\int \frac{\tan^4(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(a+b\left(\tan^2(e+fx)+1\right)\right)^3}\,d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\tan^4(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^3}\,d\tan(e+fx)}{f}$$

$$\downarrow 372$$

$$\frac{\int \frac{(a-3b)\tan^2(e+fx)+a+b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}\,d\tan(e+fx)}{4ab} - \frac{(a+b)\tan(e+fx)}{4ab(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\downarrow 402$$

$$\frac{\frac{\int \frac{(a+b)\left((a-4b)\tan^2(e+fx)+a+4b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}\,d\tan(e+fx)}{2a(a+b)} + \frac{(a-4b)\tan(e+fx)}{2a\left(a+b\tan^2(e+fx)+b\right)}}{4ab} - \frac{(a+b)\tan(e+fx)}{4ab(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\downarrow 27$$

$$\frac{\frac{\int \frac{(a-4b)\tan^2(e+fx)+a+4b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}\,d\tan(e+fx)}{2a} + \frac{(a-4b)\tan(e+fx)}{2a\left(a+b\tan^2(e+fx)+b\right)}}{4ab} - \frac{(a+b)\tan(e+fx)}{4ab(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\downarrow 397$$

$$\frac{\frac{\left(a^2-4ab-8b^2\right)\int \frac{1}{b\tan^2(e+fx)+a+b}\,d\tan(e+fx)}{a} + \frac{8b\int \frac{1}{\tan^2(e+fx)+1}\,d\tan(e+fx)}{a}}{2a} + \frac{(a-4b)\tan(e+fx)}{2a\left(a+b\tan^2(e+fx)+b\right)}}{4ab} - \frac{(a+b)\tan(e+fx)}{4ab(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\downarrow 216$$

$$\frac{\frac{\left(a^2-4ab-8b^2\right)\int \frac{1}{b\tan^2(e+fx)+a+b}\,d\tan(e+fx)}{a} + \frac{8b\arctan(\tan(e+fx))}{a}}{2a} + \frac{(a-4b)\tan(e+fx)}{2a\left(a+b\tan^2(e+fx)+b\right)}}{4ab} - \frac{(a+b)\tan(e+fx)}{4ab(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\downarrow 218$$

$$\frac{\frac{\left(a^2-4ab-8b^2\right)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{b}\sqrt{a+b}} + \frac{8b\arctan(\tan(e+fx))}{a}}{2a} + \frac{(a-4b)\tan(e+fx)}{2a\left(a+b\tan^2(e+fx)+b\right)}}{4ab} - \frac{(a+b)\tan(e+fx)}{4ab(a+b\tan^2(e+fx)+b)^2}}{f}$$

input
```
Int[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

3.370. $\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

output
```
(-1/4*((a + b)*Tan[e + f*x])/(a*b*(a + b + b*Tan[e + f*x]^2)^2) + (((8*b*A
rcTan[Tan[e + f*x]])/a + ((a^2 - 4*a*b - 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*
x])/Sqrt[a + b]])/(a*Sqrt[b]*Sqrt[a + b]))/(2*a) + ((a - 4*b)*Tan[e + f*x]
)/(2*a*(a + b + b*Tan[e + f*x]^2)))/(4*a*b))/f
```

3.370.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.370.4 Maple [A] (verified)

Time = 15.43 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{\frac{\left(\frac{1}{8}a^2-\frac{1}{2}ab\right)\tan(fx+e)^3-\frac{a\left(a^2+5ab+4b^2\right)\tan(fx+e)}{8b}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(a^2-4ab-8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8b\sqrt{(a+b)b}}}{a^3}+\frac{\arctan(\tan(fx+e))}{a^3}}{f}$
default	$\frac{\frac{\left(\frac{1}{8}a^2-\frac{1}{2}ab\right)\tan(fx+e)^3-\frac{a\left(a^2+5ab+4b^2\right)\tan(fx+e)}{8b}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(a^2-4ab-8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8b\sqrt{(a+b)b}}}{a^3}+\frac{\arctan(\tan(fx+e))}{a^3}}{f}$
risch	$\frac{x}{a^3}-\frac{i\left(a^3e^{6i(fx+e)}+12a^2be^{6i(fx+e)}+16ab^2e^{6i(fx+e)}+3a^3e^{4i(fx+e)}+26a^2be^{4i(fx+e)}+56ab^2e^{4i(fx+e)}+48b^3e^{4i(fx+$ $4a^3f\left(ae^{4i(fx+e)}+2ae^{2i(fx+e)}+4be^{2i(fx+e)}+a\right)^2$

input
```
int(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/a^3*(((1/8*a^2-1/2*a*b)*tan(f*x+e)^3-1/8*a*(a^2+5*a*b+4*b^2)/b*tan(
f*x+e))/(a+b+b*tan(f*x+e)^2)^2+1/8*(a^2-4*a*b-8*b^2)/b/((a+b)*b)^(1/2)*arc
tan(b*tan(f*x+e)/((a+b)*b)^(1/2)))+1/a^3*arctan(tan(f*x+e)))
```

3.370.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 342 vs. $2(123) = 246$.

Time $= 0.31$ (sec) , antiderivative size $= 763$, normalized size of antiderivative $= 5.57$

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$= \left[\frac{32\,(a^3b^2+a^2b^3)fx\cos(fx+e)^4+64\,(a^2b^3+ab^4)fx\cos(fx+e)^2+32\,(ab^4+b^5)fx+\left((a^4-4\,a^3b-8\right.}{\right.}$$

input
```
integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[1/32*(32*(a^3*b^2 + a^2*b^3)*f*x*cos(f*x + e)^4 + 64*(a^2*b^3 + a*b^4)*f*
x*cos(f*x + e)^2 + 32*(a*b^4 + b^5)*f*x + ((a^4 - 4*a^3*b - 8*a^2*b^2)*cos
(f*x + e)^4 + a^2*b^2 - 4*a*b^3 - 8*b^4 + 2*(a^3*b - 4*a^2*b^2 - 8*a*b^3)*
cos(f*x + e)^2)*sqrt(-a*b - b^2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4
 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(
f*x + e))*sqrt(-a*b - b^2)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b
*cos(f*x + e)^2 + b^2)) - 4*((a^4*b + 7*a^3*b^2 + 6*a^2*b^3)*cos(f*x + e)^
3 - (a^3*b^2 - 3*a^2*b^3 - 4*a*b^4)*cos(f*x + e))*sin(f*x + e))/((a^6*b^2
+ a^5*b^3)*f*cos(f*x + e)^4 + 2*(a^5*b^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a^
4*b^4 + a^3*b^5)*f), 1/16*(16*(a^3*b^2 + a^2*b^3)*f*x*cos(f*x + e)^4 + 32*
(a^2*b^3 + a*b^4)*f*x*cos(f*x + e)^2 + 16*(a*b^4 + b^5)*f*x - ((a^4 - 4*a^
3*b - 8*a^2*b^2)*cos(f*x + e)^4 + a^2*b^2 - 4*a*b^3 - 8*b^4 + 2*(a^3*b - 4
*a^2*b^2 - 8*a*b^3)*cos(f*x + e)^2)*sqrt(a*b + b^2)*arctan(1/2*((a + 2*b)*
cos(f*x + e)^2 - b)/(sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e))) - 2*((a^4
*b + 7*a^3*b^2 + 6*a^2*b^3)*cos(f*x + e)^3 - (a^3*b^2 - 3*a^2*b^3 - 4*a*b^
4)*cos(f*x + e))*sin(f*x + e))/((a^6*b^2 + a^5*b^3)*f*cos(f*x + e)^4 + 2*(
a^5*b^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a^4*b^4 + a^3*b^5)*f)]
```

3.370.6 Sympy [F]

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

input
```
integrate(tan(f*x+e)**4/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Integral(tan(e + f*x)**4/(a + b*sec(e + f*x)**2)**3, x)
```

3.370.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.19

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{(ab-4b^2)\tan(fx+e)^3-(a^2+5ab+4b^2)\tan(fx+e)}{a^2b^3\tan(fx+e)^4+a^4b+2a^3b^2+a^2b^3+2(a^3b^2+a^2b^3)\tan(fx+e)^2}+\frac{8(fx+e)}{a^3}+\frac{(a^2-4ab-8b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}a^3b}}{8f}$$

input
```
integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
1/8*(((a*b - 4*b^2)*tan(f*x + e)^3 - (a^2 + 5*a*b + 4*b^2)*tan(f*x + e))/(
a^2*b^3*tan(f*x + e)^4 + a^4*b + 2*a^3*b^2 + a^2*b^3 + 2*(a^3*b^2 + a^2*b^
3)*tan(f*x + e)^2) + 8*(f*x + e)/a^3 + (a^2 - 4*a*b - 8*b^2)*arctan(b*tan(
f*x + e)/sqrt((a + b)*b))/(sqrt((a + b)*b)*a^3*b))/f
```

3.370.8 Giac [A] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.16

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \frac{\frac{8(fx+e)}{a^3}+\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(a^2-4ab-8b^2)}{\sqrt{ab+b^2}a^3b}+\frac{ab\tan(fx+e)^3-4b^2\tan(fx+e)^3-a^2\tan(fx+e)-5ab\tan(fx+e)}{\left(b\tan(fx+e)^2+a+b\right)^2a^2b}}{8f}$$

input
```
integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
1/8*(8*(f*x + e)/a^3 + (pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan
(f*x + e)/sqrt(a*b + b^2)))*(a^2 - 4*a*b - 8*b^2)/(sqrt(a*b + b^2)*a^3*b)
+ (a*b*tan(f*x + e)^3 - 4*b^2*tan(f*x + e)^3 - a^2*tan(f*x + e) - 5*a*b*ta
n(f*x + e) - 4*b^2*tan(f*x + e))/((b*tan(f*x + e)^2 + a + b)^2*a^2*b))/f
```

3.370.9 Mupad [B] (verification not implemented)

Time = 21.39 (sec) , antiderivative size = 1117, normalized size of antiderivative = 8.15

$$\int \frac{\tan^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(tan(e + f*x)^4/(a + b/cos(e + f*x)^2)^3,x)
```

output
```
((tan(e + f*x)^3*(a - 4*b))/(8*a^2) - (tan(e + f*x)*(a + b)*(a + 4*b))/(8*
a^2*b))/(f*(2*a*b + a^2 + b^2 + tan(e + f*x)^2*(2*a*b + 2*b^2) + b^2*tan(e
 + f*x)^4)) - atan(tan(e + f*x)/(32*(b/(4*a) - 1/32)) + tan(e + f*x)/(4*(a
/(32*b) - 1/4)))/(a^3*f) + (atan(-(((((-b^3*(a + b))^(1/2)*((2*a^6*b^3 + (
a^7*b^2)/2)/(a^6*b) - (tan(e + f*x)*(512*a^6*b^4 + 256*a^7*b^3)*(-b^3*(a +
 b))^(1/2)*(4*a*b - a^2 + 8*b^2))/(512*a^4*b*(a^3*b^4 + a^4*b^3)))*(4*a*b
- a^2 + 8*b^2))/(16*(a^3*b^4 + a^4*b^3)) - (tan(e + f*x)*(64*a*b^3 - 8*a^3
*b + a^4 + 128*b^4))/(32*a^4*b))*(-b^3*(a + b))^(1/2)*(4*a*b - a^2 + 8*b^2
)*1i)/(16*(a^3*b^4 + a^4*b^3)) - ((((-b^3*(a + b))^(1/2)*((2*a^6*b^3 + (a^
7*b^2)/2)/(a^6*b) + (tan(e + f*x)*(512*a^6*b^4 + 256*a^7*b^3)*(-b^3*(a + b
))^(1/2)*(4*a*b - a^2 + 8*b^2))/(512*a^4*b*(a^3*b^4 + a^4*b^3)))*(4*a*b -
a^2 + 8*b^2))/(16*(a^3*b^4 + a^4*b^3)) + (tan(e + f*x)*(64*a*b^3 - 8*a^3*b
 + a^4 + 128*b^4))/(32*a^4*b))*(-b^3*(a + b))^(1/2)*(4*a*b - a^2 + 8*b^2)*
1i)/(16*(a^3*b^4 + a^4*b^3)))/(((a*b^2)/4 - (a^2*b)/4 + a^3/32 + b^3)/(a^6
*b) + ((((-b^3*(a + b))^(1/2)*((2*a^6*b^3 + (a^7*b^2)/2)/(a^6*b) - (tan(e
+ f*x)*(512*a^6*b^4 + 256*a^7*b^3)*(-b^3*(a + b))^(1/2)*(4*a*b - a^2 + 8*b
^2))/(512*a^4*b*(a^3*b^4 + a^4*b^3)))*(4*a*b - a^2 + 8*b^2))/(16*(a^3*b^4
+ a^4*b^3)) - (tan(e + f*x)*(64*a*b^3 - 8*a^3*b + a^4 + 128*b^4))/(32*a^4*
b))*(-b^3*(a + b))^(1/2)*(4*a*b - a^2 + 8*b^2))/(16*(a^3*b^4 + a^4*b^3)) +
 ((((-b^3*(a + b))^(1/2)*((2*a^6*b^3 + (a^7*b^2)/2)/(a^6*b) + (tan(e + ...
```

3.371 $\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

3.371.1 Optimal result 2693
3.371.2 Mathematica [C] (warning: unable to verify) 2693
3.371.3 Rubi [A] (verified) 2695
3.371.4 Maple [A] (verified) 2698
3.371.5 Fricas [B] (verification not implemented) 2699
3.371.6 Sympy [F] 2700
3.371.7 Maxima [A] (verification not implemented) 2700
3.371.8 Giac [A] (verification not implemented) 2700
3.371.9 Mupad [B] (verification not implemented) 2701

3.371.1 Optimal result

Integrand size = 23, antiderivative size = 138

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{x}{a^3} + \frac{(3a^2+12ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3\sqrt{b}(a+b)^{3/2}f} + \frac{\tan(e+fx)}{4af\left(a+b+b\tan^2(e+fx)\right)^2} + \frac{(3a+4b)\tan(e+fx)}{8a^2(a+b)f\left(a+b+b\tan^2(e+fx)\right)}$$

output
```
-x/a^3+1/8*(3*a^2+12*a*b+8*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))/a^3
/(a+b)^(3/2)/f/b^(1/2)+1/4*tan(f*x+e)/a/f/(a+b+b*tan(f*x+e)^2)^2+1/8*(3*a+
4*b)*tan(f*x+e)/a^2/(a+b)/f/(a+b+b*tan(f*x+e)^2)
```

3.371.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 13.27 (sec) , antiderivative size = 1473, normalized size of antiderivative = 10.67

$$\int \frac{\tan^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$

$$= \frac{(a+2b+a\cos(2e+2fx))^3\sec^6(e+fx)\left(\frac{\left(3a^2+8ab+8b^2\right)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} - \frac{a\sqrt{b}(3a^2+16ab+16b^2+3a(a+2b)\cos(2(e+}{(a+b)^2(a+2b+a\cos(2(e+f}\right.}{1024b^{5/2}f\left(a+b\sec^2(e+fx)\right)^3}$$

$$+ \frac{(a+2b+a\cos(2e+2fx))^3\sec^6(e+fx)\left(-\frac{3a(a+2b)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} + \frac{\sqrt{b}(3a^3+14a^2b+24ab^2+16b^3+a(3a^2+4ab}{(a+b)^2(a+2b+a\cos}\right.}{2048b^{5/2}f\left(a+b\sec^2(e+fx)\right)^3}$$

$$- \frac{(a+2b+a\cos(2e+2fx))^3\sec^6(e+fx)\left(\frac{2\left(3a^5-10a^4b+80a^3b^2+480a^2b^3+640ab^4+256b^5\right)\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))}{2\sqrt{a+b}\sqrt{}}\right.\right.}{\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}$$

$$- \frac{(a+2b+a\cos(2e+2fx))^3\sec^6(e+fx)\left(-\frac{6a^2\arctan\left(\frac{\sec(fx)(\cos(2e)-i\sin(2e))(-((a+2b)\sin(fx))+a\sin(2e+fx))}{2\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}\right)(\cos(2e)-\right.\right.}{\sqrt{a+b}\sqrt{b(\cos(e)-i\sin(e))^4}}$$

input
```
Integrate[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

output

```
((a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(((3*a^2 + 8*a*b + 8*b^2)
*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) - (a*Sqrt[b]*(3
*a^2 + 16*a*b + 16*b^2 + 3*a*(a + 2*b)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])
/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^2)))/(1024*b^(5/2)*f*(a + b*Sec
[e + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*((-3*a*
(a + 2*b)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a + b)^(5/2) + (Sqr
t[b]*(3*a^3 + 14*a^2*b + 24*a*b^2 + 16*b^3 + a*(3*a^2 + 4*a*b + 4*b^2)*Cos
[2*(e + f*x)])*Sin[2*(e + f*x)])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])
^2)))/(2048*b^(5/2)*f*(a + b*Sec[e + f*x]^2)^3) - ((a + 2*b + a*Cos[2*e +
2*f*x])^3*Sec[e + f*x]^6*((2*(3*a^5 - 10*a^4*b + 80*a^3*b^2 + 480*a^2*b^3
+ 640*a*b^4 + 256*b^5)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*Sin[2*e])*(-((a + 2*
b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e]
)^4])]*(Cos[2*e] - I*Sin[2*e]))/(Sqrt[a + b]*Sqrt[b*(Cos[e] - I*Sin[e])^4]
) + (Sec[2*e]*(256*b^2*(a + b)^2*(3*a^2 + 8*a*b + 8*b^2)*f*x*Cos[2*e] + 51
2*a*b^2*(a + b)^2*(a + 2*b)*f*x*Cos[2*f*x] + 128*a^4*b^2*f*x*Cos[2*(e + 2*
f*x)] + 256*a^3*b^3*f*x*Cos[2*(e + 2*f*x)] + 128*a^2*b^4*f*x*Cos[2*(e + 2*
f*x)] + 512*a^4*b^2*f*x*Cos[4*e + 2*f*x] + 2048*a^3*b^3*f*x*Cos[4*e + 2*f*
x] + 2560*a^2*b^4*f*x*Cos[4*e + 2*f*x] + 1024*a*b^5*f*x*Cos[4*e + 2*f*x] +
 128*a^4*b^2*f*x*Cos[6*e + 4*f*x] + 256*a^3*b^3*f*x*Cos[6*e + 4*f*x] + 128
*a^2*b^4*f*x*Cos[6*e + 4*f*x] - 9*a^6*Sin[2*e] + 12*a^5*b*Sin[2*e] + 68...
```

3.371.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.17, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4629, 2075, 373, 402, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\tan(e+fx)^2}{\left(a+b\sec(e+fx)^2\right)^3}\,dx$$

$$\downarrow 4629$$

$$\frac{\int \frac{\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(a+b\left(\tan^2(e+fx)+1\right)\right)^3}\,d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\tan^2(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^3}\,d\tan(e+fx)}{f}$$

$$\downarrow 373$$

$$\frac{\frac{\tan(e+fx)}{4a(a+b\tan^2(e+fx)+b)^2}-\frac{\int \frac{1-3\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}\,d\tan(e+fx)}{4a}}{f}$$

$$\downarrow 402$$

$$\frac{\frac{\tan(e+fx)}{4a(a+b\tan^2(e+fx)+b)^2}-\frac{\frac{\int \frac{-\left((3a+4b)\tan^2(e+fx)\right)+5a+4b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}\,d\tan(e+fx)}{2a(a+b)}-\frac{(3a+4b)\tan(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{4a}}{f}$$

$$\downarrow 397$$

$$\frac{\frac{\tan(e+fx)}{4a(a+b\tan^2(e+fx)+b)^2}-\frac{\frac{8(a+b)\int \frac{1}{\tan^2(e+fx)+1}\,d\tan(e+fx)}{a}-\frac{\left(3a^2+12ab+8b^2\right)\int \frac{1}{b\tan^2(e+fx)+a+b}\,d\tan(e+fx)}{a}}{2a(a+b)}-\frac{(3a+4b)\tan(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{4a}}{f}$$

$$\downarrow 216$$

$$\frac{\frac{\tan(e+fx)}{4a(a+b\tan^2(e+fx)+b)^2}-\frac{\frac{8(a+b)\arctan(\tan(e+fx))}{a}-\frac{\left(3a^2+12ab+8b^2\right)\int \frac{1}{b\tan^2(e+fx)+a+b}\,d\tan(e+fx)}{a}}{2a(a+b)}-\frac{(3a+4b)\tan(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{4a}}{f}$$

$$\downarrow 218$$

$$\frac{\frac{\tan(e+fx)}{4a(a+b\tan^2(e+fx)+b)^2}-\frac{\frac{8(a+b)\arctan(\tan(e+fx))}{a}-\frac{\left(3a^2+12ab+8b^2\right)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{b}\sqrt{a+b}}}{2a(a+b)}-\frac{(3a+4b)\tan(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{4a}}{f}$$

input
```
Int[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

3.371. $\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

output
```
(Tan[e + f*x]/(4*a*(a + b + b*Tan[e + f*x]^2)^2) - (((8*(a + b)*ArcTan[Tan
[e + f*x]])/a - ((3*a^2 + 12*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Tan[e + f*x])/Sq
rt[a + b]])/(a*Sqrt[b]*Sqrt[a + b]))/(2*a*(a + b)) - ((3*a + 4*b)*Tan[e +
f*x])/(2*a*(a + b)*(a + b + b*Tan[e + f*x]^2)))/(4*a))/f
```

3.371.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.371.4 Maple [A] (verified)

Time = 11.63 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{-\frac{\arctan(\tan(fx+e))}{a^3}+\frac{\frac{ab(3a+4b)\tan(fx+e)^3}{8a+8b}+\frac{(5a+4b)a\tan(fx+e)}{8}+\frac{\left(3a^2+12ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8(a+b)\sqrt{(a+b)b}}}{\left(a+b+b\tan(fx+e)^2\right)^2}}{a^3}}{f}$
default	$\frac{-\frac{\arctan(\tan(fx+e))}{a^3}+\frac{\frac{ab(3a+4b)\tan(fx+e)^3}{8a+8b}+\frac{(5a+4b)a\tan(fx+e)}{8}+\frac{\left(3a^2+12ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8(a+b)\sqrt{(a+b)b}}}{\left(a+b+b\tan(fx+e)^2\right)^2}}{a^3}}{f}$
risch	$-\frac{x}{a^3}+\frac{i\left(5a^3e^{6i(fx+e)}+20a^2be^{6i(fx+e)}+16ab^2e^{6i(fx+e)}+15a^3e^{4i(fx+e)}+58a^2be^{4i(fx+e)}+88ab^2e^{4i(fx+e)}+48b^3e^{4i(fx+e)}+\right.}{4a^3(a+b)f\left(ae^{4i(fx+e)}+2ae^{2i(fx+e)}+4be^{2i(fx+e)}+\right.}$

input
```
int(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/a^3*arctan(tan(f*x+e))+1/a^3*((1/8*a*b*(3*a+4*b)/(a+b)*tan(f*x+e)^
3+1/8*(5*a+4*b)*a*tan(f*x+e))/(a+b+b*tan(f*x+e)^2)^2+1/8*(3*a^2+12*a*b+8*b
^2)/(a+b)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))))
```

3.371.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 390 vs. $2(124) = 248$.

Time = 0.33 (sec) , antiderivative size = 860, normalized size of antiderivative = 6.23

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$= \left[-\frac{32\left(a^4b+2\,a^3b^2+a^2b^3\right)fx\cos\left(fx+e\right)^4+64\left(a^3b^2+2\,a^2b^3+ab^4\right)fx\cos\left(fx+e\right)^2+32\left(a^2b^3+2\,ab^4\right.}{} \right.$$

$$\left. -\frac{16\left(a^4b+2\,a^3b^2+a^2b^3\right)fx\cos\left(fx+e\right)^4+32\left(a^3b^2+2\,a^2b^3+ab^4\right)fx\cos\left(fx+e\right)^2+16\left(a^2b^3+2\,ab^4-\right.}{} \right.$$

input
```
integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[-1/32*(32*(a^4*b + 2*a^3*b^2 + a^2*b^3)*f*x*cos(f*x + e)^4 + 64*(a^3*b^2
+ 2*a^2*b^3 + a*b^4)*f*x*cos(f*x + e)^2 + 32*(a^2*b^3 + 2*a*b^4 + b^5)*f*x
 + ((3*a^4 + 12*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 3*a^2*b^2 + 12*a*b^3 +
 8*b^4 + 2*(3*a^3*b + 12*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*sqrt(-a*b - b^
2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x +
 e)^2 + 4*((a + 2*b)*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(-a*b - b^2)*sin
(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)) - 4*((
5*a^4*b + 11*a^3*b^2 + 6*a^2*b^3)*cos(f*x + e)^3 + (3*a^3*b^2 + 7*a^2*b^3
+ 4*a*b^4)*cos(f*x + e))*sin(f*x + e))/((a^7*b + 2*a^6*b^2 + a^5*b^3)*f*co
s(f*x + e)^4 + 2*(a^6*b^2 + 2*a^5*b^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a^5*b
^3 + 2*a^4*b^4 + a^3*b^5)*f), -1/16*(16*(a^4*b + 2*a^3*b^2 + a^2*b^3)*f*x*
cos(f*x + e)^4 + 32*(a^3*b^2 + 2*a^2*b^3 + a*b^4)*f*x*cos(f*x + e)^2 + 16*
(a^2*b^3 + 2*a*b^4 + b^5)*f*x + ((3*a^4 + 12*a^3*b + 8*a^2*b^2)*cos(f*x +
e)^4 + 3*a^2*b^2 + 12*a*b^3 + 8*b^4 + 2*(3*a^3*b + 12*a^2*b^2 + 8*a*b^3)*c
os(f*x + e)^2)*sqrt(a*b + b^2)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)/(
sqrt(a*b + b^2)*cos(f*x + e)*sin(f*x + e))) - 2*((5*a^4*b + 11*a^3*b^2 + 6
*a^2*b^3)*cos(f*x + e)^3 + (3*a^3*b^2 + 7*a^2*b^3 + 4*a*b^4)*cos(f*x + e))
*sin(f*x + e))/((a^7*b + 2*a^6*b^2 + a^5*b^3)*f*cos(f*x + e)^4 + 2*(a^6*b^
2 + 2*a^5*b^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a^5*b^3 + 2*a^4*b^4 + a^3*b^5
)*f)]
```

3.371.6 Sympy [F]

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

input
```
integrate(tan(f*x+e)**2/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Integral(tan(e + f*x)**2/(a + b*sec(e + f*x)**2)**3, x)
```

3.371.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.38

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \frac{\frac{(3a^2+12ab+8b^2)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^4+a^3b)\sqrt{(a+b)b}} + \frac{(3ab+4b^2)\tan(fx+e)^3+(5a^2+9ab+4b^2)\tan(fx+e)}{a^5+3a^4b+3a^3b^2+a^2b^3+(a^3b^2+a^2b^3)\tan(fx+e)^4+2(a^4b+2a^3b^2+a^2b^3)\tan(fx+e)^2} - \frac{8(fx+e)}{a^3}}{8f}$$

input
```
integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
1/8*((3*a^2 + 12*a*b + 8*b^2)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))/((a^4
 + a^3*b)*sqrt((a + b)*b)) + ((3*a*b + 4*b^2)*tan(f*x + e)^3 + (5*a^2 + 9*
a*b + 4*b^2)*tan(f*x + e))/(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3 + (a^3*b^2
 + a^2*b^3)*tan(f*x + e)^4 + 2*(a^4*b + 2*a^3*b^2 + a^2*b^3)*tan(f*x + e)^
2) - 8*(f*x + e)/a^3)/f
```

3.371.8 Giac [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.25

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$
$$= \frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)(3a^2+12ab+8b^2)}{(a^4+a^3b)\sqrt{ab+b^2}} + \frac{3ab\tan(fx+e)^3+4b^2\tan(fx+e)^3+5a^2\tan(fx+e)+9ab\tan(fx+e)+4b^2\tan}{(a^3+a^2b)\left(b\tan(fx+e)^2+a+b\right)^2}}{8f}$$

input
```
integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
1/8*((pi*floor((f*x + e)/pi + 1/2)*sgn(b) + arctan(b*tan(f*x + e)/sqrt(a*b
 + b^2)))*(3*a^2 + 12*a*b + 8*b^2)/((a^4 + a^3*b)*sqrt(a*b + b^2)) + (3*a*
b*tan(f*x + e)^3 + 4*b^2*tan(f*x + e)^3 + 5*a^2*tan(f*x + e) + 9*a*b*tan(f
*x + e) + 4*b^2*tan(f*x + e))/((a^3 + a^2*b)*(b*tan(f*x + e)^2 + a + b)^2)
 - 8*(f*x + e)/a^3)/f
```

3.371.9 Mupad [B] (verification not implemented)

Time = 22.82 (sec) , antiderivative size = 2405, normalized size of antiderivative = 17.43

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(tan(e + f*x)^2/(a + b/cos(e + f*x)^2)^3,x)
```

output
```
((tan(e + f*x)*(5*a + 4*b))/(8*a^2) + (tan(e + f*x)^3*(3*a*b + 4*b^2))/(8*
a^2*(a + b)))/(f*(2*a*b + a^2 + b^2 + tan(e + f*x)^2*(2*a*b + 2*b^2) + b^2
*tan(e + f*x)^4)) - atan((((((2*a^6*b^4 + (9*a^7*b^3)/2 + (5*a^8*b^2)/2)*1
i)/(2*(2*a^7*b + a^8 + a^6*b^2)) - (tan(e + f*x)*(512*a^6*b^5 + 1280*a^7*b
^4 + 1024*a^8*b^3 + 256*a^9*b^2))/(128*a^3*(2*a^5*b + a^6 + a^4*b^2)))/(2*
a^3) + (tan(e + f*x)*(320*a*b^4 + 9*a^4*b + 128*b^5 + 256*a^2*b^3 + 72*a^3
*b^2))/(64*(2*a^5*b + a^6 + a^4*b^2)))/a^3 - ((((2*a^6*b^4 + (9*a^7*b^3)/2
 + (5*a^8*b^2)/2)*1i)/(2*(2*a^7*b + a^8 + a^6*b^2)) + (tan(e + f*x)*(512*a
^6*b^5 + 1280*a^7*b^4 + 1024*a^8*b^3 + 256*a^9*b^2))/(128*a^3*(2*a^5*b + a
^6 + a^4*b^2)))/(2*a^3) - (tan(e + f*x)*(320*a*b^4 + 9*a^4*b + 128*b^5 + 2
56*a^2*b^3 + 72*a^3*b^2))/(64*(2*a^5*b + a^6 + a^4*b^2)))/a^3)/(((9*a*b^3)
/4 + (9*a^3*b)/32 + b^4 + (3*a^2*b^2)/2)/(2*a^7*b + a^8 + a^6*b^2) + (((((
2*a^6*b^4 + (9*a^7*b^3)/2 + (5*a^8*b^2)/2)*1i)/(2*(2*a^7*b + a^8 + a^6*b^2
)) - (tan(e + f*x)*(512*a^6*b^5 + 1280*a^7*b^4 + 1024*a^8*b^3 + 256*a^9*b^
2))/(128*a^3*(2*a^5*b + a^6 + a^4*b^2)))*1i)/(2*a^3) + (tan(e + f*x)*(320*
a*b^4 + 9*a^4*b + 128*b^5 + 256*a^2*b^3 + 72*a^3*b^2)*1i)/(64*(2*a^5*b + a
^6 + a^4*b^2)))/a^3 + (((((2*a^6*b^4 + (9*a^7*b^3)/2 + (5*a^8*b^2)/2)*1i)/
(2*(2*a^7*b + a^8 + a^6*b^2)) + (tan(e + f*x)*(512*a^6*b^5 + 1280*a^7*b^4
+ 1024*a^8*b^3 + 256*a^9*b^2))/(128*a^3*(2*a^5*b + a^6 + a^4*b^2)))*1i)/(2
*a^3) - (tan(e + f*x)*(320*a*b^4 + 9*a^4*b + 128*b^5 + 256*a^2*b^3 + 72...
```

3.372 $\int \frac{1}{(a+b\sec^2(e+fx))^3}\,dx$

3.372.1 Optimal result . . . 2702
3.372.2 Mathematica [C] (warning: unable to verify) . . . 2702
3.372.3 Rubi [A] (verified) . . . 2703
3.372.4 Maple [A] (verified) . . . 2706
3.372.5 Fricas [B] (verification not implemented) . . . 2706
3.372.6 Sympy [F] . . . 2707
3.372.7 Maxima [A] (verification not implemented) . . . 2707
3.372.8 Giac [A] (verification not implemented) . . . 2708
3.372.9 Mupad [B] (verification not implemented) . . . 2708

3.372.1 Optimal result

Integrand size = 14, antiderivative size = 144

$$\int \frac{1}{(a+b\sec^2(e+fx))^3}\,dx = \frac{x}{a^3} - \frac{\sqrt{b}(15a^2+20ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{5/2}f} - \frac{b\tan(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(7a+4b)\tan(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}$$

output
```
x/a^3-1/8*(15*a^2+20*a*b+8*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1/2))*b^(
1/2)/a^3/(a+b)^(5/2)/f-1/4*b*tan(f*x+e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2-1
/8*b*(7*a+4*b)*tan(f*x+e)/a^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)
```

3.372.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.52 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.31

$$\int \frac{1}{(a+b\sec^2(e+fx))^3}\,dx = \frac{(a+2b+a\cos(2(e+fx)))\sec^6(e+fx)\left(8x(a+2b+a\cos(2(e+fx)))^2 + \frac{b(15a^2+20ab+8b^2)\arctan\left(\frac{\sec(fx)(\cos}{}\right.}{}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(-3),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^6*(8*x*(a + 2*b + a*Cos[2*(e
+ f*x)])^2 + (b*(15*a^2 + 20*a*b + 8*b^2)*ArcTan[(Sec[f*x]*(Cos[2*e] - I*S
in[2*e])*(-((a + 2*b)*Sin[f*x]) + a*Sin[2*e + f*x]))/(2*Sqrt[a + b]*Sqrt[b
*(Cos[e] - I*Sin[e])^4])]*(a + 2*b + a*Cos[2*(e + f*x)])^2*(Cos[2*e] - I*S
in[2*e]))/((a + b)^(5/2)*f*Sqrt[b*(Cos[e] - I*Sin[e])^4]) - (4*b^2*((a + 2
*b)*Sin[2*e] - a*Sin[2*f*x]))/((a + b)*f*(Cos[e] - Sin[e])*(Cos[e] + Sin[e
])) + (b*(a + 2*b + a*Cos[2*(e + f*x)])*((9*a^2 + 28*a*b + 16*b^2)*Sin[2*e
] - 3*a*(3*a + 2*b)*Sin[2*f*x]))/((a + b)^2*f*(Cos[e] - Sin[e])*(Cos[e] +
Sin[e])))))/(64*a^3*(a + b*Sec[e + f*x]^2)^3)
```

3.372.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.22, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4616, 316, 402, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sec^2(e+fx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{(a+b\sec(e+fx)^2)^3}\,dx$$

$$\downarrow 4616$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^3}\,d\tan(e+fx)}{f}$$

$$\downarrow 316$$

$$\frac{\int \frac{-3b\tan^2(e+fx)+4a+b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}\,d\tan(e+fx)}{4a(a+b)} - \frac{b\tan(e+fx)}{4a(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\downarrow 402$$

$$\frac{\frac{\int \frac{8a^2+9ba+4b^2-b(7a+4b)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2a(a+b)}-\frac{b(7a+4b)\tan(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{4a(a+b)}-\frac{b\tan(e+fx)}{4a(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\downarrow 397$$

$$\frac{\frac{8(a+b)^2\int\frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}-\frac{b\left(15a^2+20ab+8b^2\right)\int\frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{2a(a+b)}-\frac{b(7a+4b)\tan(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{4a(a+b)}-\frac{b\tan(e+fx)}{4a(a+b)(a+b\tan^2(e+fx)-}}{f}$$

$$\downarrow 216$$

$$\frac{\frac{8(a+b)^2\arctan(\tan(e+fx))}{a}-\frac{b\left(15a^2+20ab+8b^2\right)\int\frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{2a(a+b)}-\frac{b(7a+4b)\tan(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{4a(a+b)}-\frac{b\tan(e+fx)}{4a(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\downarrow 218$$

$$\frac{\frac{8(a+b)^2\arctan(\tan(e+fx))}{a}-\frac{\sqrt{b}\left(15a^2+20ab+8b^2\right)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{2a(a+b)}-\frac{b(7a+4b)\tan(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{4a(a+b)}-\frac{b\tan(e+fx)}{4a(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^(-3),x]
```

output
```
(-1/4*(b*Tan[e + f*x])/(a*(a + b)*(a + b + b*Tan[e + f*x]^2)^2) + (((8*(a
+ b)^2*ArcTan[Tan[e + f*x]])/a - (Sqrt[b]*(15*a^2 + 20*a*b + 8*b^2)*ArcTan
[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(2*a*(a + b)) - (b*
(7*a + 4*b)*Tan[e + f*x])/(2*a*(a + b)*(a + b + b*Tan[e + f*x]^2)))/(4*a*(
a + b)))/f
```

3.372.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

3.372.4 Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{-\frac{b\left(\frac{ab(7a+4b)\tan(fx+e)^3}{8a^2+16ab+8b^2}+\frac{(9a+4b)a\tan(fx+e)}{8a+8b}\right)}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(15a^2+20ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\left(a^2+2ab+b^2\right)\sqrt{(a+b)b}}}{a^3}+\frac{\arctan(\tan(fx+e))}{a^3}}{f}$
default	$\frac{-\frac{b\left(\frac{ab(7a+4b)\tan(fx+e)^3}{8a^2+16ab+8b^2}+\frac{(9a+4b)a\tan(fx+e)}{8a+8b}\right)}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(15a^2+20ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\left(a^2+2ab+b^2\right)\sqrt{(a+b)b}}}{a^3}+\frac{\arctan(\tan(fx+e))}{a^3}}{f}$
risch	$\frac{x}{a^3}-\frac{ib\left(9a^3e^{6i(fx+e)}+28a^2be^{6i(fx+e)}+16ab^2e^{6i(fx+e)}+27a^3e^{4i(fx+e)}+90a^2be^{4i(fx+e)}+120ab^2e^{4i(fx+e)}+48b^3e^{4i}\right.}{4a^3(a+b)^2f\left(ae^{4i(fx+e)}+2ae^{2i(fx+e)}+4be^{2i(fx+}\right.$

input
```
int(1/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-b/a^3*((1/8*a*b*(7*a+4*b)/(a^2+2*a*b+b^2)*tan(f*x+e)^3+1/8*(9*a+4*b)
*a/(a+b)*tan(f*x+e))/(a+b+b*tan(f*x+e)^2)^2+1/8*(15*a^2+20*a*b+8*b^2)/(a^2
+2*a*b+b^2)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2)))+1/a^3*ar
ctan(tan(f*x+e)))
```

3.372.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. 2(130) = 260.

Time = 0.32 (sec) , antiderivative size = 819, normalized size of antiderivative = 5.69

$$\int \frac{1}{(a+b\sec^2(e+fx))^3}\,dx$$

$$=\left[\frac{32\left(a^4+2a^3b+a^2b^2\right)fx\cos(fx+e)^4+64\left(a^3b+2a^2b^2+ab^3\right)fx\cos(fx+e)^2+32\left(a^2b^2+2ab^3+b^4\right)}{}\right.$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output

```
[1/32*(32*(a^4 + 2*a^3*b + a^2*b^2)*f*x*cos(f*x + e)^4 + 64*(a^3*b + 2*a^2
*b^2 + a*b^3)*f*x*cos(f*x + e)^2 + 32*(a^2*b^2 + 2*a*b^3 + b^4)*f*x + ((15
*a^4 + 20*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2 + 20*a*b^3 + 8*b^
4 + 2*(15*a^3*b + 20*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*sqrt(-b/(a + b))*l
og(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^
2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sq
rt(-b/(a + b))*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e
)^2 + b^2)) - 4*(3*(3*a^3*b + 2*a^2*b^2)*cos(f*x + e)^3 + (7*a^2*b^2 + 4*a
*b^3)*cos(f*x + e))*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*x + e
)^4 + 2*(a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*a^4*
b^3 + a^3*b^4)*f), 1/16*(16*(a^4 + 2*a^3*b + a^2*b^2)*f*x*cos(f*x + e)^4 +
 32*(a^3*b + 2*a^2*b^2 + a*b^3)*f*x*cos(f*x + e)^2 + 16*(a^2*b^2 + 2*a*b^3
 + b^4)*f*x + ((15*a^4 + 20*a^3*b + 8*a^2*b^2)*cos(f*x + e)^4 + 15*a^2*b^2
 + 20*a*b^3 + 8*b^4 + 2*(15*a^3*b + 20*a^2*b^2 + 8*a*b^3)*cos(f*x + e)^2)*
sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - b)*sqrt(b/(a + b))/
(b*cos(f*x + e)*sin(f*x + e))) - 2*(3*(3*a^3*b + 2*a^2*b^2)*cos(f*x + e)^3
 + (7*a^2*b^2 + 4*a*b^3)*cos(f*x + e))*sin(f*x + e))/((a^7 + 2*a^6*b + a^5
*b^2)*f*cos(f*x + e)^4 + 2*(a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2
+ (a^5*b^2 + 2*a^4*b^3 + a^3*b^4)*f)]
```

3.372.6 Sympy [F]

$$\int \frac{1}{(a+b\sec^2(e+fx))^3}\,dx = \int \frac{1}{(a+b\sec^2(e+fx))^3}\,dx$$

input

```
integrate(1/(a+b*sec(f*x+e)**2)**3,x)
```

output

```
Integral((a + b*sec(e + f*x)**2)**(-3), x)
```

3.372.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.60

$$\int \frac{1}{(a+b\sec^2(e+fx))^3}\,dx =$$

$$-\frac{\frac{(15\,a^2b+20\,ab^2+8\,b^3)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{(a^5+2\,a^4b+a^3b^2)\sqrt{(a+b)b}} + \frac{(7\,ab^2+4\,b^3)\tan(fx+e)^3+(9\,a^2b+13\,ab^2+4\,b^3)\tan(fx+e)}{a^6+4\,a^5b+6\,a^4b^2+4\,a^3b^3+a^2b^4+(a^4b^2+2\,a^3b^3+a^2b^4)\tan(fx+e)^4+2\,(a^5b+3\,a^4b^2+3\,a^3b^3+a}}{8\,f}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
-1/8*((15*a^2*b + 20*a*b^2 + 8*b^3)*arctan(b*tan(f*x + e)/sqrt((a + b)*b))
/((a^5 + 2*a^4*b + a^3*b^2)*sqrt((a + b)*b)) + ((7*a*b^2 + 4*b^3)*tan(f*x
+ e)^3 + (9*a^2*b + 13*a*b^2 + 4*b^3)*tan(f*x + e))/(a^6 + 4*a^5*b + 6*a^4
*b^2 + 4*a^3*b^3 + a^2*b^4 + (a^4*b^2 + 2*a^3*b^3 + a^2*b^4)*tan(f*x + e)^
4 + 2*(a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*tan(f*x + e)^2) - 8*(f*x +
 e)/a^3)/f
```

3.372.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.36

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{\frac{\left(15a^2b+20ab^2+8b^3\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{\left(a^5+2a^4b+a^3b^2\right)\sqrt{ab+b^2}} + \frac{7ab^2\tan(fx+e)^3+4b^3\tan(fx+e)^3+9a^2b\tan(fx+e)+13ab^2\tan(fx}{\left(a^4+2a^3b+a^2b^2\right)\left(b\tan(fx+e)^2+a+b\right)^2}}{8f}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
-1/8*((15*a^2*b + 20*a*b^2 + 8*b^3)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b) +
 arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^5 + 2*a^4*b + a^3*b^2)*sqrt(a
*b + b^2)) + (7*a*b^2*tan(f*x + e)^3 + 4*b^3*tan(f*x + e)^3 + 9*a^2*b*tan(
f*x + e) + 13*a*b^2*tan(f*x + e) + 4*b^3*tan(f*x + e))/((a^4 + 2*a^3*b + a
^2*b^2)*(b*tan(f*x + e)^2 + a + b)^2) - 8*(f*x + e)/a^3)/f
```

3.372.9 Mupad [B] (verification not implemented)

Time = 23.81 (sec) , antiderivative size = 3271, normalized size of antiderivative = 22.72

$$\int \frac{1}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(a + b/cos(e + f*x)^2)^3,x)
```

output

```
atan(((((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^1
0*b^2)*1i)/(2*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)) - (tan(e
 + f*x)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a
^10*b^3 + 256*a^11*b^2))/(128*a^3*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6
*a^6*b^2)))/(2*a^3) + (tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 +
856*a^3*b^4 + 289*a^4*b^3))/(64*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a
^6*b^2)))/a^3 - ((((2*a^6*b^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)
/2 + 4*a^10*b^2)*1i)/(2*(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)
) + (tan(e + f*x)*(512*a^6*b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^
4 + 1536*a^10*b^3 + 256*a^11*b^2))/(128*a^3*(4*a^7*b + a^8 + a^4*b^4 + 4*a
^5*b^3 + 6*a^6*b^2)))/(2*a^3) - (tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*
a^2*b^5 + 856*a^3*b^4 + 289*a^4*b^3))/(64*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5
*b^3 + 6*a^6*b^2)))/a^3)/(((17*a*b^5)/4 + b^6 + (25*a^2*b^4)/4 + (105*a^3*
b^3)/32)/(4*a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2) + (((((2*a^6*b
^6 + (17*a^7*b^5)/2 + 15*a^8*b^4 + (25*a^9*b^3)/2 + 4*a^10*b^2)*1i)/(2*(4*
a^9*b + a^10 + a^6*b^4 + 4*a^7*b^3 + 6*a^8*b^2)) - (tan(e + f*x)*(512*a^6*
b^7 + 2304*a^7*b^6 + 4096*a^8*b^5 + 3584*a^9*b^4 + 1536*a^10*b^3 + 256*a^1
1*b^2))/(128*a^3*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)))*1i)/(
2*a^3) + (tan(e + f*x)*(576*a*b^6 + 128*b^7 + 1024*a^2*b^5 + 856*a^3*b^4 +
 289*a^4*b^3)*1i)/(64*(4*a^7*b + a^8 + a^4*b^4 + 4*a^5*b^3 + 6*a^6*b^2)...
```

3.373 $\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

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3.373.1 Optimal result

Integrand size = 23, antiderivative size = 181

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = -\frac{x}{a^3} + \frac{b^{3/2}(35a^2+28ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{7/2}f} - \frac{(8a^2-11ab-4b^2)\cot(e+fx)}{8a^2(a+b)^3 f} - \frac{b\cot(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} - \frac{b(9a+4b)\cot(e+fx)}{8a^2(a+b)^2 f\left(a+b+b\tan^2(e+fx)\right)}$$

output

```
-x/a^3+1/8*b^(3/2)*(35*a^2+28*a*b+8*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(
1/2))/a^3/(a+b)^(7/2)/f-1/8*(8*a^2-11*a*b-4*b^2)*cot(f*x+e)/a^2/(a+b)^3/f-
1/4*b*cot(f*x+e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2-1/8*b*(9*a+4*b)*cot(f*x+
e)/a^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)
```

3.373.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.53 (sec) , antiderivative size = 2089, normalized size of antiderivative = 11.54

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
((35*a^2 + 28*a*b + 8*b^2)*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6
*(-1/64*(b^2*ArcTan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*
b*Sin[4*e]]) - ((I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e
]]))*(-(a*Sin[f*x]) - 2*b*Sin[f*x] + a*Sin[2*e + f*x])]*Cos[2*e])/(a^3*Sqr
t[a + b]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) + ((I/64)*b^2*ArcTan[Sec[f*x]*
(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[2*e
])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*Sin
[f*x] + a*Sin[2*e + f*x])]*Sin[2*e])/(a^3*Sqrt[a + b]*f*Sqrt[b*Cos[4*e] -
I*b*Sin[4*e]])))/((a + b)^3*(a + b*Sec[e + f*x]^2)^3) + ((a + 2*b + a*Cos[
2*e + 2*f*x])*Csc[e]*Csc[e + f*x]*Sec[2*e]*Sec[e + f*x]^6*(8*a^5*f*x*Cos[f
*x] + 56*a^4*b*f*x*Cos[f*x] + 184*a^3*b^2*f*x*Cos[f*x] + 296*a^2*b^3*f*x*C
os[f*x] + 224*a*b^4*f*x*Cos[f*x] + 64*b^5*f*x*Cos[f*x] - 12*a^5*f*x*Cos[3*
f*x] - 68*a^4*b*f*x*Cos[3*f*x] - 132*a^3*b^2*f*x*Cos[3*f*x] - 108*a^2*b^3*
f*x*Cos[3*f*x] - 32*a*b^4*f*x*Cos[3*f*x] - 8*a^5*f*x*Cos[2*e - f*x] - 56*a
^4*b*f*x*Cos[2*e - f*x] - 184*a^3*b^2*f*x*Cos[2*e - f*x] - 296*a^2*b^3*f*x
*Cos[2*e - f*x] - 224*a*b^4*f*x*Cos[2*e - f*x] - 64*b^5*f*x*Cos[2*e - f*x]
 - 8*a^5*f*x*Cos[2*e + f*x] - 56*a^4*b*f*x*Cos[2*e + f*x] - 184*a^3*b^2*f*
x*Cos[2*e + f*x] - 296*a^2*b^3*f*x*Cos[2*e + f*x] - 224*a*b^4*f*x*Cos[2*e
+ f*x] - 64*b^5*f*x*Cos[2*e + f*x] + 8*a^5*f*x*Cos[4*e + f*x] + 56*a^4*b*f
*x*Cos[4*e + f*x] + 184*a^3*b^2*f*x*Cos[4*e + f*x] + 296*a^2*b^3*f*x*Co...
```

3.373.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.17, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4629, 2075, 374, 441, 445, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\tan(e+fx)^2\,(a+b\sec(e+fx)^2)^3}\,dx$$

$$\downarrow 4629$$

$$\frac{\int \frac{\cot^2(e+fx)}{(\tan^2(e+fx)+1)(a+b(\tan^2(e+fx)+1))^3}\,d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\cot^2(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^3}\,d\tan(e+fx)}{f}$$

$$\downarrow 374$$

$$\frac{\frac{\int \frac{\cot^2(e+fx)\left(-5b\tan^2(e+fx)+4a-b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}\,d\tan(e+fx)}{4a(a+b)} - \frac{b\cot(e+fx)}{4a(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\downarrow 441$$

$$\frac{\frac{\frac{\int \frac{\cot^2(e+fx)\left(8a^2-11ba-4b^2-3b(9a+4b)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}\,d\tan(e+fx)}{2a(a+b)} - \frac{b(9a+4b)\cot(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{4a(a+b)} - \frac{b\cot(e+fx)}{4a(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\downarrow 445$$

$$\frac{-\frac{\frac{\int \frac{8a^3+32ba^2+13b^2a+4b^3+b\left(8a^2-11ba-4b^2\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}\,d\tan(e+fx)}{a+b} - \frac{\left(8a^2-11ab-4b^2\right)\cot(e+fx)}{a+b}}{2a(a+b)} - \frac{b(9a+4b)\cot(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{4a(a+b)} - \frac{b\cot(e+}{4a(a+b)(a+b\tan^2}}{f}$$

$$\downarrow 397$$

$$\frac{-\frac{\frac{8(a+b)^3\int \frac{1}{\tan^2(e+fx)+1}\,d\tan(e+fx)}{a} - \frac{b^2\left(35a^2+28ab+8b^2\right)\int \frac{1}{b\tan^2(e+fx)+a+b}\,d\tan(e+fx)}{a}}{a+b} - \frac{\left(8a^2-11ab-4b^2\right)\cot(e+fx)}{a+b}}{2a(a+b)} - \frac{b(9a+4b)\cot(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{4a(a+b)}}{f}$$

$$\downarrow 216$$

3.373. $\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

$$\frac{-\frac{8(a+b)^3\arctan(\tan(e+fx))}{a}-\frac{b^2\left(35a^2+28ab+8b^2\right)\int\frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{2a(a+b)}-\frac{\left(8a^2-11ab-4b^2\right)\cot(e+fx)}{a+b}-\frac{b(9a+4b)\cot(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{4a(a+b)}-\frac{}{4a(a+}$$
$$f$$

$$\downarrow 218$$

$$\frac{-\frac{8(a+b)^3\arctan(\tan(e+fx))}{a}-\frac{b^{3/2}\left(35a^2+28ab+8b^2\right)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{2a(a+b)}-\frac{\left(8a^2-11ab-4b^2\right)\cot(e+fx)}{a+b}-\frac{b(9a+4b)\cot(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{4a(a+b)}-\frac{b}{4a(a+b)(a+}$$
$$f$$

input
```
Int[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
(-1/4*(b*Cot[e + f*x])/(a*(a + b)*(a + b + b*Tan[e + f*x]^2)^2) + ((-(((8*
(a + b)^3*ArcTan[Tan[e + f*x]])/a - (b^(3/2)*(35*a^2 + 28*a*b + 8*b^2)*Arc
Tan[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(a + b)) - ((8*a
^2 - 11*a*b - 4*b^2)*Cot[e + f*x])/(a + b))/(2*a*(a + b)) - (b*(9*a + 4*b)
*Cot[e + f*x])/(2*a*(a + b)*(a + b + b*Tan[e + f*x]^2)))/(4*a*(a + b)))/f
```

3.373.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.373.4 Maple [A] (verified)

Time = 23.49 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.82

method	result
derivativedivides	$-\frac{1}{(a+b)^3\tan(fx+e)}+\frac{b^2\left(\frac{\left(\frac{11}{8}a^2b+\frac{1}{2}ab^2\right)\tan(fx+e)^3+\frac{a\left(13a^2+17ab+4b^2\right)\tan(fx+e)}{8}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(35a^2+28ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right.}{8\sqrt{(a+b)b}}\right.}{a^3(a+b)^3}}{f}$
default	$-\frac{1}{(a+b)^3\tan(fx+e)}+\frac{b^2\left(\frac{\left(\frac{11}{8}a^2b+\frac{1}{2}ab^2\right)\tan(fx+e)^3+\frac{a\left(13a^2+17ab+4b^2\right)\tan(fx+e)}{8}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(35a^2+28ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right.}{8\sqrt{(a+b)b}}\right.}{a^3(a+b)^3}}{f}$
risch	$-\frac{x}{a^3}+\frac{i\left(-8a^5e^{8i(fx+e)}+13a^3b^2e^{8i(fx+e)}+36a^2b^3e^{8i(fx+e)}+16ab^4e^{8i(fx+e)}-32a^5e^{6i(fx+e)}-64a^4be^{6i(fx+e)}+26a\right.}{}$

input
```
int(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/(a+b)^3/tan(f*x+e)+b^2/a^3/(a+b)^3*(((11/8*a^2*b+1/2*a*b^2)*tan(f*
x+e)^3+1/8*a*(13*a^2+17*a*b+4*b^2)*tan(f*x+e))/(a+b+b*tan(f*x+e)^2)^2+1/8*
(35*a^2+28*a*b+8*b^2)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))
)-1/a^3*arctan(tan(f*x+e)))
```

3.373.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 486 vs. 2(165) = 330.

Time = 0.34 (sec) , antiderivative size = 1060, normalized size of antiderivative = 5.86

$$\int \frac{\cot^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output

```
[-1/32*(4*(8*a^5 + 13*a^3*b^2 + 6*a^2*b^3)*cos(f*x + e)^5 + 4*(16*a^4*b -
13*a^3*b^2 + 5*a^2*b^3 + 4*a*b^4)*cos(f*x + e)^3 - (35*a^2*b^3 + 28*a*b^4
+ 8*b^5 + (35*a^4*b + 28*a^3*b^2 + 8*a^2*b^3)*cos(f*x + e)^4 + 2*(35*a^3*b
^2 + 28*a^2*b^3 + 8*a*b^4)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*
a*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 +
 3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b)
)*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*s
in(f*x + e) + 4*(8*a^3*b^2 - 11*a^2*b^3 - 4*a*b^4)*cos(f*x + e) + 32*((a^5
 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*f*x*cos(f*x + e)^4 + 2*(a^4*b + 3*a^3*b^
2 + 3*a^2*b^3 + a*b^4)*f*x*cos(f*x + e)^2 + (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4
 + b^5)*f*x)*sin(f*x + e))/(((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*f*cos(f
*x + e)^4 + 2*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*f*cos(f*x + e)^2 +
 (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*f)*sin(f*x + e)), -1/16*(2*(8
*a^5 + 13*a^3*b^2 + 6*a^2*b^3)*cos(f*x + e)^5 + 2*(16*a^4*b - 13*a^3*b^2 +
 5*a^2*b^3 + 4*a*b^4)*cos(f*x + e)^3 + (35*a^2*b^3 + 28*a*b^4 + 8*b^5 + (3
5*a^4*b + 28*a^3*b^2 + 8*a^2*b^3)*cos(f*x + e)^4 + 2*(35*a^3*b^2 + 28*a^2*
b^3 + 8*a*b^4)*cos(f*x + e)^2)*sqrt(b/(a + b))*arctan(1/2*((a + 2*b)*cos(f
*x + e)^2 - b)*sqrt(b/(a + b))/(b*cos(f*x + e)*sin(f*x + e)))*sin(f*x + e)
 + 2*(8*a^3*b^2 - 11*a^2*b^3 - 4*a*b^4)*cos(f*x + e) + 16*((a^5 + 3*a^4*b
+ 3*a^3*b^2 + a^2*b^3)*f*x*cos(f*x + e)^4 + 2*(a^4*b + 3*a^3*b^2 + 3*a^...
```

3.373.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

input

```
integrate(cot(f*x+e)**2/(a+b*sec(f*x+e)**2)**3,x)
```

output

```
Timed out
```

3.373.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.72

$$\int \frac{\cot^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$

$$= \frac{\frac{\left(35a^2b^2+28ab^3+8b^4\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\left(a^6+3a^5b+3a^4b^2+a^3b^3\right)\sqrt{(a+b)b}} - \frac{\left(8a^2b^2-11ab^3-4b^4\right)\tan(fx+e)^4+8a^4+16a^3b+8a^2b^2+\left(16a^3b+3a^2\right.}{\left(a^5b^2+3a^4b^3+3a^3b^4+a^2b^5\right)\tan(fx+e)^5+2\left(a^6b+4a^5b^2+6a^4b^3+4a^3b^4+a^2b^5\right)\tan(fx+e)^3+(}}{8f}$$

input
```
integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
1/8*((35*a^2*b^2 + 28*a*b^3 + 8*b^4)*arctan(b*tan(f*x + e)/sqrt((a + b)*b)
)/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*sqrt((a + b)*b)) - ((8*a^2*b^2 -
11*a*b^3 - 4*b^4)*tan(f*x + e)^4 + 8*a^4 + 16*a^3*b + 8*a^2*b^2 + (16*a^3*
b + 3*a^2*b^2 - 17*a*b^3 - 4*b^4)*tan(f*x + e)^2)/((a^5*b^2 + 3*a^4*b^3 +
3*a^3*b^4 + a^2*b^5)*tan(f*x + e)^5 + 2*(a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4
*a^3*b^4 + a^2*b^5)*tan(f*x + e)^3 + (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*
b^3 + 5*a^3*b^4 + a^2*b^5)*tan(f*x + e)) - 8*(f*x + e)/a^3)/f
```

3.373.8 Giac [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.36

$$\int \frac{\cot^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$

$$= \frac{\frac{\left(35a^2b^2+28ab^3+8b^4\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{\left(a^6+3a^5b+3a^4b^2+a^3b^3\right)\sqrt{ab+b^2}} + \frac{11ab^3\tan(fx+e)^3+4b^4\tan(fx+e)^3+13a^2b^2\tan(fx+e)+17ab^3\tan(f}{\left(a^5+3a^4b+3a^3b^2+a^2b^3\right)\left(b\tan(fx+e)^2+a+b\right)^2}}{8f}$$

input
```
integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
1/8*((35*a^2*b^2 + 28*a*b^3 + 8*b^4)*(pi*floor((f*x + e)/pi + 1/2)*sgn(b)
+ arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^
3*b^3)*sqrt(a*b + b^2)) + (11*a*b^3*tan(f*x + e)^3 + 4*b^4*tan(f*x + e)^3
+ 13*a^2*b^2*tan(f*x + e) + 17*a*b^3*tan(f*x + e) + 4*b^4*tan(f*x + e))/((
a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*(b*tan(f*x + e)^2 + a + b)^2) - 8*(f*
x + e)/a^3 - 8/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*tan(f*x + e)))/f
```

3.373.9 Mupad [B] (verification not implemented)

Time = 25.32 (sec) , antiderivative size = 4890, normalized size of antiderivative = 27.02

$$\int \frac{\cot^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(cot(e + f*x)^2/(a + b/cos(e + f*x)^2)^3,x)
```

output
```
((tan(e + f*x)^4*(11*a*b^3 + 4*b^4 - 8*a^2*b^2))/(8*a^2*(a + b)^3) - 1/(a
+ b) + (tan(e + f*x)^2*(13*a*b^2 - 16*a^2*b + 4*b^3))/(8*a^2*(a + b)^2))/(
f*(tan(e + f*x)^3*(2*a*b + 2*b^2) + tan(e + f*x)*(2*a*b + a^2 + b^2) + b^2
*tan(e + f*x)^5)) - atan((286720*a^6*b^15*tan(e + f*x))/(286720*a^6*b^15 +
 3619840*a^7*b^14 + 21052416*a^8*b^13 + 74346496*a^9*b^12 + 177172480*a^10
*b^11 + 299796480*a^11*b^10 + 369346560*a^12*b^9 + 334344192*a^13*b^8 + 22
1663232*a^14*b^7 + 105978880*a^15*b^6 + 35445760*a^16*b^5 + 7864320*a^17*b
^4 + 1048576*a^18*b^3 + 65536*a^19*b^2) + (3619840*a^7*b^14*tan(e + f*x))/
(286720*a^6*b^15 + 3619840*a^7*b^14 + 21052416*a^8*b^13 + 74346496*a^9*b^1
2 + 177172480*a^10*b^11 + 299796480*a^11*b^10 + 369346560*a^12*b^9 + 33434
4192*a^13*b^8 + 221663232*a^14*b^7 + 105978880*a^15*b^6 + 35445760*a^16*b^
5 + 7864320*a^17*b^4 + 1048576*a^18*b^3 + 65536*a^19*b^2) + (21052416*a^8*
b^13*tan(e + f*x))/(286720*a^6*b^15 + 3619840*a^7*b^14 + 21052416*a^8*b^13
 + 74346496*a^9*b^12 + 177172480*a^10*b^11 + 299796480*a^11*b^10 + 3693465
60*a^12*b^9 + 334344192*a^13*b^8 + 221663232*a^14*b^7 + 105978880*a^15*b^6
 + 35445760*a^16*b^5 + 7864320*a^17*b^4 + 1048576*a^18*b^3 + 65536*a^19*b^
2) + (74346496*a^9*b^12*tan(e + f*x))/(286720*a^6*b^15 + 3619840*a^7*b^14
+ 21052416*a^8*b^13 + 74346496*a^9*b^12 + 177172480*a^10*b^11 + 299796480*
a^11*b^10 + 369346560*a^12*b^9 + 334344192*a^13*b^8 + 221663232*a^14*b^7 +
 105978880*a^15*b^6 + 35445760*a^16*b^5 + 7864320*a^17*b^4 + 1048576*a^...
```

3.374 $\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

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3.374.1 Optimal result

Integrand size = 23, antiderivative size = 230

$$\begin{aligned}\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx &= \frac{x}{a^3} - \frac{b^{5/2}(63a^2+36ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{9/2}f} \\ &+ \frac{(8a^3+32a^2b-15ab^2-4b^3)\cot(e+fx)}{8a^2(a+b)^4 f} \\ &- \frac{(8a^2-39ab-12b^2)\cot^3(e+fx)}{24a^2(a+b)^3 f} \\ &- \frac{b\cot^3(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2} \\ &- \frac{b(11a+4b)\cot^3(e+fx)}{8a^2(a+b)^2 f\left(a+b+b\tan^2(e+fx)\right)}\end{aligned}$$

output

```
x/a^3-1/8*b^(5/2)*(63*a^2+36*a*b+8*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(1
/2))/a^3/(a+b)^(9/2)/f+1/8*(8*a^3+32*a^2*b-15*a*b^2-4*b^3)*cot(f*x+e)/a^2/
(a+b)^4/f-1/24*(8*a^2-39*a*b-12*b^2)*cot(f*x+e)^3/a^2/(a+b)^3/f-1/4*b*cot(
f*x+e)^3/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2-1/8*b*(11*a+4*b)*cot(f*x+e)^3/a^
2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)
```

3.374.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.84 (sec) , antiderivative size = 3340, normalized size of antiderivative = 14.52

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
((63*a^2 + 36*a*b + 8*b^2)*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6
*((b^3*ArcTan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[
4*e]]) - ((I/2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(
-(a*Sin[f*x]) - 2*b*Sin[f*x] + a*Sin[2*e + f*x])]*Cos[2*e])/(64*a^3*Sqrt[a
 + b]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/64)*b^3*ArcTan[Sec[f*x]*(Co
s[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[2*e])/
(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*Sin[f*
x] + a*Sin[2*e + f*x])]*Sin[2*e])/(a^3*Sqrt[a + b]*f*Sqrt[b*Cos[4*e] - I*b
*Sin[4*e]])))/((a + b)^4*(a + b*Sec[e + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e
 + 2*f*x])*Csc[e]*Csc[e + f*x]^3*Sec[2*e]*Sec[e + f*x]^6*(-36*a^6*f*x*Cos[
f*x] - 336*a^5*b*f*x*Cos[f*x] - 1560*a^4*b^2*f*x*Cos[f*x] - 3600*a^3*b^3*f
*x*Cos[f*x] - 4260*a^2*b^4*f*x*Cos[f*x] - 2496*a*b^5*f*x*Cos[f*x] - 576*b^
6*f*x*Cos[f*x] + 36*a^6*f*x*Cos[3*f*x] + 240*a^5*b*f*x*Cos[3*f*x] + 408*a^
4*b^2*f*x*Cos[3*f*x] - 48*a^3*b^3*f*x*Cos[3*f*x] - 732*a^2*b^4*f*x*Cos[3*f
*x] - 672*a*b^5*f*x*Cos[3*f*x] - 192*b^6*f*x*Cos[3*f*x] + 36*a^6*f*x*Cos[2
*e - f*x] + 336*a^5*b*f*x*Cos[2*e - f*x] + 1560*a^4*b^2*f*x*Cos[2*e - f*x]
 + 3600*a^3*b^3*f*x*Cos[2*e - f*x] + 4260*a^2*b^4*f*x*Cos[2*e - f*x] + 249
6*a*b^5*f*x*Cos[2*e - f*x] + 576*b^6*f*x*Cos[2*e - f*x] + 36*a^6*f*x*Cos[2
*e + f*x] + 336*a^5*b*f*x*Cos[2*e + f*x] + 1560*a^4*b^2*f*x*Cos[2*e + f*x]
 + 3600*a^3*b^3*f*x*Cos[2*e + f*x] + 4260*a^2*b^4*f*x*Cos[2*e + f*x] + ...
```

3.374.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.14, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 4629, 2075, 374, 441, 445, 27, 445, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\tan(e+fx)^4\,(a+b\sec(e+fx)^2)^3}dx$$

$$\downarrow 4629$$

$$\frac{\int \frac{\cot^4(e+fx)}{(\tan^2(e+fx)+1)(a+b(\tan^2(e+fx)+1))^3}d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\cot^4(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^3}d\tan(e+fx)}{f}$$

$$\downarrow 374$$

$$\frac{\frac{\int \frac{\cot^4(e+fx)\left(-7b\tan^2(e+fx)+4a-3b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{4a(a+b)} - \frac{b\cot^3(e+fx)}{4a(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\downarrow 441$$

$$\frac{\frac{\frac{\int \frac{\cot^4(e+fx)\left(8a^2-39ba-12b^2-5b(11a+4b)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2a(a+b)} - \frac{b(11a+4b)\cot^3(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{4a(a+b)} - \frac{b\cot^3(e+fx)}{4a(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\downarrow 445$$

$$\frac{-\frac{\frac{\int \frac{3\cot^2(e+fx)\left(8a^3+32ba^2-15b^2a-4b^3+b\left(8a^2-39ba-12b^2\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{3(a+b)} - \frac{\left(8a^2-39ab-12b^2\right)\cot^3(e+fx)}{3(a+b)}}{2a(a+b)} - \frac{b(11a+4b)\cot^3(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{4a(a+b)} - \cdots}{f}$$

$$\downarrow 27$$

$$\frac{-\frac{\frac{\int \frac{\cot^2(e+fx)\left(8a^3+32ba^2-15b^2a-4b^3+b\left(8a^2-39ba-12b^2\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{a+b} - \frac{\left(8a^2-39ab-12b^2\right)\cot^3(e+fx)}{3(a+b)}}{2a(a+b)} - \frac{b(11a+4b)\cot^3(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{4a(a+b)} - \cdots}{f}$$

3.374. $\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

$$\downarrow 445$$

$$\frac{-\frac{\int \frac{8a^4+40ba^3+80b^2a^2+17b^3a+4b^4+b\left(8a^3+32ba^2-15b^2a-4b^3\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{a+b}-\frac{\left(8a^3+32a^2b-15ab^2-4b^3\right)\cot(e+fx)}{a+b}}{2a(a+b)}-\frac{\left(8a^2-39ab-12b^2\right)\cot^3(e+fx)}{3(a+b)}}{4a(a+b)}$$
$$f$$

$$\downarrow 397$$

$$\frac{-\frac{\frac{8(a+b)^4\int\frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)}{a}-\frac{b^3\left(63a^2+36ab+8b^2\right)\int\frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{a+b}-\frac{\left(8a^3+32a^2b-15ab^2-4b^3\right)\cot(e+fx)}{a+b}}{2a(a+b)}-\frac{\left(8a^2-39ab-12b^2\right)}{3(a+b)}}{4a(a+b)}$$
$$f$$

$$\downarrow 216$$

$$\frac{-\frac{\frac{8(a+b)^4\arctan(\tan(e+fx))}{a}-\frac{b^3\left(63a^2+36ab+8b^2\right)\int\frac{1}{b\tan^2(e+fx)+a+b}d\tan(e+fx)}{a}}{a+b}-\frac{\left(8a^3+32a^2b-15ab^2-4b^3\right)\cot(e+fx)}{a+b}}{2a(a+b)}-\frac{\left(8a^2-39ab-12b^2\right)\cot^3(e+fx)}{3(a+b)}}{4a(a+b)}$$
$$f$$

$$\downarrow 218$$

$$\frac{-\frac{\left(8a^2-39ab-12b^2\right)\cot^3(e+fx)}{3(a+b)}-\frac{-\frac{\frac{8(a+b)^4\arctan(\tan(e+fx))}{a}-\frac{b^{5/2}\left(63a^2+36ab+8b^2\right)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{a+b}}{a+b}-\frac{\left(8a^3+32a^2b-15ab^2-4b^3\right)\cot(e+fx)}{a+b}}{2a(a+b)}-\frac{}{2a(}}{4a(a+b)}$$
$$f$$

input
```
Int[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2)^3,x]
```

output
```
(-1/4*(b*Cot[e + f*x]^3)/(a*(a + b)*(a + b + b*Tan[e + f*x]^2)^2) + ((-1/3
*((8*a^2 - 39*a*b - 12*b^2)*Cot[e + f*x]^3)/(a + b) - (-(((8*(a + b)^4*Arc
Tan[Tan[e + f*x]])/a - (b^(5/2)*(63*a^2 + 36*a*b + 8*b^2)*ArcTan[(Sqrt[b]*
Tan[e + f*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(a + b)) - ((8*a^3 + 32*a^2*b
 - 15*a*b^2 - 4*b^3)*Cot[e + f*x])/(a + b))/(a + b))/(2*a*(a + b)) - (b*(1
1*a + 4*b)*Cot[e + f*x]^3)/(2*a*(a + b)*(a + b + b*Tan[e + f*x]^2)))/(4*a*
(a + b)))/f
```

3.374. $\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

3.374.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.374.4 Maple [A] (verified)

Time = 41.58 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.74

method	result
derivativedivides	$\frac{\frac{\arctan(\tan(fx+e))}{a^3}-\frac{1}{3(a+b)^3\tan(fx+e)^3}-\frac{-a-4b}{(a+b)^4\tan(fx+e)}-\frac{b^3\left(\frac{\left(\frac{15}{8}a^2b+\frac{1}{2}a\,b^2\right)\tan(fx+e)^3+\frac{a\left(17a^2+21ab+4b^2\right)\tan(fx+e)}{8}}{\left(a+b+b\tan(fx+e)^2\right)^2}\right.}{a^3(a+b)^4}}{f}$
default	$\frac{\frac{\arctan(\tan(fx+e))}{a^3}-\frac{1}{3(a+b)^3\tan(fx+e)^3}-\frac{-a-4b}{(a+b)^4\tan(fx+e)}-\frac{b^3\left(\frac{\left(\frac{15}{8}a^2b+\frac{1}{2}a\,b^2\right)\tan(fx+e)^3+\frac{a\left(17a^2+21ab+4b^2\right)\tan(fx+e)}{8}}{\left(a+b+b\tan(fx+e)^2\right)^2}\right.}{a^3(a+b)^4}}{f}$
risch	Expression too large to display

input
```
int(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/a^3*arctan(tan(f*x+e))-1/3/(a+b)^3/tan(f*x+e)^3-(-a-4*b)/(a+b)^4/ta
n(f*x+e)-b^3/a^3/(a+b)^4*(((15/8*a^2*b+1/2*a*b^2)*tan(f*x+e)^3+1/8*a*(17*a
^2+21*a*b+4*b^2)*tan(f*x+e))/(a+b+b*tan(f*x+e)^2)^2+1/8*(63*a^2+36*a*b+8*b
^2)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1/2))))
```

3.374.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 781 vs. $2(212) = 424$.

Time $= 0.37$ (sec) , antiderivative size $= 1649$, normalized size of antiderivative $= 7.17$

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output
```
[1/96*(4*(32*a^6 + 104*a^5*b + 51*a^3*b^3 + 18*a^2*b^4)*cos(f*x + e)^7 - 4
*(24*a^6 + 32*a^5*b - 208*a^4*b^2 + 102*a^3*b^3 - 9*a^2*b^4 - 12*a*b^5)*co
s(f*x + e)^5 - 4*(48*a^5*b + 160*a^4*b^2 - 155*a^3*b^3 + 72*a^2*b^4 + 24*a
*b^5)*cos(f*x + e)^3 + 3*((63*a^4*b^2 + 36*a^3*b^3 + 8*a^2*b^4)*cos(f*x +
e)^6 - 63*a^2*b^4 - 36*a*b^5 - 8*b^6 - (63*a^4*b^2 - 90*a^3*b^3 - 64*a^2*b
^4 - 16*a*b^5)*cos(f*x + e)^4 - (126*a^3*b^3 + 9*a^2*b^4 - 20*a*b^5 - 8*b^
6)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e
)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 + 4*((a^2 + 3*a*b + 2*b^2)*cos(f*x
+ e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))*sin(f*x + e) + b^2)/(a
^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*sin(f*x + e) - 12*(8*a^4*
b^2 + 32*a^3*b^3 - 15*a^2*b^4 - 4*a*b^5)*cos(f*x + e) + 96*((a^6 + 4*a^5*b
 + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*f*x*cos(f*x + e)^6 - (a^6 + 2*a^5*b -
2*a^4*b^2 - 8*a^3*b^3 - 7*a^2*b^4 - 2*a*b^5)*f*x*cos(f*x + e)^4 - (2*a^5*b
 + 7*a^4*b^2 + 8*a^3*b^3 + 2*a^2*b^4 - 2*a*b^5 - b^6)*f*x*cos(f*x + e)^2 -
 (a^4*b^2 + 4*a^3*b^3 + 6*a^2*b^4 + 4*a*b^5 + b^6)*f*x)*sin(f*x + e))/(((a
^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5*b^4)*f*cos(f*x + e)^6 - (a^9 +
2*a^8*b - 2*a^7*b^2 - 8*a^6*b^3 - 7*a^5*b^4 - 2*a^4*b^5)*f*cos(f*x + e)^4
- (2*a^8*b + 7*a^7*b^2 + 8*a^6*b^3 + 2*a^5*b^4 - 2*a^4*b^5 - a^3*b^6)*f*co
s(f*x + e)^2 - (a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 + 4*a^4*b^5 + a^3*b^6)*f)*
sin(f*x + e)), 1/48*(2*(32*a^6 + 104*a^5*b + 51*a^3*b^3 + 18*a^2*b^4)*c...
```

3.374.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)**4/(a+b*sec(f*x+e)**2)**3,x)
```

output
```
Timed out
```

3.374.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.78

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx =$$

$$-\frac{3\left(63a^2b^3+36ab^4+8b^5\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\left(a^7+4a^6b+6a^5b^2+4a^4b^3+a^3b^4\right)\sqrt{(a+b)b}} - \frac{3\left(8a^3b^2+32a^2b^3-15ab^4-4b^5\right)\tan(fx+e)^6-8a^5-24a^4b-24a^3b^2-8a^2b^3+(48a^4b+232}{\left(a^6b^2+4a^5b^3+6a^4b^4+4a^3b^5+a^2b^6\right)\tan(fx+e)^7+2\left(a^7b+5a^6b^2+10a^5b^3+10a^4b^4+5a\right.}$$
$$24f$$

input
```
integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
-1/24*(3*(63*a^2*b^3 + 36*a*b^4 + 8*b^5)*arctan(b*tan(f*x + e)/sqrt((a + b
)*b))/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*sqrt((a + b)*b))
- (3*(8*a^3*b^2 + 32*a^2*b^3 - 15*a*b^4 - 4*b^5)*tan(f*x + e)^6 - 8*a^5 -
24*a^4*b - 24*a^3*b^2 - 8*a^2*b^3 + (48*a^4*b + 232*a^3*b^2 + 133*a^2*b^3
- 63*a*b^4 - 12*b^5)*tan(f*x + e)^4 + 8*(3*a^5 + 16*a^4*b + 23*a^3*b^2 + 1
0*a^2*b^3)*tan(f*x + e)^2)/((a^6*b^2 + 4*a^5*b^3 + 6*a^4*b^4 + 4*a^3*b^5 +
 a^2*b^6)*tan(f*x + e)^7 + 2*(a^7*b + 5*a^6*b^2 + 10*a^5*b^3 + 10*a^4*b^4
+ 5*a^3*b^5 + a^2*b^6)*tan(f*x + e)^5 + (a^8 + 6*a^7*b + 15*a^6*b^2 + 20*a
^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)*tan(f*x + e)^3) - 24*(f*x + e)/
a^3)/f
```

3.374.8 Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.31

$$\int \frac{\cot^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx =$$

$$-\frac{\frac{3\left(63a^2b^3+36ab^4+8b^5\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{\left(a^7+4a^6b+6a^5b^2+4a^4b^3+a^3b^4\right)\sqrt{ab+b^2}}+\frac{3\left(15ab^4\tan(fx+e)^3+4b^5\tan(fx+e)^3+17a^2b^3\tan(fx+e)+21a\right.}{\left(a^6+4a^5b+6a^4b^2+4a^3b^3+a^2b^4\right)\left(b\tan(fx+e\right.}}{24f}$$

input
```
integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
-1/24*(3*(63*a^2*b^3 + 36*a*b^4 + 8*b^5)*(pi*floor((f*x + e)/pi + 1/2)*sgn
(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^7 + 4*a^6*b + 6*a^5*b^2
+ 4*a^4*b^3 + a^3*b^4)*sqrt(a*b + b^2)) + 3*(15*a*b^4*tan(f*x + e)^3 + 4*b
^5*tan(f*x + e)^3 + 17*a^2*b^3*tan(f*x + e) + 21*a*b^4*tan(f*x + e) + 4*b^
5*tan(f*x + e))/((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*(b*tan(
f*x + e)^2 + a + b)^2) - 24*(f*x + e)/a^3 - 8*(3*a*tan(f*x + e)^2 + 12*b*t
an(f*x + e)^2 - a - b)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*tan(f*
x + e)^3))/f
```

3.374.9 Mupad [B] (verification not implemented)

Time = 25.94 (sec) , antiderivative size = 7057, normalized size of antiderivative = 30.68

$$\int \frac{\cot^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(cot(e + f*x)^4/(a + b/cos(e + f*x)^2)^3,x)
```

output

```
atan((860160*a^6*b^20*tan(e + f*x))/(860160*a^6*b^20 + 14515200*a^7*b^19 +
 115347456*a^8*b^18 + 570587136*a^9*b^17 + 1961717760*a^10*b^16 + 49658112
00*a^11*b^15 + 9577308160*a^12*b^14 + 14379552768*a^13*b^13 + 17038737408*
a^14*b^12 + 16066462720*a^15*b^11 + 12106321920*a^16*b^10 + 7294187520*a^1
7*b^9 + 3502829568*a^18*b^8 + 1329527808*a^19*b^7 + 392232960*a^20*b^6 + 8
7162880*a^21*b^5 + 13762560*a^22*b^4 + 1376256*a^23*b^3 + 65536*a^24*b^2)
+ (14515200*a^7*b^19*tan(e + f*x))/(860160*a^6*b^20 + 14515200*a^7*b^19 +
115347456*a^8*b^18 + 570587136*a^9*b^17 + 1961717760*a^10*b^16 + 496581120
0*a^11*b^15 + 9577308160*a^12*b^14 + 14379552768*a^13*b^13 + 17038737408*a
^14*b^12 + 16066462720*a^15*b^11 + 12106321920*a^16*b^10 + 7294187520*a^17
*b^9 + 3502829568*a^18*b^8 + 1329527808*a^19*b^7 + 392232960*a^20*b^6 + 87
162880*a^21*b^5 + 13762560*a^22*b^4 + 1376256*a^23*b^3 + 65536*a^24*b^2) +
 (115347456*a^8*b^18*tan(e + f*x))/(860160*a^6*b^20 + 14515200*a^7*b^19 +
115347456*a^8*b^18 + 570587136*a^9*b^17 + 1961717760*a^10*b^16 + 496581120
0*a^11*b^15 + 9577308160*a^12*b^14 + 14379552768*a^13*b^13 + 17038737408*a
^14*b^12 + 16066462720*a^15*b^11 + 12106321920*a^16*b^10 + 7294187520*a^17
*b^9 + 3502829568*a^18*b^8 + 1329527808*a^19*b^7 + 392232960*a^20*b^6 + 87
162880*a^21*b^5 + 13762560*a^22*b^4 + 1376256*a^23*b^3 + 65536*a^24*b^2) +
 (570587136*a^9*b^17*tan(e + f*x))/(860160*a^6*b^20 + 14515200*a^7*b^19 +
115347456*a^8*b^18 + 570587136*a^9*b^17 + 1961717760*a^10*b^16 + 496581...
```

3.375 $\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

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3.375.1 Optimal result

Integrand size = 23, antiderivative size = 285

$$\begin{aligned}\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = &-\frac{x}{a^3}+\frac{b^{7/2}(99a^2+44ab+8b^2)\arctan\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{11/2}f}\\ &-\frac{(8a^4+40a^3b+80a^2b^2-19ab^3-4b^4)\cot(e+fx)}{8a^2(a+b)^5f}\\ &+\frac{(8a^3+32a^2b-51ab^2-12b^3)\cot^3(e+fx)}{24a^2(a+b)^4f}\\ &-\frac{(8a^2-75ab-20b^2)\cot^5(e+fx)}{40a^2(a+b)^3f}\\ &-\frac{b\cot^5(e+fx)}{4a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^2}\\ &-\frac{b(13a+4b)\cot^5(e+fx)}{8a^2(a+b)^2f\left(a+b+b\tan^2(e+fx)\right)}\end{aligned}$$

output

```
-x/a^3+1/8*b^(7/2)*(99*a^2+44*a*b+8*b^2)*arctan(b^(1/2)*tan(f*x+e)/(a+b)^(
1/2))/a^3/(a+b)^(11/2)/f-1/8*(8*a^4+40*a^3*b+80*a^2*b^2-19*a*b^3-4*b^4)*co
t(f*x+e)/a^2/(a+b)^5/f+1/24*(8*a^3+32*a^2*b-51*a*b^2-12*b^3)*cot(f*x+e)^3/
a^2/(a+b)^4/f-1/40*(8*a^2-75*a*b-20*b^2)*cot(f*x+e)^5/a^2/(a+b)^3/f-1/4*b*
cot(f*x+e)^5/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^2-1/8*b*(13*a+4*b)*cot(f*x+e)^
5/a^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)
```

3.375.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 12.57 (sec) , antiderivative size = 976, normalized size of antiderivative = 3.42

$$\int \frac{\cot^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = -\frac{x(a+2b+a\cos(2e+2fx))^3\sec^6(e+fx)}{8a^3\left(a+b\sec^2(e+fx)\right)^3}$$
$$+\frac{(11a\cos(e)+26b\cos(e))(a+2b+a\cos(2e+2fx))^3\csc(e)\csc^2(e+fx)\sec^6(e+fx)}{120(a+b)^4 f\left(a+b\sec^2(e+fx)\right)^3}$$
$$-\frac{(a+2b+a\cos(2e+2fx))^3\cot(e)\csc^4(e+fx)\sec^6(e+fx)}{40(a+b)^3 f\left(a+b\sec^2(e+fx)\right)^3}$$
$$+\frac{(99a^2+44ab+8b^2)(a+2b+a\cos(2e+2fx))^3\sec^6(e+fx)\left(-\frac{b^4\arctan\left(\sec(fx)\left(\frac{\cos(2e)}{2\sqrt{a+b}\sqrt{b\cos(4e)-ib\sin(4e)}}-\frac{2\sqrt{}\right.\right.}{64a^3\sqrt{}}\right.}{}$$
$$+\frac{(a+2b+a\cos(2e+2fx))^3\csc(e)\csc^5(e+fx)\sec^6(e+fx)\sin(fx)}{40(a+b)^3 f\left(a+b\sec^2(e+fx)\right)^3}$$
$$+\frac{(a+2b+a\cos(2e+2fx))^3\csc(e)\csc^3(e+fx)\sec^6(e+fx)(-11a\sin(fx)-26b\sin(fx))}{120(a+b)^4 f\left(a+b\sec^2(e+fx)\right)^3}$$
$$+\frac{(a+2b+a\cos(2e+2fx))^3\csc(e)\csc(e+fx)\sec^6(e+fx)\left(23a^2\sin(fx)+106ab\sin(fx)+173b^2\sin(\right.}{120(a+b)^5 f\left(a+b\sec^2(e+fx)\right)^3}$$
$$+\frac{(a+2b+a\cos(2e+2fx))\sec(2e)\sec^6(e+fx)\left(ab^5\sin(2e)+2b^6\sin(2e)-ab^5\sin(2fx)\right)}{16a^3(a+b)^4 f\left(a+b\sec^2(e+fx)\right)^3}$$
$$+\frac{(a+2b+a\cos(2e+2fx))^2\sec(2e)\sec^6(e+fx)\left(-21a^2b^4\sin(2e)-52ab^5\sin(2e)-16b^6\sin(2e)+21a^\right.}{64a^3(a+b)^5 f\left(a+b\sec^2(e+fx)\right)^3}$$

input
```
Integrate[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]
```

output

```
-1/8*(x*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6)/(a^3*(a + b*Sec[e
 + f*x]^2)^3) + ((11*a*Cos[e] + 26*b*Cos[e])*(a + 2*b + a*Cos[2*e + 2*f*x]
)^3*Csc[e]*Csc[e + f*x]^2*Sec[e + f*x]^6)/(120*(a + b)^4*f*(a + b*Sec[e +
f*x]^2)^3) - ((a + 2*b + a*Cos[2*e + 2*f*x])^3*Cot[e]*Csc[e + f*x]^4*Sec[e
 + f*x]^6)/(40*(a + b)^3*f*(a + b*Sec[e + f*x]^2)^3) + ((99*a^2 + 44*a*b +
 8*b^2)*(a + 2*b + a*Cos[2*e + 2*f*x])^3*Sec[e + f*x]^6*(-1/64*(b^4*ArcTan
[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/
2)*Sin[2*e])/(Sqrt[a + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x])
 - 2*b*Sin[f*x] + a*Sin[2*e + f*x])]*Cos[2*e])/(a^3*Sqrt[a + b]*f*Sqrt[b*C
os[4*e] - I*b*Sin[4*e]]) + ((I/64)*b^4*ArcTan[Sec[f*x]*(Cos[2*e]/(2*Sqrt[a
 + b]*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]]) - ((I/2)*Sin[2*e])/(Sqrt[a + b]*Sqr
t[b*Cos[4*e] - I*b*Sin[4*e]]))*(-(a*Sin[f*x]) - 2*b*Sin[f*x] + a*Sin[2*e +
 f*x])]*Sin[2*e])/(a^3*Sqrt[a + b]*f*Sqrt[b*Cos[4*e] - I*b*Sin[4*e]])))/((
a + b)^5*(a + b*Sec[e + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f*x])^3*Csc
[e]*Csc[e + f*x]^5*Sec[e + f*x]^6*Sin[f*x])/(40*(a + b)^3*f*(a + b*Sec[e +
 f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f*x])^3*Csc[e]*Csc[e + f*x]^3*Sec[
e + f*x]^6*(-11*a*Sin[f*x] - 26*b*Sin[f*x]))/(120*(a + b)^4*f*(a + b*Sec[e
 + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f*x])^3*Csc[e]*Csc[e + f*x]*Sec[
e + f*x]^6*(23*a^2*Sin[f*x] + 106*a*b*Sin[f*x] + 173*b^2*Sin[f*x]))/(120*(
a + b)^5*f*(a + b*Sec[e + f*x]^2)^3) + ((a + 2*b + a*Cos[2*e + 2*f*x])*...
```

3.375.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.12, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 4629, 2075, 374, 441, 445, 27, 445, 27, 445, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\tan(e+fx)^6\,(a+b\sec(e+fx)^2)^3}dx$$

$$\downarrow 4629$$

$$\frac{\int \frac{\cot^6(e+fx)}{(\tan^2(e+fx)+1)(a+b(\tan^2(e+fx)+1))^3}d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\cot^6(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^3}d\tan(e+fx)}{f}$$

$$\downarrow 374$$

$$\frac{\int \frac{\cot^6(e+fx)\left(-9b\tan^2(e+fx)+4a-5b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^2}d\tan(e+fx)}{4a(a+b)} - \frac{b\cot^5(e+fx)}{4a(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\downarrow 441$$

$$\frac{\frac{\int \frac{\cot^6(e+fx)\left(8a^2-75ba-20b^2-7b(13a+4b)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{2a(a+b)} - \frac{b(13a+4b)\cot^5(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{4a(a+b)} - \frac{b\cot^5(e+fx)}{4a(a+b)(a+b\tan^2(e+fx)+b)^2}}{f}$$

$$\downarrow 445$$

$$\frac{-\frac{\frac{\int \frac{5\cot^4(e+fx)\left(8a^3+32ba^2-51b^2a-12b^3+b\left(8a^2-75ba-20b^2\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{5(a+b)}}{2a(a+b)} - \frac{\left(8a^2-75ab-20b^2\right)\cot^5(e+fx)}{5(a+b)}}{2a(a+b)} - \frac{b(13a+4b)\cot^5(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{4a(a+b)} -$$

$$\downarrow 27$$

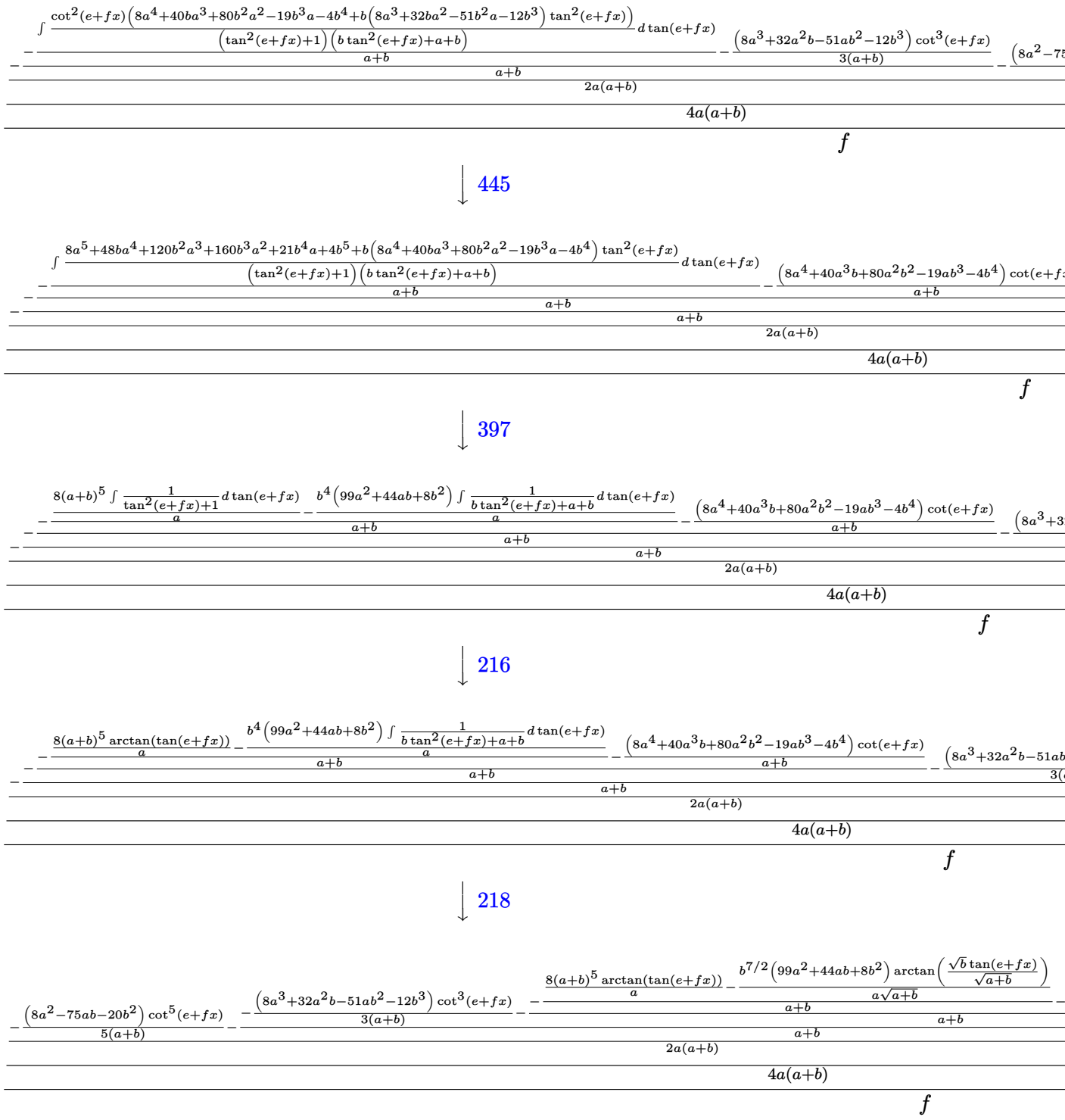
$$\frac{-\frac{\frac{\int \frac{\cot^4(e+fx)\left(8a^3+32ba^2-51b^2a-12b^3+b\left(8a^2-75ba-20b^2\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{a+b}}{2a(a+b)} - \frac{\left(8a^2-75ab-20b^2\right)\cot^5(e+fx)}{5(a+b)}}{2a(a+b)} - \frac{b(13a+4b)\cot^5(e+fx)}{2a(a+b)\left(a+b\tan^2(e+fx)+b\right)}}{4a(a+b)} -$$

$$\downarrow 445$$

$$\frac{-\frac{\frac{\int \frac{3\cot^2(e+fx)\left(8a^4+40ba^3+80b^2a^2-19b^3a-4b^4+b\left(8a^3+32ba^2-51b^2a-12b^3\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)}d\tan(e+fx)}{3(a+b)}}{a+b} - \frac{\left(8a^3+32a^2b-51ab^2-12b^3\right)\cot^3(e+fx)}{3(a+b)}}{2a(a+b)} - \frac{\left(8a^2-}{}}{4a(a+b)} -$$

$$\downarrow 27$$

3.375. $\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$



input `Int[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2)^3,x]`

3.375. $\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx$

output
```
(-1/4*(b*Cot[e + f*x]^5)/(a*(a + b)*(a + b + b*Tan[e + f*x]^2)^2) + ((-1/5
*((8*a^2 - 75*a*b - 20*b^2)*Cot[e + f*x]^5)/(a + b) - (-1/3*((8*a^3 + 32*a
^2*b - 51*a*b^2 - 12*b^3)*Cot[e + f*x]^3)/(a + b) - (-(((8*(a + b)^5*ArcTa
n[Tan[e + f*x]])/a - (b^(7/2)*(99*a^2 + 44*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Ta
n[e + f*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(a + b)) - ((8*a^4 + 40*a^3*b +
 80*a^2*b^2 - 19*a*b^3 - 4*b^4)*Cot[e + f*x])/(a + b))/(a + b))/(a + b))/(
2*a*(a + b)) - (b*(13*a + 4*b)*Cot[e + f*x]^5)/(2*a*(a + b)*(a + b + b*Tan
[e + f*x]^2)))/(4*a*(a + b)))/f
```

3.375.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.375.4 Maple [A] (verified)

Time = 69.09 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.70

method	result
derivativedivides	$\frac{-\frac{\arctan(\tan(fx+e))}{a^3}+\frac{b^4\left(\frac{\left(\frac{19}{8}a^2b+\frac{1}{2}ab^2\right)\tan(fx+e)^3+\frac{a\left(21a^2+25ab+4b^2\right)\tan(fx+e)}{8}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(99a^2+44ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\sqrt{(a+b)b}}\right)}{a^3(a+b)^5}}{f}$
default	$\frac{-\frac{\arctan(\tan(fx+e))}{a^3}+\frac{b^4\left(\frac{\left(\frac{19}{8}a^2b+\frac{1}{2}ab^2\right)\tan(fx+e)^3+\frac{a\left(21a^2+25ab+4b^2\right)\tan(fx+e)}{8}}{\left(a+b+b\tan(fx+e)^2\right)^2}+\frac{\left(99a^2+44ab+8b^2\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{8\sqrt{(a+b)b}}\right)}{a^3(a+b)^5}}{f}$
risch	Expression too large to display

input
```
int(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/a^3*arctan(tan(f*x+e))+b^4/a^3/(a+b)^5*(((19/8*a^2*b+1/2*a*b^2)*ta
n(f*x+e)^3+1/8*a*(21*a^2+25*a*b+4*b^2)*tan(f*x+e))/(a+b+b*tan(f*x+e)^2)^2+
1/8*(99*a^2+44*a*b+8*b^2)/((a+b)*b)^(1/2)*arctan(b*tan(f*x+e)/((a+b)*b)^(1
/2)))-1/5/(a+b)^3/tan(f*x+e)^5-1/3*(-a-4*b)/(a+b)^4/tan(f*x+e)^3-(a^2+5*a*
b+10*b^2)/(a+b)^5/tan(f*x+e))
```

3.375.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1071 vs. 2(265) = 530.

Time = 0.42 (sec) , antiderivative size = 2229, normalized size of antiderivative = 7.82

$$\int \frac{\cot^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="fricas")
```

output

```
[-1/480*(4*(184*a^7 + 848*a^6*b + 1384*a^5*b^2 + 315*a^3*b^4 + 90*a^2*b^5)
*cos(f*x + e)^9 - 4*(280*a^7 + 1032*a^6*b + 864*a^5*b^2 - 2768*a^4*b^3 + 9
45*a^3*b^4 - 15*a^2*b^5 - 60*a*b^6)*cos(f*x + e)^7 + 4*(120*a^7 + 40*a^6*b
 - 1416*a^5*b^2 - 4272*a^4*b^3 + 2329*a^3*b^4 - 585*a^2*b^5 - 180*a*b^6)*c
os(f*x + e)^5 + 20*(48*a^6*b + 184*a^5*b^2 + 200*a^4*b^3 - 575*a^3*b^4 + 1
53*a^2*b^5 + 36*a*b^6)*cos(f*x + e)^3 - 15*((99*a^4*b^3 + 44*a^3*b^4 + 8*a
^2*b^5)*cos(f*x + e)^8 + 99*a^2*b^5 + 44*a*b^6 + 8*b^7 - 2*(99*a^4*b^3 - 5
5*a^3*b^4 - 36*a^2*b^5 - 8*a*b^6)*cos(f*x + e)^6 + (99*a^4*b^3 - 352*a^3*b
^4 - 69*a^2*b^5 + 12*a*b^6 + 8*b^7)*cos(f*x + e)^4 + 2*(99*a^3*b^4 - 55*a^
2*b^5 - 36*a*b^6 - 8*b^7)*cos(f*x + e)^2)*sqrt(-b/(a + b))*log(((a^2 + 8*a
*b + 8*b^2)*cos(f*x + e)^4 - 2*(3*a*b + 4*b^2)*cos(f*x + e)^2 - 4*((a^2 +
3*a*b + 2*b^2)*cos(f*x + e)^3 - (a*b + b^2)*cos(f*x + e))*sqrt(-b/(a + b))
*sin(f*x + e) + b^2)/(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2))*si
n(f*x + e) + 60*(8*a^5*b^2 + 40*a^4*b^3 + 80*a^3*b^4 - 19*a^2*b^5 - 4*a*b^
6)*cos(f*x + e) + 480*((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^
4 + a^2*b^5)*f*x*cos(f*x + e)^8 - 2*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4
 - 4*a^2*b^5 - a*b^6)*f*x*cos(f*x + e)^6 + (a^7 + a^6*b - 9*a^5*b^2 - 25*a
^4*b^3 - 25*a^3*b^4 - 9*a^2*b^5 + a*b^6 + b^7)*f*x*cos(f*x + e)^4 + 2*(a^6
*b + 4*a^5*b^2 + 5*a^4*b^3 - 5*a^2*b^5 - 4*a*b^6 - b^7)*f*x*cos(f*x + e)^2
 + (a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*f*x)...
```

3.375.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^3}\,dx = \text{Timed out}$$

input

```
integrate(cot(f*x+e)**6/(a+b*sec(f*x+e)**2)**3,x)
```

output

```
Timed out
```

3.375.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 520, normalized size of antiderivative = 1.82

$$\int \frac{\cot^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$
$$= \frac{15\left(99a^2b^4+44ab^5+8b^6\right)\arctan\left(\frac{b\tan(fx+e)}{\sqrt{(a+b)b}}\right)}{\left(a^8+5a^7b+10a^6b^2+10a^5b^3+5a^4b^4+a^3b^5\right)\sqrt{(a+b)b}} - \frac{15\left(8a^4b^2+40a^3b^3+80a^2b^4-19ab^5-4b^6\right)\tan(fx+e)^8+5\left(48a^5b+280a^4b^2+680a^3b^3\right.}{\left(a^7b^2+5a^6b^3+10a^5b^4+10a^4b^5+5a^3b^6+a^2b^7\right.}$$

input
```
integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="maxima")
```

output
```
1/120*(15*(99*a^2*b^4 + 44*a*b^5 + 8*b^6)*arctan(b*tan(f*x + e)/sqrt((a +
b)*b))/((a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*sq
rt((a + b)*b)) - (15*(8*a^4*b^2 + 40*a^3*b^3 + 80*a^2*b^4 - 19*a*b^5 - 4*b
^6)*tan(f*x + e)^8 + 5*(48*a^5*b + 280*a^4*b^2 + 680*a^3*b^3 + 385*a^2*b^4
 - 75*a*b^5 - 12*b^6)*tan(f*x + e)^6 + 24*a^6 + 96*a^5*b + 144*a^4*b^2 + 9
6*a^3*b^3 + 24*a^2*b^4 + 8*(15*a^6 + 95*a^5*b + 258*a^4*b^2 + 291*a^3*b^3
+ 113*a^2*b^4)*tan(f*x + e)^4 - 8*(5*a^6 + 29*a^5*b + 57*a^4*b^2 + 47*a^3*
b^3 + 14*a^2*b^4)*tan(f*x + e)^2)/((a^7*b^2 + 5*a^6*b^3 + 10*a^5*b^4 + 10*
a^4*b^5 + 5*a^3*b^6 + a^2*b^7)*tan(f*x + e)^9 + 2*(a^8*b + 6*a^7*b^2 + 15*
a^6*b^3 + 20*a^5*b^4 + 15*a^4*b^5 + 6*a^3*b^6 + a^2*b^7)*tan(f*x + e)^7 +
(a^9 + 7*a^8*b + 21*a^7*b^2 + 35*a^6*b^3 + 35*a^5*b^4 + 21*a^4*b^5 + 7*a^3
*b^6 + a^2*b^7)*tan(f*x + e)^5) - 120*(f*x + e)/a^3)/f
```

3.375.8 Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.36

$$\int \frac{\cot^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx$$
$$= \frac{15\left(99a^2b^4+44ab^5+8b^6\right)\left(\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(fx+e)}{\sqrt{ab+b^2}}\right)\right)}{\left(a^8+5a^7b+10a^6b^2+10a^5b^3+5a^4b^4+a^3b^5\right)\sqrt{ab+b^2}} + \frac{15\left(19ab^5\tan(fx+e)^3+4b^6\tan(fx+e)^3+21a^2b^4\tan(fx+e)+25ab\right.}{\left(a^7+5a^6b+10a^5b^2+10a^4b^3+5a^3b^4+a^2b^5\right)\left(b\tan(f\right.}$$

input
```
integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

output
```
1/120*(15*(99*a^2*b^4 + 44*a*b^5 + 8*b^6)*(pi*floor((f*x + e)/pi + 1/2)*sg
n(b) + arctan(b*tan(f*x + e)/sqrt(a*b + b^2)))/((a^8 + 5*a^7*b + 10*a^6*b^
2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*sqrt(a*b + b^2)) + 15*(19*a*b^5*tan(
f*x + e)^3 + 4*b^6*tan(f*x + e)^3 + 21*a^2*b^4*tan(f*x + e) + 25*a*b^5*tan
(f*x + e) + 4*b^6*tan(f*x + e))/((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3
+ 5*a^3*b^4 + a^2*b^5)*(b*tan(f*x + e)^2 + a + b)^2) - 120*(f*x + e)/a^3 -
 8*(15*a^2*tan(f*x + e)^4 + 75*a*b*tan(f*x + e)^4 + 150*b^2*tan(f*x + e)^4
 - 5*a^2*tan(f*x + e)^2 - 25*a*b*tan(f*x + e)^2 - 20*b^2*tan(f*x + e)^2 +
3*a^2 + 6*a*b + 3*b^2)/((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4
 + b^5)*tan(f*x + e)^5))/f
```

3.375.9 Mupad [B] (verification not implemented)

Time = 27.97 (sec) , antiderivative size = 7460, normalized size of antiderivative = 26.18

$$\int \frac{\cot^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(cot(e + f*x)^6/(a + b/cos(e + f*x)^2)^3,x)
```

output
```
atan((((((65536*a^10*b^27 + 1654784*a^11*b^26 + 21954560*a^12*b^25 + 194478
080*a^13*b^24 + 1247936512*a^14*b^23 + 6060916736*a^15*b^22 + 22968795136*
a^16*b^21 + 69506170880*a^17*b^20 + 170976215040*a^18*b^19 + 346596343808*
a^19*b^18 + 585044721664*a^20*b^17 + 828584034304*a^21*b^16 + 989821665280
*a^22*b^15 + 1000564490240*a^23*b^14 + 856970493952*a^24*b^13 + 6215385743
36*a^25*b^12 + 380751118336*a^26*b^11 + 196065116160*a^27*b^10 + 842304716
80*a^28*b^9 + 29853974528*a^29*b^8 + 8588754944*a^30*b^7 + 1957904384*a^31
*b^6 + 340787200*a^32*b^5 + 42598400*a^33*b^4 + 3407872*a^34*b^3 + 131072*
a^35*b^2 + (tan(e + f*x)*(524288*a^12*b^28 + 13369344*a^13*b^27 + 16384000
0*a^14*b^26 + 1284505600*a^15*b^25 + 7235174400*a^16*b^24 + 31171543040*a^
17*b^23 + 106779115520*a^18*b^22 + 298450944000*a^19*b^21 + 693069414400*a
^20*b^20 + 1354635673600*a^21*b^19 + 2249325281280*a^22*b^18 + 31938471526
40*a^23*b^17 + 3894935552000*a^24*b^16 + 4089682329600*a^25*b^15 + 3700188
774400*a^26*b^14 + 2882252308480*a^27*b^13 + 1927993098240*a^28*b^12 + 110
2610432000*a^29*b^11 + 535553638400*a^30*b^10 + 218864025600*a^31*b^9 + 74
281123840*a^32*b^8 + 20559953920*a^33*b^7 + 4521984000*a^34*b^6 + 76021760
0*a^35*b^5 + 91750400*a^36*b^4 + 7077888*a^37*b^3 + 262144*a^38*b^2)*1i)/(
2*a^3))*1i)/(2*a^3) + tan(e + f*x)*(131072*a^6*b^28 + 3342336*a^7*b^27 + 4
0960000*a^8*b^26 + 319234048*a^9*b^25 + 1768817664*a^10*b^24 + 7390051328*
a^11*b^23 + 24132297728*a^12*b^22 + 63100984320*a^13*b^21 + 13447268454...
```

3.376 $\int \sqrt{a+b\sec^2(e+fx)}\tan^5(e+fx)\,dx$

3.376.1 Optimal result . . . 2740
3.376.2 Mathematica [A] (verified) . . . 2740
3.376.3 Rubi [A] (verified) . . . 2741
3.376.4 Maple [B] (verified) . . . 2743
3.376.5 Fricas [B] (verification not implemented) . . . 2743
3.376.6 Sympy [F] . . . 2744
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3.376.8 Giac [B] (verification not implemented) . . . 2745
3.376.9 Mupad [F(-1)] . . . 2745

3.376.1 Optimal result

Integrand size = 25, antiderivative size = 111

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^5(e+fx)\,dx = -\frac{\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} + \frac{\sqrt{a+b\sec^2(e+fx)}}{f} - \frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3b^2f} + \frac{\left(a+b\sec^2(e+fx)\right)^{5/2}}{5b^2f}$$

output

```
-1/3*(a+2*b)*(a+b*sec(f*x+e)^2)^(3/2)/b^2/f+1/5*(a+b*sec(f*x+e)^2)^(5/2)/b
^2/f-arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))*a^(1/2)/f+(a+b*sec(f*x+e)^2
)^(1/2)/f
```

3.376.2 Mathematica [A] (verified)

Time = 2.83 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.25

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^5(e+fx)\,dx$$

$$= \frac{15b^2(a+b\sec^2(e+fx)) - 5a(a+b\sec^2(e+fx))^2 - 10b(a+b\sec^2(e+fx))^2 + 3(a+b\sec^2(e+fx))^3 -}{15b^2f\sqrt{a+b\sec^2(e+fx)}}$$

input
```
Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^5,x]
```

output
```
(15*b^2*(a + b*Sec[e + f*x]^2) - 5*a*(a + b*Sec[e + f*x]^2)^2 - 10*b*(a +
b*Sec[e + f*x]^2)^2 + 3*(a + b*Sec[e + f*x]^2)^3 - 15*a*b^2*ArcTanh[Sqrt[1
 + (b*Sec[e + f*x]^2)/a]]*Sqrt[1 + (b*Sec[e + f*x]^2)/a])/(15*b^2*f*Sqrt[a
 + b*Sec[e + f*x]^2])
```

3.376.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4627, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \tan(e+fx)^5\sqrt{a+b\sec(e+fx)^2}dx$$

$$\downarrow 4627$$

$$\frac{\int \cos(e+fx)\left(1-\sec^2(e+fx)\right)^2\sqrt{b\sec^2(e+fx)+a}d\sec(e+fx)}{f}$$

$$\downarrow 354$$

$$\frac{\int \cos(e+fx)\left(1-\sec^2(e+fx)\right)^2\sqrt{b\sec^2(e+fx)+a}d\sec^2(e+fx)}{2f}$$

$$\downarrow 99$$

$$\frac{\int \left(\frac{\left(b\sec^2(e+fx)+a\right)^{3/2}}{b}+\cos(e+fx)\sqrt{b\sec^2(e+fx)+a}+\frac{(-a-2b)\sqrt{b\sec^2(e+fx)+a}}{b}\right)d\sec^2(e+fx)}{2f}$$

$$\downarrow 2009$$

$$\frac{-2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)+\frac{2\left(a+b\sec^2(e+fx)\right)^{5/2}}{5b^2}-\frac{2(a+2b)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3b^2}+2\sqrt{a+b\sec^2(e+fx)}}{2f}$$

input
```
Int[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^5,x]
```

output
```
(-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]] + 2*Sqrt[a + b*Sec
[e + f*x]^2] - (2*(a + 2*b)*(a + b*Sec[e + f*x]^2)^(3/2))/(3*b^2) + (2*(a
+ b*Sec[e + f*x]^2)^(5/2))/(5*b^2))/(2*f)
```

3.376.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.376.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 567 vs. 2(95) = 190.

Time = 12.02 (sec) , antiderivative size = 568, normalized size of antiderivative = 5.12

method	result
default	$-\frac{\sqrt{a+b\sec(fx+e)^2}\left(15\cos(fx+e)\sqrt{a}\,\ln\left(4\cos(fx+e)\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\sqrt{a}+4\cos(fx+e)a+4\sqrt{a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\right)b^2+2\cos(fx+$

input
```
int((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/15/f/b^2*(a+b*sec(f*x+e)^2)^(1/2)/(1+cos(f*x+e))/((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*(15*cos(f*x+e)*a^(1/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*b^2+2*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*a^2+10*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*a*b-15*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2*cos(f*x+e
)+2*a^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+10*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*a*b-15*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*b^2-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b*sec(f*x+e)+10*b^2*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*sec(f*x+e)-((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*a*b*sec(f*x+e)^2+10*b^2*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*sec(f*x+e)^2-3*b^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*sec(f*x+e)^3-3*b^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*sec(f
*x+e)^4)
```

3.376.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. 2(95) = 190.

Time = 1.97 (sec) , antiderivative size = 456, normalized size of antiderivative = 4.11

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^5(e+fx)\,dx$$

$$= \left[\frac{15\sqrt{a}b^2\cos(fx+e)^4\log\left(128\,a^4\cos(fx+e)^8+256\,a^3b\cos(fx+e)^6+160\,a^2b^2\cos(fx+e)^4+32\,ab^3\right.\right.$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x, algorithm="fricas")
```

output
```
[1/120*(15*sqrt(a)*b^2*cos(f*x + e)^4*log(128*a^4*cos(f*x + e)^8 + 256*a^3
*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 +
 b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f
*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*
x + e)^2)) - 8*((2*a^2 + 10*a*b - 15*b^2)*cos(f*x + e)^4 - (a*b - 10*b^2)*
cos(f*x + e)^2 - 3*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(b^2*
f*cos(f*x + e)^4), 1/60*(15*sqrt(-a)*b^2*arctan(1/4*(8*a^2*cos(f*x + e)^4
+ 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2))*cos(f*x
+ e)^4 - 4*((2*a^2 + 10*a*b - 15*b^2)*cos(f*x + e)^4 - (a*b - 10*b^2)*cos(
f*x + e)^2 - 3*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(b^2*f*co
s(f*x + e)^4)]
```

3.376.6 Sympy [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^5(e+fx)\,dx = \int \sqrt{a+b\sec^2(e+fx)}\tan^5(e+fx)\,dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**(1/2)*tan(f*x+e)**5,x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*tan(e + f*x)**5, x)
```

3.376.7 Maxima [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^5(e+fx)\,dx = \int \sqrt{b\sec(fx+e)^2+a}\tan(fx+e)^5\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*tan(f*x + e)^5, x)
```

3.376.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1280 vs. 2(95) = 190.

Time = 1.82 (sec) , antiderivative size = 1280, normalized size of antiderivative = 11.53

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^5(e+fx)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x, algorithm="giac")
```

output
```
2/15*(15*a*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/
2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 +
 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))/sqrt(-a))/sqrt(-a) - 2
*(15*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 +
 b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x +
 1/2*e)^2 + a + b))^9*a - 165*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a
*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2
*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^8*sqrt(a + b)*a + 20*(sqrt(a
+ b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*
x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a
 + b))^7*(27*a^2 - 5*a*b - 16*b^2) - 20*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^
2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2
*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^6*(33*a^2 - 83*a*b
+ 32*b^2)*sqrt(a + b) - 2*(15*a^3 + 1230*a^2*b - 625*a*b^2 - 416*b^3)*(sqr
t(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/
2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2
 + a + b))^5 + 10*(81*a^3 + 90*a^2*b - 391*a*b^2 + 256*b^3)*(sqrt(a + b)*t
an(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/
2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))
^4*sqrt(a + b) - 20*(33*a^4 - 45*a^3*b - 157*a^2*b^2 + 161*a*b^3 - 16*b...
```

3.376.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^5(e+fx)\,dx = \int \tan(e+f\,x)^5\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}\,dx$$

input
```
int(tan(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(tan(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.377 $\int \sqrt{a+b\sec^2(e+fx)}\tan^3(e+fx)\,dx$

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3.377.1 Optimal result

Integrand size = 25, antiderivative size = 80

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^3(e+fx)\,dx = \frac{\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} - \frac{\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{(a+b\sec^2(e+fx))^{3/2}}{3bf}$$

output
```
1/3*(a+b*sec(f*x+e)^2)^(3/2)/b/f+arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))
*a^(1/2)/f-(a+b*sec(f*x+e)^2)^(1/2)/f
```

3.377.2 Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.92

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^3(e+fx)\,dx = \frac{3\sqrt{a}b\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right) + \sqrt{a+b\sec^2(e+fx)}(a-3b+b\sec^2(e+fx))}{3bf}$$

input
```
Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^3,x]
```

output
```
(3*Sqrt[a]*b*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]] + Sqrt[a + b*Sec[
e + f*x]^2]*(a - 3*b + b*Sec[e + f*x]^2))/(3*b*f)
```

3.377.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4627, 25, 354, 90, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \tan(e+fx)^3\sqrt{a+b\sec(e+fx)^2}dx$$

$$\downarrow 4627$$

$$\frac{\int -\cos(e+fx)\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}d\sec(e+fx)}{f}$$

$$\downarrow 25$$

$$-\frac{\int \cos(e+fx)\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}d\sec(e+fx)}{f}$$

$$\downarrow 354$$

$$-\frac{\int \cos(e+fx)\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}d\sec^2(e+fx)}{2f}$$

$$\downarrow 90$$

$$-\frac{\int \cos(e+fx)\sqrt{b\sec^2(e+fx)+a}d\sec^2(e+fx)-\frac{2\left(a+b\sec^2(e+fx)\right)^{3/2}}{3b}}{2f}$$

$$\downarrow 60$$

$$-\frac{a\int \frac{\cos(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)-\frac{2\left(a+b\sec^2(e+fx)\right)^{3/2}}{3b}+2\sqrt{a+b\sec^2(e+fx)}}{2f}$$

$$\downarrow 73$$

$$-\frac{\frac{2a\int \frac{1}{\frac{\sec^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}-\frac{2\left(a+b\sec^2(e+fx)\right)^{3/2}}{3b}+2\sqrt{a+b\sec^2(e+fx)}}{2f}$$

$$\downarrow 221$$

$$-\frac{-2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)-\frac{2(a+b\sec^2(e+fx))^{3/2}}{3b}+2\sqrt{a+b\sec^2(e+fx)}}{2f}$$

input
```
Int[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^3,x]
```

output
```
-1/2*(-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]] + 2*Sqrt[a +
b*Sec[e + f*x]^2] - (2*(a + b*Sec[e + f*x]^2)^(3/2))/(3*b))/f
```

3.377.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.377.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(68) = 136$.

Time = 7.44 (sec) , antiderivative size = 338, normalized size of antiderivative = 4.22

method	result
default	$\frac{\sqrt{a+b\sec(fx+e)^2}\left(3\ln\left(4\cos(fx+e)\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{a}+4\cos(fx+e)a+4\sqrt{a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\right)\sqrt{a}\,b\cos(fx+e)+\sqrt{\frac{b+a\cos(fx+$ $3fb(1$

input
```
int((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/3/f/b*(a+b*sec(f*x+e)^2)^(1/2)/(1+cos(f*x+e))/((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*(3*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*a^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2))*a^(1/2)*b*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*cos(f*x+e)*a-3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*b+
a*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-3*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*b+b*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*sec(f*x+
e)+b*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*sec(f*x+e)^2)
```

3.377.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(68) = 136$.

Time = 0.59 (sec) , antiderivative size = 386, normalized size of antiderivative = 4.82

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^3(e+fx)\,dx$$

$$= \left[\frac{3\sqrt{ab}\cos(fx+e)^2 \log\left(128a^4\cos(fx+e)^8 + 256a^3b\cos(fx+e)^6 + 160a^2b^2\cos(fx+e)^4 + 32ab^3\cos(fx+e)^2 + b^4 + 8\left(16a^3\cos(fx+e)^8 + 24a^2b\cos(fx+e)^6 + 10ab^2\cos(fx+e)^4 + b^3\cos(fx+e)^2\right)\sqrt{a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\right) + 8\left((a-3b)\cos(fx+e)^2 + b\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{24bf\cos(fx+e)^2}, \right.$$

$$\left. -\frac{3\sqrt{-ab}\arctan\left(\frac{\left(8a^2\cos(fx+e)^4+8ab\cos(fx+e)^2+b^2\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\left(2a^3\cos(fx+e)^4+3a^2b\cos(fx+e)^2+ab^2\right)}\right)\cos(fx+e)^2 - 4\left((a-3b)\cos(fx+e)^2 + b\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{12bf\cos(fx+e)^2} \right]$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x, algorithm="fricas")
```

output
```
[1/24*(3*sqrt(a)*b*cos(f*x + e)^2*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*c
os(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4
 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x +
 e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)) + 8*((a - 3*b)*cos(f*x + e)^2 + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(
f*x + e)^2))/(b*f*cos(f*x + e)^2), -1/12*(3*sqrt(-a)*b*arctan(1/4*(8*a^2*c
os(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*
b^2))*cos(f*x + e)^2 - 4*((a - 3*b)*cos(f*x + e)^2 + b)*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2))/(b*f*cos(f*x + e)^2)]
```

3.377.6 Sympy [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^3(e+fx)\,dx = \int \sqrt{a+b\sec^2(e+fx)}\tan^3(e+fx)\,dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**(1/2)*tan(f*x+e)**3,x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*tan(e + f*x)**3, x)
```

3.377.7 Maxima [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^3(e+fx)\,dx = \int \sqrt{b\sec(fx+e)^2+a}\tan(fx+e)^3\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*tan(f*x + e)^3, x)
```

3.377.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 800 vs. $2(68) = 136$.

Time $= 0.76$ (sec) , antiderivative size $= 800$, normalized size of antiderivative $= 10.00$

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^3(e+fx)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x, algorithm="giac")
```

output

```
-2/3*(3*a*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2
*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 +
2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))/sqrt(-a))/sqrt(-a) - 2*
(3*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b
*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1
/2*e)^2 + a + b))^5*a - 3*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan
(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^
2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^4*(3*a - 4*b)*sqrt(a + b) + 2*(sq
rt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1
/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^
2 + a + b))^3*(3*a^2 - 9*a*b + 8*b^2) + 6*(sqrt(a + b)*tan(1/2*f*x + 1/2*e
)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1
/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2*(a^2 + a*b - 4*
b^2)*sqrt(a + b) - 3*(3*a^3 - 2*a^2*b - 5*a*b^2 + 16*b^3)*(sqrt(a + b)*tan
(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*
e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)) +
 (3*a^3 - 6*a^2*b + 19*a*b^2 - 20*b^3)*sqrt(a + b))/((sqrt(a + b)*tan(1/2*
f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4
- 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2 - 2*
(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b...
```

3.377.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^3(e+fx)\,dx = \int \tan(e+f\,x)^3\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}\,dx$$

input

```
int(tan(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2),x)
```

output

```
int(tan(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.378 $\int \sqrt{a+b\sec^2(e+fx)}\tan(e+fx)\,dx$

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3.378.1 Optimal result

Integrand size = 23, antiderivative size = 54

$$\int \sqrt{a+b\sec^2(e+fx)}\tan(e+fx)\,dx = -\frac{\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{\sqrt{a+b\sec^2(e+fx)}}{f}$$

output
```
-arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))*a^(1/2)/f+(a+b*sec(f*x+e)^2)^(1
/2)/f
```

3.378.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 119 vs. 2(54) = 108.

Time = 0.56 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.20

$$\int \sqrt{a+b\sec^2(e+fx)}\tan(e+fx)\,dx$$
$$=\frac{\left(-2\sqrt{a}\operatorname{arcsinh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{b}}\right)\cos(e+fx)+\sqrt{2}\sqrt{b}\sqrt{\frac{a+2b+a\cos(2(e+fx))}{b}}\right)\sqrt{a+b\sec^2(e+fx)}}{\sqrt{2}\sqrt{b}f\sqrt{\frac{a+2b+a\cos(2(e+fx))}{b}}}$$

input
```
Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x],x]
```

output
```
((-2*Sqrt[a]*ArcSinh[(Sqrt[a]*Cos[e + f*x])/Sqrt[b]]*Cos[e + f*x] + Sqrt[2
]*Sqrt[b]*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/b])*Sqrt[a + b*Sec[e + f*x]^
2])/(Sqrt[2]*Sqrt[b]*f*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/b])
```

3.378.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4627, 243, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \tan(e+fx)\sqrt{a+b\sec(e+fx)^2}dx$$

$$\downarrow 4627$$

$$\frac{\int \cos(e+fx)\sqrt{b\sec^2(e+fx)+a}d\sec(e+fx)}{f}$$

$$\downarrow 243$$

$$\frac{\int \cos(e+fx)\sqrt{b\sec^2(e+fx)+a}d\sec^2(e+fx)}{2f}$$

$$\downarrow 60$$

$$\frac{a\int \frac{\cos(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)+2\sqrt{a+b\sec^2(e+fx)}}{2f}$$

$$\downarrow 73$$

$$\frac{\frac{2a\int \frac{1}{\frac{\sec^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}+2\sqrt{a+b\sec^2(e+fx)}}{2f}$$

$$\downarrow 221$$

$$\frac{2\sqrt{a+b\sec^2(e+fx)}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{2f}$$

input
```
Int[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x],x]
```

output
```
(-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]] + 2*Sqrt[a + b*Sec
[e + f*x]^2])/(2*f)
```

3.378.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.378.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07

method	result	size
derivativedivides	$\frac{\sqrt{a+b\sec(fx+e)^2}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\sec(fx+e)^2}}{\sec(fx+e)}\right)}{f}$	58
default	$\frac{\sqrt{a+b\sec(fx+e)^2}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\sec(fx+e)^2}}{\sec(fx+e)}\right)}{f}$	58

input
```
int((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
1/f*((a+b*sec(f*x+e)^2)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*sec(f*x+e)^2)
^(1/2))/sec(f*x+e)))
```

3.378.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(46) = 92.

Time = 0.37 (sec) , antiderivative size = 312, normalized size of antiderivative = 5.78

$$\int \sqrt{a+b\sec^2(e+fx)}\tan(e+fx)\,dx$$

$$= \left[\frac{\sqrt{a}\log\left(128\,a^4\cos(fx+e)^8+256\,a^3b\cos(fx+e)^6+160\,a^2b^2\cos(fx+e)^4+32\,ab^3\cos(fx+e)^2+b^4\right.}{}\right.$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e),x, algorithm="fricas")
```

output
```
[1/8*(sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*
a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*a^3*cos(f*x
 + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x
+ e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + 8*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2))/f, 1/4*(sqrt(-a)*arctan(1/4*(8*a^2*cos
(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^
2)) + 4*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/f]
```

3.378.6 Sympy [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\tan(e+fx)\,dx = \int \sqrt{a+b\sec^2(e+fx)}\tan(e+fx)\,dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**(1/2)*tan(f*x+e),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*tan(e + f*x), x)
```

3.378.7 Maxima [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\tan(e+fx)\,dx = \int \sqrt{b\sec(fx+e)^2+a}\tan(fx+e)\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*tan(f*x + e), x)
```

3.378.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 377 vs. $2(46) = 92$.

Time = 0.46 (sec) , antiderivative size = 377, normalized size of antiderivative = 6.98

$$\int \sqrt{a+b\sec^2(e+fx)}\tan(e+fx)\,dx$$
$$= 2\left(\frac{a\arctan\left(-\frac{\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a+b}+\sqrt{a+b}}{2\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{}{\left(\sqrt{a+b}t\right.}\right.$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e),x, algorithm="giac")
```

output
```
2*(a*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x
+ 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*t
an(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))/sqrt(-a))/sqrt(-a) + 2*((sqr
t(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/
2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2
 + a + b))*b + sqrt(a + b)*b)/((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(
a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/
2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2 - 2*(sqrt(a + b)*tan(1/2*f
*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 -
 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*sqrt(a
+ b) + a - 3*b))*sgn(cos(f*x + e))/f
```

3.378.9 Mupad [B] (verification not implemented)

Time = 20.34 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \sqrt{a+b\sec^2(e+fx)}\tan(e+fx)\,dx = \frac{\sqrt{a+\frac{b}{\cos(e+fx)^2}}}{f} - \frac{\sqrt{a}\,\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{\cos(e+fx)^2}}}{\sqrt{a}}\right)}{f}$$

input
```
int(tan(e + f*x)*(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
(a + b/cos(e + f*x)^2)^(1/2)/f - (a^(1/2)*atanh((a + b/cos(e + f*x)^2)^(1/
2)/a^(1/2)))/f
```

3.379 $\int \cot(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

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3.379.1 Optimal result

Integrand size = 23, antiderivative size = 70

$$\int \cot(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} - \frac{\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{f}$$

output
```
arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))*a^(1/2)/f-arctanh((a+b*sec(f*x+e
)^2)^(1/2)/(a+b)^(1/2))*(a+b)^(1/2)/f
```

3.379.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \cot(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} - \frac{\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{f}$$

input
```
Integrate[Cot[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/f - (Sqrt[a + b]*Arc
Tanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/f
```

3.379.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4627, 25, 354, 94, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sqrt{a+b\sec(e+fx)^2}}{\tan(e+fx)}dx$$

$$\downarrow 4627$$

$$\frac{\int -\frac{\cos(e+fx)\sqrt{b\sec^2(e+fx)+a}}{1-\sec^2(e+fx)}d\sec(e+fx)}{f}$$

$$\downarrow 25$$

$$-\frac{\int \frac{\cos(e+fx)\sqrt{b\sec^2(e+fx)+a}}{1-\sec^2(e+fx)}d\sec(e+fx)}{f}$$

$$\downarrow 354$$

$$-\frac{\int \frac{\cos(e+fx)\sqrt{b\sec^2(e+fx)+a}}{1-\sec^2(e+fx)}d\sec^2(e+fx)}{2f}$$

$$\downarrow 94$$

$$-\frac{(a+b)\int \frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)+a\int \frac{\cos(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{2f}$$

$$\downarrow 73$$

$$-\frac{\frac{2(a+b)\int \frac{1}{\frac{a+b}{b}-\frac{\sec^4(e+fx)}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}+\frac{2a\int \frac{1}{\frac{\sec^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}}{2f}$$

$$\downarrow 221$$

$$-\frac{2\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)-2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{2f}$$

input
```
Int[Cot[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
-1/2*(-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]] + 2*Sqrt[a +
b]*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/f
```

3.379.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 94
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(a + b*x), x],
x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(c + d*x), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.379.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 539 vs. $2(58) = 116$.

Time = 1.35 (sec) , antiderivative size = 540, normalized size of antiderivative = 7.71

method	result
default	$\frac{\sqrt{a+b\sec(fx+e)^2}\left(2\sqrt{a}\,\ln\left(4\cos(fx+e)\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\sqrt{a}+4\cos(fx+e)a+4\sqrt{a}\,\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\right)\sqrt{a+b}+\ln\left(\frac{2\sqrt{\frac{b+a\cos(fx+e)}{(1+\cos(fx+e)}}\right.\right.$

input

```
int(cot(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/2/f/(a+b)^(1/2)*(a+b*sec(f*x+e)^2)^(1/2)*(2*a^(1/2)*ln(4*cos(f*x+e)*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*(a+b)^(1/2)+ln(2/(a+b)^(1/2)*(
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*
x+e)))*a+b*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(
a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1
/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))-ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*a-ln(-4*(((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*b)*cos
(f*x+e)/(1+cos(f*x+e))/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
```

3.379.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(58) = 116$.

Time = 0.42 (sec) , antiderivative size = 963, normalized size of antiderivative = 13.76

$$\int \cot(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*
a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x
 + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x
+ e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + 2*sqrt(a +
b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x
 + e)^2 + b^2 - 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x
 + e)^2 + 1)))/f, 1/8*(4*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2
 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b
)*cos(f*x + e)^2 + a*b + b^2)) + sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*
a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^
2 + b^4 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*co
s(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)))/f, -1/4*(sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*
cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - sqrt(a + b)*lo
g(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)
^2 + b^2 - 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)
^2 + 1)))/f, -1/4*(sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*co...
```

3.379.6 Sympy [F]

$$\int \cot(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{a+b\sec^2(e+fx)}\cot(e+fx)\,dx$$

input
```
integrate(cot(f*x+e)*(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*cot(e + f*x), x)
```

3.379.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 3317, normalized size of antiderivative = 47.39

$$\int \cot(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/4*(a^(3/2)*log(4*sqrt(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 +
4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4
*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 +
2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a
*b)*cos(2*f*x + 2*e))*a*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*s
in(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2
 + 4*sqrt(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b
 + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x
+ 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^
2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x
+ 2*e))*a*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e
), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + 16*(a^2*cos
(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f
*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 +
 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*
f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*(a
+ b)*sqrt(a)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x +
2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) + 16*a^2 + 3
2*a*b + 16*b^2) - 2*sqrt(a + b)*a*log(4*(4*sqrt(a^2*cos(4*f*x + 4*e)^2 + a
^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*...
```

3.379.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 402 vs. $2(58) = 116$.

Time = 0.69 (sec) , antiderivative size = 402, normalized size of antiderivative = 5.74

$$\int \cot(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx =$$

$$-\frac{\left(\frac{4\,a\arctan\left(-\frac{\sqrt{a+b}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-\sqrt{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-2\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+2\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+a+b}+\sqrt{a+b}}{2\,\sqrt{-a}}\right)}{\sqrt{-a}}+\sqrt{a+}\right.}{}$$

input
```
integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*(4*a*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2
*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 +
2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))/sqrt(-a))/sqrt(-a) + sq
rt(a + b)*log(abs(-sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 + sqrt(a*tan(1/2*f*x
 + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*
tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))) - sqrt(a + b)*log(abs(-sqr
t(a + b)*tan(1/2*f*x + 1/2*e)^2 + sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/
2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2
 + a + b) - sqrt(a + b))) + sqrt(a + b)*log(abs((sqrt(a + b)*tan(1/2*f*x +
 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a
*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*(a + b) - s
qrt(a + b)*(a - b))))*sgn(cos(f*x + e))/f
```

3.379.9 Mupad [F(-1)]

Timed out.

$$\int \cot(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \cot(e+f\,x)\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)^2}}\,dx$$

input
```
int(cot(e + f*x)*(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(cot(e + f*x)*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.380 $\int \cot^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

3.380.1 Optimal result . . . 2767
3.380.2 Mathematica [C] (warning: unable to verify) . . . 2767
3.380.3 Rubi [A] (verified) . . . 2768
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3.380.8 Giac [B] (verification not implemented) . . . 2773
3.380.9 Mupad [F(-1)] . . . 2774

3.380.1 Optimal result

Integrand size = 25, antiderivative size = 109

$$\int \cot^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = -\frac{\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} + \frac{(2a+b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2\sqrt{a+b}f} - \frac{\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}$$

output
```
-arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))*a^(1/2)/f+1/2*(2*a+b)*arctanh((
a+b*sec(f*x+e)^2)^(1/2)/(a+b)^(1/2))/f/(a+b)^(1/2)-1/2*cot(f*x+e)^2*(a+b*s
ec(f*x+e)^2)^(1/2)/f
```

3.380.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.91 (sec) , antiderivative size = 527, normalized size of antiderivative = 4.83

$$\int \cot^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$= \frac{e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos(e+fx)\left(\frac{1+e^{2i(e+fx)}}{\left(-1+e^{2i(e+fx)}\right)^2} - \frac{-2i\sqrt{a}\sqrt{a+b}fx+(2a+b)\log\left(1-e^{2i(e+fx)}\right)+\sqrt{}\right.}{}$$

input
```
Integrate[Cot[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e +
f*x))]*Cos[e + f*x]*((1 + E^((2*I)*(e + f*x)))/(-1 + E^((2*I)*(e + f*x)))^
2 - ((-2*I)*Sqrt[a]*Sqrt[a + b]*f*x + (2*a + b)*Log[1 - E^((2*I)*(e + f*x)
)] + Sqrt[a]*Sqrt[a + b]*Log[a + 2*b + a*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqr
t[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] + Sqrt[a]*Sqrt
[a + b]*Log[a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*I)*(e + f*x)) + Sqrt[a]*
Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] - 2*a*Log[a
 + b + a*E^((2*I)*(e + f*x)) + b*E^((2*I)*(e + f*x)) + Sqrt[a + b]*Sqrt[4*
b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] - b*Log[a + b + a*
E^((2*I)*(e + f*x)) + b*E^((2*I)*(e + f*x)) + Sqrt[a + b]*Sqrt[4*b*E^((2*I
)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]])/(Sqrt[a + b]*Sqrt[4*b*E^((
2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]))*Sqrt[a + b*Sec[e + f*x]
^2])/(Sqrt[2]*f*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]])
```

3.380.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4627, 354, 110, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sqrt{a+b\sec(e+fx)^2}}{\tan(e+fx)^3}dx$$

$$\downarrow 4627$$

$$\frac{\int \frac{\cos(e+fx)\sqrt{b\sec^2(e+fx)+a}}{\left(1-\sec^2(e+fx)\right)^2}d\sec(e+fx)}{f}$$

$$\downarrow 354$$

$$\frac{\int \frac{\cos(e+fx)\sqrt{b\sec^2(e+fx)+a}}{\left(1-\sec^2(e+fx)\right)^2}d\sec^2(e+fx)}{2f}$$

$$\downarrow 110$$

$$\frac{\frac{\sqrt{a+b\sec^2(e+fx)}}{1-\sec^2(e+fx)} - \int -\frac{\cos(e+fx)\left(b\sec^2(e+fx)+2a\right)}{2(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{2f}$$

$$\downarrow 27$$

$$\frac{\frac{1}{2}\int \frac{\cos(e+fx)\left(b\sec^2(e+fx)+2a\right)}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx) + \frac{\sqrt{a+b\sec^2(e+fx)}}{1-\sec^2(e+fx)}}{2f}$$

$$\downarrow 174$$

$$\frac{\frac{1}{2}\left((2a+b)\int \frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx) + 2a\int \frac{\cos(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)\right) + \frac{\sqrt{a+b\sec^2(e+fx)}}{1-\sec^2(e+fx)}}{2f}$$

$$\downarrow 73$$

$$\frac{\frac{1}{2}\left(\frac{2(2a+b)\int \frac{1}{\frac{a+b}{b}-\frac{\sec^4(e+fx)}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b} + \frac{4a\int \frac{1}{\frac{\sec^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}\right) + \frac{\sqrt{a+b\sec^2(e+fx)}}{1-\sec^2(e+fx)}}{2f}$$

$$\downarrow 221$$

$$\frac{\frac{1}{2}\left(\frac{2(2a+b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}} - 4\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)\right) + \frac{\sqrt{a+b\sec^2(e+fx)}}{1-\sec^2(e+fx)}}{2f}$$

input
```
Int[Cot[e + f*x]^3*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
((-4*Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]] + (2*(2*a + b)*Ar
cTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/Sqrt[a + b])/2 + Sqrt[a + b
*Sec[e + f*x]^2]/(1 - Sec[e + f*x]^2))/(2*f)
```

3.380.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.380.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2755 vs. $2(91) = 182$.

Time = 1.39 (sec) , antiderivative size = 2756, normalized size of antiderivative = 25.28

method	result	size
default	Expression too large to display	2756

input

```
int(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/8/f/(a+b)^(5/2)*((a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc
(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e
)^2+a+b)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^2)^(1/2)*((1-cos(f*x+e))^2*csc(
f*x+e)^2-1)*(8*a^(3/2)*ln(4*(-a*(1-cos(f*x+e))^2*csc(f*x+e)^2+a^(1/2)*(a*(
1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*
x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)+a)/((1-c
os(f*x+e))^2*csc(f*x+e)^2+1))*(1-cos(f*x+e))^2*(a+b)^(3/2)*csc(f*x+e)^2-(a
+b)^(3/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4
-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^
(1/2)*a*(1-cos(f*x+e))^4*csc(f*x+e)^4-(a+b)^(3/2)*(a*(1-cos(f*x+e))^4*csc(
f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2
+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*b*(1-cos(f*x+e))^4*csc(f*x+e
)^4+8*a^(1/2)*ln(4*(-a*(1-cos(f*x+e))^2*csc(f*x+e)^2+a^(1/2)*(a*(1-cos(f*x
+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*c
sc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)+a)/((1-cos(f*x+e)
)^2*csc(f*x+e)^2+1))*(1-cos(f*x+e))^2*(a+b)^(3/2)*b*csc(f*x+e)^2+a*(a*(1-c
os(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e
))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(1-cos(f*x+
e))^2*(a+b)^(3/2)*csc(f*x+e)^2-3*b*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-c
os(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(...
```

3.380.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. 2(91) = 182.

Time = 0.56 (sec) , antiderivative size = 1342, normalized size of antiderivative = 12.31

$$\int \cot^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(4*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2
 + ((a + b)*cos(f*x + e)^2 - a - b)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 2
56*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x +
e)^2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2
*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/
cos(f*x + e)^2)) + ((2*a + b)*cos(f*x + e)^2 - 2*a - b)*sqrt(a + b)*log(2*
((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 +
 b^2 + 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a
*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 +
 1)))/((a + b)*f*cos(f*x + e)^2 - (a + b)*f), 1/8*(4*(a + b)*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2 - 2*((2*a + b)*cos(f*x + e)^
2 - 2*a - b)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-
a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x +
e)^2 + a*b + b^2)) + ((a + b)*cos(f*x + e)^2 - a - b)*sqrt(a)*log(128*a^4*
cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 3
2*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x
 + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2)))/((a + b)*f*cos(f*x + e)^2 - (a + b)*f)
, 1/8*(4*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^
2 + 2*((a + b)*cos(f*x + e)^2 - a - b)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f...
```

3.380.6 Sympy [F]

$$\int \cot^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{a+b\sec^2(e+fx)}\cot^3(e+fx)\,dx$$

input
```
integrate(cot(f*x+e)**3*(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*cot(e + f*x)**3, x)
```

3.380.7 Maxima [F]

$$\int \cot^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\cot(fx+e)^3\,dx$$

input
```
integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*cot(f*x + e)^3, x)
```

3.380.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 574 vs. 2(91) = 182.

Time = 0.90 (sec) , antiderivative size = 574, normalized size of antiderivative = 5.27

$$\int \cot^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$
$$= \left(\frac{16\,a\arctan\left(-\frac{\sqrt{a+b}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a+b}+\sqrt{a+b}}{2\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{4\,(2a+b)}{}\right.$$

input
```
integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
1/8*(16*a*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2
*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 +
2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))/sqrt(-a))/sqrt(-a) - 4*
(2*a + b)*arctan(-(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x
 + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*
tan(1/2*f*x + 1/2*e)^2 + a + b))/sqrt(-a - b))/sqrt(-a - b) + 2*(2*a + b)*
log(abs(-(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)
^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f
*x + 1/2*e)^2 + a + b))*(a + b) + sqrt(a + b)*(a - b)))/sqrt(a + b) + sqrt
(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1
/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) - 2*((sqrt(a + b)*tan(1/2*f*
x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 -
2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*(a - b)
- (a + b)^(3/2))/((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x
 + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*
tan(1/2*f*x + 1/2*e)^2 + a + b))^2 - a - b))*sgn(cos(f*x + e))/f
```

3.380.9 Mupad [F(-1)]

Timed out.

$$\int \cot^3(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \cot(e+f\,x)^3\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}\,dx$$

input
```
int(cot(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(cot(e + f*x)^3*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.381 $\int \cot^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

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3.381.1 Optimal result

Integrand size = 25, antiderivative size = 161

$$\int \cot^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} - \frac{(8a^2+12ab+3b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8(a+b)^{3/2}f} + \frac{(4a+3b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)f} - \frac{\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f}$$

output

```
-1/8*(8*a^2+12*a*b+3*b^2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a
+b)^(3/2)/f+arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))*a^(1/2)/f+1/8*(4*a+3
*b)*cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2)/(a+b)/f-1/4*cot(f*x+e)^4*(a+b*se
c(f*x+e)^2)^(1/2)/f
```

3.381.2 Mathematica [F]

$$\int \cot^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \cot^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

input
```
Integrate[Cot[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
Integrate[Cot[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2], x]
```

3.381.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.13, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 4627, 25, 354, 110, 27, 168, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sqrt{a+b\sec(e+fx)^2}}{\tan(e+fx)^5}\,dx$$

$$\downarrow 4627$$

$$\frac{\int -\frac{\cos(e+fx)\sqrt{b\sec^2(e+fx)+a}}{(1-\sec^2(e+fx))^3}\,d\sec(e+fx)}{f}$$

$$\downarrow 25$$

$$-\frac{\int \frac{\cos(e+fx)\sqrt{b\sec^2(e+fx)+a}}{(1-\sec^2(e+fx))^3}\,d\sec(e+fx)}{f}$$

$$\downarrow 354$$

$$-\frac{\int \frac{\cos(e+fx)\sqrt{b\sec^2(e+fx)+a}}{(1-\sec^2(e+fx))^3}\,d\sec^2(e+fx)}{2f}$$

$$\downarrow 110$$

$$-\frac{\frac{\sqrt{a+b\sec^2(e+fx)}}{2(1-\sec^2(e+fx))^2}-\frac{1}{2}\int-\frac{\cos(e+fx)(3b\sec^2(e+fx)+4a)}{2(1-\sec^2(e+fx))^2\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{2f}$$

$$\downarrow 27$$

$$-\frac{\frac{1}{4}\int\frac{\cos(e+fx)(3b\sec^2(e+fx)+4a)}{(1-\sec^2(e+fx))^2\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)+\frac{\sqrt{a+b\sec^2(e+fx)}}{2(1-\sec^2(e+fx))^2}}{2f}$$

$$\downarrow 168$$

$$-\frac{\frac{1}{4}\left(\frac{(4a+3b)\sqrt{a+b\sec^2(e+fx)}}{(a+b)(1-\sec^2(e+fx))}-\frac{\int-\frac{\cos(e+fx)\left(b(4a+3b)\sec^2(e+fx)+8a(a+b)\right)}{2\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{a+b}\right)+\frac{\sqrt{a+b\sec^2(e+fx)}}{2(1-\sec^2(e+fx))^2}}{2f}$$

$$\downarrow 27$$

$$-\frac{\frac{1}{4}\left(\frac{\int\frac{\cos(e+fx)\left(b(4a+3b)\sec^2(e+fx)+8a(a+b)\right)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{2(a+b)}+\frac{(4a+3b)\sqrt{a+b\sec^2(e+fx)}}{(a+b)(1-\sec^2(e+fx))}\right)+\frac{\sqrt{a+b\sec^2(e+fx)}}{2(1-\sec^2(e+fx))^2}}{2f}$$

$$\downarrow 174$$

$$-\frac{\frac{1}{4}\left(\frac{\left(8a^2+12ab+3b^2\right)\int\frac{1}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)+8a(a+b)\int\frac{\cos(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{2(a+b)}+\frac{(4a+3b)\sqrt{a+b\sec^2(e+fx)}}{(a+b)(1-\sec^2(e+fx))}\right)}{2f}$$

$$\downarrow 73$$

$$-\frac{\frac{1}{4}\left(\frac{2\left(8a^2+12ab+3b^2\right)\int\frac{1}{\frac{a+b}{b}-\frac{\sec^4(e+fx)}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}+\frac{16a(a+b)\int\frac{1}{\frac{\sec^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}}{2(a+b)}+\frac{(4a+3b)\sqrt{a+b\sec^2(e+fx)}}{(a+b)(1-\sec^2(e+fx))}\right)+\frac{\sqrt{}}{2(}}{2f}$$

$$\downarrow 221$$

$$-\frac{\frac{1}{4}\left(\frac{2\left(8a^2+12ab+3b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-16\sqrt{a}(a+b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{2(a+b)}+\frac{(4a+3b)\sqrt{a+b\sec^2(e+fx)}}{(a+b)(1-\sec^2(e+fx))}\right)+\frac{\sqrt{a+b\sec}}{2(1-\sec^2}}{2f}$$

3.381. $\int \cot^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

input
```
Int[Cot[e + f*x]^5*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
-1/2*(Sqrt[a + b*Sec[e + f*x]^2]/(2*(1 - Sec[e + f*x]^2)^2) + ((-16*Sqrt[a
]*(a + b)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]] + (2*(8*a^2 + 12*a*b
 + 3*b^2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/Sqrt[a + b])/(2
*(a + b)) + ((4*a + 3*b)*Sqrt[a + b*Sec[e + f*x]^2])/((a + b)*(1 - Sec[e +
 f*x]^2)))/4)/f
```

3.381.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.381.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 4322 vs. $2(139) = 278$.

Time = 1.36 (sec) , antiderivative size = 4323, normalized size of antiderivative = 26.85

method	result	size
default	Expression too large to display	4323

input
```
int(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/64/f/(a+b)^(9/2)*((a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*c
sc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x
+e)^2+a+b)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^2)^(1/2)*((1-cos(f*x+e))^2*cs
c(f*x+e)^2-1)*(-18*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc
(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e
)^2+a+b)^(1/2)*(a+b)^(5/2)*a*b*(1-cos(f*x+e))^6*csc(f*x+e)^6-14*a*b*(a*(1-
cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+
e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(5/2
)*(1-cos(f*x+e))^4*csc(f*x+e)^4+64*(a+b)^(5/2)*a^(1/2)*ln(4*(-a*(1-cos(f*x
+e))^2*csc(f*x+e)^2+a^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+
e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*
csc(f*x+e)^2+a+b)^(1/2)+a)/((1-cos(f*x+e))^2*csc(f*x+e)^2+1))*b^2*(1-cos(f
*x+e))^4*csc(f*x+e)^4+128*(a+b)^(5/2)*a^(3/2)*ln(4*(-a*(1-cos(f*x+e))^2*cs
c(f*x+e)^2+a^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc
(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e
)^2+a+b)^(1/2)+a)/((1-cos(f*x+e))^2*csc(f*x+e)^2+1))*b*(1-cos(f*x+e))^4*cs
c(f*x+e)^4+252*ln((a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(
f*x+e)^2+(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-
2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(
1/2)*(a+b)^(1/2)-a+b)/(a+b)^(1/2))*a^3*b^2*(1-cos(f*x+e))^4*csc(f*x+e)^...
```

3.381.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 442 vs. $2(139) = 278$.

Time = 1.39 (sec) , antiderivative size = 1953, normalized size of antiderivative = 12.13

$$\int \cot^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/32*(4*((a^2 + 2*a*b + b^2)*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f
*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^
3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2
+ b^4 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(
f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)) + ((8*a^2 + 12*a*b + 3*b^2)*cos(f*x + e)^4 - 2*(8*a^2 + 12*a*b
 + 3*b^2)*cos(f*x + e)^2 + 8*a^2 + 12*a*b + 3*b^2)*sqrt(a + b)*log(2*((8*a
^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2
- 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(
f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1))
- 4*((6*a^2 + 11*a*b + 5*b^2)*cos(f*x + e)^4 - (4*a^2 + 7*a*b + 3*b^2)*cos
(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^2 + 2*a*b +
b^2)*f*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2 + (a^2 + 2*
a*b + b^2)*f), 1/16*(((8*a^2 + 12*a*b + 3*b^2)*cos(f*x + e)^4 - 2*(8*a^2 +
 12*a*b + 3*b^2)*cos(f*x + e)^2 + 8*a^2 + 12*a*b + 3*b^2)*sqrt(-a - b)*arc
tan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) + 2*((a^2
+ 2*a*b + b^2)*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f*x + e)^2 + a^2
 + 2*a*b + b^2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e
)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(1...
```

3.381.6 Sympy [F]

$$\int \cot^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{a+b\sec^2(e+fx)}\cot^5(e+fx)\,dx$$

input
```
integrate(cot(f*x+e)**5*(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*cot(e + f*x)**5, x)
```

3.381.7 Maxima [F]

$$\int \cot^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\cot(fx+e)^5\,dx$$

input
```
integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*cot(f*x + e)^5, x)
```

3.381.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 866 vs. $2(139) = 278$.

Time = 1.30 (sec) , antiderivative size = 866, normalized size of antiderivative = 5.38

$$\int \cot^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
1/64*(sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1
/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)*(tan(1/2*f*x + 1/2
*e)^2 - (11*a + 9*b)/(a + b)) - 128*a*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x
 + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2
*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a +
 b))/sqrt(-a))/sqrt(-a) + 8*(8*a^2 + 12*a*b + 3*b^2)*arctan(-(sqrt(a + b)*
tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1
/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)
)/sqrt(-a - b))/((a + b)*sqrt(-a - b)) - 4*(8*a^2 + 12*a*b + 3*b^2)*log(ab
s((sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*
tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/
2*e)^2 + a + b))*(a + b) - sqrt(a + b)*(a - b)))/(a + b)^(3/2) + 4*(2*(sqr
t(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/
2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2
 + a + b))^3*(3*a^2 - 2*b^2) - (sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(
a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/
2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2*(7*a^2 + 10*a*b + 3*b^2)*s
qrt(a + b) - 2*(2*a^3 + 2*a^2*b - 3*a*b^2 - 3*b^3)*(sqrt(a + b)*tan(1/2*f*
x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 -
2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b)) + 5*(...
```

3.381.9 Mupad [F(-1)]

Timed out.

$$\int \cot^5(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \cot(e+f\,x)^5\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}\,dx$$

input
```
int(cot(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(cot(e + f*x)^5*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.382 $\int \sqrt{a+b\sec^2(e+fx)}\tan^6(e+fx)\,dx$

3.382.1 Optimal result . . . 2784
3.382.2 Mathematica [A] (verified) . . . 2785
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3.382.9 Mupad [F(-1)] . . . 2793

3.382.1 Optimal result

Integrand size = 25, antiderivative size = 219

$$\begin{aligned}
&\int \sqrt{a+b\sec^2(e+fx)}\tan^6(e+fx)\,dx \\
&= -\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{(a^3+5a^2b+15ab^2-5b^3)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16b^{5/2}f} \\
&\quad - \frac{(a-b)(a+5b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16b^2f} \\
&\quad + \frac{(a-5b)\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24bf} \\
&\quad + \frac{\tan^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6f}
\end{aligned}$$

output

```
1/16*(a^3+5*a^2*b+15*a*b^2-5*b^3)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*
x+e)^2)^(1/2))/b^(5/2)/f-arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1
/2))*a^(1/2)/f-1/16*(a-b)*(a+5*b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b^
2/f+1/24*(a-5*b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^3/b/f+1/6*(a+b+b*ta
n(f*x+e)^2)^(1/2)*tan(f*x+e)^5/f
```

3.382.2 Mathematica [A] (verified)

Time = 4.13 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.20

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^6(e+fx)\,dx =$$

$$-\frac{\left(16\sqrt{a}b^2\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)-\frac{(a^3+5a^2b+15ab^2-5b^3)\operatorname{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{b}}\right)\cos(e+fx)\sqrt{a+b}}{8\sqrt{2}b^2f\sqrt{a+2b+a\cos(2e+2fx)}}$$

$$-\frac{(9a^2+34ab-59b^2+4(3a^2+12ab-7b^2)\cos(2(e+fx))+(3a^2+14ab-33b^2)\cos(4(e+fx)))\sec^4(e}{384b^2f}$$

input
```
Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^6,x]
```

output
```
-1/8*((16*Sqrt[a]*b^2*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e +
 f*x]^2]] - ((a^3 + 5*a^2*b + 15*a*b^2 - 5*b^3)*ArcTanh[(Sqrt[b]*Sin[e + f
*x])/Sqrt[a + b - a*Sin[e + f*x]^2]])/Sqrt[b])*Cos[e + f*x]*Sqrt[a + b*Sec
[e + f*x]^2])/(Sqrt[2]*b^2*f*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]) - ((9*a^2
 + 34*a*b - 59*b^2 + 4*(3*a^2 + 12*a*b - 7*b^2)*Cos[2*(e + f*x)] + (3*a^2
+ 14*a*b - 33*b^2)*Cos[4*(e + f*x)])*Sec[e + f*x]^4*Sqrt[a + b*Sec[e + f*x
]^2]*Tan[e + f*x])/(384*b^2*f)
```

3.382.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.05, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 4629, 2075, 380, 444, 27, 444, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \tan(e+fx)^6\sqrt{a+b\sec(e+fx)^2}dx$$

$$\downarrow 4629$$

$$\frac{\int \frac{\tan^6(e+fx)\sqrt{a+b(\tan^2(e+fx)+1)}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\tan^6(e+fx)\sqrt{b\tan^2(e+fx)+a+b}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\downarrow 380$$

$$\frac{\frac{1}{6}\tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}-\frac{1}{6}\int \frac{\tan^4(e+fx)(5(a+b)-(a-5b)\tan^2(e+fx))}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{f}$$

$$\downarrow 444$$

$$\frac{\frac{1}{6}\left(\frac{\int -\frac{3\tan^2(e+fx)\left((a-b)(a+5b)\tan^2(e+fx)+(a-5b)(a+b)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{4b}+\frac{(a-5b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4b}\right)+\frac{1}{6}\tan^5(e+fx)\sqrt{}}{f}$$

$$\downarrow 27$$

$$\frac{\frac{1}{6}\left(\frac{(a-5b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4b}-\frac{3\int \frac{\tan^2(e+fx)\left((a-b)(a+5b)\tan^2(e+fx)+(a-5b)(a+b)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{4b}\right)+\frac{1}{6}\tan^5(e+fx)\sqrt{a}}{f}$$

$$\downarrow 444$$

$$\frac{\frac{1}{6}\left(\frac{(a-5b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4b}-\frac{3\left(\frac{(a-b)(a+5b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{\int \frac{\left(a^3+5ba^2+15b^2a-5b^3\right)\tan^2(e+fx)+(a+5b)\left(a^2-b^2\right.}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}}{2b}\right.}{4b}\right.}{f}$$

$$\downarrow 398$$

$$\frac{1}{6}\left(\frac{(a-5b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4b}-\frac{3\left(\frac{(a-b)(a+5b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{\left(a^3+5a^2b+15ab^2-5b^3\right)\int\frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e}{4b}\right.\right.$$

$$f$$

$$\downarrow 224$$

$$\frac{1}{6}\left(\frac{(a-5b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4b}-\frac{3\left(\frac{(a-b)(a+5b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{\left(a^3+5a^2b+15ab^2-5b^3\right)\int\frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{}{\sqrt{bt}}}{4b}\right.\right.$$

$$f$$

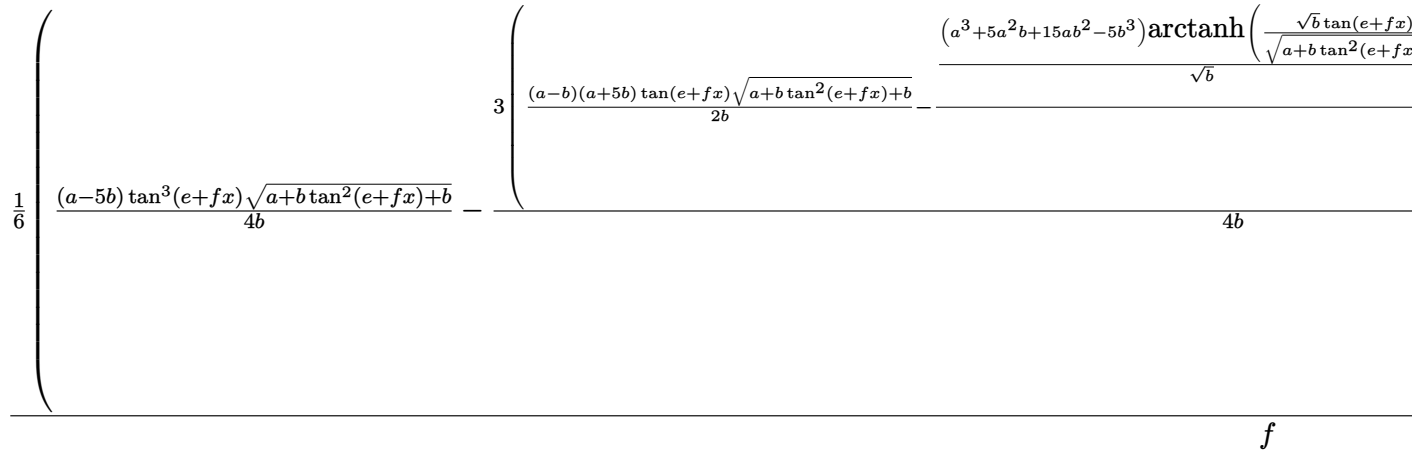
$$\downarrow 219$$

$$\frac{1}{6}\left(\frac{(a-5b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4b}-\frac{3\left(\frac{(a-b)(a+5b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{\left(a^3+5a^2b+15ab^2-5b^3\right)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx}}\right.\right.\right.}{\sqrt{b}}$$

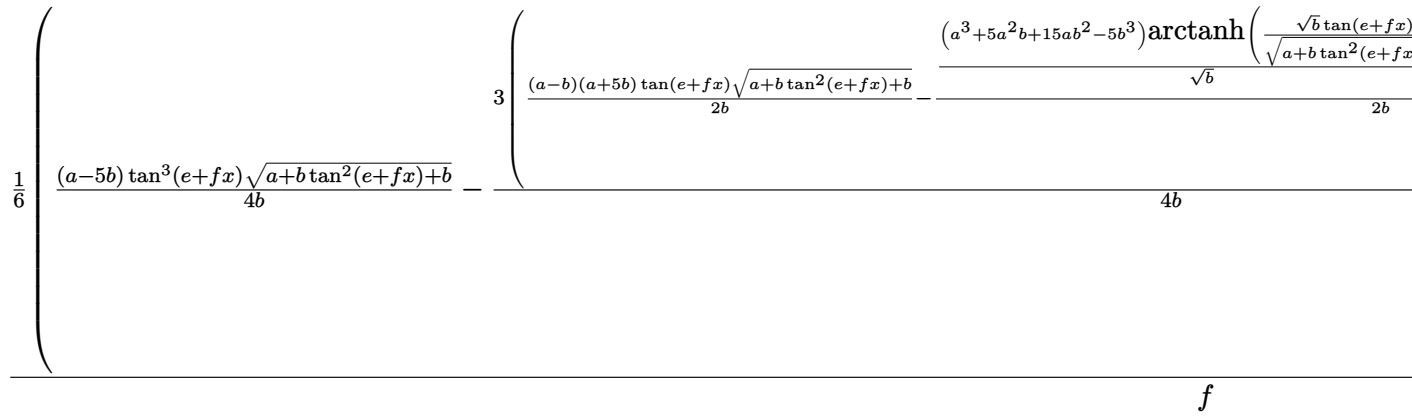
$$4b$$

$$f$$

$$\downarrow 291$$



$\downarrow$ 216



input
```
Int[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^6,x]
```

output
```
((Tan[e + f*x]^5*Sqrt[a + b + b*Tan[e + f*x]^2])/6 + (((a - 5*b)*Tan[e + f
*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(4*b) - (3*(-1/2*(-16*Sqrt[a]*b^2*Ar
cTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]] + ((a^3 + 5*a^
2*b + 15*a*b^2 - 5*b^3)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[
e + f*x]^2]])/Sqrt[b])/b + ((a - b)*(a + 5*b)*Tan[e + f*x]*Sqrt[a + b + b*
Tan[e + f*x]^2])/(2*b)))/(4*b))/6)/f
```

3.382.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 380
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*
(m + 2*(p + q) + 1))), x] - Simp[e^2/(b*(m + 2*(p + q) + 1))   Int[(e*x)^(m
 - 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[a*c*(m - 1) + (a*d*(m - 1) - 2
*q*(b*c - a*d))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && GtQ[q, 0] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p,
 q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.382.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1580 vs. $2(193) = 386$.

Time = 21.77 (sec) , antiderivative size = 1581, normalized size of antiderivative = 7.22

method	result	size
default	Expression too large to display	1581

input
```
int((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^6,x,method=_RETURNVERBOSE)
```

output
```
1/96/f/(-a)^(1/2)/b^(11/2)*(a+b*sec(f*x+e)^2)^(1/2)/(1+cos(f*x+e))/((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(66*(-a)^(1/2)*b^(11/2)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*sin(f*x+e)-96*ln(4*(-a)^(1/2)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*cos(f*x+e)*b^(11/2)*a+66*(-a)^(1/
2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(11/2)*tan(f*x+e)-28*(-a)
^(1/2)*b^(9/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*sin(f*x+e)-52
*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(11/2)*tan(f*x+e
)*sec(f*x+e)-28*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(
9/2)*a*tan(f*x+e)-6*(-a)^(1/2)*b^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*a^2*sin(f*x+e)-52*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*b^(11/2)*tan(f*x+e)*sec(f*x+e)^2+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*b^(9/2)*a*tan(f*x+e)*sec(f*x+e)-6*(-a)^(1/2)*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(7/2)*a^2*tan(f*x+e)+16*(-a)^(1/2
)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(11/2)*tan(f*x+e)*sec(f*x+
e)^3+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(9/2)*a*ta
n(f*x+e)*sec(f*x+e)^2+3*ln(4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)-a-b)/(sin(f*x+e)-1))*(-a)^(1/2)*cos(f*x+e)*a^3*b^3+15*ln(4*(-((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)...
```

3.382.5 Fricas [A] (verification not implemented)

Time = 4.22 (sec) , antiderivative size = 1775, normalized size of antiderivative = 8.11

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^6(e+fx)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^6,x, algorithm="fricas")
```

output
```
[1/192*(24*sqrt(-a)*b^3*cos(f*x + e)^5*log(128*a^4*cos(f*x + e)^8 - 256*(a
^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e
)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7
*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^
2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3
 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2
+ b)/cos(f*x + e)^2)*sin(f*x + e)) - 3*(a^3 + 5*a^2*b + 15*a*b^2 - 5*b^3)*
sqrt(b)*cos(f*x + e)^5*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b -
b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*
x + e)^4) - 4*((3*a^2*b + 14*a*b^2 - 33*b^3)*cos(f*x + e)^4 - 8*b^3 - 2*(a
*b^2 - 13*b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
*sin(f*x + e))/(b^3*f*cos(f*x + e)^5), 1/96*(12*sqrt(-a)*b^3*cos(f*x + e)^
5*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^
4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 2
8*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*
(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a
^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x +
 e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) +
3*(a^3 + 5*a^2*b + 15*a*b^2 - 5*b^3)*sqrt(-b)*arctan(-1/2*((a - b)*cos(...
```

3.382.6 Sympy [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^6(e+fx)\,dx = \int \sqrt{a+b\sec^2(e+fx)}\tan^6(e+fx)\,dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**(1/2)*tan(f*x+e)**6,x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*tan(e + f*x)**6, x)
```

3.382.7 Maxima [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^6(e+fx)\,dx = \int \sqrt{b\sec(fx+e)^2+a}\tan(fx+e)^6\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^6,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*tan(f*x + e)^6, x)
```

3.382.8 Giac [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^6(e+fx)\,dx = \int \sqrt{b\sec(fx+e)^2+a}\tan(fx+e)^6\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^6,x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*tan(f*x + e)^6, x)
```

3.382.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^6(e+fx)\,dx = \int \tan(e+f\,x)^6\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}\,dx$$

input
```
int(tan(e + f*x)^6*(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(tan(e + f*x)^6*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.383 $\int \sqrt{a+b\sec^2(e+fx)}\tan^4(e+fx)\,dx$

3.383.1 Optimal result . . . 2794
3.383.2 Mathematica [A] (verified) . . . 2795
3.383.3 Rubi [A] (verified) . . . 2795
3.383.4 Maple [B] (verified) . . . 2799
3.383.5 Fricas [B] (verification not implemented) . . . 2800
3.383.6 Sympy [F] . . . 2801
3.383.7 Maxima [F] . . . 2802
3.383.8 Giac [F] . . . 2802
3.383.9 Mupad [F(-1)] . . . 2802

3.383.1 Optimal result

Integrand size = 25, antiderivative size = 165

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^4(e+fx)\,dx = \frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{(a^2+6ab-3b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8b^{3/2}f} + \frac{(a-3b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8bf} + \frac{\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4f}$$

output

```
-1/8*(a^2+6*a*b-3*b^2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/
2))/b^(3/2)/f+arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*a^(1/2
)/f+1/8*(a-3*b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b/f+1/4*(a+b+b*tan(f
*x+e)^2)^(1/2)*tan(f*x+e)^3/f
```

3.383.2 Mathematica [A] (verified)

Time = 3.05 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.26

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^4(e+fx)\,dx$$

$$= \frac{\left(8\sqrt{a}b\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right) - \frac{(a^2+6ab-3b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{b}}\right)\cos(e+fx)\sqrt{a+b\sec^2(e+f}}{4\sqrt{2}bf\sqrt{a+2b+a\cos(2e+2fx)}}$$

$$+ \frac{(a-b+(a-5b)\cos(2(e+fx)))\sec^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\tan(e+fx)}{16bf}$$

input
```
Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^4,x]
```

output
```
((8*Sqrt[a]*b*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]
] - ((a^2 + 6*a*b - 3*b^2)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b - a*S
in[e + f*x]^2]])/Sqrt[b])*Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(4*Sqrt
[2]*b*f*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]) + ((a - b + (a - 5*b)*Cos[2*(e
 + f*x)])*Sec[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x])/(16*b*f)
```

3.383.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 4629, 2075, 380, 444, 25, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \tan(e+fx)^4\sqrt{a+b\sec(e+fx)^2}dx$$

$$\downarrow 4629$$

$$\frac{\int \frac{\tan^4(e+fx)\sqrt{a+b(\tan^2(e+fx)+1)}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\tan^4(e+fx)\sqrt{b\tan^2(e+fx)+a+b}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\downarrow 380$$

$$\frac{\frac{1}{4}\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}-\frac{1}{4}\int \frac{\tan^2(e+fx)\left(3(a+b)-(a-3b)\tan^2(e+fx)\right)}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{f}$$

$$\downarrow 444$$

$$\frac{\frac{1}{4}\left(\frac{\int -\frac{\left(a^2+6ba-3b^2\right)\tan^2(e+fx)+(a-3b)(a+b)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2b}+\frac{(a-3b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}\right)+\frac{1}{4}\tan^3(e+fx)\sqrt{a+b\tan^2(}}{f}$$

$$\downarrow 25$$

$$\frac{\frac{1}{4}\left(\frac{(a-3b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{\int \frac{\left(a^2+6ba-3b^2\right)\tan^2(e+fx)+(a-3b)(a+b)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2b}\right)+\frac{1}{4}\tan^3(e+fx)\sqrt{a+b\tan^2(e}}{f}$$

$$\downarrow 398$$

$$\frac{\frac{1}{4}\left(\frac{(a-3b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{\left(a^2+6ab-3b^2\right)\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-8ab\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan}{2b}\right)}{f}$$

$$\downarrow 224$$

$$\frac{\frac{1}{4}\left(\frac{(a-3b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{\left(a^2+6ab-3b^2\right)\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}-8ab\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+}}}{2b}\right)}{f}$$

$$\downarrow 219$$

$$\frac{\frac{1}{4}\left(\frac{(a-3b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{\frac{(a^2+6ab-3b^2)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}}-8ab\int\frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e}{2b}\right.}{f}$$

$\downarrow$ 291

$$\frac{\frac{1}{4}\left(\frac{(a-3b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{\frac{(a^2+6ab-3b^2)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}}-8ab\int\frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{2b}\right.}{f}$$

$\downarrow$ 216

$$\frac{\frac{1}{4}\left(\frac{(a-3b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{\frac{(a^2+6ab-3b^2)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}}-8\sqrt{a}b\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{2b}\right)+\frac{1}{4}\text{ta}}{f}$$

input
```
Int[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^4,x]
```

output
```
((Tan[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/4 + (-1/2*(-8*Sqrt[a]*b*A
rcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]] + ((a^2 + 6*a
*b - 3*b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]
)/Sqrt[b])/b + ((a - 3*b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*
b))/4)/f
```

3.383. $\int \sqrt{a+b\sec^2(e+fx)}\tan^4(e+fx)\,dx$

3.383.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 380
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*
(m + 2*(p + q) + 1))), x] - Simp[e^2/(b*(m + 2*(p + q) + 1))   Int[(e*x)^(m
 - 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[a*c*(m - 1) + (a*d*(m - 1) - 2
*q*(b*c - a*d))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && GtQ[q, 0] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p,
 q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.383.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1064 vs. $2(143) = 286$.

Time = 16.74 (sec) , antiderivative size = 1065, normalized size of antiderivative = 6.45

method	result	size
default	Expression too large to display	1065

input
```
int((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x,method=_RETURNVERBOSE)
```

output
```
1/16/f/(-a)^(1/2)/b^2*(a+b*sec(f*x+e)^2)^(1/2)/(1+cos(f*x+e))/((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(3*cos(f*x+e)*ln(4*(((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*(-a)^(1/2)*b^(5/2)+3*
cos(f*x+e)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(
f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+
b)/(sin(f*x+e)-1))*(-a)^(1/2)*b^(5/2)-6*cos(f*x+e)*ln(4*(((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*(-a)^(1/2)*b^(3/
2)*a-6*cos(f*x+e)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/
2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+
e)*a+a+b)/(sin(f*x+e)-1))*(-a)^(1/2)*b^(3/2)*a-cos(f*x+e)*ln(4*(((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*(-a)^(1/2
)*b^(1/2)*a^2-cos(f*x+e)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-s
in(f*x+e)*a+a+b)/(sin(f*x+e)-1))*(-a)^(1/2)*b^(1/2)*a^2+2*(-a)^(1/2)*sin(f
*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b-10*(-a)^(1/2)*sin(f*
x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2+16*cos(f*x+e)*ln(4*(-
a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^...
```

3.383.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 329 vs. $2(143) = 286$.

Time $= 1.31$ (sec) , antiderivative size $= 1621$, normalized size of antiderivative $= 9.82$

$$\int \sqrt{a + b\sec^2(e+fx)}\tan^4(e+fx)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x, algorithm="fricas")
```

output
```
[1/32*(4*sqrt(-a)*b^2*cos(f*x + e)^3*log(128*a^4*cos(f*x + e)^8 - 256*(a^4
 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^
4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a
^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*
b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 -
 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)*sin(f*x + e)) - (a^2 + 6*a*b - 3*b^2)*sqrt(b)*cos(f*x +
 e)^3*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)
^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 4*((a
*b - 5*b^2)*cos(f*x + e)^2 + 2*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*sin(f*x + e))/(b^2*f*cos(f*x + e)^3), 1/16*(2*sqrt(-a)*b^2*cos(f*x +
 e)^3*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(
5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2
 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2
- 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 -
14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f
*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)
) - (a^2 + 6*a*b - 3*b^2)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2
*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(...
```

3.383.6 Sympy [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^4(e+fx)\,dx = \int \sqrt{a+b\sec^2(e+fx)}\tan^4(e+fx)\,dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**(1/2)*tan(f*x+e)**4,x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*tan(e + f*x)**4, x)
```

3.383.7 Maxima [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^4(e+fx)\,dx = \int \sqrt{b\sec(fx+e)^2+a}\tan(fx+e)^4\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*tan(f*x + e)^4, x)
```

3.383.8 Giac [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^4(e+fx)\,dx = \int \sqrt{b\sec(fx+e)^2+a}\tan(fx+e)^4\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*tan(f*x + e)^4, x)
```

3.383.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^4(e+fx)\,dx = \int \tan(e+f\,x)^4\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}\,dx$$

input
```
int(tan(e + f*x)^4*(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(tan(e + f*x)^4*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.384 $\int \sqrt{a+b\sec^2(e+fx)}\tan^2(e+fx)\,dx$

3.384.1 Optimal result . . . 2803
3.384.2 Mathematica [C] (warning: unable to verify) . . . 2804
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3.384.1 Optimal result

Integrand size = 25, antiderivative size = 118

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^2(e+fx)\,dx = -\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{(a-b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2\sqrt{b}f} + \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f}$$

output

```
-arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*a^(1/2)/f+1/2*(a-b)
*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/b^(1/2)+1/2*(a+b
+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f
```

3.384.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.61 (sec) , antiderivative size = 526, normalized size of antiderivative = 4.46

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^2(e+fx)\,dx$$

$$= \frac{e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos(e+fx)\left(-\frac{i\left(-1+e^{2i(e+fx)}\right)}{\left(1+e^{2i(e+fx)}\right)^2}+\frac{-2\sqrt{a}\sqrt{b}fx+i\sqrt{a}\sqrt{b}\log\left(a+2b+ae^{2i(e+fx)}\right.}{}\right.}{}$$

input
```
Integrate[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^2,x]
```

output
```
(E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e +
f*x))]*Cos[e + f*x]*(((-I)*(-1 + E^((2*I)*(e + f*x))))/(1 + E^((2*I)*(e +
f*x)))^2 + (-2*Sqrt[a]*Sqrt[b]*f*x + I*Sqrt[a]*Sqrt[b]*Log[a + 2*b + a*E^(
(2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*
(e + f*x)))^2]] - I*Sqrt[a]*Sqrt[b]*Log[a + a*E^((2*I)*(e + f*x)) + 2*b*E^
((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)
*(e + f*x)))^2]] - a*Log[(2*(Sqrt[b]*(-1 + E^((2*I)*(e + f*x))) - I*Sqrt[4
*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*f)/((a - b)*(1 +
E^((2*I)*(e + f*x))))] + b*Log[(2*(Sqrt[b]*(-1 + E^((2*I)*(e + f*x))) - I*
Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*f)/((a - b)
*(1 + E^((2*I)*(e + f*x))))])/(Sqrt[b]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1
 + E^((2*I)*(e + f*x)))^2]))*Sqrt[a + b*Sec[e + f*x]^2])/(Sqrt[2]*f*Sqrt[a
 + 2*b + a*Cos[2*e + 2*f*x]])
```

3.384.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4629, 2075, 380, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \tan(e+fx)^2\sqrt{a+b\sec(e+fx)^2}dx$$

$$\downarrow 4629$$

$$\frac{\int \frac{\tan^2(e+fx)\sqrt{a+b(\tan^2(e+fx)+1)}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\tan^2(e+fx)\sqrt{b\tan^2(e+fx)+a+b}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\downarrow 380$$

$$\frac{\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}-\frac{1}{2}\int \frac{-((a-b)\tan^2(e+fx))+a+b}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{f}$$

$$\downarrow 398$$

$$\frac{\frac{1}{2}\left((a-b)\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-2a\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)\right)+\frac{1}{2}\tan(e+fx)}{f}$$

$$\downarrow 224$$

$$\frac{\frac{1}{2}\left((a-b)\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}-2a\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)\right)+\frac{1}{2}\tan(e+\right.}{f}$$

$$\downarrow 219$$

$$\frac{\frac{1}{2}\left(\frac{(a-b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}}-2a\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)\right)+\frac{1}{2}\tan(e+fx)\sqrt{a+b\t}}{f}$$

$$\downarrow 291$$

$$\frac{\frac{1}{2}\left(\frac{(a-b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}}-2a\int\frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}\right)+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+}}{f}$$

$$\downarrow 216$$

$$\frac{\frac{1}{2}\left(\frac{(a-b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}}-2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)\right)+\frac{1}{2}\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

input
```
Int[Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x]^2,x]
```

output
```
((-2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]
 + ((a - b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]
)/Sqrt[b])/2 + (Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/2)/f
```

3.384.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 380
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*
(m + 2*(p + q) + 1))), x] - Simp[e^2/(b*(m + 2*(p + q) + 1))   Int[(e*x)^(m
 - 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[a*c*(m - 1) + (a*d*(m - 1) - 2
*q*(b*c - a*d))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && GtQ[q, 0] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p,
 q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.384.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 660 vs. 2(100) = 200.

Time = 12.30 (sec) , antiderivative size = 661, normalized size of antiderivative = 5.60

method	result
default	$-\frac{\sqrt{a+b\sec(fx+e)^2}\left(\ln\left(\frac{4\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{b}\cos(fx+e)+4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a-4a-4b}{\sin(fx+e)+1}\right)\sqrt{-a}\cos(fx+e)b^{\frac{3}{2}}+b^{\frac{3}{2}}\ln\right.}{}$

input
```
int((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4/f/(-a)^(1/2)/b*(a+b*sec(f*x+e)^2)^(1/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)/(1+cos(f*x+e))*(ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*(-a)^(1/2)*cos(f*x+e)*b^(3/2)+b^(3/2
)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^
(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f
*x+e)-1))*(-a)^(1/2)*cos(f*x+e)-ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*(-a)^(1/2)*cos(f*x+e)*b^(1/2)*a-ln(
-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)
-1))*(-a)^(1/2)*cos(f*x+e)*b^(1/2)*a-2*sin(f*x+e)*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*(-a)^(1/2)*b+4*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)-4*sin(f*x+e)*a)*cos(f*x+e)*a*b-2*(-a)^(1/2)*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*b*tan(f*x+e))
```

3.384.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. 2(100) = 200.

Time = 0.62 (sec) , antiderivative size = 1471, normalized size of antiderivative = 12.47

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^2(e+fx)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x, algorithm="fricas")
```

output
```
[1/8*(sqrt(-a)*b*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*
b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4
 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2
- a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(
f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*
b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)*sin(f*x + e)) - (a - b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a*b
 + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x
 + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 4*b*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)*sin(f*x + e))/(b*f*cos(f*x + e)), 1/8*(2*(a - b)*sqrt(
-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt(
(a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x
 + e)))*cos(f*x + e) + sqrt(-a)*b*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8
- 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos
(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a
^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(
a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^
3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 4*b*sqrt((a*cos(f*x + e)^2...
```

3.384.6 Sympy [F]

$$\int \sqrt{a + b\sec^2(e + fx)}\tan^2(e + fx)\,dx = \int \sqrt{a + b\sec^2(e + fx)}\tan^2(e + fx)\,dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**(1/2)*tan(f*x+e)**2,x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*tan(e + f*x)**2, x)
```

3.384.7 Maxima [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^2(e+fx)\,dx = \int \sqrt{b\sec(fx+e)^2+a}\tan(fx+e)^2\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*tan(f*x + e)^2, x)
```

3.384.8 Giac [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^2(e+fx)\,dx = \int \sqrt{b\sec(fx+e)^2+a}\tan(fx+e)^2\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*tan(f*x + e)^2, x)
```

3.384.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec^2(e+fx)}\tan^2(e+fx)\,dx = \int \tan(e+f\,x)^2\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}\,dx$$

input
```
int(tan(e + f*x)^2*(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(tan(e + f*x)^2*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.385 $\int \sqrt{a+b\sec^2(e+fx)}\,dx$

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3.385.2 Mathematica [C] (verified) 2811
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3.385.1 Optimal result

Integrand size = 16, antiderivative size = 79

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}$$

output
```
arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*a^(1/2)/f+arctanh(b^
(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f
```

3.385.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 284, normalized size of antiderivative = 3.59

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx = -\frac{i\left(1+e^{2i(e+fx)}\right)\left(2\sqrt{b}\arctan\left(\frac{\sqrt{b}\left(-1+e^{2i(e+fx)}\right)}{\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}}\right)+\sqrt{a}\operatorname{arctanh}\left(\frac{a+2b+ae^{2i(e+fx)}}{\sqrt{a}\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}}\right)-\right.}{2\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}f}$$

input
```
Integrate[Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
((-1/2*I)*(1 + E^((2*I)*(e + f*x)))*(2*Sqrt[b]*ArcTan[(Sqrt[b]*(-1 + E^((2
*I)*(e + f*x))))/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x))
)^2]] + Sqrt[a]*ArcTanh[(a + 2*b + a*E^((2*I)*(e + f*x)))/(Sqrt[a]*Sqrt[4*
b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])] - Sqrt[a]*ArcTanh
[(a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*I)*(e + f*x)))/(Sqrt[a]*Sqrt[4*b*E
^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])])*Sqrt[a + b*Sec[e +
f*x]^2])/(Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)
```

3.385.3 Rubi [A] (verified)

Time $= 0.23$ (sec) , antiderivative size $= 77$, normalized size of antiderivative $= 0.97$, number of steps used $= 8$, number of rules used $= 7$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used $= \{3042, 4616, 301, 224, 219, 291, 216\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a + b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \sqrt{a + b\sec(e+fx)^2}dx$$

$$\downarrow 4616$$

$$\frac{\int \frac{\sqrt{b\tan^2(e+fx)+a+b}}{\tan^2(e+fx)+1} d\tan(e+fx)}{f}$$

$$\downarrow 301$$

$$\frac{b\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}} d\tan(e+fx) + a\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}} d\tan(e+fx)}{f}$$

$$\downarrow 224$$

$$\frac{a\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}} d\tan(e+fx) + b\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}} d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{f}$$

$$\downarrow 219$$

$$\frac{a\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}} d\tan(e+fx) + \sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}$$

$$\downarrow 291$$

$$\frac{a\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}+\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}$$

$$\downarrow 216$$

$$\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)+\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{f}$$

input
```
Int[Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]] + S
qrt[b]*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/f
```

3.385.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

3.385.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(67) = 134$.

Time = 4.09 (sec) , antiderivative size = 351, normalized size of antiderivative = 4.44

method	result
default	$\frac{\sqrt{a+b\sec(fx+e)^2}\left(\sqrt{b}\ln\left(-\frac{4\left(\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{b}\cos(fx+e)+\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-\sin(fx+e)a+a+b\right)}{\sin(fx+e)-1}\right)\sqrt{-a}+\sqrt{b}\ln\left(\frac{4\sqrt{\frac{b+a\cos(}{(1+\cos(f}\right.$ … 2

input
```
int((a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f/(-a)^(1/2)*(a+b*sec(f*x+e)^2)^(1/2)*(b^(1/2)*ln(-4*(((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*(-a)^(1/2)+b^(1
/2)*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b
^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(
f*x+e)+1))*(-a)^(1/2)+2*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)-4*sin(f*x+e)*a)*a)*cos(f*x+e)/(1+cos(f*x+e))/((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)
```

3.385.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(67) = 134$.

Time = 0.49 (sec) , antiderivative size = 1227, normalized size of antiderivative = 15.53

$$\int \sqrt{a + b\sec^2(e + fx)}\, dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)
^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 7
0*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*
x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*
(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b
^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin
(f*x + e)) + 2*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b -
b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*
x + e)^4))/f, 1/8*(4*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*co
s(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos
(f*x + e)^2 + b^2)*sin(f*x + e))) + sqrt(-a)*log(128*a^4*cos(f*x + e)^8 -
256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f
*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3
*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^
3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3
- (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)))/f, -1/4*(sqrt(a)*arctan(1/4*(8*a
^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos
(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*...
```

3.385.6 Sympy [F]

$$\int \sqrt{a + b\sec^2(e + fx)}\, dx = \int \sqrt{a + b\sec^2(e + fx)}\, dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2), x)
```

3.385.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.81 (sec) , antiderivative size = 3227, normalized size of antiderivative = 40.85

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*(2*sqrt(a)*b^(3/2)*arctan2(a*sin(2*f*x + 2*e) + (a^2*cos(4*f*x + 4*e)
^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 +
 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^
2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*c
os(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*sin(1/2*
arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4
*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)), a*cos(2*f*x + 2*e) + (a^2*cos(4*
f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x
 + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*
a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x
 + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a
)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos
(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) + a + 2*b) + a^(3/2)*sq
rt(b)*arctan2(2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2
 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin
(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2
 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos
(2*f*x + 2*e))^(1/4)*sqrt(a)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2
*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) +
a)), 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*...
```

3.385.8 Giac [F]

$$\int \sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec\left(fx+e\right)^2+a}\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a), x)
```

3.385.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a + b \sec^2(e + fx)}\, dx = \int \sqrt{a + \frac{b}{\cos(e + f x)^2}}\, dx$$

input
```
int((a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int((a + b/cos(e + f*x)^2)^(1/2), x)
```

3.386 $\int \cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

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3.386.1 Optimal result

Integrand size = 25, antiderivative size = 69

$$\int \cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = -\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f}$$

output
```
-arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*a^(1/2)/f-cot(f*x+e
)*(a+b+b*tan(f*x+e)^2)^(1/2)/f
```

3.386.2 Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.88

$$\int \cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = -\frac{\cot(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(\sqrt{a+b}\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}} + \sqrt{2}\sqrt{a}\arcsin\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)\sin(e+fx)\right)}{\sqrt{a+b}f\sqrt{\frac{a+2b+a\cos(2(e+fx))}{a+b}}}$$

input
```
Integrate[Cot[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
-((Cot[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*(Sqrt[a + b]*Sqrt[(a + 2*b + a*
Cos[2*(e + f*x)])/(a + b)] + Sqrt[2]*Sqrt[a]*ArcSin[(Sqrt[a]*Sin[e + f*x])
/Sqrt[a + b]]*Sin[e + f*x]))/(Sqrt[a + b]*f*Sqrt[(a + 2*b + a*Cos[2*(e + f
*x)])/(a + b)]))
```

3.386.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4629, 2075, 377, 25, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sqrt{a+b\sec(e+fx)^2}}{\tan(e+fx)^2}dx$$

$$\downarrow 4629$$

$$\frac{\int \frac{\cot^2(e+fx)\sqrt{a+b(\tan^2(e+fx)+1)}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\cot^2(e+fx)\sqrt{b\tan^2(e+fx)+a+b}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\downarrow 377$$

$$\frac{\int -\frac{a}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx) - \cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

$$\downarrow 25$$

$$\frac{\cot(e+fx)\left(-\sqrt{a+b\tan^2(e+fx)+b}\right) - \int \frac{a}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{f}$$

$$\downarrow 27$$

$$\frac{\cot(e+fx)\left(-\sqrt{a+b\tan^2(e+fx)+b}\right) - a\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{f}$$

$$\downarrow 291$$

$$\frac{\cot(e+fx)\left(-\sqrt{a+b\tan^2(e+fx)+b}\right)-a\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{f}$$

$$\downarrow 216$$

$$\frac{-\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)-\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

input
```
Int[Cot[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(-(Sqrt[a]*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])
- Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/f
```

3.386.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.386.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(61) = 122$.

Time = 3.22 (sec) , antiderivative size = 224, normalized size of antiderivative = 3.25

method	result
default	$-\frac{\left(\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)+\ln\left(4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)+4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a\right)a\sin(fx+$ $f\sqrt{-a}\,(1+\cos(fx+e))\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}$

input
```
int(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f/(-a)^(1/2)*((-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*co
s(f*x+e)+ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f
*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e
)*a)*a*sin(f*x+e)+(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*
(a+b*sec(f*x+e)^2)^(1/2)/(1+cos(f*x+e))/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*cot(f*x+e)
```

3.386.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(61) = 122$.

Time $= 0.43$ (sec) , antiderivative size $= 499$, normalized size of antiderivative $= 7.23$

$$\int \cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$= \left[\frac{\sqrt{-a}\log\left(128\,a^4\cos(fx+e)^8 - 256\,(a^4-a^3b)\cos(fx+e)^6 + 32\,(5\,a^4-14\,a^3b+5\,a^2b^2)\cos(fx+e)^4\right.}{}\right.$$

input
```
integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)
^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 7
0*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*
x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*
(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b
^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin
(f*x + e))*sin(f*x + e) - 8*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*co
s(f*x + e))/(f*sin(f*x + e)), 1/4*(sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^
5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(
a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^
2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*sin(f*x + e)
- 4*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e))/(f*sin(f*x +
 e))]
```

3.386.6 Sympy [F]

$$\int \cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{a+b\sec^2(e+fx)}\cot^2(e+fx)\,dx$$

input
```
integrate(cot(f*x+e)**2*(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*cot(e + f*x)**2, x)
```

3.386.7 Maxima [F]

$$\int \cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\cot(fx+e)^2\,dx$$

input
```
integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*cot(f*x + e)^2, x)
```

3.386.8 Giac [F]

$$\int \cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\cot(fx+e)^2\,dx$$

input
```
integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*cot(f*x + e)^2, x)
```

3.386.9 Mupad [F(-1)]

Timed out.

$$\int \cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \cot(e+f\,x)^2\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}\,dx$$

input
```
int(cot(e + f*x)^2*(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(cot(e + f*x)^2*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.387 $\int \cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

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3.387.1 Optimal result

Integrand size = 25, antiderivative size = 114

$$\int \cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{(3a+2b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)f} - \frac{\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3f}$$

output
```
arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*a^(1/2)/f+1/3*(3*a+2
*b)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)/f-1/3*cot(f*x+e)^3*(a+b+b*
tan(f*x+e)^2)^(1/2)/f
```

3.387.2 Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.54

$$\int \cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = -\frac{\sqrt{2}\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}\left(\frac{\csc^3(e+fx)(a+b-a\sin^2(e+fx))^{3/2}}{a+b} - 3\csc(e+fx)\sqrt{a+b-a\sin^2(e+f}\right.}{3f\sqrt{a+2b+a\cos(2e+2fx)}}$$

input
```
Integrate[Cot[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
-1/3*(Sqrt[2]*Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2]*((Csc[e + f*x]^3*(a
+ b - a*Sin[e + f*x]^2)^(3/2))/(a + b) - 3*Csc[e + f*x]*Sqrt[a + b - a*Sin
[e + f*x]^2]*(1 + (Sqrt[a]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b]]*Sin[
e + f*x])/(Sqrt[a + b]*Sqrt[1 - (a*Sin[e + f*x]^2)/(a + b)])))))/(f*Sqrt[a
+ 2*b + a*Cos[2*e + 2*f*x]])
```

3.387.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4629, 2075, 377, 445, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sqrt{a+b\sec(e+fx)^2}}{\tan(e+fx)^4}dx$$

$$\downarrow 4629$$

$$\frac{\int \frac{\cot^4(e+fx)\sqrt{a+b(\tan^2(e+fx)+1)}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\cot^4(e+fx)\sqrt{b\tan^2(e+fx)+a+b}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\downarrow 377$$

$$\frac{\frac{1}{3}\int \frac{\cot^2(e+fx)(-2b\tan^2(e+fx)+b-3(a+b))}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx) - \frac{1}{3}\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

$$\downarrow 445$$

$$\frac{\frac{1}{3}\left(\frac{(3a+2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}-\frac{\int -\frac{3a(a+b)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a+b}\right)-\frac{1}{3}\cot^3(e+fx)\sqrt{a+b\tan^2(e}}{f}$$

$$\downarrow 27$$

$$\frac{\frac{1}{3}\left(3a\int\frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+\frac{(3a+2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}\right)-\frac{1}{3}\cot^3(e+fx)\sqrt{a+b}}{f}$$

$$\downarrow 291$$

$$\frac{\frac{1}{3}\left(3a\int\frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}+\frac{(3a+2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}\right)-\frac{1}{3}\cot^3(e+fx)\sqrt{a+b\tan^2(e+}}{f}$$

$$\downarrow 216$$

$$\frac{\frac{1}{3}\left(3\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)+\frac{(3a+2b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}\right)-\frac{1}{3}\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

input
```
Int[Cot[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(-1/3*(Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2]) + (3*Sqrt[a]*ArcTan[
(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]] + ((3*a + 2*b)*Cot[
e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(a + b))/3)/f
```

3.387.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.387.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 595 vs. $2(100) = 200$.

Time = 4.71 (sec) , antiderivative size = 596, normalized size of antiderivative = 5.23

method	result
default	$-\frac{\left(3\sin(fx+e)a^2\ln\left(4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)+4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a\right)\cos(fx+e)+3\sin(fx+e)ab\ln($

input
```
int(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/f/(a+b)/(-a)^(1/2)*(3*sin(f*x+e)*a^2*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*cos(f*x+e)+3*sin(f*x+e)*a*b*ln(4*(-a
)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2
)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*cos(f*x+e)+4
*cos(f*x+e)^2*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a+3*c
os(f*x+e)^2*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b-3*sin
(f*x+e)*a^2*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*co
s(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*
x+e)*a)-3*sin(f*x+e)*a*b*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)-4*sin(f*x+e)*a)-3*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*a-2*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b)*(a+b*sec
(f*x+e)^2)^(1/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cot(f*x+e)*cs
c(f*x+e)^2
```

3.387.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(100) = 200$.

Time = 0.72 (sec) , antiderivative size = 629, normalized size of antiderivative = 5.52

$$\int \cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$= \left[\frac{3\left((a+b)\cos(fx+e)^2-a-b\right)\sqrt{-a}\log\left(128\,a^4\cos(fx+e)^8-256\left(a^4-a^3b\right)\cos(fx+e)^6+32\left(5\,a^4\right.\right.}{} \right.$$

$$\left. -\frac{3\left((a+b)\cos(fx+e)^2-a-b\right)\sqrt{a}\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^5-8\left(a^2-ab\right)\cos(fx+e)^3+\left(a^2-6\,ab+b^2\right)\cos(fx+e)\right)\sqrt{a}\sqrt{\frac{a\,c}{}}}{4\left(2\,a^3\cos(fx+e)^4-a^2b+ab^2-\left(a^3-3\,a^2b\right)\cos(fx+e)^2\right)\sin(fx+e)}\right.}{12\left((a+b)f\cos(fx+e)^2-(\right.} \right]$$

input
```
integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/24*(3*((a + b)*cos(f*x + e)^2 - a - b)*sqrt(-a)*log(128*a^4*cos(f*x + e
)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)
*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 -
 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 -
24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x +
 e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos
(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))*sin(f*x + e) + 8*((4*a + 3*
b)*cos(f*x + e)^3 - (3*a + 2*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/
cos(f*x + e)^2))/(((a + b)*f*cos(f*x + e)^2 - (a + b)*f)*sin(f*x + e)), -1
/12*(3*((a + b)*cos(f*x + e)^2 - a - b)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x
+ e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*
sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4
 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*sin(f*x
+ e) - 4*((4*a + 3*b)*cos(f*x + e)^3 - (3*a + 2*b)*cos(f*x + e))*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a + b)*f*cos(f*x + e)^2 - (a + b)*f
)*sin(f*x + e))]
```

3.387.6 Sympy [F]

$$\int \cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{a+b\sec^2(e+fx)}\cot^4(e+fx)\,dx$$

input
```
integrate(cot(f*x+e)**4*(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*cot(e + f*x)**4, x)
```

3.387.7 Maxima [F]

$$\int \cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\cot(fx+e)^4\,dx$$

input
```
integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*cot(f*x + e)^4, x)
```

3.387.8 Giac [F]

$$\int \cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\cot(fx+e)^4\,dx$$

input
```
integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*cot(f*x + e)^4, x)
```

3.387.9 Mupad [F(-1)]

Timed out.

$$\int \cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \cot(e+f\,x)^4\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}\,dx$$

input
```
int(cot(e + f*x)^4*(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(cot(e + f*x)^4*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.388 $\int \cot^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

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3.388.1 Optimal result

Integrand size = 25, antiderivative size = 167

$$\begin{aligned}
&\int \cot^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx \\
&= -\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} \\
&\quad -\frac{(15a^2+25ab+8b^2)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^2 f} \\
&\quad -\frac{(b-5(a+b))\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)f} \\
&\quad -\frac{\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5f}
\end{aligned}$$

output

```
-arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*a^(1/2)/f-1/15*(15*
a^2+25*a*b+8*b^2)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)^2/f-1/15*(-4
*b-5*a)*cot(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)/f-1/5*cot(f*x+e)^5*(
a+b+b*tan(f*x+e)^2)^(1/2)/f
```

3.388.2 Mathematica [A] (verified)

Time = 2.88 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.07

$$\int \cot^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$
$$= -\frac{\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)\cos(e+fx)\sqrt{a+b\sec^2(e+fx)}}{f\sqrt{a+2b+a\cos(2e+2fx)}}$$
$$-\frac{\cot(e+fx)\left(23a^2+40ab+15b^2-\left(11a^2+21ab+10b^2\right)\csc^2(e+fx)+3(a+b)^2\csc^4(e+fx)\right)\sqrt{a+b}}{15(a+b)^2 f}$$

input
```
Integrate[Cot[e + f*x]^6*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
-((Sqrt[2]*Sqrt[a]*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*
x]^2]]*Cos[e + f*x]*Sqrt[a + b*Sec[e + f*x]^2])/(f*Sqrt[a + 2*b + a*Cos[2*
e + 2*f*x]])) - (Cot[e + f*x]*(23*a^2 + 40*a*b + 15*b^2 - (11*a^2 + 21*a*b
 + 10*b^2)*Csc[e + f*x]^2 + 3*(a + b)^2*Csc[e + f*x]^4)*Sqrt[a + b*Sec[e +
 f*x]^2])/(15*(a + b)^2*f)
```

3.388.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4629, 2075, 377, 445, 25, 445, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\sqrt{a+b\sec(e+fx)^2}}{\tan(e+fx)^6}\,dx$$
$$\downarrow 4629$$
$$\frac{\int \frac{\cot^6(e+fx)\sqrt{a+b(\tan^2(e+fx)+1)}}{\tan^2(e+fx)+1}\,d\tan(e+fx)}{f}$$
$$\downarrow 2075$$

$$\frac{\int \frac{\cot^6(e+fx)\sqrt{b\tan^2(e+fx)+a+b}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\downarrow 377$$

$$\frac{\frac{1}{5}\int \frac{\cot^4(e+fx)(-4b\tan^2(e+fx)+b-5(a+b))}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-\frac{1}{5}\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

$$\downarrow 445$$

$$\frac{\frac{1}{5}\left(-\frac{\int -\frac{\cot^2(e+fx)\left(15a^2+25ba+8b^2-2b(b-5(a+b))\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{3(a+b)}-\frac{(b-5(a+b))\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)}\right)-\frac{1}{5}\cot^5(e+}{f}$$

$$\downarrow 25$$

$$\frac{\frac{1}{5}\left(\frac{\int \frac{\cot^2(e+fx)\left(15a^2+25ba+8b^2-2b(b-5(a+b))\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{3(a+b)}-\frac{(b-5(a+b))\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)}\right)-\frac{1}{5}\cot^5(e+fx}{f}$$

$$\downarrow 445$$

$$\frac{\frac{1}{5}\left(-\frac{\int \frac{15a(a+b)^2}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a+b}-\frac{\left(15a^2+25ab+8b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{3(a+b)}-\frac{(b-5(a+b))\cot^3(e+fx)\sqrt{a+b\tan^2(e+}}{3(a+b)}\right)}{f}$$

$$\downarrow 27$$

$$\frac{\frac{1}{5}\left(\frac{-15a(a+b)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-\frac{\left(15a^2+25ab+8b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{3(a+b)}-\frac{(b-5(a+b))\cot^3(e+fx)\sqrt{a-}}{3(a+b)}\right)}{f}$$

$$\downarrow 291$$

$$\frac{\frac{1}{5}\left(\frac{-15a(a+b)\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}-\frac{\left(15a^2+25ab+8b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{3(a+b)}-\frac{(b-5(a+b))\cot^3(e+fx)\sqrt{a+b\tan}}{3(a+b)}\right)}{f}$$

3.388. $\int \cot^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$

$$\downarrow 216$$

$$\frac{1}{5}\left(\frac{-\frac{\left(15a^2+25ab+8b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}-15\sqrt{a}(a+b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{3(a+b)}-\frac{(b-5(a+b))\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)}\right.$$

$$f$$

input
```
Int[Cot[e + f*x]^6*Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(-1/5*(Cot[e + f*x]^5*Sqrt[a + b + b*Tan[e + f*x]^2]) + (-1/3*((b - 5*(a +
 b))*Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(a + b) + (-15*Sqrt[a]
*(a + b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]] - (
(15*a^2 + 25*a*b + 8*b^2)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(a
+ b))/(3*(a + b)))/5)/f
```

3.388.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.388.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1015 vs. $2(149) = 298$.

Time = 5.90 (sec) , antiderivative size = 1016, normalized size of antiderivative = 6.08

method	result	size
default	Expression too large to display	1016

input
```
int(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/15/f/(a+b)^2/(-a)^(1/2)*(15*sin(f*x+e)^3*a^3*ln(4*(-a)^(1/2)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*cos(f*x+e)+30*sin(f*x+e)^3*a^2*
b*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4
*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*co
s(f*x+e)+15*sin(f*x+e)^3*a*b^2*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)-4*sin(f*x+e)*a)*cos(f*x+e)-23*cos(f*x+e)^4*(-a)^(1/2)*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2-40*cos(f*x+e)^4*(-a)^(1/2)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b-15*cos(f*x+e)^4*(-a)^(1/2)*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2-15*sin(f*x+e)^3*a^3*ln(4*(-a)^(1/
2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)-30*sin(f*x+e)^3*a
^2*b*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e
)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)
-15*sin(f*x+e)^3*a*b^2*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)-4*sin(f*x+e)*a)+35*cos(f*x+e)^2*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*a^2+59*cos(f*x+e)^2*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*a*b+20*cos(f*x+e)^2*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+...
```

3.388.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(149) = 298$.

Time = 2.30 (sec) , antiderivative size = 849, normalized size of antiderivative = 5.08

$$\int \cot^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx$$

$$= \left[\frac{15\left(\left(a^2+2\,ab+b^2\right)\cos\left(fx+e\right)^4-2\left(a^2+2\,ab+b^2\right)\cos\left(fx+e\right)^2+a^2+2\,ab+b^2\right)\sqrt{-a}\log\left(128\,a^4\,\mathrm{co}\right.\right.$$

input
```
integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/120*(15*((a^2 + 2*a*b + b^2)*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos
(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256
*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x
+ e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b
+ 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 -
 a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (
a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)
^2 + b)/cos(f*x + e)^2)*sin(f*x + e))*sin(f*x + e) - 8*((23*a^2 + 40*a*b +
 15*b^2)*cos(f*x + e)^5 - (35*a^2 + 59*a*b + 20*b^2)*cos(f*x + e)^3 + (15*
a^2 + 25*a*b + 8*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2))/(((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*f*co
s(f*x + e)^2 + (a^2 + 2*a*b + b^2)*f)*sin(f*x + e)), 1/60*(15*((a^2 + 2*a*
b + b^2)*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a
*b + b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x
 + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^
2*b)*cos(f*x + e)^2)*sin(f*x + e)))*sin(f*x + e) - 4*((23*a^2 + 40*a*b + 1
5*b^2)*cos(f*x + e)^5 - (35*a^2 + 59*a*b + 20*b^2)*cos(f*x + e)^3 + (15*a^
2 + 25*a*b + 8*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2))/(((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*f*c...
```

3.388.6 Sympy [F]

$$\int \cot^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{a+b\sec^2(e+fx)}\cot^6(e+fx)\,dx$$

input
```
integrate(cot(f*x+e)**6*(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(e + f*x)**2)*cot(e + f*x)**6, x)
```

3.388.7 Maxima [F]

$$\int \cot^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\cot(fx+e)^6\,dx$$

input
```
integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*cot(f*x + e)^6, x)
```

3.388.8 Giac [F]

$$\int \cot^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)^2+a}\cot(fx+e)^6\,dx$$

input
```
integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e)^2 + a)*cot(f*x + e)^6, x)
```

3.388.9 Mupad [F(-1)]

Timed out.

$$\int \cot^6(e+fx)\sqrt{a+b\sec^2(e+fx)}\,dx = \int \cot(e+fx)^6\sqrt{a+\frac{b}{\cos(e+fx)^2}}\,dx$$

input
```
int(cot(e + f*x)^6*(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(cot(e + f*x)^6*(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.389 $\int (a+b\sec^2(e+fx))^{3/2}\tan^5(e+fx)\,dx$

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3.389.1 Optimal result

Integrand size = 25, antiderivative size = 135

$$\int (a+b\sec^2(e+fx))^{3/2}\tan^5(e+fx)\,dx = -\frac{a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} + \frac{a\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{(a+b\sec^2(e+fx))^{3/2}}{3f} - \frac{(a+2b)(a+b\sec^2(e+fx))^{5/2}}{5b^2f} + \frac{(a+b\sec^2(e+fx))^{7/2}}{7b^2f}$$

output
```
-a^(3/2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f+1/3*(a+b*sec(f*x+e)^2
)^(3/2)/f-1/5*(a+2*b)*(a+b*sec(f*x+e)^2)^(5/2)/b^2/f+1/7*(a+b*sec(f*x+e)^2
)^(7/2)/b^2/f+a*(a+b*sec(f*x+e)^2)^(1/2)/f
```

3.389.2 Mathematica [A] (verified)

Time = 2.40 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.10

$$\int (a+b\sec^2(e+fx))^{3/2}\tan^5(e+fx)\,dx = \frac{\sqrt{a+b\sec^2(e+fx)}\left(35b^2(a+b\sec^2(e+fx)) - 42b(a+b\sec^2(e+fx))^2 + 15(a+b\sec^2(e+f\right.$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^5,x]
```

output
```
(Sqrt[a + b*Sec[e + f*x]^2]*(35*b^2*(a + b*Sec[e + f*x]^2) - 42*b*(a + b*S
ec[e + f*x]^2)^2 + 15*(a + b*Sec[e + f*x]^2)^3 - (105*a*b^2*ArcTanh[Sqrt[(
a + b*Sec[e + f*x]^2)/a]])/Sqrt[(a + b*Sec[e + f*x]^2)/a] + 21*a*(5*b^2 -
(a + b*Sec[e + f*x]^2)^2)))/(105*b^2*f)
```

3.389.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4627, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int \tan(e+fx)^5\left(a+b\sec(e+fx)^2\right)^{3/2}dx$$

$$\downarrow 4627$$

$$\frac{\int \cos(e+fx)\left(1-\sec^2(e+fx)\right)^2\left(b\sec^2(e+fx)+a\right)^{3/2}d\sec(e+fx)}{f}$$

$$\downarrow 354$$

$$\frac{\int \cos(e+fx)\left(1-\sec^2(e+fx)\right)^2\left(b\sec^2(e+fx)+a\right)^{3/2}d\sec^2(e+fx)}{2f}$$

$$\downarrow 99$$

$$\frac{\int \left(\frac{\left(b\sec^2(e+fx)+a\right)^{5/2}}{b}+\cos(e+fx)\left(b\sec^2(e+fx)+a\right)^{3/2}+\frac{(-a-2b)\left(b\sec^2(e+fx)+a\right)^{3/2}}{b}\right)d\sec^2(e+fx)}{2f}$$

$$\downarrow 2009$$

$$\frac{-2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)+\frac{2\left(a+b\sec^2(e+fx)\right)^{7/2}}{7b^2}-\frac{2(a+2b)\left(a+b\sec^2(e+fx)\right)^{5/2}}{5b^2}+\frac{2}{3}\left(a+b\sec^2(e+fx)\right)^{3/2}+2a}{2f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^5,x]
```

output
```
(-2*a^(3/2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]] + 2*a*Sqrt[a + b*S
ec[e + f*x]^2] + (2*(a + b*Sec[e + f*x]^2)^(3/2))/3 - (2*(a + 2*b)*(a + b*
Sec[e + f*x]^2)^(5/2))/(5*b^2) + (2*(a + b*Sec[e + f*x]^2)^(7/2))/(7*b^2))
/(2*f)
```

3.389.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.389.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 825 vs. $2(115) = 230$.

Time = 16.96 (sec) , antiderivative size = 826, normalized size of antiderivative = 6.12

method	result
default	$-\frac{\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}\left(105\ln\left(4\cos(fx+e)\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{a}+4\cos(fx+e)a+4\sqrt{a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\right)\cos(fx+e)^3 a^{\frac{3}{2}}b^2+6\cos($

input
```
int((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/105/f/b^2*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)/(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))*(105*ln(4*cos(f*x+e)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*cos(f*x+e)^3*a^(3/2)*b^2+6*cos(f*x+e)
^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^3+42*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)^3*a^2*b-140*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*cos(f*x+e)^3*b^2*a+6*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*a^3+42*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*a^2*b-140*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e
)^2*b^2*a-3*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*b+8
4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)*a*b^2-35*cos(f*x+
e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^3-3*a^2*b*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)+84*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*a*b^2-35*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^3-24*a*b^2*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*sec(f*x+e)+42*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*b^3*sec(f*x+e)-24*a*b^2*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*sec(f*x+e)^2+42*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*b^3*sec(f*x+e)^2-15*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^3*sec(
f*x+e)^3-15*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^3*sec(f*x+e)^4)
```

3.389.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(115) = 230$.

Time = 7.52 (sec) , antiderivative size = 527, normalized size of antiderivative = 3.90

$$\int \left(a + b\sec^2(e+fx)\right)^{3/2}\tan^5(e$$

$$+fx)\,dx = \left[\frac{105\,a^{\frac{3}{2}}b^2\cos(fx+e)^6\log\left(128\,a^4\cos(fx+e)^8 + 256\,a^3b\cos(fx+e)^6 + 160\,a^2b^2\cos(fx+e)^4\right.\right.$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^5,x, algorithm="fricas")
```

output
```
[1/840*(105*a^(3/2)*b^2*cos(f*x + e)^6*log(128*a^4*cos(f*x + e)^8 + 256*a^
3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2
 + b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(
f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)) - 8*(2*(3*a^3 + 21*a^2*b - 70*a*b^2)*cos(f*x + e)^6 - (3*a^2*b
 - 84*a*b^2 + 35*b^3)*cos(f*x + e)^4 - 15*b^3 - 6*(4*a*b^2 - 7*b^3)*cos(f*
x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(b^2*f*cos(f*x + e)
^6), 1/420*(105*sqrt(-a)*a*b^2*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*co
s(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(
2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2))*cos(f*x + e)^6 - 4
*(2*(3*a^3 + 21*a^2*b - 70*a*b^2)*cos(f*x + e)^6 - (3*a^2*b - 84*a*b^2 + 3
5*b^3)*cos(f*x + e)^4 - 15*b^3 - 6*(4*a*b^2 - 7*b^3)*cos(f*x + e)^2)*sqrt(
(a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(b^2*f*cos(f*x + e)^6)]
```

3.389.6 Sympy [F]

$$\int \left(a + b\sec^2(e+fx)\right)^{3/2}\tan^5(e+fx)\,dx = \int \left(a + b\sec^2(e+fx)\right)^{\frac{3}{2}}\tan^5(e+fx)\,dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**(3/2)*tan(f*x+e)**5,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(3/2)*tan(e + f*x)**5, x)
```

3.389.7 Maxima [F]

$$\int (a+b\sec^2(e+fx))^{3/2}\tan^5(e+fx)\,dx = \int (b\sec(fx+e)^2+a)^{\frac{3}{2}}\tan(fx+e)^5\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^5,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^5, x)
```

3.389.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2026 vs. $2(115) = 230$.

Time = 3.26 (sec) , antiderivative size = 2026, normalized size of antiderivative = 15.01

$$\int (a+b\sec^2(e+fx))^{3/2}\tan^5(e+fx)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^5,x, algorithm="giac")
```

output
```
2/105*(105*a^2*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*ta
n(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)
^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))/sqrt(-a))*sgn(cos(
f*x + e))/sqrt(-a) - 2*(105*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*t
an(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e
)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^13*a^2*sgn(cos(f*x + e)) - 1575
*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*t
an(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2
*e)^2 + a + b))^12*sqrt(a + b)*a^2*sgn(cos(f*x + e)) + 70*(129*a^3 + 21*a^
2*b - 96*a*b^2 - 32*b^3)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(
1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2
 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^11*sgn(cos(f*x + e)) - 70*(387*a^3
 - 417*a^2*b - 96*a*b^2 + 128*b^3)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - s
qrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x
+ 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^10*sqrt(a + b)*sgn(cos(f
*x + e)) + 7*(6525*a^4 - 11790*a^3*b - 2235*a^2*b^2 + 9088*a*b^3 - 1344*b^
4)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b
*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1
/2*e)^2 + a + b))^9*sgn(cos(f*x + e)) - 7*(5355*a^4 - 28290*a^3*b + 28995*
a^2*b^2 - 3008*a*b^3 - 896*b^4)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - s...
```

3.389.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\sec^2(e+fx))^{3/2}\tan^5(e+fx)\,dx = \int \tan(e+f\,x)^5\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}dx$$

input
```
int(tan(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(tan(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.390 $\int (a+b\sec^2(e+fx))^{3/2}\tan^3(e+fx)\,dx$

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3.390.1 Optimal result

Integrand size = 25, antiderivative size = 104

$$\int (a+b\sec^2(e+fx))^{3/2}\tan^3(e+fx)\,dx = \frac{a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} - \frac{a\sqrt{a+b\sec^2(e+fx)}}{f} - \frac{(a+b\sec^2(e+fx))^{3/2}}{3f} + \frac{(a+b\sec^2(e+fx))^{5/2}}{5bf}$$

output
```
a^(3/2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f-1/3*(a+b*sec(f*x+e)^2)
^(3/2)/f+1/5*(a+b*sec(f*x+e)^2)^(5/2)/b/f-a*(a+b*sec(f*x+e)^2)^(1/2)/f
```

3.390.2 Mathematica [A] (verified)

Time = 1.72 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.95

$$\int (a+b\sec^2(e+fx))^{3/2}\tan^3(e+fx)\,dx = \frac{15a^{3/2}b\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right) - 15ab\sqrt{a+b\sec^2(e+fx)} - 5b(a+b\sec^2(e+fx))^{3/2} + 3(a+\cdots}{15bf}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^3,x]
```

output
```
(15*a^(3/2)*b*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]] - 15*a*b*Sqrt[a
+ b*Sec[e + f*x]^2] - 5*b*(a + b*Sec[e + f*x]^2)^(3/2) + 3*(a + b*Sec[e +
f*x]^2)^(5/2))/(15*b*f)
```

3.390.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4627, 25, 354, 90, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int \tan(e+fx)^3\left(a+b\sec(e+fx)^2\right)^{3/2}dx$$

$$\downarrow 4627$$

$$\frac{\int -\cos(e+fx)\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{3/2}d\sec(e+fx)}{f}$$

$$\downarrow 25$$

$$-\frac{\int \cos(e+fx)\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{3/2}d\sec(e+fx)}{f}$$

$$\downarrow 354$$

$$-\frac{\int \cos(e+fx)\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{3/2}d\sec^2(e+fx)}{2f}$$

$$\downarrow 90$$

$$-\frac{\int \cos(e+fx)\left(b\sec^2(e+fx)+a\right)^{3/2}d\sec^2(e+fx)-\frac{2\left(a+b\sec^2(e+fx)\right)^{5/2}}{5b}}{2f}$$

$$\downarrow 60$$

$$-\frac{a\int \cos(e+fx)\sqrt{b\sec^2(e+fx)+a}\,d\sec^2(e+fx)-\frac{2\left(a+b\sec^2(e+fx)\right)^{5/2}}{5b}+\frac{2}{3}\left(a+b\sec^2(e+fx)\right)^{3/2}}{2f}$$

$$\downarrow 60$$

$$-\frac{a\left(a\int \frac{\cos(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)+2\sqrt{a+b\sec^2(e+fx)}\right)-\frac{2\left(a+b\sec^2(e+fx)\right)^{5/2}}{5b}+\frac{2}{3}\left(a+b\sec^2(e+fx)\right)^{3/2}}{2f}$$

$$\downarrow 73$$

$$-\frac{a\left(\frac{2a\int\frac{1}{\frac{\sec^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}+2\sqrt{a+b\sec^2(e+fx)}\right)-\frac{2(a+b\sec^2(e+fx))^{5/2}}{5b}+\frac{2}{3}(a+b\sec^2(e+fx))^{3/2}}{2f}$$

$$\downarrow 221$$

$$-\frac{a\left(2\sqrt{a+b\sec^2(e+fx)}-2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)\right)-\frac{2(a+b\sec^2(e+fx))^{5/2}}{5b}+\frac{2}{3}(a+b\sec^2(e+fx))^{3/2}}{2f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^3,x]
```

output
```
-1/2*((2*(a + b*Sec[e + f*x]^2)^(3/2))/3 - (2*(a + b*Sec[e + f*x]^2)^(5/2)
)/(5*b) + a*(-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]] + 2*Sq
rt[a + b*Sec[e + f*x]^2]))/f
```

3.390.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.390.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 517 vs. $2(88) = 176$.

Time = 14.22 (sec) , antiderivative size = 518, normalized size of antiderivative = 4.98

method	result
default	$\frac{\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}\left(15\cos(fx+e)^3 a^{\frac{3}{2}}\ln\left(4\cos(fx+e)\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{a}+4\cos(fx+e)a+4\sqrt{a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\right)b+3\cos(fx+$

input
```
int((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/15/f/b*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)/(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))*(15*cos(f*x+e)^3*a^(3/2)*ln(4*cos(f*
x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*cos(f*x+e)*a+4*
a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*b+3*cos(f*x+e)^3*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2-20*cos(f*x+e)^3*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b+3*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*a^2-20*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*a*b+6*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b-
5*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2*cos(f*x+e)+6*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b-5*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*b^2+3*b^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*sec(f*x+e)+
3*b^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*sec(f*x+e)^2)
```

3.390.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 198 vs. $2(88) = 176$.

Time $= 1.97$ (sec) , antiderivative size $= 443$, normalized size of antiderivative $= 4.26$

$$\int \left(a + b\sec^2(e+fx)\right)^{3/2}\tan^3(e+fx)\,dx = \left[\frac{15\,a^{\frac{3}{2}}b\cos(fx+e)^4\log\left(128\,a^4\cos(fx+e)^8 + 256\,a^3b\cos(fx+e)^6 + 160\,a^2b^2\cos(fx+e)^4 + \right.\right.$$

$$\left.\left. - \frac{15\sqrt{-a}ab\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^4+8\,ab\cos(fx+e)^2+b^2\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\left(2\,a^3\cos(fx+e)^4+3\,a^2b\cos(fx+e)^2+ab^2\right)}\right)\cos(fx+e)^4 - 4\left((3\,a^2-20\,ab)\cos(\right.}{60\,bf\cos(fx+e)^4}\right.$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^3,x, algorithm="fricas")
```

output
```
[1/120*(15*a^(3/2)*b*cos(f*x + e)^4*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b
*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b
^4 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x
 + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)) + 8*((3*a^2 - 20*a*b)*cos(f*x + e)^4 + (6*a*b - 5*b^2)*cos(f*x +
e)^2 + 3*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(b*f*cos(f*x +
e)^4), -1/60*(15*sqrt(-a)*a*b*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos
(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2
*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2))*cos(f*x + e)^4 - 4*
((3*a^2 - 20*a*b)*cos(f*x + e)^4 + (6*a*b - 5*b^2)*cos(f*x + e)^2 + 3*b^2)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(b*f*cos(f*x + e)^4)]
```

3.390.6 Sympy [F]

$$\int (a+b\sec^2(e+fx))^{3/2}\tan^3(e+fx)\,dx = \int (a+b\sec^2(e+fx))^{\frac{3}{2}}\tan^3(e+fx)\,dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**(3/2)*tan(f*x+e)**3,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(3/2)*tan(e + f*x)**3, x)
```

3.390.7 Maxima [F]

$$\int (a+b\sec^2(e+fx))^{3/2}\tan^3(e+fx)\,dx = \int (b\sec(fx+e)^2+a)^{\frac{3}{2}}\tan(fx+e)^3\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^3,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^3, x)
```

3.390.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1431 vs. 2(88) = 176.

Time = 1.67 (sec) , antiderivative size = 1431, normalized size of antiderivative = 13.76

$$\int (a+b\sec^2(e+fx))^{3/2}\tan^3(e+fx)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^3,x, algorithm="giac")
```

output
```
-2/15*(15*a^2*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan
(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^
2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))/sqrt(-a))*sgn(cos(f
*x + e))/sqrt(-a) - 2*(15*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan
(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^
2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^9*a^2*sgn(cos(f*x + e)) - 15*(sqr
t(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/
2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2
 + a + b))^8*(7*a^2 - 8*a*b - 4*b^2)*sqrt(a + b)*sgn(cos(f*x + e)) + 20*(1
5*a^3 - 21*a^2*b - 12*a*b^2 + 8*b^3)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 -
 sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*
x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^7*sgn(cos(f*x + e)) -
20*(21*a^3 - 63*a^2*b + 60*a*b^2 - 4*b^3)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e
)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1
/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^6*sqrt(a + b)*sgn
(cos(f*x + e)) + 2*(105*a^4 - 630*a^3*b + 1065*a^2*b^2 + 360*a*b^3 - 16*b^
4)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b
*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1
/2*e)^2 + a + b))^5*sgn(cos(f*x + e)) + 10*(21*a^4 + 42*a^3*b - 303*a^2*b^
2 + 336*a*b^3 + 4*b^4)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan...
```

3.390.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\sec^2(e+fx))^{3/2}\tan^3(e+fx)\,dx = \int \tan(e+f\,x)^3\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}dx$$

input
```
int(tan(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(tan(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.391 $\int (a+b\sec^2(e+fx))^{3/2}\tan(e+fx)\,dx$

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3.391.2 Mathematica [C] (verified) . . . 2855
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3.391.1 Optimal result

Integrand size = 23, antiderivative size = 78

$$\int (a+b\sec^2(e+fx))^{3/2}\tan(e+fx)\,dx = -\frac{a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} + \frac{a\sqrt{a+b\sec^2(e+fx)}}{f} + \frac{(a+b\sec^2(e+fx))^{3/2}}{3f}$$

output
```
-a^(3/2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f+1/3*(a+b*sec(f*x+e)^2
)^(3/2)/f+a*(a+b*sec(f*x+e)^2)^(1/2)/f
```

3.391.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.33 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.08

$$\int (a+b\sec^2(e+fx))^{3/2}\tan(e+fx)\,dx = \frac{2b\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{2},-\frac{1}{2},-\frac{a\cos^2(e+fx)}{b}\right)(a+b\sec^2(e+fx))^{3/2}}{3f\sqrt{1+\frac{a\cos^2(e+fx)}{b}}(a+2b+a\cos(2(e+fx)))}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x],x]
```

output
```
(2*b*Hypergeometric2F1[-3/2, -3/2, -1/2, -((a*Cos[e + f*x]^2)/b)]*(a + b*S
ec[e + f*x]^2)^(3/2))/(3*f*Sqrt[1 + (a*Cos[e + f*x]^2)/b]*(a + 2*b + a*Cos
[2*(e + f*x)]))
```

3.391.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4627, 243, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int \tan(e+fx)\left(a+b\sec(e+fx)^2\right)^{3/2}dx$$

$$\downarrow 4627$$

$$\frac{\int \cos(e+fx)\left(b\sec^2(e+fx)+a\right)^{3/2}d\sec(e+fx)}{f}$$

$$\downarrow 243$$

$$\frac{\int \cos(e+fx)\left(b\sec^2(e+fx)+a\right)^{3/2}d\sec^2(e+fx)}{2f}$$

$$\downarrow 60$$

$$\frac{a\int \cos(e+fx)\sqrt{b\sec^2(e+fx)+a}d\sec^2(e+fx)+\frac{2}{3}\left(a+b\sec^2(e+fx)\right)^{3/2}}{2f}$$

$$\downarrow 60$$

$$\frac{a\left(a\int \frac{\cos(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)+2\sqrt{a+b\sec^2(e+fx)}\right)+\frac{2}{3}\left(a+b\sec^2(e+fx)\right)^{3/2}}{2f}$$

$$\downarrow 73$$

$$\frac{a\left(\frac{2a\int \frac{1}{\frac{\sec^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}+2\sqrt{a+b\sec^2(e+fx)}\right)+\frac{2}{3}\left(a+b\sec^2(e+fx)\right)^{3/2}}{2f}$$

$$\downarrow 221$$

$$\frac{a\left(2\sqrt{a+b\sec^2(e+fx)}-2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)\right)+\frac{2}{3}(a+b\sec^2(e+fx))^{3/2}}{2f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x],x]
```

output
```
((2*(a + b*Sec[e + f*x]^2)^(3/2))/3 + a*(-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sec
[e + f*x]^2]/Sqrt[a]] + 2*Sqrt[a + b*Sec[e + f*x]^2]))/(2*f)
```

3.391.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.391.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}}{3f} - \frac{a^{\frac{3}{2}}\ln\left(\frac{2a+2\sqrt{a}\sqrt{a+b\sec(fx+e)^2}}{\sec(fx+e)}\right)}{f} + \frac{a\sqrt{a+b\sec(fx+e)^2}}{f}$	81
default	$\frac{\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}}{3f} - \frac{a^{\frac{3}{2}}\ln\left(\frac{2a+2\sqrt{a}\sqrt{a+b\sec(fx+e)^2}}{\sec(fx+e)}\right)}{f} + \frac{a\sqrt{a+b\sec(fx+e)^2}}{f}$	81

input
```
int((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
1/3*(a+b*sec(f*x+e)^2)^(3/2)/f-1/f*a^(3/2)*ln((2*a+2*a^(1/2)*(a+b*sec(f*x+
e)^2)^(1/2))/sec(f*x+e))+a*(a+b*sec(f*x+e)^2)^(1/2)/f
```

3.391.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(66) = 132.

Time = 0.60 (sec) , antiderivative size = 373, normalized size of antiderivative = 4.78

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan(e$$

$$+fx)\,dx = \left[\frac{3\,a^{\frac{3}{2}}\cos(fx+e)^2\log\left(128\,a^4\cos(fx+e)^8+256\,a^3b\cos(fx+e)^6+160\,a^2b^2\cos(fx+e)^4+3\right.\right.$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e),x, algorithm="fricas")
```

output
```
[1/24*(3*a^(3/2)*cos(f*x + e)^2*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos
(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 -
 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e
)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)) + 8*(4*a*cos(f*x + e)^2 + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2))/(f*cos(f*x + e)^2), 1/12*(3*sqrt(-a)*a*arctan(1/4*(8*a^2*cos(f*x + e)
^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2))*cos(f
*x + e)^2 + 4*(4*a*cos(f*x + e)^2 + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2))/(f*cos(f*x + e)^2)]
```

3.391.6 Sympy [F]

$$\int (a + b \sec^2(e + fx))^{3/2} \tan(e + fx)\, dx = \int (a + b \sec^2(e + fx))^{\frac{3}{2}} \tan(e + fx)\, dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**(3/2)*tan(f*x+e),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(3/2)*tan(e + f*x), x)
```

3.391.7 Maxima [F]

$$\int (a + b \sec^2(e + fx))^{3/2} \tan(e + fx)\, dx = \int (b \sec(fx + e)^2 + a)^{\frac{3}{2}} \tan(fx + e)\, dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*tan(f*x + e), x)
```

3.391.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 867 vs. 2(66) = 132.

Time = 1.00 (sec) , antiderivative size = 867, normalized size of antiderivative = 11.12

$$\int \left(a + b\sec^2(e+fx)\right)^{3/2} \tan(e+fx)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e),x, algorithm="giac")
```

output
```
2/3*(3*a^2*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/
2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 +
 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))/sqrt(-a))*sgn(cos(f*x
+ e))/sqrt(-a) + 2*(3*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2
*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 +
2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^5*(2*a*b + b^2)*sgn(cos(f*x + e)) - 3
*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*t
an(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2
*e)^2 + a + b))^4*(6*a*b - b^2)*sqrt(a + b)*sgn(cos(f*x + e)) + 2*(6*a^2*b
 - 15*a*b^2 - b^3)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*
x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b
*tan(1/2*f*x + 1/2*e)^2 + a + b))^3*sgn(cos(f*x + e)) + 6*(2*a^2*b + 7*a*b
^2 + b^3)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e
)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*
f*x + 1/2*e)^2 + a + b))^2*sqrt(a + b)*sgn(cos(f*x + e)) - 3*(6*a^3*b - a^
2*b^2 - 28*a*b^3 - 5*b^4)*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan
(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^
2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*sgn(cos(f*x + e)) + (6*a^3*b - 21
*a^2*b^2 + 28*a*b^3 + 7*b^4)*sqrt(a + b)*sgn(cos(f*x + e)))/((sqrt(a + b)*
tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x ...
```

3.391.9 Mupad [B] (verification not implemented)

Time = 23.60 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.85

$$\int \left(a + b\sec^2(e+fx)\right)^{3/2} \tan(e+fx)\,dx = \frac{\left(a + \frac{b}{\cos(e+fx)^2}\right)^{3/2}}{3f} - \frac{a^{3/2}\,\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{\cos(e+fx)^2}}}{\sqrt{a}}\right)}{f} + \frac{a\sqrt{a+\frac{b}{\cos(e+fx)^2}}}{f}$$

input
```
int(tan(e + f*x)*(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
(a + b/cos(e + f*x)^2)^(3/2)/(3*f) - (a^(3/2)*atanh((a + b/cos(e + f*x)^2)
^(1/2)/a^(1/2)))/f + (a*(a + b/cos(e + f*x)^2)^(1/2))/f
```

3.392 $\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

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3.392.1 Optimal result

Integrand size = 23, antiderivative size = 91

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} - \frac{(a+b)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{f} + \frac{b\sqrt{a+b\sec^2(e+fx)}}{f}$$

output
```
a^(3/2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f-(a+b)^(3/2)*arctanh((a
+b*sec(f*x+e)^2)^(1/2)/(a+b)^(1/2))/f+b*(a+b*sec(f*x+e)^2)^(1/2)/f
```

3.392.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.08 (sec) , antiderivative size = 506, normalized size of antiderivative = 5.56

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\sqrt{2}e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos^3(e+fx)\left(\frac{2b}{1+e^{2i(e+fx)}} + \frac{-2ia^{3/2}fx+2(a+b)^{3/2}\log(1}{}\right.$$

input
```
Integrate[Cot[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[2]*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*
I)*(e + f*x))]*Cos[e + f*x]^3*((2*b)/(1 + E^((2*I)*(e + f*x))) + ((-2*I)*a
^(3/2)*f*x + 2*(a + b)^(3/2)*Log[1 - E^((2*I)*(e + f*x))] + a^(3/2)*Log[a
+ 2*b + a*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(
1 + E^((2*I)*(e + f*x)))^2]] + a^(3/2)*Log[a + a*E^((2*I)*(e + f*x)) + 2*b
*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2
*I)*(e + f*x)))^2]] - 2*a*Sqrt[a + b]*Log[a + b + a*E^((2*I)*(e + f*x)) +
b*E^((2*I)*(e + f*x)) + Sqrt[a + b]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 +
E^((2*I)*(e + f*x)))^2]] - 2*b*Sqrt[a + b]*Log[a + b + a*E^((2*I)*(e + f*x
)) + b*E^((2*I)*(e + f*x)) + Sqrt[a + b]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*
(1 + E^((2*I)*(e + f*x)))^2]])/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2
*I)*(e + f*x)))^2])*(a + b*Sec[e + f*x]^2)^(3/2))/(f*(a + 2*b + a*Cos[2*e
+ 2*f*x])^(3/2))
```

3.392.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4627, 25, 354, 95, 25, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^{3/2}}{\tan(e+fx)}dx$$

$$\downarrow 4627$$

$$\frac{\int -\frac{\cos(e+fx)\left(b\sec^2(e+fx)+a\right)^{3/2}}{1-\sec^2(e+fx)}d\sec(e+fx)}{f}$$

$$\downarrow 25$$

$$-\frac{\int \frac{\cos(e+fx)\left(b\sec^2(e+fx)+a\right)^{3/2}}{1-\sec^2(e+fx)}d\sec(e+fx)}{f}$$

$$\downarrow 354$$

$$-\frac{\int \frac{\cos(e+fx)(b\sec^2(e+fx)+a)^{3/2}}{1-\sec^2(e+fx)}d\sec^2(e+fx)}{2f}$$

$$\downarrow 95$$

$$-\frac{-\int -\frac{\cos(e+fx)(a^2+b(2a+b)\sec^2(e+fx))}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)-2b\sqrt{a+b\sec^2(e+fx)}}{2f}$$

$$\downarrow 25$$

$$-\frac{\int \frac{\cos(e+fx)(a^2+b(2a+b)\sec^2(e+fx))}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)-2b\sqrt{a+b\sec^2(e+fx)}}{2f}$$

$$\downarrow 174$$

$$-\frac{a^2\int \frac{\cos(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)+(a+b)^2\int \frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)-2b\sqrt{a+b\sec^2(e+f}}{2f}$$

$$\downarrow 73$$

$$-\frac{\frac{2a^2\int \frac{1}{\frac{\sec^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}+\frac{2(a+b)^2\int \frac{1}{\frac{a+b}{b}-\frac{\sec^4(e+fx)}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}-2b\sqrt{a+b\sec^2(e+fx)}}{2f}$$

$$\downarrow 221$$

$$-\frac{-2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)+2(a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)-2b\sqrt{a+b\sec^2(e+fx)}}{2f}$$

input
```
Int[Cot[e + f*x]*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
-1/2*(-2*a^(3/2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]] + 2*(a + b)^(
3/2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]] - 2*b*Sqrt[a + b*Sec[
e + f*x]^2])/f
```

3.392.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 95
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[f*((e + f*x)^(p - 1)/(b*d*(p - 1))), x] + Simp[1/(b*d)   Int[(b
*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*((e + f*x)^(p - 2)/((a +
b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.392.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1329 vs. $2(77) = 154$.

Time = 5.57 (sec) , antiderivative size = 1330, normalized size of antiderivative = 14.62

method	result	size
default	Expression too large to display	1330

input

```
int(cot(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
1/2/f/(a+b)^(5/2)*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)/(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))*(2*(a+b)^(5/2)*cos(f*x+e)^3
*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*cos
(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*a^(3/2)+2
*(a+b)^(5/2)*cos(f*x+e)^3*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b+2*
(a+b)^(5/2)*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b+cos
(f*x+e)^3*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a
+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/
2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^4+4*cos(f*x+e)^3*ln(2/(a+b)^(1/2)*(((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+
e)))*a^3*b+6*cos(f*x+e)^3*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^2*b^2+4*cos(f*x+e)^3*
ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*
cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x
+e)*a+b)/(1+cos(f*x+e)))*a*b^3+cos(f*x+e)^3*ln(2/(a+b)^(1/2)*(((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*b^4-c
os(f*x+e)^3*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1...
```

3.392.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(77) = 154$.

Time $= 0.58$ (sec) , antiderivative size $= 1075$, normalized size of antiderivative $= 11.81$

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(a^(3/2)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*
a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x
 + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x
+ e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + 2*(a + b)^(
3/2)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f
*x + e)^2 + b^2 - 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a +
 b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f
*x + e)^2 + 1)) + 8*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/f, 1/8*
(4*(a + b)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a
- b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)
^2 + a*b + b^2)) + a^(3/2)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x
+ e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(1
6*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 +
 b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))
+ 8*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/f, -1/4*(sqrt(-a)*a*arc
tan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt(
(a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos
(f*x + e)^2 + a*b^2)) - (a + b)^(3/2)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x
 + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*a + b)*cos(f*x +
e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*...
```

3.392.6 Sympy [F]

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(a+b\sec^2(e+fx)\right)^{\frac{3}{2}}\cot(e+fx)\,dx$$

input
```
integrate(cot(f*x+e)*(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(3/2)*cot(e + f*x), x)
```

3.392.7 Maxima [F]

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)\,dx$$

input
```
integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e), x)
```

3.392.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 718 vs. $2(77) = 154$.

Time $= 4.13$ (sec) , antiderivative size $= 718$, normalized size of antiderivative $= 7.89$

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/2*(4*a^2*arctan(-1/2*(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1
/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2
+ 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b))/sqrt(-a))*sgn(cos(f*x
 + e))/sqrt(-a) - (a + b)^(3/2)*log(abs(sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2
 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*
f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b)))*sgn(c
os(f*x + e)) + (a + b)^(3/2)*log(abs(-sqrt(a + b)*tan(1/2*f*x + 1/2*e)^2 +
 sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*
x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b) + sqrt(a + b)))*sgn(cos
(f*x + e)) + (a^2 + 2*a*b + b^2)*log(abs(-(sqrt(a + b)*tan(1/2*f*x + 1/2*e
)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1
/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*(a + b) + sqrt(a
+ b)*(a - b)))*sgn(cos(f*x + e))/sqrt(a + b) - 8*((sqrt(a + b)*tan(1/2*f*x
 + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 - 2
*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))*b^2*sgn(c
os(f*x + e)) + sqrt(a + b)*b^2*sgn(cos(f*x + e)))/((sqrt(a + b)*tan(1/2*f*
x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(1/2*f*x + 1/2*e)^4 -
2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e)^2 + a + b))^2 - 2*(s
qrt(a + b)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + b*tan(
1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2...
```

3.392.9 Mupad [F(-1)]

Timed out.

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \cot(e+f\,x)\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}\,dx$$

input
```
int(cot(e + f*x)*(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(cot(e + f*x)*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.393 $\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

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3.393.2 Mathematica [C] (warning: unable to verify) . . . 2870
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3.393.9 Mupad [F(-1)] . . . 2876

3.393.1 Optimal result

Integrand size = 25, antiderivative size = 114

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = -\frac{a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} + \frac{(2a-b)\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2f} - \frac{(a+b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2f}$$

output
```
-a^(3/2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f+1/2*(2*a-b)*arctanh((
a+b*sec(f*x+e)^2)^(1/2)/(a+b)^(1/2))*(a+b)^(1/2)/f-1/2*(a+b)*cot(f*x+e)^2*
(a+b*sec(f*x+e)^2)^(1/2)/f
```

3.393.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.31 (sec) , antiderivative size = 622, normalized size of antiderivative = 5.46

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\sqrt{2}e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos^3(e+fx)\left(\frac{(a+b)\left(1+e^{2i(e+fx)}\right)}{\left(-1+e^{2i(e+fx)}\right)^2} - \frac{-2ia^{3/2}\sqrt{a+b}fx+(}{}\right.$$

input
```
Integrate[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[2]*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*
I)*(e + f*x))]*Cos[e + f*x]^3*(((a + b)*(1 + E^((2*I)*(e + f*x))))/(-1 + E
^((2*I)*(e + f*x)))^2 - ((-2*I)*a^(3/2)*Sqrt[a + b]*f*x + (2*a^2 + a*b - b
^2)*Log[1 - E^((2*I)*(e + f*x))] + a^(3/2)*Sqrt[a + b]*Log[a + 2*b + a*E^(
(2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*
(e + f*x)))^2]] + a^(3/2)*Sqrt[a + b]*Log[a + a*E^((2*I)*(e + f*x)) + 2*b*
E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*
I)*(e + f*x)))^2]] - 2*a^2*Log[a + b + a*E^((2*I)*(e + f*x)) + b*E^((2*I)*
(e + f*x)) + Sqrt[a + b]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e
 + f*x)))^2]] - a*b*Log[a + b + a*E^((2*I)*(e + f*x)) + b*E^((2*I)*(e + f*
x)) + Sqrt[a + b]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)
))^2]] + b^2*Log[a + b + a*E^((2*I)*(e + f*x)) + b*E^((2*I)*(e + f*x)) + S
qrt[a + b]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]])
/(Sqrt[a + b]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2
]))*(a + b*Sec[e + f*x]^2)^(3/2))/(f*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2))
```

3.393.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4627, 354, 109, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^{3/2}}{\tan(e+fx)^3}dx$$

$$\downarrow 4627$$

$$\frac{\int \frac{\cos(e+fx)\left(b\sec^2(e+fx)+a\right)^{3/2}}{\left(1-\sec^2(e+fx)\right)^2}d\sec(e+fx)}{f}$$

$$\downarrow 354$$

$$\frac{\int \frac{\cos(e+fx)(b\sec^2(e+fx)+a)^{3/2}}{(1-\sec^2(e+fx))^2} d\sec^2(e+fx)}{2f}$$

$$\downarrow 109$$

$$\frac{\frac{(a+b)\sqrt{a+b\sec^2(e+fx)}}{1-\sec^2(e+fx)} - \int -\frac{\cos(e+fx)(2a^2+(a-b)b\sec^2(e+fx))}{2(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}} d\sec^2(e+fx)}{2f}$$

$$\downarrow 27$$

$$\frac{\frac{1}{2}\int \frac{\cos(e+fx)(2a^2+(a-b)b\sec^2(e+fx))}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}} d\sec^2(e+fx) + \frac{(a+b)\sqrt{a+b\sec^2(e+fx)}}{1-\sec^2(e+fx)}}{2f}$$

$$\downarrow 174$$

$$\frac{\frac{1}{2}\left(2a^2\int \frac{\cos(e+fx)}{\sqrt{b\sec^2(e+fx)+a}} d\sec^2(e+fx) + (2a-b)(a+b)\int \frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}} d\sec^2(e+fx)\right) + \frac{(a+b)\sqrt{a+}}{1-\sec^2}}{2f}$$

$$\downarrow 73$$

$$\frac{\frac{1}{2}\left(\frac{4a^2\int \frac{1}{\frac{\sec^4(e+fx)}{b}-\frac{a}{b}} d\sqrt{b\sec^2(e+fx)+a}}{b} + \frac{2(2a-b)(a+b)\int \frac{1}{\frac{a+b}{b}-\frac{\sec^4(e+fx)}{b}} d\sqrt{b\sec^2(e+fx)+a}}{b}\right) + \frac{(a+b)\sqrt{a+b\sec^2(e+fx)}}{1-\sec^2(e+fx)}}{2f}$$

$$\downarrow 221$$

$$\frac{\frac{1}{2}\left(2(2a-b)\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right) - 4a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)\right) + \frac{(a+b)\sqrt{a+b\sec^2(e+fx)}}{1-\sec^2(e+fx)}}{2f}$$

input
```
Int[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
((-4*a^(3/2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]] + 2*(2*a - b)*Sqr
t[a + b]*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/2 + ((a + b)*Sqr
t[a + b*Sec[e + f*x]^2])/(1 - Sec[e + f*x]^2))/(2*f)
```

3.393. $\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2} dx$

3.393.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.393.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1568 vs. $2(96) = 192$.

Time = 1.26 (sec) , antiderivative size = 1569, normalized size of antiderivative = 13.76

method	result	size
default	Expression too large to display	1569

input

```
int(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
-1/4/f/(a+b)^(3/2)*(4*cos(f*x+e)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*a^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2))*(a+b)^(3/2)*a^(3/2)-4*a^(3/2)*ln(4*cos(f*x+e)*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*(a+b)^(3/2)-2*cos(f*x+e)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(3/2)*a-2*cos(f*x+e)*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(3/2)*b-2*cos(f*x+e)*ln(-4*(((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*
x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)
))*a^3-3*cos(f*x+e)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+
b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2
)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*a^2*b+cos(f*x+e)*ln(-4*(((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*b^3+2*c
os(f*x+e)*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a
+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/
2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^3+3*cos(f*x+e)*ln(2/(a+b)^(1/2)*(((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)
))*a^2*b-cos(f*x+e)*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)...
```

3.393.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. 2(96) = 192.

Time = 0.60 (sec) , antiderivative size = 1300, normalized size of antiderivative = 11.40

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(4*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2
 + (a*cos(f*x + e)^2 - a)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*c
os(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4
 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x +
 e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)) - ((2*a - b)*cos(f*x + e)^2 - 2*a + b)*sqrt(a + b)*log(2*((8*a^2 +
8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*(
(2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/(f*c
os(f*x + e)^2 - f), 1/8*(4*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)*cos(f*x + e)^2 - 2*((2*a - b)*cos(f*x + e)^2 - 2*a + b)*sqrt(-a - b)*
arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e
)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) + (a*co
s(f*x + e)^2 - a)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x +
 e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16
*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 +
b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)))/
(f*cos(f*x + e)^2 - f), 1/8*(4*(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)*cos(f*x + e)^2 + 2*(a*cos(f*x + e)^2 - a)*sqrt(-a)*arctan(1/4*(8*
a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f...
```

3.393.6 Sympy [F]

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(a+b\sec^2(e+fx)\right)^{\frac{3}{2}}\cot^3(e+fx)\,dx$$

input
```
integrate(cot(f*x+e)**3*(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(3/2)*cot(e + f*x)**3, x)
```

3.393.7 Maxima [F]

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^3\,dx$$

input
```
integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^3, x)
```

3.393.8 Giac [F(-2)]

Exception generated.

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.393.9 Mupad [F(-1)]

Timed out.

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \cot(e+f\,x)^3\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}\,dx$$

input
```
int(cot(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(cot(e + f*x)^3*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.394 $\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

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3.394.1 Optimal result

Integrand size = 25, antiderivative size = 159

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{f} - \frac{\left(8a^2+4ab-b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8\sqrt{a+b}f} + \frac{(4a-b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8f} - \frac{(a+b)\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4f}$$

output

```
a^(3/2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f-1/8*(8*a^2+4*a*b-b^2)*
arctanh((a+b*sec(f*x+e)^2)^(1/2)/(a+b)^(1/2))/f/(a+b)^(1/2)+1/8*(4*a-b)*co
t(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2)/f-1/4*(a+b)*cot(f*x+e)^4*(a+b*sec(f*x+
e)^2)^(1/2)/f
```

3.394.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.91 (sec) , antiderivative size = 684, normalized size of antiderivative = 4.30

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos^3(e+fx)\left(-\frac{\left(1+e^{2i(e+fx)}\right)\left(b\left(1+6e^{2i(e+fx)}+e^{4i(e+fx)}\right)+a\right.}{\left(-1+e^{2i(e+fx)}\right)^4}\right.}{}$$

input
```
Integrate[Cot[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e +
f*x))]*Cos[e + f*x]^3*(-(((1 + E^((2*I)*(e + f*x)))*(b*(1 + 6*E^((2*I)*(e
+ f*x)) + E^((4*I)*(e + f*x))) + a*(6 - 4*E^((2*I)*(e + f*x)) + 6*E^((4*I)
*(e + f*x)))))/(-1 + E^((2*I)*(e + f*x)))^4) + ((-8*I)*a^(3/2)*Sqrt[a + b]
*f*x + (8*a^2 + 4*a*b - b^2)*Log[1 - E^((2*I)*(e + f*x))] + 4*a^(3/2)*Sqrt
[a + b]*Log[a + 2*b + a*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e
 + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] + 4*a^(3/2)*Sqrt[a + b]*Log[a +
 a*E^((2*I)*(e + f*x)) + 2*b*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*
I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] - 8*a^2*Log[a + b + a*E^((
2*I)*(e + f*x)) + b*E^((2*I)*(e + f*x)) + Sqrt[a + b]*Sqrt[4*b*E^((2*I)*(e
 + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] - 4*a*b*Log[a + b + a*E^((2*I)*
(e + f*x)) + b*E^((2*I)*(e + f*x)) + Sqrt[a + b]*Sqrt[4*b*E^((2*I)*(e + f*
x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] + b^2*Log[a + b + a*E^((2*I)*(e + f*
x)) + b*E^((2*I)*(e + f*x)) + Sqrt[a + b]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a
*(1 + E^((2*I)*(e + f*x)))^2]])/(Sqrt[a + b]*Sqrt[4*b*E^((2*I)*(e + f*x))
+ a*(1 + E^((2*I)*(e + f*x)))^2]))*(a + b*Sec[e + f*x]^2)^(3/2))/(2*Sqrt[2
]*f*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2))
```

3.394.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 4627, 25, 354, 109, 27, 168, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^{3/2}}{\tan(e+fx)^5}dx$$

$$\downarrow 4627$$

$$\frac{\int -\frac{\cos(e+fx)\left(b\sec^2(e+fx)+a\right)^{3/2}}{\left(1-\sec^2(e+fx)\right)^3}d\sec(e+fx)}{f}$$

$$\downarrow 25$$

$$-\frac{\int \frac{\cos(e+fx)\left(b\sec^2(e+fx)+a\right)^{3/2}}{\left(1-\sec^2(e+fx)\right)^3}d\sec(e+fx)}{f}$$

$$\downarrow 354$$

$$-\frac{\int \frac{\cos(e+fx)\left(b\sec^2(e+fx)+a\right)^{3/2}}{\left(1-\sec^2(e+fx)\right)^3}d\sec^2(e+fx)}{2f}$$

$$\downarrow 109$$

$$-\frac{\frac{(a+b)\sqrt{a+b\sec^2(e+fx)}}{2\left(1-\sec^2(e+fx)\right)^2}-\frac{1}{2}\int -\frac{\cos(e+fx)\left(4a^2+(3a-b)b\sec^2(e+fx)\right)}{2\left(1-\sec^2(e+fx)\right)^2\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{2f}$$

$$\downarrow 27$$

$$-\frac{\frac{1}{4}\int \frac{\cos(e+fx)\left(4a^2+(3a-b)b\sec^2(e+fx)\right)}{\left(1-\sec^2(e+fx)\right)^2\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)+\frac{(a+b)\sqrt{a+b\sec^2(e+fx)}}{2\left(1-\sec^2(e+fx)\right)^2}}{2f}$$

$$\downarrow 168$$

$$-\frac{\frac{1}{4}\left(\frac{(4a-b)\sqrt{a+b\sec^2(e+fx)}}{1-\sec^2(e+fx)}-\frac{\int -\frac{(a+b)\cos(e+fx)\left(8a^2+(4a-b)b\sec^2(e+fx)\right)}{2\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{a+b}\right)+\frac{(a+b)\sqrt{a+b\sec^2(e+fx)}}{2\left(1-\sec^2(e+fx)\right)^2}}{2f}$$

$$\downarrow 27$$

$$-\frac{\frac{1}{4}\left(\frac{1}{2}\int \frac{\cos(e+fx)\left(8a^2+(4a-b)b\sec^2(e+fx)\right)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)+\frac{(4a-b)\sqrt{a+b\sec^2(e+fx)}}{1-\sec^2(e+fx)}\right)+\frac{(a+b)\sqrt{a+b\sec^2(e+fx)}}{2\left(1-\sec^2(e+fx)\right)^2}}{2f}$$

$$\downarrow 174$$

$$-\frac{\frac{1}{4}\left(\frac{1}{2}\left(\left(8a^2+4ab-b^2\right)\int \frac{1}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)+8a^2\int \frac{\cos(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)\right)+\frac{(4a}{}\right.}{2f}$$

$$\downarrow 73$$

$$-\frac{\frac{1}{4}\left(\frac{1}{2}\left(\frac{2\left(8a^2+4ab-b^2\right)\int \frac{1}{\frac{a+b}{b}-\frac{\sec^4(e+fx)}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}+\frac{16a^2\int \frac{1}{\frac{\sec^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}\right)+\frac{(4a-b)\sqrt{a+b\sec^2(e+fx)}}{1-\sec^2(e+fx)}\right.}{2f}$$

$$\downarrow 221$$

3.394. $\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

$$-\frac{\frac{1}{4}\left(\frac{1}{2}\left(\frac{2(8a^2+4ab-b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-16a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)\right)+\frac{(4a-b)\sqrt{a+b\sec^2(e+fx)}}{1-\sec^2(e+fx)}\right)+\frac{(a+}{2f}$$

input
```
Int[Cot[e + f*x]^5*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
-1/2*(((a + b)*Sqrt[a + b*Sec[e + f*x]^2])/(2*(1 - Sec[e + f*x]^2)^2) + ((
-16*a^(3/2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]] + (2*(8*a^2 + 4*a*
b - b^2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/Sqrt[a + b])/2 +
 ((4*a - b)*Sqrt[a + b*Sec[e + f*x]^2])/(1 - Sec[e + f*x]^2))/4)/f
```

3.394.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.394.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 4838 vs. 2(137) = 274.

Time = 1.18 (sec) , antiderivative size = 4839, normalized size of antiderivative = 30.43

method	result	size
default	Expression too large to display	4839

input
```
int(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16/f/(a+b)^(5/2)*(-8*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^4+ln(2/(a+b)^(1/2)*(((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e
)))*b^4+8*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*c
os(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+
e)*a+b)/(-1+cos(f*x+e)))*a^4-ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(
a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))*b^4-8*cos(f*x+e)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(5/2)*a+12*cos(f*x+e)^3*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(5/2)*a+16*ln(4*cos(f*x+e)*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*(a+b)^(5/2)*a^(3/2)*cos(f*x+e)^2+20*
ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*
cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x
+e)*a+b)/(1+cos(f*x+e)))*a^3*b*cos(f*x+e)^2+15*ln(2/(a+b)^(1/2)*(((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^
2*b^2*cos(f*x+e)^2+2*ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e...
```

3.394.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 404 vs. 2(137) = 274.

Time = 1.61 (sec) , antiderivative size = 1801, normalized size of antiderivative = 11.33

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/32*(4*((a^2 + a*b)*cos(f*x + e)^4 - 2*(a^2 + a*b)*cos(f*x + e)^2 + a^2
+ a*b)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160
*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*
x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x
 + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) - ((8*a^2 +
4*a*b - b^2)*cos(f*x + e)^4 - 2*(8*a^2 + 4*a*b - b^2)*cos(f*x + e)^2 + 8*a
^2 + 4*a*b - b^2)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4
+ 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 + 4*((2*a + b)*cos(f*x + e)^4 + b
*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/
(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)) - 4*((6*a^2 + 7*a*b + b^2)*cos(f*
x + e)^4 - (4*a^2 + 3*a*b - b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2))/((a + b)*f*cos(f*x + e)^4 - 2*(a + b)*f*cos(f*x + e)^2
 + (a + b)*f), 1/16*(((8*a^2 + 4*a*b - b^2)*cos(f*x + e)^4 - 2*(8*a^2 + 4*
a*b - b^2)*cos(f*x + e)^2 + 8*a^2 + 4*a*b - b^2)*sqrt(-a - b)*arctan(1/2*(
(2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos
(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) + 2*((a^2 + a*b)*co
s(f*x + e)^4 - 2*(a^2 + a*b)*cos(f*x + e)^2 + a^2 + a*b)*sqrt(a)*log(128*a
^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4
+ 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(
f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt...
```

3.394.6 Sympy [F(-1)]

Timed out.

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)**5*(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

3.394.7 Maxima [F]

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^5\,dx$$

input
```
integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^5, x)
```

3.394.8 Giac [F(-2)]

Exception generated.

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.394.9 Mupad [F(-1)]

Timed out.

$$\int \cot^5(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \cot(e+f\,x)^5\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}\,dx$$

input
```
int(cot(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(cot(e + f*x)^5*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.395 $\int (a+b\sec^2(e+fx))^{3/2}\tan^6(e+fx)\,dx$

3.395.1 Optimal result 2885
3.395.2 Mathematica [A] (verified) 2886
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3.395.1 Optimal result

Integrand size = 25, antiderivative size = 290

$$\begin{aligned}\int (a+b\sec^2(e+fx))^{3/2}\tan^6(e+fx)\,dx &= -\frac{a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}\\ &+\frac{(3a^4+20a^3b+90a^2b^2-60ab^3-5b^4)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{128b^{5/2}f}\\ &-\frac{(3a^3+17a^2b-55ab^2-5b^3)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{128b^2f}\\ &+\frac{(3a^2-50ab-5b^2)\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{192bf}\\ &+\frac{(9a+b)\tan^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{48f}\\ &+\frac{b\tan^7(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f}\end{aligned}$$

output

```
-a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f+1/128*(3*
a^4+20*a^3*b+90*a^2*b^2-60*a*b^3-5*b^4)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*
tan(f*x+e)^2)^(1/2))/b^(5/2)/f-1/128*(3*a^3+17*a^2*b-55*a*b^2-5*b^3)*(a+b+
b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b^2/f+1/192*(3*a^2-50*a*b-5*b^2)*(a+b+b*t
an(f*x+e)^2)^(1/2)*tan(f*x+e)^3/b/f+1/48*(9*a+b)*(a+b+b*tan(f*x+e)^2)^(1/2
)*tan(f*x+e)^5/f+1/8*b*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^7/f
```

3.395.2 Mathematica [A] (verified)

Time = 7.88 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.22

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^6(e+fx)\,dx =$$

$$-\frac{\left(128a^{3/2}b^2\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)-\frac{(3a^4+20a^3b+90a^2b^2-60ab^3-5b^4)\operatorname{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{b}}\right)\cos^3(e+fx}{32\sqrt{2}b^2f(a+2b+a\cos(2e+2fx))^{3/2}}$$

$$-\frac{(90a^3+498a^2b-1594ab^2-626b^3+(135a^3+759a^2b-2303ab^2+513b^3)\cos(2(e+fx))+2(27a^3+159}{}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^6,x]
```

output
```
-1/32*((128*a^(3/2)*b^2*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e
 + f*x]^2]] - ((3*a^4 + 20*a^3*b + 90*a^2*b^2 - 60*a*b^3 - 5*b^4)*ArcTanh[
(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]])/Sqrt[b])*Cos[e + f
*x]^3*(a + b*Sec[e + f*x]^2)^(3/2))/(Sqrt[2]*b^2*f*(a + 2*b + a*Cos[2*e +
2*f*x])^(3/2)) - ((90*a^3 + 498*a^2*b - 1594*a*b^2 - 626*b^3 + (135*a^3 +
759*a^2*b - 2303*a*b^2 + 513*b^3)*Cos[2*(e + f*x)] + 2*(27*a^3 + 159*a^2*b
 - 523*a*b^2 - 191*b^3)*Cos[4*(e + f*x)] + 9*a^3*Cos[6*(e + f*x)] + 57*a^2
*b*Cos[6*(e + f*x)] - 337*a*b^2*Cos[6*(e + f*x)] + 15*b^3*Cos[6*(e + f*x)]
)*Sec[e + f*x]^6*Sqrt[a + b*Sec[e + f*x]^2]*Tan[e + f*x])/(12288*b^2*f)
```

3.395.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.04, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 4629, 2075, 379, 444, 27, 444, 27, 444, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int \tan(e+fx)^6\left(a+b\sec(e+fx)^2\right)^{3/2}dx$$

$$\downarrow 4629$$

$$\frac{\int \frac{\tan^6(e+fx)(a+b(\tan^2(e+fx)+1))^{3/2}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\tan^6(e+fx)(b\tan^2(e+fx)+a+b)^{3/2}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\downarrow 379$$

$$\frac{\frac{1}{8}\int \frac{\tan^6(e+fx)(b(9a+b)\tan^2(e+fx)+(a+b)(8a+b))}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+\frac{1}{8}b\tan^7(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

$$\downarrow 444$$

$$\frac{\frac{1}{8}\left(\frac{1}{6}(9a+b)\tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}-\frac{\int \frac{b\tan^4(e+fx)\left(5(a+b)(9a+b)-\left(3a^2-50ba-5b^2\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{6b}\right)+}{f}$$

$$\downarrow 27$$

$$\frac{\frac{1}{8}\left(\frac{1}{6}(9a+b)\tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}-\frac{1}{6}\int \frac{\tan^4(e+fx)(5(a+b)(9a+b)-(3a^2-50ba-5b^2)\tan^2(e+fx))}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+\right.}{f}$$

$$\downarrow 444$$

$$\frac{\frac{1}{8}\left(\frac{1}{6}\left(\frac{\int -\frac{3\tan^2(e+fx)\left(\left(3a^3+17ba^2-55b^2a-5b^3\right)\tan^2(e+fx)+(a+b)\left(3a^2-50ba-5b^2\right)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{4b}+\frac{(3a^2-50ab-5b^2)\tan^3(e+fx)\sqrt{a+b\tan^2(e-}\right.\right.}{f}$$

$$\downarrow 27$$

$$\frac{\frac{1}{8}\left(\frac{1}{6}\left(\frac{(3a^2-50ab-5b^2)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4b}-\frac{3\int \frac{\tan^2(e+fx)\left(\left(3a^3+17ba^2-55b^2a-5b^3\right)\tan^2(e+fx)+(a+b)\left(3a^2-50ba-5b^2\right)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e}{4b}\right.\right.}{f}$$

$$\downarrow 444$$

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{\left(3a^2-50ab-5b^2\right)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4b}-\frac{3\left(\frac{\left(3a^3+17a^2b-55ab^2-5b^3\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{\int\frac{\left(3a^4+20ba^3+90b^2a^2-\right.}{\left(}\right.}{4b}\right.\right.$$

$$\downarrow 398$$

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{\left(3a^2-50ab-5b^2\right)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4b}-\frac{3\left(\frac{\left(3a^3+17a^2b-55ab^2-5b^3\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{\left(3a^4+20a^3b+90a^2b^2-60\right.}{}\right.}{4b}\right.\right.$$

$$\downarrow 224$$

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{\left(3a^2-50ab-5b^2\right)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4b}-\frac{3\left(\frac{\left(3a^3+17a^2b-55ab^2-5b^3\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{\left(3a^4+20a^3b+90a^2b^2-60\right.}{}\right.}{4b}\right.\right.$$

$$\downarrow 219$$

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{\left(3a^2-50ab-5b^2\right)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4b}-\frac{3\left(\frac{\left(3a^3+17a^2b-55ab^2-5b^3\right)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{\left(3a^4+20a^3b+90a^2b^2-60\right.}{}\right.}{4b}\right.\right.$$

$$\downarrow 291$$

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{(3a^2-50ab-5b^2)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4b}-\frac{3\left(\frac{(3a^3+17a^2b-55ab^2-5b^3)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{(3a^4+20a^3b+90a^2b^2-60\cdots}{\cdots}\right)}{\cdots}\right)\right)$$

$\downarrow$ 216

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{(3a^2-50ab-5b^2)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4b}-\frac{3\left(\frac{(3a^3+17a^2b-55ab^2-5b^3)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{(3a^4+20a^3b+90a^2b^2-60\cdots}{\cdots}\right)}{\cdots}\right)\right)$$

input
```
Int[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^6,x]
```

output
```
((b*Tan[e + f*x]^7*Sqrt[a + b + b*Tan[e + f*x]^2])/8 + (((9*a + b)*Tan[e +
 f*x]^5*Sqrt[a + b + b*Tan[e + f*x]^2])/6 + (((3*a^2 - 50*a*b - 5*b^2)*Tan
[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(4*b) - (3*(-1/2*(-128*a^(3/2)
*b^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]] + ((3*a
^4 + 20*a^3*b + 90*a^2*b^2 - 60*a*b^3 - 5*b^4)*ArcTanh[(Sqrt[b]*Tan[e + f*
x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/Sqrt[b])/b + ((3*a^3 + 17*a^2*b - 55*
a*b^2 - 5*b^3)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2*b)))/(4*b))
/6)/8)/f
```

3.395. $\int (a+b\sec^2(e+fx))^{3/2}\tan^6(e+fx)\,dx$

3.395.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 379
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q -
1)/(b*e*(m + 2*(p + q) + 1))), x] + Simp[1/(b*(m + 2*(p + q) + 1))   Int[(e
*x)^m*(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*((b*c - a*d)*(m + 1) + b*c*2
*(p + q)) + (d*(b*c - a*d)*(m + 1) + d*2*(q - 1)*(b*c - a*d) + b*c*d*2*(p +
 q))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d, 0
] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.395.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2228 vs. $2(260) = 520$.

Time = 30.40 (sec) , antiderivative size = 2229, normalized size of antiderivative = 7.69

method	result	size
default	Expression too large to display	2229

input
```
int((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^6,x,method=_RETURNVERBOSE)
```

output
```
-1/768/f/(-a)^(1/2)/b^(13/2)*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)/(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))*(-674*cos(f*x+e)
*sin(f*x+e)*(-a)^(1/2)*b^(13/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*a-9*cos(f*x+e)^3*(-a)^(1/2)*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*a^4*b^4-60*cos(f*x+e)^3*(-a)^(1/2)*ln
(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)
+1))*a^3*b^5-270*cos(f*x+e)^3*(-a)^(1/2)*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*a^2*b^6+180*cos(f*x+e)^3*(
-a)^(1/2)*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*
x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)
/(sin(f*x+e)+1))*a*b^7-9*cos(f*x+e)^3*(-a)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*a^4*b^4-60*cos(f*
x+e)^3*(-a)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/
2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+
e)*a+a+b)/(sin(f*x+e)-1))*a^3*b^5-270*cos(f*x+e)^3*(-a)^(1/2)*ln(-4*(((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a...
```

3.395.5 Fricas [A] (verification not implemented)

Time = 14.98 (sec) , antiderivative size = 1973, normalized size of antiderivative = 6.80

$$\int (a + b\sec^2(e + fx))^{3/2} \tan^6(e + fx)\, dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^6,x, algorithm="fricas")
```

output
```
[1/1536*(192*sqrt(-a)*a*b^3*cos(f*x + e)^7*log(128*a^4*cos(f*x + e)^8 - 25
6*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x
 + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b
 + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3
 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 -
(a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e
)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 3*(3*a^4 + 20*a^3*b + 90*a^2*b^2
 - 60*a*b^3 - 5*b^4)*sqrt(b)*cos(f*x + e)^7*log(((a^2 - 6*a*b + b^2)*cos(f*
x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + 2*b*
cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x
+ e) + 8*b^2)/cos(f*x + e)^4) - 4*((9*a^3*b + 57*a^2*b^2 - 337*a*b^3 + 15*
b^4)*cos(f*x + e)^6 - 2*(3*a^2*b^2 - 122*a*b^3 + 59*b^4)*cos(f*x + e)^4 -
48*b^4 - 8*(9*a*b^3 - 17*b^4)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/
cos(f*x + e)^2)*sin(f*x + e))/(b^3*f*cos(f*x + e)^7), 1/768*(96*sqrt(-a)*a
*b^3*cos(f*x + e)^7*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x
 + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*
b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*c
os(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5
 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^
2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e...
```

3.395.6 Sympy [F]

$$\int (a + b\sec^2(e + fx))^{3/2} \tan^6(e + fx)\, dx = \int (a + b\sec^2(e + fx))^{\frac{3}{2}} \tan^6(e + fx)\, dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**(3/2)*tan(f*x+e)**6,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(3/2)*tan(e + f*x)**6, x)
```

3.395.7 Maxima [F]

$$\int (a+b\sec^2(e+fx))^{3/2}\tan^6(e+fx)\,dx = \int (b\sec(fx+e)^2+a)^{\frac{3}{2}}\tan(fx+e)^6\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^6,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^6, x)
```

3.395.8 Giac [F]

$$\int (a+b\sec^2(e+fx))^{3/2}\tan^6(e+fx)\,dx = \int (b\sec(fx+e)^2+a)^{\frac{3}{2}}\tan(fx+e)^6\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^6,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^6, x)
```

3.395.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\sec^2(e+fx))^{3/2}\tan^6(e+fx)\,dx = \int \tan(e+f\,x)^6\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}dx$$

input
```
int(tan(e + f*x)^6*(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(tan(e + f*x)^6*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.396 $\int (a+b\sec^2(e+fx))^{3/2}\tan^4(e+fx)\,dx$

3.396.1 Optimal result 2895
3.396.2 Mathematica [A] (verified) 2896
3.396.3 Rubi [A] (verified) 2896
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3.396.8 Giac [F] 2903
3.396.9 Mupad [F(-1)] 2903

3.396.1 Optimal result

Integrand size = 25, antiderivative size = 214

$$\begin{aligned}\int (a+b\sec^2(e+fx))^{3/2}\tan^4(e+fx)\,dx &= \frac{a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}\\ &-\frac{(a-b)\left(a^2+10ab+b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{16b^{3/2}f}\\ &+\frac{\left(a^2-8ab-b^2\right)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{16bf}\\ &+\frac{(7a+b)\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{24f}\\ &+\frac{b\tan^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{6f}\end{aligned}$$

output

```
a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f-1/16*(a-b)
*(a^2+10*a*b+b^2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b
^(3/2)/f+1/16*(a^2-8*a*b-b^2)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b/f+1/
24*(7*a+b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^3/f+1/6*b*(a+b+b*tan(f*x+
e)^2)^(1/2)*tan(f*x+e)^5/f
```

3.396.2 Mathematica [A] (verified)

Time = 5.29 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.21

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^4(e+fx)\,dx = \frac{\left(16a^{3/2}b\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)-\frac{(a-b)(a^2+10ab+b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{b}}\right)\cos^3(e+fx)\,(a+\cdots}{4\sqrt{2}bf(a+2b+a\cos(2e+2fx))^{3/2}}$$

$$+\frac{(9a^2-58ab+17b^2+4(3a^2-24ab-11b^2)\cos(2(e+fx))+(3a^2-38ab+3b^2)\cos(4(e+fx)))\sec^4(e+\cdots}{384bf}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^4,x]
```

output
```
((16*a^(3/2)*b*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2
]] - ((a - b)*(a^2 + 10*a*b + b^2)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a +
 b - a*Sin[e + f*x]^2]])/Sqrt[b])*Cos[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3
/2))/(4*Sqrt[2]*b*f*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)) + ((9*a^2 - 58*a
*b + 17*b^2 + 4*(3*a^2 - 24*a*b - 11*b^2)*Cos[2*(e + f*x)] + (3*a^2 - 38*a
*b + 3*b^2)*Cos[4*(e + f*x)])*Sec[e + f*x]^4*Sqrt[a + b*Sec[e + f*x]^2]*Ta
n[e + f*x])/(384*b*f)
```

3.396.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 4629, 2075, 379, 444, 27, 444, 25, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx$$

$$\downarrow 3042$$

$$\int \tan(e+fx)^4\left(a+b\sec(e+fx)^2\right)^{3/2}dx$$

$$\downarrow 4629$$

$$\frac{\int \frac{\tan^4(e+fx)\left(a+b\left(\tan^2(e+fx)+1\right)\right)^{3/2}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\tan^4(e+fx)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\downarrow 379$$

$$\frac{\frac{1}{6}\int \frac{\tan^4(e+fx)\left(b(7a+b)\tan^2(e+fx)+(a+b)(6a+b)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+\frac{1}{6}b\tan^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

$$\downarrow 444$$

$$\frac{\frac{1}{6}\left(\frac{1}{4}(7a+b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}-\frac{\int \frac{3b\tan^2(e+fx)\left((a+b)(7a+b)-\left(a^2-8ba-b^2\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{4b}\right)+\frac{1}{6}b}{f}$$

$$\downarrow 27$$

$$\frac{\frac{1}{6}\left(\frac{1}{4}(7a+b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}-\frac{3}{4}\int \frac{\tan^2(e+fx)\left((a+b)(7a+b)-\left(a^2-8ba-b^2\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)\right.}{f}$$

$$\downarrow 444$$

$$\frac{\frac{1}{6}\left(\frac{1}{4}(7a+b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}-\frac{3}{4}\left(-\frac{\int -\frac{(a-b)\left(a^2+10ba+b^2\right)\tan^2(e+fx)+(a+b)\left(a^2-8ba-b^2\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2b}-\right.}{f}$$

$$\downarrow 25$$

$$\frac{\frac{1}{6}\left(\frac{1}{4}(7a+b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}-\frac{3}{4}\left(\frac{\int \frac{(a-b)\left(a^2+10ba+b^2\right)\tan^2(e+fx)+(a+b)\left(a^2-8ba-b^2\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2b}-\right.}{f}$$

$$\downarrow 398$$

$$\frac{1}{6}\left(\frac{1}{4}(7a+b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}-\frac{3}{4}\left(\frac{(a-b)(a^2+10ab+b^2)\int\frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-16a^2b\int\frac{}{(\tan^2(}}{2b}\right.\right.$$

$$\downarrow 224$$

$$\frac{1}{6}\left(\frac{1}{4}(7a+b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}-\frac{3}{4}\left(\frac{(a-b)(a^2+10ab+b^2)\int\frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}-16a^2\right.\right.$$

$$\downarrow 219$$

$$\frac{1}{6}\left(\frac{1}{4}(7a+b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}-\frac{3}{4}\left(\frac{\frac{(a-b)\left(a^2+10ab+b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}}-16a^2b\int\frac{}{(\tan^2(e+}}{2b}\right.\right.$$

$$\downarrow 291$$

$$\frac{1}{6}\left(\frac{1}{4}(7a+b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}-\frac{3}{4}\left(\frac{\frac{(a-b)\left(a^2+10ab+b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}}-16a^2b\int\frac{}{\frac{a\tan^2(}{b\tan^2(e+}}}{2b}\right.\right.$$

$$f$$

$$\downarrow 216$$

$$\frac{1}{6}\left(\frac{1}{4}(7a+b)\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}-\frac{3}{4}\left(\frac{\frac{(a-b)\left(a^2+10ab+b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}}-16a^{3/2}b\arctan\left(\right.\right.\right.}{2b}\right.\right.$$

$$f$$

input
```
Int[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^4,x]
```

3.396. $\int (a+b\sec^2(e+fx))^{3/2}\tan^4(e+fx)\,dx$

output
```
((b*Tan[e + f*x]^5*Sqrt[a + b + b*Tan[e + f*x]^2])/6 + (((7*a + b)*Tan[e +
 f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/4 - (3*((-16*a^(3/2)*b*ArcTan[(Sqr
t[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]] + ((a - b)*(a^2 + 10*a*
b + b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/S
qrt[b])/(2*b) - ((a^2 - 8*a*b - b^2)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f
*x]^2])/(2*b)))/4)/6)/f
```

3.396.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 379
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q -
1)/(b*e*(m + 2*(p + q) + 1))), x] + Simp[1/(b*(m + 2*(p + q) + 1))   Int[(e
*x)^m*(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*((b*c - a*d)*(m + 1) + b*c*2
*(p + q)) + (d*(b*c - a*d)*(m + 1) + d*2*(q - 1)*(b*c - a*d) + b*c*d*2*(p +
 q))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d, 0
] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.396.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1618 vs. 2(188) = 376.

Time = 21.72 (sec) , antiderivative size = 1619, normalized size of antiderivative = 7.57

method	result	size
default	Expression too large to display	1619

input
```
int((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/96/f/(-a)^(1/2)/b^(9/2)*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)/(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))*(-6*cos(f*x+e)^2*s
in(f*x+e)*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(11/2)-
96*cos(f*x+e)^3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*si
n(f*x+e)*a)*b^(9/2)*a^2-6*cos(f*x+e)*sin(f*x+e)*(-a)^(1/2)*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(11/2)+76*cos(f*x+e)^2*sin(f*x+e)*(-a)^(1/2
)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(9/2)*a+28*(-a)^(1/2)*b^(1
1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*sin(f*x+e)+76*cos(f*x+e)*
sin(f*x+e)*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(9/2)*
a-6*cos(f*x+e)^2*sin(f*x+e)*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*b^(7/2)*a^2+28*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*b^(11/2)*tan(f*x+e)-28*(-a)^(1/2)*b^(9/2)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*a*sin(f*x+e)-6*cos(f*x+e)*sin(f*x+e)*(-a)^(1/2)*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(7/2)*a^2-16*(-a)^(1/2)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(11/2)*tan(f*x+e)*sec(f*x+e)-28*(-a)^(1/2
)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(9/2)*a*tan(f*x+e)+3*cos(f
*x+e)^3*(-a)^(1/2)*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/
2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+
e)*a-a-b)/(sin(f*x+e)+1))*a^3*b^3+27*cos(f*x+e)^3*(-a)^(1/2)*ln(4*(((b+...
```

3.396.5 Fricas [A] (verification not implemented)

Time = 4.15 (sec) , antiderivative size = 1777, normalized size of antiderivative = 8.30

$$\int (a + b\sec^2(e + fx))^{3/2} \tan^4(e + fx)\, dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^4,x, algorithm="fricas")
```

output
```
[1/192*(24*sqrt(-a)*a*b^2*cos(f*x + e)^5*log(128*a^4*cos(f*x + e)^8 - 256*
(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x +
 e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b +
 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 -
a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a
^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 3*(a^3 + 9*a^2*b - 9*a*b^2 - b^3)*s
qrt(b)*cos(f*x + e)^5*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b
^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x
 + e)^4) + 4*((3*a^2*b - 38*a*b^2 + 3*b^3)*cos(f*x + e)^4 + 8*b^3 + 14*(a*
b^2 - b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin
(f*x + e))/(b^2*f*cos(f*x + e)^5), 1/96*(12*sqrt(-a)*a*b^2*cos(f*x + e)^5*
log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4
 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*
a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(1
6*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2
*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e
))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 3*
(a^3 + 9*a^2*b - 9*a*b^2 - b^3)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x +...
```

3.396.6 Sympy [F]

$$\int (a + b \sec^2(e + fx))^{3/2} \tan^4(e + fx)\, dx = \int (a + b \sec^2(e + fx))^{\frac{3}{2}} \tan^4(e + fx)\, dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**(3/2)*tan(f*x+e)**4,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(3/2)*tan(e + f*x)**4, x)
```

3.396.7 Maxima [F]

$$\int (a+b\sec^2(e+fx))^{3/2}\tan^4(e+fx)\,dx = \int (b\sec(fx+e)^2+a)^{\frac{3}{2}}\tan(fx+e)^4\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^4,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^4, x)
```

3.396.8 Giac [F]

$$\int (a+b\sec^2(e+fx))^{3/2}\tan^4(e+fx)\,dx = \int (b\sec(fx+e)^2+a)^{\frac{3}{2}}\tan(fx+e)^4\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^4,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^4, x)
```

3.396.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\sec^2(e+fx))^{3/2}\tan^4(e+fx)\,dx = \int \tan(e+f\,x)^4\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}dx$$

input
```
int(tan(e + f*x)^4*(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(tan(e + f*x)^4*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.397 $\int (a+b\sec^2(e+fx))^{3/2}\tan^2(e+fx)\,dx$

3.397.1 Optimal result . . . 2904
3.397.2 Mathematica [C] (warning: unable to verify) . . . 2905
3.397.3 Rubi [A] (verified) . . . 2906
3.397.4 Maple [B] (verified) . . . 2909
3.397.5 Fricas [B] (verification not implemented) . . . 2910
3.397.6 Sympy [F] . . . 2911
3.397.7 Maxima [F] . . . 2912
3.397.8 Giac [F] . . . 2912
3.397.9 Mupad [F(-1)] . . . 2912

3.397.1 Optimal result

Integrand size = 25, antiderivative size = 166

$$\int (a+b\sec^2(e+fx))^{3/2}\tan^2(e+fx)\,dx = -\frac{a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{(3a^2-6ab-b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8\sqrt{b}f} + \frac{(5a+b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8f} + \frac{b\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4f}$$

output

```
-a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f+1/8*(3*a^
2-6*a*b-b^2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/b^(1
/2)+1/8*(5*a+b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f+1/4*b*(a+b+b*tan(f
*x+e)^2)^(1/2)*tan(f*x+e)^3/f
```

3.397.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.21 (sec) , antiderivative size = 703, normalized size of antiderivative = 4.23

$$\int \left(a+b\sec^2(e+fx)\right)^{3/2}\tan^2(e+fx)\,dx = \frac{e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos^3(e+fx)\left(-\frac{i\left(-1+e^{2i(e+fx)}\right)\left(5a\left(1+e^{2i(e+fx)}\right)^2-b\left(1-6e^{2i(e+f\right.\right.}{\left(1+e^{2i(e+fx)}\right)^4}\right.}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^2,x]
```

output
```
(E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e +
f*x))]*Cos[e + f*x]^3*(((-I)*(-1 + E^((2*I)*(e + f*x)))*(5*a*(1 + E^((2*I)
*(e + f*x)))^2 - b*(1 - 6*E^((2*I)*(e + f*x)) + E^((4*I)*(e + f*x)))))/(1
+ E^((2*I)*(e + f*x)))^4 + (-8*a^(3/2)*Sqrt[b]*f*x + (4*I)*a^(3/2)*Sqrt[b]
*Log[a + 2*b + a*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)
) + a*(1 + E^((2*I)*(e + f*x)))^2]] - (4*I)*a^(3/2)*Sqrt[b]*Log[a + a*E^((
2*I)*(e + f*x)) + 2*b*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e +
 f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] - 3*a^2*Log[(4*(Sqrt[b]*(-1 + E^(
(2*I)*(e + f*x))) - I*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e +
f*x)))^2])*f)/((3*a^2 - 6*a*b - b^2)*(1 + E^((2*I)*(e + f*x))))] + 6*a*b*L
og[(4*(Sqrt[b]*(-1 + E^((2*I)*(e + f*x))) - I*Sqrt[4*b*E^((2*I)*(e + f*x))
 + a*(1 + E^((2*I)*(e + f*x)))^2])*f)/((3*a^2 - 6*a*b - b^2)*(1 + E^((2*I)
*(e + f*x))))] + b^2*Log[(4*(Sqrt[b]*(-1 + E^((2*I)*(e + f*x))) - I*Sqrt[4
*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*f)/((3*a^2 - 6*a*
b - b^2)*(1 + E^((2*I)*(e + f*x))))])/(Sqrt[b]*Sqrt[4*b*E^((2*I)*(e + f*x)
) + a*(1 + E^((2*I)*(e + f*x)))^2]))*(a + b*Sec[e + f*x]^2)^(3/2))/(2*Sqrt
[2]*f*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2))
```

3.397.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 4629, 2075, 379, 444, 27, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int \tan(e+fx)^2\left(a+b\sec(e+fx)^2\right)^{3/2}dx$$

$$\downarrow 4629$$

$$\frac{\int \frac{\tan^2(e+fx)\left(a+b\left(\tan^2(e+fx)+1\right)\right)^{3/2}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\tan^2(e+fx)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\downarrow 379$$

$$\frac{\frac{1}{4}\int \frac{\tan^2(e+fx)\left(b(5a+b)\tan^2(e+fx)+(a+b)(4a+b)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+\frac{1}{4}b\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

$$\downarrow 444$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}(5a+b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}-\frac{\int \frac{b\left((a+b)(5a+b)-\left(3a^2-6ba-b^2\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2b}\right)+\frac{1}{4}b\tan^3(e+}{f}$$

$$\downarrow 27$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}(5a+b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}-\frac{1}{2}\int \frac{(a+b)(5a+b)-\left(3a^2-6ba-b^2\right)\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)\right)+\frac{1}{4}b\tan}{f}$$

$$\downarrow 398$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(\left(3a^2-6ab-b^2\right)\int\frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-8a^2\int\frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)\right)+\frac{1}{2}\right.}{f}$$

$$\downarrow 224$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(\left(3a^2-6ab-b^2\right)\int\frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}-8a^2\int\frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)\right.\right.}{f}$$

$$\downarrow 219$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(\frac{\left(3a^2-6ab-b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}}-8a^2\int\frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)\right)+\frac{1}{2}(5a+b)\tan\right.}{f}$$

$$\downarrow 291$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(\frac{\left(3a^2-6ab-b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}}-8a^2\int\frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}\right)+\frac{1}{2}(5a+b)\tan(e+f\right.}{f}$$

$$\downarrow 216$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(\frac{\left(3a^2-6ab-b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}}-8a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)\right)+\frac{1}{2}(5a+b)\tan(e+fx)\sqrt{a+\right.}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^(3/2)*Tan[e + f*x]^2,x]
```

output
```
((b*Tan[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/4 + ((-8*a^(3/2)*ArcTan
[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]] + ((3*a^2 - 6*a*b
- b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/Sqr
t[b])/2 + ((5*a + b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/2)/4)/f
```

3.397. $\int (a+b\sec^2(e+fx))^{3/2}\tan^2(e+fx)\,dx$

3.397.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 379
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q -
1)/(b*e*(m + 2*(p + q) + 1))), x] + Simp[1/(b*(m + 2*(p + q) + 1))   Int[(e
*x)^m*(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*((b*c - a*d)*(m + 1) + b*c*2
*(p + q)) + (d*(b*c - a*d)*(m + 1) + d*2*(q - 1)*(b*c - a*d) + b*c*d*2*(p +
 q))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d, 0
] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.397.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1118 vs. $2(144) = 288$.

Time = 17.36 (sec) , antiderivative size = 1119, normalized size of antiderivative = 6.74

method	result	size
default	Expression too large to display	1119

input
```
int((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/16/f/(-a)^(1/2)/b^(5/2)*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)/(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))*(-2*sin(f*x+e)*cos(
f*x+e)^2*(-a)^(1/2)*b^(7/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-16
*cos(f*x+e)^3*b^(5/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2
)-4*sin(f*x+e)*a)*a^2-2*sin(f*x+e)*cos(f*x+e)*(-a)^(1/2)*b^(7/2)*((b+a*cos
(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+10*sin(f*x+e)*cos(f*x+e)^2*(-a)^(1/2)*b
^(5/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a+4*(-a)^(1/2)*b^(7/2)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*sin(f*x+e)+10*sin(f*x+e)*cos(f
*x+e)*(-a)^(1/2)*b^(5/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a+3*c
os(f*x+e)^3*(-a)^(1/2)*ln(4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)-a-b)/(sin(f*x+e)-1))*a^2*b^2-6*cos(f*x+e)^3*(-a)^(1/2)*ln(4*(-((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-
b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-a-b)/(sin(f*x+e)-1))*a
*b^3-cos(f*x+e)^3*(-a)^(1/2)*ln(4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)-a-b)/(sin(f*x+e)-1))*b^4+3*cos(f*x+e)^3*(-a)^(1/2)*ln(-4*(
-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)
*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+a+b)/(sin(f*x+e)...
```

3.397.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(144) = 288$.

Time = 1.27 (sec) , antiderivative size = 1627, normalized size of antiderivative = 9.80

$$\int (a + b\sec^2(e + fx))^{3/2} \tan^2(e + fx)\, dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^2,x, algorithm="fricas")
```

output
```
[1/32*(4*sqrt(-a)*a*b*cos(f*x + e)^3*log(128*a^4*cos(f*x + e)^8 - 256*(a^4
 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^
4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a
^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*
b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 -
 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)*sin(f*x + e)) - (3*a^2 - 6*a*b - b^2)*sqrt(b)*cos(f*x +
 e)^3*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)
^2 - 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 4*((5
*a*b - b^2)*cos(f*x + e)^2 + 2*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*sin(f*x + e))/(b*f*cos(f*x + e)^3), 1/16*(2*sqrt(-a)*a*b*cos(f*x + e
)^3*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*
a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 -
 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 +
8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14
*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x
 + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))
+ (3*a^2 - 6*a*b - b^2)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b
*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a...
```

3.397.6 Sympy [F]

$$\int (a + b \sec^2(e + fx))^{3/2} \tan^2(e + fx)\, dx = \int (a + b \sec^2(e + fx))^{\frac{3}{2}} \tan^2(e + fx)\, dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**(3/2)*tan(f*x+e)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(3/2)*tan(e + f*x)**2, x)
```

3.397.7 Maxima [F]

$$\int (a+b\sec^2(e+fx))^{3/2}\tan^2(e+fx)\,dx = \int (b\sec(fx+e)^2+a)^{\frac{3}{2}}\tan(fx+e)^2\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^2, x)
```

3.397.8 Giac [F]

$$\int (a+b\sec^2(e+fx))^{3/2}\tan^2(e+fx)\,dx = \int (b\sec(fx+e)^2+a)^{\frac{3}{2}}\tan(fx+e)^2\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^2, x)
```

3.397.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\sec^2(e+fx))^{3/2}\tan^2(e+fx)\,dx = \int \tan(e+f\,x)^2\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}dx$$

input
```
int(tan(e + f*x)^2*(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(tan(e + f*x)^2*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.398 $\int (a+b\sec^2(e+fx))^{3/2}\,dx$

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3.398.1 Optimal result

Integrand size = 16, antiderivative size = 118

$$\int (a+b\sec^2(e+fx))^{3/2}\,dx=\frac{a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}(3a+b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2f}+\frac{b\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2f}$$

output
```
a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f+1/2*(3*a+b
)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))*b^(1/2)/f+1/2*b*(
a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/f
```

3.398.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.09 (sec) , antiderivative size = 527, normalized size of antiderivative = 4.47

$$\int (a+b\sec^2(e+fx))^{3/2}\,dx=\frac{\sqrt{2}e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos^3(e+fx)\left(-\frac{ib\left(-1+e^{2i(e+fx)}\right)}{\left(1+e^{2i(e+fx)}\right)^2}+\frac{2a^{3/2}fx-ia^{3/2}\log(\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[2]*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*
I)*(e + f*x))]*Cos[e + f*x]^3*(((-I)*b*(-1 + E^((2*I)*(e + f*x))))/(1 + E^
((2*I)*(e + f*x)))^2 + (2*a^(3/2)*f*x - I*a^(3/2)*Log[a + 2*b + a*E^((2*I)
*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e +
f*x)))^2]] + I*a^(3/2)*Log[a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*I)*(e + f
*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^
2]] - 3*a*Sqrt[b]*Log[(-2*Sqrt[b]*(-1 + E^((2*I)*(e + f*x)))*f + (2*I)*Sqr
t[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*f)/(b*(3*a + b)
*(1 + E^((2*I)*(e + f*x))))] - b^(3/2)*Log[(-2*Sqrt[b]*(-1 + E^((2*I)*(e +
 f*x)))*f + (2*I)*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)
))^2]*f)/(b*(3*a + b)*(1 + E^((2*I)*(e + f*x))))])/Sqrt[4*b*E^((2*I)*(e +
f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*(a + b*Sec[e + f*x]^2)^(3/2))/(f*(
a + 2*b + a*Cos[2*e + 2*f*x])^(3/2))
```

3.398.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4616, 318, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a + b\sec^2(e+fx)\right)^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int \left(a + b\sec(e+fx)^2\right)^{3/2} dx$$

$$\downarrow 4616$$

$$\frac{\int \frac{\left(b\tan^2(e+fx)+a+b\right)^{3/2}}{\tan^2(e+fx)+1} d\tan(e+fx)}{f}$$

$$\downarrow 318$$

$$\frac{\frac{1}{2}\int \frac{b(3a+b)\tan^2(e+fx)+(a+b)(2a+b)}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}} d\tan(e+fx) + \frac{1}{2}b\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

$$\downarrow 398$$

$$\frac{\frac{1}{2}\left(2a^2\int\frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+b(3a+b)\int\frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)\right)+\frac{1}{2}b\tan(e+}{f}$$

$$\downarrow 224$$

$$\frac{\frac{1}{2}\left(2a^2\int\frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+b(3a+b)\int\frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}\right)+\frac{1}{2}b\tan}{f}$$

$$\downarrow 219$$

$$\frac{\frac{1}{2}\left(2a^2\int\frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+\sqrt{b}(3a+b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)\right)+\frac{1}{2}b\tan(e+fx)}{f}$$

$$\downarrow 291$$

$$\frac{\frac{1}{2}\left(2a^2\int\frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}+\sqrt{b}(3a+b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)\right)+\frac{1}{2}b\tan(e+fx)\sqrt{a+b}}{f}$$

$$\downarrow 216$$

$$\frac{\frac{1}{2}\left(2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)+\sqrt{b}(3a+b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)\right)+\frac{1}{2}b\tan(e+fx)\sqrt{a+b\tan^2(e+}}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
((2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]
+ Sqrt[b]*(3*a + b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e +
f*x]^2]])/2 + (b*Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/2)/f
```

3.398.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

3.398.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 708 vs. 2(100) = 200.

Time = 8.03 (sec) , antiderivative size = 709, normalized size of antiderivative = 6.01

method	result
default	$\frac{\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}\left(\cos(fx+e)^3 b^{\frac{5}{2}}\ln\left(\frac{-4\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{b}\cos(fx+e)+4\sin(fx+e)a-4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4a-4b}{\sin(fx+e)-1}\right)\sqrt{-a}+\cos(f\right.}{}$

input

```
int((a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
1/4/f/(-a)^(1/2)/b*(a+b*sec(f*x+e)^2)^(3/2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)/(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))*(cos(f*x+e)^3*b^(5/2)*ln(4
*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+
e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-a-b)/(sin(f*x+e)-
1))*(-a)^(1/2)+cos(f*x+e)^3*b^(5/2)*ln(-4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)+a+b)/(sin(f*x+e)+1))*(-a)^(1/2)+3*cos(f*x+e)^3*b^(
3/2)*ln(4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)
+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-a-b)/(si
n(f*x+e)-1))*(-a)^(1/2)*a+3*cos(f*x+e)^3*b^(3/2)*ln(-4*(-((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+a+b)/(sin(f*x+e)+1))*(-a)^(1/2)*a+2*s
in(f*x+e)*cos(f*x+e)^2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(-a)^(1
/2)*b^2+4*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(
f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+
e)*a)*cos(f*x+e)^3*a^2*b+2*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*b^2*cos(f*x+e)*sin(f*x+e))
```

3.398.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(100) = 200$.

Time = 0.62 (sec) , antiderivative size = 1457, normalized size of antiderivative = 12.35

$$\int \left(a + b\sec^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(sqrt(-a)*a*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*
b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4
 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2
- a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(
f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*
b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(
f*x + e)^2)*sin(f*x + e)) + (3*a + b)*sqrt(b)*cos(f*x + e)*log(((a^2 - 6*a
*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f
*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 4*b*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)*sin(f*x + e))/(f*cos(f*x + e)), 1/8*(2*(3*a + b)*sqr
t(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f
*x + e)))*cos(f*x + e) + sqrt(-a)*a*cos(f*x + e)*log(128*a^4*cos(f*x + e)^
8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*c
os(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7
*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24
*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e
)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 4*b*sqrt((a*cos(f*x + e)...
```

3.398.6 Sympy [F]

$$\int \left(a + b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(a + b\sec^2(e+fx)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(3/2), x)
```

3.398.7 Maxima [F]

$$\int \left(a + b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sec\left(fx+e\right)^2 + a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.398.8 Giac [F]

$$\int \left(a + b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sec\left(fx+e\right)^2 + a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.398.9 Mupad [F(-1)]

Timed out.

$$\int \left(a + b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(a + \frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}\,dx$$

input
```
int((a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int((a + b/cos(e + f*x)^2)^(3/2), x)
```

3.399 $\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

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3.399.1 Optimal result

Integrand size = 25, antiderivative size = 111

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = -\frac{a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{b^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{(a+b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{f}$$

output
```
-a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f+b^(3/2)*a
rctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f-(a+b)*cot(f*x+e)*(
a+b+b*tan(f*x+e)^2)^(1/2)/f
```

3.399.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.50 (sec) , antiderivative size = 410, normalized size of antiderivative = 3.69

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{\sqrt{2}e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}\cos^3(e+fx)\left(-\frac{2i(a+b)}{-1+e^{2i(e+fx)}}+\frac{ia^{3/2}\log\left(a+2b+ae^{2i(e}\right.\right.}{}$$

input
```
Integrate[Cot[e + f*x]^2*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(Sqrt[2]*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*I)*(e + f*x)))^2)/E^((2*
I)*(e + f*x))]*Cos[e + f*x]^3*(((-2*I)*(a + b))/(-1 + E^((2*I)*(e + f*x)))
 + (I*a^(3/2)*Log[a + 2*b + a*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2
*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] - I*a^(3/2)*Log[a + a*E^(
(2*I)*(e + f*x)) + 2*b*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e
 + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] - 2*(a^(3/2)*f*x + b^(3/2)*Log[(
(Sqrt[b]*(-1 + E^((2*I)*(e + f*x))) - I*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(
1 + E^((2*I)*(e + f*x)))^2])*f)/(b^2*(1 + E^((2*I)*(e + f*x))))]))/Sqrt[4*
b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2])*(a + b*Sec[e + f*x
]^2)^(3/2))/(f*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2))
```

3.399.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.95, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4629, 2075, 376, 25, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^{3/2}}{\tan(e+fx)^2}dx$$

$$\downarrow 4629$$

$$\frac{\int \frac{\cot^2(e+fx)\left(a+b\left(\tan^2(e+fx)+1\right)\right)^{3/2}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\cot^2(e+fx)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\downarrow 376$$

$$\frac{\int -\frac{a^2-b^2-b^2\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-(a+b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

$$\downarrow 25$$

$$\frac{-\int \frac{a^2-b^2-b^2\tan^2(e+fx)}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-\left((a+b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)}{f}$$

$$\downarrow 398$$

$$\frac{a^2\left(-\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)\right)+b^2\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-(a+b)\cot(e+fx)}{f}$$

$$\downarrow 224$$

$$\frac{a^2\left(-\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)\right)+b^2\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}-(a+b)\cot(e+}{f}$$

$$\downarrow 219$$

$$\frac{a^2\left(-\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)\right)+b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)-(a+b)\cot(e+fx)\sqrt{a}}{f}$$

$$\downarrow 291$$

$$\frac{a^2\left(-\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}\right)+b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)-(a+b)\cot(e+fx)\sqrt{a+b\tan}}{f}$$

$$\downarrow 216$$

$$\frac{a^{3/2}\left(-\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)\right)+b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)-(a+b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)}}{f}$$

input
```
Int[Cot[e + f*x]^2*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(-(a^(3/2)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])
+ b^(3/2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]] -
 (a + b)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/f
```

3.399. $\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

3.399.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 376
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1
)/(a*e*(m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^
2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b*c - a*d)*(m + 1) + 2*c*(b*c*(p + 1) + a*
d*(q - 1)) + d*((b*c - a*d)*(m + 1) + 2*b*c*(p + q))*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && GtQ[q, 1] && LtQ[m, -1] &
& IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.399.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 538 vs. 2(97) = 194.

Time = 9.30 (sec) , antiderivative size = 539, normalized size of antiderivative = 4.86

method	result
default	$\left(\sqrt{-a}\,\ln\left(-\frac{4\left(-\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,\sqrt{b}\,\cos(fx+e)+\sin(fx+e)a-\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}+a+b\right)}{\sin(fx+e)+1}\right)b^{\frac{3}{2}}\sin(fx+e)+\sqrt{-a}\,\ln\left(\frac{-4\sqrt{\frac{b+a\cos(fx+$

input
```
int(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f/(-a)^(1/2)*((-a)^(1/2)*ln(-4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)+a+b)/(sin(f*x+e)+1))*b^(3/2)*sin(f*x+e)+(-a)^(1/2)*ln(4*(
-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)
*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-a-b)/(sin(f*x+e)-1)
)*b^(3/2)*sin(f*x+e)-2*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*a*cos(f*x+e)-2*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
b*cos(f*x+e)-2*sin(f*x+e)*a^2*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)-4*sin(f*x+e)*a)-2*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*a-2*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b)*(a+
b*sec(f*x+e)^2)^(3/2)/(b+a*cos(f*x+e)^2)/((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)/(1+cos(f*x+e))*cos(f*x+e)^2*cot(f*x+e)
```

3.399.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. 2(97) = 194.

Time = 0.65 (sec) , antiderivative size = 1446, normalized size of antiderivative = 13.03

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(sqrt(-a)*a*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x +
e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b +
 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(
f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 +
2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 -
 b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*s
in(f*x + e))*sin(f*x + e) + 2*b^(3/2)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e
)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f
*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)
+ 8*b^2)/cos(f*x + e)^4)*sin(f*x + e) - 8*(a + b)*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)*cos(f*x + e))/(f*sin(f*x + e)), 1/8*(4*sqrt(-b)*b*arct
an(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*
sin(f*x + e) + sqrt(-a)*a*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*c
os(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 2
8*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*
b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x
+ e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b +
7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)*sin(f*x + e))*sin(f*x + e) - 8*(a + b)*sqrt((a*cos(f*x + e)^2 +...
```

3.399.6 Sympy [F]

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(a+b\sec^2(e+fx)\right)^{\frac{3}{2}}\cot^2(e+fx)\,dx$$

input
```
integrate(cot(f*x+e)**2*(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(3/2)*cot(e + f*x)**2, x)
```

3.399.7 Maxima [F]

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^2\,dx$$

input
```
integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^2, x)
```

3.399.8 Giac [F]

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^2\,dx$$

input
```
integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^2, x)
```

3.399.9 Mupad [F(-1)]

Timed out.

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \cot(e+f\,x)^2\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}\,dx$$

input
```
int(cot(e + f*x)^2*(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(cot(e + f*x)^2*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.400 $\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

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3.400.1 Optimal result

Integrand size = 25, antiderivative size = 112

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \frac{a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} + \frac{(3a-b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3f} - \frac{(a+b)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3f}$$

output
```
a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f+1/3*(3*a-b
)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/f-1/3*(a+b)*cot(f*x+e)^3*(a+b+b*ta
n(f*x+e)^2)^(1/2)/f
```

3.400.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.58 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.89

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = -\frac{2(a+b)\cot^3(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{2},-\frac{1}{2},\frac{a\sin^2(e+fx)}{a+b}\right)\left(a+b\sec^2(e+fx)\right)^{3/2}}{3f(a+2b+a\cos(2(e+fx)))\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}}$$

input
```
Integrate[Cot[e + f*x]^4*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(-2*(a + b)*Cot[e + f*x]^3*Hypergeometric2F1[-3/2, -3/2, -1/2, (a*Sin[e +
f*x]^2)/(a + b)]*(a + b*Sec[e + f*x]^2)^(3/2))/(3*f*(a + 2*b + a*Cos[2*(e
+ f*x)])*Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)])
```

3.400.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4629, 2075, 376, 25, 445, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^{3/2}}{\tan(e+fx)^4}dx$$

$$\downarrow 4629$$

$$\frac{\int \frac{\cot^4(e+fx)(a+b(\tan^2(e+fx)+1))^{3/2}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\cot^4(e+fx)(b\tan^2(e+fx)+a+b)^{3/2}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\downarrow 376$$

$$\frac{\frac{1}{3}\int -\frac{\cot^2(e+fx)((2a-b)b\tan^2(e+fx)+(3a-b)(a+b))}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-\frac{1}{3}(a+b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

$$\downarrow 25$$

$$\frac{-\frac{1}{3}\int \frac{\cot^2(e+fx)((2a-b)b\tan^2(e+fx)+(3a-b)(a+b))}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-\frac{1}{3}(a+b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

$$\downarrow 445$$

$$\frac{\frac{1}{3}\left(\frac{\int \frac{3a^2(a+b)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a+b}+(3a-b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)-\frac{1}{3}(a+b)\cot^3(e+}{f}$$

$$\downarrow 27$$

$$\frac{\frac{1}{3}\left(3a^2\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+(3a-b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)-\frac{1}{3}(a+b)}{f}$$

$$\downarrow 291$$

$$\frac{\frac{1}{3}\left(3a^2\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}+(3a-b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)-\frac{1}{3}(a+b)\cot^3(e+}{f}$$

$$\downarrow 216$$

$$\frac{\frac{1}{3}\left(3a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)+(3a-b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)-\frac{1}{3}(a+b)\cot^3(e+fx)\sqrt{a}}{f}$$

input
```
Int[Cot[e + f*x]^4*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(-1/3*((a + b)*Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2]) + (3*a^(3/2)
*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]] + (3*a - b)
*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/3)/f
```

3.400.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 376
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1
)/(a*e*(m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^
2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b*c - a*d)*(m + 1) + 2*c*(b*c*(p + 1) + a*
d*(q - 1)) + d*((b*c - a*d)*(m + 1) + 2*b*c*(p + q))*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && GtQ[q, 1] && LtQ[m, -1] &
& IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.400.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 368 vs. $2(98) = 196$.

Time $= 3.74$ (sec) , antiderivative size $= 369$, normalized size of antiderivative $= 3.29$

method	result
default	$-\frac{\left(4\cos(fx+e)^2\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,a+3\sin(fx+e)a^2\ln\left(4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)+4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(\right.}{}$

input
```
int(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/f/(-a)^(1/2)*(4*cos(f*x+e)^2*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*a+3*sin(f*x+e)*a^2*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)-4*sin(f*x+e)*a)*cos(f*x+e)-3*sin(f*x+e)*a^2*ln(4*(-a)^(1/2)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)-3*(-a)^(1/2)*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a+(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*b)*(a+b*sec(f*x+e)^2)^(3/2)/(b+a*cos(f*x+e)^2)/((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cot(f*x+e)^3
```

3.400.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(98) = 196$.

Time $= 0.84$ (sec) , antiderivative size $= 597$, normalized size of antiderivative $= 5.33$

$$\int \cot^4(e+fx)\left(a+b\sec^2(e\right.$$

$$\left.+fx)\right)^{3/2}\,dx = \left[\frac{3\left(a\cos\left(fx+e\right)^2-a\right)\sqrt{-a}\log\left(128\,a^4\cos\left(fx+e\right)^8-256\left(a^4-a^3b\right)\cos\left(fx+e\right)^6+32\left(5\right.\right.\right.$$

$$-\frac{3\left(a\cos\left(fx+e\right)^2-a\right)\sqrt{a}\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^5-8\left(a^2-ab\right)\cos(fx+e)^3+\left(a^2-6\,ab+b^2\right)\cos(fx+e)\right)\sqrt{a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\left(2\,a^3\cos(fx+e)^4-a^2b+ab^2-\left(a^3-3\,a^2b\right)\cos(fx+e)^2\right)\sin(fx+e)}\right)\mathrm{s}}{12\left(f\cos\left(fx+e\right)^2-f\right)\sin\left(fx+\right.}$$

input
```
integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/24*(3*(a*cos(f*x + e)^2 - a)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*
(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x +
 e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b +
 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 -
a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a
^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)*sin(f*x + e))*sin(f*x + e) + 8*(4*a*cos(f*x + e)^3
 - (3*a - b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((f
*cos(f*x + e)^2 - f)*sin(f*x + e)), -1/12*(3*(a*cos(f*x + e)^2 - a)*sqrt(a
)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 -
 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e
)^2)*sin(f*x + e)))*sin(f*x + e) - 4*(4*a*cos(f*x + e)^3 - (3*a - b)*cos(f
*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((f*cos(f*x + e)^2 -
 f)*sin(f*x + e))]
```

3.400.6 Sympy [F(-1)]

Timed out.

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)**4*(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

3.400.7 Maxima [F]

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^4\,dx$$

input
```
integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^4, x)
```

3.400.8 Giac [F]

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^4\,dx$$

input
```
integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^4, x)
```

3.400.9 Mupad [F(-1)]

Timed out.

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \cot(e+f\,x)^4\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^{3/2}\,dx$$

input
```
int(cot(e + f*x)^4*(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(cot(e + f*x)^4*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.401 $\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx$

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3.401.1 Optimal result

Integrand size = 25, antiderivative size = 165

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = -\frac{a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{f} - \frac{(15a^2+10ab-2b^2)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)f} + \frac{(5a-b)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15f} - \frac{(a+b)\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5f}$$

output

```
-a^(3/2)*arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f-1/15*(15*
a^2+10*a*b-2*b^2)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)/f+1/15*(5*a-
b)*cot(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(1/2)/f-1/5*(a+b)*cot(f*x+e)^5*(a+b+b
*tan(f*x+e)^2)^(1/2)/f
```

3.401.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.50 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.84

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx = \frac{2\cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\left(-\frac{3}{4}(a+2b+a\cos(2(e+fx)))^2\csc^2(e+fx)+\frac{5(a+b)}{}\right.}{15(a+b)f(a+2b+a\cos(2(e+fx)))}$$

input
```
Integrate[Cot[e + f*x]^6*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(2*Cot[e + f*x]^3*(a + b*Sec[e + f*x]^2)^(3/2)*((-3*(a + 2*b + a*Cos[2*(e
+ f*x)])^2*Csc[e + f*x]^2)/4 + (5*(a + b)^2*Hypergeometric2F1[-3/2, -3/2,
-1/2, (a*Sin[e + f*x]^2)/(a + b)])/Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)
]))/(15*(a + b)*f*(a + 2*b + a*Cos[2*(e + f*x)]))
```

3.401.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 4629, 2075, 376, 25, 445, 27, 445, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^{3/2}}{\tan(e+fx)^6}dx$$

$$\downarrow 4629$$

$$\frac{\int \frac{\cot^6(e+fx)\left(a+b\left(\tan^2(e+fx)+1\right)\right)^{3/2}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\cot^6(e+fx)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\downarrow 376$$

$$\frac{\frac{1}{5}\int -\frac{\cot^4(e+fx)\left((4a-b)b\tan^2(e+fx)+(5a-b)(a+b)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-\frac{1}{5}(a+b)\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

$$\downarrow 25$$

$$\frac{-\frac{1}{5}\int \frac{\cot^4(e+fx)\left((4a-b)b\tan^2(e+fx)+(5a-b)(a+b)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-\frac{1}{5}(a+b)\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{f}$$

$$\downarrow 445$$

$$\frac{\frac{1}{5}\left(\frac{\int \frac{(a+b)\cot^2(e+fx)\left(15a^2+10ba-2b^2+2(5a-b)b\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{3(a+b)}+\frac{1}{3}(5a-b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)-\frac{1}{5}}{f}$$

$$\downarrow 27$$

$$\frac{\frac{1}{5}\left(\frac{1}{3}\int \frac{\cot^2(e+fx)\left(15a^2+10ba-2b^2+2(5a-b)b\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+\frac{1}{3}(5a-b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}\right)}{f}$$

$$\downarrow 445$$

$$\frac{\frac{1}{5}\left(\frac{1}{3}\left(-\frac{\int \frac{15a^2(a+b)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a+b}-\frac{\left(15a^2+10ab-2b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}\right)+\frac{1}{3}(5a-b)\cot^3(e+\right.}{f}$$

$$\downarrow 27$$

$$\frac{\frac{1}{5}\left(\frac{1}{3}\left(-15a^2\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-\frac{\left(15a^2+10ab-2b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}\right)+\frac{1}{3}(5a-\right.}{f}$$

$$\downarrow 291$$

3.401. $\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}dx$

$$\frac{\frac{1}{5}\left(\frac{1}{3}\left(-15a^2\int\frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}-\frac{\left(15a^2+10ab-2b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}\right)+\frac{1}{3}(5a-b)\cot^3(\right.}{f}$$

$$\downarrow 216$$

$$\frac{\frac{1}{5}\left(\frac{1}{3}\left(-15a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)-\frac{\left(15a^2+10ab-2b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}\right)+\frac{1}{3}(5a-b)\cot^3(e+fx)\right.}{f}$$

input
```
Int[Cot[e + f*x]^6*(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(-1/5*((a + b)*Cot[e + f*x]^5*Sqrt[a + b + b*Tan[e + f*x]^2]) + (((5*a - b
)*Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/3 + (-15*a^(3/2)*ArcTan[(
Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]] - ((15*a^2 + 10*a*b
- 2*b^2)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(a + b))/3)/5)/f
```

3.401.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 376
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1
)/(a*e*(m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^
2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b*c - a*d)*(m + 1) + 2*c*(b*c*(p + 1) + a*
d*(q - 1)) + d*((b*c - a*d)*(m + 1) + 2*b*c*(p + q))*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && GtQ[q, 1] && LtQ[m, -1] &
& IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.401.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 792 vs. $2(147) = 294$.

Time = 5.02 (sec) , antiderivative size = 793, normalized size of antiderivative = 4.81

method	result
default	$-\frac{\left(23\cos(fx+e)^4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,a^2+20\cos(fx+e)^4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,ab-15\sin(fx+e)^3a^3\ln\left(4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)}{(1+\cos(fx+e))}}\right.\right.}{}$

input
```
int(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15/f/(a+b)/(-a)^(1/2)*(23*cos(f*x+e)^4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*a^2+20*cos(f*x+e)^4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*a*b-15*sin(f*x+e)^3*a^3*ln(4*(-a)^(1/2)*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*cos(f*x+e)-15*sin(f*x+e)^3*a^2
*b*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+
4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*c
os(f*x+e)+15*sin(f*x+e)^3*a^3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)-4*sin(f*x+e)*a)+15*sin(f*x+e)^3*a^2*b*ln(4*(-a)^(1/2)*((b+a*cos(
f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)-35*cos(f*x+e)^2*(-a)^(1/2)*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2-24*cos(f*x+e)^2*(-a)^(1/2)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b+5*cos(f*x+e)^2*(-a)^(1/2)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2+15*(-a)^(1/2)*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2+10*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*a*b-2*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*b^2)*(a+b*sec(f*x+e)^2)^(3/2)/(b+a*cos(f*x+e)^2)/((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*cot(f*x+e)^3*csc(f*x+e)^2
```

3.401.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(147) = 294$.

Time = 3.06 (sec) , antiderivative size = 767, normalized size of antiderivative = 4.65

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \left[\frac{15\left(\left(a^2+ab\right)\cos\left(fx+e\right)^4-2\left(a^2+ab\right)\cos\left(fx+e\right)^2+a^2+ab\right)\sqrt{-a}\log\left(128\,a^4\cos\left(fx+\right.\right.\right.}{}$$

input
```
integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/120*(15*((a^2 + a*b)*cos(f*x + e)^4 - 2*(a^2 + a*b)*cos(f*x + e)^2 + a^
2 + a*b)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x +
 e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b
+ 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos
(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 +
 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2
- b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*
sin(f*x + e))*sin(f*x + e) - 8*((23*a^2 + 20*a*b)*cos(f*x + e)^5 - (35*a^2
 + 24*a*b - 5*b^2)*cos(f*x + e)^3 + (15*a^2 + 10*a*b - 2*b^2)*cos(f*x + e)
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a + b)*f*cos(f*x + e)^4
- 2*(a + b)*f*cos(f*x + e)^2 + (a + b)*f)*sin(f*x + e)), 1/60*(15*((a^2 +
a*b)*cos(f*x + e)^4 - 2*(a^2 + a*b)*cos(f*x + e)^2 + a^2 + a*b)*sqrt(a)*ar
ctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a
*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)
*sin(f*x + e)))*sin(f*x + e) - 4*((23*a^2 + 20*a*b)*cos(f*x + e)^5 - (35*a
^2 + 24*a*b - 5*b^2)*cos(f*x + e)^3 + (15*a^2 + 10*a*b - 2*b^2)*cos(f*x +
e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a + b)*f*cos(f*x + e)^
4 - 2*(a + b)*f*cos(f*x + e)^2 + (a + b)*f)*sin(f*x + e))]
```

3.401.6 Sympy [F(-1)]

Timed out.

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)**6*(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Timed out
```

3.401.7 Maxima [F]

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}\cot(fx+e)^6\,dx$$

input
```
integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^6, x)
```

3.401.8 Giac [F]

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}\cot(fx+e)^6\,dx$$

input
```
integrate(cot(f*x+e)^6*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^6, x)
```

3.401.9 Mupad [F(-1)]

Timed out.

$$\int \cot^6(e+fx)\left(a+b\sec^2(e+fx)\right)^{3/2}\,dx = \int \cot(e+f\,x)^6\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}\,dx$$

input
```
int(cot(e + f*x)^6*(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(cot(e + f*x)^6*(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.402 $\int \frac{\tan^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

3.402.1 Optimal result . . . 2942
3.402.2 Mathematica [A] (verified) . . . 2942
3.402.3 Rubi [A] (verified) . . . 2943
3.402.4 Maple [B] (verified) . . . 2945
3.402.5 Fricas [B] (verification not implemented) . . . 2945
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3.402.8 Giac [F] . . . 2947
3.402.9 Mupad [F(-1)] . . . 2947

3.402.1 Optimal result

Integrand size = 25, antiderivative size = 89

$$\int \frac{\tan^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f} - \frac{(a+2b)\sqrt{a+b\sec^2(e+fx)}}{b^2 f} + \frac{\left(a+b\sec^2(e+fx)\right)^{3/2}}{3b^2 f}$$

output
```
1/3*(a+b*sec(f*x+e)^2)^(3/2)/b^2/f-arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2
))/f/a^(1/2)-(a+2*b)*(a+b*sec(f*x+e)^2)^(1/2)/b^2/f
```

3.402.2 Mathematica [A] (verified)

Time = 1.56 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.20

$$\int \frac{\tan^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{-2a(a+3b) - b(a+6b)\sec^2(e+fx) + b^2\sec^4(e+fx) - 3b^2\operatorname{arctanh}\left(\sqrt{1+\frac{b\sec^2(e+fx)}{a}}\right)\sqrt{1+\frac{b\sec^2(e+fx)}{a}}}{3b^2 f\sqrt{a+b\sec^2(e+fx)}}$$

input
```
Integrate[Tan[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(-2*a*(a + 3*b) - b*(a + 6*b)*Sec[e + f*x]^2 + b^2*Sec[e + f*x]^4 - 3*b^2*
ArcTanh[Sqrt[1 + (b*Sec[e + f*x]^2)/a]]*Sqrt[1 + (b*Sec[e + f*x]^2)/a])/(3
*b^2*f*Sqrt[a + b*Sec[e + f*x]^2])
```

3.402.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4627, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\tan(e+fx)^5}{\sqrt{a+b\sec(e+fx)^2}}dx$$

$$\downarrow 4627$$

$$\frac{\int \frac{\cos(e+fx)\left(1-\sec^2(e+fx)\right)^2}{\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{f}$$

$$\downarrow 354$$

$$\frac{\int \frac{\cos(e+fx)\left(1-\sec^2(e+fx)\right)^2}{\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{2f}$$

$$\downarrow 99$$

$$\frac{\int \left(\frac{-a-2b}{b\sqrt{b\sec^2(e+fx)+a}}+\frac{\sqrt{b\sec^2(e+fx)+a}}{b}+\frac{\cos(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}\right)d\sec^2(e+fx)}{2f}$$

$$\downarrow 2009$$

$$\frac{-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{2\left(a+b\sec^2(e+fx)\right)^{3/2}}{3b^2}-\frac{2(a+2b)\sqrt{a+b\sec^2(e+fx)}}{b^2}}{2f}$$

input
```
Int[Tan[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
((-2*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/Sqrt[a] - (2*(a + 2*b)*S
qrt[a + b*Sec[e + f*x]^2])/b^2 + (2*(a + b*Sec[e + f*x]^2)^(3/2))/(3*b^2))
/(2*f)
```

3.402.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.402.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 302 vs. $2(77) = 154$.

Time = 8.11 (sec) , antiderivative size = 303, normalized size of antiderivative = 3.40

method	result
default	$-\frac{2a^{\frac{5}{2}}+3\ln\left(4\cos(fx+e)\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{a}+4\cos(fx+e)a+4\sqrt{a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\right)\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}b^2+6a^{\frac{3}{2}}b+3\ln\left(4\cos(fx+$

input
```
int(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/f/b^2/a^(1/2)/(a+b*sec(f*x+e)^2)^(1/2)*(2*a^(5/2)+3*ln(4*cos(f*x+e)*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*cos(f*x+e)*a+4*a^(1/2
)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*b^2+6*a^(3/2)*b+3*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*a^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2*s
ec(f*x+e)+a^(3/2)*b*sec(f*x+e)^2+6*a^(1/2)*b^2*sec(f*x+e)^2-a^(1/2)*b^2*se
c(f*x+e)^4)
```

3.402.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(77) = 154$.

Time = 0.62 (sec) , antiderivative size = 410, normalized size of antiderivative = 4.61

$$\int \frac{\tan^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \left[\frac{3\sqrt{a}b^2\cos(fx+e)^2\log\left(128a^4\cos(fx+e)^8+256a^3b\cos(fx+e)^6+160a^2b^2\cos(fx+e)^4+32ab^3\right.\right.$$

input
```
integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/24*(3*sqrt(a)*b^2*cos(f*x + e)^2*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b
*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b
^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x
 + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)) - 8*(2*(a^2 + 3*a*b)*cos(f*x + e)^2 - a*b)*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2))/(a*b^2*f*cos(f*x + e)^2), 1/12*(3*sqrt(-a)*b^2*arct
an(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((
a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(
f*x + e)^2 + a*b^2))*cos(f*x + e)^2 - 4*(2*(a^2 + 3*a*b)*cos(f*x + e)^2 -
a*b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a*b^2*f*cos(f*x + e)^2)
]
```

3.402.6 Sympy [F]

$$\int \frac{\tan^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\tan^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
integrate(tan(f*x+e)**5/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(tan(e + f*x)**5/sqrt(a + b*sec(e + f*x)**2), x)
```

3.402.7 Maxima [F]

$$\int \frac{\tan^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\tan\left(fx+e\right)^5}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(tan(f*x + e)^5/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.402.8 Giac [F]

$$\int \frac{\tan^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\tan\left(fx+e\right)^5}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.402.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\tan(e+f\,x)^5}{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

input
```
int(tan(e + f*x)^5/(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(tan(e + f*x)^5/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.403 $\int \frac{\tan^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

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3.403.1 Optimal result

Integrand size = 25, antiderivative size = 56

$$\int \frac{\tan^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f} + \frac{\sqrt{a+b\sec^2(e+fx)}}{bf}$$

output
```
arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f/a^(1/2)+(a+b*sec(f*x+e)^2)^(1/
2)/b/f
```

3.403.2 Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \frac{\tan^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f} + \frac{\sqrt{a+b\sec^2(e+fx)}}{bf}$$

input
```
Integrate[Tan[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f) + Sqrt[a + b*Sec[e
 + f*x]^2]/(b*f)
```

3.403.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4627, 25, 354, 90, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\tan(e+fx)^3}{\sqrt{a+b\sec(e+fx)^2}}\,dx$$

$$\downarrow 4627$$

$$\frac{\int -\frac{\cos(e+fx)(1-\sec^2(e+fx))}{\sqrt{b\sec^2(e+fx)+a}}\,d\sec(e+fx)}{f}$$

$$\downarrow 25$$

$$-\frac{\int \frac{\cos(e+fx)(1-\sec^2(e+fx))}{\sqrt{b\sec^2(e+fx)+a}}\,d\sec(e+fx)}{f}$$

$$\downarrow 354$$

$$-\frac{\int \frac{\cos(e+fx)(1-\sec^2(e+fx))}{\sqrt{b\sec^2(e+fx)+a}}\,d\sec^2(e+fx)}{2f}$$

$$\downarrow 90$$

$$-\frac{\int \frac{\cos(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}\,d\sec^2(e+fx) - \frac{2\sqrt{a+b\sec^2(e+fx)}}{b}}{2f}$$

$$\downarrow 73$$

$$-\frac{\frac{2\int \frac{1}{\frac{\sec^4(e+fx)}{b}-\frac{a}{b}}\,d\sqrt{b\sec^2(e+fx)+a}}{b} - \frac{2\sqrt{a+b\sec^2(e+fx)}}{b}}{2f}$$

$$\downarrow 221$$

$$-\frac{-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{2\sqrt{a+b\sec^2(e+fx)}}{b}}{2f}$$

input
```
Int[Tan[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
-1/2*((-2*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/Sqrt[a] - (2*Sqrt[a
 + b*Sec[e + f*x]^2])/b)/f
```

3.403.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.403.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(48) = 96$.

Time = 4.97 (sec) , antiderivative size = 256, normalized size of antiderivative = 4.57

method	result
default	$\frac{\ln\left(4\cos(fx+e)\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{a}+4\cos(fx+e)a+4\sqrt{a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\right)\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,b+a^{\frac{3}{2}}+\ln\left(4\cos(fx+e)\sqrt{\frac{b+a\cos(fx}{(1+\cos(fx}\right.}{fb\sqrt{a}\sqrt{a+b\sec(fx+e)^2}}$

input

```
int(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/f/b/a^(1/2)/(a+b*sec(f*x+e)^2)^(1/2)*(ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
b+a^(3/2)+ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1
/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b*sec(f*x+e)+a^(1/2)*b*sec(f*x
+e)^2)
```

3.403.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(48) = 96$.

Time = 0.37 (sec) , antiderivative size = 328, normalized size of antiderivative = 5.86

$$\int \frac{\tan^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \left[\frac{\sqrt{a}b\log\left(128\,a^4\cos(fx+e)^8 + 256\,a^3b\cos(fx+e)^6 + 160\,a^2b^2\cos(fx+e)^4 + 32\,ab^3\cos(fx+e)^2 + b\right.}{} \right.$$

$$\left. \left. -\frac{\sqrt{-a}b\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^4+8\,ab\cos(fx+e)^2+b^2\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\left(2\,a^3\cos(fx+e)^4+3\,a^2b\cos(fx+e)^2+ab^2\right)}\right) - 4\,a\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\,abf} \right]$$

input
```
integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(sqrt(a)*b*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 16
0*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f
*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*
x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + 8*a*sqrt(
(a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a*b*f), -1/4*(sqrt(-a)*b*arctan(1
/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x
+ e)^2 + a*b^2)) - 4*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a*b*f
)]
```

3.403.6 Sympy [F]

$$\int \frac{\tan^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\tan^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
integrate(tan(f*x+e)**3/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(tan(e + f*x)**3/sqrt(a + b*sec(e + f*x)**2), x)
```

3.403.7 Maxima [F]

$$\int \frac{\tan^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\tan(fx+e)^3}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(tan(f*x + e)^3/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.403.8 Giac [F]

$$\int \frac{\tan^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\tan(fx+e)^3}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.403.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\tan(e+f\,x)^3}{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

input
```
int(tan(e + f*x)^3/(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(tan(e + f*x)^3/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.404 $\int \frac{\tan(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

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3.404.9 Mupad [B] (verification not implemented) . . . 2959

3.404.1 Optimal result

Integrand size = 23, antiderivative size = 33

$$\int \frac{\tan(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}$$

output
```
-arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f/a^(1/2)
```

3.404.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{\tan(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}$$

input
```
Integrate[Tan[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
-(ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f))
```

3.404.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4627, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\tan(e+fx)}{\sqrt{a+b\sec(e+fx)^2}}\,dx$$

$$\downarrow 4627$$

$$\frac{\int \frac{\cos(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}\,d\sec(e+fx)}{f}$$

$$\downarrow 243$$

$$\frac{\int \frac{\cos(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}\,d\sec^2(e+fx)}{2f}$$

$$\downarrow 73$$

$$\frac{\int \frac{1}{\frac{\sec^4(e+fx)}{b}-\frac{a}{b}}\,d\sqrt{b\sec^2(e+fx)+a}}{bf}$$

$$\downarrow 221$$

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}$$

input
```
Int[Tan[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
-(ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f))
```

3.404.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.404.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.27

method	result	size
derivativedivides	$-\frac{\ln\left(\frac{2a+2\sqrt{a}\sqrt{a+b\sec(fx+e)^2}}{\sec(fx+e)}\right)}{f\sqrt{a}}$	42
default	$-\frac{\ln\left(\frac{2a+2\sqrt{a}\sqrt{a+b\sec(fx+e)^2}}{\sec(fx+e)}\right)}{f\sqrt{a}}$	42

input
```
int(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f/a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*sec(f*x+e)^2)^(1/2))/sec(f*x+e))
```

3.404.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(27) = 54$.

Time $= 0.32$ (sec) , antiderivative size $= 261$, normalized size of antiderivative $= 7.91$

$$\int \frac{\tan(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \left[\frac{\log\left(128\,a^4\cos\left(fx+e\right)^8 + 256\,a^3b\cos\left(fx+e\right)^6 + 160\,a^2b^2\cos\left(fx+e\right)^4 + 32\,ab^3\cos\left(fx+e\right)^2 + b^4 - 8\right.}{8\sqrt{}} \right.$$

input
```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*c
os(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 +
 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*s
qrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(sqrt(a)*f), 1/4*sqrt(
-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^
2*b*cos(f*x + e)^2 + a*b^2))/(a*f)]
```

3.404.6 Sympy [F]

$$\int \frac{\tan(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\tan\left(e+fx\right)}{\sqrt{a+b\sec^2\left(e+fx\right)}}\,dx$$

input
```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(tan(e + f*x)/sqrt(a + b*sec(e + f*x)**2), x)
```

3.404.7 Maxima [F]

$$\int \frac{\tan(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\tan(fx+e)}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(tan(f*x + e)/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.404.8 Giac [F]

$$\int \frac{\tan(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\tan(fx+e)}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.404.9 Mupad [B] (verification not implemented)

Time = 20.53 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{\tan(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}{\sqrt{a}}\right)}{\sqrt{a}\,f}$$

input
```
int(tan(e + f*x)/(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
-atanh((a + b/cos(e + f*x)^2)^(1/2)/a^(1/2))/(a^(1/2)*f)
```

3.405 $\int \frac{\cot(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

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3.405.1 Optimal result

Integrand size = 23, antiderivative size = 70

$$\int \frac{\cot(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}f}$$

output
```
arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f/a^(1/2)-arctanh((a+b*sec(f*x+e
)^2)^(1/2)/(a+b)^(1/2))/f/(a+b)^(1/2)
```

3.405.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.80 (sec) , antiderivative size = 294, normalized size of antiderivative = 4.20

$$\int \frac{\cot(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$
$$= \frac{\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\left(-2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b}\left(1+e^{2i(e+fx)}\right)}{\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}}\right)+\sqrt{a+b}\left(\operatorname{arctanh}\left(\frac{a}{\sqrt{a}\sqrt{4be^{2}}}\right.\right.\right.}{2\sqrt{a}\sqrt{a+b}\left(1+e^{2i(e+fx)}\right)f\sqrt{a+b\sec^2(e+}}$$

input
```
Integrate[Cot[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*(-2*Sqrt[a]
*ArcTanh[(Sqrt[a + b]*(1 + E^((2*I)*(e + f*x))))/Sqrt[4*b*E^((2*I)*(e + f*
x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] + Sqrt[a + b]*(ArcTanh[(a + 2*b + a*
E^((2*I)*(e + f*x)))/(Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*
I)*(e + f*x)))^2])] + ArcTanh[(a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*I)*(e
 + f*x)))/(Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x
)))^2])])))/(2*Sqrt[a]*Sqrt[a + b]*(1 + E^((2*I)*(e + f*x)))*f*Sqrt[a + b*
Sec[e + f*x]^2])
```

3.405.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4627, 25, 354, 97, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\tan(e+fx)\sqrt{a+b\sec(e+fx)^2}}dx$$

$$\downarrow 4627$$

$$\frac{\int -\frac{\cos(e+fx)}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{f}$$

$$\downarrow 25$$

$$-\frac{\int \frac{\cos(e+fx)}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{f}$$

$$\downarrow 354$$

$$-\frac{\int \frac{\cos(e+fx)}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{2f}$$

$$\downarrow 97$$

$$-\frac{\int \frac{1}{(1-\sec^2(e+fx))\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)+\int \frac{\cos(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{2f}$$

$$\downarrow 73$$

$$-\frac{\frac{2\int \frac{1}{\frac{a+b}{b}-\frac{\sec^4(e+fx)}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}+\frac{2\int \frac{1}{\frac{\sec^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}}{2f}$$

$$\downarrow 221$$

$$-\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}}{2f}$$

input
```
Int[Cot[e + f*x]/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
-1/2*((-2*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/Sqrt[a] + (2*ArcTan
h[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/Sqrt[a + b])/f
```

3.405.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.405.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 342 vs. $2(58) = 116$.

Time = 1.46 (sec) , antiderivative size = 343, normalized size of antiderivative = 4.90

method	result
default	$\left(\ln\left(\frac{2\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{a+b}\cos(fx+e)+2\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{a+b}-2\cos(fx+e)a+2b}{\sqrt{a+b}(1+\cos(fx+e))}\right)\sqrt{a}-\ln\left(-\frac{4\left(\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{a+b}\cos(fx+e)+\right.}{-1+c}\right.\right.$

input
```
int(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f/(a+b)^(1/2)/a^(1/2)*(ln(2/(a+b)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*(a+b)^(1/2)-cos(f*x+e)*a+b)/(1+cos(f*x+e)))*a^(1/2)-ln(-4*(((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)*cos(f*x+e)+((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*(a+b)^(1/2)+cos(f*x+e)*a+b)/(-1+cos(f*x+e)))
*a^(1/2)+2*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(
1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))
*(a+b)^(1/2))*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)/(a+b*sec(f*x+e)^
2)^(1/2)*(sec(f*x+e)+1)
```

3.405.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. 2(58) = 116.

Time = 0.44 (sec) , antiderivative size = 1015, normalized size of antiderivative = 14.50

$$\int \frac{\cot(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*((a + b)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^
6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3
*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*
cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + 2*s
qrt(a + b)*a*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^
2)*cos(f*x + e)^2 + b^2 - 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*
sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 -
 2*cos(f*x + e)^2 + 1)))/((a^2 + a*b)*f), 1/8*(4*a*sqrt(-a - b)*arctan(1/2
*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) + (a + b)*sqrt(a)
*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f
*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x + e)^8 + 24*
a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(
a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)))/((a^2 + a*b)*f), -1/4*(sq
rt(-a)*(a + b)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b
^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x +
e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - sqrt(a + b)*a*log(2*((8*a^2 + 8*
a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2
*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e
)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/((a...
```

3.405.6 Sympy [F]

$$\int \frac{\cot(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(cot(e + f*x)/sqrt(a + b*sec(e + f*x)**2), x)
```

3.405.7 Maxima [F]

$$\int \frac{\cot(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot(fx+e)}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cot(f*x + e)/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.405.8 Giac [F]

$$\int \frac{\cot(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot(fx+e)}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.405.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot(e+fx)}{\sqrt{a+\frac{b}{\cos(e+fx)^2}}}\,dx$$

input
```
int(cot(e + f*x)/(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(cot(e + f*x)/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.406 $\int \frac{\cot^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

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3.406.1 Optimal result

Integrand size = 25, antiderivative size = 116

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f} + \frac{(2a+3b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2(a+b)^{3/2}f} - \frac{\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{2(a+b)f}$$

output
```
1/2*(2*a+3*b)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(3/2)/f-
arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f/a^(1/2)-1/2*cot(f*x+e)^2*(a+b*
sec(f*x+e)^2)^(1/2)/(a+b)/f
```

3.406.2 Mathematica [F]

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
Integrate[Cot[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
Integrate[Cot[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2], x]
```

3.406.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4627, 354, 114, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\tan(e+fx)^3\sqrt{a+b\sec(e+fx)^2}}dx$$

$$\downarrow 4627$$

$$\frac{\int \frac{\cos(e+fx)}{(1-\sec^2(e+fx))^2\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{f}$$

$$\downarrow 354$$

$$\frac{\int \frac{\cos(e+fx)}{(1-\sec^2(e+fx))^2\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{2f}$$

$$\downarrow 114$$

$$\frac{\frac{\sqrt{a+b\sec^2(e+fx)}}{(a+b)(1-\sec^2(e+fx))} - \frac{\int -\frac{\cos(e+fx)\left(b\sec^2(e+fx)+2a+2b\right)}{2\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{a+b}}{2f}$$

$$\downarrow 27$$

$$\frac{\frac{\int \frac{\cos(e+fx)\left(b\sec^2(e+fx)+2(a+b)\right)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{2(a+b)} + \frac{\sqrt{a+b\sec^2(e+fx)}}{(a+b)(1-\sec^2(e+fx))}}{2f}$$

$$\downarrow 174$$

$$\frac{\frac{(2a+3b)\int \frac{1}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)+2(a+b)\int \frac{\cos(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{2(a+b)} + \frac{\sqrt{a+b\sec^2(e+fx)}}{(a+b)(1-\sec^2(e+fx))}}{2f}$$

$$\downarrow 73$$

$$\frac{\frac{2(2a+3b)\int \frac{1}{\frac{a+b}{b}-\frac{\sec^4(e+fx)}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}+\frac{4(a+b)\int \frac{1}{\frac{\sec^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}}{2(a+b)}+\frac{\sqrt{a+b\sec^2(e+fx)}}{(a+b)(1-\sec^2(e+fx))}}{2f}$$

$$\downarrow 221$$

$$\frac{\frac{2(2a+3b)\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{4(a+b)\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}}{2(a+b)}+\frac{\sqrt{a+b\sec^2(e+fx)}}{(a+b)(1-\sec^2(e+fx))}}{2f}$$

input
```
Int[Cot[e + f*x]^3/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(((-4*(a + b)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/Sqrt[a] + (2*(2
*a + 3*b)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/Sqrt[a + b])/(2
*(a + b)) + Sqrt[a + b*Sec[e + f*x]^2]/((a + b)*(1 - Sec[e + f*x]^2)))/(2*
f)
```

3.406.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.406.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1943 vs. $2(98) = 196$.

Time = 1.38 (sec) , antiderivative size = 1944, normalized size of antiderivative = 16.76

method	result	size
default	Expression too large to display	1944

input
```
int(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/f/(a+b)^(5/2)/a^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e)
)^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*cs
c(f*x+e)^2+a+b)^(1/2)/((a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4
*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f
*x+e)^2+a+b)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^2)^(1/2)/((1-cos(f*x+e))^2*
csc(f*x+e)^2-1)/(1-cos(f*x+e))^2*(4*a^(5/2)*ln((a*(1-cos(f*x+e))^2*csc(f*x
+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(
1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(
f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(1/2)-a+b)/(a+b)^(1/2))*(1-cos(f*x
+e))^2-4*a^(5/2)*ln(2/(1-cos(f*x+e))^2*(-a*(1-cos(f*x+e))^2+b*(1-cos(f*x+e
))^2+(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*
(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)
*(a+b)^(1/2)*sin(f*x+e)^2+a*sin(f*x+e)^2+b*sin(f*x+e)^2))*(1-cos(f*x+e))^2
+10*ln((a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+(a
*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(
f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^
(1/2)-a+b)/(a+b)^(1/2))*(1-cos(f*x+e))^2*a^(3/2)*b-10*ln(2/(1-cos(f*x+e))^
2*(-a*(1-cos(f*x+e))^2+b*(1-cos(f*x+e))^2+(a*(1-cos(f*x+e))^4*csc(f*x+e)^4
+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-
cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(1/2)*sin(f*x+e)^2+a*sin(f*...
```

3.406.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(98) = 196$.

Time $= 0.62$ (sec) , antiderivative size $= 1550$, normalized size of antiderivative $= 13.36$

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(4*(a^2 + a*b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x +
e)^2 + ((a^2 + 2*a*b + b^2)*cos(f*x + e)^2 - a^2 - 2*a*b - b^2)*sqrt(a)*lo
g(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x
+ e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2
*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + ((2*a^2 + 3*a*b)*cos(f*x +
e)^2 - 2*a^2 - 3*a*b)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e
)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 + 4*((2*a + b)*cos(f*x + e)^4
 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^
2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/((a^3 + 2*a^2*b + a*b^2)*f*c
os(f*x + e)^2 - (a^3 + 2*a^2*b + a*b^2)*f), 1/8*(4*(a^2 + a*b)*sqrt((a*cos
(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2 - 2*((2*a^2 + 3*a*b)*cos(f
*x + e)^2 - 2*a^2 - 3*a*b)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)
^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a
*b)*cos(f*x + e)^2 + a*b + b^2)) + ((a^2 + 2*a*b + b^2)*cos(f*x + e)^2 - a
^2 - 2*a*b - b^2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x +
 e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16
*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 +
b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)))/
((a^3 + 2*a^2*b + a*b^2)*f*cos(f*x + e)^2 - (a^3 + 2*a^2*b + a*b^2)*f),...
```

3.406.6 Sympy [F]

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
integrate(cot(f*x+e)**3/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(cot(e + f*x)**3/sqrt(a + b*sec(e + f*x)**2), x)
```

3.406.7 Maxima [F]

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot\left(fx+e\right)^3}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cot(f*x + e)^3/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.406.8 Giac [F]

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot\left(fx+e\right)^3}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.406.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot(e+f\,x)^3}{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

input
```
int(cot(e + f*x)^3/(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(cot(e + f*x)^3/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.407 $\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

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3.407.1 Optimal result

Integrand size = 25, antiderivative size = 166

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f} - \frac{(8a^2+20ab+15b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8(a+b)^{5/2}f} + \frac{(4a+7b)\cot^2(e+fx)\sqrt{a+b\sec^2(e+fx)}}{8(a+b)^2f} - \frac{\cot^4(e+fx)\sqrt{a+b\sec^2(e+fx)}}{4(a+b)f}$$

output

```
-1/8*(8*a^2+20*a*b+15*b^2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(
a+b)^(5/2)/f+arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/f/a^(1/2)+1/8*(4*a+
7*b)*cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(1/2)/(a+b)^2/f-1/4*cot(f*x+e)^4*(a+b
*sec(f*x+e)^2)^(1/2)/(a+b)/f
```

3.407.2 Mathematica [F]

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
Integrate[Cot[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
Integrate[Cot[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2], x]
```

3.407.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.17, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 4627, 25, 354, 114, 27, 168, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\tan(e+fx)^5\sqrt{a+b\sec(e+fx)^2}}dx$$

$$\downarrow 4627$$

$$\frac{\int -\frac{\cos(e+fx)}{(1-\sec^2(e+fx))^3\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{f}$$

$$\downarrow 25$$

$$-\frac{\int \frac{\cos(e+fx)}{(1-\sec^2(e+fx))^3\sqrt{b\sec^2(e+fx)+a}}d\sec(e+fx)}{f}$$

$$\downarrow 354$$

$$-\frac{\int \frac{\cos(e+fx)}{(1-\sec^2(e+fx))^3\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{2f}$$

$$\downarrow 114$$

$$-\frac{\frac{\sqrt{a+b\sec^2(e+fx)}}{2(a+b)(1-\sec^2(e+fx))^2}-\frac{\int -\frac{\cos(e+fx)\left(3b\sec^2(e+fx)+4(a+b)\right)}{2\left(1-\sec^2(e+fx)\right)^2\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{2(a+b)}}{2f}$$

$$\downarrow 27$$

$$-\frac{\frac{\int \frac{\cos(e+fx)\left(3b\sec^2(e+fx)+4(a+b)\right)}{\left(1-\sec^2(e+fx)\right)^2\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{4(a+b)}+\frac{\sqrt{a+b\sec^2(e+fx)}}{2(a+b)(1-\sec^2(e+fx))^2}}{2f}$$

$$\downarrow 168$$

$$-\frac{\frac{(4a+7b)\sqrt{a+b\sec^2(e+fx)}}{(a+b)\left(1-\sec^2(e+fx)\right)}-\frac{\int -\frac{\cos(e+fx)\left(8(a+b)^2+b(4a+7b)\sec^2(e+fx)\right)}{2\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{a+b}}{4(a+b)}+\frac{\sqrt{a+b\sec^2(e+fx)}}{2(a+b)(1-\sec^2(e+fx))^2}}{2f}$$

$$\downarrow 27$$

$$-\frac{\frac{\int \frac{\cos(e+fx)\left(8(a+b)^2+b(4a+7b)\sec^2(e+fx)\right)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{2(a+b)}+\frac{(4a+7b)\sqrt{a+b\sec^2(e+fx)}}{(a+b)\left(1-\sec^2(e+fx)\right)}}{4(a+b)}+\frac{\sqrt{a+b\sec^2(e+fx)}}{2(a+b)(1-\sec^2(e+fx))^2}}{2f}$$

$$\downarrow 174$$

$$-\frac{\frac{\left(8a^2+20ab+15b^2\right)\int \frac{1}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)+8(a+b)^2\int \frac{\cos(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{2(a+b)}+\frac{(4a+7b)\sqrt{a+b\sec^2(e+fx)}}{(a+b)\left(1-\sec^2(e+fx)\right)}}{4(a+b)}+\frac{\sqrt{a+b\sec}}{2(a+b)(1-}}{2f}$$

$$\downarrow 73$$

$$-\frac{\frac{2\left(8a^2+20ab+15b^2\right)\int \frac{1}{\frac{a+b}{b}-\frac{\sec^4(e+fx)}{b}}d\sqrt{b\sec^2(e+fx)+a}}{2(a+b)}+\frac{16(a+b)^2\int \frac{1}{\frac{\sec^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}+\frac{(4a+7b)\sqrt{a+b\sec^2(e+fx)}}{(a+b)\left(1-\sec^2(e+fx)\right)}}{4(a+b)}+\frac{\sqrt{a+b\sec}}{2(a+b)(1-se}}{2f}$$

$$\downarrow 221$$

$$-\frac{\frac{2\left(8a^2+20ab+15b^2\right)\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{16(a+b)^2\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}}{2(a+b)}+\frac{(4a+7b)\sqrt{a+b\sec^2(e+fx)}}{(a+b)\left(1-\sec^2(e+fx)\right)}}{4(a+b)}+\frac{\sqrt{a+b\sec^2(e+fx)}}{2(a+b)(1-\sec^2(e+fx))}}{2f}$$

3.407. $\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

input
```
Int[Cot[e + f*x]^5/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
-1/2*(Sqrt[a + b*Sec[e + f*x]^2]/(2*(a + b)*(1 - Sec[e + f*x]^2)^2) + (((-
16*(a + b)^2*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/Sqrt[a] + (2*(8*
a^2 + 20*a*b + 15*b^2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/Sq
rt[a + b])/(2*(a + b)) + ((4*a + 7*b)*Sqrt[a + b*Sec[e + f*x]^2])/((a + b)
*(1 - Sec[e + f*x]^2)))/(4*(a + b)))/f
```

3.407.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.407.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 3595 vs. $2(144) = 288$.

Time = 1.55 (sec) , antiderivative size = 3596, normalized size of antiderivative = 21.66

method	result	size
default	Expression too large to display	3596

input
```
int(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/64/f/(a+b)^(9/2)/a^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+
e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*
csc(f*x+e)^2+a+b)^(1/2)/((a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))
^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc
(f*x+e)^2+a+b)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^2)^(1/2)/((1-cos(f*x+e))^
2*csc(f*x+e)^2-1)/(1-cos(f*x+e))^4*(252*a^(5/2)*ln((a*(1-cos(f*x+e))^2*csc
(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+(a*(1-cos(f*x+e))^4*csc(f*x+e)^4
+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-
cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(1/2)-a+b)/(a+b)^(1/2))*b^2*(1
-cos(f*x+e))^4-252*a^(5/2)*ln(2/(1-cos(f*x+e))^2*(-a*(1-cos(f*x+e))^2+b*(1
-cos(f*x+e))^2+(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x
+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+
a+b)^(1/2)*(a+b)^(1/2)*sin(f*x+e)^2+a*sin(f*x+e)^2+b*sin(f*x+e)^2))*b^2*(1
-cos(f*x+e))^4+(1-cos(f*x+e))^6*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(
f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e)
)^2*csc(f*x+e)^2+a+b)^(1/2)*(a+b)^(5/2)*a^(3/2)*csc(f*x+e)^2+(1-cos(f*x+e)
)^6*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(
1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*
(a+b)^(5/2)*a^(1/2)*b*csc(f*x+e)^2-11*a^(3/2)*(a*(1-cos(f*x+e))^4*csc(f*x+
e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+...
```

3.407.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 518 vs. $2(144) = 288$.

Time $= 1.38$ (sec) , antiderivative size $= 2257$, normalized size of antiderivative $= 13.60$

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/32*(4*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)^4 + a^3 + 3*a^2*b +
 3*a*b^2 + b^3 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)^2)*sqrt(a)
*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f
*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x + e)^8 + 24*
a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(
a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + ((8*a^3 + 20*a^2*b + 15*
a*b^2)*cos(f*x + e)^4 + 8*a^3 + 20*a^2*b + 15*a*b^2 - 2*(8*a^3 + 20*a^2*b
+ 15*a*b^2)*cos(f*x + e)^2)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f
*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*a + b)*cos(f*x
+ e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)) - 4*(3*(2*a^3 + 5*a^2*b
 + 3*a*b^2)*cos(f*x + e)^4 - (4*a^3 + 11*a^2*b + 7*a*b^2)*cos(f*x + e)^2)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^4 + 3*a^3*b + 3*a^2*b^2 +
 a*b^3)*f*cos(f*x + e)^4 - 2*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f*cos(f*x
 + e)^2 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f), 1/16*(((8*a^3 + 20*a^2*b
 + 15*a*b^2)*cos(f*x + e)^4 + 8*a^3 + 20*a^2*b + 15*a*b^2 - 2*(8*a^3 + 20*
a^2*b + 15*a*b^2)*cos(f*x + e)^2)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f
*x + e)^2 + b)*sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((
a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)) + 2*((a^3 + 3*a^2*b + 3*a*b^2 + b^
3)*cos(f*x + e)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 - 2*(a^3 + 3*a^2*b + ...
```

3.407.6 Sympy [F]

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
integrate(cot(f*x+e)**5/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(cot(e + f*x)**5/sqrt(a + b*sec(e + f*x)**2), x)
```

3.407.7 Maxima [F]

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot\left(fx+e\right)^5}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cot(f*x + e)^5/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.407.8 Giac [F]

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot\left(fx+e\right)^5}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.407.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot(e+f\,x)^5}{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

input
```
int(cot(e + f*x)^5/(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(cot(e + f*x)^5/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.408 $\int \frac{\tan^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

3.408.1 Optimal result 2981
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3.408.1 Optimal result

Integrand size = 25, antiderivative size = 173

$$\begin{aligned}\int \frac{\tan^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = &-\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}f}\\ &+\frac{(3a^2+10ab+15b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{8b^{5/2}f}\\ &-\frac{(3a+7b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{8b^2f}\\ &+\frac{\tan^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{4bf}\end{aligned}$$

output

```
1/8*(3*a^2+10*a*b+15*b^2)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^
(1/2))/b^(5/2)/f-arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/a
^(1/2)-1/8*(3*a+7*b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b^2/f+1/4*(a+b+
b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)^3/b/f
```

3.408.2 Mathematica [A] (verified)

Time = 3.64 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.33

$$\int \frac{\tan^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\left(\frac{8b^2\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{a}} - \frac{(3a^2+10ab+15b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{b}}\right)\sqrt{a+2b+a\cos(2e+2fx)}\sec(}{8\sqrt{2}b^2f\sqrt{a+b\sec^2(e+fx)}}$$
$$-\frac{(a+2b+a\cos(2(e+fx)))(3a+5b+3(a+3b)\cos(2(e+fx)))\sec^4(e+fx)\tan(e+fx)}{32b^2f\sqrt{a+b\sec^2(e+fx)}}$$

input
```
Integrate[Tan[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
-1/8*(((8*b^2*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]
])/Sqrt[a] - ((3*a^2 + 10*a*b + 15*b^2)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqr
t[a + b - a*Sin[e + f*x]^2]])/Sqrt[b])*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]*
Sec[e + f*x])/(Sqrt[2]*b^2*f*Sqrt[a + b*Sec[e + f*x]^2]) - ((a + 2*b + a*C
os[2*(e + f*x)])*(3*a + 5*b + 3*(a + 3*b)*Cos[2*(e + f*x)])*Sec[e + f*x]^4
*Tan[e + f*x])/(32*b^2*f*Sqrt[a + b*Sec[e + f*x]^2])
```

3.408.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4629, 2075, 381, 444, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\tan(e+fx)^6}{\sqrt{a+b\sec(e+fx)^2}}\,dx$$
$$\downarrow 4629$$

$$\frac{\int \frac{\tan^6(e+fx)}{(\tan^2(e+fx)+1)\sqrt{a+b(\tan^2(e+fx)+1)}}d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\tan^6(e+fx)}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{f}$$

$$\downarrow 381$$

$$\frac{\frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4b}-\frac{\int \frac{\tan^2(e+fx)\left((3a+7b)\tan^2(e+fx)+3(a+b)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{4b}}{f}$$

$$\downarrow 444$$

$$\frac{\frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4b}-\frac{\frac{(3a+7b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{\int \frac{\left(3a^2+10ba+15b^2\right)\tan^2(e+fx)+(a+b)(3a+7b)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2b}}{4b}}{f}$$

$$\downarrow 398$$

$$\frac{\frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4b}-\frac{\frac{(3a+7b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{\left(3a^2+10ab+15b^2\right)\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-8b^2\int \frac{}{\left(\tan^2(e+fx)+\right.}}{2b}}{4b}}{f}$$

$$\downarrow 224$$

$$\frac{\frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4b}-\frac{\frac{(3a+7b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{\left(3a^2+10ab+15b^2\right)\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}-8b^2\int \frac{}{(t}}{2b}}{4b}}{f}$$

$$\downarrow 219$$

$$\frac{\frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4b}-\frac{\frac{(3a+7b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{\frac{\left(3a^2+10ab+15b^2\right)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}}-8b^2\int \frac{}{\left(\tan^2(e+fx)\right.}}{2b}}{4b}}{f}$$

$$\downarrow 291$$

3.408. $\int \frac{\tan^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}dx$

$$\frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4b} - \frac{(3a+7b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b} - \frac{\frac{\left(3a^2+10ab+15b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}} - 8b^2\int \frac{1}{\frac{a\tan^2(e+}{b\tan^2(e+fx}}}{2b}}{4b}$$
$$f$$

$$\downarrow 216$$

$$\frac{\tan^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{4b} - \frac{(3a+7b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b} - \frac{\frac{\left(3a^2+10ab+15b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}} - 8b^2\arctan\left(\frac{\sqrt{a}}{\sqrt{a+b}}\right.}{2b}}{4b}$$
$$f$$

input
```
Int[Tan[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
((Tan[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(4*b) - (-1/2*((-8*b^2*Ar
cTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/Sqrt[a] + ((3
*a^2 + 10*a*b + 15*b^2)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[
e + f*x]^2]])/Sqrt[b])/b + ((3*a + 7*b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e
+ f*x]^2])/(2*b))/(4*b))/f
```

3.408.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 381
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
+ 1)/(b*d*(m + 2*(p + q) + 1))), x] - Simp[e^4/(b*d*(m + 2*(p + q) + 1))
Int[(e*x)^(m - 4)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*c*(m - 3) + (a*d*(m +
2*q - 1) + b*c*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 3] && IntBinomialQ[a, b, c, d, e, m, 2
, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.408.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1929 vs. 2(151) = 302.

Time = 17.68 (sec) , antiderivative size = 1930, normalized size of antiderivative = 11.16

method	result	size
default	Expression too large to display	1930

input
```
int(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/16/f/b^(9/2)/(-a)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2)*(-16*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)-4*sin(f*x+e)*a)*b^(9/2)-16*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+
4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*b
^(9/2)*sec(f*x+e)-18*(-a)^(1/2)*b^(7/2)*a*tan(f*x+e)-6*(-a)^(1/2)*b^(5/2)*
a^2*tan(f*x+e)-18*(-a)^(1/2)*b^(9/2)*tan(f*x+e)*sec(f*x+e)^2+3*(-a)^(1/2)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*a^2*b^2+10*(-a)^(1/2)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*a*b^3+15*(-a)^(1/2)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*b^4+3*(-a)^(1/2)*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*a^2*b^2+10*(-a)^(1/2)*((b+a...
```

3.408.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 342 vs. 2(151) = 302.

Time = 1.39 (sec) , antiderivative size = 1673, normalized size of antiderivative = 9.67

$$\int \frac{\tan^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/32*(4*sqrt(-a)*b^3*cos(f*x + e)^3*log(128*a^4*cos(f*x + e)^8 - 256*(a^
4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)
^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*
a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2
*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3
 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 +
 b)/cos(f*x + e)^2)*sin(f*x + e)) - (3*a^3 + 10*a^2*b + 15*a*b^2)*sqrt(b)*
cos(f*x + e)^3*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos
(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((
a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4
) - 4*(2*a*b^2 - 3*(a^2*b + 3*a*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a*b^3*f*cos(f*x + e)^3), -1/16*(2*sq
rt(-a)*b^3*cos(f*x + e)^3*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*c
os(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 2
8*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*
b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x
+ e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b +
7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)*sin(f*x + e)) - (3*a^3 + 10*a^2*b + 15*a*b^2)*sqrt(-b)*arctan(-1/2
*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x +...
```

3.408.6 Sympy [F]

$$\int \frac{\tan^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\tan^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
integrate(tan(f*x+e)**6/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(tan(e + f*x)**6/sqrt(a + b*sec(e + f*x)**2), x)
```

3.408.7 Maxima [F]

$$\int \frac{\tan^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\tan(fx+e)^6}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(tan(f*x + e)^6/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.408.8 Giac [F]

$$\int \frac{\tan^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\tan(fx+e)^6}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.408.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\tan(e+fx)^6}{\sqrt{a+\frac{b}{\cos(e+fx)^2}}}\,dx$$

input
```
int(tan(e + f*x)^6/(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(tan(e + f*x)^6/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.409 $\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

3.409.1 Optimal result . . . 2989
3.409.2 Mathematica [A] (verified) . . . 2990
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3.409.8 Giac [F] . . . 2996
3.409.9 Mupad [F(-1)] . . . 2996

3.409.1 Optimal result

Integrand size = 25, antiderivative size = 120

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}f} - \frac{(a+3b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2b^{3/2}f} + \frac{\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2bf}$$

output

```
-1/2*(a+3*b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b^(3/2
)/f+arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/a^(1/2)+1/2*(a
+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/b/f
```

3.409.2 Mathematica [A] (verified)

Time = 2.54 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.63

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$
$$= \frac{\left(\frac{2b\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{a}} - \frac{(a+3b)\operatorname{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{b}}\right)\sqrt{a+2b+a\cos(2e+2fx)}\sec(e+fx)}{2\sqrt{2}bf\sqrt{a+b\sec^2(e+fx)}}$$
$$+ \frac{(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\tan(e+fx)}{4bf\sqrt{a+b\sec^2(e+fx)}}$$

input
```
Integrate[Tan[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(((2*b*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]])/Sqrt
[a] - ((a + 3*b)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x
]^2]])/Sqrt[b])*Sqrt[a + 2*b + a*Cos[2*e + 2*f*x]]*Sec[e + f*x])/(2*Sqrt[2
]*b*f*Sqrt[a + b*Sec[e + f*x]^2]) + ((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e
+ f*x]^2*Tan[e + f*x])/(4*b*f*Sqrt[a + b*Sec[e + f*x]^2])
```

3.409.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4629, 2075, 381, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\tan(e+fx)^4}{\sqrt{a+b\sec(e+fx)^2}}\,dx$$
$$\downarrow 4629$$

$$\frac{\int \frac{\tan^4(e+fx)}{(\tan^2(e+fx)+1)\sqrt{a+b(\tan^2(e+fx)+1)}}d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\tan^4(e+fx)}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{f}$$

$$\downarrow 381$$

$$\frac{\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b} - \frac{\int \frac{(a+3b)\tan^2(e+fx)+a+b}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2b}}{f}$$

$$\downarrow 398$$

$$\frac{\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b} - \frac{(a+3b)\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx) - 2b\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2b}}{f}$$

$$\downarrow 224$$

$$\frac{\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b} - \frac{(a+3b)\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}} - 2b\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2b}}{f}$$

$$\downarrow 219$$

$$\frac{\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b} - \frac{\frac{(a+3b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}} - 2b\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{2b}}{f}$$

$$\downarrow 291$$

$$\frac{\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b} - \frac{\frac{(a+3b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}} - 2b\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{2b}}{f}$$

$$\downarrow 216$$

$$\frac{\frac{\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b} - \frac{\frac{(a+3b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}} - \frac{2b\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}}}{2b}}{f}$$

3.409. $\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}dx$

input
```
Int[Tan[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(-1/2*((-2*b*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]
)/Sqrt[a] + ((a + 3*b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e
 + f*x]^2]])/Sqrt[b])/b + (Tan[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(2
*b))/f
```

3.409.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 381
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(b*d*(m + 2*(p + q) + 1))), x] - Simp[e^4/(b*d*(m + 2*(p + q) + 1))
Int[(e*x)^(m - 4)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*c*(m - 3) + (a*d*(m +
2*q - 1) + b*c*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 3] && IntBinomialQ[a, b, c, d, e, m, 2
, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.409.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1333 vs. $2(102) = 204$.

Time = 12.66 (sec) , antiderivative size = 1334, normalized size of antiderivative = 11.12

method	result	size
default	Expression too large to display	1334

input
```
int(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/f/(-a)^(1/2)/b^(5/2)/(a+b*sec(f*x+e)^2)^(1/2)*(4*((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^
2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)-4*sin(f*x+e)*a)*b^(5/2)+4*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*l
n(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-
a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*b^(5/
2)*sec(f*x+e)+2*(-a)^(1/2)*b^(3/2)*a*tan(f*x+e)-(-a)^(1/2)*((b+a*cos(f*x+e
)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)+a+b)/(sin(f*x+e)+1))*a*b-3*(-a)^(1/2)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)+a+b)/(sin(f*x+e)+1))*b^2-(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)*ln(4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1
/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)-a-b)/(sin(f*x+e)-1))*a*b-3*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)*ln(4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*c
os(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
-a-b)/(sin(f*x+e)-1))*b^2+2*(-a)^(1/2)*b^(5/2)*tan(f*x+e)*sec(f*x+e)^2-(-a
)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(-4*(-((b+a*cos(f...
```

3.409.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(102) = 204$.

Time = 0.67 (sec) , antiderivative size = 1507, normalized size of antiderivative = 12.56

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*(sqrt(-a)*b^2*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a
^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 +
a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b
^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*c
os(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a
^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)*sin(f*x + e)) - (a^2 + 3*a*b)*sqrt(b)*cos(f*x + e)*log(((a^
2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 - 4*((a - b
)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*a*b*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a*b^2*f*cos(f*x + e)), -1/8*
(sqrt(-a)*b^2*cos(f*x + e)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*
cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 -
28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a
*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x
 + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b +
 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)*sin(f*x + e)) + 2*(a^2 + 3*a*b)*sqrt(-b)*arctan(-1/2*((a - b)*cos
(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e)))*cos(f*x + e) - 4*...
```

3.409.6 Sympy [F]

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\tan^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
integrate(tan(f*x+e)**4/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(tan(e + f*x)**4/sqrt(a + b*sec(e + f*x)**2), x)
```

3.409.7 Maxima [F]

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\tan(fx+e)^4}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(tan(f*x + e)^4/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.409.8 Giac [F]

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\tan(fx+e)^4}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.409.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\tan(e+f\,x)^4}{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

input
```
int(tan(e + f*x)^4/(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(tan(e + f*x)^4/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.410 $\int \frac{\tan^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

3.410.1 Optimal result . . . 2997
3.410.2 Mathematica [C] (verified) . . . 2997
3.410.3 Rubi [A] (verified) . . . 2998
3.410.4 Maple [B] (warning: unable to verify) . . . 3000
3.410.5 Fricas [B] (verification not implemented) . . . 3001
3.410.6 Sympy [F] . . . 3002
3.410.7 Maxima [F] . . . 3002
3.410.8 Giac [F] . . . 3002
3.410.9 Mupad [F(-1)] . . . 3003

3.410.1 Optimal result

Integrand size = 25, antiderivative size = 80

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}f} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{b}f}$$

output
```
-arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/a^(1/2)+arctanh(b
^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/b^(1/2)
```

3.410.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.48 (sec) , antiderivative size = 296, normalized size of antiderivative = 3.70

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$
$$= \frac{i\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}\left(-2\sqrt{a}\arctan\left(\frac{\sqrt{b}\left(-1+e^{2i(e+fx)}\right)}{\sqrt{4be^{2i(e+fx)}+a\left(1+e^{2i(e+fx)}\right)^2}}\right)+\sqrt{b}\operatorname{arctanh}\left(\frac{a+2b+a}{\sqrt{a}\sqrt{4be^{2i(e+fx)}}}\right.\right.}{2\sqrt{a}\sqrt{b}\left(1+e^{2i(e+fx)}\right)f\sqrt{a+b\sec^2(e+fx)}}$$

input
```
Integrate[Tan[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
((I/2)*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]*(-2*S
qrt[a]*ArcTan[(Sqrt[b]*(-1 + E^((2*I)*(e + f*x))))/Sqrt[4*b*E^((2*I)*(e +
f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] + Sqrt[b]*ArcTanh[(a + 2*b + a*E^(
(2*I)*(e + f*x)))/(Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*
(e + f*x)))^2])] - Sqrt[b]*ArcTanh[(a + a*E^((2*I)*(e + f*x)) + 2*b*E^((2*
I)*(e + f*x)))/(Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e
+ f*x)))^2])]))/(Sqrt[a]*Sqrt[b]*(1 + E^((2*I)*(e + f*x)))*f*Sqrt[a + b*Se
c[e + f*x]^2])
```

3.410.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4629, 2075, 385, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\tan(e+fx)^2}{\sqrt{a+b\sec(e+fx)^2}}\,dx$$

$$\downarrow 4629$$

$$\frac{\int \frac{\tan^2(e+fx)}{(\tan^2(e+fx)+1)\sqrt{a+b(\tan^2(e+fx)+1)}}\,d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\tan^2(e+fx)}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}\,d\tan(e+fx)}{f}$$

$$\downarrow 385$$

$$\frac{\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}\,d\tan(e+fx) - \int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}\,d\tan(e+fx)}{f}$$

$$\downarrow 224$$

$$\frac{\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}\,d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}} - \int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}\,d\tan(e+fx)}{f}$$

$$\downarrow 219$$

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}} - \int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}} d\tan(e+fx)$$
$$f$$

$$\downarrow 291$$

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}} - \int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1} d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}$$
$$f$$

$$\downarrow 216$$

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}} - \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}}$$
$$f$$

input
```
Int[Tan[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(-(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/Sqrt[a])
+ ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/Sqrt[b])/
f
```

3.410.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 385
```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2),
x_Symbol] :> Simp[e^2/b   Int[(e*x)^(m - 2)*(c + d*x^2)^q, x], x] - Simp[a*
(e^2/b)   Int[(e*x)^(m - 2)*((c + d*x^2)^q/(a + b*x^2)), x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LeQ[2, m, 3] && IntBinomial
Q[a, b, c, d, e, m, 2, -1, q, x]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.410.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(68) = 136$.

Time = 8.97 (sec) , antiderivative size = 344, normalized size of antiderivative = 4.30

method	result
default	$\frac{\left(\ln\left(\frac{-4\sqrt{\frac{b+a\cos(fx+e)}{(1+\cos(fx+e))^2}}\sqrt{b}\cos(fx+e)+4\sin(fx+e)a-4\sqrt{b}\sqrt{\frac{b+a\cos(fx+e)}{(1+\cos(fx+e))^2}}-4a-4b}{\sin(fx+e)-1}\right)\sqrt{-a}+\ln\left(-\frac{4\left(-\sqrt{\frac{b+a\cos(fx+e)}{(1+\cos(fx+e))^2}}\sqrt{b}\cos(fx+e)-\right.\right.\right.}{\sin($

input
```
int(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f/(-a)^(1/2)/b^(1/2)*(ln(4*(-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)-a-b)/(sin(f*x+e)-1))*(-a)^(1/2)+ln(-4*(-((b+a*cos(f*x+e)^2)/(
1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+sin(f*x+e)*a-b^(1/2)*((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)+a+b)/(sin(f*x+e)+1))*(-a)^(1/2)-2*ln(4*(-
a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/
2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*b^(1/2))*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2)*(sec(f*
x+e)+1)
```

3.410.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(68) = 136$.

Time = 0.50 (sec) , antiderivative size = 1259, normalized size of antiderivative = 15.74

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*(sqrt(-a)*b*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x +
 e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b
+ 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos
(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 +
 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2
- b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*
sin(f*x + e)) - 2*a*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a
*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*s
qrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/c
os(f*x + e)^4))/(a*b*f), 1/8*(4*a*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x +
e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e))) - sqrt(-a)*b*log(128*a^4*co
s(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5
*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 -
32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x
+ e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*
cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)))/(a*b*f), 1/4*(sqr
t(a)*b*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (
a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/co...
```

3.410.6 Sympy [F]

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\tan^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
integrate(tan(f*x+e)**2/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(tan(e + f*x)**2/sqrt(a + b*sec(e + f*x)**2), x)
```

3.410.7 Maxima [F]

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\tan(fx+e)^2}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(tan(f*x + e)^2/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.410.8 Giac [F]

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\tan(fx+e)^2}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.410.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\tan(e+f\,x)^2}{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

input
```
int(tan(e + f*x)^2/(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(tan(e + f*x)^2/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.411 $\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

3.411.1 Optimal result . . . 3004
3.411.2 Mathematica [B] (verified) . . . 3004
3.411.3 Rubi [A] (verified) . . . 3005
3.411.4 Maple [B] (verified) . . . 3006
3.411.5 Fricas [B] (verification not implemented) . . . 3007
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3.411.7 Maxima [B] (verification not implemented) . . . 3008
3.411.8 Giac [F] . . . 3009
3.411.9 Mupad [F(-1)] . . . 3009

3.411.1 Optimal result

Integrand size = 16, antiderivative size = 39

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}f}$$

output
```
arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/a^(1/2)
```

3.411.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 87 vs. 2(39) = 78.

Time = 0.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.23

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)\sqrt{a+2b+a\cos(2e+2fx)}\sec(e+fx)}{\sqrt{2}\sqrt{a}f\sqrt{a+b\sec^2(e+fx)}}$$

input
```
Integrate[1/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*Sqrt[a + 2*
b + a*Cos[2*e + 2*f*x]]*Sec[e + f*x])/(Sqrt[2]*Sqrt[a]*f*Sqrt[a + b*Sec[e
+ f*x]^2])
```

3.411.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 4616, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)^2}}dx$$

$$\downarrow 4616$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{f}$$

$$\downarrow 291$$

$$\frac{\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{f}$$

$$\downarrow 216$$

$$\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}f}$$

input
```
Int[1/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/(Sqrt[a]*f)
```

3.411.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

3.411.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(33) = 66$.

Time = 3.32 (sec) , antiderivative size = 138, normalized size of antiderivative = 3.54

method	result	size
default	$\frac{\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\ln\left(4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)+4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a\right)(\sec(fx+e)+1)}{f\sqrt{-a}\sqrt{a+b\sec(fx+e)^2}}$	138

input
```
int(1/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f/(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)/(a+b*sec(f*x+e)^2)^
(1/2)*(sec(f*x+e)+1)
```

3.411.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(33) = 66$.

Time = 0.37 (sec) , antiderivative size = 408, normalized size of antiderivative = 10.46

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \left[-\frac{\sqrt{-a}\log\left(128\,a^4\cos(fx+e)^8 - 256\,(a^4-a^3b)\cos(fx+e)^6 + 32\,(5\,a^4-14\,a^3b+5\,a^2b^2)\cos(fx+e)\right.}{} \right.$$

$$\left. \left. -\frac{\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^5-8\,(a^2-ab)\cos(fx+e)^3+(a^2-6\,ab+b^2)\cos(fx+e)\right)\sqrt{a}\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}}{4\left(2\,a^3\cos(fx+e)^4-a^2b+ab^2-(a^3-3\,a^2b)\cos(fx+e)^2\right)\sin(fx+e)}\right)}{4\sqrt{a}f} \right]$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)
^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 7
0*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*
x + e)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*
(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b
^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin
(f*x + e))/(a*f), -1/4*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*co
s(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 -
 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))/(sqrt(a)*f)]
```

3.411.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*sec(e + f*x)**2), x)
```

3.411.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 992 vs. $2(33) = 66$.

Time = 0.42 (sec) , antiderivative size = 992, normalized size of antiderivative = 25.44

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*(arctan2(2*a*sin(2*f*x + 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*
f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b
)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x +
2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e)
 + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*sin(1/2*arctan2(a*sin(4
*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*
b)*cos(2*f*x + 2*e) + a)), 2*a*cos(2*f*x + 2*e) + 2*(a^2*cos(4*f*x + 4*e)^
2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 +
4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2
)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*co
s(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*cos(1/2*a
rctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*
e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) + 2*a + 4*b) - arctan2(2*(a^2*cos(
4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*
x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 +
4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f
*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt
(a)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*c
os(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)), 2*(a^2*cos(4*f*x + 4
*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*...
```

3.411.8 Giac [F]

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.411.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{\cos(e+fx)^2}}}\,dx$$

input
```
int(1/(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(1/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.412 $\int \frac{\cot^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

3.412.1 Optimal result . . . 3010
3.412.2 Mathematica [A] (verified) . . . 3010
3.412.3 Rubi [A] (verified) . . . 3011
3.412.4 Maple [B] (verified) . . . 3013
3.412.5 Fricas [B] (verification not implemented) . . . 3014
3.412.6 Sympy [F] . . . 3015
3.412.7 Maxima [F] . . . 3015
3.412.8 Giac [F] . . . 3016
3.412.9 Mupad [F(-1)] . . . 3016

3.412.1 Optimal result

Integrand size = 25, antiderivative size = 74

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}f} - \frac{\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{(a+b)f}$$

output
```
-arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/a^(1/2)-cot(f*x+e
)*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)/f
```

3.412.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.72

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = -\frac{\sqrt{a+2b+a\cos(2(e+fx))}\sec(e+fx)\left((a+b)\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)+\sqrt{a}\csc(e+fx)\sqrt{a+b-}\right.}{\sqrt{2}\sqrt{a}(a+b)f\sqrt{a+b\sec^2(e+fx)}}$$

input
```
Integrate[Cot[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
-((Sqrt[a + 2*b + a*Cos[2*(e + f*x)]]*Sec[e + f*x]*((a + b)*ArcTan[(Sqrt[a
]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]] + Sqrt[a]*Csc[e + f*x]*Sqr
t[a + b - a*Sin[e + f*x]^2]))/(Sqrt[2]*Sqrt[a]*(a + b)*f*Sqrt[a + b*Sec[e
+ f*x]^2]))
```

3.412.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4629, 2075, 382, 25, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\tan(e+fx)^2\sqrt{a+b\sec(e+fx)^2}}dx$$

$$\downarrow 4629$$

$$\frac{\int \frac{\cot^2(e+fx)}{(\tan^2(e+fx)+1)\sqrt{a+b(\tan^2(e+fx)+1)}}d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\cot^2(e+fx)}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{f}$$

$$\downarrow 382$$

$$\frac{\int -\frac{\frac{a+b}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a+b} - \frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{f}$$

$$\downarrow 25$$

$$\frac{-\frac{\int \frac{\frac{a+b}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a+b} - \frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{f}$$

$$\downarrow 27$$

$$\frac{-\int \frac{1}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx) - \frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{f}$$

$$\downarrow 291$$

$$\frac{-\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}} - \frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{f}$$

$$\downarrow 216$$

$$\frac{-\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}} - \frac{\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{f}$$

input
```
Int[Cot[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(-(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/Sqrt[a])
- (Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(a + b))/f
```

3.412.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.412.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 506 vs. $2(66) = 132$.

Time = 4.12 (sec) , antiderivative size = 507, normalized size of antiderivative = 6.85

method	result
default	$-\frac{\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\ln\left(4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)+4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a\right)a+\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\ln\left(4\sqrt{-}\right.$

input
```
int(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f/(a+b)/(-a)^(1/2)/(a+b*sec(f*x+e)^2)^(1/2)*(((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*
sin(f*x+e)*a)*a+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2
)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*b+(-a)^(1/2)*a*cot
(f*x+e)+sec(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(
1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*(
(b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a+((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(
f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)-4*sin(f*x+e)*a)*b*sec(f*x+e)+(-a)^(1/2)*b*sec(f*x+e)*csc(f*x+e)
)
```

3.412.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(66) = 132$.

Time $= 0.44$ (sec) , antiderivative size $= 525$, normalized size of antiderivative $= 7.09$

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \left[-\frac{\sqrt{-a}(a+b)\log\left(128\,a^4\cos(fx+e)^8 - 256\,(a^4-a^3b)\cos(fx+e)^6 + 32\,(5\,a^4-14\,a^3b+5\,a^2b^2)\cos(\right.}{} \right.$$

input
```
integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*(sqrt(-a)*(a + b)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos
(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*
a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^
3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x +
e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*
a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2)*sin(f*x + e))*sin(f*x + e) + 8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
 + e)^2)*cos(f*x + e))/((a^2 + a*b)*f*sin(f*x + e)), 1/4*((a + b)*sqrt(a)*
arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6
*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^
2)*sin(f*x + e)))*sin(f*x + e) - 4*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2)*cos(f*x + e))/((a^2 + a*b)*f*sin(f*x + e))]
```

3.412.6 Sympy [F]

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
integrate(cot(f*x+e)**2/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(cot(e + f*x)**2/sqrt(a + b*sec(e + f*x)**2), x)
```

3.412.7 Maxima [F]

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot(fx+e)^2}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cot(f*x + e)^2/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.412.8 Giac [F]

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot\left(fx+e\right)^2}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.412.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot(e+f\,x)^2}{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

input
```
int(cot(e + f*x)^2/(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(cot(e + f*x)^2/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.413 $\int \frac{\cot^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

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3.413.2 Mathematica [A] (verified) . . . 3017
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3.413.1 Optimal result

Integrand size = 25, antiderivative size = 119

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}f} + \frac{(3a+5b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)^2 f} - \frac{\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3(a+b)f}$$

output
```
arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/a^(1/2)+1/3*(3*a+5
*b)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)^2/f-1/3*cot(f*x+e)^3*(a+b+
b*tan(f*x+e)^2)^(1/2)/(a+b)/f
```

3.413.2 Mathematica [A] (verified)

Time = 1.49 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.41

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)\sqrt{a+2b+a\cos(2e+2fx)}\sec(e+fx)}{\sqrt{2}\sqrt{a}f\sqrt{a+b\sec^2(e+fx)}} - \frac{(a+2b+a\cos(2(e+fx)))(-a-2b+(2a+3b)\cos(2(e+fx)))\csc^3(e+fx)\sec(e+fx)}{6(a+b)^2 f\sqrt{a+b\sec^2(e+fx)}}$$

input
```
Integrate[Cot[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*Sqrt[a + 2*
b + a*Cos[2*e + 2*f*x]]*Sec[e + f*x])/(Sqrt[2]*Sqrt[a]*f*Sqrt[a + b*Sec[e
+ f*x]^2]) - ((a + 2*b + a*Cos[2*(e + f*x)])*(-a - 2*b + (2*a + 3*b)*Cos[2
*(e + f*x)])*Csc[e + f*x]^3*Sec[e + f*x])/(6*(a + b)^2*f*Sqrt[a + b*Sec[e
+ f*x]^2])
```

3.413.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4629, 2075, 382, 25, 445, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\tan(e+fx)^4\sqrt{a+b\sec(e+fx)^2}}dx$$

$$\downarrow 4629$$

$$\frac{\int \frac{\cot^4(e+fx)}{(\tan^2(e+fx)+1)\sqrt{a+b(\tan^2(e+fx)+1)}}d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\cot^4(e+fx)}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{f}$$

$$\downarrow 382$$

$$\frac{\int -\frac{\cot^2(e+fx)\left(2b\tan^2(e+fx)+3a+5b\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{3(a+b)} - \frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)}}{f}$$

$$\downarrow 25$$

$$\frac{-\frac{\int \frac{\cot^2(e+fx)\left(2b\tan^2(e+fx)+3a+5b\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{3(a+b)}-\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)}}{f}$$

$$\downarrow 445$$

$$\frac{-\frac{\int \frac{3(a+b)^2}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a+b}-\frac{(3a+5b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{3(a+b)}-\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)}}{f}$$

$$\downarrow 27$$

$$\frac{-\frac{-3(a+b)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-\frac{(3a+5b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{3(a+b)}-\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)}}{f}$$

$$\downarrow 291$$

$$\frac{-\frac{-3(a+b)\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}-\frac{(3a+5b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{3(a+b)}-\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)}}{f}$$

$$\downarrow 216$$

$$\frac{-\frac{-\frac{3(a+b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}}-\frac{(3a+5b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{3(a+b)}-\frac{\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)}}{f}$$

input
```
Int[Cot[e + f*x]^4/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(-1/3*(Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(a + b) - ((-3*(a +
b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/Sqrt[a]
- ((3*a + 5*b)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(a + b))/(3*(a
 + b)))/f
```

3.413. $\int \frac{\cot^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

3.413.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.413.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 890 vs. $2(105) = 210$.

Time = 5.28 (sec) , antiderivative size = 891, normalized size of antiderivative = 7.49

method	result	size
default	Expression too large to display	891

input

```
int(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-1/3/f/(a+b)^2/(-a)^(1/2)*(3*sin(f*x+e)^3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2
)^(1/2)*a^2*cos(f*x+e)+6*sin(f*x+e)^3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*a*b*cos(f*x+e)+3*sin(f*x+e)^3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x
+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*
b^2*cos(f*x+e)+3*sin(f*x+e)^3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f
*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2+
6*sin(f*x+e)^3*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin
(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b+3*sin(f*x+e)^3*
ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(
-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+
a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^2-4*cos(f*x+e)^4*(-a)^(1/2)*a^2-
6*cos(f*x+e)^4*(-a)^(1/2)*a*b+3*cos(f*x+e)^2*(-a)^(1/2)*a^2+cos(f*x+e)^2*(
-a)^(1/2)*a*b-6*cos(f*x+e)^2*(-a)^(1/2)*b^2+3*(-a)^(1/2)*a*b+5*(-a)^(1/...
```

3.413.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(105) = 210$.

Time = 0.68 (sec) , antiderivative size = 723, normalized size of antiderivative = 6.08

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

$$= \left[-\frac{3\left((a^2+2ab+b^2)\cos(fx+e)^2 - a^2 - 2ab - b^2\right)\sqrt{-a}\log\left(128a^4\cos(fx+e)^8 - 256\left(a^4 - a^3b\right)\cos(\right.\right.$$

$$\left. -\frac{3\left((a^2+2ab+b^2)\cos(fx+e)^2 - a^2 - 2ab - b^2\right)\sqrt{a}\arctan\left(\frac{\left(8a^2\cos(fx+e)^5 - 8\left(a^2-ab\right)\cos(fx+e)^3 + (a^2-6ab+\right.}{4\left(2a^3\cos(fx+e)^4 - a^2b + ab^2 - (a^3-3a^2b)\right.}\right.}{12\left((a^3+2a^2b+ab^2)f\cos(\right.}\right]$$

input
```
integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/24*(3*((a^2 + 2*a*b + b^2)*cos(f*x + e)^2 - a^2 - 2*a*b - b^2)*sqrt(-a
)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^
4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 2
8*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*
(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a
^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x +
 e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))*si
n(f*x + e) - 8*(2*(2*a^2 + 3*a*b)*cos(f*x + e)^3 - (3*a^2 + 5*a*b)*cos(f*x
 + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^3 + 2*a^2*b + a*b
^2)*f*cos(f*x + e)^2 - (a^3 + 2*a^2*b + a*b^2)*f)*sin(f*x + e)), -1/12*(3*
((a^2 + 2*a*b + b^2)*cos(f*x + e)^2 - a^2 - 2*a*b - b^2)*sqrt(a)*arctan(1/
4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^
2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a
^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*
x + e)))*sin(f*x + e) - 4*(2*(2*a^2 + 3*a*b)*cos(f*x + e)^3 - (3*a^2 + 5*a
*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^3 + 2*
a^2*b + a*b^2)*f*cos(f*x + e)^2 - (a^3 + 2*a^2*b + a*b^2)*f)*sin(f*x + e))
]
```

3.413.6 Sympy [F]

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
integrate(cot(f*x+e)**4/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(cot(e + f*x)**4/sqrt(a + b*sec(e + f*x)**2), x)
```

3.413.7 Maxima [F]

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot(fx+e)^4}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cot(f*x + e)^4/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.413.8 Giac [F]

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot(fx+e)^4}{\sqrt{b\sec(fx+e)^2+a}}\,dx$$

input
```
integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.413.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot(e+f\,x)^4}{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

input
```
int(cot(e + f*x)^4/(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(cot(e + f*x)^4/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.414 $\int \frac{\cot^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$

3.414.1 Optimal result . . . 3025
3.414.2 Mathematica [A] (verified) . . . 3026
3.414.3 Rubi [A] (verified) . . . 3026
3.414.4 Maple [B] (verified) . . . 3029
3.414.5 Fricas [B] (verification not implemented) . . . 3030
3.414.6 Sympy [F] . . . 3031
3.414.7 Maxima [F] . . . 3032
3.414.8 Giac [F] . . . 3032
3.414.9 Mupad [F(-1)] . . . 3032

3.414.1 Optimal result

Integrand size = 25, antiderivative size = 172

$$\begin{aligned}\int \frac{\cot^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = &-\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{\sqrt{a}f}\\ &-\frac{(15a^2+40ab+33b^2)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^3f}\\ &+\frac{(5a+9b)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15(a+b)^2f}\\ &-\frac{\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5(a+b)f}\end{aligned}$$

output

```
-arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/f/a^(1/2)-1/15*(15*
a^2+40*a*b+33*b^2)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)^3/f+1/15*(5
*a+9*b)*cot(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)^2/f-1/5*cot(f*x+e)^5
*(a+b+b*tan(f*x+e)^2)^(1/2)/(a+b)/f
```

3.414.2 Mathematica [A] (verified)

Time = 3.34 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.16

$$\int \frac{\cot^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$
$$= -\frac{\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)\sqrt{a+2b+a\cos(2e+2fx)}\sec(e+fx)}{\sqrt{2}\sqrt{a}f\sqrt{a+b\sec^2(e+fx)}}$$
$$-\frac{(a+2b+a\cos(2(e+fx)))\csc(e+fx)\left(23a^2+60ab+45b^2-(11a^2+26ab+15b^2)\csc^2(e+fx)+3(a\right.}{30(a+b)^3f\sqrt{a+b\sec^2(e+fx)}}$$

input
```
Integrate[Cot[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
-((ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*Sqrt[a +
2*b + a*Cos[2*e + 2*f*x]]*Sec[e + f*x])/(Sqrt[2]*Sqrt[a]*f*Sqrt[a + b*Sec[
e + f*x]^2])) - ((a + 2*b + a*Cos[2*(e + f*x)])*Csc[e + f*x]*(23*a^2 + 60*
a*b + 45*b^2 - (11*a^2 + 26*a*b + 15*b^2)*Csc[e + f*x]^2 + 3*(a + b)^2*Csc
[e + f*x]^4)*Sec[e + f*x])/(30*(a + b)^3*f*Sqrt[a + b*Sec[e + f*x]^2])
```

3.414.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4629, 2075, 382, 25, 445, 445, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{\tan(e+fx)^6\sqrt{a+b\sec(e+fx)^2}}dx$$
$$\downarrow 4629$$
$$\frac{\int \frac{\cot^6(e+fx)}{(\tan^2(e+fx)+1)\sqrt{a+b(\tan^2(e+fx)+1)}}d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\cot^6(e+fx)}{(\tan^2(e+fx)+1)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{f}$$

$$\downarrow 382$$

$$\frac{\int -\frac{\cot^4(e+fx)\left(4b\tan^2(e+fx)+5a+9b\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{5(a+b)} - \frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{5(a+b)}}{f}$$

$$\downarrow 25$$

$$\frac{-\frac{\int \frac{\cot^4(e+fx)\left(4b\tan^2(e+fx)+5a+9b\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{5(a+b)} - \frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{5(a+b)}}{f}$$

$$\downarrow 445$$

$$\frac{-\frac{\int \frac{\cot^2(e+fx)\left(15a^2+40ba+33b^2+2b(5a+9b)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{3(a+b)} - \frac{(5a+9b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)}}{5(a+b)} - \frac{\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{5(a+b)}}{f}$$

$$\downarrow 445$$

$$\frac{-\frac{\frac{\int \frac{15(a+b)^3}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a+b} - \frac{\left(15a^2+40ab+33b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{3(a+b)} - \frac{(5a+9b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)}}{5(a+b)} -$$

$$\downarrow 27$$

$$\frac{-\frac{-15(a+b)^2\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx) - \frac{\left(15a^2+40ab+33b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{3(a+b)} - \frac{(5a+9b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+f}}{3(a+b)}}{5(a+b)}$$

$$\downarrow 291$$

$$\frac{-\frac{-15(a+b)^2\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}} - \frac{\left(15a^2+40ab+33b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{3(a+b)} - \frac{(5a+9b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)}}{5(a+b)}$$

$$\downarrow 216$$

3.414. $\int \frac{\cot^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}dx$

$$\frac{-\frac{-\frac{\left(15a^2+40ab+33b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}-\frac{15(a+b)^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}}}{3(a+b)}-\frac{(5a+9b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)}}{5(a+b)}-\frac{\cot^5(e+f}{f}$$

input
```
Int[Cot[e + f*x]^6/Sqrt[a + b*Sec[e + f*x]^2],x]
```

output
```
(-1/5*(Cot[e + f*x]^5*Sqrt[a + b + b*Tan[e + f*x]^2])/(a + b) - (-1/3*((5*
a + 9*b)*Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(a + b) - ((-15*(a
 + b)^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/Sqr
t[a] - ((15*a^2 + 40*a*b + 33*b^2)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x
]^2])/(a + b))/(3*(a + b)))/(5*(a + b)))/f
```

3.414.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.414.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1278 vs. $2(154) = 308$.

Time = 6.78 (sec) , antiderivative size = 1279, normalized size of antiderivative = 7.44

method	result	size
default	Expression too large to display	1279

input
```
int(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15/f/(a+b)^3/(-a)^(1/2)*(15*sin(f*x+e)^5*ln(4*(-a)^(1/2)*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)*a^3*cos(f*x+e)+45*sin(f*x+e)^5*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^
2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+
cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)
^(1/2)*a^2*b*cos(f*x+e)+45*sin(f*x+e)^5*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2
)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+c
os(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^
(1/2)*a*b^2*cos(f*x+e)+15*sin(f*x+e)^5*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(
1/2)*b^3*cos(f*x+e)+23*cos(f*x+e)^6*(-a)^(1/2)*a^3+60*cos(f*x+e)^6*(-a)^(1
/2)*a^2*b+45*cos(f*x+e)^6*(-a)^(1/2)*a*b^2+15*sin(f*x+e)^5*ln(4*(-a)^(1/2)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*a^3+45*sin(f*x+e)^5*ln(4*(-a)^(1/2)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e)
)^2)^(1/2)*a^2*b+45*sin(f*x+e)^5*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1...
```

3.414.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(154) = 308$.

Time = 1.91 (sec) , antiderivative size = 987, normalized size of antiderivative = 5.74

$$\int \frac{\cot^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/120*(15*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)^4 + a^3 + 3*a^2*
b + 3*a*b^2 + b^3 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)^2)*sqrt
(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5
*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2
- 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 -
 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 1
4*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*
x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))
*sin(f*x + e) + 8*((23*a^3 + 60*a^2*b + 45*a*b^2)*cos(f*x + e)^5 - (35*a^3
 + 94*a^2*b + 75*a*b^2)*cos(f*x + e)^3 + (15*a^3 + 40*a^2*b + 33*a*b^2)*co
s(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^4 + 3*a^3*b
+ 3*a^2*b^2 + a*b^3)*f*cos(f*x + e)^4 - 2*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b
^3)*f*cos(f*x + e)^2 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f)*sin(f*x + e)
), 1/60*(15*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)^4 + a^3 + 3*a^2*
b + 3*a*b^2 + b^3 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)^2)*sqrt
(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2
 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x +
 e)^2)*sin(f*x + e)))*sin(f*x + e) - 4*((23*a^3 + 60*a^2*b + 45*a*b^2)*cos
(f*x + e)^5 - (35*a^3 + 94*a^2*b + 75*a*b^2)*cos(f*x + e)^3 + (15*a^3 +...
```

3.414.6 Sympy [F]

$$\int \frac{\cot^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx$$

input
```
integrate(cot(f*x+e)**6/(a+b*sec(f*x+e)**2)**(1/2),x)
```

output
```
Integral(cot(e + f*x)**6/sqrt(a + b*sec(e + f*x)**2), x)
```

3.414.7 Maxima [F]

$$\int \frac{\cot^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot\left(fx+e\right)^6}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cot(f*x + e)^6/sqrt(b*sec(f*x + e)^2 + a), x)
```

3.414.8 Giac [F]

$$\int \frac{\cot^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot\left(fx+e\right)^6}{\sqrt{b\sec\left(fx+e\right)^2+a}}\,dx$$

input
```
integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.414.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^6(e+fx)}{\sqrt{a+b\sec^2(e+fx)}}\,dx = \int \frac{\cot(e+f\,x)^6}{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}\,dx$$

input
```
int(cot(e + f*x)^6/(a + b/cos(e + f*x)^2)^(1/2),x)
```

output
```
int(cot(e + f*x)^6/(a + b/cos(e + f*x)^2)^(1/2), x)
```

3.415 $\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

3.415.1 Optimal result . . . 3033
3.415.2 Mathematica [A] (verified) . . . 3033
3.415.3 Rubi [A] (verified) . . . 3034
3.415.4 Maple [B] (verified) . . . 3036
3.415.5 Fricas [B] (verification not implemented) . . . 3036
3.415.6 Sympy [F] . . . 3037
3.415.7 Maxima [F] . . . 3037
3.415.8 Giac [F] . . . 3038
3.415.9 Mupad [F(-1)] . . . 3038

3.415.1 Optimal result

Integrand size = 25, antiderivative size = 88

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} + \frac{(a+b)^2}{ab^2f\sqrt{a+b\sec^2(e+fx)}} + \frac{\sqrt{a+b\sec^2(e+fx)}}{b^2f}$$

output
```
-arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f+(a+b)^2/a/b^2/f/(a+b*
sec(f*x+e)^2)^(1/2)+(a+b*sec(f*x+e)^2)^(1/2)/b^2/f
```

3.415.2 Mathematica [A] (verified)

Time = 5.46 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.38

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} + \frac{(a+2b+a\cos(2(e+fx)))\left(2a^2+4ab+b^2+(2a^2+2ab+b^2)\cos(2(e+fx))\right)\sec^4(e+fx)}{4ab^2f\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

input
```
Integrate[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
-(ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]]/(a^(3/2)*f)) + ((a + 2*b + a
*Cos[2*(e + f*x)])*(2*a^2 + 4*a*b + b^2 + (2*a^2 + 2*a*b + b^2)*Cos[2*(e +
 f*x)])*Sec[e + f*x]^4)/(4*a*b^2*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

3.415.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4627, 354, 98, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\tan(e+fx)^5}{(a+b\sec(e+fx)^2)^{3/2}}\,dx$$

$$\downarrow 4627$$

$$\frac{\int \frac{\cos(e+fx)\left(1-\sec^2(e+fx)\right)^2}{(b\sec^2(e+fx)+a)^{3/2}}\,d\sec(e+fx)}{f}$$

$$\downarrow 354$$

$$\frac{\int \frac{\cos(e+fx)\left(1-\sec^2(e+fx)\right)^2}{(b\sec^2(e+fx)+a)^{3/2}}\,d\sec^2(e+fx)}{2f}$$

$$\downarrow 98$$

$$\frac{\int \left(-\frac{(a+b)^2}{ab(b\sec^2(e+fx)+a)^{3/2}}+\frac{\cos(e+fx)}{a\sqrt{b\sec^2(e+fx)+a}}+\frac{1}{b\sqrt{b\sec^2(e+fx)+a}}\right)d\sec^2(e+fx)}{2f}$$

$$\downarrow 2009$$

$$\frac{-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{2(a+b)^2}{ab^2\sqrt{a+b\sec^2(e+fx)}}+\frac{2\sqrt{a+b\sec^2(e+fx)}}{b^2}}{2f}$$

input
```
Int[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
((-2*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/a^(3/2) + (2*(a + b)^2)/
(a*b^2*Sqrt[a + b*Sec[e + f*x]^2]) + (2*Sqrt[a + b*Sec[e + f*x]^2])/b^2)/(
2*f)
```

3.415.3.1 Defintions of rubi rules used

rule 98
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(x
_)), x_] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*x)^n*((
e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.415.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 561 vs. $2(78) = 156$.

Time = 5.38 (sec) , antiderivative size = 562, normalized size of antiderivative = 6.39

method	result
default	$-\frac{-2a^{\frac{9}{2}}-2a^{\frac{7}{2}}b-b^2a^{\frac{5}{2}}+\ln\left(4\cos(fx+e)\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\sqrt{a}+4\cos(fx+e)a+4\sqrt{a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\right)\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\,a^2b^2-3a^{\frac{7}{2}}$

input
```
int(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f/b^2/a^(5/2)/(a+b*sec(f*x+e)^2)^(3/2)*(-2*a^(9/2)-2*a^(7/2)*b-b^2*a^(5
/2)+ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*
cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*b^2-3*a^(7/2)*b*sec(f*x+e)^2+ln(
4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*cos(f*x
+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*b^2*sec(f*x+e)-2*b^2*a^(5/2)*sec(f*x+e)
^2+ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*c
os(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*((b+a*c
os(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b^3*sec(f*x+e)^2-a^(3/2)*b^3*sec(f*
x+e)^2+ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)
+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b^3*sec(f*x+e)^3-b^2*a^(5/2)*se
c(f*x+e)^4)
```

3.415.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(78) = 156$.

Time = 0.83 (sec) , antiderivative size = 458, normalized size of antiderivative = 5.20

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \left[\frac{\left(ab^2\cos(fx+e)^2+b^3\right)\sqrt{a}\log\left(128\,a^4\cos(fx+e)^8+256\,a^3b\cos(fx+e)\right.}{\right.}\right.$$

input
```
integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/8*((a*b^2*cos(f*x + e)^2 + b^3)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 25
6*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e
)^2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*
cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)) + 8*(a^2*b + (2*a^3 + 2*a^2*b + a*b^2)*cos(f*x + e)^2)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a^3*b^2*f*cos(f*x + e)^2 + a^2*
b^3*f), 1/4*((a*b^2*cos(f*x + e)^2 + b^3)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f
*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 +
b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)
) + 4*(a^2*b + (2*a^3 + 2*a^2*b + a*b^2)*cos(f*x + e)^2)*sqrt((a*cos(f*x +
 e)^2 + b)/cos(f*x + e)^2))/(a^3*b^2*f*cos(f*x + e)^2 + a^2*b^3*f)]
```

3.415.6 Sympy [F]

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)**5/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(tan(e + f*x)**5/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.415.7 Maxima [F]

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\tan(fx+e)^5}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(tan(f*x + e)^5/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.415.8 Giac [F]

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\tan(fx+e)^5}{\left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.415.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\tan(e+fx)^5}{\left(a+\frac{b}{\cos(e+fx)^2}\right)^{3/2}}\,dx$$

input
```
int(tan(e + f*x)^5/(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(tan(e + f*x)^5/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.416 $\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

3.416.1 Optimal result . . . 3039
3.416.2 Mathematica [A] (verified) . . . 3039
3.416.3 Rubi [A] (verified) . . . 3040
3.416.4 Maple [B] (verified) . . . 3042
3.416.5 Fricas [B] (verification not implemented) . . . 3043
3.416.6 Sympy [F] . . . 3044
3.416.7 Maxima [B] (verification not implemented) . . . 3044
3.416.8 Giac [F] . . . 3045
3.416.9 Mupad [F(-1)] . . . 3046

3.416.1 Optimal result

Integrand size = 25, antiderivative size = 63

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} - \frac{a+b}{abf\sqrt{a+b\sec^2(e+fx)}}$$

output
```
arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f+(-a-b)/a/b/f/(a+b*sec(
f*x+e)^2)^(1/2)
```

3.416.2 Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.98

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{a+b}{a\sqrt{a+b\sec^2(e+fx)}}}{bf}$$

input
```
Integrate[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
((b*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/a^(3/2) - (a + b)/(a*Sqrt
[a + b*Sec[e + f*x]^2]))/(b*f)
```

3.416.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4627, 25, 354, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\tan(e+fx)^3}{(a+b\sec(e+fx)^2)^{3/2}}\,dx$$

$$\downarrow 4627$$

$$\frac{\int -\frac{\cos(e+fx)(1-\sec^2(e+fx))}{(b\sec^2(e+fx)+a)^{3/2}}\,d\sec(e+fx)}{f}$$

$$\downarrow 25$$

$$-\frac{\int \frac{\cos(e+fx)(1-\sec^2(e+fx))}{(b\sec^2(e+fx)+a)^{3/2}}\,d\sec(e+fx)}{f}$$

$$\downarrow 354$$

$$-\frac{\int \frac{\cos(e+fx)(1-\sec^2(e+fx))}{(b\sec^2(e+fx)+a)^{3/2}}\,d\sec^2(e+fx)}{2f}$$

$$\downarrow 87$$

$$-\frac{\frac{\int \frac{\cos(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}\,d\sec^2(e+fx)}{a}+\frac{2(a+b)}{ab\sqrt{a+b\sec^2(e+fx)}}}{2f}$$

$$\downarrow 73$$

$$-\frac{\frac{2\int \frac{1}{\frac{\sec^4(e+fx)}{b}-\frac{a}{b}}\,d\sqrt{b\sec^2(e+fx)+a}}{ab}+\frac{2(a+b)}{ab\sqrt{a+b\sec^2(e+fx)}}}{2f}$$

$$\downarrow 221$$

$$-\frac{\frac{2(a+b)}{ab\sqrt{a+b\sec^2(e+fx)}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}}}{2f}$$

input
```
Int[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
-1/2*((-2*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/a^(3/2) + (2*(a + b
))/(a*b*Sqrt[a + b*Sec[e + f*x]^2]))/f
```

3.416.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.416.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2292 vs. $2(58) = 116$.

Time = 1.59 (sec) , antiderivative size = 2293, normalized size of antiderivative = 36.40

method	result	size
default	Expression too large to display	2293

input
```
int(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
-1/f/a^(3/2)/((-a*b)^(1/2)-a)/((-a*b)^(1/2)+a)/b/(a+b*sec(f*x+e)^2)^(3/2)/
(b+a*cos(f*x+e)^2)/(a+b)*(2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln
(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*cos(f*
x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*a^4*b^2+4*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)
^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x
+e)^2)/(1+cos(f*x+e))^2)^(1/2))*a^3*b^3+2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e
))^2)^(1/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^
(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
)*a^2*b^4-a^(9/2)*b^2*sec(f*x+e)^2-3*a^(7/2)*b^3*sec(f*x+e)^2-3*a^(5/2)*b^
4*sec(f*x+e)^2-a^(3/2)*b^5*sec(f*x+e)^2-3*cos(f*x+e)^2*a^(11/2)*b-3*cos(f*
x+e)^2*a^(9/2)*b^2-cos(f*x+e)^2*a^(7/2)*b^3-2*a^(11/2)*b-6*a^(9/2)*b^2-6*a
^(7/2)*b^3-2*a^(5/2)*b^4+2*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(
4*cos(f*x+e)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*cos(f*x
+e)*a+4*a^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*a^2*b^4*sec(f
*x+e)^3+((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos(f*x+e)*((b+a*
cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b
+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2))*a*b^5*sec(f*x+e)^3+2*cos(f*x+e)*
((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*cos(f*x+e)*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^(1/2)+4*cos(f*x+e)*a+4*a^(1/2)*((b+a*co...
```

3.416.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(55) = 110$.

Time = 0.39 (sec) , antiderivative size = 417, normalized size of antiderivative = 6.62

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \left[-\frac{8\left(a^2+ab\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos\left(fx+e\right)^2 - \left(ab\cos\left(fx+e\right)^2+b^2\right)\sqrt{a}\log \right.$$

$$\left. -\frac{4\left(a^2+ab\right)\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos\left(fx+e\right)^2 + \left(ab\cos\left(fx+e\right)^2+b^2\right)\sqrt{-a}\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^4+8\,ab\cos(fx+e)^2\right.}{4\left(2\,a^3\cos(fx+e)^4+3\,a^2b\right.}\right.}{4\left(a^3bf\cos\left(fx+e\right)^2+a^2b^2f\right)} \right]$$

input

```
integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(8*(a^2 + a*b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x +
 e)^2 - (a*b*cos(f*x + e)^2 + b^2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 25
6*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e
)^2 + b^4 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*
cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)))/(a^3*b*f*cos(f*x + e)^2 + a^2*b^2*f), -1/4*(4*(a^2 + a*b)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2 + (a*b*cos(f*x
 + e)^2 + b^2)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x +
 e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*c
os(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)))/(a^3*b*f*cos(f*x + e)^2
+ a^2*b^2*f)]
```

3.416.6 Sympy [F]

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)**3/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(tan(e + f*x)**3/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.416.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2532 vs. $2(55) = 110$.

Time $= 0.63$ (sec) , antiderivative size $= 2532$, normalized size of antiderivative $= 40.19$

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(4*a*b*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*
e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^3 + 4*a*b*cos(
1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x
 + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))*sin(1/2*arctan2(a*sin(4*f*x +
 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos
(2*f*x + 2*e) + a))^2 - 4*(a^2 + a*b)*sin(2*f*x + 2*e)*sin(1/2*arctan2(a*s
in(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a
+ 2*b)*cos(2*f*x + 2*e) + a)) - 4*(a^2 + 2*a*b + (a^2 + a*b)*cos(2*f*x + 2
*e))*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*
cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) + (a^2*cos(4*f*x + 4
*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^
2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4
*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e)
)*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*((b*cos(1/2*a
rctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*
e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 + b*sin(1/2*arctan2(a*sin(4*f*x
+ 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*co
s(2*f*x + 2*e) + a))^2)*log(4*a^2*cos(2*f*x + 2*e)^2 + 4*a^2*sin(2*f*x + 2
*e)^2 + 4*a^2 + 16*a*b + 16*b^2 + 8*(a^2 + 2*a*b)*cos(2*f*x + 2*e) + 8*(a^
2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)...
```

3.416.8 Giac [F]

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\tan\left(fx+e\right)^3}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.416.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\tan(e+f\,x)^3}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

input
```
int(tan(e + f*x)^3/(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(tan(e + f*x)^3/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.417 $\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

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3.417.1 Optimal result

Integrand size = 23, antiderivative size = 57

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} + \frac{1}{af\sqrt{a+b\sec^2(e+fx)}}$$

output
```
-arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f+1/a/f/(a+b*sec(f*x+e)
^2)^(1/2)
```

3.417.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.66 (sec) , antiderivative size = 382, normalized size of antiderivative = 6.70

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = (a+2b+a\cos(2(e+fx)))^{3/2}\sec^2(e+fx)\left(-\frac{2}{b\sqrt{a+2b+a\cos(2(e+fx))}} + \frac{\sqrt{2}e^{i(e+}}{}\right.$$

input
```
Integrate[Tan[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])^(3/2)*Sec[e + f*x]^2*(-2/(b*Sqrt[a + 2*b +
 a*Cos[2*(e + f*x)]]) + (Sqrt[2]*E^(I*(e + f*x))*Sqrt[4*b + (a*(1 + E^((2*
I)*(e + f*x)))^2)/E^((2*I)*(e + f*x))]*((Sqrt[a]*(a + 4*b)*(1 + E^((2*I)*(
e + f*x))))/(b*(4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2))
+ ((4*I)*f*x - 2*Log[a + 2*b + a*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^
((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2]] - 2*Log[a + a*E^((2*I)
*(e + f*x)) + 2*b*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x
)) + a*(1 + E^((2*I)*(e + f*x)))^2]])/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1
+ E^((2*I)*(e + f*x)))^2])*Sec[e + f*x])/a^(3/2)))/(16*f*(a + b*Sec[e + f*
x]^2)^(3/2))
```

3.417.3 Rubi [A] (verified)

Time $= 0.25$ (sec) , antiderivative size $= 59$, normalized size of antiderivative $= 1.04$, number of steps used $= 7$, number of rules used $= 6$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used $= \{3042, 4627, 243, 61, 73, 221\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\tan(e+fx)}{(a+b\sec(e+fx)^2)^{3/2}}\,dx$$

$$\downarrow 4627$$

$$\frac{\int \frac{\cos(e+fx)}{(b\sec^2(e+fx)+a)^{3/2}}\,d\sec(e+fx)}{f}$$

$$\downarrow 243$$

$$\frac{\int \frac{\cos(e+fx)}{(b\sec^2(e+fx)+a)^{3/2}}\,d\sec^2(e+fx)}{2f}$$

$$\downarrow 61$$

$$\frac{\frac{\int \frac{\cos(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}\,d\sec^2(e+fx)}{a} + \frac{2}{a\sqrt{a+b\sec^2(e+fx)}}}{2f}$$

$$\downarrow 73$$

$$\frac{\frac{2\int \frac{1}{\frac{\sec^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sec^2(e+fx)+a}}{ab}+\frac{2}{a\sqrt{a+b\sec^2(e+fx)}}}{2f}$$

$$\downarrow 221$$

$$\frac{\frac{2}{a\sqrt{a+b\sec^2(e+fx)}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}}}{2f}$$

input
```
Int[Tan[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
((-2*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/a^(3/2) + 2/(a*Sqrt[a +
b*Sec[e + f*x]^2]))/(2*f)
```

3.417.3.1 Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.417.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\frac{\frac{1}{a\sqrt{a+b\sec(fx+e)^2}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\sqrt{a+b\sec(fx+e)^2}}{\sec(fx+e)}\right)}{a^{\frac{3}{2}}}}{f}$	62
default	$\frac{\frac{1}{a\sqrt{a+b\sec(fx+e)^2}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\sqrt{a+b\sec(fx+e)^2}}{\sec(fx+e)}\right)}{a^{\frac{3}{2}}}}{f}$	62

input
```
int(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/a/(a+b*sec(f*x+e)^2)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*(a+b*sec(f*x
+e)^2)^(1/2))/sec(f*x+e)))
```

3.417.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(49) = 98$.

Time = 0.37 (sec) , antiderivative size = 392, normalized size of antiderivative = 6.88

$$\int \frac{\tan(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \left[\frac{8\,a\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos\left(fx+e\right)^2+\left(a\cos\left(fx+e\right)^2+b\right)\sqrt{a}\log\left(128\,a^4\cos\left(\right.\right.\right.\right.$$

input
```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2 + (a*
cos(f*x + e)^2 + b)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x
 + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(
16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4
 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))
)/(a^3*f*cos(f*x + e)^2 + a^2*b*f), 1/4*(4*a*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)*cos(f*x + e)^2 + (a*cos(f*x + e)^2 + b)*sqrt(-a)*arctan(1/4
*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(
f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x +
e)^2 + a*b^2)))/(a^3*f*cos(f*x + e)^2 + a^2*b*f)]
```

3.417.6 Sympy [A] (verification not implemented)

Time = 5.89 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.32

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \begin{cases} \dfrac{2\left(\dfrac{b}{2af\sqrt{a+b\sec^2(e+fx)}} + \dfrac{b\operatorname{atan}\left(\dfrac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{-a}}\right)}{2af\sqrt{-a}}\right)}{b} & \text{for } b \neq 0 \\ \dfrac{\log\left(\sec^2(e+fx)\right)}{2a^{\frac{3}{2}}f} & \text{otherwise} \end{cases}$$

input
```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Piecewise((2*(b/(2*a*f*sqrt(a + b*sec(e + f*x)**2)) + b*atan(sqrt(a + b*se
c(e + f*x)**2)/sqrt(-a))/(2*a*f*sqrt(-a)))/b, Ne(b, 0)), (log(sec(e + f*x)
**2)/(2*a**(3/2)*f), True))
```

3.417.7 Maxima [F]

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\tan(fx+e)}{\left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(tan(f*x + e)/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.417.8 Giac [F]

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\tan\left(fx+e\right)}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.417.9 Mupad [B] (verification not implemented)

Time = 21.31 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{1}{a\,f\,\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}} - \frac{\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{\cos(e+f\,x)^2}}}{\sqrt{a}}\right)}{a^{3/2}\,f}$$

input
```
int(tan(e + f*x)/(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
1/(a*f*(a + b/cos(e + f*x)^2)^(1/2)) - atanh((a + b/cos(e + f*x)^2)^(1/2)/
a^(1/2))/(a^(3/2)*f)
```

3.418 $\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

3.418.1 Optimal result . . . 3053
3.418.2 Mathematica [F] . . . 3053
3.418.3 Rubi [A] (verified) . . . 3054
3.418.4 Maple [B] (warning: unable to verify) . . . 3056
3.418.5 Fricas [B] (verification not implemented) . . . 3057
3.418.6 Sympy [F] . . . 3057
3.418.7 Maxima [F] . . . 3058
3.418.8 Giac [F] . . . 3058
3.418.9 Mupad [F(-1)] . . . 3058

3.418.1 Optimal result

Integrand size = 23, antiderivative size = 100

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}f} - \frac{b}{a(a+b)f\sqrt{a+b\sec^2(e+fx)}}$$

output
```
arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f-arctanh((a+b*sec(f*x+e
)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(3/2)/f-b/a/(a+b)/f/(a+b*sec(f*x+e)^2)^(1/2)
```

3.418.2 Mathematica [F]

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

input
```
Integrate[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
Integrate[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2), x]
```

3.418.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.13, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4627, 25, 354, 96, 25, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\tan(e+fx)\,(a+b\sec(e+fx)^2)^{3/2}}dx$$

$$\downarrow 4627$$

$$\frac{\int -\frac{\cos(e+fx)}{(1-\sec^2(e+fx))(b\sec^2(e+fx)+a)^{3/2}}d\sec(e+fx)}{f}$$

$$\downarrow 25$$

$$-\frac{\int \frac{\cos(e+fx)}{(1-\sec^2(e+fx))(b\sec^2(e+fx)+a)^{3/2}}d\sec(e+fx)}{f}$$

$$\downarrow 354$$

$$-\frac{\int \frac{\cos(e+fx)}{(1-\sec^2(e+fx))(b\sec^2(e+fx)+a)^{3/2}}d\sec^2(e+fx)}{2f}$$

$$\downarrow 96$$

$$-\frac{\frac{2b}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}-\frac{\int -\frac{\cos(e+fx)\left(-b\sec^2(e+fx)+a+b\right)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{a(a+b)}}{2f}$$

$$\downarrow 25$$

$$-\frac{\frac{\int \frac{\cos(e+fx)\left(-b\sec^2(e+fx)+a+b\right)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{a(a+b)}+\frac{2b}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{2f}$$

$$\downarrow 174$$

$$-\frac{\frac{a\int \frac{1}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)+(a+b)\int \frac{\cos(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{a(a+b)}+\frac{2b}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{2f}$$

$$\downarrow 73$$

$$-\frac{\frac{2a\int \frac{1}{\frac{a+b}{b}-\frac{\sec^4(e+fx)}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}+\frac{2(a+b)\int \frac{1}{\frac{\sec^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}}{a(a+b)}+\frac{2b}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{2f}$$

$$\downarrow 221$$

$$-\frac{\frac{2a\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{2(a+b)\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}}{a(a+b)}+\frac{2b}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{2f}$$

input
```
Int[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
-1/2*(((-2*(a + b)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/Sqrt[a] +
(2*a*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/Sqrt[a + b])/(a*(a +
 b)) + (2*b)/(a*(a + b)*Sqrt[a + b*Sec[e + f*x]^2]))/f
```

3.418.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 96
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))), x] + S
imp[1/((b*e - a*f)*(d*e - c*f))   Int[(b*d*e - b*c*f - a*d*f - b*d*f*x)*((e
 + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && LtQ[p, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.418.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 9461 vs. $2(86) = 172$.

Time = 1.81 (sec) , antiderivative size = 9462, normalized size of antiderivative = 94.62

method	result	size
default	Expression too large to display	9462

input
```
int(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.418.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. 2(86) = 172.

Time = 0.61 (sec) , antiderivative size = 1569, normalized size of antiderivative = 15.69

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(8*(a^2*b + a*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f
*x + e)^2 - (a^2*b + 2*a*b^2 + b^3 + (a^3 + 2*a^2*b + a*b^2)*cos(f*x + e)^
2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2
*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x +
e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e
)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) - 2*(a^3*cos(f*x
 + e)^2 + a^2*b)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 +
 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*a + b)*cos(f*x + e)^4 + b*
cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(
cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/((a^5 + 2*a^4*b + a^3*b^2)*f*cos(
f*x + e)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*f), -1/8*(8*(a^2*b + a*b^2)*sqr
t((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)^2 - 4*(a^3*cos(f*x +
 e)^2 + a^2*b)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt
(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x
+ e)^2 + a*b + b^2)) - (a^2*b + 2*a*b^2 + b^3 + (a^3 + 2*a^2*b + a*b^2)*co
s(f*x + e)^2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^
6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3
*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*
cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)))/((a^
5 + 2*a^4*b + a^3*b^2)*f*cos(f*x + e)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)...
```

3.418.6 Sympy [F]

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(cot(e + f*x)/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.418.7 Maxima [F]

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cot(fx+e)}{\left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(cot(f*x + e)/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.418.8 Giac [F]

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cot(fx+e)}{\left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.418.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cot(e+f\,x)}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

input
```
int(cot(e + f*x)/(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(cot(e + f*x)/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.419 $\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

3.419.1 Optimal result . . . 3059
3.419.2 Mathematica [F] . . . 3060
3.419.3 Rubi [A] (verified) . . . 3060
3.419.4 Maple [B] (warning: unable to verify) . . . 3063
3.419.5 Fricas [B] (verification not implemented) . . . 3064
3.419.6 Sympy [F] . . . 3064
3.419.7 Maxima [F(-1)] . . . 3065
3.419.8 Giac [F] . . . 3065
3.419.9 Mupad [F(-1)] . . . 3065

3.419.1 Optimal result

Integrand size = 25, antiderivative size = 153

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} + \frac{(2a+5b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2(a+b)^{5/2}f} - \frac{(a-2b)b}{2a(a+b)^2f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cot^2(e+fx)}{2(a+b)f\sqrt{a+b\sec^2(e+fx)}}$$

output

```
-arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f+1/2*(2*a+5*b)*arctanh
((a+b*sec(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(5/2)/f-1/2*(a-2*b)*b/a/(a+b)
^2/f/(a+b*sec(f*x+e)^2)^(1/2)-1/2*cot(f*x+e)^2/(a+b)/f/(a+b*sec(f*x+e)^2)^
(1/2)
```

3.419.2 Mathematica [F]

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

input
```
Integrate[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
Integrate[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2), x]
```

3.419.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.13, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4627, 354, 114, 27, 169, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{\tan(e+fx)^3\left(a+b\sec(e+fx)^2\right)^{3/2}}dx$$
$$\downarrow 4627$$
$$\frac{\int \frac{\cos(e+fx)}{(1-\sec^2(e+fx))^2(b\sec^2(e+fx)+a)^{3/2}}d\sec(e+fx)}{f}$$
$$\downarrow 354$$
$$\frac{\int \frac{\cos(e+fx)}{(1-\sec^2(e+fx))^2(b\sec^2(e+fx)+a)^{3/2}}d\sec^2(e+fx)}{2f}$$
$$\downarrow 114$$
$$\frac{\frac{1}{(a+b)(1-\sec^2(e+fx))\sqrt{a+b\sec^2(e+fx)}} - \frac{\int -\frac{\cos(e+fx)\left(3b\sec^2(e+fx)+2a+2b\right)}{2\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec^2(e+fx)}{a+b}}{2f}$$
$$\downarrow 27$$

$$\frac{\frac{\int \frac{\cos(e+fx)\left(3b\sec^2(e+fx)+2(a+b)\right)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec^2(e+fx)}{2(a+b)}+\frac{1}{(a+b)(1-\sec^2(e+fx))\sqrt{a+b\sec^2(e+fx)}}}{2f}$$

$$\downarrow 169$$

$$\frac{\frac{2\int \frac{\cos(e+fx)\left(2(a+b)^2+(a-2b)b\sec^2(e+fx)\right)}{2\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{a(a+b)}-\frac{2b(a-2b)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{2(a+b)}+\frac{1}{(a+b)(1-\sec^2(e+fx))\sqrt{a+b\sec^2(e+fx)}}}{2f}$$

$$\downarrow 27$$

$$\frac{\frac{\int \frac{\cos(e+fx)\left(2(a+b)^2+(a-2b)b\sec^2(e+fx)\right)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{a(a+b)}-\frac{2b(a-2b)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{2(a+b)}+\frac{1}{(a+b)(1-\sec^2(e+fx))\sqrt{a+b\sec^2(e+fx)}}}{2f}$$

$$\downarrow 174$$

$$\frac{\frac{a(2a+5b)\int \frac{1}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)+2(a+b)^2\int \frac{\cos(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{a(a+b)}-\frac{2b(a-2b)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{2(a+b)}+\frac{1}{(a+b)(1-\sec^2(e+fx))}}{2f}$$

$$\downarrow 73$$

$$\frac{\frac{\frac{4(a+b)^2\int \frac{1}{\frac{\sec^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}+\frac{2a(2a+5b)\int \frac{1}{\frac{a+b}{b}-\frac{\sec^4(e+fx)}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}}{a(a+b)}-\frac{2b(a-2b)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{2(a+b)}+\frac{1}{(a+b)(1-\sec^2(e+fx))\sqrt{a}}}{2f}$$

$$\downarrow 221$$

$$\frac{\frac{\frac{2a(2a+5b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{4(a+b)^2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}}{a(a+b)}-\frac{2b(a-2b)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{2(a+b)}+\frac{1}{(a+b)(1-\sec^2(e+fx))\sqrt{a+b\sec^2(e-}}}{2f}$$

input
```
Int[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

3.419. $\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

output
```
(1/((a + b)*(1 - Sec[e + f*x]^2)*Sqrt[a + b*Sec[e + f*x]^2]) + (((-4*(a +
b)^2*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/Sqrt[a] + (2*a*(2*a + 5*
b)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/Sqrt[a + b])/(a*(a + b
)) - (2*(a - 2*b)*b)/(a*(a + b)*Sqrt[a + b*Sec[e + f*x]^2]))/(2*(a + b)))/
(2*f)
```

3.419.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.419.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 13609 vs. $2(131) = 262$.

Time = 1.16 (sec) , antiderivative size = 13610, normalized size of antiderivative = 88.95

method	result	size
default	Expression too large to display	13610

input
```
int(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.419.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 540 vs. 2(131) = 262.

Time = 1.49 (sec) , antiderivative size = 2347, normalized size of antiderivative = 15.34

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cos(f*x + e)^4 - a^3*b - 3*a^2*
b^2 - 3*a*b^3 - b^4 - (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cos(f*x + e)^2)*sqrt
(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*co
s(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 +
24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sq
rt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + ((2*a^4 + 5*a^3*b)*co
s(f*x + e)^4 - 2*a^3*b - 5*a^2*b^2 - (2*a^4 + 3*a^3*b - 5*a^2*b^2)*cos(f*x
 + e)^2)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*
b + 3*b^2)*cos(f*x + e)^2 + b^2 + 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x
+ e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x
+ e)^4 - 2*cos(f*x + e)^2 + 1)) + 4*((a^4 + a^3*b + 2*a^2*b^2 + 2*a*b^3)*c
os(f*x + e)^4 + (a^3*b - a^2*b^2 - 2*a*b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*
x + e)^2 + b)/cos(f*x + e)^2))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*f*co
s(f*x + e)^4 - (a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*f*cos(f*x + e)^2 - (a
^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*f), -1/8*(2*((2*a^4 + 5*a^3*b)*cos
(f*x + e)^4 - 2*a^3*b - 5*a^2*b^2 - (2*a^4 + 3*a^3*b - 5*a^2*b^2)*cos(f*x
+ e)^2)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*sqrt(-a - b
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(f*x + e)^2
+ a*b + b^2)) - ((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cos(f*x + e)^4 - a^3*
b - 3*a^2*b^2 - 3*a*b^3 - b^4 - (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cos(f*x...
```

3.419.6 Sympy [F]

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)**3/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(cot(e + f*x)**3/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.419.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.419.8 Giac [F]

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cot\left(fx+e\right)^3}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.419.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cot(e+f\,x)^3}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

input
```
int(cot(e + f*x)^3/(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(cot(e + f*x)^3/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.420 $\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

3.420.1 Optimal result . . . 3066
3.420.2 Mathematica [F] . . . 3067
3.420.3 Rubi [A] (verified) . . . 3067
3.420.4 Maple [B] (warning: unable to verify) . . . 3071
3.420.5 Fricas [B] (verification not implemented) . . . 3072
3.420.6 Sympy [F] . . . 3072
3.420.7 Maxima [F(-1)] . . . 3073
3.420.8 Giac [F] . . . 3073
3.420.9 Mupad [F(-1)] . . . 3073

3.420.1 Optimal result

Integrand size = 25, antiderivative size = 213

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} - \frac{(8a^2+28ab+35b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8(a+b)^{7/2}f} + \frac{b(4a^2+11ab-8b^2)}{8a(a+b)^3 f\sqrt{a+b\sec^2(e+fx)}} + \frac{(4a+9b)\cot^2(e+fx)}{8(a+b)^2 f\sqrt{a+b\sec^2(e+fx)}} - \frac{\cot^4(e+fx)}{4(a+b)f\sqrt{a+b\sec^2(e+fx)}}$$

output

```
arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f-1/8*(8*a^2+28*a*b+35*b
^2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(7/2)/f+1/8*b*(4*a
^2+11*a*b-8*b^2)/a/(a+b)^3/f/(a+b*sec(f*x+e)^2)^(1/2)+1/8*(4*a+9*b)*cot(f*
x+e)^2/(a+b)^2/f/(a+b*sec(f*x+e)^2)^(1/2)-1/4*cot(f*x+e)^4/(a+b)/f/(a+b*se
c(f*x+e)^2)^(1/2)
```

3.420.2 Mathematica [F]

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

input
```
Integrate[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
Integrate[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2), x]
```

3.420.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.16, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 4627, 25, 354, 114, 27, 168, 27, 169, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\tan(e+fx)^5\,(a+b\sec(e+fx)^2)^{3/2}}dx$$

$$\downarrow 4627$$

$$\frac{\int -\frac{\cos(e+fx)}{(1-\sec^2(e+fx))^3(b\sec^2(e+fx)+a)^{3/2}}d\sec(e+fx)}{f}$$

$$\downarrow 25$$

$$-\frac{\int \frac{\cos(e+fx)}{(1-\sec^2(e+fx))^3(b\sec^2(e+fx)+a)^{3/2}}d\sec(e+fx)}{f}$$

$$\downarrow 354$$

$$-\frac{\int \frac{\cos(e+fx)}{(1-\sec^2(e+fx))^3(b\sec^2(e+fx)+a)^{3/2}}d\sec^2(e+fx)}{2f}$$

$$\downarrow 114$$

$$-\frac{\frac{1}{2(a+b)(1-\sec^2(e+fx))^2\sqrt{a+b\sec^2(e+fx)}}-\frac{\int -\frac{\cos(e+fx)\left(5b\sec^2(e+fx)+4(a+b)\right)}{2\left(1-\sec^2(e+fx)\right)^2\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec^2(e+fx)}{2(a+b)}}{2f}$$

$$\downarrow 27$$

$$-\frac{\frac{\int \frac{\cos(e+fx)\left(5b\sec^2(e+fx)+4(a+b)\right)}{\left(1-\sec^2(e+fx)\right)^2\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec^2(e+fx)}{4(a+b)}+\frac{1}{2(a+b)(1-\sec^2(e+fx))^2\sqrt{a+b\sec^2(e+fx)}}}{2f}$$

$$\downarrow 168$$

$$-\frac{\frac{\frac{4a+9b}{(a+b)\left(1-\sec^2(e+fx)\right)\sqrt{a+b\sec^2(e+fx)}}-\frac{\int -\frac{\cos(e+fx)\left(8(a+b)^2+3b(4a+9b)\sec^2(e+fx)\right)}{2\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec^2(e+fx)}{a+b}}{4(a+b)}+\frac{1}{2(a+b)(1-\sec^2(e+fx))^2\sqrt{a+b\sec^2(e+fx)}}}{2f}$$

$$\downarrow 27$$

$$-\frac{\frac{\frac{\int \frac{\cos(e+fx)\left(8(a+b)^2+3b(4a+9b)\sec^2(e+fx)\right)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec^2(e+fx)}{2(a+b)}+\frac{4a+9b}{(a+b)\left(1-\sec^2(e+fx)\right)\sqrt{a+b\sec^2(e+fx)}}}{4(a+b)}+\frac{1}{2(a+b)(1-\sec^2(e+fx))^2\sqrt{a+b\sec^2(e+fx)}}}{2f}$$

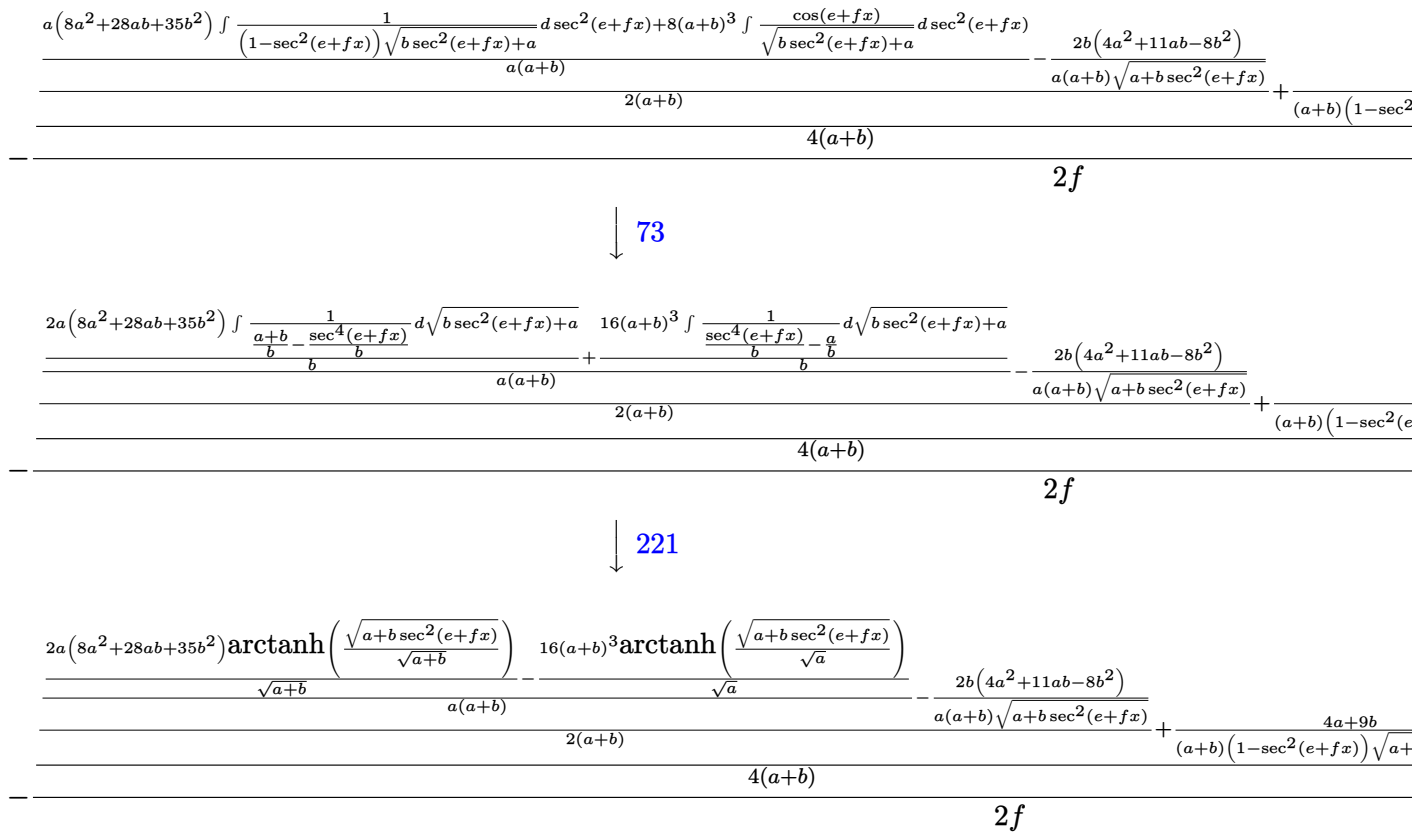
$$\downarrow 169$$

$$-\frac{\frac{\frac{2\int \frac{\cos(e+fx)\left(8(a+b)^3+b\left(4a^2+11ba-8b^2\right)\sec^2(e+fx)\right)}{2\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{a(a+b)}-\frac{2b\left(4a^2+11ab-8b^2\right)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{2(a+b)}+\frac{4a+9b}{(a+b)\left(1-\sec^2(e+fx)\right)\sqrt{a+b\sec^2(e+fx)}}}{4(a+b)}+\frac{}{2(a+b)(1-}}{2f}$$

$$\downarrow 27$$

$$-\frac{\frac{\frac{\int \frac{\cos(e+fx)\left(8(a+b)^3+b\left(4a^2+11ba-8b^2\right)\sec^2(e+fx)\right)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{a(a+b)}-\frac{2b\left(4a^2+11ab-8b^2\right)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{2(a+b)}+\frac{4a+9b}{(a+b)\left(1-\sec^2(e+fx)\right)\sqrt{a+b\sec^2(e+fx)}}}{4(a+b)}+\frac{}{2(a+b)(1-se}}{2f}$$

$$\downarrow 174$$



input
```
Int[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
-1/2*(1/(2*(a + b)*(1 - Sec[e + f*x]^2)^2*Sqrt[a + b*Sec[e + f*x]^2]) + ((
4*a + 9*b)/((a + b)*(1 - Sec[e + f*x]^2)*Sqrt[a + b*Sec[e + f*x]^2]) + (((
-16*(a + b)^3*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/Sqrt[a] + (2*a*
(8*a^2 + 28*a*b + 35*b^2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])
/Sqrt[a + b])/(a*(a + b)) - (2*b*(4*a^2 + 11*a*b - 8*b^2))/(a*(a + b)*Sqrt
[a + b*Sec[e + f*x]^2]))/(2*(a + b)))/(4*(a + b)))/f
```

3.420.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.420.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 17867 vs. $2(187) = 374$.

Time = 2.34 (sec) , antiderivative size = 17868, normalized size of antiderivative = 83.89

method	result	size
default	Expression too large to display	17868

input
```
int(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.420.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 829 vs. 2(187) = 374.

Time = 4.80 (sec) , antiderivative size = 3501, normalized size of antiderivative = 16.44

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/32*(4*((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cos(f*x + e)^6 +
 a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5 - (2*a^5 + 7*a^4*b + 8*a^3*
b^2 + 2*a^2*b^3 - 2*a*b^4 - b^5)*cos(f*x + e)^4 + (a^5 + 2*a^4*b - 2*a^3*b
^2 - 8*a^2*b^3 - 7*a*b^4 - 2*b^5)*cos(f*x + e)^2)*sqrt(a)*log(128*a^4*cos(
f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*
b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e
)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*
x + e)^2 + b)/cos(f*x + e)^2)) + ((8*a^5 + 28*a^4*b + 35*a^3*b^2)*cos(f*x
+ e)^6 + 8*a^4*b + 28*a^3*b^2 + 35*a^2*b^3 - (16*a^5 + 48*a^4*b + 42*a^3*b
^2 - 35*a^2*b^3)*cos(f*x + e)^4 + (8*a^5 + 12*a^4*b - 21*a^3*b^2 - 70*a^2*
b^3)*cos(f*x + e)^2)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)
^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*a + b)*cos(f*x + e)^4
+ b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)) - 4*((6*a^5 + 19*a^4*b + 13*a^
3*b^2 + 8*a^2*b^3 + 8*a*b^4)*cos(f*x + e)^6 - (4*a^5 + 9*a^4*b - 8*a^3*b^2
 + 3*a^2*b^3 + 16*a*b^4)*cos(f*x + e)^4 - (4*a^4*b + 15*a^3*b^2 + 3*a^2*b^
3 - 8*a*b^4)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/
((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^6 - (2*a
^7 + 7*a^6*b + 8*a^5*b^2 + 2*a^4*b^3 - 2*a^3*b^4 - a^2*b^5)*f*cos(f*x + e)
^4 + (a^7 + 2*a^6*b - 2*a^5*b^2 - 8*a^4*b^3 - 7*a^3*b^4 - 2*a^2*b^5)*f*...
```

3.420.6 Sympy [F]

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)**5/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(cot(e + f*x)**5/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.420.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.420.8 Giac [F]

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cot\left(fx+e\right)^5}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.420.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Hanged}$$

input
```
int(cot(e + f*x)^5/(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
\text{Hanged}
```

3.421 $\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

3.421.1 Optimal result . . . 3074
3.421.2 Mathematica [A] (verified) . . . 3075
3.421.3 Rubi [A] (verified) . . . 3075
3.421.4 Maple [B] (verified) . . . 3079
3.421.5 Fricas [B] (verification not implemented) . . . 3080
3.421.6 Sympy [F] . . . 3080
3.421.7 Maxima [F(-1)] . . . 3081
3.421.8 Giac [F] . . . 3081
3.421.9 Mupad [F(-1)] . . . 3081

3.421.1 Optimal result

Integrand size = 25, antiderivative size = 172

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f} - \frac{(3a+5b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{2b^{5/2}f} - \frac{(a+b)\tan^3(e+fx)}{abf\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(3a+2b)\tan(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{2ab^2f}$$

output

```
-arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f-1/2*(3*a+
5*b)*arctanh(b^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b^(5/2)/f+1/2*
(3*a+2*b)*(a+b+b*tan(f*x+e)^2)^(1/2)*tan(f*x+e)/a/b^2/f-(a+b)*tan(f*x+e)^3
/a/b/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

3.421.2 Mathematica [A] (verified)

Time = 9.40 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.44

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx =$$

$$-\frac{\left(\frac{2b^2\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{a}}+\frac{a(3a+5b)\operatorname{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{b}}\right)(a+2b+a\cos(2e+2fx))^{3/2}\sec^3(e+fx}{4\sqrt{2}ab^2f\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

$$+\frac{(a+2b+a\cos(2(e+fx)))\left(3a^2+6ab+2b^2+(3a^2+4ab+2b^2)\cos(2(e+fx))\right)\sec^4(e+fx)\tan(e+fx}{8ab^2f\left(a+b\sec^2(e+fx)\right)^{3/2}}$$

input
```
Integrate[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
-1/4*(((2*b^2*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]
])/Sqrt[a] + (a*(3*a + 5*b)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b - a*
Sin[e + f*x]^2]])/Sqrt[b])*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sec[e + f*
x]^3)/(Sqrt[2]*a*b^2*f*(a + b*Sec[e + f*x]^2)^(3/2)) + ((a + 2*b + a*Cos[2
*(e + f*x)])*(3*a^2 + 6*a*b + 2*b^2 + (3*a^2 + 4*a*b + 2*b^2)*Cos[2*(e + f
*x)])*Sec[e + f*x]^4*Tan[e + f*x])/(8*a*b^2*f*(a + b*Sec[e + f*x]^2)^(3/2)
)
```

3.421.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4629, 2075, 372, 444, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\tan(e+fx)^6}{(a+b\sec(e+fx)^2)^{3/2}}\,dx$$

$$\downarrow 4629$$

$$\frac{\int \frac{\tan^6(e+fx)}{(\tan^2(e+fx)+1)(a+b(\tan^2(e+fx)+1))^{3/2}}\,d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\tan^6(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^{3/2}}\,d\tan(e+fx)}{f}$$

$$\downarrow 372$$

$$\frac{\int \frac{\tan^2(e+fx)\left((3a+2b)\tan^2(e+fx)+3(a+b)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}\,d\tan(e+fx)}{ab} - \frac{(a+b)\tan^3(e+fx)}{ab\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\downarrow 444$$

$$\frac{\frac{(3a+2b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b} - \frac{\int \frac{a(3a+5b)\tan^2(e+fx)+(a+b)(3a+2b)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}\,d\tan(e+fx)}{2b}}{ab} - \frac{(a+b)\tan^3(e+fx)}{ab\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\downarrow 398$$

$$\frac{\frac{(3a+2b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b} - \frac{2b^2\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}\,d\tan(e+fx)+a(3a+5b)\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}\,d\tan(e+fx)}{2b}}{ab} - \frac{(a+b)}{ab\sqrt{a+b}}}{f}$$

$$\downarrow 224$$

$$\frac{\frac{(3a+2b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b} - \frac{2b^2\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}\,d\tan(e+fx)+a(3a+5b)\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}\,d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{2b}}{ab} -}{f}$$

$$\downarrow 219$$

$$\frac{\frac{(3a+2b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b} - \frac{2b^2\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}\,d\tan(e+fx)+\frac{a(3a+5b)\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}}}{2b}}{ab} - \frac{(a+b)}{ab\sqrt{a+b}}}{f}$$

$$\downarrow 291$$

3.421. $\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

$$\frac{\frac{(3a+2b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{2b^2\int\frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}+\frac{a(3a+5b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}}}{2b}}{ab}-\frac{(a+b)\tan}{ab\sqrt{a+b\tan}}}{f}$$

$$\downarrow 216$$

$$\frac{\frac{(3a+2b)\tan(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{2b}-\frac{\frac{2b^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}}+\frac{a(3a+5b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}}}{2b}}{ab}-\frac{(a+b)\tan^3(e+fx)}{ab\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

input
```
Int[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(-(((a + b)*Tan[e + f*x]^3)/(a*b*Sqrt[a + b + b*Tan[e + f*x]^2])) + (-1/2*
((2*b^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/Sqr
t[a] + (a*(3*a + 5*b)*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e
+ f*x]^2]])/Sqrt[b])/b + ((3*a + 2*b)*Tan[e + f*x]*Sqrt[a + b + b*Tan[e +
f*x]^2])/(2*b))/(a*b))/f
```

3.421.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.421.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2791 vs. $2(152) = 304$.

Time = 13.01 (sec) , antiderivative size = 2792, normalized size of antiderivative = 16.23

method	result	size
default	Expression too large to display	2792

input
```
int(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/f/b^(7/2)/(-a)^(1/2)/a/(a+b*sec(f*x+e)^2)^(3/2)*(3*ln(-4*(((b+a*cos(f
*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+
e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*(-a)^(1/2)
*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*b^2*sec(f*x+e)^2+5*ln(-4*
(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1)
)*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a*b^3*sec(f*x+e)^
2+3*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b
^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*a-a-b)/(sin(
f*x+e)+1))*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*a^2*b^2*
sec(f*x+e)^2+3*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*b^(1/2)*
cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-sin(f*x+e)*
a+a+b)/(sin(f*x+e)-1))*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*a^2*b^2*sec(f*x+e)^3+5*ln(4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/
2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-
sin(f*x+e)*a-a-b)/(sin(f*x+e)+1))*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*
x+e))^2)^(1/2)*a*b^3*sec(f*x+e)^2+5*ln(-4*(((b+a*cos(f*x+e)^2)/(1+cos(f*x+
e))^2)^(1/2)*b^(1/2)*cos(f*x+e)+b^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))
^2)^(1/2)-sin(f*x+e)*a+a+b)/(sin(f*x+e)-1))*(-a)^(1/2)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*a*b^3*sec(f*x+e)^3+5*ln(4*(((b+a*cos(f*x+e)^2)...
```

3.421.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 398 vs. 2(152) = 304.

Time = 1.76 (sec) , antiderivative size = 1895, normalized size of antiderivative = 11.02

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*((a*b^3*cos(f*x + e)^3 + b^4*cos(f*x + e))*sqrt(-a)*log(128*a^4*cos(
f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a
^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32
*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x +
e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*co
s(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt
((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - ((3*a^4 + 5*a^3*b)
*cos(f*x + e)^3 + (3*a^3*b + 5*a^2*b^2)*cos(f*x + e))*sqrt(b)*log(((a^2 -
6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*co
s(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) - 4*(a^2*b^2 + (3*a^3*b +
 4*a^2*b^2 + 2*a*b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)*sin(f*x + e))/(a^3*b^3*f*cos(f*x + e)^3 + a^2*b^4*f*cos(f*x + e)),
 -1/8*(2*((3*a^4 + 5*a^3*b)*cos(f*x + e)^3 + (3*a^3*b + 5*a^2*b^2)*cos(f*x
 + e))*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sq
rt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 +
b^2)*sin(f*x + e))) + (a*b^3*cos(f*x + e)^3 + b^4*cos(f*x + e))*sqrt(-a)*l
og(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 -
 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a
*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*...
```

3.421.6 Sympy [F]

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)**6/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(tan(e + f*x)**6/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.421.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.421.8 Giac [F]

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\tan\left(fx+e\right)^6}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.421.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\tan(e+f\,x)^6}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

input
```
int(tan(e + f*x)^6/(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(tan(e + f*x)^6/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.422 $\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

3.422.1 Optimal result . . . 3082
3.422.2 Mathematica [A] (verified) . . . 3082
3.422.3 Rubi [A] (verified) . . . 3083
3.422.4 Maple [B] (warning: unable to verify) . . . 3086
3.422.5 Fricas [B] (verification not implemented) . . . 3087
3.422.6 Sympy [F] . . . 3088
3.422.7 Maxima [F] . . . 3089
3.422.8 Giac [F] . . . 3089
3.422.9 Mupad [F(-1)] . . . 3089

3.422.1 Optimal result

Integrand size = 25, antiderivative size = 116

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{b^{3/2}f} - \frac{(a+b)\tan(e+fx)}{abf\sqrt{a+b+b\tan^2(e+fx)}}$$

output
```
arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f+arctanh(b^
(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b^(3/2)/f-(a+b)*tan(f*x+e)/a/
b/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

3.422.2 Mathematica [A] (verified)

Time = 4.61 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.73

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\left(\frac{b\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{a}} + \frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{b}}\right)(a+2b+a\cos(2e+$$
$$2\sqrt{2}abf\,(a+b\sec^2(e+fx))^{3/2}$$
$$-\frac{(a+b)(a+2b+a\cos(2(e+fx)))\sec^2(e+fx)\tan(e+fx)}{2abf\,(a+b\sec^2(e+fx))^{3/2}}$$

input
```
Integrate[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(((b*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]])/Sqrt[a
] + (a*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]])/Sqr
t[b])*(a + 2*b + a*Cos[2*e + 2*f*x])^(3/2)*Sec[e + f*x]^3)/(2*Sqrt[2]*a*b*
f*(a + b*Sec[e + f*x]^2)^(3/2)) - ((a + b)*(a + 2*b + a*Cos[2*(e + f*x)])*
Sec[e + f*x]^2*Tan[e + f*x])/(2*a*b*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

3.422.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4629, 2075, 372, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\tan(e+fx)^4}{(a+b\sec(e+fx)^2)^{3/2}}\,dx$$

$$\downarrow 4629$$

$$\frac{\int \frac{\tan^4(e+fx)}{(\tan^2(e+fx)+1)(a+b(\tan^2(e+fx)+1))^{3/2}}\,d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\tan^4(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^{3/2}}\,d\tan(e+fx)}{f}$$

$$\downarrow 372$$

$$\frac{\int \frac{a\tan^2(e+fx)+a+b}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}\,d\tan(e+fx)}{ab} - \frac{(a+b)\tan(e+fx)}{ab\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\downarrow 398$$

$$\frac{\frac{a\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+b\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{ab}-\frac{(a+b)\tan(e+fx)}{ab\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\downarrow 224$$

$$\frac{\frac{b\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+a\int \frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{ab}-\frac{(a+b)\tan(e+fx)}{ab\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\downarrow 219$$

$$\frac{\frac{b\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)+\frac{{}_a\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}}}{ab}-\frac{(a+b)\tan(e+fx)}{ab\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\downarrow 291$$

$$\frac{\frac{b\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}+\frac{{}_a\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}}}{ab}-\frac{(a+b)\tan(e+fx)}{ab\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\downarrow 216$$

$$\frac{\frac{\frac{b\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}}+\frac{{}_a\text{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}}}{ab}-\frac{(a+b)\tan(e+fx)}{ab\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

input
```
Int[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(((b*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/Sqrt[a
] + (a*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/Sqr
t[b])/(a*b) - ((a + b)*Tan[e + f*x])/(a*b*Sqrt[a + b + b*Tan[e + f*x]^2]))
/f
```

3.422.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.422.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1185 vs. 2(102) = 204.

Time = 10.59 (sec) , antiderivative size = 1186, normalized size of antiderivative = 10.22

method	result	size
default	Expression too large to display	1186

input
```
int(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f/b^(5/2)/(-a)^(1/2)/a*(4*a*(-a)^(1/2)*b^(3/2)*(csc(f*x+e)-cot(f*x+e))
+4*(-a)^(1/2)*b^(5/2)*(csc(f*x+e)-cot(f*x+e))-2*ln(4*((-a)^(1/2)*(a*(1-cos
(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))
^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)-2*a*(csc(f*x+
e)-cot(f*x+e)))/((1-cos(f*x+e))^2*csc(f*x+e)^2+1))*(a*(1-cos(f*x+e))^4*csc
(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^
2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*b^(5/2)-ln(4*(a*(1-cos(f*x+
e))^2*csc(f*x+e)^2+b*(1-cos(f*x+e))^2*csc(f*x+e)^2+b^(1/2)*(a*(1-cos(f*x+e
))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc
(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot
(f*x+e))+a+b)/((1-cos(f*x+e))^2*csc(f*x+e)^2-2*csc(f*x+e)+2*cot(f*x+e)+1))
*a*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1
-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(
-a)^(1/2)*b-ln(4*(-a*(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(
f*x+e)^2+b^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f
*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^
2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(f*x+e))-a-b)/((1-cos(f*x+e))^2*csc(f*x+e)
^2+2*csc(f*x+e)-2*cot(f*x+e)+1))*a*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-c
os(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x
+e))^2*csc(f*x+e)^2+a+b)^(1/2)*(-a)^(1/2)*b)*(a*(1-cos(f*x+e))^4*csc(f*...
```

3.422.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(102) = 204$.

Time $= 0.71$ (sec) , antiderivative size $= 1655$, normalized size of antiderivative $= 14.27$

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(8*(a^2*b + a*b^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f
*x + e)*sin(f*x + e) + (a*b^2*cos(f*x + e)^2 + b^3)*sqrt(-a)*log(128*a^4*c
os(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b +
5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 -
 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x
 + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)
*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 2*(a^3*cos(f*x
+ e)^2 + a^2*b)*sqrt(b)*log(((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b -
 b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(
b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f
*x + e)^4))/(a^3*b^2*f*cos(f*x + e)^2 + a^2*b^3*f), -1/8*(8*(a^2*b + a*b^2
)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x + e) -
4*(a^3*cos(f*x + e)^2 + a^2*b)*sqrt(-b)*arctan(-1/2*((a - b)*cos(f*x + e)^
3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)
/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e))) + (a*b^2*cos(f*x + e)^2 + b^3)
*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 +
32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2
*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e
)^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5...
```

3.422.6 Sympy [F]

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)**4/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(tan(e + f*x)**4/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.422.7 Maxima [F]

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\tan(fx+e)^4}{\left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(tan(f*x + e)^4/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.422.8 Giac [F]

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\tan(fx+e)^4}{\left(b\sec(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.422.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\tan(e+f\,x)^4}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

input
```
int(tan(e + f*x)^4/(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(tan(e + f*x)^4/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.423 $\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

3.423.1 Optimal result . . . 3090
3.423.2 Mathematica [B] (verified) . . . 3090
3.423.3 Rubi [A] (verified) . . . 3091
3.423.4 Maple [B] (verified) . . . 3093
3.423.5 Fricas [B] (verification not implemented) . . . 3093
3.423.6 Sympy [F] . . . 3094
3.423.7 Maxima [B] (verification not implemented) . . . 3094
3.423.8 Giac [F] . . . 3095
3.423.9 Mupad [F(-1)] . . . 3096

3.423.1 Optimal result

Integrand size = 25, antiderivative size = 71

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f} + \frac{\tan(e+fx)}{af\sqrt{a+b+b\tan^2(e+fx)}}$$

output
```
-arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f+tan(f*x+e
)/a/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

3.423.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 169 vs. 2(71) = 142.

Time = 2.61 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.38

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx =$$
$$-\frac{(a+2b+a\cos(2(e+fx)))\sec^3(e+fx)\left(\arcsin\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)(a+2b+a\cos(2(e+fx))) - \sqrt{2}\sqrt{a}\sqrt{a+}\right.}{4a^{3/2}\sqrt{a+b}f\,(a+b\sec^2(e+fx))^{3/2}\sqrt{\frac{a+b-a\sin^2(e+fx)}{a+b}}}$$

input
```
Integrate[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
-1/4*((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(ArcSin[(Sqrt[a]*Sin[e
 + f*x])/Sqrt[a + b]]*(a + 2*b + a*Cos[2*(e + f*x)]) - Sqrt[2]*Sqrt[a]*Sqr
t[a + b]*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*Sin[e + f*x]))/(a^(3
/2)*Sqrt[a + b]*f*(a + b*Sec[e + f*x]^2)^(3/2)*Sqrt[(a + b - a*Sin[e + f*x
]^2)/(a + b)])
```

3.423.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 4629, 2075, 373, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\tan(e+fx)^2}{(a+b\sec(e+fx)^2)^{3/2}}\,dx$$

$$\downarrow 4629$$

$$\frac{\int \frac{\tan^2(e+fx)}{(\tan^2(e+fx)+1)(a+b(\tan^2(e+fx)+1))^{3/2}}\,d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\tan^2(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^{3/2}}\,d\tan(e+fx)}{f}$$

$$\downarrow 373$$

$$\frac{\frac{\tan(e+fx)}{a\sqrt{a+b\tan^2(e+fx)+b}} - \frac{\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}\,d\tan(e+fx)}{a}}{f}$$

$$\downarrow 291$$

$$\frac{\frac{\tan(e+fx)}{a\sqrt{a+b\tan^2(e+fx)+b}} - \frac{\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}\,d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{a}}{f}$$

$$\downarrow 216$$

$$\frac{\frac{\tan(e+fx)}{a\sqrt{a+b\tan^2(e+fx)+b}}-\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}}}{f}$$

input
```
Int[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(-(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/a^(3/2))
+ Tan[e + f*x]/(a*Sqrt[a + b + b*Tan[e + f*x]^2]))/f
```

3.423.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.423.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 281 vs. $2(63) = 126$.

Time = 4.09 (sec) , antiderivative size = 282, normalized size of antiderivative = 3.97

method	result
default	$-\frac{\left(b+a\cos(fx+e)^2\right)\left(\ln\left(4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)+4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a\right)\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+$ $fa\sqrt{-a}\left(a-$

input

```
int(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
-1/f/a/(-a)^(1/2)*(b+a*cos(f*x+e)^2)*(ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/
(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos
(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1
/2)*cos(f*x+e)+ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)
*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin
(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-(-a)^(1/2)*sin(f*x+
e))/(a+b*sec(f*x+e)^2)^(3/2)*sec(f*x+e)^3
```

3.423.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(63) = 126$.

Time = 0.43 (sec) , antiderivative size = 548, normalized size of antiderivative = 7.72

$$\int \frac{\tan^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \left[\frac{8\,a\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)\sin(fx+e) - \left(a\cos(fx+e)^2+b\right)\sqrt{-a}\log\right.$$

input
```
integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(8*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x
 + e) - (a*cos(f*x + e)^2 + b)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(
a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x +
e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b +
7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a
^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^
3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)*sin(f*x + e)))/(a^3*f*cos(f*x + e)^2 + a^2*b*f), 1/4
*(4*a*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x + e
) + (a*cos(f*x + e)^2 + b)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a
^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt(
(a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*
b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))))/(a^3*f*cos(f*x + e)^
2 + a^2*b*f)]
```

3.423.6 Sympy [F]

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)**2/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(tan(e + f*x)**2/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.423.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2005 vs. $2(63) = 126$.

Time $= 0.50$ (sec) , antiderivative size $= 2005$, normalized size of antiderivative $= 28.24$

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/2*(2*a*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e)
, a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^3 + 2*a*cos(1/2*
arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4
*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))*sin(2*f*x + 2*e) + 2*(a*cos(1/2*a
rctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*
e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 - a*cos(2*f*x + 2*e))*sin(1/2*ar
ctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e
) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)) - (a^2*cos(4*f*x + 4*e)^2 + a^2*sin
(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*
a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x
 + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4
*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*((cos(1/2*arctan2(a*sin(4*f*
x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*
cos(2*f*x + 2*e) + a))^2 + sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b
)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a)
)^2)*arctan2(2*a*sin(2*f*x + 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*
f*x + 4*e)^2 + 4*(a^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b
)*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x +
2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e)
 + 4*(a^2 + 2*a*b)*cos(2*f*x + 2*e))^(1/4)*sqrt(a)*sin(1/2*arctan2(a*si...
```

3.423.8 Giac [F]

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\tan\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.423.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\tan(e+f\,x)^2}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

input
```
int(tan(e + f*x)^2/(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(tan(e + f*x)^2/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.424 $\int \frac{1}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

3.424.1 Optimal result . . . 3097
3.424.2 Mathematica [B] (verified) . . . 3097
3.424.3 Rubi [A] (verified) . . . 3098
3.424.4 Maple [B] (verified) . . . 3100
3.424.5 Fricas [B] (verification not implemented) . . . 3100
3.424.6 Sympy [F] . . . 3102
3.424.7 Maxima [B] (verification not implemented) . . . 3102
3.424.8 Giac [F] . . . 3103
3.424.9 Mupad [F(-1)] . . . 3103

3.424.1 Optimal result

Integrand size = 16, antiderivative size = 77

$$\int \frac{1}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f} - \frac{b\tan(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

output
```
arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f-b*tan(f*x+
e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)
```

3.424.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 168 vs. 2(77) = 154.

Time = 1.45 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.18

$$\int \frac{1}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{(a+2b+a\cos(2(e+fx)))\sec^3(e+fx)\left(\sqrt{a+b}\arcsin\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b}}\right)(a+\right.}{4a^{3/2}(a+b)f\,(a+b\sec^2(e+fx$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(-3/2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*Sec[e + f*x]^3*(Sqrt[a + b]*ArcSin[(Sqrt[a
]*Sin[e + f*x])/Sqrt[a + b]]*(a + 2*b + a*Cos[2*(e + f*x)]) - Sqrt[2]*Sqrt
[a]*b*Sqrt[(a + 2*b + a*Cos[2*(e + f*x)])/(a + b)]*Sin[e + f*x]))/(4*a^(3/
2)*(a + b)*f*(a + b*Sec[e + f*x]^2)^(3/2)*Sqrt[(a + b - a*Sin[e + f*x]^2)/
(a + b)])
```

3.424.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3042, 4616, 296, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{(a+b\sec(e+fx)^2)^{3/2}}\,dx$$

$$\downarrow 4616$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^{3/2}}\,d\tan(e+fx)}{f}$$

$$\downarrow 296$$

$$\frac{\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}\,d\tan(e+fx)}{a} - \frac{b\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\downarrow 291$$

$$\frac{\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}\,d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{a} - \frac{b\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\downarrow 216$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}} - \frac{b\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^(-3/2),x]
```

output
```
(ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]/a^(3/2) - (
b*Tan[e + f*x])/(a*(a + b)*Sqrt[a + b + b*Tan[e + f*x]^2]))/f
```

3.424.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

3.424.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 514 vs. $2(69) = 138$.

Time = 4.43 (sec) , antiderivative size = 515, normalized size of antiderivative = 6.69

method	result
default	$-\frac{\left(b+a\cos(fx+e)^2\right)\left(-\cos(fx+e)\ln\left(4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}\cos(fx+e)+4\sqrt{-a}\sqrt{\frac{b+a\cos(fx+e)^2}{(1+\cos(fx+e))^2}}-4\sin(fx+e)a\right)\sqrt{\frac{b+a\cos(fx-}{(1+\cos(fx+}$

input
```
int(1/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f/(a+b)/a/(-a)^(1/2)*(b+a*cos(f*x+e)^2)*(-cos(f*x+e)*ln(4*(-a)^(1/2)*((
b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*co
s(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*((b+a*cos(f*x+e)^2)/(1
+cos(f*x+e))^2)^(1/2)*a-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(
-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1
/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*b*cos(f*x+
e)+(-a)^(1/2)*b*sin(f*x+e)-((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(
4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)
^(1/2)*((b+a*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*a-((b+a
*cos(f*x+e)^2)/(1+cos(f*x+e))^2)^(1/2)*ln(4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)
/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)+4*(-a)^(1/2)*((b+a*cos(f*x+e)^2)/(1+co
s(f*x+e))^2)^(1/2)-4*sin(f*x+e)*a)*b)/(a+b*sec(f*x+e)^2)^(3/2)*sec(f*x+e)^
3
```

3.424.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(69) = 138$.

Time = 0.43 (sec) , antiderivative size = 601, normalized size of antiderivative = 7.81

$$\int \frac{1}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \left[-\frac{8\,ab\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)\sin(fx+e) + \left((a^2+ab)\cos(fx+e)^2 + \right.\right.$$

$$\left. -\frac{4\,ab\sqrt{\frac{a\cos(fx+e)^2+b}{\cos(fx+e)^2}}\cos(fx+e)\sin(fx+e) + \left((a^2+ab)\cos(fx+e)^2 + ab + b^2\right)\sqrt{a}\arctan\left(\frac{\left(8\,a^2\cos(fx+e\right.}{4\left(2\right.}\right.}{4\left((a^4+a^3b)f\cos(fx+e)^2 + (a^3b+a^2b^2)f\right)}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(8*a*b*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(
f*x + e) + ((a^2 + a*b)*cos(f*x + e)^2 + a*b + b^2)*sqrt(-a)*log(128*a^4*c
os(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b +
5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 -
 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x
 + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)
*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)))/((a^4 + a^3*b)*f
*cos(f*x + e)^2 + (a^3*b + a^2*b^2)*f), -1/4*(4*a*b*sqrt((a*cos(f*x + e)^2
 + b)/cos(f*x + e)^2)*cos(f*x + e)*sin(f*x + e) + ((a^2 + a*b)*cos(f*x + e
)^2 + a*b + b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*
cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x
 + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3
 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))))/((a^4 + a^3*b)*f*cos(f*x + e)^
2 + (a^3*b + a^2*b^2)*f)]
```

3.424.6 Sympy [F]

$$\int \frac{1}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{1}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(-3/2), x)
```

3.424.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2055 vs. 2(69) = 138.

Time = 0.54 (sec) , antiderivative size = 2055, normalized size of antiderivative = 26.69

$$\int \frac{1}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/2*(2*a*b*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2
*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))*sin(2*f*x + 2
*e) - 2*(a^2 + a*b)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2
*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^3 - 2
*(a*b*cos(2*f*x + 2*e) + (a^2 + a*b)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) +
2*(a + 2*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x +
 2*e) + a))^2 - a^2 - 2*a*b)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2
*b)*sin(2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) +
a)) - (a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a^2 + 4*a*b +
4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*sin(2*f*x + 2
*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a^2 + 2*(a^2 +
 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*b)*cos(2*f*x + 2
*e))^(1/4)*(((a + b)*cos(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(
2*f*x + 2*e), a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2 +
(a + b)*sin(1/2*arctan2(a*sin(4*f*x + 4*e) + 2*(a + 2*b)*sin(2*f*x + 2*e),
 a*cos(4*f*x + 4*e) + 2*(a + 2*b)*cos(2*f*x + 2*e) + a))^2)*arctan2(2*a*si
n(2*f*x + 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*(a
^2 + 4*a*b + 4*b^2)*cos(2*f*x + 2*e)^2 + 4*(a^2 + 2*a*b)*sin(4*f*x + 4*e)*
sin(2*f*x + 2*e) + 4*(a^2 + 4*a*b + 4*b^2)*sin(2*f*x + 2*e)^2 + a^2 + 2*(a
^2 + 2*(a^2 + 2*a*b)*cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + 4*(a^2 + 2*a*...
```

3.424.8 Giac [F]

$$\int \frac{1}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{1}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(-3/2), x)
```

3.424.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(e+fx)^2}\right)^{3/2}}\,dx$$

input
```
int(1/(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(1/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.425 $\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

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3.425.1 Optimal result

Integrand size = 25, antiderivative size = 119

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f} - \frac{b\cot(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(a-b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{a(a+b)^2 f}$$

output

```
-arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f-b*cot(f*x
+e)/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)-(a-b)*cot(f*x+e)*(a+b+b*tan(f*x+e
)^2)^(1/2)/a/(a+b)^2/f
```

3.425.2 Mathematica [A] (verified)

Time = 4.46 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.53

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)(a+2b+a\cos(2e+2fx))^{3/2}\sec^3(e+fx)}{2\sqrt{2}a^{3/2}f\,(a+b\sec^2(e+fx))^{3/2}} - \frac{(a+2b+a\cos(2(e+fx)))\,(a^2+2ab-b^2+(a^2+b^2)\cos(2(e+fx)))\csc(e+fx)\sec^3(e+fx)}{4a(a+b)^2 f\,(a+b\sec^2(e+fx))^{3/2}}$$

input
```
Integrate[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
-1/2*(ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*(a + 2
*b + a*Cos[2*e + 2*f*x])^(3/2)*Sec[e + f*x]^3)/(Sqrt[2]*a^(3/2)*f*(a + b*S
ec[e + f*x]^2)^(3/2)) - ((a + 2*b + a*Cos[2*(e + f*x)])*(a^2 + 2*a*b - b^2
 + (a^2 + b^2)*Cos[2*(e + f*x)])*Csc[e + f*x]*Sec[e + f*x]^3)/(4*a*(a + b)
^2*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

3.425.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4629, 2075, 374, 445, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\tan(e+fx)^2\,(a+b\sec(e+fx)^2)^{3/2}}dx$$

$$\downarrow 4629$$

$$\frac{\int \frac{\cot^2(e+fx)}{(\tan^2(e+fx)+1)(a+b(\tan^2(e+fx)+1))^{3/2}}d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\cot^2(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^{3/2}}d\tan(e+fx)}{f}$$

$$\downarrow 374$$

$$\frac{\int \frac{\cot^2(e+fx)\left(-2b\tan^2(e+fx)+a-b\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a(a+b)} - \frac{b\cot(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\downarrow 445$$

$$\frac{-\frac{\int \frac{(a+b)^2}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a+b}-\frac{(a-b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{a(a+b)}-\frac{b\cot(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\downarrow 27$$

$$\frac{\frac{-(a+b)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-\frac{(a-b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{a(a+b)}-\frac{b\cot(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\downarrow 291$$

$$\frac{\frac{-(a+b)\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}-\frac{(a-b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{a(a+b)}-\frac{b\cot(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\downarrow 216$$

$$\frac{-\frac{(a+b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}}-\frac{(a-b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{a(a+b)}-\frac{b\cot(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

input
```
Int[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(-((b*Cot[e + f*x])/(a*(a + b)*Sqrt[a + b + b*Tan[e + f*x]^2])) + (-(((a +
 b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/Sqrt[a]
) - ((a - b)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(a + b))/(a*(a +
 b)))/f
```

3.425.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.425.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1227 vs. 2(109) = 218.

Time = 5.74 (sec) , antiderivative size = 1228, normalized size of antiderivative = 10.32

method	result	size
default	Expression too large to display	1228

input
```
int(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f/(a+b)^2/a/(-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e
))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*c
sc(f*x+e)^2+a+b)*((-a)^(1/2)*a^2*(1-cos(f*x+e))^4*csc(f*x+e)^4+(-a)^(1/2)*
a*b*(1-cos(f*x+e))^4*csc(f*x+e)^4+2*ln(4*((-a)^(1/2)*(a*(1-cos(f*x+e))^4*c
sc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e
)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(f*x+e
)))/((1-cos(f*x+e))^2*csc(f*x+e)^2+1))*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*
(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos
(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*a^2*(csc(f*x+e)-cot(f*x+e))+4*ln(4*((-a
)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2
*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1
/2)-2*a*(csc(f*x+e)-cot(f*x+e)))/((1-cos(f*x+e))^2*csc(f*x+e)^2+1))*(a*(1-
cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+
e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*a*b*(csc(f
*x+e)-cot(f*x+e))+2*ln(4*((-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1
-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f
*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(f*x+e)))/((1-cos(f*x+
e))^2*csc(f*x+e)^2+1))*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4
*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f
*x+e)^2+a+b)^(1/2)*b^2*(csc(f*x+e)-cot(f*x+e))-2*a^2*(1-cos(f*x+e))^2*(...
```

3.425.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(109) = 218$.

Time = 0.73 (sec) , antiderivative size = 741, normalized size of antiderivative = 6.23

$$\int \frac{\cot^2(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{3/2}}\,dx = \left[-\frac{\left(a^2b+2\,ab^2+b^3+\left(a^3+2\,a^2b+ab^2\right)\cos\left(fx+e\right)^2\right)\sqrt{-a}\log\left(128\,a^4\cos\right.\right.$$

input
```
integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*((a^2*b + 2*a*b^2 + b^3 + (a^3 + 2*a^2*b + a*b^2)*cos(f*x + e)^2)*sq
rt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*
(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^
2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2
 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 -
 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(
f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e
))*sin(f*x + e) + 8*((a^3 + a*b^2)*cos(f*x + e)^3 + (a^2*b - a*b^2)*cos(f*
x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^5 + 2*a^4*b + a^
3*b^2)*f*cos(f*x + e)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*f)*sin(f*x + e)),
1/4*((a^2*b + 2*a*b^2 + b^3 + (a^3 + 2*a^2*b + a*b^2)*cos(f*x + e)^2)*sqrt
(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2
 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x
+ e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x +
 e)^2)*sin(f*x + e)))*sin(f*x + e) - 4*((a^3 + a*b^2)*cos(f*x + e)^3 + (a^
2*b - a*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((
(a^5 + 2*a^4*b + a^3*b^2)*f*cos(f*x + e)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)
*f)*sin(f*x + e))]
```

3.425.6 Sympy [F]

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)**2/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(cot(e + f*x)**2/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.425.7 Maxima [F]

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cot(fx+e)^2}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(cot(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(3/2), x)
```

3.425.8 Giac [F]

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cot(fx+e)^2}{(b\sec(fx+e)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.425.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cot(e+f\,x)^2}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

input
```
int(cot(e + f*x)^2/(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(cot(e + f*x)^2/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.426 $\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

3.426.1 Optimal result . . . 3112
3.426.2 Mathematica [A] (verified) . . . 3113
3.426.3 Rubi [A] (verified) . . . 3113
3.426.4 Maple [B] (warning: unable to verify) . . . 3116
3.426.5 Fricas [B] (verification not implemented) . . . 3117
3.426.6 Sympy [F] . . . 3118
3.426.7 Maxima [F(-1)] . . . 3119
3.426.8 Giac [F] . . . 3119
3.426.9 Mupad [F(-1)] . . . 3119

3.426.1 Optimal result

Integrand size = 25, antiderivative size = 174

$$\begin{aligned}\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx &= \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f}\\ &- \frac{b\cot^3(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}\\ &+ \frac{(3a-b)(a+3b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3a(a+b)^3f}\\ &- \frac{(a-3b)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3a(a+b)^2f}\end{aligned}$$

output

```
arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f-b*cot(f*x+
e)^3/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/3*(3*a-b)*(a+3*b)*cot(f*x+e)*(
a+b+b*tan(f*x+e)^2)^(1/2)/a/(a+b)^3/f-1/3*(a-3*b)*cot(f*x+e)^3*(a+b+b*tan(
f*x+e)^2)^(1/2)/a/(a+b)^2/f
```

3.426.2 Mathematica [A] (verified)

Time = 4.24 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.29

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)(a+2b+a\cos(2e+2fx))^{3/2}\sec^3(e+fx)}{2\sqrt{2}a^{3/2}f\,(a+b\sec^2(e+fx))^{3/2}} + \frac{(a+2b+a\cos(2e+2fx))^2\sec^3(e+fx)\left(\frac{(4a+9b)\csc(e+fx)}{12(a+b)^3f} - \frac{\csc^3(e+fx)}{12(a+b)^2f} - \frac{b^3\sin(e+fx)}{2a(a+b)^3f(a+2b+a\cos(2e+2fx))}\right)}{(a+b\sec^2(e+fx))^{3/2}}$$

input
```
Integrate[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*(a + 2*b +
a*Cos[2*e + 2*f*x])^(3/2)*Sec[e + f*x]^3)/(2*Sqrt[2]*a^(3/2)*f*(a + b*Sec[
e + f*x]^2)^(3/2)) + ((a + 2*b + a*Cos[2*e + 2*f*x])^2*Sec[e + f*x]^3*(((4
*a + 9*b)*Csc[e + f*x])/(12*(a + b)^3*f) - Csc[e + f*x]^3/(12*(a + b)^2*f)
 - (b^3*Sin[e + f*x])/(2*a*(a + b)^3*f*(a + 2*b + a*Cos[2*e + 2*f*x]))))/(
a + b*Sec[e + f*x]^2)^(3/2)
```

3.426.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4629, 2075, 374, 445, 445, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{\tan(e+fx)^4\,(a+b\sec(e+fx)^2)^{3/2}}\,dx$$
$$\downarrow 4629$$
$$\frac{\int \frac{\cot^4(e+fx)}{(\tan^2(e+fx)+1)(a+b(\tan^2(e+fx)+1))^{3/2}}\,d\tan(e+fx)}{f}$$
$$\downarrow 2075$$

$$\frac{\int \frac{\cot^4(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^{3/2}}d\tan(e+fx)}{f}$$

$$\downarrow 374$$

$$\frac{\frac{\int \frac{\cot^4(e+fx)\left(-4b\tan^2(e+fx)+a-3b\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a(a+b)} - \frac{b\cot^3(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\downarrow 445$$

$$\frac{-\frac{\int \frac{\cot^2(e+fx)\left(2(a-3b)b\tan^2(e+fx)+(3a-b)(a+3b)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{3(a+b)} - \frac{(a-3b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)}}{a(a+b)} - \frac{b\cot^3(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\downarrow 445$$

$$\frac{-\frac{-\frac{\int \frac{3(a+b)^3}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a+b} - \frac{(3a-b)(a+3b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{3(a+b)} - \frac{(a-3b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)}}{a(a+b)} - \frac{}{a(a+b)}}{f}$$

$$\downarrow 27$$

$$\frac{-\frac{-3(a+b)^2\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx) - \frac{(3a-b)(a+3b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{3(a+b)} - \frac{(a-3b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)}}{a(a+b)} - \frac{}{}}{f}$$

$$\downarrow 291$$

$$\frac{-\frac{-3(a+b)^2\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}} - \frac{(3a-b)(a+3b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{3(a+b)} - \frac{(a-3b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)}}{a(a+b)} - \frac{}{a(a+}}{f}$$

$$\downarrow 216$$

$$\frac{-\frac{-\frac{3(a+b)^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}} - \frac{(3a-b)(a+3b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{3(a+b)} - \frac{(a-3b)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)}}{a(a+b)} - \frac{b\cot^3(e+f}{a(a+b)\sqrt{a+b\tan^2(}}}{f}$$

3.426. $\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

input
```
Int[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(-((b*Cot[e + f*x]^3)/(a*(a + b)*Sqrt[a + b + b*Tan[e + f*x]^2])) + (-1/3*
((a - 3*b)*Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(a + b) - ((-3*(
a + b)^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/Sq
rt[a] - ((3*a - b)*(a + 3*b)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/
(a + b))/(3*(a + b)))/(a*(a + b)))/f
```

3.426.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.426.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1778 vs. $2(158) = 316$.

Time = 6.69 (sec) , antiderivative size = 1779, normalized size of antiderivative = 10.22

method	result	size
default	Expression too large to display	1779

input
```
int(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/24/f/(a+b)^3/a/(-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+
e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*
csc(f*x+e)^2+a+b)*((-a)^(1/2)*a^3*(1-cos(f*x+e))^8*csc(f*x+e)^8+2*(-a)^(1/
2)*a^2*b*(1-cos(f*x+e))^8*csc(f*x+e)^8+(-a)^(1/2)*a*b^2*(1-cos(f*x+e))^8*c
sc(f*x+e)^8-16*(-a)^(1/2)*a^3*(1-cos(f*x+e))^6*csc(f*x+e)^6-48*(-a)^(1/2)*
a^2*b*(1-cos(f*x+e))^6*csc(f*x+e)^6-32*(-a)^(1/2)*a*b^2*(1-cos(f*x+e))^6*c
sc(f*x+e)^6+30*a^3*(1-cos(f*x+e))^4*(-a)^(1/2)*csc(f*x+e)^4+44*a^2*(1-cos(
f*x+e))^4*(-a)^(1/2)*b*csc(f*x+e)^4-66*(-a)^(1/2)*a*b^2*(1-cos(f*x+e))^4*c
sc(f*x+e)^4+48*(-a)^(1/2)*b^3*(1-cos(f*x+e))^4*csc(f*x+e)^4-24*ln(4*((-a)^
(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a
*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2
)-2*a*(csc(f*x+e)-cot(f*x+e)))/((1-cos(f*x+e))^2*csc(f*x+e)^2+1))*(a*(1-co
s(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e)
)^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*a^3*(1-cos(f
*x+e))^3*csc(f*x+e)^3-72*ln(4*((-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4
+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-
cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(f*x+e)))/((1-cos
(f*x+e))^2*csc(f*x+e)^2+1))*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+
e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*
csc(f*x+e)^2+a+b)^(1/2)*a^2*b*(1-cos(f*x+e))^3*csc(f*x+e)^3-72*ln(4*((-...
```

3.426.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 470 vs. $2(158) = 316$.

Time $= 2.11$ (sec) , antiderivative size $= 1061$, normalized size of antiderivative $= 6.10$

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/24*(3*((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cos(f*x + e)^4 - a^3*b - 3*
a^2*b^2 - 3*a*b^3 - b^4 - (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cos(f*x + e)^2)*
sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 3
2*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*
b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)
^2 + 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3
 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*co
s(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x +
 e))*sin(f*x + e) - 8*((4*a^4 + 9*a^3*b + 3*a*b^3)*cos(f*x + e)^5 - (3*a^4
 + 4*a^3*b - 9*a^2*b^2 + 6*a*b^3)*cos(f*x + e)^3 - (3*a^3*b + 8*a^2*b^2 -
3*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^6
 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*f*cos(f*x + e)^4 - (a^6 + 2*a^5*b - 2*a^
3*b^3 - a^2*b^4)*f*cos(f*x + e)^2 - (a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b
^4)*f)*sin(f*x + e)), -1/12*(3*((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cos(f*
x + e)^4 - a^3*b - 3*a^2*b^2 - 3*a*b^3 - b^4 - (a^4 + 2*a^3*b - 2*a*b^3 -
b^4)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a
*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos
(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 -
(a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e)))*sin(f*x + e) - 4*((4*a^4 +
9*a^3*b + 3*a*b^3)*cos(f*x + e)^5 - (3*a^4 + 4*a^3*b - 9*a^2*b^2 + 6*a*...
```

3.426.6 Sympy [F]

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)**4/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(cot(e + f*x)**4/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.426.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.426.8 Giac [F]

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cot\left(fx+e\right)^4}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.426.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cot(e+f\,x)^4}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{3/2}}\,dx$$

input
```
int(cot(e + f*x)^4/(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
int(cot(e + f*x)^4/(a + b/cos(e + f*x)^2)^(3/2), x)
```

3.427 $\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

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3.427.9 Mupad [F(-1)] . . . 3127

3.427.1 Optimal result

Integrand size = 25, antiderivative size = 241

$$\begin{aligned}\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx &= -\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{3/2}f}\\ &-\frac{b\cot^5(e+fx)}{a(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}\\ &-\frac{(15a^3+55a^2b+73ab^2-15b^3)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15a(a+b)^4f}\\ &+\frac{(5a^2+14ab-15b^2)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15a(a+b)^3f}\\ &-\frac{(a-5b)\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5a(a+b)^2f}\end{aligned}$$

output

```
-arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(3/2)/f-b*cot(f*x
+e)^5/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/15*(15*a^3+55*a^2*b+73*a*b^2-
15*b^3)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/a/(a+b)^4/f+1/15*(5*a^2+14*a
*b-15*b^2)*cot(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(1/2)/a/(a+b)^3/f-1/5*(a-5*b)
*cot(f*x+e)^5*(a+b+b*tan(f*x+e)^2)^(1/2)/a/(a+b)^2/f
```

3.427.2 Mathematica [A] (verified)

Time = 6.09 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.98

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx =$$
$$-\frac{\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)(a+2b+a\cos(2e+2fx))^{3/2}\sec^3(e+fx)}{2\sqrt{2}a^{3/2}f\,(a+b\sec^2(e+fx))^{3/2}}$$
$$+\frac{(a+2b+a\cos(2(e+fx)))^2\left(\frac{30b^4}{a(a+2b+a\cos(2(e+fx)))}-(23a^2+80ab+90b^2)\csc^2(e+fx)+(a+b)(11a+20\right.}{60(a+b)^4f\,(a+b\sec^2(e+fx))^{3/2}}$$

input
```
Integrate[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
-1/2*(ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*(a + 2
*b + a*Cos[2*e + 2*f*x])^(3/2)*Sec[e + f*x]^3)/(Sqrt[2]*a^(3/2)*f*(a + b*S
ec[e + f*x]^2)^(3/2)) + ((a + 2*b + a*Cos[2*(e + f*x)])^2*((30*b^4)/(a*(a
+ 2*b + a*Cos[2*(e + f*x)])) - (23*a^2 + 80*a*b + 90*b^2)*Csc[e + f*x]^2 +
 (a + b)*(11*a + 20*b)*Csc[e + f*x]^4 - 3*(a + b)^2*Csc[e + f*x]^6)*Sec[e
+ f*x]^2*Tan[e + f*x])/(60*(a + b)^4*f*(a + b*Sec[e + f*x]^2)^(3/2))
```

3.427.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4629, 2075, 374, 445, 445, 445, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{\tan(e+fx)^6\,(a+b\sec(e+fx)^2)^{3/2}}dx$$
$$\downarrow 4629$$
$$\frac{\int \frac{\cot^6(e+fx)}{(\tan^2(e+fx)+1)(a+b(\tan^2(e+fx)+1))^{3/2}}d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\cot^6(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^{3/2}}d\tan(e+fx)}{f}$$

$$\downarrow 374$$

$$\frac{\int \frac{\cot^6(e+fx)\left(-6b\tan^2(e+fx)+a-5b\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a(a+b)} - \frac{b\cot^5(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\downarrow 445$$

$$\frac{-\frac{\int \frac{\cot^4(e+fx)\left(5a^2+14ba-15b^2+4(a-5b)b\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{5(a+b)} - \frac{(a-5b)\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{5(a+b)}}{a(a+b)} - \frac{b\cot^5(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{f}$$

$$\downarrow 445$$

$$\frac{-\frac{-\frac{\int \frac{\cot^2(e+fx)\left(15a^3+55ba^2+73b^2a-15b^3+2b\left(5a^2+14ba-15b^2\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{3(a+b)} - \frac{\left(5a^2+14ab-15b^2\right)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)}}{5(a+b)} - \frac{(a-5b)\cot}{}}{a(a+b)}}{f}$$

$$\downarrow 445$$

$$\frac{-\frac{-\frac{\int \frac{15(a+b)^4}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a+b} - \frac{\left(15a^3+55a^2b+73ab^2-15b^3\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{3(a+b)} - \frac{\left(5a^2+14ab-15b^2\right)\cot^3(e+fx)\sqrt{a+b}}{3(a+b)}}{5(a+b)}}{a(a+b)}}{f}$$

$$\downarrow 27$$

$$\frac{-\frac{-\frac{-15(a+b)^3\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx) - \frac{\left(15a^3+55a^2b+73ab^2-15b^3\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{3(a+b)} - \frac{\left(5a^2+14ab-15b^2\right)\cot^3(e+f}{3(a+b}}{5(a+b)}}{a(a+b)}}{f}$$

$$\downarrow 291$$

3.427. $\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx$

$$\frac{-\frac{\left(5a^2+14ab-15b^2\right)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)}-\frac{\left(15a^3+55a^2b+73ab^2-15b^3\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}-\frac{15(a+b)^3\int\frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{3(a+b)}}{5(a+b)}}{a(a+b)}}{f}$$

$$\downarrow 216$$

$$\frac{-\frac{\left(5a^2+14ab-15b^2\right)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)}-\frac{\left(15a^3+55a^2b+73ab^2-15b^3\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}-\frac{15(a+b)^3\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}}}{3(a+b)}}{5(a+b)}}{a(a+b)}}{f}$$

input
```
Int[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(3/2),x]
```

output
```
(-((b*Cot[e + f*x]^5)/(a*(a + b)*Sqrt[a + b + b*Tan[e + f*x]^2])) + (-1/5*
((a - 5*b)*Cot[e + f*x]^5*Sqrt[a + b + b*Tan[e + f*x]^2])/(a + b) - (-1/3*
((5*a^2 + 14*a*b - 15*b^2)*Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/
(a + b) - ((-15*(a + b)^3*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan
[e + f*x]^2]])/Sqrt[a] - ((15*a^3 + 55*a^2*b + 73*a*b^2 - 15*b^3)*Cot[e +
f*x]*Sqrt[a + b + b*Tan[e + f*x]^2])/(a + b))/(3*(a + b)))/(5*(a + b)))/(a
*(a + b)))/f
```

3.427.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.427.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2448 vs. $2(221) = 442$.

Time = 8.82 (sec) , antiderivative size = 2449, normalized size of antiderivative = 10.16

method	result	size
default	Expression too large to display	2449

input
```
int(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/480/f/(a+b)^4/a/(-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x
+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2
*csc(f*x+e)^2+a+b)*(3*(-a)^(1/2)*a*b^3+1920*ln(4*((-a)^(1/2)*(a*(1-cos(f*x
+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*c
sc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-c
ot(f*x+e)))/((1-cos(f*x+e))^2*csc(f*x+e)^2+1))*(a*(1-cos(f*x+e))^4*csc(f*x
+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*
b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*a^3*b*(1-cos(f*x+e))^5*csc(f*x+
e)^5+2880*ln(4*((-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e
))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*c
sc(f*x+e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(f*x+e)))/((1-cos(f*x+e))^2*csc(
f*x+e)^2+1))*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e
)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+
b)^(1/2)*a^2*b^2*(1-cos(f*x+e))^5*csc(f*x+e)^5+1920*ln(4*((-a)^(1/2)*(a*(1
-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x
+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)-2*a*(csc(
f*x+e)-cot(f*x+e)))/((1-cos(f*x+e))^2*csc(f*x+e)^2+1))*(a*(1-cos(f*x+e))^4
*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x
+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)*a*b^3*(1-cos(f*x+e))^5*
csc(f*x+e)^5-1804*a^3*(1-cos(f*x+e))^6*(-a)^(1/2)*b*csc(f*x+e)^6-796*(-...
```

3.427.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 698 vs. $2(221) = 442$.

Time = 7.21 (sec) , antiderivative size = 1517, normalized size of antiderivative = 6.29

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/120*(15*((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cos(f*x + e)^
6 + a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5 - (2*a^5 + 7*a^4*b + 8*a
^3*b^2 + 2*a^2*b^3 - 2*a*b^4 - b^5)*cos(f*x + e)^4 + (a^5 + 2*a^4*b - 2*a^
3*b^2 - 8*a^2*b^3 - 7*a*b^4 - 2*b^5)*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*
cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b +
 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4
- 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*
x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2
)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))*sin(f*x + e) + 8
*((23*a^5 + 80*a^4*b + 90*a^3*b^2 + 15*a*b^4)*cos(f*x + e)^7 - (35*a^5 + 1
06*a^4*b + 80*a^3*b^2 - 90*a^2*b^3 + 45*a*b^4)*cos(f*x + e)^5 + (15*a^5 +
20*a^4*b - 56*a^3*b^2 - 160*a^2*b^3 + 45*a*b^4)*cos(f*x + e)^3 + (15*a^4*b
 + 55*a^3*b^2 + 73*a^2*b^3 - 15*a*b^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2))/(((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4
)*f*cos(f*x + e)^6 - (2*a^7 + 7*a^6*b + 8*a^5*b^2 + 2*a^4*b^3 - 2*a^3*b^4
- a^2*b^5)*f*cos(f*x + e)^4 + (a^7 + 2*a^6*b - 2*a^5*b^2 - 8*a^4*b^3 - 7*a
^3*b^4 - 2*a^2*b^5)*f*cos(f*x + e)^2 + (a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*
a^3*b^4 + a^2*b^5)*f)*sin(f*x + e)), 1/60*(15*((a^5 + 4*a^4*b + 6*a^3*b^2
+ 4*a^2*b^3 + a*b^4)*cos(f*x + e)^6 + a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4...
```

3.427.6 Sympy [F]

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)**6/(a+b*sec(f*x+e)**2)**(3/2),x)
```

output
```
Integral(cot(e + f*x)**6/(a + b*sec(e + f*x)**2)**(3/2), x)
```

3.427.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.427.8 Giac [F]

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \int \frac{\cot\left(fx+e\right)^6}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.427.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{3/2}}\,dx = \text{Hanged}$$

input
```
int(cot(e + f*x)^6/(a + b/cos(e + f*x)^2)^(3/2),x)
```

output
```
\text{Hanged}
```

3.428 $\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

3.428.1 Optimal result . . . 3128
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3.428.1 Optimal result

Integrand size = 25, antiderivative size = 97

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} + \frac{(a+b)^2}{3ab^2f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{\frac{1}{a^2}-\frac{1}{b^2}}{f\sqrt{a+b\sec^2(e+fx)}}$$

output
```
-arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2)/f+1/3*(a+b)^2/a/b^2/f/(
a+b*sec(f*x+e)^2)^(3/2)+(1/a^2-1/b^2)/f/(a+b*sec(f*x+e)^2)^(1/2)
```

3.428.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 8.91 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.93

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{4(a+}{3f\left(a+b\sec^2(e+fx)\right)^{5/2}\left(8(a+b)\operatorname{AppellF1}\left(3,\frac{1}{2},\frac{5}{2},4,\sin^2(e+fx),\frac{a\sin^2(e}{a+}\right.}$$

input
```
Integrate[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(4*(a + b)*AppellF1[3, 1/2, 5/2, 4, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a
+ b)]*Tan[e + f*x]^6)/(3*f*(a + b*Sec[e + f*x]^2)^(5/2)*(8*(a + b)*AppellF
1[3, 1/2, 5/2, 4, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*Appel
lF1[4, 1/2, 7/2, 5, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (a + b)*
AppellF1[4, 3/2, 5/2, 5, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[
e + f*x]^2))
```

3.428.3 Rubi [A] (verified)

Time $= 0.32$ (sec) , antiderivative size $= 96$, normalized size of antiderivative $= 0.99$, number of steps used $= 6$, number of rules used $= 5$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used $= \{3042, 4627, 354, 98, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\tan(e+fx)^5}{(a+b\sec(e+fx)^2)^{5/2}}\,dx$$

$$\downarrow 4627$$

$$\frac{\int \frac{\cos(e+fx)\left(1-\sec^2(e+fx)\right)^2}{(b\sec^2(e+fx)+a)^{5/2}}\,d\sec(e+fx)}{f}$$

$$\downarrow 354$$

$$\frac{\int \frac{\cos(e+fx)\left(1-\sec^2(e+fx)\right)^2}{(b\sec^2(e+fx)+a)^{5/2}}\,d\sec^2(e+fx)}{2f}$$

$$\downarrow 98$$

$$\frac{\int \left(-\frac{(a+b)^2}{ab(b\sec^2(e+fx)+a)^{5/2}}+\frac{\cos(e+fx)}{a^2\sqrt{b\sec^2(e+fx)+a}}+\frac{a^2-b^2}{a^2b(b\sec^2(e+fx)+a)^{3/2}}\right)d\sec^2(e+fx)}{2f}$$

$$\downarrow 2009$$

$$\frac{-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{2\left(\frac{1}{a^2}-\frac{1}{b^2}\right)}{\sqrt{a+b\sec^2(e+fx)}}+\frac{2(a+b)^2}{3ab^2(a+b\sec^2(e+fx))^{3/2}}}{2f}$$

input
```
Int[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
((-2*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/a^(5/2) + (2*(a + b)^2)/
(3*a*b^2*(a + b*Sec[e + f*x]^2)^(3/2)) + (2*(a^(-2) - b^(-2)))/Sqrt[a + b*
Sec[e + f*x]^2])/(2*f)
```

3.428.3.1 Defintions of rubi rules used

rule 98
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(x
_)), x_] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*x)^n*((
e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.428.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 6372 vs. 2(85) = 170.

Time = 2.94 (sec) , antiderivative size = 6373, normalized size of antiderivative = 65.70

method	result	size
default	Expression too large to display	6373

input
```
int(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.428.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. 2(85) = 170.

Time = 0.97 (sec) , antiderivative size = 564, normalized size of antiderivative = 5.81

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \left[\frac{3\left(a^2b^2\cos(fx+e)^4+2ab^3\cos(fx+e)^2+b^4\right)\sqrt{a}\log\left(128a^4\cos(fx+e)\right.\right.}{}$$

input
```
integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/24*(3*(a^2*b^2*cos(f*x + e)^4 + 2*a*b^3*cos(f*x + e)^2 + b^4)*sqrt(a)*l
og(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x
 + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^
2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)
*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) - 8*(2*(a^4 - a^3*b - 2*a^2*
b^2)*cos(f*x + e)^4 + 3*(a^3*b - a*b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2))/(a^5*b^2*f*cos(f*x + e)^4 + 2*a^4*b^3*f*cos(f*x
 + e)^2 + a^3*b^4*f), 1/12*(3*(a^2*b^2*cos(f*x + e)^4 + 2*a*b^3*cos(f*x +
e)^2 + b^4)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)
^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(
f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) - 4*(2*(a^4 - a^3*b - 2*a^2*
b^2)*cos(f*x + e)^4 + 3*(a^3*b - a*b^3)*cos(f*x + e)^2)*sqrt((a*cos(f*x +
e)^2 + b)/cos(f*x + e)^2))/(a^5*b^2*f*cos(f*x + e)^4 + 2*a^4*b^3*f*cos(f*x
 + e)^2 + a^3*b^4*f)]
```

3.428.6 Sympy [F]

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)**5/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral(tan(e + f*x)**5/(a + b*sec(e + f*x)**2)**(5/2), x)
```

3.428.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.428.8 Giac [F]

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\tan(fx+e)^5}{\left(b\sec(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.428.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\tan(e+fx)^5}{\left(a+\frac{b}{\cos(e+fx)^2}\right)^{5/2}}\,dx$$

input
```
int(tan(e + f*x)^5/(a + b/cos(e + f*x)^2)^(5/2),x)
```

output
```
int(tan(e + f*x)^5/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.429 $\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

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3.429.2 Mathematica [C] (warning: unable to verify) . . . 3135
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3.429.1 Optimal result

Integrand size = 25, antiderivative size = 89

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} - \frac{a+b}{3abf\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{1}{a^2f\sqrt{a+b\sec^2(e+fx)}}$$

output

```
arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2)/f+1/3*(-a-b)/a/b/f/(a+b*
sec(f*x+e)^2)^(3/2)-1/a^2/f/(a+b*sec(f*x+e)^2)^(1/2)
```

3.429.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.45 (sec) , antiderivative size = 613, normalized size of antiderivative = 6.89

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx =$$
$$-\frac{(a+3b+a\cos(2(e+fx)))(a+2b+a\cos(2e+2fx))^{5/2}\sec^4(e+fx)}{48b^2f(a+2b+a\cos(2(e+fx)))^{3/2}(a+b\sec^2(e+fx))^{5/2}}$$
$$+\frac{(a+b+(a-2b)\cos(2(e+fx)))(a+2b+a\cos(2e+2fx))^{5/2}\sec^4(e+fx)}{96b^2f(a+2b+a\cos(2(e+fx)))^{3/2}(a+b\sec^2(e+fx))^{5/2}}$$
$$-\frac{e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}(a+2b+a\cos(2e+2fx))^{5/2}\left(-\frac{\sqrt{a}(1+e^{2i(e+fx)})\left(-96b^3e^{2i(e+fx)}+a^3(1-\right.\right.}{b^2\left(4\right.}$$

input
```
Integrate[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
-1/48*((a + 3*b + a*Cos[2*(e + f*x)])*(a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)
*Sec[e + f*x]^4)/(b^2*f*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2)*(a + b*Sec[e
+ f*x]^2)^(5/2)) + ((a + b + (a - 2*b)*Cos[2*(e + f*x)])*(a + 2*b + a*Cos[
2*e + 2*f*x])^(5/2)*Sec[e + f*x]^4)/(96*b^2*f*(a + 2*b + a*Cos[2*(e + f*x)
])^(3/2)*(a + b*Sec[e + f*x]^2)^(5/2)) - (E^(I*(e + f*x))*Sqrt[4*b + (a*(1
 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f*x))]*(a + 2*b + a*Cos[2*e + 2*f
*x])^(5/2)*(-((Sqrt[a]*(1 + E^((2*I)*(e + f*x)))*(-96*b^3*E^((2*I)*(e + f*
x)) + a^3*(1 + E^((2*I)*(e + f*x)))^2 - 32*a*b^2*(1 + E^((2*I)*(e + f*x)))
^2 - 6*a^2*b*(1 + E^((2*I)*(e + f*x)) + E^((4*I)*(e + f*x)))))/(b^2*(4*b*E
^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2)^2)) + ((24*I)*f*x - 12
*Log[a + 2*b + a*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)
) + a*(1 + E^((2*I)*(e + f*x)))^2]] - 12*Log[a + a*E^((2*I)*(e + f*x)) + 2
*b*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^(
(2*I)*(e + f*x)))^2]])/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e +
 f*x)))^2])*Sec[e + f*x]^5)/(96*Sqrt[2]*a^(5/2)*f*(a + b*Sec[e + f*x]^2)^(
5/2))
```

3.429.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4627, 25, 354, 87, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\tan(e+fx)^3}{(a+b\sec(e+fx)^2)^{5/2}}\,dx$$

$$\downarrow 4627$$

$$\frac{\int -\frac{\cos(e+fx)(1-\sec^2(e+fx))}{(b\sec^2(e+fx)+a)^{5/2}}\,d\sec(e+fx)}{f}$$

$$\downarrow 25$$

$$-\frac{\int \frac{\cos(e+fx)(1-\sec^2(e+fx))}{(b\sec^2(e+fx)+a)^{5/2}}\,d\sec(e+fx)}{f}$$

$$\downarrow 354$$

$$-\frac{\int \frac{\cos(e+fx)(1-\sec^2(e+fx))}{(b\sec^2(e+fx)+a)^{5/2}}\,d\sec^2(e+fx)}{2f}$$

$$\downarrow 87$$

$$-\frac{\frac{\int \frac{\cos(e+fx)}{\left(b\sec^2(e+fx)+a\right)^{3/2}}\,d\sec^2(e+fx)}{a}+\frac{2(a+b)}{3ab(a+b\sec^2(e+fx))^{3/2}}}{2f}$$

$$\downarrow 61$$

$$-\frac{\frac{\int \frac{\cos(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}\,d\sec^2(e+fx)}{a}+\frac{2}{a\sqrt{a+b\sec^2(e+fx)}}}{a}+\frac{2(a+b)}{3ab(a+b\sec^2(e+fx))^{3/2}}}{2f}$$

$$\downarrow 73$$

$$-\frac{\frac{2\int \frac{1}{\frac{\sec^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sec^2(e+fx)+a}}{ab}+\frac{2}{a\sqrt{a+b\sec^2(e+fx)}}}{a}+\frac{2(a+b)}{3ab(a+b\sec^2(e+fx))^{3/2}}}{2f}$$

$$\downarrow 221$$

$$-\frac{\frac{\frac{2}{a\sqrt{a+b\sec^2(e+fx)}}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}}}{a}+\frac{2(a+b)}{3ab(a+b\sec^2(e+fx))^{3/2}}}{2f}$$

input
```
Int[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
-1/2*((2*(a + b))/(3*a*b*(a + b*Sec[e + f*x]^2)^(3/2)) + ((-2*ArcTanh[Sqrt
[a + b*Sec[e + f*x]^2]/Sqrt[a]])/a^(3/2) + 2/(a*Sqrt[a + b*Sec[e + f*x]^2]
))/a)/f
```

3.429.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.429.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 6312 vs. $2(81) = 162$.

Time = 2.75 (sec) , antiderivative size = 6313, normalized size of antiderivative = 70.93

method	result	size
default	Expression too large to display	6313

input
```
int(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.429.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(77) = 154.

Time = 0.84 (sec) , antiderivative size = 522, normalized size of antiderivative = 5.87

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \left[\frac{3\left(a^2b\cos(fx+e)^4+2ab^2\cos(fx+e)^2+b^3\right)\sqrt{a}\log\left(128a^4\cos(fx+e)^8\right.}{} \right.$$

$$\left. -\frac{3\left(a^2b\cos(fx+e)^4+2ab^2\cos(fx+e)^2+b^3\right)\sqrt{-a}\arctan\left(\frac{\left(8a^2\cos(fx+e)^4+8ab\cos(fx+e)^2+b^2\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)^2}{\cos(fx+e)^2}}}{4\left(2a^3\cos(fx+e)^4+3a^2b\cos(fx+e)^2+ab^2\right)}\right.}{12\left(a^5bf\cos(fx+e)^4+2a^4b^2f\cos(fx+e)\right.} \right]$$

input
```
integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/24*(3*(a^2*b*cos(f*x + e)^4 + 2*a*b^2*cos(f*x + e)^2 + b^3)*sqrt(a)*log
(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x +
 e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*
b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) - 8*(3*a*b^2*cos(f*x + e)^2 +
(a^3 + 4*a^2*b)*cos(f*x + e)^4)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
))/(a^5*b*f*cos(f*x + e)^4 + 2*a^4*b^2*f*cos(f*x + e)^2 + a^3*b^3*f), -1/1
2*(3*(a^2*b*cos(f*x + e)^4 + 2*a*b^2*cos(f*x + e)^2 + b^3)*sqrt(-a)*arctan
(1/4*(8*a^2*cos(f*x + e)^4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*
cos(f*x + e)^2 + b)/cos(f*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*
x + e)^2 + a*b^2)) + 4*(3*a*b^2*cos(f*x + e)^2 + (a^3 + 4*a^2*b)*cos(f*x +
 e)^4)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a^5*b*f*cos(f*x + e)^
4 + 2*a^4*b^2*f*cos(f*x + e)^2 + a^3*b^3*f)]
```

3.429.6 Sympy [F]

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)**3/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral(tan(e + f*x)**3/(a + b*sec(e + f*x)**2)**(5/2), x)
```

3.429.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.429.8 Giac [F]

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\tan\left(fx+e\right)^3}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.429.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\tan(e+f\,x)^3}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

input
```
int(tan(e + f*x)^3/(a + b/cos(e + f*x)^2)^(5/2),x)
```

output
```
int(tan(e + f*x)^3/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.430 $\int \frac{\tan(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

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3.430.1 Optimal result

Integrand size = 23, antiderivative size = 83

$$\int \frac{\tan(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} + \frac{1}{3af\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{1}{a^2f\sqrt{a+b\sec^2(e+fx)}}$$

output

```
-arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2)/f+1/3/a/f/(a+b*sec(f*x+
e)^2)^(3/2)+1/a^2/f/(a+b*sec(f*x+e)^2)^(1/2)
```

3.430.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.12 (sec) , antiderivative size = 613, normalized size of antiderivative = 7.39

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx =$$
$$-\frac{(a+3b+a\cos(2(e+fx)))(a+2b+a\cos(2e+2fx))^{5/2}\sec^4(e+fx)}{48b^2f(a+2b+a\cos(2(e+fx)))^{3/2}(a+b\sec^2(e+fx))^{5/2}}$$
$$+\frac{(a+b+(a-2b)\cos(2(e+fx)))(a+2b+a\cos(2e+2fx))^{5/2}\sec^4(e+fx)}{32b^2f(a+2b+a\cos(2(e+fx)))^{3/2}(a+b\sec^2(e+fx))^{5/2}}$$
$$+\frac{e^{i(e+fx)}\sqrt{4b+ae^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2}(a+2b+a\cos(2e+2fx))^{5/2}\left(-\frac{\sqrt{a}(1+e^{2i(e+fx)})\left(-96b^3e^{2i(e+fx)}+a^3(1+\right.\right.}{b^2\left(4b\right.}$$

input
```
Integrate[Tan[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
-1/48*((a + 3*b + a*Cos[2*(e + f*x)])*(a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)
*Sec[e + f*x]^4)/(b^2*f*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2)*(a + b*Sec[e
+ f*x]^2)^(5/2)) + ((a + b + (a - 2*b)*Cos[2*(e + f*x)])*(a + 2*b + a*Cos[
2*e + 2*f*x])^(5/2)*Sec[e + f*x]^4)/(32*b^2*f*(a + 2*b + a*Cos[2*(e + f*x)
])^(3/2)*(a + b*Sec[e + f*x]^2)^(5/2)) + (E^(I*(e + f*x))*Sqrt[4*b + (a*(1
 + E^((2*I)*(e + f*x)))^2)/E^((2*I)*(e + f*x))]*(a + 2*b + a*Cos[2*e + 2*f
*x])^(5/2)*(-((Sqrt[a]*(1 + E^((2*I)*(e + f*x)))*(-96*b^3*E^((2*I)*(e + f*
x)) + a^3*(1 + E^((2*I)*(e + f*x)))^2 - 32*a*b^2*(1 + E^((2*I)*(e + f*x)))
^2 - 6*a^2*b*(1 + E^((2*I)*(e + f*x)) + E^((4*I)*(e + f*x)))))/(b^2*(4*b*E
^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e + f*x)))^2)^2)) + ((24*I)*f*x - 12
*Log[a + 2*b + a*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)
) + a*(1 + E^((2*I)*(e + f*x)))^2]] - 12*Log[a + a*E^((2*I)*(e + f*x)) + 2
*b*E^((2*I)*(e + f*x)) + Sqrt[a]*Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^(
(2*I)*(e + f*x)))^2]])/Sqrt[4*b*E^((2*I)*(e + f*x)) + a*(1 + E^((2*I)*(e +
 f*x)))^2])*Sec[e + f*x]^5)/(96*Sqrt[2]*a^(5/2)*f*(a + b*Sec[e + f*x]^2)^(
5/2))
```

3.430.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4627, 243, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\tan(e+fx)}{(a+b\sec(e+fx)^2)^{5/2}}\,dx$$

$$\downarrow 4627$$

$$\frac{\int \frac{\cos(e+fx)}{(b\sec^2(e+fx)+a)^{5/2}}\,d\sec(e+fx)}{f}$$

$$\downarrow 243$$

$$\frac{\int \frac{\cos(e+fx)}{(b\sec^2(e+fx)+a)^{5/2}}\,d\sec^2(e+fx)}{2f}$$

$$\downarrow 61$$

$$\frac{\frac{\int \frac{\cos(e+fx)}{\left(b\sec^2(e+fx)+a\right)^{3/2}}\,d\sec^2(e+fx)}{a}+\frac{2}{3a(a+b\sec^2(e+fx))^{3/2}}}{2f}$$

$$\downarrow 61$$

$$\frac{\frac{\int \frac{\cos(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}\,d\sec^2(e+fx)}{a}+\frac{2}{a\sqrt{a+b\sec^2(e+fx)}}}{a}+\frac{2}{3a(a+b\sec^2(e+fx))^{3/2}}}{2f}$$

$$\downarrow 73$$

$$\frac{\frac{2\int \frac{1}{\frac{\sec^4(e+fx)}{b}-\frac{a}{b}}\,d\sqrt{b\sec^2(e+fx)+a}}{ab}+\frac{2}{a\sqrt{a+b\sec^2(e+fx)}}}{a}+\frac{2}{3a(a+b\sec^2(e+fx))^{3/2}}}{2f}$$

$$\downarrow 221$$

$$\frac{\frac{2}{a\sqrt{a+b\sec^2(e+fx)}}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}}}{a}+\frac{2}{3a(a+b\sec^2(e+fx))^{3/2}}}{2f}$$

input
```
Int[Tan[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(2/(3*a*(a + b*Sec[e + f*x]^2)^(3/2)) + ((-2*ArcTanh[Sqrt[a + b*Sec[e + f*
x]^2]/Sqrt[a]])/a^(3/2) + 2/(a*Sqrt[a + b*Sec[e + f*x]^2]))/a)/(2*f)
```

3.430.3.1 Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.430.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{1}{3af\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}}+\frac{1}{a^2f\sqrt{a+b\sec(fx+e)^2}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\sqrt{a+b\sec(fx+e)^2}}{\sec(fx+e)}\right)}{fa^{\frac{5}{2}}}$	86
default	$\frac{1}{3af\left(a+b\sec(fx+e)^2\right)^{\frac{3}{2}}}+\frac{1}{a^2f\sqrt{a+b\sec(fx+e)^2}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\sqrt{a+b\sec(fx+e)^2}}{\sec(fx+e)}\right)}{fa^{\frac{5}{2}}}$	86

input
```
int(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/a/f/(a+b*sec(f*x+e)^2)^(3/2)+1/a^2/f/(a+b*sec(f*x+e)^2)^(1/2)-1/f/a^(5
/2)*ln((2*a+2*a^(1/2)*(a+b*sec(f*x+e)^2)^(1/2))/sec(f*x+e))
```

3.430.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. 2(71) = 142.

Time = 0.71 (sec) , antiderivative size = 494, normalized size of antiderivative = 5.95

$$\int \frac{\tan(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \left[\frac{3\left(a^2\cos(fx+e)^4+2ab\cos(fx+e)^2+b^2\right)\sqrt{a}\log\left(128a^4\cos(fx+e)^8-\right.\right.$$

input
```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output

```
[1/24*(3*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*sqrt(a)*log(128
*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^
4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*co
s(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt(
(a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + 8*(4*a^2*cos(f*x + e)^4 + 3*a*b*
cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(a^5*f*cos(f*
x + e)^4 + 2*a^4*b*f*cos(f*x + e)^2 + a^3*b^2*f), 1/12*(3*(a^2*cos(f*x + e
)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*arctan(1/4*(8*a^2*cos(f*x + e)^
4 + 8*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f
*x + e)^2)/(2*a^3*cos(f*x + e)^4 + 3*a^2*b*cos(f*x + e)^2 + a*b^2)) + 4*(4
*a^2*cos(f*x + e)^4 + 3*a*b*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2))/(a^5*f*cos(f*x + e)^4 + 2*a^4*b*f*cos(f*x + e)^2 + a^3*b^2*
f)]
```

3.430.6 Sympy [A] (verification not implemented)

Time = 9.79 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.20

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \begin{cases} \dfrac{2\left(\frac{b}{6af\left(a+b\sec^2(e+fx)\right)^{\frac{3}{2}}}+\frac{b}{2a^2f\sqrt{a+b\sec^2(e+fx)}}+\frac{b\operatorname{atan}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{-a}}\right)}{2a^2f\sqrt{-a}}\right)}{b} & \text{for } b\neq 0 \\ \dfrac{\log\left(\sec^2(e+fx)\right)}{2a^{\frac{5}{2}}f} & \text{otherwise} \end{cases}$$

input

```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output

```
Piecewise((2*(b/(6*a*f*(a + b*sec(e + f*x)**2)**(3/2)) + b/(2*a**2*f*sqrt(
a + b*sec(e + f*x)**2)) + b*atan(sqrt(a + b*sec(e + f*x)**2)/sqrt(-a))/(2*
a**2*f*sqrt(-a)))/b, Ne(b, 0)), (log(sec(e + f*x)**2)/(2*a**(5/2)*f), True
))
```

3.430.7 Maxima [F]

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\tan(fx+e)}{\left(b\sec(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(tan(f*x + e)/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

3.430.8 Giac [F]

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\tan(fx+e)}{\left(b\sec(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.430.9 Mupad [B] (verification not implemented)

Time = 24.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.82

$$\int \frac{\tan(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\frac{a+\frac{b}{\cos(e+fx)^2}}{a^2}+\frac{1}{3a}}{f\left(a+\frac{b}{\cos(e+fx)^2}\right)^{3/2}} - \frac{\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{\cos(e+fx)^2}}}{\sqrt{a}}\right)}{a^{5/2}f}$$

input
```
int(tan(e + f*x)/(a + b/cos(e + f*x)^2)^(5/2),x)
```

output
```
((a + b/cos(e + f*x)^2)/a^2 + 1/(3*a))/(f*(a + b/cos(e + f*x)^2)^(3/2)) -
atanh((a + b/cos(e + f*x)^2)^(1/2)/a^(1/2))/(a^(5/2)*f)
```

3.431 $\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

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3.431.1 Optimal result

Integrand size = 23, antiderivative size = 137

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}f} - \frac{b}{3a(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{b(2a+b)}{a^2(a+b)^2f\sqrt{a+b\sec^2(e+fx)}}$$

output
```
arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2)/f-arctanh((a+b*sec(f*x+e
)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(5/2)/f-1/3*b/a/(a+b)/f/(a+b*sec(f*x+e)^2)^(
3/2)-b*(2*a+b)/a^2/(a+b)^2/f/(a+b*sec(f*x+e)^2)^(1/2)
```

3.431.2 Mathematica [F]

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

input
```
Integrate[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
Integrate[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2), x]
```

3.431.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.18, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 4627, 25, 354, 96, 25, 169, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\tan(e+fx)\,(a+b\sec(e+fx)^2)^{5/2}}dx$$

$$\downarrow 4627$$

$$\frac{\int -\frac{\cos(e+fx)}{(1-\sec^2(e+fx))(b\sec^2(e+fx)+a)^{5/2}}d\sec(e+fx)}{f}$$

$$\downarrow 25$$

$$-\frac{\int \frac{\cos(e+fx)}{(1-\sec^2(e+fx))(b\sec^2(e+fx)+a)^{5/2}}d\sec(e+fx)}{f}$$

$$\downarrow 354$$

$$-\frac{\int \frac{\cos(e+fx)}{(1-\sec^2(e+fx))(b\sec^2(e+fx)+a)^{5/2}}d\sec^2(e+fx)}{2f}$$

$$\downarrow 96$$

$$-\frac{\frac{2b}{3a(a+b)(a+b\sec^2(e+fx))^{3/2}}-\frac{\int -\frac{\cos(e+fx)\left(-b\sec^2(e+fx)+a+b\right)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec^2(e+fx)}{a(a+b)}}{2f}$$

$$\downarrow 25$$

$$-\frac{\frac{\int \frac{\cos(e+fx)\left(-b\sec^2(e+fx)+a+b\right)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec^2(e+fx)}{a(a+b)}+\frac{2b}{3a(a+b)(a+b\sec^2(e+fx))^{3/2}}}{2f}$$

$$\downarrow 169$$

$$-\frac{\frac{2\int \frac{\cos(e+fx)\left((a+b)^2-b(2a+b)\sec^2(e+fx)\right)}{2\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{a(a+b)}+\frac{2b(2a+b)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{a(a+b)}+\frac{2b}{3a(a+b)(a+b\sec^2(e+fx))^{3/2}}}{2f}$$

$$\downarrow 27$$

$$-\frac{\frac{\int \frac{\cos(e+fx)\left((a+b)^2-b(2a+b)\sec^2(e+fx)\right)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{a(a+b)}+\frac{2b(2a+b)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{a(a+b)}+\frac{2b}{3a(a+b)(a+b\sec^2(e+fx))^{3/2}}}{2f}$$

$$\downarrow 174$$

$$-\frac{\frac{a^2\int \frac{1}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)+(a+b)^2\int \frac{\cos(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{a(a+b)}+\frac{2b(2a+b)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{a(a+b)}+\frac{2b}{3a(a+b)(a+b\sec^2(e+fx))^3}}{2f}$$

$$\downarrow 73$$

$$-\frac{\frac{2a^2\int \frac{1}{\frac{a+b}{b}-\frac{\sec^4(e+fx)}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}+\frac{2(a+b)^2\int \frac{1}{\frac{\sec^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}}{a(a+b)}+\frac{2b(2a+b)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{a(a+b)}+\frac{2b}{3a(a+b)(a+b\sec^2(e+fx))^{3/2}}}{2f}$$

$$\downarrow 221$$

$$-\frac{\frac{2a^2\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{2(a+b)^2\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}}{a(a+b)}+\frac{2b(2a+b)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{a(a+b)}+\frac{2b}{3a(a+b)(a+b\sec^2(e+fx))^{3/2}}}{2f}$$

input
```
Int[Cot[e + f*x]/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
-1/2*((2*b)/(3*a*(a + b)*(a + b*Sec[e + f*x]^2)^(3/2)) + (((-2*(a + b)^2*A
rcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/Sqrt[a] + (2*a^2*ArcTanh[Sqrt[
a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/Sqrt[a + b])/(a*(a + b)) + (2*b*(2*a +
 b))/(a*(a + b)*Sqrt[a + b*Sec[e + f*x]^2]))/(a*(a + b)))/f
```

3.431. $\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

3.431.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 96
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))), x] + S
imp[1/((b*e - a*f)*(d*e - c*f))   Int[(b*d*e - b*c*f - a*d*f - b*d*f*x)*((e
 + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && LtQ[p, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.431.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 43854 vs. $2(119) = 238$.

Time = 4.92 (sec) , antiderivative size = 43855, normalized size of antiderivative = 320.11

method	result	size
default	Expression too large to display	43855

input
```
int(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.431.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 523 vs. $2(119) = 238$.

Time = 1.48 (sec) , antiderivative size = 2279, normalized size of antiderivative = 16.64

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/24*(3*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5 + (a^5 + 3*a^4*b + 3*a^3*b^2
 + a^2*b^3)*cos(f*x + e)^4 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*cos
(f*x + e)^2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6
 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 + 8*(16*a^3*
cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*c
os(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + 6*(a
^5*cos(f*x + e)^4 + 2*a^4*b*cos(f*x + e)^2 + a^3*b^2)*sqrt(a + b)*log(2*((
8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b
^2 - 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1
)) - 8*((7*a^4*b + 11*a^3*b^2 + 4*a^2*b^3)*cos(f*x + e)^4 + 3*(2*a^3*b^2 +
 3*a^2*b^3 + a*b^4)*cos(f*x + e)^2)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
e)^2))/((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*f*cos(f*x + e)^4 + 2*(a^7*b
+ 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a^6*b^2 + 3*a^5*b^3
 + 3*a^4*b^4 + a^3*b^5)*f), 1/24*(12*(a^5*cos(f*x + e)^4 + 2*a^4*b*cos(f*x
 + e)^2 + a^3*b^2)*sqrt(-a - b)*arctan(1/2*((2*a + b)*cos(f*x + e)^2 + b)*
sqrt(-a - b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((a^2 + a*b)*cos(
f*x + e)^2 + a*b + b^2)) + 3*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5 + (a^5 +
 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cos(f*x + e)^4 + 2*(a^4*b + 3*a^3*b^2 + 3*
a^2*b^3 + a*b^4)*cos(f*x + e)^2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 + 2...
```

3.431.6 Sympy [F]

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral(cot(e + f*x)/(a + b*sec(e + f*x)**2)**(5/2), x)
```

3.431.7 Maxima [F]

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cot(fx+e)}{\left(b\sec(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(cot(f*x + e)/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

3.431.8 Giac [F]

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cot(fx+e)}{\left(b\sec(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.431.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cot(e+f\,x)}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

input
```
int(cot(e + f*x)/(a + b/cos(e + f*x)^2)^(5/2),x)
```

output
```
int(cot(e + f*x)/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.432 $\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

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3.432.1 Optimal result

Integrand size = 25, antiderivative size = 200

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} + \frac{(2a+7b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{2(a+b)^{7/2}f} - \frac{(3a-2b)b}{6a(a+b)^2 f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^2(e+fx)}{2(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{b(a^2-6ab-2b^2)}{2a^2(a+b)^3 f\sqrt{a+b\sec^2(e+fx)}}$$

output

```
-arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2)/f+1/2*(2*a+7*b)*arctanh
((a+b*sec(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(7/2)/f-1/6*(3*a-2*b)*b/a/(a+
b)^2/f/(a+b*sec(f*x+e)^2)^(3/2)-1/2*cot(f*x+e)^2/(a+b)/f/(a+b*sec(f*x+e)^2
)^(3/2)-1/2*b*(a^2-6*a*b-2*b^2)/a^2/(a+b)^3/f/(a+b*sec(f*x+e)^2)^(1/2)
```

3.432.2 Mathematica [F]

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

input
```
Integrate[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
Integrate[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2), x]
```

3.432.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.14, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 4627, 354, 114, 27, 169, 27, 169, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\tan(e+fx)^3\left(a+b\sec(e+fx)^2\right)^{5/2}}dx$$

$$\downarrow 4627$$

$$\frac{\int \frac{\cos(e+fx)}{(1-\sec^2(e+fx))^2(b\sec^2(e+fx)+a)^{5/2}}d\sec(e+fx)}{f}$$

$$\downarrow 354$$

$$\frac{\int \frac{\cos(e+fx)}{(1-\sec^2(e+fx))^2(b\sec^2(e+fx)+a)^{5/2}}d\sec^2(e+fx)}{2f}$$

$$\downarrow 114$$

$$\frac{\frac{1}{(a+b)(1-\sec^2(e+fx))(a+b\sec^2(e+fx))^{3/2}} - \frac{\int -\frac{\cos(e+fx)\left(5b\sec^2(e+fx)+2a+2b\right)}{2\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{5/2}}d\sec^2(e+fx)}{a+b}}{2f}$$

$$\downarrow 27$$

$$\frac{\frac{\int \frac{\cos(e+fx)\left(5b\sec^2(e+fx)+2(a+b)\right)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{5/2}}d\sec^2(e+fx)}{2(a+b)}+\frac{1}{(a+b)(1-\sec^2(e+fx))(a+b\sec^2(e+fx))^{3/2}}}{2f}$$

$$\downarrow 169$$

$$\frac{\frac{2\int \frac{3\cos(e+fx)\left(2(a+b)^2+(3a-2b)b\sec^2(e+fx)\right)}{2\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec^2(e+fx)}{3a(a+b)}+\frac{2b\left(\frac{2}{a}-\frac{5}{a+b}\right)}{3\left(a+b\sec^2(e+fx)\right)^{3/2}}}{2(a+b)}+\frac{1}{(a+b)(1-\sec^2(e+fx))(a+b\sec^2(e+fx))^{3/2}}}{2f}$$

$$\downarrow 27$$

$$\frac{\frac{\int \frac{\cos(e+fx)\left(2(a+b)^2+(3a-2b)b\sec^2(e+fx)\right)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec^2(e+fx)}{a(a+b)}+\frac{2b\left(\frac{2}{a}-\frac{5}{a+b}\right)}{3\left(a+b\sec^2(e+fx)\right)^{3/2}}}{2(a+b)}+\frac{1}{(a+b)(1-\sec^2(e+fx))(a+b\sec^2(e+fx))^{3/2}}}{2f}$$

$$\downarrow 169$$

$$\frac{\frac{2\int \frac{\cos(e+fx)\left(2(a+b)^3+b\left(a^2-6ba-2b^2\right)\sec^2(e+fx)\right)}{2\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{a(a+b)}-\frac{2b\left(a^2-6ab-2b^2\right)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{a(a+b)}+\frac{\frac{2b\left(\frac{2}{a}-\frac{5}{a+b}\right)}{3\left(a+b\sec^2(e+fx)\right)^{3/2}}}{2(a+b)}+\frac{1}{(a+b)(1-\sec^2(e+fx))(a+b\sec^2}}{2f}$$

$$\downarrow 27$$

$$\frac{\frac{\int \frac{\cos(e+fx)\left(2(a+b)^3+b\left(a^2-6ba-2b^2\right)\sec^2(e+fx)\right)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{a(a+b)}-\frac{2b\left(a^2-6ab-2b^2\right)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{a(a+b)}+\frac{\frac{2b\left(\frac{2}{a}-\frac{5}{a+b}\right)}{3\left(a+b\sec^2(e+fx)\right)^{3/2}}}{2(a+b)}+\frac{1}{(a+b)(1-\sec^2(e+fx))(a+b\sec^2(}}{2f}$$

$$\downarrow 174$$

$$\frac{\frac{a^2(2a+7b)\int \frac{1}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)+2(a+b)^3\int \frac{\cos(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{a(a+b)}-\frac{2b\left(a^2-6ab-2b^2\right)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{a(a+b)}+\frac{\frac{2b\left(\frac{2}{a}-\frac{5}{a+b}\right)}{3\left(a+b\sec^2(e+fx)\right)^{3/2}}}{2(a+b)}+$$

$$\downarrow 73$$

3.432. $\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

$$\frac{\frac{2a^2(2a+7b)\int \frac{1}{\frac{a+b}{b}-\frac{\sec^4(e+fx)}{b}}d\sqrt{b\sec^2(e+fx)+a}}{a(a+b)}+\frac{4(a+b)^3\int \frac{1}{\frac{\sec^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}}{a(a+b)}-\frac{2b\left(a^2-6ab-2b^2\right)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}+\frac{2b\left(\frac{2}{a}-\frac{5}{a+b}\right)}{3\left(a+b\sec^2(e+fx)\right)^{3/2}}}{2(a+b)}+$$

$$\Big\downarrow 221$$

$$\frac{\frac{2a^2(2a+7b)\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{4(a+b)^3\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}}{a(a+b)}-\frac{2b\left(a^2-6ab-2b^2\right)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}+\frac{2b\left(\frac{2}{a}-\frac{5}{a+b}\right)}{3\left(a+b\sec^2(e+fx)\right)^{3/2}}}{2(a+b)}+$$

input
```
Int[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(1/((a + b)*(1 - Sec[e + f*x]^2)*(a + b*Sec[e + f*x]^2)^(3/2)) + ((2*b*(2/
a - 5/(a + b)))/(3*(a + b*Sec[e + f*x]^2)^(3/2)) + (((-4*(a + b)^3*ArcTanh
[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/Sqrt[a] + (2*a^2*(2*a + 7*b)*ArcTanh
[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a + b]])/Sqrt[a + b])/(a*(a + b)) - (2*b*
(a^2 - 6*a*b - 2*b^2))/(a*(a + b)*Sqrt[a + b*Sec[e + f*x]^2]))/(a*(a + b))
)/(2*(a + b)))/(2*f)
```

3.432.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.432.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 73510 vs. $2(174) = 348$.

Time = 8.71 (sec) , antiderivative size = 73511, normalized size of antiderivative = 367.56

method	result	size
default	Expression too large to display	73511

input
```
int(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.432.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 830 vs. $2(174) = 348$.

Time = 4.92 (sec) , antiderivative size = 3507, normalized size of antiderivative = 17.54

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output

```
[1/24*(3*((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*cos(f*x + e)^6
 - a^4*b^2 - 4*a^3*b^3 - 6*a^2*b^4 - 4*a*b^5 - b^6 - (a^6 + 2*a^5*b - 2*a^
4*b^2 - 8*a^3*b^3 - 7*a^2*b^4 - 2*a*b^5)*cos(f*x + e)^4 - (2*a^5*b + 7*a^4
*b^2 + 8*a^3*b^3 + 2*a^2*b^4 - 2*a*b^5 - b^6)*cos(f*x + e)^2)*sqrt(a)*log(
128*a^4*cos(f*x + e)^8 + 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x +
e)^4 + 32*a*b^3*cos(f*x + e)^2 + b^4 - 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b
*cos(f*x + e)^6 + 10*a*b^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sq
rt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)) + 3*((2*a^6 + 7*a^5*b)*cos(f*x
+ e)^6 - 2*a^4*b^2 - 7*a^3*b^3 - (2*a^6 + 3*a^5*b - 14*a^4*b^2)*cos(f*x +
e)^4 - (4*a^5*b + 12*a^4*b^2 - 7*a^3*b^3)*cos(f*x + e)^2)*sqrt(a + b)*log(
2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2
 + b^2 + 4*((2*a + b)*cos(f*x + e)^4 + b*cos(f*x + e)^2)*sqrt(a + b)*sqrt(
(a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2
 + 1)) + 4*((3*a^6 + 3*a^5*b + 20*a^4*b^2 + 28*a^3*b^3 + 8*a^2*b^4)*cos(f*
x + e)^6 + 2*(3*a^5*b - 7*a^4*b^2 - 5*a^3*b^3 + 8*a^2*b^4 + 3*a*b^5)*cos(f
*x + e)^4 + 3*(a^4*b^2 - 5*a^3*b^3 - 8*a^2*b^4 - 2*a*b^5)*cos(f*x + e)^2)*
sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2))/((a^9 + 4*a^8*b + 6*a^7*b^2 +
 4*a^6*b^3 + a^5*b^4)*f*cos(f*x + e)^6 - (a^9 + 2*a^8*b - 2*a^7*b^2 - 8*a^
6*b^3 - 7*a^5*b^4 - 2*a^4*b^5)*f*cos(f*x + e)^4 - (2*a^8*b + 7*a^7*b^2 + 8
*a^6*b^3 + 2*a^5*b^4 - 2*a^4*b^5 - a^3*b^6)*f*cos(f*x + e)^2 - (a^7*b^2...
```

3.432.6 Sympy [F]

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

input

```
integrate(cot(f*x+e)**3/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output

```
Integral(cot(e + f*x)**3/(a + b*sec(e + f*x)**2)**(5/2), x)
```

3.432.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.432.8 Giac [F]

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cot\left(fx+e\right)^3}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.432.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^3(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cot(e+f\,x)^3}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

input
```
int(cot(e + f*x)^3/(a + b/cos(e + f*x)^2)^(5/2),x)
```

output
```
int(cot(e + f*x)^3/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.433 $\int \frac{\cot^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

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3.433.1 Optimal result

Integrand size = 25, antiderivative size = 268

$$\int \frac{\cot^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} - \frac{(8a^2+36ab+63b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{8(a+b)^{9/2}f} + \frac{b(12a^2+39ab-8b^2)}{24a(a+b)^3 f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{(4a+11b)\cot^2(e+fx)}{8(a+b)^2 f\left(a+b\sec^2(e+fx)\right)^{3/2}} - \frac{\cot^4(e+fx)}{4(a+b)f\left(a+b\sec^2(e+fx)\right)^{3/2}} + \frac{b(4a^3+15a^2b-32ab^2-8b^3)}{8a^2(a+b)^4 f\sqrt{a+b\sec^2(e+fx)}}$$

output

```
arctanh((a+b*sec(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2)/f-1/8*(8*a^2+36*a*b+63*b
^2)*arctanh((a+b*sec(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(9/2)/f+1/24*b*(12
*a^2+39*a*b-8*b^2)/a/(a+b)^3/f/(a+b*sec(f*x+e)^2)^(3/2)+1/8*(4*a+11*b)*cot
(f*x+e)^2/(a+b)^2/f/(a+b*sec(f*x+e)^2)^(3/2)-1/4*cot(f*x+e)^4/(a+b)/f/(a+b
*sec(f*x+e)^2)^(3/2)+1/8*b*(4*a^3+15*a^2*b-32*a*b^2-8*b^3)/a^2/(a+b)^4/f/(
a+b*sec(f*x+e)^2)^(1/2)
```

3.433.2 Mathematica [F]

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

input
```
Integrate[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
Integrate[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2), x]
```

3.433.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.16, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 4627, 25, 354, 114, 27, 168, 27, 169, 27, 169, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\tan(e+fx)^5\,(a+b\sec(e+fx)^2)^{5/2}}dx$$

$$\downarrow 4627$$

$$\frac{\int -\frac{\cos(e+fx)}{(1-\sec^2(e+fx))^3(b\sec^2(e+fx)+a)^{5/2}}d\sec(e+fx)}{f}$$

$$\downarrow 25$$

$$-\frac{\int \frac{\cos(e+fx)}{(1-\sec^2(e+fx))^3(b\sec^2(e+fx)+a)^{5/2}}d\sec(e+fx)}{f}$$

$$\downarrow 354$$

$$-\frac{\int \frac{\cos(e+fx)}{(1-\sec^2(e+fx))^3(b\sec^2(e+fx)+a)^{5/2}}d\sec^2(e+fx)}{2f}$$

$$\downarrow 114$$

$$-\frac{\frac{1}{2(a+b)(1-\sec^2(e+fx))^2(a+b\sec^2(e+fx))^{3/2}}-\frac{\int -\frac{\cos(e+fx)\left(7b\sec^2(e+fx)+4(a+b)\right)}{2\left(1-\sec^2(e+fx)\right)^2\left(b\sec^2(e+fx)+a\right)^{5/2}}d\sec^2(e+fx)}{2(a+b)}}{2f}$$

$$\downarrow 27$$

$$-\frac{\frac{\int \frac{\cos(e+fx)\left(7b\sec^2(e+fx)+4(a+b)\right)}{\left(1-\sec^2(e+fx)\right)^2\left(b\sec^2(e+fx)+a\right)^{5/2}}d\sec^2(e+fx)}{4(a+b)}+\frac{1}{2(a+b)(1-\sec^2(e+fx))^2(a+b\sec^2(e+fx))^{3/2}}}{2f}$$

$$\downarrow 168$$

$$-\frac{\frac{\frac{4a+11b}{(a+b)\left(1-\sec^2(e+fx)\right)\left(a+b\sec^2(e+fx)\right)^{3/2}}-\frac{\int -\frac{\cos(e+fx)\left(8(a+b)^2+5b(4a+11b)\sec^2(e+fx)\right)}{2\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{5/2}}d\sec^2(e+fx)}{a+b}}{4(a+b)}+\frac{1}{2(a+b)(1-\sec^2(e+fx))^2(a+b\sec^2(e+f}}{2f}$$

$$\downarrow 27$$

$$-\frac{\frac{\frac{\int \frac{\cos(e+fx)\left(8(a+b)^2+5b(4a+11b)\sec^2(e+fx)\right)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{5/2}}d\sec^2(e+fx)}{2(a+b)}+\frac{4a+11b}{(a+b)\left(1-\sec^2(e+fx)\right)\left(a+b\sec^2(e+fx)\right)^{3/2}}}{4(a+b)}+\frac{1}{2(a+b)(1-\sec^2(e+fx))^2(a+b\sec^2(e+fx}}{2f}$$

$$\downarrow 169$$

$$-\frac{\frac{\frac{2\int \frac{3\cos(e+fx)\left(8(a+b)^3+b\left(12a^2+39ba-8b^2\right)\sec^2(e+fx)\right)}{2\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec^2(e+fx)}{3a(a+b)}-\frac{2b\left(12a^2+39ab-8b^2\right)}{3a(a+b)\left(a+b\sec^2(e+fx)\right)^{3/2}}}{2(a+b)}+\frac{4a+11b}{(a+b)\left(1-\sec^2(e+fx)\right)\left(a+b\sec^2(e+fx)\right)^{3/2}}}{4(a+b)}+$$

$$\downarrow 27$$

$$-\frac{\frac{\frac{\int \frac{\cos(e+fx)\left(8(a+b)^3+b\left(12a^2+39ba-8b^2\right)\sec^2(e+fx)\right)}{\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^{3/2}}d\sec^2(e+fx)}{a(a+b)}-\frac{2b\left(12a^2+39ab-8b^2\right)}{3a(a+b)\left(a+b\sec^2(e+fx)\right)^{3/2}}}{2(a+b)}+\frac{4a+11b}{(a+b)\left(1-\sec^2(e+fx)\right)\left(a+b\sec^2(e+fx)\right)^{3/2}}}{4(a+b)}+\frac{}{2(a}}{2f}$$

$$\downarrow 169$$

$$-\frac{\frac{2\int \frac{\cos(e+fx)\left(8(a+b)^4+b\left(4a^3+15ba^2-32b^2a-8b^3\right)\sec^2(e+fx)\right)}{2\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{a(a+b)}-\frac{2b\left(4a^3+15a^2b-32ab^2-8b^3\right)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{2(a+b)}-\frac{2b\left(12a^2+39ab-8b^2\right)}{3a(a+b)\left(a+b\sec^2(e+fx)\right)^{3/2}}+\frac{}{(a+b)\left(1-\sec}}{4(a+b)}}{2f}$$

$$\downarrow 27$$

$$-\frac{\frac{\int \frac{\cos(e+fx)\left(8(a+b)^4+b\left(4a^3+15ba^2-32b^2a-8b^3\right)\sec^2(e+fx)\right)}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{a(a+b)}-\frac{2b\left(4a^3+15a^2b-32ab^2-8b^3\right)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{2(a+b)}-\frac{2b\left(12a^2+39ab-8b^2\right)}{3a(a+b)\left(a+b\sec^2(e+fx)\right)^{3/2}}+\frac{}{(a+b)\left(1-\sec^2}}{4(a+b)}}{2f}$$

$$\downarrow 174$$

$$-\frac{\frac{a^2\left(8a^2+36ab+63b^2\right)\int \frac{1}{\left(1-\sec^2(e+fx)\right)\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)+8(a+b)^4\int \frac{\cos(e+fx)}{\sqrt{b\sec^2(e+fx)+a}}d\sec^2(e+fx)}{a(a+b)}-\frac{2b\left(4a^3+15a^2b-32ab^2-8b^3\right)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{2(a+b)}-\frac{2b(1}{3a(a+b)}}{4(a+b)}}{2f}$$

$$\downarrow 73$$

$$-\frac{\frac{2a^2\left(8a^2+36ab+63b^2\right)\int \frac{1}{\frac{a+b}{b}-\frac{\sec^4(e+fx)}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}+\frac{16(a+b)^4\int \frac{1}{\frac{\sec^4(e+fx)}{b}-\frac{a}{b}}d\sqrt{b\sec^2(e+fx)+a}}{b}}{a(a+b)}-\frac{2b\left(4a^3+15a^2b-32ab^2-8b^3\right)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{2(a+b)}-\frac{2b(12a}{3a(a+b)(a}}{4(a+b)}}{2f}$$

$$\downarrow 221$$

$$-\frac{\frac{2a^2\left(8a^2+36ab+63b^2\right)\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{16(a+b)^4\text{arctanh}\left(\frac{\sqrt{a+b\sec^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}}}{a(a+b)}-\frac{2b\left(4a^3+15a^2b-32ab^2-8b^3\right)}{a(a+b)\sqrt{a+b\sec^2(e+fx)}}}{2(a+b)}-\frac{2b\left(12a^2+39ab-8\right.}{3a(a+b)\left(a+b\sec^2(e+f\right.}}{4(a+b)}}{2f}$$

3.433. $\int \frac{\cot^5(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

input
```
Int[Cot[e + f*x]^5/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
-1/2*(1/(2*(a + b)*(1 - Sec[e + f*x]^2)^2*(a + b*Sec[e + f*x]^2)^(3/2)) +
((4*a + 11*b)/((a + b)*(1 - Sec[e + f*x]^2)*(a + b*Sec[e + f*x]^2)^(3/2))
+ ((-2*b*(12*a^2 + 39*a*b - 8*b^2))/(3*a*(a + b)*(a + b*Sec[e + f*x]^2)^(3
/2)) + (((-16*(a + b)^4*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/Sqrt[a]])/Sqrt[
a] + (2*a^2*(8*a^2 + 36*a*b + 63*b^2)*ArcTanh[Sqrt[a + b*Sec[e + f*x]^2]/S
qrt[a + b]])/Sqrt[a + b])/(a*(a + b)) - (2*b*(4*a^3 + 15*a^2*b - 32*a*b^2
- 8*b^3))/(a*(a + b)*Sqrt[a + b*Sec[e + f*x]^2]))/(a*(a + b)))/(2*(a + b))
)/(4*(a + b)))/f
```

3.433.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.433.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 77708 vs. $2(238) = 476$.

Time = 10.43 (sec) , antiderivative size = 77709, normalized size of antiderivative = 289.96

method	result	size
default	Expression too large to display	77709

input
```
int(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.433.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1141 vs. $2(238) = 476$.

Time = 18.70 (sec) , antiderivative size = 4751, normalized size of antiderivative = 17.73

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/96*(12*((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)
*cos(f*x + e)^8 + a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6
+ b^7 - 2*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*cos(
f*x + e)^6 + (a^7 + a^6*b - 9*a^5*b^2 - 25*a^4*b^3 - 25*a^3*b^4 - 9*a^2*b^
5 + a*b^6 + b^7)*cos(f*x + e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 - 5*a^2
*b^5 - 4*a*b^6 - b^7)*cos(f*x + e)^2)*sqrt(a)*log(128*a^4*cos(f*x + e)^8 +
 256*a^3*b*cos(f*x + e)^6 + 160*a^2*b^2*cos(f*x + e)^4 + 32*a*b^3*cos(f*x
+ e)^2 + b^4 + 8*(16*a^3*cos(f*x + e)^8 + 24*a^2*b*cos(f*x + e)^6 + 10*a*b
^2*cos(f*x + e)^4 + b^3*cos(f*x + e)^2)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b
)/cos(f*x + e)^2)) + 3*((8*a^7 + 36*a^6*b + 63*a^5*b^2)*cos(f*x + e)^8 + 8
*a^5*b^2 + 36*a^4*b^3 + 63*a^3*b^4 - 2*(8*a^7 + 28*a^6*b + 27*a^5*b^2 - 63
*a^4*b^3)*cos(f*x + e)^6 + (8*a^7 + 4*a^6*b - 73*a^5*b^2 - 216*a^4*b^3 + 6
3*a^3*b^4)*cos(f*x + e)^4 + 2*(8*a^6*b + 28*a^5*b^2 + 27*a^4*b^3 - 63*a^3*
b^4)*cos(f*x + e)^2)*sqrt(a + b)*log(2*((8*a^2 + 8*a*b + b^2)*cos(f*x + e)
^4 + 2*(4*a*b + 3*b^2)*cos(f*x + e)^2 + b^2 - 4*((2*a + b)*cos(f*x + e)^4
+ b*cos(f*x + e)^2)*sqrt(a + b)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2
))/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)) - 4*((18*a^7 + 69*a^6*b + 51*a
^5*b^2 + 104*a^4*b^3 + 136*a^3*b^4 + 32*a^2*b^5)*cos(f*x + e)^8 - (12*a^7
+ 21*a^6*b - 93*a^5*b^2 + 106*a^4*b^3 + 176*a^3*b^4 - 56*a^2*b^5 - 24*a*b^
6)*cos(f*x + e)^6 - (24*a^6*b + 96*a^5*b^2 - 83*a^4*b^3 + 5*a^3*b^4 + 2...
```

3.433.6 Sympy [F]

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)**5/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral(cot(e + f*x)**5/(a + b*sec(e + f*x)**2)**(5/2), x)
```

3.433.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.433.8 Giac [F]

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cot\left(fx+e\right)^5}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.433.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^5(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Hanged}$$

input
```
int(cot(e + f*x)^5/(a + b/cos(e + f*x)^2)^(5/2),x)
```

output
```
\text{Hanged}
```

3.434 $\int \frac{\tan^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$

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3.434.1 Optimal result

Integrand size = 25, antiderivative size = 157

$$\int \frac{\tan^6(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{b^{5/2}f} - \frac{(a+b)\tan^3(e+fx)}{3abf\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\tan(e+fx)}{f\sqrt{a+b+b\tan^2(e+fx)}}$$

output

```
-arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/f+arctanh(b
^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/b^(5/2)/f+(1/a^2-1/b^2)*tan(
f*x+e)/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/3*(a+b)*tan(f*x+e)^3/a/b/f/(a+b+b*ta
n(f*x+e)^2)^(3/2)
```

3.434.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 316 vs. 2(157) = 314.

Time = 11.15 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.01

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx =$$

$$-\frac{\left(\frac{b^2\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{a}}-\frac{a^2\text{arctanh}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)}{\sqrt{b}}\right)(a+2b+a\cos(2e+2fx))^{5/2}\sec^5(e+fx)}{4\sqrt{2}a^2b^2f\,(a+b\sec^2(e+fx))^{5/2}}$$

$$+\frac{(a+2b+a\cos(2e+2fx))^3\sec^5(e+fx)\left(\frac{-a^2\sin(e+fx)-2ab\sin(e+fx)-b^2\sin(e+fx)}{6a^2bf(a+2b+a\cos(2e+2fx))^2}+\frac{-3a^2\sin(e+fx)+ab\sin(e+fx)+4b^2}{12a^2b^2f(a+2b+a\cos(2e+2f}\right.}{(a+b\sec^2(e+fx))^{5/2}}$$

input
```
Integrate[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
-1/4*(((b^2*ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]])
/Sqrt[a] - (a^2*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]
^2]])/Sqrt[b])*(a + 2*b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^5)/(Sqrt[
2]*a^2*b^2*f*(a + b*Sec[e + f*x]^2)^(5/2)) + ((a + 2*b + a*Cos[2*e + 2*f*x
])^3*Sec[e + f*x]^5*((-(a^2*Sin[e + f*x]) - 2*a*b*Sin[e + f*x] - b^2*Sin[e
 + f*x])/(6*a^2*b*f*(a + 2*b + a*Cos[2*e + 2*f*x])^2) + (-3*a^2*Sin[e + f*
x] + a*b*Sin[e + f*x] + 4*b^2*Sin[e + f*x])/(12*a^2*b^2*f*(a + 2*b + a*Cos
[2*e + 2*f*x]))))/(a + b*Sec[e + f*x]^2)^(5/2)
```

3.434.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.13, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 4629, 2075, 372, 27, 440, 25, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\tan(e+fx)^6}{(a+b\sec(e+fx)^2)^{5/2}}\,dx$$

$$\downarrow 4629$$

$$\frac{\int \frac{\tan^6(e+fx)}{(\tan^2(e+fx)+1)(a+b(\tan^2(e+fx)+1))^{5/2}}\,d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\tan^6(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^{5/2}}\,d\tan(e+fx)}{f}$$

$$\downarrow 372$$

$$\frac{\int \frac{3\tan^2(e+fx)\left(a\tan^2(e+fx)+a+b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}\,d\tan(e+fx)}{3ab} - \frac{(a+b)\tan^3(e+fx)}{3ab(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\downarrow 27$$

$$\frac{\int \frac{\tan^2(e+fx)\left(a\tan^2(e+fx)+a+b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}\,d\tan(e+fx)}{ab} - \frac{(a+b)\tan^3(e+fx)}{3ab(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\downarrow 440$$

$$\frac{-\frac{\int -\frac{\tan^2(e+fx)a^2+a^2-b^2}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}\,d\tan(e+fx)}{ab} - \frac{\left(a^2-b^2\right)\tan(e+fx)}{ab\sqrt{a+b\tan^2(e+fx)+b}}}{ab} - \frac{(a+b)\tan^3(e+fx)}{3ab(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\downarrow 25$$

$$\frac{\frac{\int \frac{\tan^2(e+fx)a^2+a^2-b^2}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}\,d\tan(e+fx)}{ab} - \frac{\left(a^2-b^2\right)\tan(e+fx)}{ab\sqrt{a+b\tan^2(e+fx)+b}}}{ab} - \frac{(a+b)\tan^3(e+fx)}{3ab(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\downarrow 398$$

$$\frac{\frac{a^2\int \frac{1}{\sqrt{b\tan^2(e+fx)+a+b}}\,d\tan(e+fx) - b^2\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}\,d\tan(e+fx)}{ab} - \frac{\left(a^2-b^2\right)\tan(e+fx)}{ab\sqrt{a+b\tan^2(e+fx)+b}}}{ab} - \frac{(a+b)\tan^3(e+fx)}{3ab(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\downarrow 224$$

3.434. $\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

$$\frac{\frac{a^2\int\frac{1}{1-\frac{b\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}-b^2\int\frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{ab}-\frac{\left(a^2-b^2\right)\tan(e+fx)}{ab\sqrt{a+b\tan^2(e+fx)+b}}}{ab}-\frac{(a+b)\tan^3(e+f}{3ab(a+b\tan^2(e+fx)}$$

f

$\downarrow$ 219

$$\frac{\frac{a^2\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}}-b^2\int\frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{ab}-\frac{\left(a^2-b^2\right)\tan(e+fx)}{ab\sqrt{a+b\tan^2(e+fx)+b}}}{ab}-\frac{(a+b)\tan^3(e+fx)}{3ab(a+b\tan^2(e+fx)+b)^{3/2}}$$

f

$\downarrow$ 291

$$\frac{\frac{a^2\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}}-b^2\int\frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{ab}-\frac{\left(a^2-b^2\right)\tan(e+fx)}{ab\sqrt{a+b\tan^2(e+fx)+b}}}{ab}-\frac{(a+b)\tan^3(e+fx)}{3ab(a+b\tan^2(e+fx)+b)^{3/2}}$$

f

$\downarrow$ 216

$$\frac{\frac{a^2\operatorname{arctanh}\left(\frac{\sqrt{b}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{b}}-\frac{b^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}}}{ab}-\frac{\left(a^2-b^2\right)\tan(e+fx)}{ab\sqrt{a+b\tan^2(e+fx)+b}}}{ab}-\frac{(a+b)\tan^3(e+fx)}{3ab(a+b\tan^2(e+fx)+b)^{3/2}}$$

f

input
```
Int[Tan[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(-1/3*((a + b)*Tan[e + f*x]^3)/(a*b*(a + b + b*Tan[e + f*x]^2)^(3/2)) + ((
-((b^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/Sqrt
[a]) + (a^2*ArcTanh[(Sqrt[b]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]]
)/Sqrt[b])/(a*b) - ((a^2 - b^2)*Tan[e + f*x])/(a*b*Sqrt[a + b + b*Tan[e +
f*x]^2]))/(a*b))/f
```

3.434.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.434.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1499 vs. 2(139) = 278.

Time = 9.86 (sec) , antiderivative size = 1500, normalized size of antiderivative = 9.55

method	result	size
default	Expression too large to display	1500

input
```
int(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
-1/6/f/b^(7/2)/(-a)^(1/2)/a^2*(12*(-a)^(1/2)*b^(7/2)*a*(1-cos(f*x+e))^5*cs
c(f*x+e)^5+12*(-a)^(1/2)*b^(9/2)*(1-cos(f*x+e))^5*csc(f*x+e)^5-40*(1-cos(f
*x+e))^3*a*(-a)^(1/2)*b^(7/2)*csc(f*x+e)^3+24*(-a)^(1/2)*b^(9/2)*(1-cos(f*
x+e))^3*csc(f*x+e)^3-12*(-a)^(1/2)*b^(3/2)*a^3*(1-cos(f*x+e))^5*csc(f*x+e)
^5-12*(-a)^(1/2)*b^(5/2)*a^2*(1-cos(f*x+e))^5*csc(f*x+e)^5-40*(1-cos(f*x+e
))^3*a^2*(-a)^(1/2)*b^(5/2)*csc(f*x+e)^3-6*ln(4*((-a)^(1/2)*(a*(1-cos(f*x+
e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*cs
c(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-co
t(f*x+e)))/((1-cos(f*x+e))^2*csc(f*x+e)^2+1))*b^(7/2)*(a*(1-cos(f*x+e))^4*
csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+
e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(3/2)+12*(-a)^(1/2)*b^(7/2)*a*
(csc(f*x+e)-cot(f*x+e))+12*(-a)^(1/2)*b^(9/2)*(csc(f*x+e)-cot(f*x+e))+24*(
-a)^(1/2)*b^(3/2)*a^3*(1-cos(f*x+e))^3*csc(f*x+e)^3+3*(-a)^(1/2)*ln(4*(-a*
(1-cos(f*x+e))^2*csc(f*x+e)^2-b*(1-cos(f*x+e))^2*csc(f*x+e)^2+b^(1/2)*(a*(
1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*
x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)-2*a*(csc
(f*x+e)-cot(f*x+e))-a-b)/((1-cos(f*x+e))^2*csc(f*x+e)^2+2*csc(f*x+e)-2*cot
(f*x+e)+1))*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)
^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b
)^(3/2)*a^2*b+3*(-a)^(1/2)*ln(4*(a*(1-cos(f*x+e))^2*csc(f*x+e)^2+b*(1-c...
```

3.434.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(139) = 278$.

Time = 2.15 (sec) , antiderivative size = 2035, normalized size of antiderivative = 12.96

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input

```
integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[-1/24*(3*(a^2*b^3*cos(f*x + e)^4 + 2*a*b^4*cos(f*x + e)^2 + b^5)*sqrt(-a)
*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4
 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28
*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(
16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^
2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x +
e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) - 6
*(a^5*cos(f*x + e)^4 + 2*a^4*b*cos(f*x + e)^2 + a^3*b^2)*sqrt(b)*log(((a^2
 - 6*a*b + b^2)*cos(f*x + e)^4 + 8*(a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)
*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(b)*sqrt((a*cos(f*x + e)^2 + b)/co
s(f*x + e)^2)*sin(f*x + e) + 8*b^2)/cos(f*x + e)^4) + 8*((3*a^4*b - a^3*b^
2 - 4*a^2*b^3)*cos(f*x + e)^3 + (4*a^3*b^2 + a^2*b^3 - 3*a*b^4)*cos(f*x +
e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^5*b^3*f*c
os(f*x + e)^4 + 2*a^4*b^4*f*cos(f*x + e)^2 + a^3*b^5*f), 1/24*(12*(a^5*cos
(f*x + e)^4 + 2*a^4*b*cos(f*x + e)^2 + a^3*b^2)*sqrt(-b)*arctan(-1/2*((a -
 b)*cos(f*x + e)^3 + 2*b*cos(f*x + e))*sqrt(-b)*sqrt((a*cos(f*x + e)^2 + b
)/cos(f*x + e)^2)/((a*b*cos(f*x + e)^2 + b^2)*sin(f*x + e))) - 3*(a^2*b^3*
cos(f*x + e)^4 + 2*a*b^4*cos(f*x + e)^2 + b^5)*sqrt(-a)*log(128*a^4*cos(f*
x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2
*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 3...
```

3.434.6 Sympy [F]

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)**6/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral(tan(e + f*x)**6/(a + b*sec(e + f*x)**2)**(5/2), x)
```

3.434.7 Maxima [F]

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\tan(fx+e)^6}{\left(b\sec(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(tan(f*x + e)^6/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

3.434.8 Giac [F]

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\tan(fx+e)^6}{\left(b\sec(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.434.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\tan(e+fx)^6}{\left(a+\frac{b}{\cos(e+fx)^2}\right)^{5/2}}\,dx$$

input
```
int(tan(e + f*x)^6/(a + b/cos(e + f*x)^2)^(5/2),x)
```

output
```
int(tan(e + f*x)^6/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.435 $\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

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3.435.1 Optimal result

Integrand size = 25, antiderivative size = 120

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} - \frac{(a+b)\tan(e+fx)}{3abf\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(a-3b)\tan(e+fx)}{3a^2bf\sqrt{a+b+b\tan^2(e+fx)}}$$

output

```
arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/f+1/3*(a-3*b
)*tan(f*x+e)/a^2/b/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/3*(a+b)*tan(f*x+e)/a/b/f
/(a+b+b*tan(f*x+e)^2)^(3/2)
```

3.435.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 409 vs. $2(120) = 240$.

Time = 6.55 (sec) , antiderivative size = 409, normalized size of antiderivative = 3.41

$$\int \frac{\tan^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx = \frac{(a+2b+a\cos(2(e+fx)))^{5/2}\sec^4(e+fx)\left(\frac{\sqrt{2}\csc(e+fx)\sec(e+fx)\left(\frac{\sin^2(e+fx)}{a+b}+(\right.\right.}{}$$

input
```
Integrate[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])^(5/2)*Sec[e + f*x]^4*((Sqrt[2]*Csc[e + f*x
]*Sec[e + f*x]*(Sin[e + f*x]^2/(a + b) + ((a + 2*b + a*Cos[2*(e + f*x)])*S
in[e + f*x]^2)/(a + b)^2 - (12*Sin[e + f*x]^4)/(a + b) + (16*(a + b - a*Si
n[e + f*x]^2)*(1 - (a*Sin[e + f*x]^2)/(a + b))*((-6*a*(a + b)*Sin[e + f*x]
^2)/(a + 2*b + a*Cos[2*(e + f*x)]) + (a^2*(a + b)*Sin[e + f*x]^4)/(a + b -
 a*Sin[e + f*x]^2)^2 + (3*Sqrt[a]*Sqrt[a + b]*ArcSin[(Sqrt[a]*Sin[e + f*x]
)/Sqrt[a + b]]*Sin[e + f*x])/Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)]))/a^
3))/(a + b - a*Sin[e + f*x]^2)^(3/2) + (8*(2*a + 3*b + a*Cos[2*(e + f*x)])
*Tan[e + f*x])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2)) - (12*(b +
 (3*a + 2*b)*Cos[2*(e + f*x)])*Tan[e + f*x])/((a + b)^2*(a + 2*b + a*Cos[2
*(e + f*x)])^(3/2))))/(384*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

3.435.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4629, 2075, 372, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^4(e+fx)}{\left(a+b\sec^2(e+fx)\right)^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\tan(e+fx)^4}{(a+b\sec(e+fx)^2)^{5/2}}dx$$

$$\downarrow 4629$$

$$\frac{\int \frac{\tan^4(e+fx)}{(\tan^2(e+fx)+1)(a+b(\tan^2(e+fx)+1))^{5/2}}d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\tan^4(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^{5/2}}d\tan(e+fx)}{f}$$

$$\downarrow 372$$

$$\frac{\int \frac{(a-2b)\tan^2(e+fx)+a+b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{3ab} - \frac{(a+b)\tan(e+fx)}{3ab(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\downarrow 402$$

$$\frac{\int \frac{3b(a+b)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a(a+b)} + \frac{(a-3b)\tan(e+fx)}{a\sqrt{a+b\tan^2(e+fx)+b}}}{3ab} - \frac{(a+b)\tan(e+fx)}{3ab(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\downarrow 27$$

$$\frac{3b\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a} + \frac{(a-3b)\tan(e+fx)}{a\sqrt{a+b\tan^2(e+fx)+b}}}{3ab} - \frac{(a+b)\tan(e+fx)}{3ab(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\downarrow 291$$

$$\frac{3b\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{a} + \frac{(a-3b)\tan(e+fx)}{a\sqrt{a+b\tan^2(e+fx)+b}}}{3ab} - \frac{(a+b)\tan(e+fx)}{3ab(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\downarrow 216$$

$$\frac{\frac{3b\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}} + \frac{(a-3b)\tan(e+fx)}{a\sqrt{a+b\tan^2(e+fx)+b}}}{3ab} - \frac{(a+b)\tan(e+fx)}{3ab(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

input
```
Int[Tan[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(-1/3*((a + b)*Tan[e + f*x])/(a*b*(a + b + b*Tan[e + f*x]^2)^(3/2)) + ((3*
b*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/a^(3/2) +
 ((a - 3*b)*Tan[e + f*x])/(a*Sqrt[a + b + b*Tan[e + f*x]^2]))/(3*a*b))/f
```

3.435.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.435.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 656 vs. $2(106) = 212$.

Time = 4.59 (sec) , antiderivative size = 657, normalized size of antiderivative = 5.48

method	result
default	$-\frac{\left(a(1-\cos(fx+e))^4\csc(fx+e)^4+b(1-\cos(fx+e))^4\csc(fx+e)^4-2a(1-\cos(fx+e))^2\csc(fx+e)^2+2b(1-\cos(fx+e))^2\csc(fx+e)^2+$

input
```
int(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/f/a^2/(-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*
csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*
x+e)^2+a+b)*(-6*(-a)^(1/2)*a*(1-cos(f*x+e))^5*csc(f*x+e)^5-6*(-a)^(1/2)*b*
(1-cos(f*x+e))^5*csc(f*x+e)^5+20*(1-cos(f*x+e))^3*a*(-a)^(1/2)*csc(f*x+e)^
3-12*(-a)^(1/2)*b*(1-cos(f*x+e))^3*csc(f*x+e)^3+3*ln(4*((-a)^(1/2)*(a*(1-c
os(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e
))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)-2*a*(csc(f*
x+e)-cot(f*x+e)))/((1-cos(f*x+e))^2*csc(f*x+e)^2+1))*(a*(1-cos(f*x+e))^4*c
sc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e
)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(3/2)-6*(-a)^(1/2)*a*(csc(f*x+e
)-cot(f*x+e))-6*(-a)^(1/2)*b*(csc(f*x+e)-cot(f*x+e)))/((a*(1-cos(f*x+e))^4
*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x
+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/((1-cos(f*x+e))^2*csc(f*x+e)^
2-1)^2)^(5/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^5
```

3.435.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(106) = 212$.

Time = 0.95 (sec) , antiderivative size = 661, normalized size of antiderivative = 5.51

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \left[-\frac{3\left(a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2\right)\sqrt{-a}\log\left(128\,a^4\cos(fx+e\right.}{}\right.$$

$$-\frac{3\left(a^2\cos(fx+e)^4+2\,ab\cos(fx+e)^2+b^2\right)\sqrt{a}\arctan\left(\frac{\left(8\,a^2\cos(fx+e)^5-8\,(a^2-ab)\cos(fx+e)^3+(a^2-6\,ab+b^2)\cos(fx+\right.}{4\left(2\,a^3\cos(fx+e)^4-a^2b+ab^2-(a^3-3\,a^2b)\cos(fx+e)^2\right.}\right.}{12\left(a^5f\cos(fx+e)^4+2\,a^4bf\right.}$$

input
```
integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[-1/24*(3*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 + b^2)*sqrt(-a)*log(1
28*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*
a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3
 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3
*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b +
5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sq
rt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(4*a^
2*cos(f*x + e)^3 - (a^2 - 3*a*b)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)*sin(f*x + e))/(a^5*f*cos(f*x + e)^4 + 2*a^4*b*f*cos(f*x +
 e)^2 + a^3*b^2*f), -1/12*(3*(a^2*cos(f*x + e)^4 + 2*a*b*cos(f*x + e)^2 +
b^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)
^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*
cos(f*x + e)^2)*sin(f*x + e))) + 4*(4*a^2*cos(f*x + e)^3 - (a^2 - 3*a*b)*c
os(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/(a^
5*f*cos(f*x + e)^4 + 2*a^4*b*f*cos(f*x + e)^2 + a^3*b^2*f)]
```

3.435.6 Sympy [F]

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)**4/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral(tan(e + f*x)**4/(a + b*sec(e + f*x)**2)**(5/2), x)
```

3.435.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.435.8 Giac [F]

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\tan(fx+e)^4}{\left(b\sec(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.435.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\tan(e+f\,x)^4}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

input
```
int(tan(e + f*x)^4/(a + b/cos(e + f*x)^2)^(5/2),x)
```

output
```
int(tan(e + f*x)^4/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.436 $\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

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3.436.1 Optimal result

Integrand size = 25, antiderivative size = 119

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} + \frac{\tan(e+fx)}{3af\left(a+b+b\tan^2(e+fx)\right)^{3/2}} + \frac{(2a+3b)\tan(e+fx)}{3a^2(a+b)f\sqrt{a+b+b\tan^2(e+fx)}}$$

output
```
-arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/f+1/3*(2*a+
3*b)*tan(f*x+e)/a^2/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/3*tan(f*x+e)/a/f/
(a+b+b*tan(f*x+e)^2)^(3/2)
```

3.436.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 410 vs. $2(119) = 238$.

Time = 4.69 (sec) , antiderivative size = 410, normalized size of antiderivative = 3.45

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{(a+2b+a\cos(2(e+fx)))^{5/2}\sec^4(e+fx)\left(-\frac{\sqrt{2}\csc(e+fx)\sec(e+fx)\left(\frac{\sin^2(e+fx)}{a+b}-\right.\right.}{}$$

input
```
Integrate[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])^(5/2)*Sec[e + f*x]^4*(-((Sqrt[2]*Csc[e + f
*x]*Sec[e + f*x]*(Sin[e + f*x]^2/(a + b) + ((a + 2*b + a*Cos[2*(e + f*x)])
*Sin[e + f*x]^2)/(a + b)^2 - (12*Sin[e + f*x]^4)/(a + b) + (16*(a + b - a*
Sin[e + f*x]^2)*(1 - (a*Sin[e + f*x]^2)/(a + b))*((-6*a*(a + b)*Sin[e + f*
x]^2)/(a + 2*b + a*Cos[2*(e + f*x)]) + (a^2*(a + b)*Sin[e + f*x]^4)/(a + b
 - a*Sin[e + f*x]^2)^2 + (3*Sqrt[a]*Sqrt[a + b]*ArcSin[(Sqrt[a]*Sin[e + f*
x])/Sqrt[a + b]]*Sin[e + f*x])/Sqrt[(a + b - a*Sin[e + f*x]^2)/(a + b)]))/
a^3))/(a + b - a*Sin[e + f*x]^2)^(3/2)) + (8*(2*a + 3*b + a*Cos[2*(e + f*x
)])*Tan[e + f*x])/((a + b)^2*(a + 2*b + a*Cos[2*(e + f*x)])^(3/2)) - (4*(b
 + (3*a + 2*b)*Cos[2*(e + f*x)])*Tan[e + f*x])/((a + b)^2*(a + 2*b + a*Cos
[2*(e + f*x)])^(3/2))))/(384*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

3.436.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4629, 2075, 373, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\tan(e+fx)^2}{(a+b\sec(e+fx)^2)^{5/2}}dx$$

$$\downarrow 4629$$

$$\frac{\int \frac{\tan^2(e+fx)}{(\tan^2(e+fx)+1)(a+b(\tan^2(e+fx)+1))^{5/2}}d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\tan^2(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^{5/2}}d\tan(e+fx)}{f}$$

$$\downarrow 373$$

$$\frac{\frac{\tan(e+fx)}{3a(a+b\tan^2(e+fx)+b)^{3/2}} - \frac{\int \frac{1-2\tan^2(e+fx)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{3a}}{f}$$

$$\downarrow 402$$

$$\frac{\frac{\tan(e+fx)}{3a(a+b\tan^2(e+fx)+b)^{3/2}} - \frac{\frac{\int \frac{3(a+b)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a(a+b)} - \frac{(2a+3b)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a}}{f}$$

$$\downarrow 27$$

$$\frac{\frac{\tan(e+fx)}{3a(a+b\tan^2(e+fx)+b)^{3/2}} - \frac{\frac{3\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a} - \frac{(2a+3b)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a}}{f}$$

$$\downarrow 291$$

$$\frac{\frac{\tan(e+fx)}{3a(a+b\tan^2(e+fx)+b)^{3/2}} - \frac{\frac{3\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{a} - \frac{(2a+3b)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a}}{f}$$

$$\downarrow 216$$

$$\frac{\frac{\tan(e+fx)}{3a(a+b\tan^2(e+fx)+b)^{3/2}} - \frac{\frac{3\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}} - \frac{(2a+3b)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a}}{f}$$

input
```
Int[Tan[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(Tan[e + f*x]/(3*a*(a + b + b*Tan[e + f*x]^2)^(3/2)) - ((3*ArcTan[(Sqrt[a]
*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/a^(3/2) - ((2*a + 3*b)*Tan
[e + f*x])/(a*(a + b)*Sqrt[a + b + b*Tan[e + f*x]^2]))/(3*a))/f
```

3.436.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.436.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1002 vs. $2(105) = 210$.

Time = 5.60 (sec) , antiderivative size = 1003, normalized size of antiderivative = 8.43

method	result	size
default	Expression too large to display	1003

input
```
int(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
1/3/f/(a+b)/a^2/(-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e
))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*c
sc(f*x+e)^2+a+b)*(-6*(-a)^(1/2)*a^2*(1-cos(f*x+e))^5*csc(f*x+e)^5-12*(-a)^
(1/2)*a*b*(1-cos(f*x+e))^5*csc(f*x+e)^5-6*(-a)^(1/2)*b^2*(1-cos(f*x+e))^5*
csc(f*x+e)^5+12*(1-cos(f*x+e))^3*a^2*(-a)^(1/2)*csc(f*x+e)^3+8*(1-cos(f*x+
e))^3*a*(-a)^(1/2)*b*csc(f*x+e)^3-12*(-a)^(1/2)*b^2*(1-cos(f*x+e))^3*csc(f
*x+e)^3+3*ln(4*((-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e
))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*c
sc(f*x+e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(f*x+e)))/((1-cos(f*x+e))^2*csc(
f*x+e)^2+1))*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e
)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+
b)^(3/2)*a+3*ln(4*((-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*
x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^
2*csc(f*x+e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(f*x+e)))/((1-cos(f*x+e))^2*c
sc(f*x+e)^2+1))*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*
x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2
+a+b)^(3/2)*b-6*(-a)^(1/2)*a^2*(csc(f*x+e)-cot(f*x+e))-12*(-a)^(1/2)*a*b*(
csc(f*x+e)-cot(f*x+e))-6*(-a)^(1/2)*b^2*(csc(f*x+e)-cot(f*x+e)))/((a*(1-co
s(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e)
)^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)/((1-cos(f*x+e))...
```

3.436.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(105) = 210$.

Time $= 0.82$ (sec) , antiderivative size $= 773$, normalized size of antiderivative $= 6.50$

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \left[-\frac{3\left(\left(a^3+a^2b\right)\cos\left(fx+e\right)^4+ab^2+b^3+2\left(a^2b+ab^2\right)\cos\left(fx+e\right)^2\right)\sqrt{-a}\right.$$

input

```
integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[-1/24*(3*((a^3 + a^2*b)*cos(f*x + e)^4 + a*b^2 + b^3 + 2*(a^2*b + a*b^2)*
cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*co
s(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28
*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b
^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x +
 e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7
*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x +
 e)^2)*sin(f*x + e)) - 8*((3*a^3 + 4*a^2*b)*cos(f*x + e)^3 + (2*a^2*b + 3*
a*b^2)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x +
 e))/((a^6 + a^5*b)*f*cos(f*x + e)^4 + 2*(a^5*b + a^4*b^2)*f*cos(f*x + e)^
2 + (a^4*b^2 + a^3*b^3)*f), 1/12*(3*((a^3 + a^2*b)*cos(f*x + e)^4 + a*b^2
+ b^3 + 2*(a^2*b + a*b^2)*cos(f*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*
x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e)
)*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)
^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^2)*sin(f*x + e))) + 4*((
3*a^3 + 4*a^2*b)*cos(f*x + e)^3 + (2*a^2*b + 3*a*b^2)*cos(f*x + e))*sqrt((
a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^6 + a^5*b)*f*cos(f
*x + e)^4 + 2*(a^5*b + a^4*b^2)*f*cos(f*x + e)^2 + (a^4*b^2 + a^3*b^3)*f)]
```

3.436.6 Sympy [F]

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)**2/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral(tan(e + f*x)**2/(a + b*sec(e + f*x)**2)**(5/2), x)
```

3.436.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.436.8 Giac [F]

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\tan(fx+e)^2}{\left(b\sec(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.436.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\tan(e+f\,x)^2}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

input
```
int(tan(e + f*x)^2/(a + b/cos(e + f*x)^2)^(5/2),x)
```

output
```
int(tan(e + f*x)^2/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.437 $\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

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3.437.1 Optimal result

Integrand size = 16, antiderivative size = 125

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} - \frac{b\tan(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(5a+3b)\tan(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}$$

output
```
arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/f-1/3*b*(5*a
+3*b)*tan(f*x+e)/a^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/3*b*tan(f*x+e)
/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(3/2)
```

3.437.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 6.48 (sec) , antiderivative size = 1927, normalized size of antiderivative = 15.42

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^(-5/2),x]
```

output

```
(3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/
(a + b)]*Cos[e + f*x]^4*Sin[e + f*x])/(4*Sqrt[2]*f*(a + b*Sec[e + f*x]^2)^
(5/2)*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2, 5/2, 3
/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -2, 7
/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[
3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*
x]^2)*((15*a*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e
+ f*x]^2)/(a + b)]*Cos[e + f*x]^5*Sin[e + f*x]^2)/(4*Sqrt[2]*(a + b - a*Si
n[e + f*x]^2)^(7/2)*(3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2,
 (a*Sin[e + f*x]^2)/(a + b)] + (5*a*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*
x]^2, (a*Sin[e + f*x]^2)/(a + b)] - 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2,
Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)])*Sin[e + f*x]^2)) + (3*(a + b)
*AppellF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*C
os[e + f*x]^5)/(4*Sqrt[2]*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*Appe
llF1[1/2, -2, 5/2, 3/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] + (5*a
*AppellF1[3/2, -2, 7/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/(a + b)] -
 4*(a + b)*AppellF1[3/2, -1, 5/2, 5/2, Sin[e + f*x]^2, (a*Sin[e + f*x]^2)/
(a + b)])*Sin[e + f*x]^2)) - (3*(a + b)*AppellF1[1/2, -2, 5/2, 3/2, Sin[e
+ f*x]^2, (a*Sin[e + f*x]^2)/(a + b)]*Cos[e + f*x]^3*Sin[e + f*x]^2)/(Sqrt
[2]*(a + b - a*Sin[e + f*x]^2)^(5/2)*(3*(a + b)*AppellF1[1/2, -2, 5/2, ...
```

3.437.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {3042, 4616, 316, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{(a+b\sec(e+fx)^2)^{5/2}}dx$$

$$\downarrow 4616$$

$$\frac{\int \frac{1}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^{5/2}}d\tan(e+fx)}{f}$$

$$\downarrow 316$$

$$\frac{\int \frac{-2b\tan^2(e+fx)+3a+b}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}\,d\tan(e+fx)}{3a(a+b)} - \frac{b\tan(e+fx)}{3a(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}$$

$$\Big\downarrow f$$

$$\Big\downarrow 402$$

$$\frac{\int \frac{3(a+b)^2}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}\,d\tan(e+fx)}{a(a+b)} - \frac{b(5a+3b)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a(a+b)} - \frac{b\tan(e+fx)}{3a(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}$$

$$\Big\downarrow f$$

$$\Big\downarrow 27$$

$$\frac{3(a+b)\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}\,d\tan(e+fx)}{a} - \frac{b(5a+3b)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a(a+b)} - \frac{b\tan(e+fx)}{3a(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}$$

$$\Big\downarrow f$$

$$\Big\downarrow 291$$

$$\frac{3(a+b)\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}\,d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}}{a} - \frac{b(5a+3b)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a(a+b)} - \frac{b\tan(e+fx)}{3a(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}$$

$$\Big\downarrow f$$

$$\Big\downarrow 216$$

$$\frac{3(a+b)\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{a^{3/2}} - \frac{b(5a+3b)\tan(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a(a+b)} - \frac{b\tan(e+fx)}{3a(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}$$

$$\Big\downarrow f$$

input

```
Int[(a + b*Sec[e + f*x]^2)^(-5/2),x]
```

output

```
(-1/3*(b*Tan[e + f*x])/(a*(a + b)*(a + b + b*Tan[e + f*x]^2)^(3/2)) + ((3*
(a + b)*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/a^(
3/2) - (b*(5*a + 3*b)*Tan[e + f*x])/(a*(a + b)*Sqrt[a + b + b*Tan[e + f*x]
^2]))/(3*a*(a + b)))/f
```

3.437.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

3.437.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1262 vs. 2(111) = 222.

Time = 6.40 (sec) , antiderivative size = 1263, normalized size of antiderivative = 10.10

method	result	size
default	Expression too large to display	1263

input
```
int(1/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/f/(a+b)^2/a^2/(-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x
+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2
*csc(f*x+e)^2+a+b)*(12*(-a)^(1/2)*a^2*b*(1-cos(f*x+e))^5*csc(f*x+e)^5+18*(
-a)^(1/2)*a*b^2*(1-cos(f*x+e))^5*csc(f*x+e)^5+6*(-a)^(1/2)*b^3*(1-cos(f*x+
e))^5*csc(f*x+e)^5-24*(1-cos(f*x+e))^3*a^2*(-a)^(1/2)*b*csc(f*x+e)^3+4*(-a
)^(1/2)*a*b^2*(1-cos(f*x+e))^3*csc(f*x+e)^3+12*(-a)^(1/2)*b^3*(1-cos(f*x+e
))^3*csc(f*x+e)^3-3*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*cs
c(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+
e)^2+a+b)^(3/2)*ln(4*((-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos
(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e
))^2*csc(f*x+e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(f*x+e)))/((1-cos(f*x+e))^
2*csc(f*x+e)^2+1))*a^2-6*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))
^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc
(f*x+e)^2+a+b)^(3/2)*ln(4*((-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(
1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(
f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(f*x+e)))/((1-cos(f*x
+e))^2*csc(f*x+e)^2+1))*a*b-3*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*
x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^
2*csc(f*x+e)^2+a+b)^(3/2)*ln(4*((-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^
4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b...
```

3.437.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(111) = 222$.

Time $= 0.73$ (sec) , antiderivative size $= 881$, normalized size of antiderivative $= 7.05$

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \left[-\frac{3\left(\left(a^4+2\,a^3b+a^2b^2\right)\cos\left(fx+e\right)^4+a^2b^2+2\,ab^3+b^4+2\left(a^3b+2\,a^2b^2\right.\right.}{}\right.$$

$$\left. -\frac{3\left(\left(a^4+2\,a^3b+a^2b^2\right)\cos\left(fx+e\right)^4+a^2b^2+2\,ab^3+b^4+2\left(a^3b+2\,a^2b^2+ab^3\right)\cos\left(fx+e\right)^2\right)\sqrt{a}\arctan\left(\right.}{12\left(\left(a^7+2\,a^6b+a^5b^2\right).\right.}\right.$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[-1/24*(3*((a^4 + 2*a^3*b + a^2*b^2)*cos(f*x + e)^4 + a^2*b^2 + 2*a*b^3 +
b^4 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*c
os(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b +
5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 -
 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*a^3*cos(f*x
 + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)
*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*s
qrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e)) + 8*(2*(3*a^3*b +
 2*a^2*b^2)*cos(f*x + e)^3 + (5*a^2*b^2 + 3*a*b^3)*cos(f*x + e))*sqrt((a*c
os(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2
)*f*cos(f*x + e)^4 + 2*(a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a
^5*b^2 + 2*a^4*b^3 + a^3*b^4)*f), -1/12*(3*((a^4 + 2*a^3*b + a^2*b^2)*cos(
f*x + e)^4 + a^2*b^2 + 2*a*b^3 + b^4 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*cos(f
*x + e)^2)*sqrt(a)*arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*
x + e)^3 + (a^2 - 6*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^
2 + b)/cos(f*x + e)^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a
^2*b)*cos(f*x + e)^2)*sin(f*x + e))) + 4*(2*(3*a^3*b + 2*a^2*b^2)*cos(f*x
+ e)^3 + (5*a^2*b^2 + 3*a*b^3)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)/c
os(f*x + e)^2)*sin(f*x + e))/((a^7 + 2*a^6*b + a^5*b^2)*f*cos(f*x + e)^4 +
 2*(a^6*b + 2*a^5*b^2 + a^4*b^3)*f*cos(f*x + e)^2 + (a^5*b^2 + 2*a^4*b^...
```

3.437.6 Sympy [F]

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{1}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**(-5/2), x)
```

3.437.7 Maxima [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.437.8 Giac [F]

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{1}{(b\sec(fx+e)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^(-5/2), x)
```

3.437.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(e+fx)^2}\right)^{5/2}}\,dx$$

input
```
int(1/(a + b/cos(e + f*x)^2)^(5/2),x)
```

output
```
int(1/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.438 $\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

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3.438.1 Optimal result

Integrand size = 25, antiderivative size = 174

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} - \frac{b\cot(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(7a+3b)\cot(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} - \frac{(a-3b)(3a+b)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3a^2(a+b)^3f}$$

output

```
-arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/f-1/3*b*(7*
a+3*b)*cot(f*x+e)/a^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/3*(a-3*b)*(3*
a+b)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/a^2/(a+b)^3/f-1/3*b*cot(f*x+e)/
a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(3/2)
```

3.438.2 Mathematica [A] (verified)

Time = 7.77 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.42

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx =$$
$$-\frac{\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)(a+2b+a\cos(2e+2fx))^{5/2}\sec^5(e+fx)}{4\sqrt{2}a^{5/2}f\,(a+b\sec^2(e+fx))^{5/2}}$$
$$-\frac{(a+2b+a\cos(2(e+fx)))\,(3(3a^4+8a^3b+5a^2b^2-12ab^3-4b^4)+4(3a^4+6a^3b+8ab^3+3b^4)\cos(2(e+$$
$$48a^2(a+b)^3f\,(a+b\sec^2(e+fx))^{5/2}$$

input
```
Integrate[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
-1/4*(ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*(a + 2
*b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^5)/(Sqrt[2]*a^(5/2)*f*(a + b*S
ec[e + f*x]^2)^(5/2)) - ((a + 2*b + a*Cos[2*(e + f*x)])*(3*(3*a^4 + 8*a^3*
b + 5*a^2*b^2 - 12*a*b^3 - 4*b^4) + 4*(3*a^4 + 6*a^3*b + 8*a*b^3 + 3*b^4)*
Cos[2*(e + f*x)] + a*(3*a^3 + 9*a*b^2 + 4*b^3)*Cos[4*(e + f*x)])*Csc[e + f
*x]*Sec[e + f*x]^5)/(48*a^2*(a + b)^3*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

3.438.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4629, 2075, 374, 441, 445, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{\tan(e+fx)^2\,(a+b\sec(e+fx)^2)^{5/2}}dx$$
$$\downarrow 4629$$
$$\frac{\int \frac{\cot^2(e+fx)}{(\tan^2(e+fx)+1)(a+b(\tan^2(e+fx)+1))^{5/2}}d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\cot^2(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^{5/2}}d\tan(e+fx)}{f}$$

$$\downarrow 374$$

$$\frac{\int \frac{\cot^2(e+fx)\left(-4b\tan^2(e+fx)+3a-b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{3a(a+b)} - \frac{b\cot(e+fx)}{3a(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\downarrow 441$$

$$\frac{\frac{\int \frac{\cot^2(e+fx)\left((a-3b)(3a+b)-2b(7a+3b)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a(a+b)} - \frac{b(7a+3b)\cot(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a(a+b)} - \frac{b\cot(e+fx)}{3a(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\downarrow 445$$

$$\frac{-\frac{\int \frac{3(a+b)^3}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a+b} - \frac{(a-3b)(3a+b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{a(a+b)} - \frac{b(7a+3b)\cot(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a(a+b)} - \frac{b\cot(e+f}{3a(a+b)(a+b\tan^2(e}}{f}$$

$$\downarrow 27$$

$$\frac{\frac{-3(a+b)^2\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx) - \frac{(a-3b)(3a+b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{a(a+b)} - \frac{b(7a+3b)\cot(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a(a+b)} - \frac{b\cot(e+f}{3a(a+b)(a+b}}{f}$$

$$\downarrow 291$$

$$\frac{\frac{-3(a+b)^2\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}} - \frac{(a-3b)(3a+b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{a(a+b)} - \frac{b(7a+3b)\cot(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a(a+b)} - \frac{b\cot(e-}{3a(a+b)(a+b\tan^2}}{f}$$

$$\downarrow 216$$

$$\frac{-\frac{3(a+b)^2\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b\tan^2(e+fx)+b}}\right)}{\sqrt{a}}-\frac{(a-3b)(3a+b)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{a(a+b)}-\frac{b(7a+3b)\cot(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a(a+b)}-\frac{b\cot(e+fx)}{3a(a+b)(a+b\tan^2(e+fx)+b)^{3/}}$$
$$f$$

input
```
Int[Cot[e + f*x]^2/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(-1/3*(b*Cot[e + f*x])/(a*(a + b)*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (-((
b*(7*a + 3*b)*Cot[e + f*x])/(a*(a + b)*Sqrt[a + b + b*Tan[e + f*x]^2])) +
((-3*(a + b)^2*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2
]])/Sqrt[a] - ((a - 3*b)*(3*a + b)*Cot[e + f*x]*Sqrt[a + b + b*Tan[e + f*x
]^2])/(a + b))/(a*(a + b)))/(3*a*(a + b)))/f
```

3.438.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.438.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1861 vs. $2(156) = 312$.

Time = 7.59 (sec) , antiderivative size = 1862, normalized size of antiderivative = 10.70

method	result	size
default	Expression too large to display	1862

input
```
int(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/6/f/(a+b)^3/a^2/(-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x
+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2
*csc(f*x+e)^2+a+b)*(3*(-a)^(1/2)*a^4*(1-cos(f*x+e))^8*csc(f*x+e)^8+6*(-a)^
(1/2)*a^3*b*(1-cos(f*x+e))^8*csc(f*x+e)^8+3*(-a)^(1/2)*a^2*b^2*(1-cos(f*x+
e))^8*csc(f*x+e)^8-12*a^4*(1-cos(f*x+e))^6*(-a)^(1/2)*csc(f*x+e)^6-24*(-a)
^(1/2)*a^2*b^2*(1-cos(f*x+e))^6*csc(f*x+e)^6-48*(-a)^(1/2)*a*b^3*(1-cos(f*
x+e))^6*csc(f*x+e)^6-12*(-a)^(1/2)*b^4*(1-cos(f*x+e))^6*csc(f*x+e)^6+18*(-
a)^(1/2)*a^4*(1-cos(f*x+e))^4*csc(f*x+e)^4-12*(-a)^(1/2)*a^3*b*(1-cos(f*x+
e))^4*csc(f*x+e)^4+90*(-a)^(1/2)*a^2*b^2*(1-cos(f*x+e))^4*csc(f*x+e)^4-32*
(-a)^(1/2)*a*b^3*(1-cos(f*x+e))^4*csc(f*x+e)^4-24*(-a)^(1/2)*b^4*(1-cos(f*
x+e))^4*csc(f*x+e)^4+6*ln(4*((-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b
*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-co
s(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(f*x+e)))/((1-cos(f
*x+e))^2*csc(f*x+e)^2+1))*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e)
)^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*cs
c(f*x+e)^2+a+b)^(3/2)*a^3*(csc(f*x+e)-cot(f*x+e))+18*ln(4*((-a)^(1/2)*(a*(
1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*
x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)-2*a*(csc
(f*x+e)-cot(f*x+e)))/((1-cos(f*x+e))^2*csc(f*x+e)^2+1))*(a*(1-cos(f*x+e))^
4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc...
```

3.438.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 488 vs. $2(156) = 312$.

Time = 2.15 (sec) , antiderivative size = 1097, normalized size of antiderivative = 6.30

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[-1/24*(3*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5 + (a^5 + 3*a^4*b + 3*a^3*b^
2 + a^2*b^3)*cos(f*x + e)^4 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*co
s(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(
f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a
^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3
)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e
)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a
*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e
)^2)*sin(f*x + e))*sin(f*x + e) + 8*((3*a^5 + 9*a^3*b^2 + 4*a^2*b^3)*cos(f
*x + e)^5 + (6*a^4*b - 9*a^3*b^2 + 4*a^2*b^3 + 3*a*b^4)*cos(f*x + e)^3 + (
3*a^3*b^2 - 8*a^2*b^3 - 3*a*b^4)*cos(f*x + e))*sqrt((a*cos(f*x + e)^2 + b)
/cos(f*x + e)^2))/(((a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3)*f*cos(f*x + e)^4
 + 2*(a^7*b + 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*f*cos(f*x + e)^2 + (a^6*b^2
 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*f)*sin(f*x + e)), 1/12*(3*(a^3*b^2 + 3
*a^2*b^3 + 3*a*b^4 + b^5 + (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cos(f*x +
 e)^4 + 2*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*cos(f*x + e)^2)*sqrt(a)*
arctan(1/4*(8*a^2*cos(f*x + e)^5 - 8*(a^2 - a*b)*cos(f*x + e)^3 + (a^2 - 6
*a*b + b^2)*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)
^2)/((2*a^3*cos(f*x + e)^4 - a^2*b + a*b^2 - (a^3 - 3*a^2*b)*cos(f*x + e)^
2)*sin(f*x + e)))*sin(f*x + e) - 4*((3*a^5 + 9*a^3*b^2 + 4*a^2*b^3)*cos...
```

3.438.6 Sympy [F]

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)**2/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral(cot(e + f*x)**2/(a + b*sec(e + f*x)**2)**(5/2), x)
```

3.438.7 Maxima [F]

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cot\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(cot(f*x + e)^2/(b*sec(f*x + e)^2 + a)^(5/2), x)
```

3.438.8 Giac [F]

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cot\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.438.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^2(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cot(e+f\,x)^2}{\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^{5/2}}\,dx$$

input
```
int(cot(e + f*x)^2/(a + b/cos(e + f*x)^2)^(5/2),x)
```

output
```
int(cot(e + f*x)^2/(a + b/cos(e + f*x)^2)^(5/2), x)
```

3.439 $\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

3.439.1 Optimal result . . . 3214
3.439.2 Mathematica [A] (verified) . . . 3215
3.439.3 Rubi [A] (verified) . . . 3215
3.439.4 Maple [B] (warning: unable to verify) . . . 3219
3.439.5 Fricas [B] (verification not implemented) . . . 3220
3.439.6 Sympy [F] . . . 3220
3.439.7 Maxima [F(-1)] . . . 3221
3.439.8 Giac [F] . . . 3221
3.439.9 Mupad [F(-1)] . . . 3221

3.439.1 Optimal result

Integrand size = 25, antiderivative size = 236

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f} - \frac{b\cot^3(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}} - \frac{b(3a+b)\cot^3(e+fx)}{a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}} + \frac{(a-b)\left(3a^2+14ab+3b^2\right)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3a^2(a+b)^4f} - \frac{\left(a^2-10ab-3b^2\right)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{3a^2(a+b)^3f}$$

output

```
arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/f-b*(3*a+b)*
cot(f*x+e)^3/a^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)+1/3*(a-b)*(3*a^2+14*
a*b+3*b^2)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/2)/a^2/(a+b)^4/f-1/3*(a^2-10
*a*b-3*b^2)*cot(f*x+e)^3*(a+b+b*tan(f*x+e)^2)^(1/2)/a^2/(a+b)^3/f-1/3*b*co
t(f*x+e)^3/a/(a+b)/f/(a+b+b*tan(f*x+e)^2)^(3/2)
```

3.439.2 Mathematica [A] (verified)

Time = 11.19 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.99

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)(a+2b+a\cos(2e+2fx))^{5/2}\sec^5(e+fx)}{4\sqrt{2}a^{5/2}f\,(a+b\sec^2(e+fx))^{5/2}}$$
$$-\frac{(a+2b+a\cos(2(e+fx)))^3\sec^5(e+fx)\left(-4(a+3b)\csc(e+fx)+(a+b)\csc^3(e+fx)+\frac{4b^3(6a^2+13ab+3}{a^2(}\right.}{24(a+b)^4f\,(a+b\sec^2(e+fx))^{5/2}}$$

input
```
Integrate[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*(a + 2*b +
a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^5)/(4*Sqrt[2]*a^(5/2)*f*(a + b*Sec[
e + f*x]^2)^(5/2)) - ((a + 2*b + a*Cos[2*(e + f*x)])^3*Sec[e + f*x]^5*(-4*
(a + 3*b)*Csc[e + f*x] + (a + b)*Csc[e + f*x]^3 + (4*b^3*(6*a^2 + 13*a*b +
 3*b^2 + 2*a*(3*a + b)*Cos[2*(e + f*x)])*Sin[e + f*x])/(a^2*(a + 2*b + a*C
os[2*(e + f*x)])^2)))/(24*(a + b)^4*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

3.439.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 4629, 2075, 374, 27, 441, 445, 445, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{\tan(e+fx)^4\,(a+b\sec(e+fx)^2)^{5/2}}dx$$
$$\downarrow 4629$$
$$\frac{\int \frac{\cot^4(e+fx)}{(\tan^2(e+fx)+1)(a+b(\tan^2(e+fx)+1))^{5/2}}d\tan(e+fx)}{f}$$
$$\downarrow 2075$$

$$\frac{\int \frac{\cot^4(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^{5/2}}d\tan(e+fx)}{f}$$

$$\downarrow 374$$

$$\frac{\int \frac{3\cot^4(e+fx)\left(-2b\tan^2(e+fx)+a-b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{3a(a+b)} - \frac{b\cot^3(e+fx)}{3a(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\downarrow 27$$

$$\frac{\int \frac{\cot^4(e+fx)\left(-2b\tan^2(e+fx)+a-b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{a(a+b)} - \frac{b\cot^3(e+fx)}{3a(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\downarrow 441$$

$$\frac{\int \frac{\cot^4(e+fx)\left(a^2-10ba-3b^2-4b(3a+b)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a(a+b)} - \frac{b(3a+b)\cot^3(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{a(a+b)} - \frac{b\cot^3(e+fx)}{3a(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\downarrow 445$$

$$\frac{-\frac{\int \frac{\cot^2(e+fx)\left(2b\left(a^2-10ba-3b^2\right)\tan^2(e+fx)+(a-b)\left(3a^2+14ba+3b^2\right)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{3(a+b)} - \frac{\left(a^2-10ab-3b^2\right)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)}}{a(a+b)} - \frac{b(3a+b)\cot^3(e+}{a(a+b)\sqrt{a+b\tan^2(e}}}{a(a+b)}}{f}$$

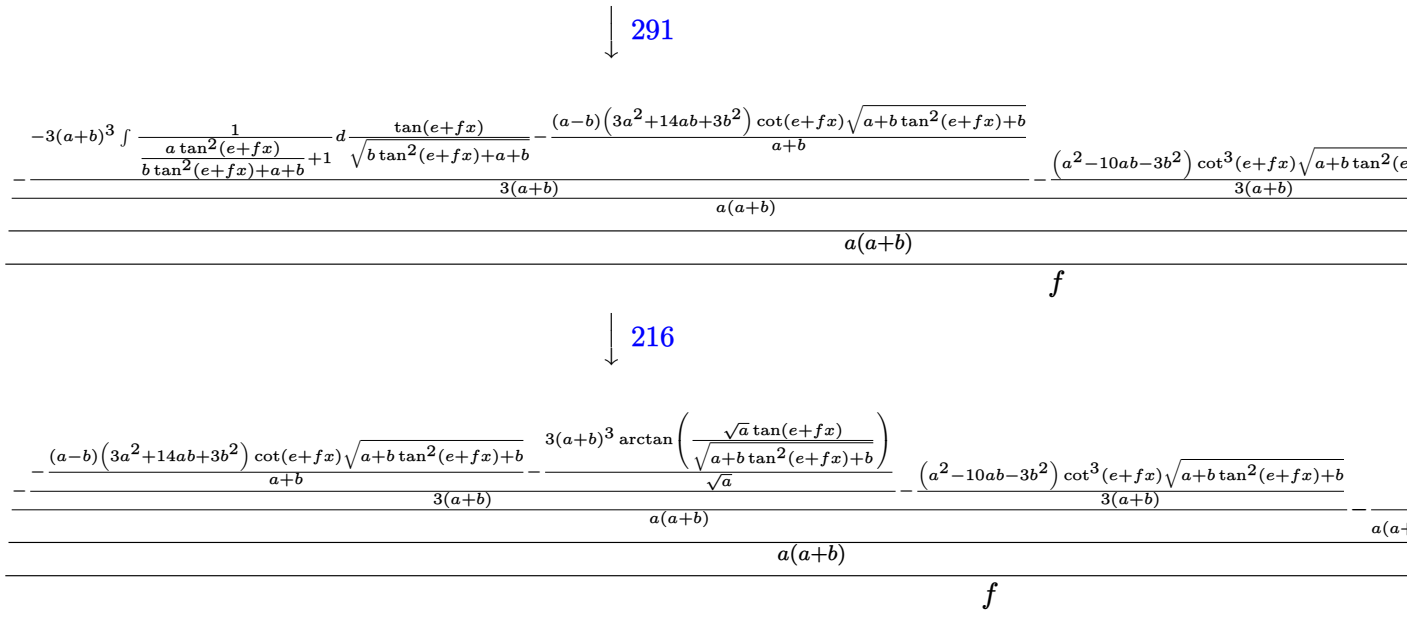
$$\downarrow 445$$

$$\frac{-\frac{\int \frac{3(a+b)^4}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a+b} - \frac{(a-b)\left(3a^2+14ab+3b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{3(a+b)} - \frac{\left(a^2-10ab-3b^2\right)\cot^3(e+fx)\sqrt{a+b\tan^2(e+f}}{3(a+b)}}{a(a+b)} - \frac{b(3a+b)\cot^3(e+}{a(a+b)\sqrt{a+b\tan^2(e}}}{a(a+b)}}{f}$$

$$\downarrow 27$$

$$\frac{-\frac{-3(a+b)^3\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx) - \frac{(a-b)\left(3a^2+14ab+3b^2\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{3(a+b)} - \frac{\left(a^2-10ab-3b^2\right)\cot^3(e+fx)\sqrt{a+b\ta}}{3(a+b)}}{a(a+b)} - \frac{b(3a+b)\cot^3(e+}{a(a+b)\sqrt{a+b\tan^2(e}}}{a(a+b)}}{f}$$

3.439. $\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$



input
```
Int[Cot[e + f*x]^4/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(-1/3*(b*Cot[e + f*x]^3)/(a*(a + b)*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (-
((b*(3*a + b)*Cot[e + f*x]^3)/(a*(a + b)*Sqrt[a + b + b*Tan[e + f*x]^2]))
+ (-1/3*((a^2 - 10*a*b - 3*b^2)*Cot[e + f*x]^3*Sqrt[a + b + b*Tan[e + f*x]
^2])/(a + b) - ((-3*(a + b)^3*ArcTan[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b
*Tan[e + f*x]^2]])/Sqrt[a] - ((a - b)*(3*a^2 + 14*a*b + 3*b^2)*Cot[e + f*x
]*Sqrt[a + b + b*Tan[e + f*x]^2])/(a + b))/(3*(a + b)))/(a*(a + b)))/(a*(a
 + b)))/f
```

3.439.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.439.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2611 vs. 2(216) = 432.

Time = 9.65 (sec) , antiderivative size = 2612, normalized size of antiderivative = 11.07

method	result	size
default	Expression too large to display	2612

input
```
int(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/24/f/(a+b)^4/a^2/(-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*
x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^
2*csc(f*x+e)^2+a+b)*(-18*(-a)^(1/2)*a^5*(1-cos(f*x+e))^2*csc(f*x+e)^2+48*(
-a)^(1/2)*b^5*(1-cos(f*x+e))^8*csc(f*x+e)^8+96*(-a)^(1/2)*b^5*(1-cos(f*x+e
))^6*csc(f*x+e)^6+48*(-a)^(1/2)*b^5*(1-cos(f*x+e))^4*csc(f*x+e)^4+(-a)^(1/
2)*a^5*(1-cos(f*x+e))^12*csc(f*x+e)^12-18*(-a)^(1/2)*a^5*(1-cos(f*x+e))^10
*csc(f*x+e)^10+63*(-a)^(1/2)*a^5*(1-cos(f*x+e))^8*csc(f*x+e)^8-92*a^5*(1-c
os(f*x+e))^6*(-a)^(1/2)*csc(f*x+e)^6+63*(-a)^(1/2)*a^5*(1-cos(f*x+e))^4*cs
c(f*x+e)^4+(-a)^(1/2)*a^5+240*(-a)^(1/2)*a*b^4*(1-cos(f*x+e))^8*csc(f*x+e)
^8+240*(-a)^(1/2)*a*b^4*(1-cos(f*x+e))^4*csc(f*x+e)^4-42*(-a)^(1/2)*a^2*b^
3*(1-cos(f*x+e))^2*csc(f*x+e)^2+108*(-a)^(1/2)*a^3*b^2*(1-cos(f*x+e))^6*cs
c(f*x+e)^6-652*(-a)^(1/2)*a^2*b^3*(1-cos(f*x+e))^6*csc(f*x+e)^6+224*(-a)^(
1/2)*a*b^4*(1-cos(f*x+e))^6*csc(f*x+e)^6-24*ln(4*((-a)^(1/2)*(a*(1-cos(f*x
+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*c
sc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-c
ot(f*x+e)))/((1-cos(f*x+e))^2*csc(f*x+e)^2+1))*(a*(1-cos(f*x+e))^4*csc(f*x
+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*
b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(3/2)*a^4*(1-cos(f*x+e))^3*csc(f*x+e)
^3-24*ln(4*((-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4
*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*cs...
```

3.439.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 729 vs. 2(216) = 432.

Time = 7.36 (sec) , antiderivative size = 1579, normalized size of antiderivative = 6.69

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output
```
[-1/24*(3*((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*cos(f*x + e)^
6 - a^4*b^2 - 4*a^3*b^3 - 6*a^2*b^4 - 4*a*b^5 - b^6 - (a^6 + 2*a^5*b - 2*a
^4*b^2 - 8*a^3*b^3 - 7*a^2*b^4 - 2*a*b^5)*cos(f*x + e)^4 - (2*a^5*b + 7*a^
4*b^2 + 8*a^3*b^3 + 2*a^2*b^4 - 2*a*b^5 - b^6)*cos(f*x + e)^2)*sqrt(-a)*lo
g(128*a^4*cos(f*x + e)^8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 -
14*a^3*b + 5*a^2*b^2)*cos(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*
b^3 + b^4 - 32*(a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 + 8*(16*
a^3*cos(f*x + e)^7 - 24*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b
 + 5*a*b^2)*cos(f*x + e)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))
*sqrt(-a)*sqrt((a*cos(f*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))*sin(f*
x + e) - 8*(4*(a^6 + 3*a^5*b + 3*a^3*b^3 + a^2*b^4)*cos(f*x + e)^7 - 3*(a^
6 + a^5*b - 8*a^4*b^2 + 8*a^3*b^3 - a^2*b^4 - a*b^5)*cos(f*x + e)^5 - 6*(a
^5*b + 3*a^4*b^2 - 4*a^3*b^3 + 3*a^2*b^4 + a*b^5)*cos(f*x + e)^3 - (3*a^4*
b^2 + 11*a^3*b^3 - 11*a^2*b^4 - 3*a*b^5)*cos(f*x + e))*sqrt((a*cos(f*x + e
)^2 + b)/cos(f*x + e)^2))/(((a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^3 + a^5*b
^4)*f*cos(f*x + e)^6 - (a^9 + 2*a^8*b - 2*a^7*b^2 - 8*a^6*b^3 - 7*a^5*b^4
- 2*a^4*b^5)*f*cos(f*x + e)^4 - (2*a^8*b + 7*a^7*b^2 + 8*a^6*b^3 + 2*a^5*b
^4 - 2*a^4*b^5 - a^3*b^6)*f*cos(f*x + e)^2 - (a^7*b^2 + 4*a^6*b^3 + 6*a^5*
b^4 + 4*a^4*b^5 + a^3*b^6)*f)*sin(f*x + e)), -1/12*(3*((a^6 + 4*a^5*b + 6*
a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*cos(f*x + e)^6 - a^4*b^2 - 4*a^3*b^3 - 6...
```

3.439.6 Sympy [F]

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)**4/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output
```
Integral(cot(e + f*x)**4/(a + b*sec(e + f*x)**2)**(5/2), x)
```

3.439.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.439.8 Giac [F]

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cot\left(fx+e\right)^4}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.439.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^4(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Hanged}$$

input
```
int(cot(e + f*x)^4/(a + b/cos(e + f*x)^2)^(5/2),x)
```

output
```
\text{Hanged}
```

3.440 $\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

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3.440.1 Optimal result

Integrand size = 25, antiderivative size = 315

$$\begin{aligned}\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx &= -\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+b+b\tan^2(e+fx)}}\right)}{a^{5/2}f}\\ &-\frac{b\cot^5(e+fx)}{3a(a+b)f\left(a+b+b\tan^2(e+fx)\right)^{3/2}}-\frac{b(11a+3b)\cot^5(e+fx)}{3a^2(a+b)^2f\sqrt{a+b+b\tan^2(e+fx)}}\\ &-\frac{\left(15a^4+70a^3b+128a^2b^2-70ab^3-15b^4\right)\cot(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15a^2(a+b)^5f}\\ &+\frac{\left(5a^3+19a^2b-65ab^2-15b^3\right)\cot^3(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{15a^2(a+b)^4f}\\ &-\frac{\left(a^2-20ab-5b^2\right)\cot^5(e+fx)\sqrt{a+b+b\tan^2(e+fx)}}{5a^2(a+b)^3f}\end{aligned}$$

output

```
-arctan(a^(1/2)*tan(f*x+e)/(a+b+b*tan(f*x+e)^2)^(1/2))/a^(5/2)/f-1/3*b*(11
*a+3*b)*cot(f*x+e)^5/a^2/(a+b)^2/f/(a+b+b*tan(f*x+e)^2)^(1/2)-1/15*(15*a^4
+70*a^3*b+128*a^2*b^2-70*a*b^3-15*b^4)*cot(f*x+e)*(a+b+b*tan(f*x+e)^2)^(1/
2)/a^2/(a+b)^5/f+1/15*(5*a^3+19*a^2*b-65*a*b^2-15*b^3)*cot(f*x+e)^3*(a+b+b
*tan(f*x+e)^2)^(1/2)/a^2/(a+b)^4/f-1/5*(a^2-20*a*b-5*b^2)*cot(f*x+e)^5*(a+
b+b*tan(f*x+e)^2)^(1/2)/a^2/(a+b)^3/f-1/3*b*cot(f*x+e)^5/a/(a+b)/f/(a+b+b*
tan(f*x+e)^2)^(3/2)
```

3.440.2 Mathematica [A] (verified)

Time = 16.99 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.86

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx =$$
$$-\frac{\arctan\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+b-a\sin^2(e+fx)}}\right)(a+2b+a\cos(2e+2fx))^{5/2}\sec^5(e+fx)}{4\sqrt{2}a^{5/2}f\,(a+b\sec^2(e+fx))^{5/2}}$$
$$+\frac{(a+2b+a\cos(2(e+fx)))^3\left(-\frac{20b^5(a+b)}{a^2(a+2b+a\cos(2(e+fx)))^2}+\frac{10b^4(15a+4b)}{a^2(a+2b+a\cos(2(e+fx)))}-(23a^2+100ab+150b^2)\csc^2\right.}{120(a+b)^5f\,(a+b\sec$$

input
```
Integrate[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
-1/4*(ArcTan[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + b - a*Sin[e + f*x]^2]]*(a + 2
*b + a*Cos[2*e + 2*f*x])^(5/2)*Sec[e + f*x]^5)/(Sqrt[2]*a^(5/2)*f*(a + b*S
ec[e + f*x]^2)^(5/2)) + ((a + 2*b + a*Cos[2*(e + f*x)])^3*((-20*b^5*(a + b
))/(a^2*(a + 2*b + a*Cos[2*(e + f*x)])^2) + (10*b^4*(15*a + 4*b))/(a^2*(a
+ 2*b + a*Cos[2*(e + f*x)])) - (23*a^2 + 100*a*b + 150*b^2)*Csc[e + f*x]^2
 + (a + b)*(11*a + 25*b)*Csc[e + f*x]^4 - 3*(a + b)^2*Csc[e + f*x]^6)*Sec[
e + f*x]^4*Tan[e + f*x])/(120*(a + b)^5*f*(a + b*Sec[e + f*x]^2)^(5/2))
```

3.440.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.07, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 4629, 2075, 374, 441, 27, 445, 445, 445, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{\tan(e+fx)^6\,(a+b\sec(e+fx)^2)^{5/2}}dx$$
$$\downarrow 4629$$

$$\frac{\int \frac{\cot^6(e+fx)}{(\tan^2(e+fx)+1)(a+b(\tan^2(e+fx)+1))^{5/2}}d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\cot^6(e+fx)}{(\tan^2(e+fx)+1)(b\tan^2(e+fx)+a+b)^{5/2}}d\tan(e+fx)}{f}$$

$$\downarrow 374$$

$$\frac{\int \frac{\cot^6(e+fx)\left(-8b\tan^2(e+fx)+3a-5b\right)}{\left(\tan^2(e+fx)+1\right)\left(b\tan^2(e+fx)+a+b\right)^{3/2}}d\tan(e+fx)}{3a(a+b)} - \frac{b\cot^5(e+fx)}{3a(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\downarrow 441$$

$$\frac{\frac{\int \frac{3\cot^6(e+fx)\left(a^2-20ba-5b^2-2b(11a+3b)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a(a+b)} - \frac{b(11a+3b)\cot^5(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a(a+b)} - \frac{b\cot^5(e+fx)}{3a(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\downarrow 27$$

$$\frac{\frac{3\int \frac{\cot^6(e+fx)\left(a^2-20ba-5b^2-2b(11a+3b)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a(a+b)} - \frac{b(11a+3b)\cot^5(e+fx)}{a(a+b)\sqrt{a+b\tan^2(e+fx)+b}}}{3a(a+b)} - \frac{b\cot^5(e+fx)}{3a(a+b)(a+b\tan^2(e+fx)+b)^{3/2}}}{f}$$

$$\downarrow 445$$

$$\frac{3\left(-\frac{\int \frac{\cot^4(e+fx)\left(5a^3+19ba^2-65b^2a-15b^3+4b\left(a^2-20ba-5b^2\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{5(a+b)} - \frac{\left(a^2-20ab-5b^2\right)\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{5(a+b)}\right)}{a(a+b)} - \frac{b(11a+3b)}{a(a+b)\sqrt{a+b}}}{3a(a+b)}}{f}$$

$$\downarrow 445$$

3.440. $\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

$$\frac{3\left(-\frac{\int \frac{\cot^2(e+fx)\left(15a^4+70ba^3+128b^2a^2-70b^3a-15b^4+2b\left(5a^3+19ba^2-65b^2a-15b^3\right)\tan^2(e+fx)\right)}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{3(a+b)}-\frac{\left(5a^3+19a^2b-65ab^2-15b^3\right)\cot^3(e+fx)\sqrt{a-}}{3(a+b)}}{5(a+b)}}{a(a+b)}}{3a(a+b)}$$

f

$\downarrow$ 445

$$\frac{3\left(-\frac{\int \frac{15(a+b)^5}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)}{a+b}-\frac{\left(15a^4+70a^3b+128a^2b^2-70ab^3-15b^4\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{3(a+b)}-\frac{\left(5a^3+19a^2b-65ab^2-15\right.}{5(a+b)}}{a(a+b)}}{3a(a+b)}$$

$\downarrow$ 27

$$\frac{3\left(-\frac{-15(a+b)^4\int \frac{1}{\left(\tan^2(e+fx)+1\right)\sqrt{b\tan^2(e+fx)+a+b}}d\tan(e+fx)-\frac{\left(15a^4+70a^3b+128a^2b^2-70ab^3-15b^4\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{3(a+b)}-\frac{\left(5a^3+19a^2b-6\right.}{5(a+b)}}{a(a+b)}}{3a(a+b)}$$

$\downarrow$ 291

$$\frac{3\left(-\frac{-15(a+b)^4\int \frac{1}{\frac{a\tan^2(e+fx)}{b\tan^2(e+fx)+a+b}+1}d\frac{\tan(e+fx)}{\sqrt{b\tan^2(e+fx)+a+b}}-\frac{\left(15a^4+70a^3b+128a^2b^2-70ab^3-15b^4\right)\cot(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{a+b}}{3(a+b)}-\frac{\left(5a^3+19a^2b-65ab^2\right.}{5(a+b)}}{a(a+b)}}{3a(a+b)}$$

$\downarrow$ 216

3.440. $\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx$

$$\frac{3\left(-\frac{\left(a^2-20ab-5b^2\right)\cot^5(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{5(a+b)}-\frac{\left(5a^3+19a^2b-65ab^2-15b^3\right)\cot^3(e+fx)\sqrt{a+b\tan^2(e+fx)+b}}{3(a+b)}-\frac{\left(15a^4+70a^3b+128a^2b^2-70ab^3-15b^4\right.}{a+b}\right)}{a(a+b)}$$
$$\frac{}{3a(a+b)}$$
$$f$$

input
```
Int[Cot[e + f*x]^6/(a + b*Sec[e + f*x]^2)^(5/2),x]
```

output
```
(-1/3*(b*Cot[e + f*x]^5)/(a*(a + b)*(a + b + b*Tan[e + f*x]^2)^(3/2)) + (-
((b*(11*a + 3*b)*Cot[e + f*x]^5)/(a*(a + b)*Sqrt[a + b + b*Tan[e + f*x]^2]
)) + (3*(-1/5*((a^2 - 20*a*b - 5*b^2)*Cot[e + f*x]^5*Sqrt[a + b + b*Tan[e
+ f*x]^2])/(a + b) - (-1/3*((5*a^3 + 19*a^2*b - 65*a*b^2 - 15*b^3)*Cot[e +
 f*x]^3*Sqrt[a + b + b*Tan[e + f*x]^2])/(a + b) - ((-15*(a + b)^4*ArcTan[(
Sqrt[a]*Tan[e + f*x])/Sqrt[a + b + b*Tan[e + f*x]^2]])/Sqrt[a] - ((15*a^4
+ 70*a^3*b + 128*a^2*b^2 - 70*a*b^3 - 15*b^4)*Cot[e + f*x]*Sqrt[a + b + b*
Tan[e + f*x]^2])/(a + b))/(3*(a + b)))/(5*(a + b))))/(a*(a + b)))/(3*a*(a
+ b)))/f
```

3.440.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.440.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 3417 vs. 2(289) = 578.

Time = 11.21 (sec) , antiderivative size = 3418, normalized size of antiderivative = 10.85

method	result	size
default	Expression too large to display	3418

input
```
int(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/480/f/(a+b)^5/a^2/(-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f
*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))
^2*csc(f*x+e)^2+a+b)*(-960*(-a)^(1/2)*b^6*(1-cos(f*x+e))^10*csc(f*x+e)^10-
1920*(-a)^(1/2)*b^6*(1-cos(f*x+e))^8*csc(f*x+e)^8-960*(-a)^(1/2)*b^6*(1-co
s(f*x+e))^6*csc(f*x+e)^6+3*(-a)^(1/2)*a^6*(1-cos(f*x+e))^16*csc(f*x+e)^16-
44*(-a)^(1/2)*a^6*(1-cos(f*x+e))^14*csc(f*x+e)^14+444*(-a)^(1/2)*a^6*(1-co
s(f*x+e))^12*csc(f*x+e)^12-1428*(-a)^(1/2)*a^6*(1-cos(f*x+e))^10*csc(f*x+e
)^10+2050*a^6*(1-cos(f*x+e))^8*(-a)^(1/2)*csc(f*x+e)^8-1428*(-a)^(1/2)*a^6
*(1-cos(f*x+e))^6*csc(f*x+e)^6+444*(-a)^(1/2)*a^6*(1-cos(f*x+e))^4*csc(f*x
+e)^4-44*(-a)^(1/2)*a^6*(1-cos(f*x+e))^2*csc(f*x+e)^2+3*(-a)^(1/2)*a^6+12*
(-a)^(1/2)*a^5*b+18*(-a)^(1/2)*a^4*b^2+12*(-a)^(1/2)*a^3*b^3+2400*ln(4*((-
a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-
2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(
1/2)-2*a*(csc(f*x+e)-cot(f*x+e)))/((1-cos(f*x+e))^2*csc(f*x+e)^2+1))*(a*(1
-cos(f*x+e))^4*csc(f*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x
+e))^2*csc(f*x+e)^2+2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(3/2)*a^4*b*(1-
cos(f*x+e))^5*csc(f*x+e)^5+4800*ln(4*((-a)^(1/2)*(a*(1-cos(f*x+e))^4*csc(f
*x+e)^4+b*(1-cos(f*x+e))^4*csc(f*x+e)^4-2*a*(1-cos(f*x+e))^2*csc(f*x+e)^2+
2*b*(1-cos(f*x+e))^2*csc(f*x+e)^2+a+b)^(1/2)-2*a*(csc(f*x+e)-cot(f*x+e)))/
((1-cos(f*x+e))^2*csc(f*x+e)^2+1))*(a*(1-cos(f*x+e))^4*csc(f*x+e)^4+b*(...
```

3.440.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 969 vs. 2(289) = 578.

Time = 24.64 (sec) , antiderivative size = 2059, normalized size of antiderivative = 6.54

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

output

```
[-1/120*(15*((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^
5)*cos(f*x + e)^8 + a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^
6 + b^7 - 2*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*co
s(f*x + e)^6 + (a^7 + a^6*b - 9*a^5*b^2 - 25*a^4*b^3 - 25*a^3*b^4 - 9*a^2*
b^5 + a*b^6 + b^7)*cos(f*x + e)^4 + 2*(a^6*b + 4*a^5*b^2 + 5*a^4*b^3 - 5*a
^2*b^5 - 4*a*b^6 - b^7)*cos(f*x + e)^2)*sqrt(-a)*log(128*a^4*cos(f*x + e)^
8 - 256*(a^4 - a^3*b)*cos(f*x + e)^6 + 32*(5*a^4 - 14*a^3*b + 5*a^2*b^2)*c
os(f*x + e)^4 + a^4 - 28*a^3*b + 70*a^2*b^2 - 28*a*b^3 + b^4 - 32*(a^4 - 7
*a^3*b + 7*a^2*b^2 - a*b^3)*cos(f*x + e)^2 - 8*(16*a^3*cos(f*x + e)^7 - 24
*(a^3 - a^2*b)*cos(f*x + e)^5 + 2*(5*a^3 - 14*a^2*b + 5*a*b^2)*cos(f*x + e
)^3 - (a^3 - 7*a^2*b + 7*a*b^2 - b^3)*cos(f*x + e))*sqrt(-a)*sqrt((a*cos(f
*x + e)^2 + b)/cos(f*x + e)^2)*sin(f*x + e))*sin(f*x + e) + 8*((23*a^7 + 1
00*a^6*b + 150*a^5*b^2 + 75*a^3*b^4 + 20*a^2*b^5)*cos(f*x + e)^9 - (35*a^7
 + 118*a^6*b + 75*a^5*b^2 - 300*a^4*b^3 + 225*a^3*b^4 - 10*a^2*b^5 - 15*a*
b^6)*cos(f*x + e)^7 + 3*(5*a^7 - 59*a^5*b^2 - 150*a^4*b^3 + 125*a^3*b^4 -
50*a^2*b^5 - 15*a*b^6)*cos(f*x + e)^5 + (30*a^6*b + 105*a^5*b^2 + 92*a^4*b
^3 - 350*a^3*b^4 + 190*a^2*b^5 + 45*a*b^6)*cos(f*x + e)^3 + (15*a^5*b^2 +
70*a^4*b^3 + 128*a^3*b^4 - 70*a^2*b^5 - 15*a*b^6)*cos(f*x + e))*sqrt((a*co
s(f*x + e)^2 + b)/cos(f*x + e)^2))/(((a^10 + 5*a^9*b + 10*a^8*b^2 + 10*a^7
*b^3 + 5*a^6*b^4 + a^5*b^5)*f*cos(f*x + e)^8 - 2*(a^10 + 4*a^9*b + 5*a^...
```

3.440.6 Sympy [F]

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{\frac{5}{2}}}\,dx$$

input

```
integrate(cot(f*x+e)**6/(a+b*sec(f*x+e)**2)**(5/2),x)
```

output

```
Integral(cot(e + f*x)**6/(a + b*sec(e + f*x)**2)**(5/2), x)
```

3.440.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.440.8 Giac [F]

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \int \frac{\cot\left(fx+e\right)^6}{\left(b\sec\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)^6/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.440.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^6(e+fx)}{(a+b\sec^2(e+fx))^{5/2}}\,dx = \text{Hanged}$$

input
```
int(cot(e + f*x)^6/(a + b/cos(e + f*x)^2)^(5/2),x)
```

output
```
\text{Hanged}
```

3.441 $\int (a+b\sec^2(e+fx))^p (d\tan(e+fx))^m\,dx$

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3.441.2 Mathematica [B] (warning: unable to verify) . . . 3231
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3.441.8 Giac [F] . . . 3235
3.441.9 Mupad [F(-1)] . . . 3235

3.441.1 Optimal result

Integrand size = 25, antiderivative size = 105

$$\int (a+b\sec^2(e+fx))^p (d\tan(e+fx))^m\,dx$$
$$= \frac{\text{AppellF1}\left(\frac{1+m}{2},1,-p,\frac{3+m}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)(d\tan(e+fx))^{1+m}(a+b+b\tan^2(e+fx))^p\left(\right.}{df(1+m)}$$

output
```
AppellF1(1/2+1/2*m,1,-p,3/2+1/2*m,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*(d*
tan(f*x+e))^(1+m)*(a+b+b*tan(f*x+e)^2)^p/d/f/(1+m)/((1+b*tan(f*x+e)^2/(a+b
))^p)
```

3.441.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 259 vs. 2(105) = 210.

Time = 5.25 (sec) , antiderivative size = 259, normalized size of antiderivative = 2.47

$$\int (a+b\sec^2(e+fx))^p (d\tan(e+fx))^m\,dx$$
$$= \frac{\text{AppellF1}\left(\frac{1+m}{2},-p,1,\frac{3+m}{2},-\frac{b\tan^2(e+fx)}{a+b},-\tan^2(e+fx)\right)\cos(e+fx)(a+}{f(1+m)\left(\text{AppellF1}\left(\frac{1+m}{2},-p,1,\frac{3+m}{2},-\frac{b\tan^2(e+fx)}{a+b},-\tan^2(e+fx)\right)+\frac{2\left(bp\,\text{AppellF1}\left(\frac{3+m}{2},1-p,1,\frac{5+m}{2},-\frac{b\tan^2(e+}{a+b}\right.\right.\right.}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^p*(d*Tan[e + f*x])^m,x]
```

output
```
(AppellF1[(1 + m)/2, -p, 1, (3 + m)/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan
[e + f*x]^2]*Cos[e + f*x]*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x]*(d*Tan[e +
 f*x])^m)/(f*(1 + m)*(AppellF1[(1 + m)/2, -p, 1, (3 + m)/2, -((b*Tan[e + f
*x]^2)/(a + b)), -Tan[e + f*x]^2] + (2*(b*p*AppellF1[(3 + m)/2, 1 - p, 1,
(5 + m)/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*Appel
lF1[(3 + m)/2, -p, 2, (5 + m)/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f
*x]^2])*Tan[e + f*x]^2)/((a + b)*(3 + m))))
```

3.441.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4629, 2075, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\tan(e+fx))^m \left(a+b\sec^2(e+fx)\right)^p dx$$

$$\downarrow 3042$$

$$\int (d\tan(e+fx))^m \left(a+b\sec(e+fx)^2\right)^p dx$$

$$\downarrow 4629$$

$$\frac{\int \frac{(d\tan(e+fx))^m (a+b(\tan^2(e+fx)+1))^p}{\tan^2(e+fx)+1} d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{(d\tan(e+fx))^m (b\tan^2(e+fx)+a+b)^p}{\tan^2(e+fx)+1} d\tan(e+fx)}{f}$$

$$\downarrow 395$$

$$\frac{(a+b\tan^2(e+fx)+b)^p \left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p} \int \frac{(d\tan(e+fx))^m \left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^p}{\tan^2(e+fx)+1} d\tan(e+fx)}{f}$$

$$\downarrow 394$$

$$\frac{(d\tan(e+fx))^{m+1}\left(a+b\tan^2(e+fx)+b\right)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\text{AppellF1}\left(\frac{m+1}{2},1,-p,\frac{m+3}{2},-\tan^2(e+fx),-\right.}{df(m+1)}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^p*(d*Tan[e + f*x])^m,x]
```

output
```
(AppellF1[(1 + m)/2, 1, -p, (3 + m)/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^
2)/(a + b))]*(d*Tan[e + f*x])^(1 + m)*(a + b + b*Tan[e + f*x]^2)^p)/(d*f*(
1 + m)*(1 + (b*Tan[e + f*x]^2)/(a + b))^p)
```

3.441.3.1 Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f  Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.441.4 Maple [F]

$$\int \left(a + b \sec\left(fx + e\right)^2\right)^p \left(d \tan\left(fx + e\right)\right)^m dx$$

input

```
int((a+b*sec(f*x+e)^2)^p*(d*tan(f*x+e))^m,x)
```

output

```
int((a+b*sec(f*x+e)^2)^p*(d*tan(f*x+e))^m,x)
```

3.441.5 Fricas [F]

$$\int \left(a + b \sec^2(e + fx)\right)^p \left(d \tan(e + fx)\right)^m dx = \int \left(b \sec\left(fx + e\right)^2 + a\right)^p \left(d \tan\left(fx + e\right)\right)^m dx$$

input

```
integrate((a+b*sec(f*x+e)^2)^p*(d*tan(f*x+e))^m,x, algorithm="fricas")
```

output

```
integral((b*sec(f*x + e)^2 + a)^p*(d*tan(f*x + e))^m, x)
```

3.441.6 Sympy [F(-1)]

Timed out.

$$\int \left(a + b \sec^2(e + fx)\right)^p \left(d \tan(e + fx)\right)^m dx = \text{Timed out}$$

input

```
integrate((a+b*sec(f*x+e)**2)**p*(d*tan(f*x+e))**m,x)
```

output

```
Timed out
```

3.441.7 Maxima [F]

$$\int (a+b\sec^2(e+fx))^p (d\tan(e+fx))^m\,dx = \int (b\sec(fx+e)^2+a)^p (d\tan(fx+e))^m\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*(d*tan(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*(d*tan(f*x + e))^m, x)
```

3.441.8 Giac [F]

$$\int (a+b\sec^2(e+fx))^p (d\tan(e+fx))^m\,dx = \int (b\sec(fx+e)^2+a)^p (d\tan(fx+e))^m\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*(d*tan(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*(d*tan(f*x + e))^m, x)
```

3.441.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\sec^2(e+fx))^p (d\tan(e+fx))^m\,dx = \int (d\tan(e+fx))^m \left(a+\frac{b}{\cos(e+fx)^2}\right)^p dx$$

input
```
int((d*tan(e + f*x))^m*(a + b/cos(e + f*x)^2)^p,x)
```

output
```
int((d*tan(e + f*x))^m*(a + b/cos(e + f*x)^2)^p, x)
```

3.442 $\int (a+b\sec^2(e+fx))^p \tan^5(e+fx)\,dx$

3.442.1 Optimal result . . . 3236
3.442.2 Mathematica [A] (verified) . . . 3236
3.442.3 Rubi [A] (verified) . . . 3237
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3.442.7 Maxima [F] . . . 3240
3.442.8 Giac [F] . . . 3240
3.442.9 Mupad [F(-1)] . . . 3240

3.442.1 Optimal result

Integrand size = 23, antiderivative size = 122

$$\begin{aligned}
&\int (a+b\sec^2(e+fx))^p \tan^5(e+fx)\,dx \\
&= -\frac{(a+2b)\,(a+b\sec^2(e+fx))^{1+p}}{2b^2 f(1+p)} \\
&\quad - \frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b\sec^2(e+fx)}{a}\right)(a+b\sec^2(e+fx))^{1+p}}{2af(1+p)} \\
&\quad + \frac{(a+b\sec^2(e+fx))^{2+p}}{2b^2 f(2+p)}
\end{aligned}$$

output

```
-1/2*(a+2*b)*(a+b*sec(f*x+e)^2)^(p+1)/b^2/f/(p+1)-1/2*hypergeom([1, p+1],[
2+p],1+b*sec(f*x+e)^2/a)*(a+b*sec(f*x+e)^2)^(p+1)/a/f/(p+1)+1/2*(a+b*sec(f
*x+e)^2)^(2+p)/b^2/f/(2+p)
```

3.442.2 Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.77

$$\begin{aligned}
&\int (a+b\sec^2(e+fx))^p \tan^5(e+fx)\,dx = \\
&-\frac{(a+b\sec^2(e+fx))^{1+p}\left(b^2(2+p)\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b\sec^2(e+fx)}{a}\right)+a(a+2b(2+\right.}{2ab^2 f(1+p)(2+p)}
\end{aligned}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]^5,x]
```

output
```
-1/2*((a + b*Sec[e + f*x]^2)^(1 + p)*(b^2*(2 + p)*Hypergeometric2F1[1, 1 +
 p, 2 + p, 1 + (b*Sec[e + f*x]^2)/a] + a*(a + 2*b*(2 + p) - b*(1 + p)*Sec[
e + f*x]^2)))/(a*b^2*f*(1 + p)*(2 + p))
```

3.442.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4627, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^5(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$

$$\downarrow 3042$$

$$\int \tan(e+fx)^5\left(a+b\sec(e+fx)^2\right)^p\,dx$$

$$\downarrow 4627$$

$$\frac{\int \cos(e+fx)\left(1-\sec^2(e+fx)\right)^2\left(b\sec^2(e+fx)+a\right)^p\,d\sec(e+fx)}{f}$$

$$\downarrow 354$$

$$\frac{\int \cos(e+fx)\left(1-\sec^2(e+fx)\right)^2\left(b\sec^2(e+fx)+a\right)^p\,d\sec^2(e+fx)}{2f}$$

$$\downarrow 99$$

$$\frac{\int \left(\cos(e+fx)\left(b\sec^2(e+fx)+a\right)^p+\frac{(-a-2b)\left(b\sec^2(e+fx)+a\right)^p}{b}+\frac{\left(b\sec^2(e+fx)+a\right)^{p+1}}{b}\right)d\sec^2(e+fx)}{2f}$$

$$\downarrow 2009$$

$$\frac{-\frac{(a+2b)\left(a+b\sec^2(e+fx)\right)^{p+1}}{b^2(p+1)}+\frac{\left(a+b\sec^2(e+fx)\right)^{p+2}}{b^2(p+2)}-\frac{\left(a+b\sec^2(e+fx)\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\sec^2(e+fx)}{a}+1\right)}{a(p+1)}}{2f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]^5,x]
```

output
```
(-(((a + 2*b)*(a + b*Sec[e + f*x]^2)^(1 + p))/(b^2*(1 + p))) - (Hypergeome
tric2F1[1, 1 + p, 2 + p, 1 + (b*Sec[e + f*x]^2)/a]*(a + b*Sec[e + f*x]^2)^
(1 + p))/(a*(1 + p)) + (a + b*Sec[e + f*x]^2)^(2 + p)/(b^2*(2 + p)))/(2*f)
```

3.442.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.442.4 Maple [F]

$$\int (a + b \sec (fx + e)^2)^p \tan (fx + e)^5 \, dx$$

input
```
int((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^5,x)
```

output
```
int((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^5,x)
```

3.442.5 Fricas [F]

$$\int (a + b \sec^2(e + fx))^p \tan^5(e + fx) \, dx = \int (b \sec (fx + e)^2 + a)^p \tan (fx + e)^5 \, dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^5,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^5, x)
```

3.442.6 Sympy [F(-1)]

Timed out.

$$\int (a + b \sec^2(e + fx))^p \tan^5(e + fx) \, dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e)**2)**p*tan(f*x+e)**5,x)
```

output
```
Timed out
```

3.442.7 Maxima [F]

$$\int (a+b\sec^2(e+fx))^p \tan^5(e+fx)\,dx = \int (b\sec(fx+e)^2+a)^p \tan(fx+e)^5\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^5,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^5, x)
```

3.442.8 Giac [F]

$$\int (a+b\sec^2(e+fx))^p \tan^5(e+fx)\,dx = \int (b\sec(fx+e)^2+a)^p \tan(fx+e)^5\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^5,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^5, x)
```

3.442.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\sec^2(e+fx))^p \tan^5(e+fx)\,dx = \int \tan(e+f\,x)^5 \left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p dx$$

input
```
int(tan(e + f*x)^5*(a + b/cos(e + f*x)^2)^p,x)
```

output
```
int(tan(e + f*x)^5*(a + b/cos(e + f*x)^2)^p, x)
```

3.443 $\int (a+b\sec^2(e+fx))^p \tan^3(e+fx)\,dx$

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3.443.1 Optimal result

Integrand size = 23, antiderivative size = 86

$$\int (a+b\sec^2(e+fx))^p \tan^3(e+fx)\,dx = \frac{(a+b\sec^2(e+fx))^{1+p}}{2bf(1+p)} + \frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b\sec^2(e+fx)}{a}\right)(a+b\sec^2(e+fx))^{1+p}}{2af(1+p)}$$

output
```
1/2*(a+b*sec(f*x+e)^2)^(p+1)/b/f/(p+1)+1/2*hypergeom([1, p+1],[2+p],1+b*se
c(f*x+e)^2/a)*(a+b*sec(f*x+e)^2)^(p+1)/a/f/(p+1)
```

3.443.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.71

$$\int (a+b\sec^2(e+fx))^p \tan^3(e+fx)\,dx = \frac{\left(a+b\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b\sec^2(e+fx)}{a}\right)\right)(a+b\sec^2(e+fx))^{1+p}}{2abf(1+p)}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]^3,x]
```

output
```
((a + b*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sec[e + f*x]^2)/a])*(a +
 b*Sec[e + f*x]^2)^(1 + p))/(2*a*b*f*(1 + p))
```

3.443.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4627, 25, 354, 90, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$

$$\downarrow 3042$$

$$\int \tan(e+fx)^3\left(a+b\sec(e+fx)^2\right)^p\,dx$$

$$\downarrow 4627$$

$$\frac{\int -\cos(e+fx)\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^p\,d\sec(e+fx)}{f}$$

$$\downarrow 25$$

$$-\frac{\int \cos(e+fx)\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^p\,d\sec(e+fx)}{f}$$

$$\downarrow 354$$

$$-\frac{\int \cos(e+fx)\left(1-\sec^2(e+fx)\right)\left(b\sec^2(e+fx)+a\right)^p\,d\sec^2(e+fx)}{2f}$$

$$\downarrow 90$$

$$-\frac{\int \cos(e+fx)\left(b\sec^2(e+fx)+a\right)^p\,d\sec^2(e+fx)-\frac{\left(a+b\sec^2(e+fx)\right)^{p+1}}{b(p+1)}}{2f}$$

$$\downarrow 75$$

$$-\frac{-\frac{\left(a+b\sec^2(e+fx)\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\sec^2(e+fx)}{a}+1\right)}{a(p+1)}-\frac{\left(a+b\sec^2(e+fx)\right)^{p+1}}{b(p+1)}}{2f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]^3,x]
```

output
```
-1/2*(-((a + b*Sec[e + f*x]^2)^(1 + p)/(b*(1 + p))) - (Hypergeometric2F1[1
, 1 + p, 2 + p, 1 + (b*Sec[e + f*x]^2)/a]*(a + b*Sec[e + f*x]^2)^(1 + p))/
(a*(1 + p)))/f
```

3.443.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.443.4 Maple [F]

$$\int (a + b \sec (fx + e)^2)^p \tan (fx + e)^3 \, dx$$

input
```
int((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^3,x)
```

output
```
int((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^3,x)
```

3.443.5 Fricas [F]

$$\int (a + b \sec^2(e + fx))^p \tan^3(e + fx) \, dx = \int (b \sec (fx + e)^2 + a)^p \tan (fx + e)^3 \, dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^3,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^3, x)
```

3.443.6 Sympy [F]

$$\int (a + b \sec^2(e + fx))^p \tan^3(e + fx) \, dx = \int (a + b \sec^2 (e + fx))^p \tan^3 (e + fx) \, dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**p*tan(f*x+e)**3,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**p*tan(e + f*x)**3, x)
```

3.443.7 Maxima [F]

$$\int (a+b\sec^2(e+fx))^p \tan^3(e+fx)\,dx = \int (b\sec(fx+e)^2+a)^p \tan(fx+e)^3\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^3,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^3, x)
```

3.443.8 Giac [F]

$$\int (a+b\sec^2(e+fx))^p \tan^3(e+fx)\,dx = \int (b\sec(fx+e)^2+a)^p \tan(fx+e)^3\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^3,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^3, x)
```

3.443.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\sec^2(e+fx))^p \tan^3(e+fx)\,dx = \int \tan(e+f\,x)^3 \left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p dx$$

input
```
int(tan(e + f*x)^3*(a + b/cos(e + f*x)^2)^p,x)
```

output
```
int(tan(e + f*x)^3*(a + b/cos(e + f*x)^2)^p, x)
```

3.444 $\int (a+b\sec^2(e+fx))^p \tan(e+fx)\,dx$

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3.444.9 Mupad [F(-1)] . . . 3250

3.444.1 Optimal result

Integrand size = 21, antiderivative size = 54

$$\int (a+b\sec^2(e+fx))^p \tan(e+fx)\,dx = -\frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b\sec^2(e+fx)}{a}\right)(a+b\sec^2(e+fx))^{1+p}}{2af(1+p)}$$

output
```
-1/2*hypergeom([1, p+1],[2+p],1+b*sec(f*x+e)^2/a)*(a+b*sec(f*x+e)^2)^(p+1)
/a/f/(p+1)
```

3.444.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int (a+b\sec^2(e+fx))^p \tan(e+fx)\,dx = -\frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b\sec^2(e+fx)}{a}\right)(a+b\sec^2(e+fx))^{1+p}}{2af(1+p)}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x],x]
```

output
```
-1/2*(Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sec[e + f*x]^2)/a]*(a + b*
Sec[e + f*x]^2)^(1 + p))/(a*f*(1 + p))
```

3.444.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4627, 243, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$

$$\downarrow 3042$$

$$\int \tan(e+fx)\left(a+b\sec(e+fx)^2\right)^p\,dx$$

$$\downarrow 4627$$

$$\frac{\int \cos(e+fx)\left(b\sec^2(e+fx)+a\right)^p\,d\sec(e+fx)}{f}$$

$$\downarrow 243$$

$$\frac{\int \cos(e+fx)\left(b\sec^2(e+fx)+a\right)^p\,d\sec^2(e+fx)}{2f}$$

$$\downarrow 75$$

$$-\frac{\left(a+b\sec^2(e+fx)\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\sec^2(e+fx)}{a}+1\right)}{2af(p+1)}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x],x]
```

output
```
-1/2*(Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sec[e + f*x]^2)/a]*(a + b*
Sec[e + f*x]^2)^(1 + p))/(a*f*(1 + p))
```

3.444.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.444.4 Maple [F]

$$\int \left(a + b \sec\left(fx + e\right)^2\right)^p \tan\left(fx + e\right) dx$$

input
```
int((a+b*sec(f*x+e)^2)^p*tan(f*x+e),x)
```

output
```
int((a+b*sec(f*x+e)^2)^p*tan(f*x+e),x)
```

3.444.5 Fricas [F]

$$\int (a + b \sec^2(e + fx))^p \tan(e + fx)\, dx = \int (b \sec(fx + e)^2 + a)^p \tan(fx + e)\, dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e),x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*tan(f*x + e), x)
```

3.444.6 Sympy [F]

$$\int (a + b \sec^2(e + fx))^p \tan(e + fx)\, dx = \int (a + b \sec^2(e + fx))^p \tan(e + fx)\, dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**p*tan(f*x+e),x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**p*tan(e + f*x), x)
```

3.444.7 Maxima [F]

$$\int (a + b \sec^2(e + fx))^p \tan(e + fx)\, dx = \int (b \sec(fx + e)^2 + a)^p \tan(fx + e)\, dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e), x)
```

3.444.8 Giac [F]

$$\int (a+b\sec^2(e+fx))^p \tan(e+fx)\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p \tan\left(fx+e\right)\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e), x)
```

3.444.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\sec^2(e+fx))^p \tan(e+fx)\,dx = \int \tan(e+f\,x)\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p dx$$

input
```
int(tan(e + f*x)*(a + b/cos(e + f*x)^2)^p,x)
```

output
```
int(tan(e + f*x)*(a + b/cos(e + f*x)^2)^p, x)
```

3.445 $\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

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3.445.9 Mupad [F(-1)] . . . 3256

3.445.1 Optimal result

Integrand size = 21, antiderivative size = 114

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$
$$= -\frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+b\sec^2(e+fx)}{a+b}\right)\left(a+b\sec^2(e+fx)\right)^{1+p}}{2(a+b)f(1+p)}$$
$$+\frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b\sec^2(e+fx)}{a}\right)\left(a+b\sec^2(e+fx)\right)^{1+p}}{2af(1+p)}$$

output
```
-1/2*hypergeom([1, p+1],[2+p],(a+b*sec(f*x+e)^2)/(a+b))*(a+b*sec(f*x+e)^2)
^(p+1)/(a+b)/f/(p+1)+1/2*hypergeom([1, p+1],[2+p],1+b*sec(f*x+e)^2/a)*(a+b
*sec(f*x+e)^2)^(p+1)/a/f/(p+1)
```

3.445.2 Mathematica [A] (verified)

Time = 1.90 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.01

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$
$$= \frac{(a+2b+a\cos(2(e+fx)))\left((a+b)\,\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+b+b\tan^2(e+fx)}{a}\right)-a\,\text{Hypergeom}\right.}{4a(a+b)f(1+p)}$$

input
```
Integrate[Cot[e + f*x]*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
((a + 2*b + a*Cos[2*(e + f*x)])*((a + b)*Hypergeometric2F1[1, 1 + p, 2 + p
, (a + b + b*Tan[e + f*x]^2)/a] - a*Hypergeometric2F1[1, 1 + p, 2 + p, 1 +
 (b*Tan[e + f*x]^2)/(a + b)])*Sec[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p)/(4*
a*(a + b)*f*(1 + p))
```

3.445.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4627, 25, 354, 97, 75, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^p}{\tan(e+fx)}dx$$

$$\downarrow 4627$$

$$\frac{\int -\frac{\cos(e+fx)\left(b\sec^2(e+fx)+a\right)^p}{1-\sec^2(e+fx)}d\sec(e+fx)}{f}$$

$$\downarrow 25$$

$$-\frac{\int \frac{\cos(e+fx)\left(b\sec^2(e+fx)+a\right)^p}{1-\sec^2(e+fx)}d\sec(e+fx)}{f}$$

$$\downarrow 354$$

$$-\frac{\int \frac{\cos(e+fx)\left(b\sec^2(e+fx)+a\right)^p}{1-\sec^2(e+fx)}d\sec^2(e+fx)}{2f}$$

$$\downarrow 97$$

$$-\frac{\int \frac{\left(b\sec^2(e+fx)+a\right)^p}{1-\sec^2(e+fx)}d\sec^2(e+fx)+\int \cos(e+fx)\left(b\sec^2(e+fx)+a\right)^p d\sec^2(e+fx)}{2f}$$

$$\downarrow 75$$

$$-\frac{\int \frac{(b\sec^2(e+fx)+a)^p}{1-\sec^2(e+fx)} d\sec^2(e+fx) - \frac{(a+b\sec^2(e+fx))^{p+1}\,\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\sec^2(e+fx)}{a}+1\right)}{a(p+1)}}{2f}$$

$$\downarrow 78$$

$$-\frac{\frac{(a+b\sec^2(e+fx))^{p+1}\,\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\sec^2(e+fx)+a}{a+b}\right)}{(p+1)(a+b)} - \frac{(a+b\sec^2(e+fx))^{p+1}\,\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\sec^2(e+fx)}{a}\right.}{a(p+1)}}{2f}$$

input
```
Int[Cot[e + f*x]*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
-1/2*((Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sec[e + f*x]^2)/(a + b)]*
(a + b*Sec[e + f*x]^2)^(1 + p))/((a + b)*(1 + p)) - (Hypergeometric2F1[1,
1 + p, 2 + p, 1 + (b*Sec[e + f*x]^2)/a]*(a + b*Sec[e + f*x]^2)^(1 + p))/(a
*(1 + p)))/f
```

3.445.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.445.4 Maple [F]

$$\int \cot(fx+e)\left(a+b\sec(fx+e)^2\right)^p dx$$

input
```
int(cot(f*x+e)*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int(cot(f*x+e)*(a+b*sec(f*x+e)^2)^p,x)
```

3.445.5 Fricas [F]

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^p dx = \int \left(b\sec(fx+e)^2+a\right)^p \cot(fx+e)\, dx$$

input
```
integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*cot(f*x + e), x)
```

3.445.6 Sympy [F]

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(a+b\sec^2\left(e+fx\right)\right)^p\cot\left(e+fx\right)dx$$

input
```
integrate(cot(f*x+e)*(a+b*sec(f*x+e)**2)**p,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**p*cot(e + f*x), x)
```

3.445.7 Maxima [F]

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p\cot\left(fx+e\right)\,dx$$

input
```
integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*cot(f*x + e), x)
```

3.445.8 Giac [F]

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec\left(fx+e\right)^2+a\right)^p\cot\left(fx+e\right)\,dx$$

input
```
integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*cot(f*x + e), x)
```

3.445.9 Mupad [F(-1)]

Timed out.

$$\int \cot(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \cot(e+f\,x)\left(a+\frac{b}{\cos\left(e+f\,x\right)^2}\right)^p dx$$

input
```
int(cot(e + f*x)*(a + b/cos(e + f*x)^2)^p,x)
```

output
```
int(cot(e + f*x)*(a + b/cos(e + f*x)^2)^p, x)
```

3.446 $\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

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3.446.1 Optimal result

Integrand size = 23, antiderivative size = 157

$$\begin{aligned}
&\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx \\
&= -\frac{\cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^{1+p}}{2(a+b)f} \\
&\quad + \frac{(a+b-bp)\,\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+b\sec^2(e+fx)}{a+b}\right)\left(a+b\sec^2(e+fx)\right)^{1+p}}{2(a+b)^2 f(1+p)} \\
&\quad - \frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b\sec^2(e+fx)}{a}\right)\left(a+b\sec^2(e+fx)\right)^{1+p}}{2af(1+p)}
\end{aligned}$$

output

```
-1/2*cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^(p+1)/(a+b)/f+1/2*(-b*p+a+b)*hypergeo
m([1, p+1],[2+p],(a+b*sec(f*x+e)^2)/(a+b))*(a+b*sec(f*x+e)^2)^(p+1)/(a+b)^
2/f/(p+1)-1/2*hypergeom([1, p+1],[2+p],1+b*sec(f*x+e)^2/a)*(a+b*sec(f*x+e)
^2)^(p+1)/a/f/(p+1)
```

3.446.2 Mathematica [A] (verified)

Time = 3.96 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.89

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx =$$

$$-\frac{(b+(a+b)\cot^2(e+fx))\left(a(a+b)(1+p)\cot^2(e+fx)+(a+b)^2\,\text{Hypergeometric2F1}\left(1,1+p,2+p\right.\right.}{}$$

input
```
Integrate[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
-1/2*((b + (a + b)*Cot[e + f*x]^2)*(a*(a + b)*(1 + p)*Cot[e + f*x]^2 + (a
+ b)^2*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b + b*Tan[e + f*x]^2)/a] -
a*(a + b - b*p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Tan[e + f*x]^2)/
(a + b)])*(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]^2)/(a*(a + b)^2*f*(1 + p))
```

3.446.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4627, 354, 114, 25, 174, 75, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^p}{\tan(e+fx)^3}\,dx$$

$$\downarrow 4627$$

$$\frac{\int \frac{\cos(e+fx)\left(b\sec^2(e+fx)+a\right)^p}{\left(1-\sec^2(e+fx)\right)^2}\,d\sec(e+fx)}{f}$$

$$\downarrow 354$$

$$\frac{\int \frac{\cos(e+fx)\left(b\sec^2(e+fx)+a\right)^p}{\left(1-\sec^2(e+fx)\right)^2}\,d\sec^2(e+fx)}{2f}$$

$$\downarrow 114$$

$$\frac{\frac{(a+b\sec^2(e+fx))^{p+1}}{(a+b)(1-\sec^2(e+fx))}-\frac{\int -\frac{\cos(e+fx)\left(b\sec^2(e+fx)+a\right)^p\left(-bp\sec^2(e+fx)+a+b\right)}{1-\sec^2(e+fx)}d\sec^2(e+fx)}{a+b}}{2f}$$

$$\downarrow 25$$

$$\frac{\frac{\int \frac{\cos(e+fx)\left(b\sec^2(e+fx)+a\right)^p\left(-bp\sec^2(e+fx)+a+b\right)}{1-\sec^2(e+fx)}d\sec^2(e+fx)}{a+b}+\frac{(a+b\sec^2(e+fx))^{p+1}}{(a+b)(1-\sec^2(e+fx))}}{2f}$$

$$\downarrow 174$$

$$\frac{\frac{(a-bp+b)\int \frac{\left(b\sec^2(e+fx)+a\right)^p}{1-\sec^2(e+fx)}d\sec^2(e+fx)+(a+b)\int \cos(e+fx)\left(b\sec^2(e+fx)+a\right)^p d\sec^2(e+fx)}{a+b}+\frac{(a+b\sec^2(e+fx))^{p+1}}{(a+b)(1-\sec^2(e+fx))}}{2f}$$

$$\downarrow 75$$

$$\frac{\frac{(a-bp+b)\int \frac{\left(b\sec^2(e+fx)+a\right)^p}{1-\sec^2(e+fx)}d\sec^2(e+fx)-\frac{(a+b)\left(a+b\sec^2(e+fx)\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\sec^2(e+fx)}{a}+1\right)}{a(p+1)}}{a+b}+\frac{(a+b\sec^2(e+fx))^{p+1}}{(a+b)(1-\sec^2(e+fx))}}{2f}$$

$$\downarrow 78$$

$$\frac{\frac{(a-bp+b)\left(a+b\sec^2(e+fx)\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\sec^2(e+fx)+a}{a+b}\right)}{(p+1)(a+b)}-\frac{(a+b)\left(a+b\sec^2(e+fx)\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b\sec^2(e+fx)}{a}+\right.}{a(p+1)}}{a+b}}{2f}$$

input
```
Int[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
((a + b*Sec[e + f*x]^2)^(1 + p)/((a + b)*(1 - Sec[e + f*x]^2)) + (((a + b
- b*p)*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sec[e + f*x]^2)/(a + b)]*
(a + b*Sec[e + f*x]^2)^(1 + p))/((a + b)*(1 + p)) - ((a + b)*Hypergeometri
c2F1[1, 1 + p, 2 + p, 1 + (b*Sec[e + f*x]^2)/a]*(a + b*Sec[e + f*x]^2)^(1
+ p))/(a*(1 + p)))/(a + b))/(2*f)
```

3.446.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.446.4 Maple [F]

$$\int \cot(fx+e)^3 \left(a+b\sec(fx+e)^2\right)^p dx$$

input
```
int(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x)
```

3.446.5 Fricas [F]

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p dx = \int \left(b\sec(fx+e)^2+a\right)^p \cot(fx+e)^3 dx$$

input
```
integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*cot(f*x + e)^3, x)
```

3.446.6 Sympy [F(-1)]

Timed out.

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)**3*(a+b*sec(f*x+e)**2)**p,x)
```

output
```
Timed out
```

3.446.7 Maxima [F]

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec(fx+e)^2+a\right)^p \cot(fx+e)^3\,dx$$

input
```
integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*cot(f*x + e)^3, x)
```

3.446.8 Giac [F]

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec(fx+e)^2+a\right)^p \cot(fx+e)^3\,dx$$

input
```
integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*cot(f*x + e)^3, x)
```

3.446.9 Mupad [F(-1)]

Timed out.

$$\int \cot^3(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \cot(e+f\,x)^3\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p\,dx$$

input
```
int(cot(e + f*x)^3*(a + b/cos(e + f*x)^2)^p,x)
```

output
```
int(cot(e + f*x)^3*(a + b/cos(e + f*x)^2)^p, x)
```

3.447 $\int (a+b\sec^2(e+fx))^p \tan^4(e+fx)\,dx$

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3.447.1 Optimal result

Integrand size = 23, antiderivative size = 88

$$\int (a+b\sec^2(e+fx))^p \tan^4(e+fx)\,dx$$
$$= \frac{\text{AppellF1}\left(\frac{5}{2},1,-p,\frac{7}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan^5(e+fx)\,(a+b+b\tan^2(e+fx))^p\left(1+\frac{b\tan^2(e+}{a+b}\right.}{5f}$$

output
```
1/5*AppellF1(5/2,1,-p,7/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)^
5*(a+b+b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

3.447.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2777 vs. 2(88) = 176.

Time = 18.57 (sec) , antiderivative size = 2777, normalized size of antiderivative = 31.56

$$\int (a+b\sec^2(e+fx))^p \tan^4(e+fx)\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]^4,x]
```

output

```
((a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*(a + b*Sec[e + f*x]^2
)^p*Tan[e + f*x]^5*((9*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]
^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p
, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF
1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a
 + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*
x]^2])*Tan[e + f*x]^2) + (-3*Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e +
f*x]^2)/(a + b))] + Hypergeometric2F1[3/2, -p, 5/2, -((b*Tan[e + f*x]^2)/(
a + b))]*Tan[e + f*x]^2)/(1 + (b*Tan[e + f*x]^2)/(a + b))^p))/(3*f*(((a +
2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^(1 + p)*((9*(a + b)*AppellF1[
1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f
*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)),
 -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^
2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan
[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) + (-3*Hypergeomet
ric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(a + b))] + Hypergeometric2F1[3/
2, -p, 5/2, -((b*Tan[e + f*x]^2)/(a + b))]*Tan[e + f*x]^2)/(1 + (b*Tan[e +
 f*x]^2)/(a + b))^p))/3 - (2*a*p*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*(
Sec[e + f*x]^2)^p*Sin[2*(e + f*x)]*Tan[e + f*x]*((9*(a + b)*AppellF1[1/2,
-p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x...
```

3.447.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4629, 2075, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$

$$\downarrow 3042$$

$$\int \tan(e+fx)^4\left(a+b\sec(e+fx)^2\right)^p\,dx$$

$$\downarrow 4629$$

$$\frac{\int \frac{\tan^4(e+fx)(a+b(\tan^2(e+fx)+1))^p}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\tan^4(e+fx)(b\tan^2(e+fx)+a+b)^p}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\downarrow 395$$

$$\frac{(a+b\tan^2(e+fx)+b)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\int \frac{\tan^4(e+fx)\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^p}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\downarrow 394$$

$$\frac{\tan^5(e+fx)\,(a+b\tan^2(e+fx)+b)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\text{AppellF1}\left(\frac{5}{2},1,-p,\frac{7}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{5f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]^4,x]
```

output
```
(AppellF1[5/2, 1, -p, 7/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
*Tan[e + f*x]^5*(a + b + b*Tan[e + f*x]^2)^p)/(5*f*(1 + (b*Tan[e + f*x]^2)
/(a + b))^p)
```

3.447.3.1 Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.447.4 Maple [F]

$$\int (a + b \sec (fx + e)^2)^p \tan (fx + e)^4 \, dx$$

input
```
int((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^4,x)
```

output
```
int((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^4,x)
```

3.447.5 Fricas [F]

$$\int (a + b \sec^2(e + fx))^p \tan^4(e + fx) \, dx = \int (b \sec (fx + e)^2 + a)^p \tan (fx + e)^4 \, dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^4,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^4, x)
```

3.447.6 Sympy [F(-1)]

Timed out.

$$\int (a + b \sec^2(e + fx))^p \tan^4(e + fx) \, dx = \text{Timed out}$$

input
```
integrate((a+b*sec(f*x+e)**2)**p*tan(f*x+e)**4,x)
```

output
```
Timed out
```

3.447.7 Maxima [F]

$$\int (a+b\sec^2(e+fx))^p \tan^4(e+fx)\,dx = \int (b\sec(fx+e)^2+a)^p \tan(fx+e)^4\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^4,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^4, x)
```

3.447.8 Giac [F]

$$\int (a+b\sec^2(e+fx))^p \tan^4(e+fx)\,dx = \int (b\sec(fx+e)^2+a)^p \tan(fx+e)^4\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^4,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^4, x)
```

3.447.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\sec^2(e+fx))^p \tan^4(e+fx)\,dx = \int \tan(e+f\,x)^4 \left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p dx$$

input
```
int(tan(e + f*x)^4*(a + b/cos(e + f*x)^2)^p,x)
```

output
```
int(tan(e + f*x)^4*(a + b/cos(e + f*x)^2)^p, x)
```

3.448 $\int (a+b\sec^2(e+fx))^p \tan^2(e+fx)\,dx$

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3.448.1 Optimal result

Integrand size = 23, antiderivative size = 88

$$\int (a+b\sec^2(e+fx))^p \tan^2(e+fx)\,dx$$
$$= \frac{\text{AppellF1}\left(\frac{3}{2},1,-p,\frac{5}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan^3(e+fx)\,(a+b+b\tan^2(e+fx))^p\left(1+\frac{b\tan^2(e+}{a+b}\right.}{3f}$$

output
```
1/3*AppellF1(3/2,1,-p,5/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)^
3*(a+b+b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

3.448.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2465 vs. 2(88) = 176.

Time = 17.52 (sec) , antiderivative size = 2465, normalized size of antiderivative = 28.01

$$\int (a+b\sec^2(e+fx))^p \tan^2(e+fx)\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]^2,x]
```

output

```
((a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*(a + b*Sec[e + f*x]^2
)^p*Tan[e + f*x]^3*(Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(
a + b))]/(1 + (b*Tan[e + f*x]^2)/(a + b))^p - (3*(a + b)*AppellF1[1/2, -p,
 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]^2)/(
3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e
+ f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a +
b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x
]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2)))/(f*((a + 2*b + a*Cos[2*
(e + f*x)])^p*(Sec[e + f*x]^2)^(1 + p)*(Hypergeometric2F1[1/2, -p, 3/2, -(
(b*Tan[e + f*x]^2)/(a + b))]/(1 + (b*Tan[e + f*x]^2)/(a + b))^p - (3*(a +
b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^
2]*Cos[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^
2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*T
an[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5
/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2)) - 2*
a*p*(a + 2*b + a*Cos[2*(e + f*x)])^(-1 + p)*(Sec[e + f*x]^2)^p*Sin[2*(e +
f*x)]*Tan[e + f*x]*(Hypergeometric2F1[1/2, -p, 3/2, -((b*Tan[e + f*x]^2)/(
a + b))]/(1 + (b*Tan[e + f*x]^2)/(a + b))^p - (3*(a + b)*AppellF1[1/2, -p,
 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Cos[e + f*x]^2)/(
3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan...
```

3.448.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4629, 2075, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p dx$$

$$\downarrow 3042$$

$$\int \tan(e+fx)^2\left(a+b\sec(e+fx)^2\right)^p dx$$

$$\downarrow 4629$$

$$\frac{\int \frac{\tan^2(e+fx)(a+b(\tan^2(e+fx)+1))^p}{\tan^2(e+fx)+1} d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\tan^2(e+fx)(b\tan^2(e+fx)+a+b)^p}{\tan^2(e+fx)+1} d\tan(e+fx)}{f}$$

$$\downarrow 395$$

$$\frac{(a+b\tan^2(e+fx)+b)^p \left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p} \int \frac{\tan^2(e+fx)\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^p}{\tan^2(e+fx)+1} d\tan(e+fx)}{f}$$

$$\downarrow 394$$

$$\frac{\tan^3(e+fx)\,(a+b\tan^2(e+fx)+b)^p \left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p} \text{AppellF1}\left(\frac{3}{2},1,-p,\frac{5}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{3f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^p*Tan[e + f*x]^2,x]
```

output
```
(AppellF1[3/2, 1, -p, 5/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
*Tan[e + f*x]^3*(a + b + b*Tan[e + f*x]^2)^p)/(3*f*(1 + (b*Tan[e + f*x]^2)
/(a + b))^p)
```

3.448.3.1 Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.448.4 Maple [F]

$$\int (a + b \sec (fx + e)^2)^p \tan (fx + e)^2 \, dx$$

input
```
int((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^2,x)
```

output
```
int((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^2,x)
```

3.448.5 Fricas [F]

$$\int (a + b \sec^2(e + fx))^p \tan^2(e + fx) \, dx = \int (b \sec (fx + e)^2 + a)^p \tan (fx + e)^2 \, dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^2,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^2, x)
```

3.448.6 Sympy [F]

$$\int (a + b \sec^2(e + fx))^p \tan^2(e + fx) \, dx = \int (a + b \sec^2 (e + fx))^p \tan^2 (e + fx) \, dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**p*tan(f*x+e)**2,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**p*tan(e + f*x)**2, x)
```

3.448.7 Maxima [F]

$$\int (a+b\sec^2(e+fx))^p \tan^2(e+fx)\,dx = \int (b\sec(fx+e)^2+a)^p \tan(fx+e)^2\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^2, x)
```

3.448.8 Giac [F]

$$\int (a+b\sec^2(e+fx))^p \tan^2(e+fx)\,dx = \int (b\sec(fx+e)^2+a)^p \tan(fx+e)^2\,dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p*tan(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*tan(f*x + e)^2, x)
```

3.448.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\sec^2(e+fx))^p \tan^2(e+fx)\,dx = \int \tan(e+fx)^2 \left(a+\frac{b}{\cos(e+fx)^2}\right)^p dx$$

input
```
int(tan(e + f*x)^2*(a + b/cos(e + f*x)^2)^p,x)
```

output
```
int(tan(e + f*x)^2*(a + b/cos(e + f*x)^2)^p, x)
```

3.449 $\int (a+b\sec^2(e+fx))^p\,dx$

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3.449.1 Optimal result

Integrand size = 14, antiderivative size = 83

$$\int (a+b\sec^2(e+fx))^p\,dx$$
$$= \frac{\text{AppellF1}\left(\frac{1}{2},1,-p,\frac{3}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\tan(e+fx)\,(a+b+b\tan^2(e+fx))^p\left(1+\frac{b\tan^2(e+f}{a+b}\right.}{f}$$

output
```
AppellF1(1/2,1,-p,3/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*tan(f*x+e)*(a+b
+b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

3.449.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2137 vs. 2(83) = 166.

Time = 6.25 (sec) , antiderivative size = 2137, normalized size of antiderivative = 25.75

$$\int (a+b\sec^2(e+fx))^p\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Sec[e + f*x]^2)^p,x]
```

output

```
(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e
 + f*x]^2]*Cos[e + f*x]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^
p*(a + b*Sec[e + f*x]^2)^p*Sin[e + f*x])/(f*(3*(a + b)*AppellF1[1/2, -p, 1
, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3
/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a +
b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^
2])*Tan[e + f*x]^2)*((3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x
]^2)/(a + b)), -Tan[e + f*x]^2]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e +
f*x]^2)^(-1 + p))/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2
)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Ta
n[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/
2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) - (3*(
a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f
*x]^2]*(a + 2*b + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]^2)
/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[
e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a
+ b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f
*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) + (6*(a + b)*p*AppellF1
[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*(a + 2*b
 + a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*Sin[e + f*x]^2)/(3*(a + b)*...
```

3.449.3 Rubi [A] (verified)

Time $= 0.22$ (sec) , antiderivative size $= 83$, normalized size of antiderivative $= 1.00$, number of steps used $= 5$, number of rules used $= 4$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used $= \{3042, 4616, 334, 333\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + b\sec^2(e + fx))^p \, dx$$

$$\downarrow 3042$$

$$\int (a + b\sec(e + fx)^2)^p \, dx$$

$$\downarrow 4616$$

$$\frac{\int \frac{(b\tan^2(e+fx)+a+b)^p}{\tan^2(e+fx)+1} d\tan(e + fx)}{f}$$

$$\downarrow 334$$

$$\frac{(a+b\tan^2(e+fx)+b)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\int\frac{\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^p}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\downarrow 333$$

$$\frac{\tan(e+fx)\,(a+b\tan^2(e+fx)+b)^p\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},1,-p,\frac{3}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
(AppellF1[1/2, 1, -p, 3/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b))]
*Tan[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/(a
+ b))^p)
```

3.449.3.1 Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4616
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[(a + b + b*ff^2*x^2)^p
/(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& NeQ[a + b, 0] && NeQ[p, -1]
```

3.449.4 Maple [F]

$$\int (a + b \sec (fx + e)^2)^p \, dx$$

input
```
int((a+b*sec(f*x+e)^2)^p,x)
```

output
```
int((a+b*sec(f*x+e)^2)^p,x)
```

3.449.5 Fricas [F]

$$\int (a + b \sec^2(e + fx))^p \, dx = \int (b \sec (fx + e)^2 + a)^p \, dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p, x)
```

3.449.6 Sympy [F]

$$\int (a + b \sec^2(e + fx))^p \, dx = \int (a + b \sec^2 (e + fx))^p \, dx$$

input
```
integrate((a+b*sec(f*x+e)**2)**p,x)
```

output
```
Integral((a + b*sec(e + f*x)**2)**p, x)
```

3.449.7 Maxima [F]

$$\int (a + b \sec^2(e + fx))^p \, dx = \int (b \sec(fx + e)^2 + a)^p \, dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p, x)
```

3.449.8 Giac [F]

$$\int (a + b \sec^2(e + fx))^p \, dx = \int (b \sec(fx + e)^2 + a)^p \, dx$$

input
```
integrate((a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p, x)
```

3.449.9 Mupad [F(-1)]

Timed out.

$$\int (a + b \sec^2(e + fx))^p \, dx = \int \left(a + \frac{b}{\cos(e + fx)^2}\right)^p dx$$

input
```
int((a + b/cos(e + f*x)^2)^p,x)
```

output
```
int((a + b/cos(e + f*x)^2)^p, x)
```

3.450 $\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

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3.450.9 Mupad [F(-1)] . . . 3282

3.450.1 Optimal result

Integrand size = 23, antiderivative size = 84

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = -\frac{\text{AppellF1}\left(-\frac{1}{2},1,-p,\frac{1}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\cot(e+fx)\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{b\tan^2}{a}\right.}{f}$$

output
```
-AppellF1(-1/2,1,-p,1/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*cot(f*x+e)*(a
+b+b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

3.450.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2469 vs. 2(84) = 168.

Time = 17.48 (sec) , antiderivative size = 2469, normalized size of antiderivative = 29.39

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \text{Result too large to show}$$

input
```
Integrate[Cot[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p,x]
```

output

```
((a + 2*b + a*Cos[2*(e + f*x)])^p*Cot[e + f*x]^3*(Sec[e + f*x]^2)^p*(a + b
*Sec[e + f*x]^2)^p*(-(Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2
)/(a + b))]/(1 + (b*Tan[e + f*x]^2)/(a + b))^p) - (3*(a + b)*AppellF1[1/2,
 -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sin[e + f*x]^
2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Ta
n[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(
a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e +
 f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2)))/(f*(2*p*(a + 2*b +
a*Cos[2*(e + f*x)])^p*(Sec[e + f*x]^2)^p*(-(Hypergeometric2F1[-1/2, -p, 1/
2, -((b*Tan[e + f*x]^2)/(a + b))]/(1 + (b*Tan[e + f*x]^2)/(a + b))^p) - (3
*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e +
 f*x]^2]*Sin[e + f*x]^2)/(3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e +
 f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2,
-((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] - (a + b)*AppellF1[3/2, -p
, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2])*Tan[e + f*x]^2)
) - (a + 2*b + a*Cos[2*(e + f*x)])^p*Csc[e + f*x]^2*(Sec[e + f*x]^2)^p*(-(
Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*x]^2)/(a + b))]/(1 + (b*Ta
n[e + f*x]^2)/(a + b))^p) - (3*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[
e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]*Sin[e + f*x]^2)/(3*(a + b)*AppellF1
[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(...
```

3.450.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4629, 2075, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^p}{\tan(e+fx)^2}dx$$

$$\downarrow 4629$$

$$\frac{\int \frac{\cot^2(e+fx)\left(a+b\left(\tan^2(e+fx)+1\right)\right)^p}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\cot^2(e+fx)(b\tan^2(e+fx)+a+b)^p}{\tan^2(e+fx)+1} d\tan(e+fx)}{f}$$

$$\downarrow 395$$

$$\frac{(a+b\tan^2(e+fx)+b)^p \left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p} \int \frac{\cot^2(e+fx)\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^p}{\tan^2(e+fx)+1} d\tan(e+fx)}{f}$$

$$\downarrow 394$$

$$-\frac{\cot(e+fx)\,(a+b\tan^2(e+fx)+b)^p \left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p} \text{AppellF1}\left(-\frac{1}{2},1,-p,\frac{1}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right.}{f}$$

input
```
Int[Cot[e + f*x]^2*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
-((AppellF1[-1/2, 1, -p, 1/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a + b
))]*Cot[e + f*x]*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*x]^2)/
(a + b))^p))
```

3.450.3.1 Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.450.4 Maple [F]

$$\int \cot(fx+e)^2 \left(a+b\sec(fx+e)^2\right)^p dx$$

input
```
int(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x)
```

3.450.5 Fricas [F]

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p dx = \int \left(b\sec(fx+e)^2+a\right)^p \cot(fx+e)^2 dx$$

input
```
integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*cot(f*x + e)^2, x)
```

3.450.6 Sympy [F(-1)]

Timed out.

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)**2*(a+b*sec(f*x+e)**2)**p,x)
```

output
```
Timed out
```

3.450.7 Maxima [F]

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec(fx+e)^2+a\right)^p \cot(fx+e)^2\,dx$$

input
```
integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*cot(f*x + e)^2, x)
```

3.450.8 Giac [F]

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec(fx+e)^2+a\right)^p \cot(fx+e)^2\,dx$$

input
```
integrate(cot(f*x+e)^2*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*cot(f*x + e)^2, x)
```

3.450.9 Mupad [F(-1)]

Timed out.

$$\int \cot^2(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \cot(e+f\,x)^2\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p\,dx$$

input
```
int(cot(e + f*x)^2*(a + b/cos(e + f*x)^2)^p,x)
```

output
```
int(cot(e + f*x)^2*(a + b/cos(e + f*x)^2)^p, x)
```

3.451 $\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$

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3.451.2 Mathematica [B] (warning: unable to verify) . . . 3283
3.451.3 Rubi [A] (verified) . . . 3284
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3.451.5 Fricas [F] . . . 3286
3.451.6 Sympy [F(-1)] . . . 3286
3.451.7 Maxima [F] . . . 3287
3.451.8 Giac [F] . . . 3287
3.451.9 Mupad [F(-1)] . . . 3287

3.451.1 Optimal result

Integrand size = 23, antiderivative size = 88

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx =$$

$$-\frac{\text{AppellF1}\left(-\frac{3}{2},1,-p,-\frac{1}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+fx)}{a+b}\right)\cot^3(e+fx)\left(a+b+b\tan^2(e+fx)\right)^p\left(1+\frac{bt}{3f}\right.}$$

output
```
-1/3*AppellF1(-3/2,1,-p,-1/2,-tan(f*x+e)^2,-b*tan(f*x+e)^2/(a+b))*cot(f*x+
e)^3*(a+b+b*tan(f*x+e)^2)^p/f/((1+b*tan(f*x+e)^2/(a+b))^p)
```

3.451.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 3033 vs. 2(88) = 176.

Time = 18.98 (sec) , antiderivative size = 3033, normalized size of antiderivative = 34.47

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \text{Result too large to show}$$

input
```
Integrate[Cot[e + f*x]^4*(a + b*Sec[e + f*x]^2)^p,x]
```

output

```
((a + 2*b + a*Cos[2*(e + f*x)])^p*Cot[e + f*x]^7*(Sec[e + f*x]^2)^p*(a + b
*Sec[e + f*x]^2)^p*((9*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]
^2)/(a + b)), -Tan[e + f*x]^2]*Sin[e + f*x]^2*Tan[e + f*x]^2)/(3*(a + b)*A
ppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] +
 2*(b*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e
 + f*x]^2] - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b
)), -Tan[e + f*x]^2])*Tan[e + f*x]^2) - (Hypergeometric2F1[-3/2, -p, -1/2,
 -((b*Tan[e + f*x]^2)/(a + b))] - 3*Hypergeometric2F1[-1/2, -p, 1/2, -((b*
Tan[e + f*x]^2)/(a + b))]*Tan[e + f*x]^2)/(1 + (b*Tan[e + f*x]^2)/(a + b))
^p))/(3*f*((2*p*(a + 2*b + a*Cos[2*(e + f*x)])^p*Cot[e + f*x]^2*(Sec[e + f
*x]^2)^p*((9*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b
)), -Tan[e + f*x]^2]*Sin[e + f*x]^2*Tan[e + f*x]^2)/(3*(a + b)*AppellF1[1/
2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2] + 2*(b*p*Ap
pellF1[3/2, 1 - p, 1, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e + f*x]^2]
 - (a + b)*AppellF1[3/2, -p, 2, 5/2, -((b*Tan[e + f*x]^2)/(a + b)), -Tan[e
 + f*x]^2])*Tan[e + f*x]^2) - (Hypergeometric2F1[-3/2, -p, -1/2, -((b*Tan[
e + f*x]^2)/(a + b))] - 3*Hypergeometric2F1[-1/2, -p, 1/2, -((b*Tan[e + f*
x]^2)/(a + b))]*Tan[e + f*x]^2)/(1 + (b*Tan[e + f*x]^2)/(a + b))^p))/3 - (
a + 2*b + a*Cos[2*(e + f*x)])^p*Cot[e + f*x]^2*Csc[e + f*x]^2*(Sec[e + f*x
]^2)^p*((9*(a + b)*AppellF1[1/2, -p, 1, 3/2, -((b*Tan[e + f*x]^2)/(a + ...
```

3.451.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4629, 2075, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+b\sec(e+fx)^2\right)^p}{\tan(e+fx)^4}dx$$

$$\downarrow 4629$$

$$\frac{\int \frac{\cot^4(e+fx)\left(a+b\left(\tan^2(e+fx)+1\right)\right)^p}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\downarrow 2075$$

$$\frac{\int \frac{\cot^4(e+fx)(b\tan^2(e+fx)+a+b)^p}{\tan^2(e+fx)+1} d\tan(e+fx)}{f}$$

$$\downarrow 395$$

$$\frac{(a+b\tan^2(e+fx)+b)^p \left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p} \int \frac{\cot^4(e+fx)\left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^p}{\tan^2(e+fx)+1} d\tan(e+fx)}{f}$$

$$\downarrow 394$$

$$-\frac{\cot^3(e+fx)\,(a+b\tan^2(e+fx)+b)^p \left(\frac{b\tan^2(e+fx)}{a+b}+1\right)^{-p} \text{AppellF1}\left(-\frac{3}{2},1,-p,-\frac{1}{2},-\tan^2(e+fx),-\frac{b\tan^2(e+}{a+b}\right.}{3f}$$

input
```
Int[Cot[e + f*x]^4*(a + b*Sec[e + f*x]^2)^p,x]
```

output
```
-1/3*(AppellF1[-3/2, 1, -p, -1/2, -Tan[e + f*x]^2, -((b*Tan[e + f*x]^2)/(a
 + b))]*Cot[e + f*x]^3*(a + b + b*Tan[e + f*x]^2)^p)/(f*(1 + (b*Tan[e + f*
x]^2)/(a + b))^p)
```

3.451.3.1 Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2075
```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_))^(m_.), x_Symbol] :> Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] &&  !
BinomialMatchQ[{u, v}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4629
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*((d_.)*tan[(e_.) + (f
_.)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/f   Subst[Int[(d*ff*x)^m*((a + b*(1 + ff^2*x^2)^(n/2))^p/(1 + ff^2*x^2
)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && Inte
gerQ[n/2] && (IntegerQ[m/2] || EqQ[n, 2])
```

3.451.4 Maple [F]

$$\int \cot(fx+e)^4 \left(a+b\sec(fx+e)^2\right)^p dx$$

input
```
int(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x)
```

output
```
int(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x)
```

3.451.5 Fricas [F]

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p dx = \int \left(b\sec(fx+e)^2+a\right)^p \cot(fx+e)^4 dx$$

input
```
integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e)^2 + a)^p*cot(f*x + e)^4, x)
```

3.451.6 Sympy [F(-1)]

Timed out.

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)**4*(a+b*sec(f*x+e)**2)**p,x)
```

output
```
Timed out
```

3.451.7 Maxima [F]

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec(fx+e)^2+a\right)^p \cot(fx+e)^4\,dx$$

input
```
integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*cot(f*x + e)^4, x)
```

3.451.8 Giac [F]

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \left(b\sec(fx+e)^2+a\right)^p \cot(fx+e)^4\,dx$$

input
```
integrate(cot(f*x+e)^4*(a+b*sec(f*x+e)^2)^p,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e)^2 + a)^p*cot(f*x + e)^4, x)
```

3.451.9 Mupad [F(-1)]

Timed out.

$$\int \cot^4(e+fx)\left(a+b\sec^2(e+fx)\right)^p\,dx = \int \cot(e+f\,x)^4\left(a+\frac{b}{\cos(e+f\,x)^2}\right)^p\,dx$$

input
```
int(cot(e + f*x)^4*(a + b/cos(e + f*x)^2)^p,x)
```

output
```
int(cot(e + f*x)^4*(a + b/cos(e + f*x)^2)^p, x)
```

3.452 $\int (a + b\sec^3(e+fx))\tan^5(e+fx)\,dx$

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3.452.1 Optimal result

Integrand size = 21, antiderivative size = 92

$$\int (a + b\sec^3(e+fx))\tan^5(e+fx)\,dx = -\frac{a\log(\cos(e+fx))}{f} - \frac{a\sec^2(e+fx)}{f} + \frac{b\sec^3(e+fx)}{3f} + \frac{a\sec^4(e+fx)}{4f} - \frac{2b\sec^5(e+fx)}{5f} + \frac{b\sec^7(e+fx)}{7f}$$

output
```
-a*ln(cos(f*x+e))/f-a*sec(f*x+e)^2/f+1/3*b*sec(f*x+e)^3/f+1/4*a*sec(f*x+e)
^4/f-2/5*b*sec(f*x+e)^5/f+1/7*b*sec(f*x+e)^7/f
```

3.452.2 Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.95

$$\int (a + b\sec^3(e+fx))\tan^5(e+fx)\,dx = \frac{b\sec^3(e+fx)}{3f} - \frac{2b\sec^5(e+fx)}{5f} + \frac{b\sec^7(e+fx)}{7f} - \frac{a(4\log(\cos(e+fx)) + 2\tan^2(e+fx) - \tan^4(e+fx))}{4f}$$

input
```
Integrate[(a + b*Sec[e + f*x]^3)*Tan[e + f*x]^5,x]
```

output
```
(b*Sec[e + f*x]^3)/(3*f) - (2*b*Sec[e + f*x]^5)/(5*f) + (b*Sec[e + f*x]^7)
/(7*f) - (a*(4*Log[Cos[e + f*x]] + 2*Tan[e + f*x]^2 - Tan[e + f*x]^4))/(4*
f)
```

3.452.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4626, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^5(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \tan(e+fx)^5\left(a+b\sec(e+fx)^3\right)dx$$

$$\downarrow 4626$$

$$-\frac{\int \left(1-\cos^2(e+fx)\right)^2\left(a\cos^3(e+fx)+b\right)\sec^8(e+fx)d\cos(e+fx)}{f}$$

$$\downarrow 2333$$

$$-\frac{\int \left(b\sec^8(e+fx)-2b\sec^6(e+fx)+a\sec^5(e+fx)+b\sec^4(e+fx)-2a\sec^3(e+fx)+a\sec(e+fx)\right)d\cos(e}{f}$$

$$\downarrow 2009$$

$$-\frac{-\frac{1}{4}a\sec^4(e+fx)+a\sec^2(e+fx)+a\log(\cos(e+fx))-\frac{1}{7}b\sec^7(e+fx)+\frac{2}{5}b\sec^5(e+fx)-\frac{1}{3}b\sec^3(e+fx)}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^3)*Tan[e + f*x]^5,x]
```

output
```
-((a*Log[Cos[e + f*x]] + a*Sec[e + f*x]^2 - (b*Sec[e + f*x]^3)/3 - (a*Sec[
e + f*x]^4)/4 + (2*b*Sec[e + f*x]^5)/5 - (b*Sec[e + f*x]^7)/7)/f)
```

3.452.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)  Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.452.4 Maple [A] (verified)

Time = 3.18 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.76

method	result
derivativedivides	$\frac{\frac{b\sec(fx+e)^7}{7}-\frac{2b\sec(fx+e)^5}{5}+\frac{a\sec(fx+e)^4}{4}+\frac{b\sec(fx+e)^3}{3}-a\sec(fx+e)^2+a\ln(\sec(fx+e))}{f}$
default	$\frac{\frac{b\sec(fx+e)^7}{7}-\frac{2b\sec(fx+e)^5}{5}+\frac{a\sec(fx+e)^4}{4}+\frac{b\sec(fx+e)^3}{3}-a\sec(fx+e)^2+a\ln(\sec(fx+e))}{f}$
parts	$\frac{a\left(\frac{\tan(fx+e)^4}{4}-\frac{\tan(fx+e)^2}{2}+\frac{\ln\left(1+\tan(fx+e)^2\right)}{2}\right)}{f}+\frac{b\left(\frac{\sec(fx+e)^7}{7}-\frac{2\sec(fx+e)^5}{5}+\frac{\sec(fx+e)^3}{3}\right)}{f}$
risch	$iax+\frac{2iae}{f}-\frac{4\left(105a\,e^{12i(fx+e)}-70b\,e^{11i(fx+e)}+420a\,e^{10i(fx+e)}+56b\,e^{9i(fx+e)}+735a\,e^{8i(fx+e)}-228b\,e^{7i(fx+e)}+\right.}{105f\left(e^{2i(fx+e)}+1\right)^7}$

input
```
int((a+b*sec(f*x+e)^3)*tan(f*x+e)^5,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/7*b*sec(f*x+e)^7-2/5*b*sec(f*x+e)^5+1/4*a*sec(f*x+e)^4+1/3*b*sec(f*
x+e)^3-a*sec(f*x+e)^2+a*ln(sec(f*x+e)))
```

3.452.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.88

$$\int \left(a + b\sec^3(e+fx)\right)\tan^5(e+fx)\,dx =$$
$$-\frac{420\,a\cos(fx+e)^7\log(-\cos(fx+e)) + 420\,a\cos(fx+e)^5 - 140\,b\cos(fx+e)^4 - 105\,a\cos(fx+e)}{420\,f\cos(fx+e)^7}$$

input
```
integrate((a+b*sec(f*x+e)^3)*tan(f*x+e)^5,x, algorithm="fricas")
```

output
```
-1/420*(420*a*cos(f*x + e)^7*log(-cos(f*x + e)) + 420*a*cos(f*x + e)^5 - 1
40*b*cos(f*x + e)^4 - 105*a*cos(f*x + e)^3 + 168*b*cos(f*x + e)^2 - 60*b)/
(f*cos(f*x + e)^7)
```

3.452.6 Sympy [A] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.29

$$\int \left(a + b\sec^3(e+fx)\right)\tan^5(e+fx)\,dx$$
$$= \begin{cases} \frac{a\log\left(\tan^2(e+fx)+1\right)}{2f} + \frac{a\tan^4(e+fx)}{4f} - \frac{a\tan^2(e+fx)}{2f} + \frac{b\tan^4(e+fx)\sec^3(e+fx)}{7f} - \frac{4b\tan^2(e+fx)\sec^3(e+fx)}{35f} + \frac{8b\sec^3(e+fx)}{105f} & \\ x(a+b\sec^3(e))\tan^5(e) & \end{cases}$$

input
```
integrate((a+b*sec(f*x+e)**3)*tan(f*x+e)**5,x)
```

output
```
Piecewise((a*log(tan(e + f*x)**2 + 1)/(2*f) + a*tan(e + f*x)**4/(4*f) - a*
tan(e + f*x)**2/(2*f) + b*tan(e + f*x)**4*sec(e + f*x)**3/(7*f) - 4*b*tan(
e + f*x)**2*sec(e + f*x)**3/(35*f) + 8*b*sec(e + f*x)**3/(105*f), Ne(f, 0)
), (x*(a + b*sec(e)**3)*tan(e)**5, True))
```

3.452.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.79

$$\int \left(a + b\sec^3(e+fx)\right)\tan^5(e+fx)\,dx$$
$$= -\frac{420\,a\log\left(\cos\left(fx+e\right)\right) + \frac{420\,a\cos(fx+e)^5 - 140\,b\cos(fx+e)^4 - 105\,a\cos(fx+e)^3 + 168\,b\cos(fx+e)^2 - 60\,b}{\cos(fx+e)^7}}{420\,f}$$

input
```
integrate((a+b*sec(f*x+e)^3)*tan(f*x+e)^5,x, algorithm="maxima")
```

output
```
-1/420*(420*a*log(cos(f*x + e)) + (420*a*cos(f*x + e)^5 - 140*b*cos(f*x +
e)^4 - 105*a*cos(f*x + e)^3 + 168*b*cos(f*x + e)^2 - 60*b)/cos(f*x + e)^7)
/f
```

3.452.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 340 vs. 2(84) = 168.

Time = 1.63 (sec) , antiderivative size = 340, normalized size of antiderivative = 3.70

$$\int \left(a + b\sec^3(e+fx)\right)\tan^5(e+fx)\,dx$$
$$= \frac{420\,a\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right) - 420\,a\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}-1\right|\right) + \frac{1089\,a+64\,b+\frac{8463\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{448\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}}{}}{}$$

input
```
integrate((a+b*sec(f*x+e)^3)*tan(f*x+e)^5,x, algorithm="giac")
```

output
```
1/420*(420*a*log(abs(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1)) - 420*a*
log(abs(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 1)) + (1089*a + 64*b + 84
63*a*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 448*b*(cos(f*x + e) - 1)/(cos
(f*x + e) + 1) + 28749*a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 1344*
b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 51555*a*(cos(f*x + e) - 1)^3
/(cos(f*x + e) + 1)^3 - 2240*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 +
 51555*a*(cos(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 4480*b*(cos(f*x + e)
- 1)^4/(cos(f*x + e) + 1)^4 + 28749*a*(cos(f*x + e) - 1)^5/(cos(f*x + e) +
 1)^5 + 8463*a*(cos(f*x + e) - 1)^6/(cos(f*x + e) + 1)^6 + 1089*a*(cos(f*x
 + e) - 1)^7/(cos(f*x + e) + 1)^7)/((cos(f*x + e) - 1)/(cos(f*x + e) + 1)
+ 1)^7)/f
```

3.452.9 Mupad [B] (verification not implemented)

Time = 24.34 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.47

$$\int \left(a+b\sec^3(e+fx)\right)\tan^5(e+fx)\,dx = \frac{2\,a\,\operatorname{atanh}\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\right)}{f}$$
$$-\frac{2\,a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{12}-14\,a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}+\left(32\,a+\frac{32\,b}{3}\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8+\left(\frac{16\,b}{3}-32\,a\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6+(}{f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{14}-7\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{12}+21\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}-35\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8+35\tan\left(\frac{e}{2}+\right.\right.}$$

input
```
int(tan(e + f*x)^5*(a + b/cos(e + f*x)^3),x)
```

output
```
(2*a*atanh(tan(e/2 + (f*x)/2)^2))/f - ((16*b)/105 - tan(e/2 + (f*x)/2)^2*(
2*a + (16*b)/15) + tan(e/2 + (f*x)/2)^4*(14*a + (16*b)/5) - tan(e/2 + (f*x
)/2)^6*(32*a - (16*b)/3) + tan(e/2 + (f*x)/2)^8*(32*a + (32*b)/3) - 14*a*t
an(e/2 + (f*x)/2)^10 + 2*a*tan(e/2 + (f*x)/2)^12)/(f*(7*tan(e/2 + (f*x)/2)
^2 - 21*tan(e/2 + (f*x)/2)^4 + 35*tan(e/2 + (f*x)/2)^6 - 35*tan(e/2 + (f*x
)/2)^8 + 21*tan(e/2 + (f*x)/2)^10 - 7*tan(e/2 + (f*x)/2)^12 + tan(e/2 + (f
*x)/2)^14 - 1))
```

3.453 $\int (a + b\sec^3(e+fx))\tan^3(e+fx)\,dx$

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3.453.2 Mathematica [A] (verified) . . . 3294
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3.453.1 Optimal result

Integrand size = 21, antiderivative size = 61

$$\int (a + b\sec^3(e+fx))\tan^3(e+fx)\,dx = \frac{a\log(\cos(e+fx))}{f} + \frac{a\sec^2(e+fx)}{2f} - \frac{b\sec^3(e+fx)}{3f} + \frac{b\sec^5(e+fx)}{5f}$$

output
```
a*ln(cos(f*x+e))/f+1/2*a*sec(f*x+e)^2/f-1/3*b*sec(f*x+e)^3/f+1/5*b*sec(f*x
+e)^5/f
```

3.453.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

$$\int (a + b\sec^3(e+fx))\tan^3(e+fx)\,dx = -\frac{b\sec^3(e+fx)}{3f} + \frac{b\sec^5(e+fx)}{5f} + \frac{a(2\log(\cos(e+fx)) + \tan^2(e+fx))}{2f}$$

input
```
Integrate[(a + b*Sec[e + f*x]^3)*Tan[e + f*x]^3,x]
```

output
```
-1/3*(b*Sec[e + f*x]^3)/f + (b*Sec[e + f*x]^5)/(5*f) + (a*(2*Log[Cos[e + f
*x]] + Tan[e + f*x]^2))/(2*f)
```

3.453.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4626, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \tan(e+fx)^3\left(a+b\sec(e+fx)^3\right)dx$$

$$\downarrow 4626$$

$$-\frac{\int \left(1-\cos^2(e+fx)\right)\left(a\cos^3(e+fx)+b\right)\sec^6(e+fx)d\cos(e+fx)}{f}$$

$$\downarrow 2333$$

$$-\frac{\int \left(b\sec^6(e+fx)-b\sec^4(e+fx)+a\sec^3(e+fx)-a\sec(e+fx)\right)d\cos(e+fx)}{f}$$

$$\downarrow 2009$$

$$-\frac{-\frac{1}{2}a\sec^2(e+fx)-a\log(\cos(e+fx))-\frac{1}{5}b\sec^5(e+fx)+\frac{1}{3}b\sec^3(e+fx)}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^3)*Tan[e + f*x]^3,x]
```

output
```
-((-(a*Log[Cos[e + f*x]]) - (a*Sec[e + f*x]^2)/2 + (b*Sec[e + f*x]^3)/3 -
(b*Sec[e + f*x]^5)/5)/f)
```

3.453.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)  Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.453.4 Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.80

method	result
derivativedivides	$\frac{\frac{b\sec(fx+e)^5}{5}-\frac{b\sec(fx+e)^3}{3}+\frac{a\sec(fx+e)^2}{2}-a\ln(\sec(fx+e))}{f}$
default	$\frac{\frac{b\sec(fx+e)^5}{5}-\frac{b\sec(fx+e)^3}{3}+\frac{a\sec(fx+e)^2}{2}-a\ln(\sec(fx+e))}{f}$
parts	$\frac{a\left(\frac{\tan(fx+e)^2}{2}-\frac{\ln\left(1+\tan(fx+e)^2\right)}{2}\right)}{f}+\frac{b\left(\frac{\sec(fx+e)^5}{5}-\frac{\sec(fx+e)^3}{3}\right)}{f}$
risch	$-iax-\frac{2iae}{f}+\frac{2a\,\mathrm{e}^{8i(fx+e)}-\frac{8b\,\mathrm{e}^{7i(fx+e)}}{3}+6a\,\mathrm{e}^{6i(fx+e)}+\frac{16b\,\mathrm{e}^{5i(fx+e)}}{15}+6a\,\mathrm{e}^{4i(fx+e)}-\frac{8b\,\mathrm{e}^{3i(fx+e)}}{3}+2a\,\mathrm{e}^{2i(fx+e)}}{f\left(\mathrm{e}^{2i(fx+e)}+1\right)^5}$

input
```
int((a+b*sec(f*x+e)^3)*tan(f*x+e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/5*b*sec(f*x+e)^5-1/3*b*sec(f*x+e)^3+1/2*a*sec(f*x+e)^2-a*ln(sec(f*x
+e)))
```

3.453.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

$$\int \left(a+b\sec^3(e+fx)\right)\tan^3(e+fx)\,dx$$
$$=\frac{30\,a\cos(fx+e)^5\log(-\cos(fx+e))+15\,a\cos(fx+e)^3-10\,b\cos(fx+e)^2+6\,b}{30\,f\cos(fx+e)^5}$$

input
```
integrate((a+b*sec(f*x+e)^3)*tan(f*x+e)^3,x, algorithm="fricas")
```

output
```
1/30*(30*a*cos(f*x + e)^5*log(-cos(f*x + e)) + 15*a*cos(f*x + e)^3 - 10*b*
cos(f*x + e)^2 + 6*b)/(f*cos(f*x + e)^5)
```

3.453.6 Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.34

$$\int (a + b\sec^3(e+fx))\tan^3(e+fx)\,dx$$
$$= \begin{cases} -\frac{a\log(\tan^2(e+fx)+1)}{2f} + \frac{a\tan^2(e+fx)}{2f} + \frac{b\tan^2(e+fx)\sec^3(e+fx)}{5f} - \frac{2b\sec^3(e+fx)}{15f} & \text{for } f \neq 0 \\ x(a+b\sec^3(e))\tan^3(e) & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*sec(f*x+e)**3)*tan(f*x+e)**3,x)
```

output
```
Piecewise((-a*log(tan(e + f*x)**2 + 1)/(2*f) + a*tan(e + f*x)**2/(2*f) + b
*tan(e + f*x)**2*sec(e + f*x)**3/(5*f) - 2*b*sec(e + f*x)**3/(15*f), Ne(f,
 0)), (x*(a + b*sec(e)**3)*tan(e)**3, True))
```

3.453.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int (a+b\sec^3(e+fx))\tan^3(e+fx)\,dx = \frac{30\,a\log(\cos(fx+e)) + \frac{15\,a\cos(fx+e)^3 - 10\,b\cos(fx+e)^2 + 6\,b}{\cos(fx+e)^5}}{30\,f}$$

input
```
integrate((a+b*sec(f*x+e)^3)*tan(f*x+e)^3,x, algorithm="maxima")
```

output
```
1/30*(30*a*log(cos(f*x + e)) + (15*a*cos(f*x + e)^3 - 10*b*cos(f*x + e)^2
+ 6*b)/cos(f*x + e)^5)/f
```

3.453.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(55) = 110$.

Time = 0.69 (sec) , antiderivative size = 271, normalized size of antiderivative = 4.44

$$\int (a + b\sec^3(e+fx))\tan^3(e+fx)\,dx =$$

$$-\frac{60\,a\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right) - 60\,a\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}-1\right|\right) + \frac{137\,a+16\,b+\frac{805\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{80\,b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{1}{}}{60\,f}$$

input
```
integrate((a+b*sec(f*x+e)^3)*tan(f*x+e)^3,x, algorithm="giac")
```

output
```
-1/60*(60*a*log(abs(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1)) - 60*a*lo
g(abs(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 1)) + (137*a + 16*b + 805*a
*(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 80*b*(cos(f*x + e) - 1)/(cos(f*x
+ e) + 1) + 1730*a*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 - 80*b*(cos(f
*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 1730*a*(cos(f*x + e) - 1)^3/(cos(f*x
 + e) + 1)^3 + 240*b*(cos(f*x + e) - 1)^3/(cos(f*x + e) + 1)^3 + 805*a*(co
s(f*x + e) - 1)^4/(cos(f*x + e) + 1)^4 + 137*a*(cos(f*x + e) - 1)^5/(cos(f
*x + e) + 1)^5)/((cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1)^5)/f
```

3.453.9 Mupad [B] (verification not implemented)

Time = 24.52 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.74

$$\int (a + b\sec^3(e+fx))\tan^3(e+fx)\,dx$$

$$= \frac{2\,a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8 + (-6\,a-4\,b)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6 + \left(6\,a-\frac{4\,b}{3}\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4 + \left(-2\,a-\frac{4\,b}{3}\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2 +}{f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{10} - 5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8 + 10\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6 - 10\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4 + 5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2 - 1\right)}$$

$$-\frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\right)}{f}$$

input
```
int(tan(e + f*x)^3*(a + b/cos(e + f*x)^3),x)
```

output
```
((4*b)/15 - tan(e/2 + (f*x)/2)^2*(2*a + (4*b)/3) - tan(e/2 + (f*x)/2)^6*(6
*a + 4*b) + tan(e/2 + (f*x)/2)^4*(6*a - (4*b)/3) + 2*a*tan(e/2 + (f*x)/2)^
8)/(f*(5*tan(e/2 + (f*x)/2)^2 - 10*tan(e/2 + (f*x)/2)^4 + 10*tan(e/2 + (f*
x)/2)^6 - 5*tan(e/2 + (f*x)/2)^8 + tan(e/2 + (f*x)/2)^10 - 1)) - (2*a*atan
h(tan(e/2 + (f*x)/2)^2))/f
```

3.454 $\int (a + b\sec^3(e + fx))\tan(e + fx)\,dx$

3.454.1 Optimal result . . . 3300
3.454.2 Mathematica [A] (verified) . . . 3300
3.454.3 Rubi [A] (verified) . . . 3301
3.454.4 Maple [A] (verified) . . . 3302
3.454.5 Fricas [A] (verification not implemented) . . . 3302
3.454.6 Sympy [A] (verification not implemented) . . . 3303
3.454.7 Maxima [A] (verification not implemented) . . . 3303
3.454.8 Giac [B] (verification not implemented) . . . 3303
3.454.9 Mupad [B] (verification not implemented) . . . 3304

3.454.1 Optimal result

Integrand size = 19, antiderivative size = 30

$$\int (a + b\sec^3(e + fx))\tan(e + fx)\,dx = -\frac{a\log(\cos(e + fx))}{f} + \frac{b\sec^3(e + fx)}{3f}$$

output
```
-a*ln(cos(f*x+e))/f+1/3*b*sec(f*x+e)^3/f
```

3.454.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (a + b\sec^3(e + fx))\tan(e + fx)\,dx = -\frac{a\log(\cos(e + fx))}{f} + \frac{b\sec^3(e + fx)}{3f}$$

input
```
Integrate[(a + b*Sec[e + f*x]^3)*Tan[e + f*x],x]
```

output
```
-((a*Log[Cos[e + f*x]])/f) + (b*Sec[e + f*x]^3)/(3*f)
```

3.454.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 4626, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \tan(e+fx)\left(a+b\sec(e+fx)^3\right)\,dx$$

$$\downarrow 4626$$

$$-\frac{\int \left(a\cos^3(e+fx)+b\right)\sec^4(e+fx)d\cos(e+fx)}{f}$$

$$\downarrow 802$$

$$-\frac{\int \left(b\sec^4(e+fx)+a\sec(e+fx)\right)d\cos(e+fx)}{f}$$

$$\downarrow 2009$$

$$-\frac{a\log(\cos(e+fx))-\frac{1}{3}b\sec^3(e+fx)}{f}$$

input
```
Int[(a + b*Sec[e + f*x]^3)*Tan[e + f*x],x]
```

output
```
-((a*Log[Cos[e + f*x]] - (b*Sec[e + f*x]^3)/3)/f)
```

3.454.3.1 Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)  Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.454.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

method	result	size
derivativedivides	$\frac{\frac{b\sec(fx+e)^3}{3}+a\ln(\sec(fx+e))}{f}$	26
default	$\frac{\frac{b\sec(fx+e)^3}{3}+a\ln(\sec(fx+e))}{f}$	26
parts	$\frac{a\ln\left(1+\tan(fx+e)^2\right)}{2f}+\frac{b\sec(fx+e)^3}{3f}$	33
risch	$iax+\frac{2iae}{f}+\frac{8b\mathrm{e}^{3i(fx+e)}}{3f\left(\mathrm{e}^{2i(fx+e)}+1\right)^3}-\frac{a\ln\left(\mathrm{e}^{2i(fx+e)}+1\right)}{f}$	61

input
```
int((a+b*sec(f*x+e)^3)*tan(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/3*b*sec(f*x+e)^3+a*ln(sec(f*x+e)))
```

3.454.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.23

$$\int \left(a+b\sec^3(e+fx)\right)\tan(e+fx)\,dx = -\frac{3\,a\cos\left(fx+e\right)^3\log\left(-\cos\left(fx+e\right)\right)-b}{3\,f\cos\left(fx+e\right)^3}$$

input
```
integrate((a+b*sec(f*x+e)^3)*tan(f*x+e),x, algorithm="fricas")
```

output
```
-1/3*(3*a*cos(f*x + e)^3*log(-cos(f*x + e)) - b)/(f*cos(f*x + e)^3)
```

3.454.6 Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.40

$$\int \left(a + b\sec^3(e+fx)\right)\tan(e+fx)\,dx = \begin{cases} \frac{a\log\left(\tan^2(e+fx)+1\right)}{2f} + \frac{b\sec^3(e+fx)}{3f} & \text{for } f \neq 0 \\ x(a+b\sec^3(e))\tan(e) & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*sec(f*x+e)**3)*tan(f*x+e),x)
```

output
```
Piecewise((a*log(tan(e + f*x)**2 + 1)/(2*f) + b*sec(e + f*x)**3/(3*f), Ne(
f, 0)), (x*(a + b*sec(e)**3)*tan(e), True))
```

3.454.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \left(a + b\sec^3(e+fx)\right)\tan(e+fx)\,dx = -\frac{a\log\left(\cos\left(fx+e\right)^3\right) - \frac{b}{\cos(fx+e)^3}}{3\,f}$$

input
```
integrate((a+b*sec(f*x+e)^3)*tan(f*x+e),x, algorithm="maxima")
```

output
```
-1/3*(a*log(cos(f*x + e)^3) - b/cos(f*x + e)^3)/f
```

3.454.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. 2(28) = 56.

Time = 0.35 (sec) , antiderivative size = 181, normalized size of antiderivative = 6.03

$$\int \left(a + b\sec^3(e+fx)\right)\tan(e+fx)\,dx$$

$$= \frac{6\,a\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right) - 6\,a\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}-1\right|\right) + \frac{11\,a+4\,b+\frac{33\,a(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{33\,a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{12\,b(\cos(fx}{(\cos(fx+}}{\left(\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right)^3}}{6\,f}$$

input
```
integrate((a+b*sec(f*x+e)^3)*tan(f*x+e),x, algorithm="giac")
```

output
```
1/6*(6*a*log(abs(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1)) - 6*a*log(ab
s(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) - 1)) + (11*a + 4*b + 33*a*(cos(f
*x + e) - 1)/(cos(f*x + e) + 1) + 33*a*(cos(f*x + e) - 1)^2/(cos(f*x + e)
+ 1)^2 + 12*b*(cos(f*x + e) - 1)^2/(cos(f*x + e) + 1)^2 + 11*a*(cos(f*x +
e) - 1)^3/(cos(f*x + e) + 1)^3)/((cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1
)^3)/f
```

3.454.9 Mupad [B] (verification not implemented)

Time = 20.76 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.77

$$\int \left(a + b\sec^3(e+fx)\right)\tan(e+fx)\,dx$$
$$= \frac{2\,a\,\operatorname{atanh}\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\right)}{f} - \frac{2\,b\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4 + \frac{2b}{3}}{f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6 - 3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4 + 3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2 - 1\right)}$$

input
```
int(tan(e + f*x)*(a + b/cos(e + f*x)^3),x)
```

output
```
(2*a*atanh(tan(e/2 + (f*x)/2)^2))/f - ((2*b)/3 + 2*b*tan(e/2 + (f*x)/2)^4)
/(f*(3*tan(e/2 + (f*x)/2)^2 - 3*tan(e/2 + (f*x)/2)^4 + tan(e/2 + (f*x)/2)^
6 - 1))
```

3.455 $\int \cot(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx$

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3.455.1 Optimal result

Integrand size = 19, antiderivative size = 54

$$\int \cot(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx = \frac{(a+b)\log(1-\cos(e+fx))}{2f} + \frac{(a-b)\log(1+\cos(e+fx))}{2f} + \frac{b\sec(e+fx)}{f}$$

output
```
1/2*(a+b)*ln(1-cos(f*x+e))/f+1/2*(a-b)*ln(1+cos(f*x+e))/f+b*sec(f*x+e)/f
```

3.455.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.28

$$\int \cot(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx = -\frac{b\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{f} + \frac{a\log(\cos(e+fx))}{f} + \frac{b\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f} + \frac{a\log(\tan(e+fx))}{f} + \frac{b\sec(e+fx)}{f}$$

input
```
Integrate[Cot[e + f*x]*(a + b*Sec[e + f*x]^3),x]
```

output
```
-((b*Log[Cos[(e + f*x)/2]])/f) + (a*Log[Cos[e + f*x]])/f + (b*Log[Sin[(e +
 f*x)/2]])/f + (a*Log[Tan[e + f*x]])/f + (b*Sec[e + f*x])/f
```

3.455.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 4626, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \frac{a+b\sec(e+fx)^3}{\tan(e+fx)}dx$$

$$\downarrow 4626$$

$$-\frac{\int \frac{\left(a\cos^3(e+fx)+b\right)\sec^2(e+fx)}{1-\cos^2(e+fx)}d\cos(e+fx)}{f}$$

$$\downarrow 2333$$

$$-\frac{\int \left(b\sec^2(e+fx)+\frac{-a-b}{2(\cos(e+fx)-1)}+\frac{b-a}{2(\cos(e+fx)+1)}\right)d\cos(e+fx)}{f}$$

$$\downarrow 2009$$

$$-\frac{-\frac{1}{2}(a+b)\log(1-\cos(e+fx))-\frac{1}{2}(a-b)\log(\cos(e+fx)+1)-b\sec(e+fx)}{f}$$

input
```
Int[Cot[e + f*x]*(a + b*Sec[e + f*x]^3),x]
```

output
```
-((-1/2*((a + b)*Log[1 - Cos[e + f*x]]) - ((a - b)*Log[1 + Cos[e + f*x]])/
2 - b*Sec[e + f*x])/f)
```

3.455.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)  Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.455.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{a\ln(\sin(fx+e))+b\left(\frac{1}{\cos(fx+e)}+\ln(\csc(fx+e)-\cot(fx+e))\right)}{f}$
default	$\frac{a\ln(\sin(fx+e))+b\left(\frac{1}{\cos(fx+e)}+\ln(\csc(fx+e)-\cot(fx+e))\right)}{f}$
risch	$-iax-\frac{2iae}{f}+\frac{2b\,e^{i(fx+e)}}{f\left(e^{2i(fx+e)}+1\right)}+\frac{\ln\left(e^{i(fx+e)}-1\right)a}{f}+\frac{\ln\left(e^{i(fx+e)}-1\right)b}{f}+\frac{\ln\left(e^{i(fx+e)}+1\right)a}{f}-\frac{\ln\left(e^{i(fx+e)}+1\right.}{f}$

input
```
int(cot(f*x+e)*(a+b*sec(f*x+e)^3),x,method=_RETURNVERBOSE)
```

output
```
1/f*(a*ln(sin(f*x+e))+b*(1/cos(f*x+e)+ln(csc(f*x+e)-cot(f*x+e))))
```

3.455.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.13

$$\int \cot(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx$$
$$=\frac{(a-b)\cos(fx+e)\log\left(\frac{1}{2}\cos(fx+e)+\frac{1}{2}\right)+(a+b)\cos(fx+e)\log\left(-\frac{1}{2}\cos(fx+e)+\frac{1}{2}\right)+2b}{2f\cos(fx+e)}$$

input
```
integrate(cot(f*x+e)*(a+b*sec(f*x+e)^3),x, algorithm="fricas")
```

output
```
1/2*((a - b)*cos(f*x + e)*log(1/2*cos(f*x + e) + 1/2) + (a + b)*cos(f*x +
e)*log(-1/2*cos(f*x + e) + 1/2) + 2*b)/(f*cos(f*x + e))
```

3.455.6 Sympy [F]

$$\int \cot(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx = \int \left(a+b\sec^3(e+fx)\right)\cot(e+fx)\,dx$$

input
```
integrate(cot(f*x+e)*(a+b*sec(f*x+e)**3),x)
```

output
```
Integral((a + b*sec(e + f*x)**3)*cot(e + f*x), x)
```

3.455.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int \cot(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx$$
$$=\frac{(a-b)\log(\cos(fx+e)+1)+(a+b)\log(\cos(fx+e)-1)+\frac{2b}{\cos(fx+e)}}{2f}$$

input
```
integrate(cot(f*x+e)*(a+b*sec(f*x+e)^3),x, algorithm="maxima")
```

output
```
1/2*((a - b)*log(cos(f*x + e) + 1) + (a + b)*log(cos(f*x + e) - 1) + 2*b/c
os(f*x + e))/f
```

3.455.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.61

$$\int \cot(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx = \frac{(a+b)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right) - 2\,a\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right) + \frac{4\,b}{\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1}}{2\,f}$$

input
```
integrate(cot(f*x+e)*(a+b*sec(f*x+e)^3),x, algorithm="giac")
```

output
```
1/2*((a + b)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1)) - 2*a*log(a
bs(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1)) + 4*b/((cos(f*x + e) - 1)/
(cos(f*x + e) + 1) + 1))/f
```

3.455.9 Mupad [B] (verification not implemented)

Time = 20.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.33

$$\int \cot(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx = \frac{a\,\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f} - \frac{a\,\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2+1\right)}{f} - \frac{2\,b}{f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2-1\right)} + \frac{b\,\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f}$$

input
```
int(cot(e + f*x)*(a + b/cos(e + f*x)^3),x)
```

output
```
(a*log(tan(e/2 + (f*x)/2)))/f - (a*log(tan(e/2 + (f*x)/2)^2 + 1))/f - (2*b
)/(f*(tan(e/2 + (f*x)/2)^2 - 1)) + (b*log(tan(e/2 + (f*x)/2)))/f
```

3.456 $\int \cot^3(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx$

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3.456.1 Optimal result

Integrand size = 21, antiderivative size = 72

$$\int \cot^3(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx = -\frac{(a+b\cos(e+fx))\csc^2(e+fx)}{2f} - \frac{(2a-b)\log(1-\cos(e+fx))}{4f} - \frac{(2a+b)\log(1+\cos(e+fx))}{4f}$$

output
```
-1/2*(a+b*cos(f*x+e))*csc(f*x+e)^2/f-1/4*(2*a-b)*ln(1-cos(f*x+e))/f-1/4*(2
*a+b)*ln(1+cos(f*x+e))/f
```

3.456.2 Mathematica [A] (verified)

Time = 1.18 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.58

$$\begin{aligned}&\int \cot^3(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx \\ &= -\frac{b\csc^2\left(\frac{1}{2}(e+fx)\right)}{8f} - \frac{b\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{2f} + \frac{b\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{2f} \\ &\quad - \frac{a(\cot^2(e+fx)+2\log(\cos(e+fx))+2\log(\tan(e+fx)))}{2f} + \frac{b\sec^2\left(\frac{1}{2}(e+fx)\right)}{8f}\end{aligned}$$

input
```
Integrate[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^3),x]
```

output
```
-1/8*(b*Csc[(e + f*x)/2]^2)/f - (b*Log[Cos[(e + f*x)/2]])/(2*f) + (b*Log[S
in[(e + f*x)/2]])/(2*f) - (a*(Cot[e + f*x]^2 + 2*Log[Cos[e + f*x]] + 2*Log
[Tan[e + f*x]]))/(2*f) + (b*Sec[(e + f*x)/2]^2)/(8*f)
```

3.456.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.88, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4626, 2345, 25, 452, 219, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \frac{a+b\sec(e+fx)^3}{\tan(e+fx)^3}dx$$

$$\downarrow 4626$$

$$-\frac{\int \frac{a\cos^3(e+fx)+b}{\left(1-\cos^2(e+fx)\right)^2}d\cos(e+fx)}{f}$$

$$\downarrow 2345$$

$$-\frac{\frac{a+b\cos(e+fx)}{2\left(1-\cos^2(e+fx)\right)}-\frac{1}{2}\int -\frac{b-2a\cos(e+fx)}{1-\cos^2(e+fx)}d\cos(e+fx)}{f}$$

$$\downarrow 25$$

$$-\frac{\frac{1}{2}\int \frac{b-2a\cos(e+fx)}{1-\cos^2(e+fx)}d\cos(e+fx)+\frac{a+b\cos(e+fx)}{2\left(1-\cos^2(e+fx)\right)}}{f}$$

$$\downarrow 452$$

$$-\frac{\frac{1}{2}\left(b\int \frac{1}{1-\cos^2(e+fx)}d\cos(e+fx)-2a\int \frac{\cos(e+fx)}{1-\cos^2(e+fx)}d\cos(e+fx)\right)+\frac{a+b\cos(e+fx)}{2\left(1-\cos^2(e+fx)\right)}}{f}$$

$$\downarrow 219$$

$$-\frac{\frac{1}{2}\left(b\operatorname{arctanh}(\cos(e+fx))-2a\int\frac{\cos(e+fx)}{1-\cos^2(e+fx)}d\cos(e+fx)\right)+\frac{a+b\cos(e+fx)}{2(1-\cos^2(e+fx))}}{f}$$

$$\downarrow 240$$

$$-\frac{\frac{1}{2}\left(a\log\left(1-\cos^2(e+fx)\right)+b\operatorname{arctanh}(\cos(e+fx))\right)+\frac{a+b\cos(e+fx)}{2(1-\cos^2(e+fx))}}{f}$$

input
```
Int[Cot[e + f*x]^3*(a + b*Sec[e + f*x]^3),x]
```

output
```
-(((a + b*Cos[e + f*x])/(2*(1 - Cos[e + f*x]^2)) + (b*ArcTanh[Cos[e + f*x]
] + a*Log[1 - Cos[e + f*x]^2])/2)/f)
```

3.456.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)  Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.456.4 Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{a\left(-\frac{\cot(fx+e)^2}{2}-\ln(\sin(fx+e))\right)+b\left(-\frac{\csc(fx+e)\cot(fx+e)}{2}+\frac{\ln(\csc(fx+e)-\cot(fx+e))}{2}\right)}{f}$
default	$\frac{a\left(-\frac{\cot(fx+e)^2}{2}-\ln(\sin(fx+e))\right)+b\left(-\frac{\csc(fx+e)\cot(fx+e)}{2}+\frac{\ln(\csc(fx+e)-\cot(fx+e))}{2}\right)}{f}$
risch	$iax+\frac{2iae}{f}+\frac{b\,e^{3i(fx+e)}+2a\,e^{2i(fx+e)}+b\,e^{i(fx+e)}}{f\left(e^{2i(fx+e)}-1\right)^2}-\frac{\ln\left(e^{i(fx+e)}+1\right)a}{f}-\frac{\ln\left(e^{i(fx+e)}+1\right)b}{2f}-\frac{\ln\left(e^{i(fx+e)}-1\right)a}{f}$

input
```
int(cot(f*x+e)^3*(a+b*sec(f*x+e)^3),x,method=_RETURNVERBOSE)
```

output
```
1/f*(a*(-1/2*cot(f*x+e)^2-ln(sin(f*x+e)))+b*(-1/2*csc(f*x+e)*cot(f*x+e)+1/
2*ln(csc(f*x+e)-cot(f*x+e))))
```

3.456.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.38

$$\int \cot^3(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx$$
$$=\frac{2\,b\cos\left(fx+e\right)-\left(\left(2\,a+b\right)\cos\left(fx+e\right)^2-2\,a-b\right)\log\left(\frac{1}{2}\,\cos\left(fx+e\right)+\frac{1}{2}\right)-\left(\left(2\,a-b\right)\cos\left(fx+e\right)^2-\right.}{4\left(f\cos\left(fx+e\right)^2-f\right)}$$

input
```
integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^3),x, algorithm="fricas")
```

output
```
1/4*(2*b*cos(f*x + e) - ((2*a + b)*cos(f*x + e)^2 - 2*a - b)*log(1/2*cos(f
*x + e) + 1/2) - ((2*a - b)*cos(f*x + e)^2 - 2*a + b)*log(-1/2*cos(f*x + e
) + 1/2) + 2*a)/(f*cos(f*x + e)^2 - f)
```

3.456.6 Sympy [F]

$$\int \cot^3(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx = \int \left(a+b\sec^3(e+fx)\right)\cot^3(e+fx)\,dx$$

input
```
integrate(cot(f*x+e)**3*(a+b*sec(f*x+e)**3),x)
```

output
```
Integral((a + b*sec(e + f*x)**3)*cot(e + f*x)**3, x)
```

3.456.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.86

$$\int \cot^3(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx$$
$$= -\frac{(2a+b)\log(\cos(fx+e)+1)+(2a-b)\log(\cos(fx+e)-1)-\frac{2(b\cos(fx+e)+a)}{\cos(fx+e)^2-1}}{4f}$$

input
```
integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^3),x, algorithm="maxima")
```

output
```
-1/4*((2*a + b)*log(cos(f*x + e) + 1) + (2*a - b)*log(cos(f*x + e) - 1) -
2*(b*cos(f*x + e) + a)/(cos(f*x + e)^2 - 1))/f
```

3.456.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(66) = 132$.

Time = 0.30 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.39

$$\int \cot^3(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx =$$
$$-\frac{2(2a-b)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)-8a\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)-\frac{\left(a+b+\frac{4a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2b(\cos(fx+e)-1)}{\cos(fx+e)+1}\right)(\cos(fx+e}{\cos(fx+e)-1}}{8f}$$

input
```
integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^3),x, algorithm="giac")
```

output
```
-1/8*(2*(2*a - b)*log(abs(-cos(f*x + e) + 1)/abs(cos(f*x + e) + 1)) - 8*a*
log(abs(-(cos(f*x + e) - 1)/(cos(f*x + e) + 1) + 1)) - (a + b + 4*a*(cos(f
*x + e) - 1)/(cos(f*x + e) + 1) - 2*b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1
))*(cos(f*x + e) + 1)/(cos(f*x + e) - 1) - a*(cos(f*x + e) - 1)/(cos(f*x +
 e) + 1) + b*(cos(f*x + e) - 1)/(cos(f*x + e) + 1))/f
```

3.456.9 Mupad [B] (verification not implemented)

Time = 20.23 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.19

$$\int \cot^3(e+fx)\left(a+b\sec^3(e+fx)\right)\,dx = \frac{a\,\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2+1\right)}{f} - \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\left(\frac{a}{8}-\frac{b}{8}\right)}{f} - \frac{\cot\left(\frac{e}{2}+\frac{fx}{2}\right)^2\left(\frac{a}{8}+\frac{b}{8}\right)}{f} - \frac{\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)\left(a-\frac{b}{2}\right)}{f}$$

input
```
int(cot(e + f*x)^3*(a + b/cos(e + f*x)^3),x)
```

output
```
(a*log(tan(e/2 + (f*x)/2)^2 + 1))/f - (tan(e/2 + (f*x)/2)^2*(a/8 - b/8))/f
 - (cot(e/2 + (f*x)/2)^2*(a/8 + b/8))/f - (log(tan(e/2 + (f*x)/2))*(a - b/
2))/f
```

3.457 $\int \frac{\tan^5(e+fx)}{a+b\sec^3(e+fx)}\,dx$

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3.457.1 Optimal result

Integrand size = 23, antiderivative size = 219

$$\begin{aligned}
&\int \frac{\tan^5(e+fx)}{a+b\sec^3(e+fx)}\,dx \\
&= -\frac{\left(a^{2/3}+2b^{2/3}\right)\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\cos(e+fx)}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{a}b^{4/3}f} \\
&\quad -\frac{\left(a^{2/3}-2b^{2/3}\right)\log\left(\sqrt[3]{b}+\sqrt[3]{a}\cos(e+fx)\right)}{3\sqrt[3]{a}b^{4/3}f} \\
&\quad +\frac{\left(a^{2/3}-2b^{2/3}\right)\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+a^{2/3}\cos^2(e+fx)\right)}{6\sqrt[3]{a}b^{4/3}f} \\
&\quad -\frac{\log\left(b+a\cos^3(e+fx)\right)}{3af}+\frac{\sec(e+fx)}{bf}
\end{aligned}$$

output

```
-1/3*(a^(2/3)-2*b^(2/3))*ln(b^(1/3)+a^(1/3)*cos(f*x+e))/a^(1/3)/b^(4/3)/f+
1/6*(a^(2/3)-2*b^(2/3))*ln(b^(2/3)-a^(1/3)*b^(1/3)*cos(f*x+e)+a^(2/3)*cos(
f*x+e)^2)/a^(1/3)/b^(4/3)/f-1/3*ln(b+a*cos(f*x+e)^3)/a/f+sec(f*x+e)/b/f-1/
3*(a^(2/3)+2*b^(2/3))*arctan(1/3*(b^(1/3)-2*a^(1/3)*cos(f*x+e))/b^(1/3)*3^
(1/2))/a^(1/3)/b^(4/3)/f*3^(1/2)
```

3.457.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.89 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.15

$$\int \frac{\tan^5(e+fx)}{a+b\sec^3(e+fx)}\,dx$$
$$= \frac{3b\log\left(\sec^2\left(\frac{1}{2}(e+fx)\right)\right) - \text{RootSum}\left[-8a+12a\#1-6a\#1^2+a\#1^3-b\#1^3\&,\ \frac{-4a^2\log\left(1-\#1+\tan^2\left(\frac{1}{2}(e+f\right.\right.}$$

input
```
Integrate[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^3),x]
```

output
```
(3*b*Log[Sec[(e + f*x)/2]^2] - RootSum[-8*a + 12*a*#1 - 6*a*#1^2 + a*#1^3
- b*#1^3 & , (-4*a^2*Log[1 - #1 + Tan[(e + f*x)/2]^2] + 4*a*b*Log[1 - #1 +
 Tan[(e + f*x)/2]^2] + 2*a^2*Log[1 - #1 + Tan[(e + f*x)/2]^2]*#1 - 8*a*b*L
og[1 - #1 + Tan[(e + f*x)/2]^2]*#1 + a*b*Log[1 - #1 + Tan[(e + f*x)/2]^2]*
#1^2 - b^2*Log[1 - #1 + Tan[(e + f*x)/2]^2]*#1^2)/(4*a - 4*a*#1 + a*#1^2 -
 b*#1^2) & ] + 3*a*Sec[e + f*x])/(3*a*b*f)
```

3.457.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4626, 2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^5(e+fx)}{a+b\sec^3(e+fx)}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\tan(e+fx)^5}{a+b\sec(e+fx)^3}\,dx$$
$$\downarrow 4626$$
$$-\frac{\int \frac{\left(1-\cos^2(e+fx)\right)^2\sec^2(e+fx)}{a\cos^3(e+fx)+b}\,d\cos(e+fx)}{f}$$

$$\downarrow 2373$$

$$-\frac{\int \left(\frac{\sec^2(e+fx)}{b}+\frac{b\cos^2(e+fx)-a\cos(e+fx)-2b}{b(a\cos^3(e+fx)+b)}\right) d\cos(e+fx)}{f}$$

$$\downarrow 2009$$

$$-\frac{\frac{\left(a^{2/3}+2b^{2/3}\right)\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\cos(e+fx)}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{a}b^{4/3}}-\frac{\left(a^{2/3}-2b^{2/3}\right)\log\left(a^{2/3}\cos^2(e+fx)-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+b^{2/3}\right)}{6\sqrt[3]{a}b^{4/3}}+\frac{\left(a^{2/3}-2b^{2/3}\right)\log\left(\sqrt[3]{a}\right.}{3\sqrt[3]{a}b}}{f}$$

input
```
Int[Tan[e + f*x]^5/(a + b*Sec[e + f*x]^3),x]
```

output
```
-((((a^(2/3) + 2*b^(2/3))*ArcTan[(b^(1/3) - 2*a^(1/3)*Cos[e + f*x])/(Sqrt[
3]*b^(1/3))])/(Sqrt[3]*a^(1/3)*b^(4/3)) + ((a^(2/3) - 2*b^(2/3))*Log[b^(1/
3) + a^(1/3)*Cos[e + f*x]])/(3*a^(1/3)*b^(4/3)) - ((a^(2/3) - 2*b^(2/3))*L
og[b^(2/3) - a^(1/3)*b^(1/3)*Cos[e + f*x] + a^(2/3)*Cos[e + f*x]^2])/(6*a^
(1/3)*b^(4/3)) + Log[b + a*Cos[e + f*x]^3]/(3*a) - Sec[e + f*x]/b)/f)
```

3.457.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)  Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.457.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.02 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.06

method	result
risch	$\frac{ix}{a}+\frac{2ie}{af}+\frac{2e^{i(fx+e)}}{fb\left(e^{2i(fx+e)}+1\right)}-i\left(\sum_{\_R=\mathrm{RootOf}\left(27a^3b^4f^3\_Z^3+27ia^2b^4f^2\_Z^2+(-18a^3b^2f-9ab^4f)\_Z-ia^4+2ia\right.}\right.$
derivativedivides	$\frac{1}{b\cos(fx+e)}+\left(2b\left(\frac{\ln\left(\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{\ln\left(\cos(fx+e)^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2\cos(fx+e)}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}\right)\right.$
default	$\frac{1}{b\cos(fx+e)}+\left(2b\left(\frac{\ln\left(\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{\ln\left(\cos(fx+e)^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2\cos(fx+e)}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}\right)\right.$

input
```
int(tan(f*x+e)^5/(a+b*sec(f*x+e)^3),x,method=_RETURNVERBOSE)
```

output
```
I*x/a+2*I/a/f*e+2*exp(I*(f*x+e))/f/b/(exp(2*I*(f*x+e))+1)-I*sum(_R*ln(exp(
2*I*(f*x+e))+(-18/(a^3+8*a*b^2)*a^2*b^3*f^2*_R^2-36*I/(a^3+8*a*b^2)*b^3*f*
a*_R+8/(a^3+8*a*b^2)*a^2*b+10/(a^3+8*a*b^2)*b^3)*exp(I*(f*x+e))+1),_R=Root
Of(27*a^3*b^4*f^3*_Z^3+27*I*a^2*b^4*f^2*_Z^2+(-18*a^3*b^2*f-9*a*b^4*f)*_Z-
I*a^4+2*I*a^2*b^2-I*b^4))
```

3.457.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 4427, normalized size of antiderivative = 20.21

$$\int \frac{\tan^5(e+fx)}{a+b\sec^3(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^3),x, algorithm="fricas")
```

output
```
-1/36*(2*((-I*sqrt(3) + 1)*(1/(a^2*f^2) - (2*a^2 + b^2)/(a^2*b^2*f^2))/(-1
/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b
^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 9*(I*sqrt(3)
 + 1)*(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^
2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 6/(
a*f))*a*b*f*cos(f*x + e)*log(1/36*((-I*sqrt(3) + 1)*(1/(a^2*f^2) - (2*a^2
+ b^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) +
1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f
^3))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^
4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(
a^3*b^4*f^3))^(1/3) + 6/(a*f))^2*a^2*b^3*f^2 - ((-I*sqrt(3) + 1)*(1/(a^2*f
^2) - (2*a^2 + b^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(
a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^
4)/(a^3*b^4*f^3))^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27/(a^3*f^3) + 1/54*(a^2 +
 8*b^2)/(a*b^4*f^3) + 1/18*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2
*b^2 + b^4)/(a^3*b^4*f^3))^(1/3) + 6/(a*f))*a*b^3*f + 4*a^2*b + 5*b^3 + (a
^3 + 8*a*b^2)*cos(f*x + e)) - (((-I*sqrt(3) + 1)*(1/(a^2*f^2) - (2*a^2 + b
^2)/(a^2*b^2*f^2))/(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^4*f^3) + 1/1
8*(2*a^2 + b^2)/(a^3*b^2*f^3) - 1/54*(a^4 - 2*a^2*b^2 + b^4)/(a^3*b^4*f^3)
)^(1/3) + 9*(I*sqrt(3) + 1)*(-1/27/(a^3*f^3) + 1/54*(a^2 + 8*b^2)/(a*b^...
```

3.457.6 Sympy [F]

$$\int \frac{\tan^5(e+fx)}{a+b\sec^3(e+fx)}\,dx = \int \frac{\tan^5(e+fx)}{a+b\sec^3(e+fx)}\,dx$$

input
```
integrate(tan(f*x+e)**5/(a+b*sec(f*x+e)**3),x)
```

output
```
Integral(tan(e + f*x)**5/(a + b*sec(e + f*x)**3), x)
```

3.457.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.00

$$\int \frac{\tan^5(e+fx)}{a+b\sec^3(e+fx)}\,dx$$

$$= \frac{\frac{2\sqrt{3}\left(2ab\left(3\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{b}{a}\right)+3a^2\left(\frac{b}{a}\right)^{\frac{2}{3}}+2b^2\right)\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{b}{a}\right)^{\frac{1}{3}}-2\cos(fx+e)\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{ab^2} - \frac{3\left(2b\left(\left(\frac{b}{a}\right)^{\frac{2}{3}}+1\right)-a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)\log\left(\cos(fx+e)^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}c\right.}{ab\left(\frac{b}{a}\right)^{\frac{2}{3}}}}{18\,f}$$

input
```
integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^3),x, algorithm="maxima")
```

output
```
1/18*(2*sqrt(3)*(2*a*b*(3*(b/a)^(1/3) - b/a) + 3*a^2*(b/a)^(2/3) + 2*b^2)*
arctan(-1/3*sqrt(3)*((b/a)^(1/3) - 2*cos(f*x + e))/(b/a)^(1/3))/(a*b^2) -
3*(2*b*((b/a)^(2/3) + 1) - a*(b/a)^(1/3))*log(cos(f*x + e)^2 - (b/a)^(1/3)
*cos(f*x + e) + (b/a)^(2/3))/(a*b*(b/a)^(2/3)) - 6*(b*((b/a)^(2/3) - 2) +
a*(b/a)^(1/3))*log((b/a)^(1/3) + cos(f*x + e))/(a*b*(b/a)^(2/3)) + 18/(b*c
os(f*x + e)))/f
```

3.457.8 Giac [F]

$$\int \frac{\tan^5(e+fx)}{a+b\sec^3(e+fx)}\,dx = \int \frac{\tan\left(fx+e\right)^5}{b\sec\left(fx+e\right)^3+a}\,dx$$

input
```
integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^3),x, algorithm="giac")
```

output
```
sage0*x
```

3.457.9 Mupad [B] (verification not implemented)

Time = 22.95 (sec) , antiderivative size = 7402, normalized size of antiderivative = 33.80

$$\int \frac{\tan^5(e+fx)}{a+b\sec^3(e+fx)}\,dx = \text{Too large to display}$$

input
```
int(tan(e + f*x)^5/(a + b/cos(e + f*x)^3),x)
```

output
```
symsum(log(-(262144*(148*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z
+ 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*b^17 - 1920*a*b^15 - 156*b^1
6*cos(e + f*x) + 300*b^16 + 16*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*
b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)^2*b^18 + 5232*a^2*b^14
 - 7872*a^3*b^13 + 7080*a^4*b^12 - 3840*a^5*b^11 + 1200*a^6*b^10 - 192*a^7
*b^9 + 12*a^8*b^8 - 5916*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z
+ 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*a^2*b^15 + 4820*root(27*a^3*
b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^
4, z, k)*a^3*b^14 + 5933*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z
+ 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*a^4*b^13 - 12882*root(27*a^3
*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a
^4, z, k)*a^5*b^12 + 8891*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z
 + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*a^6*b^11 - 2872*root(27*a^3
*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a
^4, z, k)*a^7*b^10 + 447*root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z
+ 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*a^8*b^9 - 26*root(27*a^3*b^4
*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4,
z, k)*a^9*b^8 + root(27*a^3*b^4*z^3 + 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*
b^2*z - 2*a^2*b^2 + b^4 + a^4, z, k)*a^10*b^7 + 1396*root(27*a^3*b^4*z^3 +
 27*a^2*b^4*z^2 + 9*a*b^4*z + 18*a^3*b^2*z - 2*a^2*b^2 + b^4 + a^4, z, ...
```

3.458 $\int \frac{\tan^3(e+fx)}{a+b\sec^3(e+fx)}\,dx$

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3.458.1 Optimal result

Integrand size = 23, antiderivative size = 166

$$\int \frac{\tan^3(e+fx)}{a+b\sec^3(e+fx)}\,dx = \frac{\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\cos(e+fx)}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}f} - \frac{\log\left(\sqrt[3]{b}+\sqrt[3]{a}\cos(e+fx)\right)}{3\sqrt[3]{a}b^{2/3}f} + \frac{\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+a^{2/3}\cos^2(e+fx)\right)}{6\sqrt[3]{a}b^{2/3}f} + \frac{\log\left(b+a\cos^3(e+fx)\right)}{3af}$$

output

```
-1/3*ln(b^(1/3)+a^(1/3)*cos(f*x+e))/a^(1/3)/b^(2/3)/f+1/6*ln(b^(2/3)-a^(1/
3)*b^(1/3)*cos(f*x+e)+a^(2/3)*cos(f*x+e)^2)/a^(1/3)/b^(2/3)/f+1/3*ln(b+a*c
os(f*x+e)^3)/a/f+1/3*arctan(1/3*(b^(1/3)-2*a^(1/3)*cos(f*x+e))/b^(1/3)*3^(
1/2))/a^(1/3)/b^(2/3)/f*3^(1/2)
```

3.458.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.64 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.46

$$\int \frac{\tan^3(e+fx)}{a+b\sec^3(e+fx)}\,dx$$

$$= \frac{-3\log\left(\sec^2\left(\frac{1}{2}(e+fx)\right)\right) + \text{RootSum}\left[-a-b+3a\#1-3b\#1-3a\#1^2-3b\#1^2+a\#1^3-b\#1^3\&, \frac{-a\log}{}\right.}{}$$

input
```
Integrate[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^3),x]
```

output
```
(-3*Log[Sec[(e + f*x)/2]^2] + RootSum[-a - b + 3*a*#1 - 3*b*#1 - 3*a*#1^2
- 3*b*#1^2 + a*#1^3 - b*#1^3 & , (-(a*Log[-#1 + Tan[(e + f*x)/2]^2]) - b*L
og[-#1 + Tan[(e + f*x)/2]^2] - 4*a*Log[-#1 + Tan[(e + f*x)/2]^2]*#1 - 2*b*
Log[-#1 + Tan[(e + f*x)/2]^2]*#1 + a*Log[-#1 + Tan[(e + f*x)/2]^2]*#1^2 -
b*Log[-#1 + Tan[(e + f*x)/2]^2]*#1^2)/(a - b - 2*a*#1 - 2*b*#1 + a*#1^2 -
b*#1^2) & ])/(3*a*f)
```

3.458.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.94, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 4626, 2410, 750, 16, 792, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(e+fx)}{a+b\sec^3(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\tan(e+fx)^3}{a+b\sec(e+fx)^3}\,dx$$

$$\downarrow 4626$$

$$-\frac{\int \frac{1-\cos^2(e+fx)}{a\cos^3(e+fx)+b}\,d\cos(e+fx)}{f}$$

$$\downarrow 2410$$

$$-\frac{\int \frac{1}{a\cos^3(e+fx)+b}\,d\cos(e+fx) - \int \frac{\cos^2(e+fx)}{a\cos^3(e+fx)+b}\,d\cos(e+fx)}{f}$$

$$\downarrow 750$$

$$-\frac{\int \frac{2\sqrt[3]{b}-\sqrt[3]{a}\cos(e+fx)}{a^{2/3}\cos^2(e+fx)-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+b^{2/3}}\,d\cos(e+fx)}{3b^{2/3}} + \frac{\int \frac{1}{\sqrt[3]{a}\cos(e+fx)+\sqrt[3]{b}}\,d\cos(e+fx)}{3b^{2/3}} - \int \frac{\cos^2(e+fx)}{a\cos^3(e+fx)+b}\,d\cos(e+fx)}{f}$$

$$\downarrow 16$$

$$-\frac{\frac{\int \frac{2\sqrt[3]{b}-\sqrt[3]{a}\cos(e+fx)}{a^{2/3}\cos^2(e+fx)-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+b^{2/3}}d\cos(e+fx)}{3b^{2/3}}-\int\frac{\cos^2(e+fx)}{a\cos^3(e+fx)+b}d\cos(e+fx)+\frac{\log\left(\sqrt[3]{a}\cos(e+fx)+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}}{f}$$

$$\downarrow 792$$

$$-\frac{\frac{\int \frac{2\sqrt[3]{b}-\sqrt[3]{a}\cos(e+fx)}{a^{2/3}\cos^2(e+fx)-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+b^{2/3}}d\cos(e+fx)}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}\cos(e+fx)+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}-\frac{\log\left(a\cos^3(e+fx)+b\right)}{3a}}{f}$$

$$\downarrow 1142$$

$$-\frac{\frac{\frac{3}{2}\sqrt[3]{b}\int\frac{1}{a^{2/3}\cos^2(e+fx)-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+b^{2/3}}d\cos(e+fx)-\frac{\int-\frac{\sqrt[3]{a}\left(\sqrt[3]{b}-2\sqrt[3]{a}\cos(e+fx)\right)}{a^{2/3}\cos^2(e+fx)-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+b^{2/3}}d\cos(e+fx)}{2\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}\cos(e+fx)\right.}{3\sqrt[3]{a}b^{2/3}}}{f}$$

$$\downarrow 25$$

$$-\frac{\frac{\frac{3}{2}\sqrt[3]{b}\int\frac{1}{a^{2/3}\cos^2(e+fx)-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+b^{2/3}}d\cos(e+fx)+\frac{\int\frac{\sqrt[3]{a}\left(\sqrt[3]{b}-2\sqrt[3]{a}\cos(e+fx)\right)}{a^{2/3}\cos^2(e+fx)-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+b^{2/3}}d\cos(e+fx)}{2\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}\cos(e+fx)-\right.}{3\sqrt[3]{a}b^{2/3}}}{f}$$

$$\downarrow 27$$

$$-\frac{\frac{\frac{3}{2}\sqrt[3]{b}\int\frac{1}{a^{2/3}\cos^2(e+fx)-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+b^{2/3}}d\cos(e+fx)+\frac{1}{2}\int\frac{\sqrt[3]{b}-2\sqrt[3]{a}\cos(e+fx)}{a^{2/3}\cos^2(e+fx)-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+b^{2/3}}d\cos(e+fx)}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}\cos(e+f\right.}{3\sqrt[3]{a}b^{2/}}}{f}$$

$$\downarrow 1082$$

$$-\frac{\frac{\frac{1}{2}\int\frac{\sqrt[3]{b}-2\sqrt[3]{a}\cos(e+fx)}{a^{2/3}\cos^2(e+fx)-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+b^{2/3}}d\cos(e+fx)+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{a}\cos(e+fx)}{\sqrt[3]{b}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{a}\cos(e+fx)}{\sqrt[3]{b}}\right)}{\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}\cos(e+fx)+\sqrt[3]{}\right.}{3\sqrt[3]{a}b^{2/3}}}{f}$$

$$\downarrow 217$$

$$-\frac{\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{a}\cos(e+fx)}{a^{2/3}\cos^2(e+fx)-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+b^{2/3}}d\cos(e+fx)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}\cos(e+fx)}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}\cos(e+fx)+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}-\frac{\log(a\cos^3(e+fx)+b)}{3a}}{f}$$

$$\downarrow 1103$$

$$-\frac{-\frac{\log\left(a^{2/3}\cos^2(e+fx)-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+b^{2/3}\right)}{2\sqrt[3]{a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}\cos(e+fx)}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}\cos(e+fx)+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}-\frac{\log(a\cos^3(e+fx)+b)}{3a}}{f}$$

input
```
Int[Tan[e + f*x]^3/(a + b*Sec[e + f*x]^3),x]
```

output
```
-((Log[b^(1/3) + a^(1/3)*Cos[e + f*x]]/(3*a^(1/3)*b^(2/3)) + (-((Sqrt[3]*A
rcTan[(1 - (2*a^(1/3)*Cos[e + f*x])/b^(1/3))/Sqrt[3]])/a^(1/3)) - Log[b^(2
/3) - a^(1/3)*b^(1/3)*Cos[e + f*x] + a^(2/3)*Cos[e + f*x]^2]/(2*a^(1/3)))/
(3*b^(2/3)) - Log[b + a*Cos[e + f*x]^3]/(3*a))/f)
```

3.458.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2410
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Si
mp[C   Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[
a/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.458.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.38 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.68

method	result
risch	$-\frac{ix}{a}-\frac{2ie}{af}+i\left(\sum_{\_R=\mathrm{RootOf}\left(27b^2a^3f^3\_Z^3+27ib^2a^2f^2\_Z^2-9\_Zab^2f+ia^2-ib^2\right)}\_R\ln\left(e^{2i(fx+e)}+(-6\right.\right.$
derivativedivides	$\frac{-\frac{\ln\left(\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{\ln\left(\cos(fx+e)^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2\cos(fx+e)}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{\ln\left(b+a\cos(fx+e\right.}{3a}}{f}$
default	$\frac{-\frac{\ln\left(\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{\ln\left(\cos(fx+e)^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2\cos(fx+e)}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{\ln\left(b+a\cos(fx+e\right.}{3a}}{f}$

input
```
int(tan(f*x+e)^3/(a+b*sec(f*x+e)^3),x,method=_RETURNVERBOSE)
```

output
```
-I*x/a-2*I/a/f*e+I*sum(_R*ln(exp(2*I*(f*x+e))+(-6*I*b*f*_R+2*b/a)*exp(I*(f
*x+e))+1),_R=RootOf(27*b^2*a^3*f^3*_Z^3+27*I*b^2*a^2*f^2*_Z^2-9*_Z*a*b^2*f
+I*a^2-I*b^2))
```

3.458.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.89 (sec) , antiderivative size = 1052, normalized size of antiderivative = 6.34

$$\int \frac{\tan^3(e+fx)}{a+b\sec^3(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^3),x, algorithm="fricas")
```

output
```
-1/12*(2*(3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^
2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))*a*f*log(-1/2*(3*(I*sqrt(3) + 1)*(
-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3)
 - 2/(a*f))*a*b*f - a*cos(f*x + e) - b) - ((3*(I*sqrt(3) + 1)*(-1/54/(a^3*
f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))
*a*f + 3*sqrt(1/3)*a*f*sqrt(-((3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(
a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))^2*a^2*f^2 +
4*(3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2
)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))*a*f + 4)/(a^2*f^2)) + 6)*log(1/2*(3*(I*s
qrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^
2*f^3))^(1/3) - 2/(a*f))*a*b*f + 3/2*sqrt(1/3)*a*b*f*sqrt(-((3*(I*sqrt(3)
+ 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))
^(1/3) - 2/(a*f))^2*a^2*f^2 + 4*(3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54
/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))*a*f + 4)/(
a^2*f^2)) - 2*a*cos(f*x + e) + b) - ((3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) +
 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))*a*f -
 3*sqrt(1/3)*a*f*sqrt(-((3*(I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*
f^3) - 1/54*(a^2 - b^2)/(a^3*b^2*f^3))^(1/3) - 2/(a*f))^2*a^2*f^2 + 4*(3*(
I*sqrt(3) + 1)*(-1/54/(a^3*f^3) + 1/54/(a*b^2*f^3) - 1/54*(a^2 - b^2)/(a^3
*b^2*f^3))^(1/3) - 2/(a*f))*a*f + 4)/(a^2*f^2)) + 6)*log(-1/2*(3*(I*sqr...
```

3.458.6 Sympy [F]

$$\int \frac{\tan^3(e+fx)}{a+b\sec^3(e+fx)}\,dx = \int \frac{\tan^3(e+fx)}{a+b\sec^3(e+fx)}\,dx$$

input
```
integrate(tan(f*x+e)**3/(a+b*sec(f*x+e)**3),x)
```

output
```
Integral(tan(e + f*x)**3/(a + b*sec(e + f*x)**3), x)
```

3.458.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.96

$$\int \frac{\tan^3(e+fx)}{a+b\sec^3(e+fx)}\,dx = \frac{2\sqrt{3}\left(a\left(3\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{2b}{a}\right)+2b\right)\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{b}{a}\right)^{\frac{1}{3}}-2\cos(fx+e)\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{ab} - \frac{3\left(2\left(\frac{b}{a}\right)^{\frac{2}{3}}+1\right)\log\left(\cos(fx+e)^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{a\left(\frac{b}{a}\right)^{\frac{2}{3}}} - \frac{6}{18\,f}$$

input
```
integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^3),x, algorithm="maxima")
```

output
```
-1/18*(2*sqrt(3)*(a*(3*(b/a)^(1/3) - 2*b/a) + 2*b)*arctan(-1/3*sqrt(3)*((b
/a)^(1/3) - 2*cos(f*x + e))/(b/a)^(1/3))/(a*b) - 3*(2*(b/a)^(2/3) + 1)*log
(cos(f*x + e)^2 - (b/a)^(1/3)*cos(f*x + e) + (b/a)^(2/3))/(a*(b/a)^(2/3))
- 6*((b/a)^(2/3) - 1)*log((b/a)^(1/3) + cos(f*x + e))/(a*(b/a)^(2/3)))/f
```

3.458.8 Giac [F]

$$\int \frac{\tan^3(e+fx)}{a+b\sec^3(e+fx)}\,dx = \int \frac{\tan\left(fx+e\right)^3}{b\sec\left(fx+e\right)^3+a}\,dx$$

input
```
integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^3),x, algorithm="giac")
```

output
```
sage0*x
```

3.458.9 Mupad [B] (verification not implemented)

Time = 24.32 (sec) , antiderivative size = 1620, normalized size of antiderivative = 9.76

$$\int \frac{\tan^3(e+fx)}{a+b\sec^3(e+fx)}\,dx = \text{Too large to display}$$

input
```
int(tan(e + f*x)^3/(a + b/cos(e + f*x)^3),x)
```

output
```
symsum(log(262144*(a - b)^2*(8*a - 8*b + 4*root(27*a^3*b^2*z^3 - 27*a^2*b^
2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)*a^2 + 4*root(27*a^3*b^2*z^3 - 27*a^2*
b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)*b^2 - 3*root(27*a^3*b^2*z^3 - 27*a^
2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)^2*a^3 - 24*root(27*a^3*b^2*z^3 -
27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)^2*a*b^2 - 36*root(27*a^3*b^2
*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)^3*a^3*b + 28*root(27*
a^3*b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)*a*b + 36*root(
27*a^3*b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)^3*a^2*b^2)*
(16*a^2*tan(e/2 + (f*x)/2)^2 + 32*b^2*tan(e/2 + (f*x)/2)^2 - 4*root(27*a^3
*b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)*a^3 - 4*root(27*a
^3*b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)*b^3 - 8*a^2 + 8
*b^2 + 3*root(27*a^3*b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z,
k)^2*a^4 - 3*root(27*a^3*b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2,
 z, k)^2*a^4*tan(e/2 + (f*x)/2)^2 + 24*root(27*a^3*b^2*z^3 - 27*a^2*b^2*z^
2 + 9*a*b^2*z + a^2 - b^2, z, k)^2*a*b^3 + 3*root(27*a^3*b^2*z^3 - 27*a^2*
b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)^2*a^3*b + 36*root(27*a^3*b^2*z^3 -
27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^2, z, k)^3*a^4*b - 48*a*b*tan(e/2 + (
f*x)/2)^2 + 24*root(27*a^3*b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z + a^2 - b^
2, z, k)^2*a^2*b^2 - 36*root(27*a^3*b^2*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z +
 a^2 - b^2, z, k)^3*a^2*b^3 + 14*root(27*a^3*b^2*z^3 - 27*a^2*b^2*z^2 +...
```

3.459 $\int \frac{\tan(e+fx)}{a+b\sec^3(e+fx)}\,dx$

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3.459.1 Optimal result

Integrand size = 21, antiderivative size = 23

$$\int \frac{\tan(e+fx)}{a+b\sec^3(e+fx)}\,dx = -\frac{\log\left(b+a\cos^3(e+fx)\right)}{3af}$$

output
```
-1/3*ln(b+a*cos(f*x+e)^3)/a/f
```

3.459.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\tan(e+fx)}{a+b\sec^3(e+fx)}\,dx = -\frac{\log\left(b+a\cos^3(e+fx)\right)}{3af}$$

input
```
Integrate[Tan[e + f*x]/(a + b*Sec[e + f*x]^3),x]
```

output
```
-1/3*Log[b + a*Cos[e + f*x]^3]/(a*f)
```

3.459.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4626, 792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(e+fx)}{a+b\sec^3(e+fx)}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\tan(e+fx)}{a+b\sec(e+fx)^3}\,dx$$
$$\downarrow 4626$$
$$-\frac{\int \frac{\cos^2(e+fx)}{a\cos^3(e+fx)+b}\,d\cos(e+fx)}{f}$$
$$\downarrow 792$$
$$-\frac{\log\left(a\cos^3(e+fx)+b\right)}{3af}$$

input
```
Int[Tan[e + f*x]/(a + b*Sec[e + f*x]^3),x]
```

output
```
-1/3*Log[b + a*Cos[e + f*x]^3]/(a*f)
```

3.459.3.1 Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

3.459.4 Maple [A] (verified)

Time = 1.70 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.52

method	result	size
derivativedivides	$\frac{\frac{\ln(\sec(fx+e))}{a}-\frac{\ln\left(a+b\sec(fx+e)^3\right)}{3a}}{f}$	35
default	$\frac{\frac{\ln(\sec(fx+e))}{a}-\frac{\ln\left(a+b\sec(fx+e)^3\right)}{3a}}{f}$	35
risch	$\frac{ix}{a}+\frac{2ie}{af}-\frac{\ln\left(e^{6i(fx+e)}+3e^{4i(fx+e)}+\frac{8be^{3i(fx+e)}}{a}+3e^{2i(fx+e)}+1\right)}{3af}$	76

input
```
int(tan(f*x+e)/(a+b*sec(f*x+e)^3),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/a*ln(sec(f*x+e))-1/3/a*ln(a+b*sec(f*x+e)^3))
```

3.459.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\tan(e+fx)}{a+b\sec^3(e+fx)}\,dx = -\frac{\log\left(a\cos\left(fx+e\right)^3+b\right)}{3af}$$

input
```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)^3),x, algorithm="fricas")
```

output
```
-1/3*log(a*cos(f*x + e)^3 + b)/(a*f)
```

3.459.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(19) = 38$.

Time = 18.05 (sec) , antiderivative size = 139, normalized size of antiderivative = 6.04

$$\int \frac{\tan(e+fx)}{a+b\sec^3(e+fx)}\,dx$$

$$= \begin{cases} \frac{\tilde{\infty} x \tan(e)}{\sec^3(e)} & \text{for } a = 0 \wedge b = \\ -\frac{1}{3bf\sec^3(e+fx)} & \text{for } a = 0 \\ \frac{\log(\tan^2(e+fx)+1)}{2af} & \text{for } b = 0 \\ \frac{x\tan(e)}{a+b\sec^3(e)} & \text{for } f = 0 \\ -\frac{\log\left(-\sqrt[3]{-\frac{a}{b}}+\sec(e+fx)\right)}{3af} + \frac{\log(\tan^2(e+fx)+1)}{2af} - \frac{\log\left(4\left(-\frac{a}{b}\right)^{\frac{2}{3}}+4\sqrt[3]{-\frac{a}{b}}\sec(e+fx)+4\sec^2(e+fx)\right)}{3af} & \text{otherwise} \end{cases}$$

input
```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)**3),x)
```

output
```
Piecewise((zoo*x*tan(e)/sec(e)**3, Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), (-1/(3
*b*f*sec(e + f*x)**3), Eq(a, 0)), (log(tan(e + f*x)**2 + 1)/(2*a*f), Eq(b,
 0)), (x*tan(e)/(a + b*sec(e)**3), Eq(f, 0)), (-log(-(-a/b)**(1/3) + sec(e
 + f*x))/(3*a*f) + log(tan(e + f*x)**2 + 1)/(2*a*f) - log(4*(-a/b)**(2/3)
+ 4*(-a/b)**(1/3)*sec(e + f*x) + 4*sec(e + f*x)**2)/(3*a*f), True))
```

3.459.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\tan(e+fx)}{a+b\sec^3(e+fx)}\,dx = -\frac{\log\left(a\cos(fx+e)^3+b\right)}{3af}$$

input
```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)^3),x, algorithm="maxima")
```

output
```
-1/3*log(a*cos(f*x + e)^3 + b)/(a*f)
```

3.459.8 Giac [F]

$$\int \frac{\tan(e+fx)}{a+b\sec^3(e+fx)}\,dx = \int \frac{\tan(fx+e)}{b\sec(fx+e)^3+a}\,dx$$

input
```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)^3),x, algorithm="giac")
```

output
```
sage0*x
```

3.459.9 Mupad [B] (verification not implemented)

Time = 20.76 (sec) , antiderivative size = 114, normalized size of antiderivative = 4.96

$$\int \frac{\tan(e+fx)}{a+b\sec^3(e+fx)}\,dx$$
$$= \frac{3\ln\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2+1\right)-\ln\left(a+b-3a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2+3a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4-a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6+3b\tan\right.}{3af}$$

input
```
int(tan(e + f*x)/(a + b/cos(e + f*x)^3),x)
```

output
```
(3*log(tan(e/2 + (f*x)/2)^2 + 1) - log(a + b - 3*a*tan(e/2 + (f*x)/2)^2 +
3*a*tan(e/2 + (f*x)/2)^4 - a*tan(e/2 + (f*x)/2)^6 + 3*b*tan(e/2 + (f*x)/2)
^2 + 3*b*tan(e/2 + (f*x)/2)^4 + b*tan(e/2 + (f*x)/2)^6))/(3*a*f)
```

3.460 $\int \frac{\cot(e+fx)}{a+b\sec^3(e+fx)}\,dx$

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3.460.8 Giac [F] . . . 3342
3.460.9 Mupad [B] (verification not implemented) . . . 3342

3.460.1 Optimal result

Integrand size = 21, antiderivative size = 295

$$\begin{aligned}
&\int \frac{\cot(e+fx)}{a+b\sec^3(e+fx)}\,dx \\
&= -\frac{b^{2/3}\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\cos(e+fx)}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{a}\left(a^{4/3}+a^{2/3}b^{2/3}+b^{4/3}\right)f} + \frac{\log(1-\cos(e+fx))}{2(a+b)f} \\
&\quad + \frac{\log(1+\cos(e+fx))}{2(a-b)f} - \frac{\left(a^{2/3}+b^{2/3}\right)b^{2/3}\log\left(\sqrt[3]{b}+\sqrt[3]{a}\cos(e+fx)\right)}{3\sqrt[3]{a}\left(a^2-b^2\right)f} \\
&\quad + \frac{\left(a^{2/3}+b^{2/3}\right)b^{2/3}\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+a^{2/3}\cos^2(e+fx)\right)}{6\sqrt[3]{a}\left(a^2-b^2\right)f} \\
&\quad - \frac{b^2\log\left(b+a\cos^3(e+fx)\right)}{3a\left(a^2-b^2\right)f}
\end{aligned}$$

output

```
1/2*ln(1-cos(f*x+e))/(a+b)/f+1/2*ln(1+cos(f*x+e))/(a-b)/f-1/3*(a^(2/3)+b^(
2/3))*b^(2/3)*ln(b^(1/3)+a^(1/3)*cos(f*x+e))/a^(1/3)/(a^2-b^2)/f+1/6*(a^(2
/3)+b^(2/3))*b^(2/3)*ln(b^(2/3)-a^(1/3)*b^(1/3)*cos(f*x+e)+a^(2/3)*cos(f*x
+e)^2)/a^(1/3)/(a^2-b^2)/f-1/3*b^2*ln(b+a*cos(f*x+e)^3)/a/(a^2-b^2)/f-1/3*
b^(2/3)*arctan(1/3*(b^(1/3)-2*a^(1/3)*cos(f*x+e))/b^(1/3)*3^(1/2))/a^(1/3)
/(a^(4/3)+a^(2/3)*b^(2/3)+b^(4/3))/f*3^(1/2)
```

3.460.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.69 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.73

$$\int \frac{\cot(e+fx)}{a+b\sec^3(e+fx)}\,dx = \frac{\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{(a-b)f} + \frac{\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{(a+b)f}$$
$$+ \frac{b\left(3b\log\left(\sec^2\left(\frac{1}{2}(e+fx)\right)\right) + (-a+b)\text{RootSum}\left[-8a+12a\#1-6a\#1^2+a\#1^3-b\#1^3\&, \frac{-4a\log\left(1-\#\right.}{3\left(a^3-ab^2\right)f}\right.$$

input
```
Integrate[Cot[e + f*x]/(a + b*Sec[e + f*x]^3),x]
```

output
```
Log[Cos[(e + f*x)/2]]/((a - b)*f) + Log[Sin[(e + f*x)/2]]/((a + b)*f) + (b
*(3*b*Log[Sec[(e + f*x)/2]^2] + (-a + b)*RootSum[-8*a + 12*a*#1 - 6*a*#1^2
 + a*#1^3 - b*#1^3 & , (-4*a*Log[1 - #1 + Tan[(e + f*x)/2]^2] + 2*a*Log[1
- #1 + Tan[(e + f*x)/2]^2]*#1 + b*Log[1 - #1 + Tan[(e + f*x)/2]^2]*#1^2)/(
4*a - 4*a*#1 + a*#1^2 - b*#1^2) & ]))/(3*(a^3 - a*b^2)*f)
```

3.460.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4626, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(e+fx)}{a+b\sec^3(e+fx)}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{\tan(e+fx)\left(a+b\sec(e+fx)^3\right)}dx$$
$$\downarrow 4626$$
$$-\frac{\int \frac{\cos^4(e+fx)}{(1-\cos^2(e+fx))(a\cos^3(e+fx)+b)}d\cos(e+fx)}{f}$$
$$\downarrow 7276$$

$$-\frac{\int \left(-\frac{b(b\cos^2(e+fx)-a\cos(e+fx)+b)}{(b^2-a^2)(a\cos^3(e+fx)+b)}-\frac{1}{2(a+b)(\cos(e+fx)-1)}-\frac{1}{2(a-b)(\cos(e+fx)+1)}\right)d\cos(e+fx)}{f}$$

$$\downarrow \text{2009}$$

$$-\frac{\frac{b^{2/3}\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\cos(e+fx)}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{a}(a^{2/3}b^{2/3}+a^{4/3}+b^{4/3})}+\frac{b^2\log(a\cos^3(e+fx)+b)}{3a(a^2-b^2)}-\frac{b^{2/3}(a^{2/3}+b^{2/3})\log\left(a^{2/3}\cos^2(e+fx)-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+b^{2/3}\right)}{6\sqrt[3]{a}(a^2-b^2)}+\frac{b^{}}{}}{f}$$

input
```
Int[Cot[e + f*x]/(a + b*Sec[e + f*x]^3),x]
```

output
```
-(((b^(2/3)*ArcTan[(b^(1/3) - 2*a^(1/3)*Cos[e + f*x])/(Sqrt[3]*b^(1/3))])/
(Sqrt[3]*a^(1/3)*(a^(4/3) + a^(2/3)*b^(2/3) + b^(4/3))) - Log[1 - Cos[e +
f*x]]/(2*(a + b)) - Log[1 + Cos[e + f*x]]/(2*(a - b)) + ((a^(2/3) + b^(2/3
))*b^(2/3)*Log[b^(1/3) + a^(1/3)*Cos[e + f*x]])/(3*a^(1/3)*(a^2 - b^2)) -
((a^(2/3) + b^(2/3))*b^(2/3)*Log[b^(2/3) - a^(1/3)*b^(1/3)*Cos[e + f*x] +
a^(2/3)*Cos[e + f*x]^2])/(6*a^(1/3)*(a^2 - b^2)) + (b^2*Log[b + a*Cos[e +
f*x]^3])/(3*a*(a^2 - b^2)))/f)
```

3.460.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)  Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.460.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.43 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.98

method	result
risch	$\frac{ix}{a}-\frac{ix}{a+b}-\frac{ie}{f(a+b)}-\frac{ix}{a-b}-\frac{ie}{f(a-b)}-\frac{2ia^2b^2f^3x}{-a^5f^3+a^3b^2f^3}-\frac{2ia^2b^2f^2e}{-a^5f^3+a^3b^2f^3}+\frac{\ln\left(e^{i(fx+e)}-1\right)}{f(a+b)}+\frac{\ln\left(e^{i(fx+}\right.}{f(a-}$
derivativedivides	$\frac{\ln(1+\cos(fx+e))}{2a-2b}+\frac{\ln(-1+\cos(fx+e))}{2a+2b}+\left(\right.$ $-b\left(\right.$ $\frac{\ln\left(\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{\ln\left(\cos(fx+e)^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan$
default	$\frac{\ln(1+\cos(fx+e))}{2a-2b}+\frac{\ln(-1+\cos(fx+e))}{2a+2b}+\left(\right.$ $-b\left(\right.$ $\frac{\ln\left(\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{\ln\left(\cos(fx+e)^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan$

input
```
int(cot(f*x+e)/(a+b*sec(f*x+e)^3),x,method=_RETURNVERBOSE)
```

output
```
I*x/a-I/(a+b)*x-I/f/(a+b)*e-I/(a-b)*x-I/f/(a-b)*e-2*I*a^2*b^2*f^3/(-a^5*f^
3+a^3*b^2*f^3)*x-2*I*a^2*b^2*f^2/(-a^5*f^3+a^3*b^2*f^3)*e+1/f/(a+b)*ln(exp
(I*(f*x+e))-1)+1/f/(a-b)*ln(exp(I*(f*x+e))+1)+I*sum(_R*ln(exp(2*I*(f*x+e))
+((-18*a^3/b*f^2+18*a*b*f^2)*_R^2+18*I*b*f*_R-4*b/a)*exp(I*(f*x+e))+1),_R=
RootOf((27*a^5*f^3-27*a^3*b^2*f^3)*_Z^3-27*I*b^2*a^2*f^2*_Z^2+9*_Z*a*b^2*f
+I*b^2))
```

3.460.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.06 (sec) , antiderivative size = 6482, normalized size of antiderivative = 21.97

$$\int \frac{\cot(e+fx)}{a+b\sec^3(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

3.460.6 Sympy [F]

$$\int \frac{\cot(e+fx)}{a+b\sec^3(e+fx)}\,dx = \int \frac{\cot(e+fx)}{a+b\sec^3(e+fx)}\,dx$$

input
```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)**3),x)
```

output
```
Integral(cot(e + f*x)/(a + b*sec(e + f*x)**3), x)
```

3.460.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.04

$$\int \frac{\cot(e+fx)}{a+b\sec^3(e+fx)}\,dx =$$

$$\frac{2\sqrt{3}\left(ab^2\left(3\left(\frac{b}{a}\right)^{\frac{1}{3}}+\frac{2b}{a}\right)-3a^2b\left(\frac{b}{a}\right)^{\frac{2}{3}}-2b^3\right)\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{b}{a}\right)^{\frac{1}{3}}-2\cos(fx+e)\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{\left(a^4\left(\frac{b}{a}\right)^{\frac{2}{3}}-a^2b^2\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}} + \frac{3\left(b^2\left(2\left(\frac{b}{a}\right)^{\frac{2}{3}}-1\right)-ab\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)\log\left(\cos(fx+e)^2-\right.}{a^3\left(\frac{b}{a}\right)^{\frac{2}{3}}-ab^2\left(\frac{b}{a}\right)^{\frac{2}{3}}}$$

$$-\frac{}{18}$$

input
```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)^3),x, algorithm="maxima")
```

output
```
-1/18*(2*sqrt(3)*(a*b^2*(3*(b/a)^(1/3) + 2*b/a) - 3*a^2*b*(b/a)^(2/3) - 2*
b^3)*arctan(-1/3*sqrt(3)*((b/a)^(1/3) - 2*cos(f*x + e))/(b/a)^(1/3))/((a^4
*(b/a)^(2/3) - a^2*b^2*(b/a)^(2/3))*(b/a)^(1/3)) + 3*(b^2*(2*(b/a)^(2/3) -
 1) - a*b*(b/a)^(1/3))*log(cos(f*x + e)^2 - (b/a)^(1/3)*cos(f*x + e) + (b/
a)^(2/3))/(a^3*(b/a)^(2/3) - a*b^2*(b/a)^(2/3)) + 6*(b^2*((b/a)^(2/3) + 1)
 + a*b*(b/a)^(1/3))*log((b/a)^(1/3) + cos(f*x + e))/(a^3*(b/a)^(2/3) - a*b
^2*(b/a)^(2/3)) - 9*log(cos(f*x + e) + 1)/(a - b) - 9*log(cos(f*x + e) - 1
)/(a + b))/f
```

3.460.8 Giac [F]

$$\int \frac{\cot(e+fx)}{a+b\sec^3(e+fx)}\,dx = \int \frac{\cot(fx+e)}{b\sec(fx+e)^3+a}\,dx$$

input
```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)^3),x, algorithm="giac")
```

output
```
sage0*x
```

3.460.9 Mupad [B] (verification not implemented)

Time = 23.78 (sec) , antiderivative size = 11182, normalized size of antiderivative = 37.91

$$\int \frac{\cot(e+fx)}{a+b\sec^3(e+fx)}\,dx = \text{Too large to display}$$

input
```
int(cot(e + f*x)/(a + b/cos(e + f*x)^3),x)
```

output

```
log(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2))/(f*(a + b)) - log(1/cos(e/2 + (
f*x)/2)^2)/(f*(a + b)) + (a*symsum(log((262144*(832*root(27*a^3*b^2*z^3 -
27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)*b^7 - 22*a*b^5 - 840*
b^6*cos(e + f*x) + 440*b^6 - 264*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2
*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^2*b^8 + 16*root(27*a^3*b^2*z^3 - 27*a^5*
z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^3*b^9 + 1823*root(27*a^3*b^2
*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)*a^2*b^5 - 21*r
oot(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)*
a^3*b^4 - 8864*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2
*z - b^2, z, k)^2*a*b^7 + 3092*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b
^2*z^2 + 9*a*b^2*z - b^2, z, k)^3*a*b^8 - 192*root(27*a^3*b^2*z^3 - 27*a^5
*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^4*a*b^9 + 88*root(27*a^3*b^
2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^2*b^8*cos(e +
 f*x) - a^2*b^4*cos(e + f*x) + 65221*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27
*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^2*a^2*b^6 - 32708*root(27*a^3*b^2*z^
3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^2*a^3*b^5 + 2859*
root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)
^2*a^4*b^4 - 9*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^2*b^2*z^2 + 9*a*b^2
*z - b^2, z, k)^2*a^5*b^3 + 26274*root(27*a^3*b^2*z^3 - 27*a^5*z^3 - 27*a^
2*b^2*z^2 + 9*a*b^2*z - b^2, z, k)^3*a^2*b^7 - 212230*root(27*a^3*b^2*z...
```

3.461 $\int \frac{\cot^3(e+fx)}{a+b\sec^3(e+fx)}\,dx$

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3.461.1 Optimal result

Integrand size = 23, antiderivative size = 393

$$\begin{aligned}
&\int \frac{\cot^3(e+fx)}{a+b\sec^3(e+fx)}\,dx \\
&= \frac{b^{4/3}\left(a^2-3a^{2/3}b^{4/3}+2b^2\right)\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\cos(e+fx)}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{a}\left(a^2-b^2\right)^2 f} \\
&\quad - \frac{1}{4(a+b)f(1-\cos(e+fx))} - \frac{1}{4(a-b)f(1+\cos(e+fx))} \\
&\quad - \frac{(2a+5b)\log(1-\cos(e+fx))}{4(a+b)^2 f} - \frac{(2a-5b)\log(1+\cos(e+fx))}{4(a-b)^2 f} \\
&\quad - \frac{b^{4/3}\left(a^2+3a^{2/3}b^{4/3}+2b^2\right)\log\left(\sqrt[3]{b}+\sqrt[3]{a}\cos(e+fx)\right)}{3\sqrt[3]{a}\left(a^2-b^2\right)^2 f} \\
&\quad + \frac{b^{4/3}\left(a^2+3a^{2/3}b^{4/3}+2b^2\right)\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\cos(e+fx)+a^{2/3}\cos^2(e+fx)\right)}{6\sqrt[3]{a}\left(a^2-b^2\right)^2 f} \\
&\quad - \frac{b^2(2a^2+b^2)\log\left(b+a\cos^3(e+fx)\right)}{3a\left(a^2-b^2\right)^2 f}
\end{aligned}$$

output
```
-1/4/(a+b)/f/(1-cos(f*x+e))-1/4/(a-b)/f/(1+cos(f*x+e))-1/4*(2*a+5*b)*ln(1-
cos(f*x+e))/(a+b)^2/f-1/4*(2*a-5*b)*ln(1+cos(f*x+e))/(a-b)^2/f-1/3*b^(4/3)
*(a^2+3*a^(2/3)*b^(4/3)+2*b^2)*ln(b^(1/3)+a^(1/3)*cos(f*x+e))/a^(1/3)/(a^2
-b^2)^2/f+1/6*b^(4/3)*(a^2+3*a^(2/3)*b^(4/3)+2*b^2)*ln(b^(2/3)-a^(1/3)*b^(
1/3)*cos(f*x+e)+a^(2/3)*cos(f*x+e)^2)/a^(1/3)/(a^2-b^2)^2/f-1/3*b^2*(2*a^2
+b^2)*ln(b+a*cos(f*x+e)^3)/a/(a^2-b^2)^2/f+1/3*b^(4/3)*(a^2-3*a^(2/3)*b^(4
/3)+2*b^2)*arctan(1/3*(b^(1/3)-2*a^(1/3)*cos(f*x+e))/b^(1/3)*3^(1/2))/a^(1
/3)/(a^2-b^2)^2/f*3^(1/2)
```

3.461.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 2.65 (sec) , antiderivative size = 336, normalized size of antiderivative = 0.85

$$\int \frac{\cot^3(e+fx)}{a+b\sec^3(e+fx)}\,dx$$

$$= \frac{-\frac{3\csc^2\left(\frac{1}{2}(e+fx)\right)}{a+b}+\frac{12(-2a+5b)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)}{(a-b)^2}-\frac{12(2a+5b)\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)}{(a+b)^2}+\frac{8b^2\left(3(2a^2+b^2)\log\left(\sec^2\left(\frac{1}{2}(e+fx)\right)\right)+(-a+b)\right.}{}}{}$$

input
```
Integrate[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^3),x]
```

output
```
((-3*Csc[(e + f*x)/2]^2)/(a + b) + (12*(-2*a + 5*b)*Log[Cos[(e + f*x)/2]])
/(a - b)^2 - (12*(2*a + 5*b)*Log[Sin[(e + f*x)/2]])/(a + b)^2 + (8*b^2*(3*
(2*a^2 + b^2)*Log[Sec[(e + f*x)/2]^2] + (-a + b)*RootSum[-8*a + 12*a*#1 -
6*a*#1^2 + a*#1^3 - b*#1^3 & , (8*a^2*Log[1 - #1 + Tan[(e + f*x)/2]^2] - 4
*a*b*Log[1 - #1 + Tan[(e + f*x)/2]^2] - 6*a^2*Log[1 - #1 + Tan[(e + f*x)/2
]^2]*#1 + 2*a^2*Log[1 - #1 + Tan[(e + f*x)/2]^2]*#1^2 + b^2*Log[1 - #1 + T
an[(e + f*x)/2]^2]*#1^2)/(4*a - 4*a*#1 + a*#1^2 - b*#1^2) & ]))/(a*(a^2 -
b^2)^2) - (3*Sec[(e + f*x)/2]^2)/(a - b))/(24*f)
```

3.461.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 375, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4626, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(e+fx)}{a+b\sec^3(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\tan(e+fx)^3\,(a+b\sec(e+fx)^3)}dx$$

$$\downarrow 4626$$

$$-\frac{\int \frac{\cos^6(e+fx)}{(1-\cos^2(e+fx))^2(a\cos^3(e+fx)+b)}d\cos(e+fx)}{f}$$

$$\downarrow 7276$$

$$-\frac{\int \left(\frac{(a^2-3b\cos(e+fx)a+2b^2+(2a^2+b^2)\cos^2(e+fx))b^2}{(a^2-b^2)^2(a\cos^3(e+fx)+b)}+\frac{2a+5b}{4(a+b)^2(\cos(e+fx)-1)}+\frac{2a-5b}{4(a-b)^2(\cos(e+fx)+1)}+\frac{1}{4(a+b)(\cos(e+fx)-1)^2}-\right.}{f}$$

$$\downarrow 2009$$

$$-\frac{b^2(2a^2+b^2)\log(a\cos^3(e+fx)+b)}{3a(a^2-b^2)^2}-\frac{b^{4/3}(-3a^{2/3}b^{4/3}+a^2+2b^2)\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\cos(e+fx)}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{a}(a^2-b^2)^2}-\frac{b^{4/3}(3a^{2/3}b^{4/3}+a^2+2b^2)\log\left(a^{2/3}\cos^2(\right.}{6\sqrt[3]{a}(a^2-}$$

input
```
Int[Cot[e + f*x]^3/(a + b*Sec[e + f*x]^3),x]
```

output
```
-((-((b^(4/3)*(a^2 - 3*a^(2/3)*b^(4/3) + 2*b^2)*ArcTan[(b^(1/3) - 2*a^(1/3
)*Cos[e + f*x])/(Sqrt[3]*b^(1/3))])/(Sqrt[3]*a^(1/3)*(a^2 - b^2)^2)) + 1/(
4*(a + b)*(1 - Cos[e + f*x])) + 1/(4*(a - b)*(1 + Cos[e + f*x])) + ((2*a +
 5*b)*Log[1 - Cos[e + f*x]])/(4*(a + b)^2) + ((2*a - 5*b)*Log[1 + Cos[e +
f*x]])/(4*(a - b)^2) + (b^(4/3)*(a^2 + 3*a^(2/3)*b^(4/3) + 2*b^2)*Log[b^(1
/3) + a^(1/3)*Cos[e + f*x]])/(3*a^(1/3)*(a^2 - b^2)^2) - (b^(4/3)*(a^2 + 3
*a^(2/3)*b^(4/3) + 2*b^2)*Log[b^(2/3) - a^(1/3)*b^(1/3)*Cos[e + f*x] + a^(
2/3)*Cos[e + f*x]^2])/(6*a^(1/3)*(a^2 - b^2)^2) + (b^2*(2*a^2 + b^2)*Log[b
 + a*Cos[e + f*x]^3])/(3*a*(a^2 - b^2)^2))/f)
```

3.461.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4626
```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> Module[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-(f
*ff^(m + n*p - 1))^(-1)   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*((b + a*(ff*
x)^n)^p/x^(m + n*p)), x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, n}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n] && IntegerQ[p]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.461.4 Maple [A] (verified)

Time = 8.55 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{1}{(4a+4b)(-1+\cos(fx+e))}+\frac{(-2a-5b)\ln(-1+\cos(fx+e))}{4(a+b)^2}-\left(\left(a^2+2b^2\right)\left(\frac{\ln\left(\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{\ln\left(\cos(fx+e)^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}\cos(fx+\right.\right.\right.$
default	$\frac{1}{(4a+4b)(-1+\cos(fx+e))}+\frac{(-2a-5b)\ln(-1+\cos(fx+e))}{4(a+b)^2}-\left(\left(a^2+2b^2\right)\left(\frac{\ln\left(\cos(fx+e)+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{\ln\left(\cos(fx+e)^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}\cos(fx+\right.\right.\right.$
risch	$\frac{iae}{f(a^2+2ab+b^2)}+\frac{4ia^4b^2f^2e}{a^7f^3-2a^5b^2f^3+a^3b^4f^3}+\frac{5ibe}{2f(a^2+2ab+b^2)}+\frac{iae}{f(a^2-2ab+b^2)}-\frac{5ibe}{2f(a^2-2ab+b^2)}-i\left(\underset{\_R=}{}\right.$

input
```
int(cot(f*x+e)^3/(a+b*sec(f*x+e)^3),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/(4*a+4*b)/(-1+cos(f*x+e))+1/4/(a+b)^2*(-2*a-5*b)*ln(-1+cos(f*x+e))-
((a^2+2*b^2)*(1/3/a/(b/a)^(2/3)*ln(cos(f*x+e)+(b/a)^(1/3))-1/6/a/(b/a)^(2/
3)*ln(cos(f*x+e)^2-(b/a)^(1/3)*cos(f*x+e)+(b/a)^(2/3))+1/3/a/(b/a)^(2/3)*3
^(1/2)*arctan(1/3*3^(1/2)*(2/(b/a)^(1/3)*cos(f*x+e)-1)))-3*a*b*(-1/3/a/(b/
a)^(1/3)*ln(cos(f*x+e)+(b/a)^(1/3))+1/6/a/(b/a)^(1/3)*ln(cos(f*x+e)^2-(b/a
)^(1/3)*cos(f*x+e)+(b/a)^(2/3))+1/3*3^(1/2)/a/(b/a)^(1/3)*arctan(1/3*3^(1/
2)*(2/(b/a)^(1/3)*cos(f*x+e)-1)))+1/3*(2*a^2+b^2)/a*ln(b+a*cos(f*x+e)^3))*
b^2/(a-b)^2/(a+b)^2-1/(4*a-4*b)/(1+cos(f*x+e))+1/4/(a-b)^2*(-2*a+5*b)*ln(1
+cos(f*x+e)))
```

3.461.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.21 (sec) , antiderivative size = 10746, normalized size of antiderivative = 27.34

$$\int \frac{\cot^3(e+fx)}{a+b\sec^3(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

3.461.6 Sympy [F]

$$\int \frac{\cot^3(e+fx)}{a+b\sec^3(e+fx)}\,dx = \int \frac{\cot^3(e+fx)}{a+b\sec^3(e+fx)}\,dx$$

input
```
integrate(cot(f*x+e)**3/(a+b*sec(f*x+e)**3),x)
```

output
```
Integral(cot(e + f*x)**3/(a + b*sec(e + f*x)**3), x)
```

3.461.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.24

$$\int \frac{\cot^3(e+fx)}{a+b\sec^3(e+fx)}\,dx$$
$$= \frac{4\sqrt{3}\left(a^2b^3\left(9\left(\frac{b}{a}\right)^{\frac{2}{3}}+4\right)-a^3b^2\left(3\left(\frac{b}{a}\right)^{\frac{1}{3}}+\frac{4b}{a}\right)-2ab^4\left(3\left(\frac{b}{a}\right)^{\frac{1}{3}}+\frac{b}{a}\right)+2b^5\right)\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{b}{a}\right)^{\frac{1}{3}}-2\cos(fx+e)\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{\left(a^6\left(\frac{b}{a}\right)^{\frac{2}{3}}-2a^4b^2\left(\frac{b}{a}\right)^{\frac{2}{3}}+a^2b^4\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}} - \frac{6\left(a^2b^2\left(4\left(\frac{b}{a}\right)^{\frac{2}{3}}-1\right)+$$

input
```
integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^3),x, algorithm="maxima")
```

output
```
1/36*(4*sqrt(3)*(a^2*b^3*(9*(b/a)^(2/3) + 4) - a^3*b^2*(3*(b/a)^(1/3) + 4*
b/a) - 2*a*b^4*(3*(b/a)^(1/3) + b/a) + 2*b^5)*arctan(-1/3*sqrt(3)*((b/a)^(
1/3) - 2*cos(f*x + e))/(b/a)^(1/3))/((a^6*(b/a)^(2/3) - 2*a^4*b^2*(b/a)^(2
/3) + a^2*b^4*(b/a)^(2/3))*(b/a)^(1/3)) - 6*(a^2*b^2*(4*(b/a)^(2/3) - 1) +
 2*b^4*((b/a)^(2/3) - 1) - 3*a*b^3*(b/a)^(1/3))*log(cos(f*x + e)^2 - (b/a)
^(1/3)*cos(f*x + e) + (b/a)^(2/3))/(a^5*(b/a)^(2/3) - 2*a^3*b^2*(b/a)^(2/3
) + a*b^4*(b/a)^(2/3)) - 12*(a^2*b^2*(2*(b/a)^(2/3) + 1) + b^4*((b/a)^(2/3
) + 2) + 3*a*b^3*(b/a)^(1/3))*log((b/a)^(1/3) + cos(f*x + e))/(a^5*(b/a)^(
2/3) - 2*a^3*b^2*(b/a)^(2/3) + a*b^4*(b/a)^(2/3)) - 9*(2*a - 5*b)*log(cos(
f*x + e) + 1)/(a^2 - 2*a*b + b^2) - 9*(2*a + 5*b)*log(cos(f*x + e) - 1)/(a
^2 + 2*a*b + b^2) - 18*(b*cos(f*x + e) - a)/((a^2 - b^2)*cos(f*x + e)^2 -
a^2 + b^2))/f
```

3.461.8 Giac [F]

$$\int \frac{\cot^3(e+fx)}{a+b\sec^3(e+fx)}\,dx = \int \frac{\cot\left(fx+e\right)^3}{b\sec\left(fx+e\right)^3+a}\,dx$$

input
```
integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^3),x, algorithm="giac")
```

output
```
sage0*x
```

3.461.9 Mupad [B] (verification not implemented)

Time = 35.83 (sec) , antiderivative size = 58699, normalized size of antiderivative = 149.36

$$\int \frac{\cot^3(e+fx)}{a+b\sec^3(e+fx)}\,dx = \text{Too large to display}$$

input
```
int(cot(e + f*x)^3/(a + b/cos(e + f*x)^3),x)
```

output

```
-(a^3*cos(e/2 + (f*x)/2)^4 + a^3*sin(e/2 + (f*x)/2)^4 - a*b^2*cos(e/2 + (f
*x)/2)^4 + a*b^2*sin(e/2 + (f*x)/2)^4 + 2*a^2*b*sin(e/2 + (f*x)/2)^4 - 8*a
^3*log((cos(e/2 + (f*x)/2)^2 + sin(e/2 + (f*x)/2)^2)/cos(e/2 + (f*x)/2)^2)
*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^2 + 8*b^3*log((cos(e/2 + (f*x)/2)
^2 + sin(e/2 + (f*x)/2)^2)/cos(e/2 + (f*x)/2)^2)*cos(e/2 + (f*x)/2)^2*sin(
e/2 + (f*x)/2)^2 + 8*a^3*cos(e/2 + (f*x)/2)^2*log(sin(e/2 + (f*x)/2)/cos(e
/2 + (f*x)/2))*sin(e/2 + (f*x)/2)^2 - 8*a^4*cos(e/2 + (f*x)/2)^2*sin(e/2 +
 (f*x)/2)^2*symsum(log((131072*(980*b^11*cos(e/2 + (f*x)/2)^2 + 336*b^11*s
in(e/2 + (f*x)/2)^2 + 1764*a^2*b^9*cos(e/2 + (f*x)/2)^2 + 392*a^3*b^8*cos(
e/2 + (f*x)/2)^2 + 640*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 -
 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^2*b^13*sin(e/2 +
 (f*x)/2)^2 + 32*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^
4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)^3*b^14*sin(e/2 + (f*x)
/2)^2 - 1176*a^2*b^9*sin(e/2 + (f*x)/2)^2 - 784*a^3*b^8*sin(e/2 + (f*x)/2)
^2 + 952*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^
2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*b^12*cos(e/2 + (f*x)/2)^2 + 23
52*a*b^10*cos(e/2 + (f*x)/2)^2 + 1944*root(54*a^5*b^2*z^3 - 27*a^3*b^4*z^3
 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z^2 - 9*a*b^4*z - b^4, z, k)*b
^12*sin(e/2 + (f*x)/2)^2 - 56*a*b^10*sin(e/2 + (f*x)/2)^2 + 304*root(54*a^
5*b^2*z^3 - 27*a^3*b^4*z^3 - 27*a^7*z^3 - 54*a^4*b^2*z^2 - 27*a^2*b^4*z...
```

3.462 $\int (a+b(c\sec(e+fx))^n)^p (d\tan(e+fx))^m\,dx$

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3.462.3 Rubi [N/A] . . . 3353
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3.462.9 Mupad [N/A] . . . 3355

3.462.1 Optimal result

Integrand size = 27, antiderivative size = 27

$$\int (a+b(c\sec(e+fx))^n)^p (d\tan(e+fx))^m\,dx = \text{Int}((a+b(c\sec(e+fx))^n)^p (d\tan(e+fx))^m, x)$$

output
```
Unintegrable((a+b*(c*sec(f*x+e))^n)^p*(d*tan(f*x+e))^m,x)
```

3.462.2 Mathematica [N/A]

Not integrable

Time = 5.73 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int (a+b(c\sec(e+fx))^n)^p (d\tan(e+fx))^m\,dx = \int (a+b(c\sec(e+fx))^n)^p (d\tan(e+fx))^m\,dx$$

input
```
Integrate[(a + b*(c*Sec[e + f*x])^n)^p*(d*Tan[e + f*x])^m,x]
```

output
```
Integrate[(a + b*(c*Sec[e + f*x])^n)^p*(d*Tan[e + f*x])^m, x]
```

3.462.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4632}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\tan(e+fx))^m (a+b(c\sec(e+fx))^n)^p \, dx$$

$$\downarrow 3042$$

$$\int (d\tan(e+fx))^m (a+b(c\sec(e+fx))^n)^p \, dx$$

$$\downarrow 4632$$

$$\int (d\tan(e+fx))^m (a+b(c\sec(e+fx))^n)^p \, dx$$

input
```
Int[(a + b*(c*Sec[e + f*x])^n)^p*(d*Tan[e + f*x])^m,x]
```

output
```
$Aborted
```

3.462.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4632
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*((d_.)*tan[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Unintegrable[(a + b*(c*Sec[e + f*x])
^n)^p*(d*Tan[e + f*x])^m, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

3.462.4 Maple [N/A] (verified)

Not integrable

Time = 0.83 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int (a + b(c\sec(fx+e))^n)^p (d\tan(fx+e))^m \, dx$$

input
```
int((a+b*(c*sec(f*x+e))^n)^p*(d*tan(f*x+e))^m,x)
```

output
```
int((a+b*(c*sec(f*x+e))^n)^p*(d*tan(f*x+e))^m,x)
```

3.462.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int (a + b(c\sec(e+fx))^n)^p (d\tan(e+fx))^m \, dx$$
$$= \int ((c\sec(fx+e))^n b + a)^p (d\tan(fx+e))^m \, dx$$

input
```
integrate((a+b*(c*sec(f*x+e))^n)^p*(d*tan(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral(((c*sec(f*x + e))^n*b + a)^p*(d*tan(f*x + e))^m, x)
```

3.462.6 Sympy [N/A]

Not integrable

Time = 104.48 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int (a + b(c\sec(e+fx))^n)^p (d\tan(e+fx))^m \, dx$$
$$= \int (d\tan(e+fx))^m (a + b(c\sec(e+fx))^n)^p \, dx$$

input
```
integrate((a+b*(c*sec(f*x+e))**n)**p*(d*tan(f*x+e))**m,x)
```

output
```
Integral((d*tan(e + f*x))**m*(a + b*(c*sec(e + f*x))**n)**p, x)
```

3.462.7 Maxima [N/A]

Not integrable

Time = 5.46 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int (a+b(c\sec(e+fx))^n)^p\,(d\tan(e+fx))^m\,dx$$
$$= \int ((c\sec(fx+e))^n\,b+a)^p(d\tan(fx+e))^m\,dx$$

input
```
integrate((a+b*(c*sec(f*x+e))^n)^p*(d*tan(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate(((c*sec(f*x + e))^n*b + a)^p*(d*tan(f*x + e))^m, x)
```

3.462.8 Giac [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int (a+b(c\sec(e+fx))^n)^p\,(d\tan(e+fx))^m\,dx$$
$$= \int ((c\sec(fx+e))^n\,b+a)^p(d\tan(fx+e))^m\,dx$$

input
```
integrate((a+b*(c*sec(f*x+e))^n)^p*(d*tan(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate(((c*sec(f*x + e))^n*b + a)^p*(d*tan(f*x + e))^m, x)
```

3.462.9 Mupad [N/A]

Not integrable

Time = 21.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int (a+b(c\sec(e+fx))^n)^p\,(d\tan(e+fx))^m\,dx$$
$$= \int (d\tan(e+fx))^m\left(a+b\left(\frac{c}{\cos(e+fx)}\right)^n\right)^p dx$$

input
```
int((d*tan(e + f*x))^m*(a + b*(c/cos(e + f*x))^n)^p,x)
```

output
```
int((d*tan(e + f*x))^m*(a + b*(c/cos(e + f*x))^n)^p, x)
```

3.463 $\int (a+b(c\sec(e+fx))^n)^p \tan^5(e+fx)\,dx$

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3.463.9 Mupad [F(-1)] . . . 3361

3.463.1 Optimal result

Integrand size = 25, antiderivative size = 226

$$\int (a+b(c\sec(e+fx))^n)^p \tan^5(e+fx)\,dx$$
$$= -\frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b(c\sec(e+fx))^n}{a}\right)(a+b(c\sec(e+fx))^n)^{1+p}}{afn(1+p)}$$
$$-\frac{\text{Hypergeometric2F1}\left(\frac{2}{n},-p,\frac{2+n}{n},-\frac{b(c\sec(e+fx))^n}{a}\right)\sec^2(e+fx)(a+b(c\sec(e+fx))^n)^p\left(1+\frac{b(c\sec(e+fx)}{a}\right.}{f}$$
$$+\frac{\text{Hypergeometric2F1}\left(\frac{4}{n},-p,\frac{4+n}{n},-\frac{b(c\sec(e+fx))^n}{a}\right)\sec^4(e+fx)(a+b(c\sec(e+fx))^n)^p\left(1+\frac{b(c\sec(e+fx)}{a}\right.}{4f}$$

output

```
-hypergeom([1, p+1],[2+p],1+b*(c*sec(f*x+e))^n/a)*(a+b*(c*sec(f*x+e))^n)^(
p+1)/a/f/n/(p+1)-hypergeom([-p, 2/n],[(2+n)/n],-b*(c*sec(f*x+e))^n/a)*sec(
f*x+e)^2*(a+b*(c*sec(f*x+e))^n)^p/f/((1+b*(c*sec(f*x+e))^n/a)^p)+1/4*hyper
geom([-p, 4/n],[(4+n)/n],-b*(c*sec(f*x+e))^n/a)*sec(f*x+e)^4*(a+b*(c*sec(f
*x+e))^n)^p/f/((1+b*(c*sec(f*x+e))^n/a)^p)
```

3.463.2 Mathematica [A] (warning: unable to verify)

Time = 13.76 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.08

$$\int (a+b(c\sec(e+fx))^n)^p \tan^5(e+fx)\,dx$$

$$= \frac{(a+b(c\sec(e+fx))^n)^p \left(1+\frac{b\left(c\sqrt{\sec^2(e+fx)}\right)^n}{a}\right)^{-p} \left(-4an(1+p)\,\text{Hypergeometric2F1}\left(\frac{2}{n},-p,\frac{2+n}{n},-\frac{b\left(c\sqrt{}\right.\right.\right.}{}$$

input
```
Integrate[(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x]^5,x]
```

output
```
((a + b*(c*Sec[e + f*x])^n)^p*(-4*a*n*(1 + p)*Hypergeometric2F1[2/n, -p, (
2 + n)/n, -((b*(c*Sqrt[Sec[e + f*x]^2])^n)/a)]*Sec[e + f*x]^2 + a*n*(1 + p
)*Hypergeometric2F1[4/n, -p, (4 + n)/n, -((b*(c*Sqrt[Sec[e + f*x]^2])^n)/a
)]*Sec[e + f*x]^4 - 4*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*(c*Sqrt[Se
c[e + f*x]^2])^n)/a]*(a + b*(c*Sqrt[Sec[e + f*x]^2])^n)*(1 + (b*(c*Sqrt[Se
c[e + f*x]^2])^n)/a)^p))/(4*a*f*n*(1 + p)*(1 + (b*(c*Sqrt[Sec[e + f*x]^2])
^n)/a)^p)
```

3.463.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 4627, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^5(e+fx)\,(a+b(c\sec(e+fx))^n)^p\,dx$$

$$\downarrow 3042$$

$$\int \tan(e+fx)^5\,(a+b(c\sec(e+fx))^n)^p\,dx$$

$$\downarrow 4627$$

$$\frac{\int \cos(e+fx)\,(1-\sec^2(e+fx))^2\,(b(c\sec(e+fx))^n+a)^p\,d\sec(e+fx)}{f}$$

$$\downarrow 7293$$

$$\frac{\int \left(\sec^3(e+fx)\left(b(c\sec(e+fx))^n+a\right)^p+\cos(e+fx)\left(b(c\sec(e+fx))^n+a\right)^p-2\sec(e+fx)\left(b(c\sec(e+fx))^n\right.\right.}{f}$$

$$\downarrow 2009$$

$$\frac{\frac{1}{4}\sec^4(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\left(\frac{b(c\sec(e+fx))^n}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{4}{n},-p,\frac{n+4}{n},-\frac{b(c\sec(e+fx))^n}{a}\right)-}{}$$

input
```
Int[(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x]^5,x]
```

output
```
(-((Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*(c*Sec[e + f*x])^n)/a]*(a +
b*(c*Sec[e + f*x])^n)^(1 + p))/(a*n*(1 + p))) - (Hypergeometric2F1[2/n, -p
, (2 + n)/n, -((b*(c*Sec[e + f*x])^n)/a)]*Sec[e + f*x]^2*(a + b*(c*Sec[e +
 f*x])^n)^p)/(1 + (b*(c*Sec[e + f*x])^n)/a)^p + (Hypergeometric2F1[4/n, -p
, (4 + n)/n, -((b*(c*Sec[e + f*x])^n)/a)]*Sec[e + f*x]^4*(a + b*(c*Sec[e +
 f*x])^n)^p)/(4*(1 + (b*(c*Sec[e + f*x])^n)/a)^p))/f
```

3.463.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.463.4 Maple [F]

$$\int (a + b(c\sec(fx+e))^n)^p \tan(fx+e)^5 \, dx$$

input
```
int((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^5,x)
```

output
```
int((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^5,x)
```

3.463.5 Fricas [F]

$$\int (a + b(c\sec(e+fx))^n)^p \tan^5(e+fx) \, dx = \int ((c\sec(fx+e))^n b + a)^p \tan(fx+e)^5 \, dx$$

input
```
integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^5,x, algorithm="fricas")
```

output
```
integral(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e)^5, x)
```

3.463.6 Sympy [F(-1)]

Timed out.

$$\int (a + b(c\sec(e+fx))^n)^p \tan^5(e+fx) \, dx = \text{Timed out}$$

input
```
integrate((a+b*(c*sec(f*x+e))**n)**p*tan(f*x+e)**5,x)
```

output
```
Timed out
```

3.463.7 Maxima [F]

$$\int (a+b(c\sec(e+fx))^n)^p \tan^5(e+fx)\,dx = \int ((c\sec(fx+e))^n b+a)^p \tan(fx+e)^5\,dx$$

input
```
integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^5,x, algorithm="maxima")
```

output
```
integrate(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e)^5, x)
```

3.463.8 Giac [F]

$$\int (a+b(c\sec(e+fx))^n)^p \tan^5(e+fx)\,dx = \int ((c\sec(fx+e))^n b+a)^p \tan(fx+e)^5\,dx$$

input
```
integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^5,x, algorithm="giac")
```

output
```
integrate(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e)^5, x)
```

3.463.9 Mupad [F(-1)]

Timed out.

$$\int (a+b(c\sec(e+fx))^n)^p \tan^5(e+fx)\,dx = \int \tan(e+f\,x)^5 \left(a+b\left(\frac{c}{\cos(e+f\,x)}\right)^n\right)^p dx$$

input
```
int(tan(e + f*x)^5*(a + b*(c/cos(e + f*x))^n)^p,x)
```

output
```
int(tan(e + f*x)^5*(a + b*(c/cos(e + f*x))^n)^p, x)
```

3.464 $\int (a+b(c\sec(e+fx))^n)^p \tan^3(e+fx)\,dx$

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3.464.8 Giac [F] . . . 3366
3.464.9 Mupad [F(-1)] . . . 3366

3.464.1 Optimal result

Integrand size = 25, antiderivative size = 143

$$\int (a+b(c\sec(e+fx))^n)^p \tan^3(e+fx)\,dx$$
$$= \frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b(c\sec(e+fx))^n}{a}\right)(a+b(c\sec(e+fx))^n)^{1+p}}{afn(1+p)}$$
$$+\frac{\text{Hypergeometric2F1}\left(\frac{2}{n},-p,\frac{2+n}{n},-\frac{b(c\sec(e+fx))^n}{a}\right)\sec^2(e+fx)\,(a+b(c\sec(e+fx))^n)^p\left(1+\frac{b(c\sec(e+fx)}{a}\right.}{2f}$$

output
```
hypergeom([1, p+1],[2+p],1+b*(c*sec(f*x+e))^n/a)*(a+b*(c*sec(f*x+e))^n)^(p
+1)/a/f/n/(p+1)+1/2*hypergeom([-p, 2/n],[(2+n)/n],-b*(c*sec(f*x+e))^n/a)*s
ec(f*x+e)^2*(a+b*(c*sec(f*x+e))^n)^p/f/((1+b*(c*sec(f*x+e))^n/a)^p)
```

3.464.2 Mathematica [A] (warning: unable to verify)

Time = 6.43 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.13

$$\int (a+b(c\sec(e+fx))^n)^p \tan^3(e+fx)\,dx$$
$$= (a+b(c\sec(e+fx))^n)^p \left(\frac{2\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b\left(c\sqrt{\sec^2(e+fx)}\right)^n}{a}\right)\left(a+b\left(c\sqrt{\sec^2(e+fx)}\right)^n\right)}{an(1+p)} + \text{Hypergeomet}\right.$$
$$\left. \right) \Big/ 2f$$

input
```
Integrate[(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x]^3,x]
```

output
```
((a + b*(c*Sec[e + f*x])^n)^p*((2*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (
b*(c*Sqrt[Sec[e + f*x]^2])^n)/a]*(a + b*(c*Sqrt[Sec[e + f*x]^2])^n))/(a*n*
(1 + p)) + (Hypergeometric2F1[2/n, -p, (2 + n)/n, -((b*(c*Sqrt[Sec[e + f*x
]^2])^n)/a)]*Sec[e + f*x]^2)/(1 + (b*(c*Sqrt[Sec[e + f*x]^2])^n)/a)^p))/(2
*f)
```

3.464.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4627, 25, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(e+fx)\,(a+b(c\sec(e+fx))^n)^p\,dx$$

$$\downarrow 3042$$

$$\int \tan(e+fx)^3\,(a+b(c\sec(e+fx))^n)^p\,dx$$

$$\downarrow 4627$$

$$\frac{\int -\cos(e+fx)\,(1-\sec^2(e+fx))\,(b(c\sec(e+fx))^n+a)^p\,d\sec(e+fx)}{f}$$

$$\downarrow 25$$

$$-\frac{\int \cos(e+fx)\,(1-\sec^2(e+fx))\,(b(c\sec(e+fx))^n+a)^p\,d\sec(e+fx)}{f}$$

$$\downarrow 7293$$

$$-\frac{\int (\cos(e+fx)\,(b(c\sec(e+fx))^n+a)^p-\sec(e+fx)\,(b(c\sec(e+fx))^n+a)^p)\,d\sec(e+fx)}{f}$$

$$\downarrow 2009$$

$$\frac{\frac{1}{2}\sec^2(e+fx)\,(a+b(c\sec(e+fx))^n)^p\left(\frac{b(c\sec(e+fx))^n}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{2}{n},-p,\frac{n+2}{n},-\frac{b(c\sec(e+fx))^n}{a}\right)+}{f}$$

input
```
Int[(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x]^3,x]
```

output
```
((Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*(c*Sec[e + f*x])^n)/a]*(a + b*
(c*Sec[e + f*x])^n)^(1 + p))/(a*n*(1 + p)) + (Hypergeometric2F1[2/n, -p, (
2 + n)/n, -((b*(c*Sec[e + f*x])^n)/a)]*Sec[e + f*x]^2*(a + b*(c*Sec[e + f*
x])^n)^p)/(2*(1 + (b*(c*Sec[e + f*x])^n)/a)^p))/f
```

3.464.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.464.4 Maple [F]

$$\int (a + b(c \sec(fx + e))^n)^p \tan(fx + e)^3 \, dx$$

input
```
int((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^3,x)
```

output
```
int((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^3,x)
```

3.464.5 Fricas [F]

$$\int (a+b(c\sec(e+fx))^n)^p \tan^3(e+fx)\,dx = \int ((c\sec(fx+e))^n b+a)^p \tan(fx+e)^3\,dx$$

input
```
integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^3,x, algorithm="fricas")
```

output
```
integral(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e)^3, x)
```

3.464.6 Sympy [F]

$$\int (a+b(c\sec(e+fx))^n)^p \tan^3(e+fx)\,dx = \int (a+b(c\sec(e+fx))^n)^p \tan^3(e+fx)\,dx$$

input
```
integrate((a+b*(c*sec(f*x+e))**n)**p*tan(f*x+e)**3,x)
```

output
```
Integral((a + b*(c*sec(e + f*x))**n)**p*tan(e + f*x)**3, x)
```

3.464.7 Maxima [F]

$$\int (a+b(c\sec(e+fx))^n)^p \tan^3(e+fx)\,dx = \int ((c\sec(fx+e))^n b+a)^p \tan(fx+e)^3\,dx$$

input
```
integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^3,x, algorithm="maxima")
```

output
```
integrate(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e)^3, x)
```

3.464.8 Giac [F]

$$\int (a+b(c\sec(e+fx))^n)^p \tan^3(e+fx)\,dx = \int ((c\sec(fx+e))^n b+a)^p \tan(fx+e)^3\,dx$$

input
```
integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^3,x, algorithm="giac")
```

output
```
integrate(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e)^3, x)
```

3.464.9 Mupad [F(-1)]

Timed out.

$$\int (a+b(c\sec(e+fx))^n)^p \tan^3(e+fx)\,dx = \int \tan(e+f\,x)^3 \left(a+b\left(\frac{c}{\cos(e+f\,x)}\right)^n\right)^p dx$$

input
```
int(tan(e + f*x)^3*(a + b*(c/cos(e + f*x))^n)^p,x)
```

output
```
int(tan(e + f*x)^3*(a + b*(c/cos(e + f*x))^n)^p, x)
```

3.465 $\int (a+b(c\sec(e+fx))^n)^p \tan(e+fx)\,dx$

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3.465.1 Optimal result

Integrand size = 23, antiderivative size = 59

$$\int (a+b(c\sec(e+fx))^n)^p \tan(e+fx)\,dx = -\frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b(c\sec(e+fx))^n}{a}\right)(a+b(c\sec(e+fx))^n)^{1+p}}{afn(1+p)}$$

output
```
-hypergeom([1, p+1],[2+p],1+b*(c*sec(f*x+e))^n/a)*(a+b*(c*sec(f*x+e))^n)^(
p+1)/a/f/n/(p+1)
```

3.465.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

$$\int (a+b(c\sec(e+fx))^n)^p \tan(e+fx)\,dx = -\frac{\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b(c\sec(e+fx))^n}{a}\right)(a+b(c\sec(e+fx))^n)^{1+p}}{afn(1+p)}$$

input
```
Integrate[(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x],x]
```

output
```
-((Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*(c*Sec[e + f*x])^n)/a]*(a + b
*(c*Sec[e + f*x])^n)^(1 + p))/(a*f*n*(1 + p)))
```

3.465.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4627, 891, 27, 798, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(e+fx)\,(a+b(c\sec(e+fx))^n)^p\,dx$$

$$\downarrow 3042$$

$$\int \tan(e+fx)\,(a+b(c\sec(e+fx))^n)^p\,dx$$

$$\downarrow 4627$$

$$\frac{\int \cos(e+fx)\,(b(c\sec(e+fx))^n+a)^p\,d\sec(e+fx)}{f}$$

$$\downarrow 891$$

$$\frac{\int c\cos(e+fx)\,(b(c\sec(e+fx))^n+a)^p\,d(c\sec(e+fx))}{cf}$$

$$\downarrow 27$$

$$\frac{\int \cos(e+fx)\,(b(c\sec(e+fx))^n+a)^p\,d(c\sec(e+fx))}{f}$$

$$\downarrow 798$$

$$\frac{\int \cos(e+fx)\,(b(c\sec(e+fx))^n+a)^p\,d(c\sec(e+fx))^n}{fn}$$

$$\downarrow 75$$

$$-\frac{(a+b(c\sec(e+fx))^n)^{p+1}\,\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b(c\sec(e+fx))^n}{a}+1\right)}{afn(p+1)}$$

input
```
Int[(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x],x]
```

output
```
-((Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*(c*Sec[e + f*x])^n)/a]*(a + b
*(c*Sec[e + f*x])^n)^(1 + p))/(a*f*n*(1 + p)))
```

3.465.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 891
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Simp[1/c   Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a
, b, c, d, m, n, p}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4627
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Si
mp[1/f   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x]
, x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(
m - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || Integers
Q[2*n, p])
```

3.465.4 Maple [F]

$$\int (a+b(c\sec(fx+e))^n)^p \tan(fx+e)\,dx$$

input
```
int((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e),x)
```

output
```
int((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e),x)
```

3.465.5 Fricas [F]

$$\int (a+b(c\sec(e+fx))^n)^p \tan(e+fx)\,dx = \int ((c\sec(fx+e))^n b+a)^p \tan(fx+e)\,dx$$

input
```
integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e),x, algorithm="fricas")
```

output
```
integral(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e), x)
```

3.465.6 Sympy [F]

$$\int (a+b(c\sec(e+fx))^n)^p \tan(e+fx)\,dx = \int (a+b(c\sec(e+fx))^n)^p \tan(e+fx)\,dx$$

input
```
integrate((a+b*(c*sec(f*x+e))**n)**p*tan(f*x+e),x)
```

output
```
Integral((a + b*(c*sec(e + f*x))**n)**p*tan(e + f*x), x)
```

3.465.7 Maxima [F]

$$\int (a+b(c\sec(e+fx))^n)^p \tan(e+fx)\,dx = \int ((c\sec(fx+e))^n b+a)^p \tan(fx+e)\,dx$$

input
```
integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e),x, algorithm="maxima")
```

output
```
integrate(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e), x)
```

3.465.8 Giac [F]

$$\int (a+b(c\sec(e+fx))^n)^p \tan(e+fx)\,dx = \int ((c\sec(fx+e))^n b+a)^p \tan(fx+e)\,dx$$

input
```
integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e),x, algorithm="giac")
```

output
```
integrate(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e), x)
```

3.465.9 Mupad [F(-1)]

Timed out.

$$\int (a+b(c\sec(e+fx))^n)^p \tan(e+fx)\,dx = \int \tan(e+fx)\left(a+b\left(\frac{c}{\cos(e+fx)}\right)^n\right)^p dx$$

input
```
int(tan(e + f*x)*(a + b*(c/cos(e + f*x))^n)^p,x)
```

output
```
int(tan(e + f*x)*(a + b*(c/cos(e + f*x))^n)^p, x)
```

3.466 $\int \cot(e+fx)\,(a+b(c\sec(e+fx))^n)^p\,dx$

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3.466.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \cot(e+fx)\,(a+b(c\sec(e+fx))^n)^p\,dx = \text{Int}(\cot(e+fx)\,(a+b(c\sec(e+fx))^n)^p, x)$$

output
```
Unintegrable(cot(f*x+e)*(a+b*(c*sec(f*x+e))^n)^p,x)
```

3.466.2 Mathematica [N/A]

Not integrable

Time = 4.53 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cot(e+fx)\,(a+b(c\sec(e+fx))^n)^p\,dx = \int \cot(e+fx)\,(a+b(c\sec(e+fx))^n)^p\,dx$$

input
```
Integrate[Cot[e + f*x]*(a + b*(c*Sec[e + f*x])^n)^p,x]
```

output
```
Integrate[Cot[e + f*x]*(a + b*(c*Sec[e + f*x])^n)^p, x]
```

3.466.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4632}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(e+fx)\,(a+b(c\sec(e+fx))^n)^p\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a+b(c\sec(e+fx))^n)^p}{\tan(e+fx)}dx$$

$$\downarrow 4632$$

$$\int \cot(e+fx)\,(a+b(c\sec(e+fx))^n)^p\,dx$$

input
```
Int[Cot[e + f*x]*(a + b*(c*Sec[e + f*x])^n)^p,x]
```

output
```
$Aborted
```

3.466.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4632
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*((d_.)*tan[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Unintegrable[(a + b*(c*Sec[e + f*x])
^n)^p*(d*Tan[e + f*x])^m, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

3.466.4 Maple [N/A] (verified)

Not integrable

Time = 0.60 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \cot(fx+e)\left(a+b(c\sec(fx+e))^n\right)^p dx$$

input
```
int(cot(f*x+e)*(a+b*(c*sec(f*x+e))^n)^p,x)
```

output
```
int(cot(f*x+e)*(a+b*(c*sec(f*x+e))^n)^p,x)
```

3.466.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cot(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\, dx = \int \left((c\sec(fx+e))^n b+a\right)^p \cot(fx+e)\, dx$$

input
```
integrate(cot(f*x+e)*(a+b*(c*sec(f*x+e))^n)^p,x, algorithm="fricas")
```

output
```
integral(((c*sec(f*x + e))^n*b + a)^p*cot(f*x + e), x)
```

3.466.6 Sympy [N/A]

Not integrable

Time = 12.66 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \cot(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\, dx = \int \left(a+b(c\sec(e+fx))^n\right)^p \cot(e+fx)\, dx$$

input
```
integrate(cot(f*x+e)*(a+b*(c*sec(f*x+e))**n)**p,x)
```

output
```
Integral((a + b*(c*sec(e + f*x))**n)**p*cot(e + f*x), x)
```

3.466.7 Maxima [N/A]

Not integrable

Time = 3.78 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cot(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx = \int \left((c\sec(fx+e))^n b+a\right)^p \cot(fx+e)\,dx$$

input
```
integrate(cot(f*x+e)*(a+b*(c*sec(f*x+e))^n)^p,x, algorithm="maxima")
```

output
```
integrate(((c*sec(f*x + e))^n*b + a)^p*cot(f*x + e), x)
```

3.466.8 Giac [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \cot(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx = \int \left((c\sec(fx+e))^n b+a\right)^p \cot(fx+e)\,dx$$

input
```
integrate(cot(f*x+e)*(a+b*(c*sec(f*x+e))^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*sec(f*x + e))^n*b + a)^p*cot(f*x + e), x)
```

3.466.9 Mupad [N/A]

Not integrable

Time = 20.71 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \cot(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx = \int \cot(e+f\,x)\left(a+b\left(\frac{c}{\cos(e+f\,x)}\right)^n\right)^p dx$$

input
```
int(cot(e + f*x)*(a + b*(c/cos(e + f*x))^n)^p,x)
```

output
```
int(cot(e + f*x)*(a + b*(c/cos(e + f*x))^n)^p, x)
```

3.467 $\int \cot^3(e+fx)\,(a+b(c\sec(e+fx))^n)^p\,dx$

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3.467.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \cot^3(e+fx)\,(a+b(c\sec(e+fx))^n)^p\,dx = \text{Int}\left(\cot^3(e+fx)\,(a+b(c\sec(e+fx))^n)^p,x\right)$$

output
```
Unintegrable(cot(f*x+e)^3*(a+b*(c*sec(f*x+e))^n)^p,x)
```

3.467.2 Mathematica [N/A]

Not integrable

Time = 31.51 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \cot^3(e+fx)\,(a+b(c\sec(e+fx))^n)^p\,dx = \int \cot^3(e+fx)\,(a+b(c\sec(e+fx))^n)^p\,dx$$

input
```
Integrate[Cot[e + f*x]^3*(a + b*(c*Sec[e + f*x])^n)^p,x]
```

output
```
Integrate[Cot[e + f*x]^3*(a + b*(c*Sec[e + f*x])^n)^p, x]
```

3.467.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4632}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(e+fx)\,(a+b(c\sec(e+fx))^n)^p\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a+b(c\sec(e+fx))^n)^p}{\tan(e+fx)^3}dx$$

$$\downarrow 4632$$

$$\int \cot^3(e+fx)\,(a+b(c\sec(e+fx))^n)^p\,dx$$

input
```
Int[Cot[e + f*x]^3*(a + b*(c*Sec[e + f*x])^n)^p,x]
```

output
```
$Aborted
```

3.467.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4632
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*((d_.)*tan[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Unintegrable[(a + b*(c*Sec[e + f*x])
^n)^p*(d*Tan[e + f*x])^m, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

3.467.4 Maple [N/A] (verified)

Not integrable

Time = 0.62 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \cot(fx+e)^3 (a+b(c\sec(fx+e))^n)^p \, dx$$

input
```
int(cot(f*x+e)^3*(a+b*(c*sec(f*x+e))^n)^p,x)
```

output
```
int(cot(f*x+e)^3*(a+b*(c*sec(f*x+e))^n)^p,x)
```

3.467.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \cot^3(e+fx) (a+b(c\sec(e+fx))^n)^p \, dx = \int ((c\sec(fx+e))^n b+a)^p \cot(fx+e)^3 \, dx$$

input
```
integrate(cot(f*x+e)^3*(a+b*(c*sec(f*x+e))^n)^p,x, algorithm="fricas")
```

output
```
integral(((c*sec(f*x + e))^n*b + a)^p*cot(f*x + e)^3, x)
```

3.467.6 Sympy [F(-1)]

Timed out.

$$\int \cot^3(e+fx) (a+b(c\sec(e+fx))^n)^p \, dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)**3*(a+b*(c*sec(f*x+e))**n)**p,x)
```

output
```
Timed out
```

3.467.7 Maxima [N/A]

Not integrable

Time = 9.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \cot^3(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx = \int \left((c\sec(fx+e))^n b+a\right)^p \cot(fx+e)^3\,dx$$

input
```
integrate(cot(f*x+e)^3*(a+b*(c*sec(f*x+e))^n)^p,x, algorithm="maxima")
```

output
```
integrate(((c*sec(f*x + e))^n*b + a)^p*cot(f*x + e)^3, x)
```

3.467.8 Giac [N/A]

Not integrable

Time = 1.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \cot^3(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx = \int \left((c\sec(fx+e))^n b+a\right)^p \cot(fx+e)^3\,dx$$

input
```
integrate(cot(f*x+e)^3*(a+b*(c*sec(f*x+e))^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*sec(f*x + e))^n*b + a)^p*cot(f*x + e)^3, x)
```

3.467.9 Mupad [N/A]

Not integrable

Time = 21.97 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \cot^3(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx = \int \cot(e+f\,x)^3\left(a+b\left(\frac{c}{\cos(e+f\,x)}\right)^n\right)^p dx$$

input
```
int(cot(e + f*x)^3*(a + b*(c/cos(e + f*x))^n)^p,x)
```

output
```
int(cot(e + f*x)^3*(a + b*(c/cos(e + f*x))^n)^p, x)
```

3.468 $\int (a + b(c\sec(e+fx))^n)^p \tan^2(e+fx)\,dx$

3.468.1 Optimal result . . . 3380
3.468.2 Mathematica [N/A] . . . 3380
3.468.3 Rubi [N/A] . . . 3381
3.468.4 Maple [N/A] (verified) . . . 3382
3.468.5 Fricas [N/A] . . . 3382
3.468.6 Sympy [N/A] . . . 3382
3.468.7 Maxima [N/A] . . . 3383
3.468.8 Giac [N/A] . . . 3383
3.468.9 Mupad [N/A] . . . 3383

3.468.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int (a + b(c\sec(e+fx))^n)^p \tan^2(e+fx)\,dx = \text{Int}\left((a + b(c\sec(e+fx))^n)^p \tan^2(e+fx), x\right)$$

output
```
Unintegrable((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^2,x)
```

3.468.2 Mathematica [N/A]

Not integrable

Time = 3.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (a + b(c\sec(e+fx))^n)^p \tan^2(e+fx)\,dx = \int (a + b(c\sec(e+fx))^n)^p \tan^2(e+fx)\,dx$$

input
```
Integrate[(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x]^2,x]
```

output
```
Integrate[(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x]^2, x]
```

3.468.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4632}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(e+fx)\,(a+b(c\sec(e+fx))^n)^p\,dx$$

$$\downarrow 3042$$

$$\int \tan(e+fx)^2\,(a+b(c\sec(e+fx))^n)^p\,dx$$

$$\downarrow 4632$$

$$\int \tan^2(e+fx)\,(a+b(c\sec(e+fx))^n)^p\,dx$$

input
```
Int[(a + b*(c*Sec[e + f*x])^n)^p*Tan[e + f*x]^2,x]
```

output
```
$Aborted
```

3.468.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4632
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*((d_.)*tan[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Unintegrable[(a + b*(c*Sec[e + f*x])
^n)^p*(d*Tan[e + f*x])^m, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

3.468.4 Maple [N/A] (verified)

Not integrable

Time = 0.46 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (a+b(c\sec(fx+e))^n)^p \tan(fx+e)^2\,dx$$

input
```
int((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^2,x)
```

output
```
int((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^2,x)
```

3.468.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (a+b(c\sec(e+fx))^n)^p \tan^2(e+fx)\,dx = \int ((c\sec(fx+e))^n b+a)^p \tan(fx+e)^2\,dx$$

input
```
integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^2,x, algorithm="fricas")
```

output
```
integral(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e)^2, x)
```

3.468.6 Sympy [N/A]

Not integrable

Time = 19.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int (a+b(c\sec(e+fx))^n)^p \tan^2(e+fx)\,dx = \int (a+b(c\sec(e+fx))^n)^p \tan^2(e+fx)\,dx$$

input
```
integrate((a+b*(c*sec(f*x+e))**n)**p*tan(f*x+e)**2,x)
```

output
```
Integral((a + b*(c*sec(e + f*x))**n)**p*tan(e + f*x)**2, x)
```

3.468.7 Maxima [N/A]

Not integrable

Time = 5.86 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (a+b(c\sec(e+fx))^n)^p \tan^2(e+fx)\,dx = \int ((c\sec(fx+e))^n b+a)^p \tan(fx+e)^2\,dx$$

input
```
integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e)^2, x)
```

3.468.8 Giac [N/A]

Not integrable

Time = 1.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (a+b(c\sec(e+fx))^n)^p \tan^2(e+fx)\,dx = \int ((c\sec(fx+e))^n b+a)^p \tan(fx+e)^2\,dx$$

input
```
integrate((a+b*(c*sec(f*x+e))^n)^p*tan(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate(((c*sec(f*x + e))^n*b + a)^p*tan(f*x + e)^2, x)
```

3.468.9 Mupad [N/A]

Not integrable

Time = 21.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int (a+b(c\sec(e+fx))^n)^p \tan^2(e+fx)\,dx = \int \tan(e+f\,x)^2 \left(a+b\left(\frac{c}{\cos(e+f\,x)}\right)^n\right)^p dx$$

input
```
int(tan(e + f*x)^2*(a + b*(c/cos(e + f*x))^n)^p,x)
```

output
```
int(tan(e + f*x)^2*(a + b*(c/cos(e + f*x))^n)^p, x)
```

3.469 $\int (a + b(c\sec(e + fx))^n)^p \, dx$

3.469.1 Optimal result . . . 3384
3.469.2 Mathematica [N/A] . . . 3384
3.469.3 Rubi [N/A] . . . 3385
3.469.4 Maple [N/A] (verified) . . . 3386
3.469.5 Fricas [N/A] . . . 3386
3.469.6 Sympy [N/A] . . . 3386
3.469.7 Maxima [N/A] . . . 3387
3.469.8 Giac [N/A] . . . 3387
3.469.9 Mupad [N/A] . . . 3387

3.469.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int (a + b(c\sec(e + fx))^n)^p \, dx = \text{Int}((a + b(c\sec(e + fx))^n)^p, x)$$

output
```
Unintegrable((a+b*(c*sec(f*x+e))^n)^p,x)
```

3.469.2 Mathematica [N/A]

Not integrable

Time = 1.74 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (a + b(c\sec(e + fx))^n)^p \, dx = \int (a + b(c\sec(e + fx))^n)^p \, dx$$

input
```
Integrate[(a + b*(c*Sec[e + f*x])^n)^p,x]
```

output
```
Integrate[(a + b*(c*Sec[e + f*x])^n)^p, x]
```

3.469.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4619}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + b(c\sec(e+fx))^n)^p \, dx$$

$$\downarrow 3042$$

$$\int (a + b(c\sec(e+fx))^n)^p \, dx$$

$$\downarrow 4619$$

$$\int (a + b(c\sec(e+fx))^n)^p \, dx$$

input
```
Int[(a + b*(c*Sec[e + f*x])^n)^p,x]
```

output
```
$Aborted
```

3.469.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4619
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Unintegrable[(a + b*(c*Sec[e + f*x])^n)^p, x] /; FreeQ[{a, b, c, e, f, n, p
}, x]
```

3.469.4 Maple [N/A] (verified)

Not integrable

Time = 0.48 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (a+b(c\sec(fx+e))^n)^p\,dx$$

input
```
int((a+b*(c*sec(f*x+e))^n)^p,x)
```

output
```
int((a+b*(c*sec(f*x+e))^n)^p,x)
```

3.469.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (a+b(c\sec(e+fx))^n)^p\,dx = \int ((c\sec(fx+e))^n b+a)^p\,dx$$

input
```
integrate((a+b*(c*sec(f*x+e))^n)^p,x, algorithm="fricas")
```

output
```
integral(((c*sec(f*x + e))^n*b + a)^p, x)
```

3.469.6 Sympy [N/A]

Not integrable

Time = 1.73 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int (a+b(c\sec(e+fx))^n)^p\,dx = \int (a+b(c\sec(e+fx))^n)^p\,dx$$

input
```
integrate((a+b*(c*sec(f*x+e))**n)**p,x)
```

output
```
Integral((a + b*(c*sec(e + f*x))**n)**p, x)
```

3.469.7 Maxima [N/A]

Not integrable

Time = 3.95 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (a+b(c\sec(e+fx))^n)^p\,dx = \int ((c\sec(fx+e))^n b+a)^p\,dx$$

input
```
integrate((a+b*(c*sec(f*x+e))^n)^p,x, algorithm="maxima")
```

output
```
integrate(((c*sec(f*x + e))^n*b + a)^p, x)
```

3.469.8 Giac [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (a+b(c\sec(e+fx))^n)^p\,dx = \int ((c\sec(fx+e))^n b+a)^p\,dx$$

input
```
integrate((a+b*(c*sec(f*x+e))^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*sec(f*x + e))^n*b + a)^p, x)
```

3.469.9 Mupad [N/A]

Not integrable

Time = 20.78 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int (a+b(c\sec(e+fx))^n)^p\,dx = \int \left(a+b\left(\frac{c}{\cos(e+fx)}\right)^n\right)^p dx$$

input
```
int((a + b*(c/cos(e + f*x))^n)^p,x)
```

output
```
int((a + b*(c/cos(e + f*x))^n)^p, x)
```

3.470 $\int \cot^2(e+fx)\,(a+b(c\sec(e+fx))^n)^p\,dx$

3.470.1 Optimal result . . . 3388
3.470.2 Mathematica [N/A] . . . 3388
3.470.3 Rubi [N/A] . . . 3389
3.470.4 Maple [N/A] (verified) . . . 3390
3.470.5 Fricas [N/A] . . . 3390
3.470.6 Sympy [N/A] . . . 3390
3.470.7 Maxima [N/A] . . . 3391
3.470.8 Giac [N/A] . . . 3391
3.470.9 Mupad [N/A] . . . 3391

3.470.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \cot^2(e+fx)\,(a+b(c\sec(e+fx))^n)^p\,dx = \text{Int}\left(\cot^2(e+fx)\,(a+b(c\sec(e+fx))^n)^p,x\right)$$

output
```
Unintegrable(cot(f*x+e)^2*(a+b*(c*sec(f*x+e))^n)^p,x)
```

3.470.2 Mathematica [N/A]

Not integrable

Time = 2.79 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \cot^2(e+fx)\,(a+b(c\sec(e+fx))^n)^p\,dx = \int \cot^2(e+fx)\,(a+b(c\sec(e+fx))^n)^p\,dx$$

input
```
Integrate[Cot[e + f*x]^2*(a + b*(c*Sec[e + f*x])^n)^p,x]
```

output
```
Integrate[Cot[e + f*x]^2*(a + b*(c*Sec[e + f*x])^n)^p, x]
```

3.470.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4632}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(e+fx)\,(a+b(c\sec(e+fx))^n)^p\,dx$$

$$\downarrow 3042$$

$$\int \frac{(a+b(c\sec(e+fx))^n)^p}{\tan(e+fx)^2}dx$$

$$\downarrow 4632$$

$$\int \cot^2(e+fx)\,(a+b(c\sec(e+fx))^n)^p\,dx$$

input
```
Int[Cot[e + f*x]^2*(a + b*(c*Sec[e + f*x])^n)^p,x]
```

output
```
$Aborted
```

3.470.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4632
```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*((d_.)*tan[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Unintegrable[(a + b*(c*Sec[e + f*x])
^n)^p*(d*Tan[e + f*x])^m, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

3.470.4 Maple [N/A] (verified)

Not integrable

Time = 0.60 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \cot(fx+e)^2 (a+b(c\sec(fx+e))^n)^p \, dx$$

input
```
int(cot(f*x+e)^2*(a+b*(c*sec(f*x+e))^n)^p,x)
```

output
```
int(cot(f*x+e)^2*(a+b*(c*sec(f*x+e))^n)^p,x)
```

3.470.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \cot^2(e+fx) (a+b(c\sec(e+fx))^n)^p \, dx = \int ((c\sec(fx+e))^n b + a)^p \cot(fx+e)^2 \, dx$$

input
```
integrate(cot(f*x+e)^2*(a+b*(c*sec(f*x+e))^n)^p,x, algorithm="fricas")
```

output
```
integral(((c*sec(f*x + e))^n*b + a)^p*cot(f*x + e)^2, x)
```

3.470.6 Sympy [N/A]

Not integrable

Time = 73.55 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \cot^2(e+fx) (a+b(c\sec(e+fx))^n)^p \, dx = \int (a+b(c\sec(e+fx))^n)^p \cot^2(e+fx) \, dx$$

input
```
integrate(cot(f*x+e)**2*(a+b*(c*sec(f*x+e))**n)**p,x)
```

output
```
Integral((a + b*(c*sec(e + f*x))**n)**p*cot(e + f*x)**2, x)
```

3.470.7 Maxima [N/A]

Not integrable

Time = 6.35 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \cot^2(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx = \int \left((c\sec(fx+e))^n b+a\right)^p \cot(fx+e)^2\,dx$$

input
```
integrate(cot(f*x+e)^2*(a+b*(c*sec(f*x+e))^n)^p,x, algorithm="maxima")
```

output
```
integrate(((c*sec(f*x + e))^n*b + a)^p*cot(f*x + e)^2, x)
```

3.470.8 Giac [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \cot^2(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx = \int \left((c\sec(fx+e))^n b+a\right)^p \cot(fx+e)^2\,dx$$

input
```
integrate(cot(f*x+e)^2*(a+b*(c*sec(f*x+e))^n)^p,x, algorithm="giac")
```

output
```
integrate(((c*sec(f*x + e))^n*b + a)^p*cot(f*x + e)^2, x)
```

3.470.9 Mupad [N/A]

Not integrable

Time = 21.32 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \cot^2(e+fx)\left(a+b(c\sec(e+fx))^n\right)^p\,dx = \int \cot(e+f\,x)^2\left(a+b\left(\frac{c}{\cos(e+f\,x)}\right)^n\right)^p dx$$

input
```
int(cot(e + f*x)^2*(a + b*(c/cos(e + f*x))^n)^p,x)
```

output
```
int(cot(e + f*x)^2*(a + b*(c/cos(e + f*x))^n)^p, x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions 3392

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                               convert(leaf_count_result,string)," vs. $2 (",
                               convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                           convert(leaf_count_result,string),"$ vs. $2(",
                           convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#           Port of original Maple grading function by
#           Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#           added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
    return 1
  elif isinstance(expn,list):
    return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
  elif  is_sqrt(expn):
    if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
      return 1
    else:
      return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
  elif isinstance(expn,Pow):   #type(expn,'`^`')
    if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
      return expnType(expn.args[0])   #ExpnType(op(1,expn))
    elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
        return 1
      else:
        return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    else:
      return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
  elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
    m1 = expnType(expn.args[0])
    m2 = expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
    return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
  elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
  elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  elif is_appell_function(expn.func):
    m1 = max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  elif isinstance(expn,RootSum):
    m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
    return max(7,m1)
  elif str(expn).find("Integral") != -1:
    m1 = max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  else:
    return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:    #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if  not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif is_sqrt(expn):
      if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])   #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(4,m1)   #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(5,m1)   #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(6,m1)   #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```